Plenary papers

Long, Caroline: What can we learn from TIMSS 2003? 1
Moremedi, G. M.: Challenges in implementing the new mathematics curriculum 23
Sethole, Godfrey: Innocence of realistic tasks: teachers’ perspectives 30
Venkat, Hamsa: Living in the world of numbers – being mathematically literate: Panel Discussion, AMESA 2007 39
De Villiers, Michael: Mathematical applications, modeling and technology 47

Keynotes lectures

Sethole, Godfrey: Dialogical engagement between theory and observations with reference to weakly classified activities 65
Venkat, Hamsa & Graven, Mellony: Insights into the implementation of Mathematical Literacy 72
De Villiers, Michael: Border patterns, tessellations & problem solving with transformations 84

Long papers

Boshoff, Hennie: Generate graphs and generalise the effects of parameters on graphs by means of Autograph and Sketchpad 96
Butgereit, Laurie: Math on Mxit: The medium is the message 107
Davis, Zain & Johnson, Yusuf: Failing by example: initial remarks on the constitution of school mathematics, with special reference to the teaching and learning of mathematics in five secondary schools 121
Diniz, Paulo: Challenge to students’ perceptions of lines relative positions in spatial geometry in Pedagogical University – Beira (Mozambique) 137
Du Plessis, Jacques & Setati, Mamokgethi: Cognitive architecture for algebraic division: base ten decomposition – an emulation of the ‘real thing’ 146
Essien, Anthony A.: Between understanding the language and understanding the maths? Translating from written to symbolic form in a multilingual algebra classroom 161
Govender, V. G.: A multiple case study of parental involvement with Grade 8 learners of mathematics 169
Cyril, Julie: Critique of mathematical models and applications: A necessary component of Mathematical Literacy 191
Langa, Mampho & Setati, Mamokgethi: Investigating the use of learners’ home languages to support mathematics learning 198
Machaba, Frans: Learning mathematics through integration with contexts that draw on learners’ everyday experiences both in mathematics and mathematical literacy classes: A pilot study 207
Mbekwa, Monde: An exploratory study into the introduction of mathematical literacy in selected Cape Peninsula high schools 220
Mhlolo, Mike: “Trying to describe the air” as a metaphor in conceptualising the mathematics-for-teaching 233
Molefe, Terence Baron: Using multiple languages to support mathematics proficiency in a grade 11 multilingual classroom of second language learners: An action research 241
Neser, Carolyn & Els, Nic: Maths fear in second-year B. Compt. students: the effect on examination results and study behaviour, with reference to long term personality attributes, gender and ethnicity 258
North, Marc: The Midmar mile, mixing concrete, and other anomalies in my Maths Literacy classroom 270
Mwakapenda, Willy: Understanding connections in school mathematics texts 282
Phoshoko, Moshe: Teacher’s selection of contexts in mathematics learning support materials for teaching the concept variable 293
Tshabalala, Lindiwe: Looking at how a grade six educator promotes conceptual understanding when introducing mixed and improper fractions 303
Venkat, Hamsa & Graven, Mellony: Learners’ experiences of Mathematical Literacy in grade 10 324

Short papers

Govender, V. G: Preparing learners for a Mathematical Literacy Olympiad 332
Graven, Mellony & Venkat, Hamsa: Teaching Mathematical Literacy: A spectrum of agendas 338
Neser, Carolyn & Els, Nic: The need for research into maths fear in South Africa: A physiologically-based assessment of the legitimacy of the concept of maths fear and how it affects different aspects of mathematical learning 345
Roberts, Anthea & Johnson, Yusuf: Reflecting on a baseline test 352
Southwood, Sue: Some Sums 362
Vermeulen, Nelis: Mathematical Literacy: Terminator or perpetuator of Mathematical Anxiety? 368
South Africa participated in the Trends in Mathematics and Science Study (TIMSS) in 1995, 1999 and 2003, but not in 2007. Critics have accused the National Department of Education (DoE) of opting out because of previous poor results (Business Day 25th April, 2007). The defense from the DoE is that the interventions in place have not had time to take effect. The major question guiding this paper is “What can teachers learn from South Africa’s participation in TIMSS 2003?”

The purpose of the TIMSS study is the improvement of teaching and learning through evaluating different aspects of the education system from educational policy to implementation in schools and finally to learner attainment. One of the biggest limitations of large-scale international studies such as TIMSS is that the information obtained does not reach teachers in a form, which enables them to interpret the evidence and take action.

In this paper I describe how the conceptual resources from the TIMSS study were used to investigate the performance of a sample of South African children on ratio and proportional reasoning.

INTRODUCTION

South Africa participated at the Grade 8 level in the Trends in Mathematics and Science Study (TIMSS) in 1995, 1999 and 2003, but not in 2007. There has been strong criticism from the press and other interested watchdogs accusing the National Department of Education (DoE) of opting out because of poor results (Blaine, 2007). The defense from the DoE has been that the interventions that have been put in place have not had time to take effect. The questions the general public may ask is what interventions have been put in place, how effective are the interventions and what time period are we envisaging for the interventions to take effect.

Teachers and teacher educators may ask, “What have we learnt from the TIMSS study?” Our answer could be “Not very much”, as our results have remained largely the same over a period of eight years. The responsibility for the poor performance is generally attributed to teachers. The role of the teacher, however, is critically affected by the education system that is in place. A responsibility therefore lies with the policy makers, the curriculum planners and educational officials, who decide on curriculum and make resources available to schools. The TIMSS studies are designed specifically to tell us about the overall system and not to per school and within a school.

There are both benefits and pitfalls to participating in large-scale international studies
for all stakeholders in the system (Howie, 2001). For the policy makers, curriculum planners and educational officials, external evaluation provides objective benchmarking of aspects of the education system and comparative information regarding the effectiveness of educational practices and learner performance. Problem areas are highlighted so that funds and resources can be redistributed to these areas. A problem that arises is that instead of reflecting on the system as a whole a scapegoat is targeted and this is nearly always the teacher or the study itself. The problem of lack of adequate understanding of the conceptual, methodological and technical underpinnings of studies like TIMSS leads to a disregard of the results, and so loss of an opportunity for learning.

For schools and teachers the results may lead to professional development and improved instruction. A problem may of course be that the unmediated release of bad results can result in unthinking and shortsighted decision making about what should be done. The direct value of large-scale evaluation for individual teachers is questionable as only a small percentage of mathematics teachers have access to the details of the study, which could inform classroom practice. The major question guiding this paper is “What can teachers learn from the Trends in Mathematics and Science Study (TIMSS) 2003?”

In this paper I describe how a research study, using TIMSS 2003 data, and related conceptual and technical resources, was designed to investigate the performance of a sample of South African learners on ratio and proportional reasoning. Description of the responses on selected items, analysis of the concepts underpinning the items and the predominant errors made by learners give deeper insight into the mathematical topic, but also into aspects of the TIMSS research design thereby placing educational officials, teachers and researchers in a position to give a reasoned response as to whether participation in the TIMSS studies could assist in the improvement of mathematics and science teaching and learning.

THE TIMSS STUDY

The purpose of large scale testing in countries around the world is to improve the quality of educational outcomes. Four purposes have been described by Nasser Abu-Alhija: to monitor educational systems for public accountability, assure quality control, provide information to teachers about teaching and learning and to “identify needs and allocate resources” (2007:52). The TIMSS study, one of many studies conducted by the International Association for the Evaluation of Educational Achievement (IEA), performs the function of monitoring educational systems for the purpose of improving education systems, through a carefully constructed research design that compares the intended curriculum, the implemented curriculum and attained curricula of education systems across the world with regard to mathematics and science.

Information on the attained curriculum, the aspect of the study that gets the most publicity in the media, is obtained through an extensive battery of items based on the mathematical frameworks that incorporate the “goals of mathematics and science education regarded as important in a significant number of countries” (Mullis, Martin,

The extensive cross-country comparisons in the TIMSS 2003 Report (Mullis et al., 2004) show differences in the intended curriculum\footnote{Problems with the South African intended curriculum and the reporting thereof have been reported elsewhere (Long, 2006a)} among countries. The implemented curriculum is reported to have the least overlap with the TIMSS mathematical framework (Reddy, 2007). This is cause for concern as the TIMSS mathematical framework was constructed from the “goals of mathematics and science education regarded as important in a significant number of countries” (Mullis et al., 2003).

The South African national report (Reddy, 2006) gives results by province and by former department. The Western Cape, the Northern Cape and Gauteng performed better than the national average, with the other provinces lagging behind. Former Model C schools performed around the international average, while schools formerly under the control of other departments scored lower. Results are also given for individual content domains and cognitive domains. The performance overall was relatively better in the content domain of measurement and data, but poor in geometry.

General information of this sort is critically important for education officials, but for teachers what is of greater value is providing information that can inform them about their practice. This third purpose listed above is “the most controversial function attributed to large scale assessment” (Nasser Abu-Alhija, 2007). The argument is made that while the information provided by large-scale external evaluation might result in motivation for teachers it does not necessarily provide the means to improve, especially in a conceptually complex subject like mathematics.

It is widely acknowledged that it is formative assessment, which provides feedback on teaching and learning to both teachers and learners, that leads to the improvement of learning (Black & Wiliam, 1998). This feedback has to be informed by a detailed knowledge of the curriculum and indeed a deep knowledge of the mathematical field. The situation at present is that not all teachers of mathematics have an adequate knowledge of the field. External evaluations such as the TIMSS study bring the expertise of the international community together in the design of mathematical frameworks and the construction of items (Mullis et al., 2003). Access to both the framework and the items alert the teachers to the need for more knowledge in a particular area. The research study reported below offers a model for using the conceptual and technical resources provided by the TIMSS study to investigate the mathematical domain of ratio and proportional reasoning.
RATIO AND PROPORTIONAL REASONING²

This study forms part of a larger research study into the epistemological and conceptual underpinnings of ratio and proportional reasoning and related subtopics within the multiplicative conceptual field (MCF) (Vergnaud, 1988). From a conceptual perspective MCF includes multiplication and division, ratio, rate, fractions and rational numbers. It also includes the mathematical processes required to solve problems associated with this domain. Vergnaud argues that the reason for mathematics education researchers to study conceptual fields it that concepts develop in relationship with other concepts rather than in isolation (Vergnaud, 1988). Mastery of these concepts, processes and ability to engage with associated problems, form the basis of more advanced mathematics topics in secondary school. Failure to engage with this field, which is generally agreed to be complex and to be cognitively challenging, undermines the mathematical progress of learners.

The theoretical focus of this research involves a conceptual analysis using the work of Vergnaud and others. An added dimension is the investigation of pivotal changes in the historical development of concepts within this field. A requirement from an empirical perspective was to understand the present level of attainment of South African learners in this field. The identification of learners mathematical processing and significant errors provided an empirical link towards the broader understanding of this mathematical domain.

The performance of the South African sample on the TIMSS items designed to test this content domain and aligned cognitive processes within this field confirmed that South African learners in general found these items difficult. Much of the low performance can be attributed to lack of properly functioning schools and associated poor teaching, but a large measure of difficulty can be attributed to the complexity of the field. In order to assess the complexity of the topic apart from adverse school factors, it was decided to use the same items to test a sample of learners from two well functioning schools. In addition to testing Grade 8s, the grade on either side, the Grade 7s and 9s, were included in order to track the progression of cognitive development across this phase.

The sample consisted of 330 learners from two schools. One of the schools serves learners from a lower socio-economic level, including street children, children from an orphanage, and boarders from rural areas. This is a private fee-paying school where 20% of learners are funded by bursaries. The second school catered for more affluent learners with a proportion of professional parents. The claim cannot be made that the sample is representative of the South African population, however as the purpose of the study was

² The title ratio and proportional reasoning is used in this paper as it embraces most of the constructs of interest in this study, and is familiar to teachers. It is however qualified as other related constructs are included in the study.
an investigation of a mathematical field it was felt that the range of abilities in these two schools would adequately support the purpose.

**ASSESSING LEARNER ABILITY**

Wright and Stone (1979) stipulate four steps to be taken in the process of measuring learners’ understanding. The first requirement in measuring is to get a clear idea of the domain to be tested. As mentioned above this included concepts, processes, and specific contexts which embody the problems associated with this field, as outlined in the conceptual framework (Long, in progress). The sub-components in the TIMSS mathematical framework were found to be a ‘good enough’ match for the relevant sub constructs in the multiplicative conceptual field, the focus of this study.

The second requirement in measuring constructs within a domain, is the use of items which “are believable realizations” of the particular mathematical domain and which can elicit signs of the variable in the person we want to measure (Wright and Stone, 1979:3). Items were initially selected from the mathematical domain *Number* and the subtopic *ratio, proportion and percent*. Additional items which included concepts, processes and contexts requiring multiplicative structures (Vergnaud, 1988) were selected from the mathematical domains Algebra, Measurement, Geometry and Data. The highlighted topics in the table below were selected as these included elements of the topic under investigation.

*Table 1: TIMSS 2003 Content domain (Mullis et al., 2003) showing subtopics*

<table>
<thead>
<tr>
<th>Number</th>
<th>Algebra</th>
<th>Measurement</th>
<th>Geometry</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>whole numbers</td>
<td>patterns</td>
<td>attributes and units</td>
<td>lines and angles</td>
<td>data collection and organisation</td>
</tr>
<tr>
<td>fractions and</td>
<td>algebraic expressions</td>
<td>tools, techniques and</td>
<td>two- and three-dimensional shapes</td>
<td>data representation and interpretation</td>
</tr>
<tr>
<td>decimals</td>
<td>equations and formulas</td>
<td>formula</td>
<td>congruence and similarity</td>
<td>uncertainty and probability</td>
</tr>
<tr>
<td>integers</td>
<td>relationship</td>
<td></td>
<td>locations and spatial relationships</td>
<td></td>
</tr>
<tr>
<td>ratio, proportion and percent</td>
<td>algebraic expressions</td>
<td>attributes and units</td>
<td>symmetry and transformations</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In addition to the content domain, aligned cognitive processes are included in the TIMSS mathematical framework. To respond correctly to mathematical problems learners need to be familiar with the mathematics concepts being assessed, but they also need to draw on a range of cognitive skills. The first set of skills, *knowing facts, procedures and concepts*, covers what students need to know, while the second, *applying knowledge and conceptual understanding*, focuses on the ability of the learner...
to apply what he or she knows to solve problems or answer questions. The third sub
domain, reasoning, goes beyond solving of routine problems to encompass unfamiliar
situations, complex contexts, and multi-step problems (Mullis, Martin and Foy, 2005: 65). The cognitive skills above, most notably the construct reasoning is critical for
successful development of concepts within MCF.

Table 2 shows the extent of the content and cognitive domains from which the items
were selected, indicating the complexity of the multiplicative conceptual field, of which
the subtopic ratio and proportional reasoning is a part. Appendix A gives brief
descriptions of the items. Detailed descriptions of the items and the items themselves
can be found in the research report (Long, 2007).

\textit{Table 2: Items in content and cognitive domains}

<table>
<thead>
<tr>
<th>Cognitive domain</th>
<th>Content domain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number</td>
</tr>
<tr>
<td>Knowing facts,</td>
<td>8</td>
</tr>
<tr>
<td>procedures and concepts</td>
<td></td>
</tr>
<tr>
<td>Applying knowledge and</td>
<td>10</td>
</tr>
<tr>
<td>concepts</td>
<td></td>
</tr>
<tr>
<td>Reasoning</td>
<td>2</td>
</tr>
</tbody>
</table>

The third and fourth requirements are essentially a test of the reliability of the test and
the consistency with which learners respond to the items. This requires that we
demonstrate that the items when taken by suitable persons are consistent with our
expectations and that the patterns of learner responses are consistent with our
expectations. Finding unexpected responses in either the items or the learners require an
investigation of the \textit{items} that we are using to measure the construct and an investigation
of the \textit{construct} we are aiming to measure. The process of converting both items and
learner measures to the same linear units requires a level of consistency in both the third
and fourth requirement.

The above four steps are requirements of the Rasch Measurement Model, the statistical
tool used in many large scale studies, which allows the relationship between learner
ability and item difficulty to be investigated. The mathematical model converts both
learner ability and item difficulty into the same linear units of measurement and
therefore allows measurement and comparison of ability and difficulty on the same
scale. The model presupposes that the higher a person's ability relative to the difficulty
of a question, the higher the probability of a correct response on that question. It defines
the difficulty of answering a question correctly, as 0.5 probability of success, when a
person's location on the construct being measured is equal to the difficulty of the item.
In keeping with the steps outlined above additional criteria for the selection of items were that the items include a range of difficulty, and that the items provide a benchmark against which to compare the sample of learners in this study. Using the TIMSS items enabled national and international comparison.

Figure 1: Item difficulty level and learner ability shown on the same scale
The initial selection of items was checked against the national sample for difficulty level. Because most of the items had a high difficulty index as determined by the national sample, making it “too difficult” in terms of item response theory, six items from the Grade 4 test were added to make a balanced test.

As in the TIMSS studies matrix sampling was used in order to allow for extensive coverage of the curriculum without overburdening the individual learner (Martin, Mullis and Chrostowski, 2004). The 36 items were allocated to 4 booklets. 12 core items and 6 additional items made up each booklet. 24 items were multiple choice and 12 were constructed response format. As in the TIMSS study, probabilistic procedures were used to establish scores for individual students over the test as a whole despite the fact that individual students had not answered all the items. Three of the constructed response items required 2 responses. In this case each of the responses was allocated a difficulty index, making 39 items.

**INTERPRETING RESULTS**

The results from the 330 learners, the total population of Grades 7, 8 and 9 at School A and School B, were coded and scored. Information was obtained on both the item and the learner. The graph (Figure 1) shows the items on the right ranging from high difficulty level at the top and low difficulty level at the bottom. The graph on the left shows the estimated ability of learners across the test, with learners with estimated low ability as measured on this test at the bottom and learners with estimated high ability at the top. The item difficulty indices were scaled to give a mean of 100 and a standard deviation of 30. Note that Item 22 has a difficulty index of 116. The learners located on the scale to have a estimated person ability of 116 have a 0.5 probability of getting Item 22 correct, a greater probability of getting the easier items correct and a lesser probability of getting the more difficult items correct.

A conceptual analysis of the items led to the categorization of items into the subtopics fractions, ratio, rate, percentage and algebra. The working definitions were as follows:

**Fraction** refers to a divided quantity, for example given a region or a quantity divided into equal parts, the learner is required to determine the fraction.

**Ratio** relates two or more quantities and projects that relationship onto a second quantity, using proportional reasoning.

**Percent** refers to a fraction sense, for example comparing parts to wholes, or a ratio sense, for example finds a percentage given original and new quantities.

**Rate** is a relational quantity that relates quantities of different types and is a compound measure, for example cost per hour.

**Algebra** involves relational thinking, the recognition of patterns, unknowns and equations.
Figure 2: Item difficulty level and learner ability, showing means of subtopics

The conceptual complexities of the content categories outlined above have been described elsewhere (Long, in progress), but in general, the items selected involved...
comparative relationships. The categories are not mutually exclusive, for example rate is a type of ratio, and the percent category involves both the fraction concept and the ratio concept (Parker and Leinhardt 1998). The items that have been categorized as algebra, involve relational thinking but in addition require recognition of a pattern, or working with an unknown in an equation. These items can sometimes be solved using counting procedures, hence making the solving of them not necessarily reliant on complex cognitive structures. As said previously the multiplicative conceptual field is complex and includes concepts, cognitive processes and problem situations. The difficulty of the item is dependent on all three parameters.

Nevertheless analysis of the above groupings elicits interesting results, from a conceptual and a psychometric perspective. Figure 2 above shows the items categorized into these subtopics. The means of performance on these subtopics shows the relative difficulty. For example rate problems are more difficult on the whole than ratio problems, given the number levels and the contextual problems in this test. What is also of significance is that the relative difficulty remains the same across the two schools, and the three grades (see table) with fraction and ratio problems easier than percent and rate. The relative difficulty decreases from Grades 7 to 9 showing progressive development in the understanding of the concept. Fraction is the notable exception, as the difficulty index has remained the same across the three grades.

Table 3: Analysis of subtopics within MCF by grade and school showing means of difficulty index

<table>
<thead>
<tr>
<th>Subtopic</th>
<th>Grade 7</th>
<th>Grade 8</th>
<th>Grade 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction</td>
<td>83</td>
<td>80</td>
<td>82</td>
</tr>
<tr>
<td>Ratio</td>
<td>101</td>
<td>91</td>
<td>81</td>
</tr>
<tr>
<td>Percent</td>
<td>109</td>
<td>100</td>
<td>90</td>
</tr>
<tr>
<td>Algebra</td>
<td>112</td>
<td>97</td>
<td>92</td>
</tr>
<tr>
<td>Rate</td>
<td>121</td>
<td>115</td>
<td>112</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>School A</th>
<th>School B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction</td>
<td>92</td>
</tr>
<tr>
<td>Ratio</td>
<td>100</td>
</tr>
<tr>
<td>Percent</td>
<td>108</td>
</tr>
<tr>
<td>Algebra</td>
<td>107</td>
</tr>
<tr>
<td>Rate</td>
<td>125</td>
</tr>
</tbody>
</table>

The cognitive domain categories defined by TIMSS, and discussed earlier in the paper, increase in complexity, though this is influenced by the number range, the kind of number, for example rational numbers are more complex than whole numbers, and the context of the problem. Again what is of interest is that the relative difficulty of these cognitive domains remains the same across the two schools and the three classes (see Table 4).
Table 4: Analysis of cognitive domains within MCF by grade and school showing means of difficulty index

<table>
<thead>
<tr>
<th></th>
<th>Grade 7</th>
<th>Grade 8</th>
<th>Grade 9</th>
<th>School A</th>
<th>School B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowing</td>
<td>85</td>
<td>75</td>
<td>73</td>
<td>87</td>
<td>66</td>
</tr>
<tr>
<td>Applying</td>
<td>117</td>
<td>109</td>
<td>100</td>
<td>117</td>
<td>96</td>
</tr>
<tr>
<td>Reasoning</td>
<td>120</td>
<td>112</td>
<td>106</td>
<td>118</td>
<td>102</td>
</tr>
</tbody>
</table>

The above results show a level of consistency, which implies that the broad construct being investigated is coherent and that the assessment is consistent. While the overall results by school and grade are interesting, what is of greater importance in this study is the analysis of responses to selected items. For discussion and analysis I have chosen four items from relatively low difficulty to high difficulty (see Figure 2). All four items were selected from the TIMSS content domain Number and subtopic ratio, proportion and percent.

**Item Analysis**

Item 2 (Figure 5) was empirically verified as having a low difficulty level

For every cool drink bottle that Zanele collected, Modiegi collected 3. Zanele collected a total of 9 cool drink bottles. How many did Modiegi collect?

| A | 3  |
| B | 12 |
| C | 13 |
| D | 27 |

Answer n %

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>28</td>
<td>8.48</td>
</tr>
<tr>
<td>B</td>
<td>26</td>
<td>7.88</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>D</td>
<td>272</td>
<td>82.42</td>
</tr>
<tr>
<td>X</td>
<td>3</td>
<td>0.91</td>
</tr>
</tbody>
</table>

Figure 5: Item 2 - Solves a word problem showing simple proportional reasoning

for the population being tested. This was to be expected as it was selected from the TIMSS Grade 4 mathematics items. The cognitive domain was knowing facts, procedures and concepts. As can be seen from the table above 82 % of the sample answered this item correctly (D). The selection of A by eight percent indicates a possible language difficulty. The selection of (B) can be explained by the use of additive reasoning instead of multiplicative reasoning. The learner may have reasoned that Modiegi collected three more bottles rather than three times more bottles.
The graph (Figure 6) shows the probability of a learner getting this item correct on the vertical axis. The horizontal axis shows learner ability as measured by the test on a scale from –2 (estimated low ability on the test as a whole) to +2 (estimated high ability on the test as a whole). So for example learners with high ‘ratio attainment’, will be shown to the right of the graph, while learners with low ‘ratio attainment’ will show to the left of the horizontal axis. On this item it can be seen that learners at the upper end, between 1 and 2 learner ability measures had a 90% probability of getting this item correct. Even those at the low end of ratio attainment were more likely to choose the correct answer (D), than any of the distracters. There was a reasonable chance (20-30%) of the learners with ‘low attainment’ selecting the distracters (A) and (B).

Figure 6: Characteristic curve by category for Item 2

Item 12 (Figure 7) was empirically verified as being of moderate difficult. The cognitive domain in which this item was categorized was knowing facts, procedures and concepts, the same as Item 2. 62% of the sample from the study got this item correct. The most common errors were (B), 15%, and (C), 14%. In the case of (B) it appears that the learners had not understood the concept of percent and selected the response where the % sign was added. The choice of (D) was in fact “more right” than either (B) or (C), as it takes account of the equivalence of 3/25 and 12/100.

Looking at the graph below we see that at the top end of the learner ability scale there is an 80% probability of getting the item correct. At the lower end there is a less than 50% probability of getting this correct. The tendency is for students at the lower end of the learner ability scale to select (B) and (C).
At a play, \( \frac{3}{25} \) of the people in the audience were children.

What percent of the audience was this?

- (A) 12%
- (B) 3%
- (C) 0.3%
- (D) 0.12%

<table>
<thead>
<tr>
<th>Answer</th>
<th>n</th>
<th>( % )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>206</td>
<td>62.42</td>
</tr>
<tr>
<td>B</td>
<td>48</td>
<td>14.55</td>
</tr>
<tr>
<td>C</td>
<td>47</td>
<td>14.24</td>
</tr>
<tr>
<td>D</td>
<td>19</td>
<td>5.76</td>
</tr>
<tr>
<td>X</td>
<td>10</td>
<td>3.03</td>
</tr>
</tbody>
</table>

**Figure 7: Item 12: Identifies a percent equivalent to a given fraction when the denominator is a factor of 100.**

![Figure 7: Item 12](image)

**Figure 8: Characteristic curve by category for Item 12**

Item 22 (Figure 9) below was of moderate to high difficulty level as measured in this test. The cognitive domain was applying knowledge and concepts. The understanding of percent to have a ratio meaning rather than the part-whole meaning is required here. The average correct for this sample is 37%. Of interest to mathematics teachers is that 34% selected response (D). The learner who selected answer 1000, (D), used a part-whole understanding of percentage rather than a ratio understanding. Their reasoning may have been 800 is to 1000, as 80% is to 100%. The part-whole understanding of percent is reinforced when converting between fractions and percent; the more difficult move to a ratio understanding is required for this item.
A shop increased its prices by 20%. What is the new price of an item which previously sold for 800 zeds?

- A 640 zeds
- B 900 zeds
- C 560 zeds
- D 1000 zeds

<table>
<thead>
<tr>
<th>Answer</th>
<th>n</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>22</td>
<td>6.67</td>
</tr>
<tr>
<td>B</td>
<td>59</td>
<td>17.88</td>
</tr>
<tr>
<td>C</td>
<td>122</td>
<td>36.97</td>
</tr>
<tr>
<td>D</td>
<td>111</td>
<td>33.64</td>
</tr>
<tr>
<td>NR</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>X</td>
<td>15</td>
<td>4.55</td>
</tr>
</tbody>
</table>

**Figure 9: Item 22—Calculates new price given percentage increase**

For learners with a higher measure of attainment, greater than 0.5 there is a 50-70% chance of getting this item correct. For learners with low attainment, less than 0.5, on the test as a whole the likelihood of choosing the response C and D is greater.

**Figure 10: Characteristic curve by category of person measure for Item 22**

Item 36, the final item was very difficult. This item was from the cognitive domain reasoning, which as defined by the TIMSS study (see earlier) includes more complex contexts and multi-step processing. An additional factor leading to the difficulty was that it was a constructed response item, which for South African learners have proved to be more difficult. There are two percentage related concepts, the first is “finding a percentage of” (What is 60% of 40?); the second requires finding the percent when given two quantities (Given 24/50, what is the percent?). The item proved to be very
difficult for the sample tested, with 8% getting the item partially correct. The international mean for this item was 12%, also relatively low, indicating that this type of problem is complex.

A computer club had 40 members, and 60% of the members were girls. Later, 10 boys joined the club. What percent of the members now are girls? Show the calculations that lead to your answer.

Answer: ________________

<table>
<thead>
<tr>
<th>Answer</th>
<th>n</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
<td>7.58</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>NR</td>
<td>7</td>
<td>2.12</td>
</tr>
<tr>
<td>X</td>
<td>297</td>
<td>90</td>
</tr>
</tbody>
</table>

1. Correct scored if: 48% with calculations own
2. Partial correct scored if: 24 girls or correct method but computational error or 48% with no calculations shown

Figure 11: Item 36- Solves a multi-step non routine problem involving percents

The graph shows the learners with high person ability having a 30% probability of getting this item correct whereas there was little understanding among learners with low ability on the overall test.

Figure 12: Characteristic curve by category of person ability for Item 22
OVERVIEW OF RESULTS

Four of the 36 items have been discussed in this paper. Preliminary analysis of all 36 items, in relation to the conceptual framework and current literature, is in process.

Many of the errors, observed in the item analysis, suggested that learners were relying on informal estimation skills, and intuitive responses (Long, in process). While these skills work with simpler concepts, the multiplicative conceptual field requires more complex cognitive skills. Another significant error was the use of additive reasoning where multiplicative reasoning was required. From research on this topic it has been noted that additive reasoning is the precursor to multiplicative reasoning (Misaildou and Williams, 2003), suggesting that this is a developmental issue, in addition to being an instructional issue. It appeared from some of the responses that incomplete reasoning (Kaput and West, 1994; Misaildou and Williams, 2003) was the cause of error in a number of learners. Their reasoning took them part way towards the correct answer. What was also evident was that there was little understanding of percentage problems and speed, distance and time problems. Both of these areas have been identified as highly complex (Parker and Leinhardt, 1998). And finally language and reading cannot be discounted among the causes of errors in this study as well as in the larger South African study. These preliminary findings will be further investigated through the teacher and learner interviews.

At the time of writing the results on selected items had been discussed with one of the teachers. This teacher had been responsible for the Grade 8 class, but had experience from Grade 3 to 12. Questions were put as to whether the item is at the level expected for Grade 7s, 8s and 9s, how the learner might have reasoned to find the correct answer and what errors in thinking may have led to incorrect answer. Plans were in process for interviews with the other teachers. The teacher from School A reported that it was by Grade 9, rather than Grade 8, that more than half of the learners would be able to grasp the concepts required in this test. This preliminary information together with the TIMSS data collected on curriculum indicates the need for a more thorough investigation of the curriculum requirements at Grades 7, 8 and 9, and even perhaps at Grades 4, 5 and 6.

Also at the time of writing interviews were being held with learners with high ability, medium ability and low ability as measured by the test, on selected items. The purpose of these interviews was to gain further insight into the concepts and processes used in solving the particular item. The plan was for learners to redo the selected items showing their working. An interview would then be conducted to investigate their strategies for solving these problems, thereby gaining insight into this field.

CONCLUSION

The investigation of learner attainment on a specific mathematical topic reinforces the need to address systemic issues facing education in South Africa. Two major problems are the specification of the intended curriculum, and the related implementation by the teachers. In addition the detailed analysis of specific problem areas by researchers, motivated by the
TIMSS results and supported by analysis of the specific conceptual field provides insightful information for curriculum planners and teacher educators.

The value of participating in international external evaluation studies such as TIMSS is that we are alerted to problems in the education system as a whole. The TIMSS 2003 report, and this secondary analysis using the TIMSS items, has brought attention to the South African curriculum. In comparison with the TIMSS curriculum framework, which includes generally agreed upon goals of mathematics, our curriculum is underspecified and in implementation is lagging a year behind when it comes to problems related to the multiplicative conceptual field. These topics are dealt with in Grade 5 in the Chinese curriculum and a great deal of time is devoted to mastery of these concepts at this phase (Cai & Sun, 2002). By contrast the South African curriculum is underspecified, with the same topic repeated in a number of grades, with little elaboration of content.

In the case of ratio and proportional reasoning and related topics within MCF, the TIMSS curriculum framework shows levels of detail, which are not apparent in the South African curriculum. The expectation is either that the textbook writers will fill in the gaps or that teachers themselves will be able to elaborate the curriculum statements for use in the classroom.

A focused investigation of items designed to test ratio and proportional reasoning and related concepts in MCF highlight some of the errors. The errors are not new, in fact the literature points to the same significant errors that are apparent among South African learners’ responses. This points to the complexity in the multiplicative conceptual field and alerts us to the necessity of giving this section of the curriculum more time, and perhaps engaging the learners on these topics in earlier grades. We might learn from the University of Chicago Schools Mathematics Project (UCSMP) where learners are expected to master concepts over three years of instruction. This means implementing a program of concepts within this mathematical field that progress with increasing complexity starting at Grade 6, so that by Grade 8 the procedures and concepts are in place.

The debate about whether South Africa should have participated in TIMSS 2007 will resurface when the decision to participate in TIMSS 2011 has to be made. To get the most benefit out of participation in the TIMSS studies more people at different levels of the education system, but especially the mathematics and science teachers at the cutting edge could be involved. They could, for example, engage with the research design, compare the TIMSS mathematical frameworks with our own curriculum documents with a view to enriching our curriculum, or justifying the differences between them. The released items could also be used as part of class tests, for both formative and diagnostic purposes.

Finally, well-designed and researched international studies such as TIMSS are evaluative in nature, informing the educational system as a whole, but they also have an important function in providing information and resources for improving teaching and learning.
REFERENCES


<table>
<thead>
<tr>
<th>No</th>
<th>Content domain</th>
<th>Cognitive domain</th>
<th>Item description: Example Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Number: Fractions and decimals</td>
<td>Knowing facts, procedures and concepts</td>
<td>Recognises one-half of a set of objects</td>
</tr>
<tr>
<td>2</td>
<td>Number: Ratio, proportion, percent</td>
<td>Applying knowledge and concepts</td>
<td>Solves a word problem involving simple proportional reasoning</td>
</tr>
<tr>
<td>3</td>
<td>Number: Fractions and decimals</td>
<td>Knowing facts, procedures and concepts</td>
<td>Recognises a figure that illustrates a simple ratio</td>
</tr>
<tr>
<td>4</td>
<td>Number: Fractions and decimals</td>
<td>Applying knowledge and concepts</td>
<td>Word problem: adding $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{4}$</td>
</tr>
<tr>
<td>5</td>
<td>Number: Fractions and decimals</td>
<td>Knowing facts, procedures and concepts</td>
<td>Recognises a familiar fraction represented by a figure with shaded parts</td>
</tr>
<tr>
<td>6</td>
<td>Number: Fractions and decimals</td>
<td>Applying knowledge and concepts</td>
<td>Selects a fraction representing the comparison of part to whole, given two parts in a word problem setting</td>
</tr>
<tr>
<td>7</td>
<td>Number: Fractions and decimals</td>
<td>Reasoning</td>
<td>Selects the statement that describes the effect of adding the same to both terms of a ratio</td>
</tr>
<tr>
<td>8</td>
<td>Algebra: Equations and formulas</td>
<td>Knowing facts, procedures and concepts</td>
<td>Using properties of a balance, reasons to find an unknown weight (mass)</td>
</tr>
<tr>
<td>9</td>
<td>Algebra: Equations and formulas</td>
<td>Knowing facts, procedures and concepts</td>
<td>Solves equation for missing number in a proportion</td>
</tr>
<tr>
<td>10</td>
<td>Data: Uncertainty and probability</td>
<td>Reasoning</td>
<td>Given the set of possible outcomes as fractions of all outcomes, recognizes probability is associated with size of fraction</td>
</tr>
<tr>
<td>11</td>
<td>Data: Data interpretation</td>
<td>Reasoning</td>
<td>Solves a comparison problem by associating elements of a bar graph with the terms, most often, fewer, more</td>
</tr>
<tr>
<td>12</td>
<td>Number: Ratio, proportion, percent</td>
<td>Knowing facts, procedures and concepts</td>
<td>Identifies a percent equivalent to a given fraction with denominator a factor of 100</td>
</tr>
<tr>
<td>13</td>
<td>Number: Ratio, proportion, percent</td>
<td>Applying knowledge and concepts</td>
<td>Solves a word problem by finding the missing term in a proportion</td>
</tr>
<tr>
<td>14</td>
<td>Algebra: Patterns</td>
<td>Reasoning</td>
<td>Finds a specified term in a sequence given the first three terms pictorially</td>
</tr>
<tr>
<td>15</td>
<td>Data: Data interpretation</td>
<td>Reasoning</td>
<td>Interprets data from a table, draws and justifies conclusions</td>
</tr>
<tr>
<td></td>
<td>Subject</td>
<td>Knowledge &amp; Concepts</td>
<td>Application</td>
</tr>
<tr>
<td>---</td>
<td>----------------------------------------</td>
<td>----------------------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>16</td>
<td>Number: Ratio, proportion, percent</td>
<td>Applying knowledge and concepts</td>
<td>Identifies proportional share of an amount divided into three unequal parts</td>
</tr>
<tr>
<td>17</td>
<td>Algebra: Patterns</td>
<td>Reasoning</td>
<td>Identifies numbers common to two different arithmetic sequences</td>
</tr>
<tr>
<td>18</td>
<td>Data: Uncertainty and probability</td>
<td>Applying knowledge and concepts</td>
<td>Given possible number of outcomes and probability of successful outcomes, solves for the number of successful outcomes</td>
</tr>
<tr>
<td>19</td>
<td>Algebra: Patterns</td>
<td>Applying knowledge and concepts</td>
<td>Given a sequence of diagrams growing in two dimensions and a partially completed table, finds the next two terms in a table.</td>
</tr>
<tr>
<td>20</td>
<td>Number: Fractions and decimals</td>
<td>Applying knowledge and concepts</td>
<td>Solves a multi step problem involving multiplication of whole numbers by fractions</td>
</tr>
<tr>
<td>21</td>
<td>Number: Ratio, proportion, percent</td>
<td>Knowing facts, procedures and concepts</td>
<td>Determines the simplified ratio of shaded to unshaded parts of a shape</td>
</tr>
<tr>
<td>22</td>
<td>Number: Ratio, proportion, percent</td>
<td>Applying knowledge and concepts</td>
<td>Calculates the new price of an item given the percent increase in price</td>
</tr>
<tr>
<td>23</td>
<td>Geometry: Congruence and similarity</td>
<td>Knowing facts, procedures and concepts</td>
<td>Identifies a triangle similar to a specific triangle given the lengths of all sides</td>
</tr>
<tr>
<td>24</td>
<td>Number: Fractions and decimals</td>
<td>Knowing facts, procedures and concepts</td>
<td>Finds 4/5 of a region divided into 10 equal parts</td>
</tr>
<tr>
<td>25</td>
<td>Data: Uncertainty and probability</td>
<td>Reasoning</td>
<td>Uses size of a group with a given characteristic in a sample to estimate size of a group with that characteristic in a population</td>
</tr>
<tr>
<td>26</td>
<td>Data: Data interpretation</td>
<td>Reasoning</td>
<td>Interprets data from a table, draws and justifies conclusions.</td>
</tr>
<tr>
<td>27</td>
<td>Algebra: Patterns</td>
<td>Reasoning</td>
<td>Generalising from the first several terms of a sequence growing in two dimensions, explains a way to find a specified term,</td>
</tr>
<tr>
<td>28</td>
<td>Number: Ratio, proportion, percent</td>
<td>Applying knowledge and concepts</td>
<td>Solves a word problem with decimals involving a proportion 2.4: 30 : X = 100</td>
</tr>
<tr>
<td>29</td>
<td>Number: Fractions and decimals</td>
<td>Knowing facts, procedures and concepts</td>
<td>Solves a one step word problem involving division of a whole number by a unit fraction</td>
</tr>
<tr>
<td>30</td>
<td>Algebra: Patterns</td>
<td>Reasoning</td>
<td>Generalising from the first several terms of a sequence growing in two dimensions, explains a way to find a specified term,</td>
</tr>
<tr>
<td>31</td>
<td>Algebra: Patterns</td>
<td>Applying knowledge and concepts</td>
<td>Knowing the first three terms of a sequence growing in two dimension finds the seventh term</td>
</tr>
<tr>
<td>32</td>
<td>Number: Ratio, proportion, percent</td>
<td>Knowing facts, procedures and concepts</td>
<td>Finds the percent change given the original and the new quantities</td>
</tr>
<tr>
<td>33</td>
<td>Measurement: Tools, techniques and formulas</td>
<td>Applying knowledge and concepts</td>
<td>Solves a word problem to find average speed given distance and time.</td>
</tr>
<tr>
<td>34</td>
<td>Data: Data interpretation</td>
<td>Applying knowledge and concepts</td>
<td>Interprets the data from a table to make calculations to solve a problem.</td>
</tr>
<tr>
<td>35</td>
<td>Measurement: Tools, techniques and formulas</td>
<td>Reasoning</td>
<td>Solves a multi step problem involving time distance and average speed</td>
</tr>
<tr>
<td>36</td>
<td>Number: Ratio, proportion, percent</td>
<td>Reasoning</td>
<td>Solves a multi step non-routine problem involving percents</td>
</tr>
<tr>
<td>37</td>
<td>Data: Data interpretation</td>
<td>Applying knowledge and concepts</td>
<td>Interprets the data from a table to make calculations to solve a problem</td>
</tr>
<tr>
<td>38</td>
<td>Number: Ratio, proportion, percent</td>
<td>Reasoning</td>
<td>Solves a multi step non-routine problem involving percents</td>
</tr>
<tr>
<td>39</td>
<td>Data: Data interpretation</td>
<td>Applying knowledge and concepts</td>
<td>Interprets the data from a table to make calculations to solve a problem</td>
</tr>
</tbody>
</table>
CHALLENGES IN IMPLEMENTING THE NEW MATHEMATICS CURRICULUM

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The challenges concerning implementation of the new mathematics curriculum at school level are considered in this paper. More emphasis is placed on those challenges that in the author’s view can be best handled by AMESA and SAMS (South African Mathematical Society) together.

INTRODUCTION

According to the Department of Education (DoE) documents listed in the website [1], the imperatives show an adequately developed curriculum relevant to the needs of the 21st century. The aims and objectives contained therein are good.

POLICY

The policy for the mathematics curriculum is good and is detailed as follows:

Our vision is of a South Africa in which all our people will have access to lifelong learning, education and training opportunities, which will, in turn, contribute towards improving the quality of life and building a peaceful, prosperous and democratic South Africa.

Our mission is to provide leadership in the establishment of a South African education system for the 21st century.

The Department of Education adheres to the following values:

People

Upholding the Constitution, being accountable to the Minister, the government and the people of South Africa

Excellence

Maintaining high standards of performance and professionalism by aiming for excellence in everything we do, including being fair, ethical and trustworthy in all that we do.
Teamwork

Co-operating with one another and with our partners in education in an open and supportive way to achieve shared goals.

Learning

Creating a learning organization in which staff members seek and share knowledge and information, while committing themselves to personal growth.

Innovation

Striving to address the training needs for high quality service and seeking ways to achieve our goals.

We are at the implementation phase of the policies governing the new curriculum. It is mentioned in the Director-General’s introductory remarks that the department’s focus is on the full and effective implementation of policies that have a positive impact on the lives of South Africans and furthermore to review those policies that show little or no impact. The government has committed itself to deliver quality education for all South Africans. Where huge cash injection is being made, we often hear corporate statements like: “Return on investments”. Likewise, government is also expecting improvements in the education sector as a return on the huge financial investment of taxpayer’s money.

In order to implement the new curriculum, we look at the following stakeholders:

1. Government
2. Educators
3. Learners
4. Parents
5. Environment

Sincere, committed and constructive interaction among all the above stakeholders, is critically important to guarantee the success of this new curriculum implementation. The roles that the different stakeholders play in the whole scenario are themselves different, but complement one another i.e. one cannot successfully work without the others. For example, government must supply the necessary resources such as teaching aids, subject materials, software materials, infrastructure etc, educators must dedicate themselves to quality teaching and learning etc, learners must exert themselves to study and work hard to their maximum potential etc.

PROBLEMS AND CHALLENGES

As things are at present, there is a huge wastage in terms of dropout rates, failure and pass rates and this necessitates immediate action. Problems experienced at school level can be summarized as follows:
Educators/teachers

In mathematics, it is absolutely unrealistic to expect a teacher with grade 12 mathematical knowledge to teach mathematics at high school level. Do our educators actually have the required **content knowledge in mathematics** as a subject in order for them to be able to teach the mathematics **content** at the required level? Focus is on those teachers who already have a professional qualification. It is even worse if the said teacher is expected to adjust to new curriculum developments. Mind you, we are doing pretty badly as a country in the international mathematics olympiad. According to Rudy Rucker [3]:

> unlike chess or astrology, mathematics has the curious property of being an intellectual game that really matters. Mathematics is a language whose form is universal. There is no such a thing as Chinese mathematics or American mathematics.

Learners

The problems of the teachers cannot be divorced from those of the learners. Teachers in primary schools are sometimes expected to teach everything resulting in teachers with NO knowledge and background of mathematics ending up teaching mathematics. This has happened in the past and still continues to happen, with NO visible sign of the trend ending. This results in learners being **pushed** to higher levels. Some of us do interact with teachers and learners in an endeavour to help. Teachers who are proficient in the subject, especially at high school level will tell you that these learners are **blank** in mathematics albeit they can express themselves well in English. The reality is that there is no correlation between speaking good English and mathematical proficiency.

Learners with shining results from school, more often than not they struggle in mathematics at tertiary level. The struggle is based on lack of understanding the very basic concepts.

Government

Government fails to supply adequate resources for a meaningful teaching and learning environment, e.g. some schools still teach under trees. For the successful implementation of the new curriculum, it is expected that REAL subject specialists are employed across all levels of education. However this expectation is far from being a reality.

Adequately qualified mathematics teachers leave the teaching service **en masse** as they feel undervalued and unappreciated by the government and other stakeholders. This is evidenced by the remuneration level of the teachers which is demoralizing. Hence there are few adequately qualified mathematics teachers, who themselves leave as soon as opportunities present themselves. It is a fact that the employment scope of mathematics teachers is large. It is also a fact that bright students who study mathematics and science
in general, are altogether NOT interested in becoming teachers. What does this say about us as a developing country and its future?

It is already mentioned somewhere above that the major obstacle in the teaching and learning of mathematics is associated with the deficiencies in the teachers’ mathematical proficiency. The practice of learners being pushed to higher levels is going to be exacerbated since the teachers’ remuneration will be linked to the pass rate of learners as was recently announced by the National Minister of Education. However, such a practice will definitely defeat the good intentions of the new curriculum. The practice of learners being pushed to higher levels leads to semi-literate graduates.

**Tertiary Institutions**

The government funding policies for higher education, necessitate further pushing of students to graduate in order for the institutions to access the subsidy from government of such students. In particular, higher education institutions are entrusted with producing well-skilled graduates for the development of the country. These institutions have found ways to come up with programmes like “ACE” which are fully subsidized by the government, but DO NOT contribute to improving the situation and do not even add value to the knowledge of the mathematics subject that the teachers already have. These programmes are lifelines for certain sections within the tertiary education institutions. The irony of the situation is that people who were previously excluded from studying science related disciplines, are the ones who are registering for these programmes *en masse*. In view of the legacies of the past, these institutions have a responsibility to make sure that government resources which are taxpayers’ money are not wasted, by directing their limited resources to producing good quality programmes which add value. The lack of proficiency in the mathematics content of the teachers, should precisely be attributed to tertiary institutions as they are the ones that train teachers.

The tertiary institutions have established foundation programmes to address the under preparedness of school leavers in science and engineering. It is alluded that people who go into teaching cannot go anywhere else and so the science teachers would probably not be the best school leavers in science. These teacher-trainees are not put through foundation programmes which are aimed at correcting the content deficiencies from high school. Is it a real wonder that when they graduate they still are not proficient content wise, but they are professional educators.

**Parents**

Parents have to provide support to their children by ensuring that they do homeworks, go to school everyday, be respectful to the teachers, wear uniform to school, provide a conducive studying environment at home, make sure that their children do not carry weapons e.g. guns to school etc. It is not easy for parents to be visibly involved in their children’s educational activities because in most cases, both parents are working and also a vast majority of parents are illiterate due to apartheid legacy. Despite all the other
activities like working taking most of the parents’ time, parents must NOT abdicate their roles in their children’s educational activities. In fact the successful implantation of the new curriculum demands more of the parents’ involvement than ever before.

MATHEMATICAL LITERACY

With the current situation in most schools that mathematics teachers lack proficiency in mathematics, it stands to reason that those who will be tasked with teaching Mathematics Literacy may have never had any numeracy exposure in their lives. This being the case, what is expected of these teachers to teach and learners to learn. This is a complex problem!

When mathematics literacy learners do well and later want to join the mainstream mathematics stream, serious problems will certainly arise. Presently, the trend is that people are ill-advised and want to take “shortcuts” to obtain higher qualifications. However, in mathematics, shortcuts certainly **do not exist.**

In the South African context, it is a fact that the learners who take Mathematical Literacy are assumed will follow the social sciences. When these learners pursue research in their respectively chosen disciplines, they will be seriously disadvantaged. Internationally, even social scientists are well-grounded in mathematics (See “The Origins of Finite Mathematics: The Social Science Connection, pages 106-118” [2]).

COMPARISON BETWEEN MATHEMATICS AND MATHEMATICS LITERACY

The comparison is based on the information obtained from the government document entitled National Curriculum Statement, Learning Programme Guidelines, January 2007. According to the Mathematics Literacy document, the purpose for Mathematics Literacy is as follows:

The inclusion of Mathematical Literacy as a fundamental subject in the Grade 10-12 curriculum, will ensure that learners are highly numerate consumers of mathematics. In the teaching and learning of Mathematical Literacy learners will be provided with opportunities to engage with real life problems in different contexts and so consolidate and extend basis mathematical skills. Mathematical Literacy will thus result in the ability to understand mathematical terminology and make sense of numerical and spatial information communicated in tables, graphs, diagrams and texts. Mathematical Literacy will, furthermore, develop the use of basis mathematical skills in critically analyzing situations and creatively solving everyday problems.

The subject Mathematical Literacy should enable the learner to become a self-managing person, a contributing worker and a participating citizen in a developing democracy. Mathematical Literacy “will ensure a broadening of the education of the learner that is suited to the modern world.”

According to the Mathematics document, the purpose for Mathematics is as follows:

*Mathematics provides powerful conceptual tools to:*
• Analyze situations and arguments;
• Make and justify critical decisions, and
• Take transformative action, thereby empowering people to:
  Work towards the reconstruction and development of society.
  Develop equal opportunities and choice.
  Contribute towards the widest development of society’s cultures, in a rapidly changing technological global context.
  Derive pleasure and satisfaction through the pursuit of rigour, elegance and the analysis of patterns and relationships.
  Engage with political, organizational and socio-economic relations.”

In the statement of intent for Mathematical Literacy, it should have the same effects as Mathematics (which appears to be quite noble). However the syllabus for the Mathematical Literacy seems not to be in agreement with the noble statement of intent. According to the statements of intent for both Mathematical Literacy and Mathematics, the syllabi of the two should overlap and thus raising a curious question as to why the need for Mathematical Literacy.

RECOMMENDATIONS

At present it is a reality that there is a serious deficiency in the content knowledge of the teachers. In order to effectively address the shortcomings of the teachers, SAMS and AMESA must both work together to come up with a new curriculum for teacher training in mathematics, where SAMS is solely responsible for the content and AMESA is solely responsible for the pedagogy.

It would be ideal if in the future everybody were to be well-grounded in Mathematics for their whole schooling years, i.e. before tertiary level. This would help grow the economy (positively) at a faster rate than currently is the case and everybody would benefit from such a growth.

There should be minimum mathematics requirements for admission into mathematics teacher training.

Mathematics professionals have a critical role to play in designing the curriculum. Amongst other things, they are supposed to be role models in the real sense as they are involved in developing the discipline. As specialists (researchers) in the discipline, they are better placed to identify problem areas such as deficiencies.
REFERENCES


INNOCENCE OF REALISTIC TASKS:
TEACHERS’ PERSPECTIVES

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Teachers are, as it were, central to the success of classroom events. The success to which the use of realistic tasks will advance is largely determined by teachers’ resolve about what has to go on in class. But what concerns do teachers have? This paper presents and reflects on two studies which make it possible to explore the teachers’ perspectives on the use of realistic tasks. The current finding is that there is a range of concerns, which include the nature of tasks and the type of learners.

Realistic tasks (mathematics tasks which incorporate the everyday) function in a space between mathematics and the everyday. It is hardly surprising that the post-apartheid education policy presents these tasks as enabling access to mathematics and also creating awareness of everyday aspects which call for the use of the mathematics. This point is especially made explicit in the policy document for mathematical literacy (DoE, 2005:7).

Mathematical Literacy will thus result in the ability to understand mathematical terminology and make sense of numerical and spatial information communicated in tables, graphs, diagrams and texts. Mathematical Literacy will, furthermore, develop the use of basic mathematical skills in critically analyzing situations and creatively solving problems.

A number of mathematics educators have also registered their discomfort with what appears to be a wedge between mathematics and everyday life experiences of learners. Calls have been made, mainly (but not only) in areas of mathematics education such as critical mathematics education (Skovsmose, 1994; Volmink, 1993) and ethnomathematics (D’Ambrosio, 1991; Mosimege, 1998) to relate mathematics with learners’ everyday experiences. It would therefore appear that post-apartheid South African policy on education is politically and pedagogically credible.

However, using Bernsteinian theoretical tools, a number of mathematics educators have cautioned against the uncritical inclusion of the everyday in mathematical tasks (Lerman and Zevenbergen, 2004 & Cooper and Dunne, 2000). The thesis of their argument is that realistic tasks tend to benefit middle class learners than working class learners (Cooper and Dunne, 2000). The rationale for this argument is grounded on Bernstein’s notion of recognition rules. Bernstein (1996:34) argues that middle-class learners apply specialized recognition rules when they engage a task which incorporates the everyday. Thus, they focus on the mathematical demands of the task and in turn do not become distracted by the everyday embedded in the task. In contrast, working-class learners apply non-specialized recognition rules. They use the everyday (embedded in the task)
as their frame of reference for engaging the task and therefore become distracted away from the mathematical demands of the task. The essence of this argument is that the way learners engage and make sense of realistic tasks can be explained in terms of their socio-economic background.

My aim is to enter this everyday-mathematics debate from the perspective of the teacher. In South Africa, as highlighted before, calls for the incorporation of the everyday in mathematics were made within the context of a new curriculum, Curriculum 2005. Yet, the take-up and demands of the new curriculum were experienced differently by different teachers. As Vithal and Volmink (2005: 10) observe:

In the (human and physically) well-resourced schools, mathematics teachers claimed to have been implementing the progressive pedagogy implied in the new curriculum all along, whilst in poorer schools with poorly trained teachers, lack of explicit direction and resources created great confusion and uncertainty about the new requirements.

It therefore seems important to explore and reflect on the teachers’ views regarding the incorporation of the everyday into mathematics. To this end, two studies, with different research methodologies and contexts were used to access and document teachers’ views on what the challenges of incorporating the everyday into the mathematics were. These views are discussed in sections three and four of this paper. In section four I will reflect on the implications of the teachers’ views. I begin this discussion with a suggestive reflection on what can possibly happen when realistic tasks are engaged in a mathematics classroom.

REALISTIC TASKS IN THE MATHEMATICS CLASSROOM

To explore the possible conversations that might emerge from the use of realistic tasks, an analysis of the following example, adapted from Cooper and Dunne (2000:36) shall be used as a vehicle:

In the morning rush, 269 want to take a lift. The lift can take a maximum of 14 people. How many times must it go up?

Focusing on the mathematical demands of the task, a learner might produce 19.2 as the answer obtained from an operation 269/14. However, as Cooper and Dunne (2000:37) admit, this answer might not be accepted as correct (in spite of its mathematical correctness) because a lift cannot go up 19.2 times. Even so, another learner might realize that the answer needs to be a whole number and may then decide to round it down to 19. In this case, as well, the answer remains incorrect because 3 people would be left. In both cases, we observe that failure to invite realistic considerations may compromise the learners’ response.

On the contrary, another learner may provide 10 as an answer on the grounds that “some people might get impatient and use the stairs”. (Cooper and Dunne, 2000:36). After all, the situation is that of “a morning rush” and because people are in a hurry, waiting for
the lift may be considered time-wasting. In this case, invitation of realistic considerations may compromise the learners’ response.

The use of realistic tasks certainly brings new pedagogic challenges. Ignoring or inviting realistic considerations does not necessarily guarantee correctness of an answer. It is also not obvious when and why should realistic considerations be suspended or considered by learners? Given these possible challenges, what are teachers’ main concerns with the everyday in mathematics, the next section reflects on this question with reference to two teachers.

**SUMMONING THE TEACHERS’ PERSPECTIVES: TWO TEACHERS**

Through the international Learners’ Perspectives Study (LPS), an international study conducted in ten countries, including South Africa (Clarke, 2006), we were able to observe consecutive mathematics lessons in different schools and then ask teachers and learners about these lessons. A comprehensive description of the rationale for the methodology and how the research unfolded in different countries is documented elsewhere (for example, see Sethole, Goba, Adler and Vithal 2006; Clarke, 2001). For purposes of this discussion I will limit myself only to our experiences in South Africa.

The South African team focused on three schools located in Durban. The three schools were chosen on the bases of having a ‘good’ mathematics teacher and on the type of communities they served. Thus, the one school was predominately Indian, the other predominantly black and the third predominantly white. For each school we focused on and observed at least nine grade 8 mathematics lessons. Each lesson was followed by a post lesson interview with learners and after each data collection period (i.e. after at least nine days) a teacher interview was conducted. It is during these interviews that questions about the use of the everyday in mathematics were raised. The two teachers whose views we summoned on this matter are Bulelwa (a mathematics teacher at a predominantly black school) and Mr. Smith *(a mathematics teacher at a predominantly white school).

It became clear, during the interview, that the two teachers used the everyday differently and therefore experienced different challenges. In one of her lessons, Bulelwa, for example, used AIDS as the everyday. For her, the everyday served as an object that warranted attention on and by itself. However, she also sensed that in the process, AIDS became some form of distractor, in other words it became so visible that the mathematics became concealed. Such a concern can be deduced from her closing remarks at the end of the lesson where AIDS was summoned (lesson 7).
B (to the learners): Thank you for the discussion we have had on activity 7. I just hope that in the end it will help you and your communities. Please spread the message, write those posters. Most unfortunately, I thought we would have sometime to discuss patterns and nature. Now did you enjoy the lesson?

Learners: Yes

B: What did you enjoy most, the AIDS part or the maths part?

L’s: AIDS

B: So AIDS is more interesting than mathematics

Mr. Smith invited the everyday in his mathematics classrooms through the use of word problems. In this regard, he appealed to a wider range of contexts. For him, the everyday served as a see-through, in other words, a vehicle through which the mathematics could be accessed. In this regard, the everyday became invisible. However, rendering mathematics invisible also let to the trivialising of the everyday. This became apparent when, during the interview, Mr. Smith tried to explain the extent to which the everyday should be realistic. In particular, I asked Mr. Smith whether it mattered to him if a mathematical answer to the price of one CD was R2.30*. He responded

Mr. Smith: I will probably want it to be more accurate… I want them to pick up that if 3 CD’s cost R9.00 one would cost R3.00. For me, the procedure is important, not the actual cost. It’s just that I know these kids buy CD’s or they buy shoes with exorbitant price. I want to relate to an everyday experience to bring it …(he doesn’t finish the sentence). What I am actually saying: “this is where you are using maths in real life without saying this is math in real life”.

The dilemma of using inauthentic everyday contexts is apparent in Mr. Smith’s response; on the one hand he wishes for learners to “relate to the real world contexts” and on the other he asserts the “procedure is more important”.

For Bulelwa and Mr. Smith, the everyday embedded in the task serves two different purposes. Bulelwa sees the everyday as a tool to create an awareness of beyond-the-classroom challenges. For this reason, it is important for the context to be as authentic and close to learners’ experiences as possible. AIDS talk does not end in the classroom. Mr. Smith regards the everyday as a means to access the mathematics. It just has to be good enough to assist the production of correct mathematical calculations. The authenticity of the everyday is not a priority to him.

In spite of these differences, it appears that central to Bulelwa and Mr. Smith is the nature of the everyday they would incorporate in a task. To them, the nature of the everyday embedded in a task plays an important role in determining whether mathematics or the everyday will be accessed. Could this be a general trend? To answer this question, we embarked on a study that would allow participation of more teachers. The following section focuses closely on this study.

* An acceptable low price for a CD is R100. R2.30 is a ridiculously low price.
SUMMONING MORE TEACHERS’ PERSPECTIVES: MANY TEACHERS

The second phase of the research project started in February 2006 and it involved a group of 35 teachers who were attending a workshop in the Soshanguve area. In January 2007, another set of data, involving 46 teachers in the Mpumalanga area was also gathered. In both cases, the teachers had come for a mathematics workshop on Linear Programming, Probability, Differential Calculus and Measures of Central tendencies. In this regard, this data collection exercise was opportunistic, motivated by the presence of a group of mathematics teachers. On the first day of the workshop, we explained the purpose of the data-gathering exercise and highlighted that participation and completion of the questionnaire was voluntary. The questionnaire itself consisted of 17 items and took about 30 – 45 minutes to complete.

The question was itself not innocent; it assumed that teachers would experience problems in their attempt to incorporate the everyday. Yet, it was informed by Bulelwa and Mr. Smith’s cases and a range of literature that reflected on the incorporation of the everyday in mathematics. Indeed, Ole Skovsmose, a critical mathematics educator has encountered a case in which the teacher felt that the everyday considerations concealed the mathematics (Skovsmose, 1994). In South Africa, researchers such as Vithal (2001), Mogari (2001) and Mosimege (1998) have also pointed to the possible ways in which the everyday may prevent access to mathematics. In the following sections I provide responses obtained from the two groups separately.

The Soshanguve group: A total of 35 teachers responded to the questionnaire. 27 (about 75%) of the teachers possessed at least a four year teaching qualification. Of significance to the study is that 28 (80%) had majored in mathematics. All participants taught mathematics in their schools and the majority of teachers (75%) had a teaching experience of over six years. Therefore the participants had been exposed to a reasonable amount of mathematics content either at a pre-service level or at an in-service level or at both levels.

One of the items in the 19-items questionnaire asked teachers to identify what they saw as “challenges regarding the incorporation of the everyday in mathematics”. 39 responses were generated by this item. The following table 1 summarises the categories into which their responses were grouped.

<table>
<thead>
<tr>
<th>Response</th>
<th>Type of the everyday to draw</th>
<th>Learners’ background</th>
<th>General schooling problems</th>
<th>Difficulty of drawing relations within the mathematics</th>
<th>Difficulty in linking mathematics topics</th>
<th>No responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>11</td>
<td>09</td>
<td>08</td>
<td>06</td>
<td>01</td>
<td>05</td>
</tr>
<tr>
<td>Percentage</td>
<td>28</td>
<td>23</td>
<td>20</td>
<td>15</td>
<td>2</td>
<td>13</td>
</tr>
</tbody>
</table>

Table 1: Responses on challenges of incorporating the everyday - Soshanguve
**The Mpumalanga teachers**: A total of 46 teachers participated in the study. 32 of the participants (69%) had a three year teaching qualification and the rest possessed at least a four year teaching qualification. All participants had majored in mathematics and 41 (89%) had a teaching experience of five years or more. Similar to the Soshanguve group, all the participants have had exposure to mathematics both at a pre-service and in-service level.

The following table reflects responses to the item on ‘challenges regarding the incorporation of the everyday in mathematics’.

<table>
<thead>
<tr>
<th>Response</th>
<th>Type of the everyday to draw</th>
<th>Learners’ background</th>
<th>Proper teaching method</th>
<th>Lack of physical resources</th>
<th>Deliberately leaves out the everyday</th>
<th>No problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>12</td>
<td>08</td>
<td>08</td>
<td>06</td>
<td>02</td>
<td>04</td>
</tr>
<tr>
<td>Percentage</td>
<td>24</td>
<td>16</td>
<td>16</td>
<td>12</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>Reasons</td>
<td>Not categorized</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number</td>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percentage</td>
<td>21</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Response on challenges of incorporating the everyday-Mpumalanga

Teachers, as Lerman (1993) notes, are more powerful at a classroom level than they realize. In other words, teachers actually ‘approve’ what goes into the classroom. Yet, their concerns over the everyday in mathematics can be categorized into two: teacher-proof concerns and teacher-manageable concerns. I use the term teacher-proof concerns in reference to factors which affect the schooling or classroom environment and about which teachers have little influence. With respect to the responses above, teacher-proof concerns would be “learners’ background” and “lack of physical resources”. Teacher-manageable concerns refer to factors which affect the schooling or classroom environment and about which teachers have some or full control. These would be the following responses: “type of the everyday to draw”, “general schooling problems” like late coming, “difficulty of drawing relations within mathematics”, “difficulty in linking mathematics topics”, “proper teaching method” and “deliberately leaves out the everyday”. Even though these responses reflect teachers’ concerns, it is possible that through workshops, further studies, communication and co-operation with other teachers, teachers may gain skills to engage these concerns.

There does not seem to be any consistent or comparable trend in table 1 and table 2. However, the high frequency of responses which referenced the “type of the everyday to draw” and “learners’ background” as teachers’ major concerns was noticeable. These particular concerns constitute over 50% of categorizable data in each of the tables. It is too early to argue that this high frequency will be observed in other cases. However, it is possible to suggest the high frequency is not surprising at all, learners and the everyday are two important constituents of a classroom in which realistic items are engaged.
REFLECTIONS AND CONCLUSIONS

What data from table 1 and table 2 suggests is that the type of learners in the class is considered a concern and a possible ‘obstacle’ towards bringing the everyday into the mathematics. In some respect, this position resonates with Bernstein’s recognition rules highlighted before as far as it positions the nature of learners as a function of their classroom performance. It is important to point out that none of the submissions obtained made specific mention learners’ socio-economic conditions, instead learners’ different types of behaviour or habits were implied as possible causes of why the everyday was considered a challenge. The following are examples of comments in this respect: ‘learners’ negative attitude’, ‘learners are lazy’ and ‘learners’ different backgrounds’. So the reasons for which learners will struggle with realistic tasks are varied.

From both studies, the nature of the everyday is implied as being central to using realistic tasks. Examples of comments submitted by teachers in the second study included “the type of examples to use”, “topics not related to mathematics” and “having a situation that will be relevant to content that is in real life”. As pointed out in the first section of this paper through an analysis of a lift item, the everyday is itself not a bystander. On the one hand, the use of authentic contexts with which learners are familiar (like AIDS) may distract and slow learners’ access to mathematics and, on the other, the use of inauthentic context with which learners are unfamiliar trivialize the context and create a view of mathematics as a subject which has no place elsewhere in the society. In her observation of how learners engage activities which incorporate the everyday, Vithal (2003) also cautions that the everyday embedded in the mathematics can be deeply gendered.

It would seem the bringing in of the everyday or the use of realistic contexts in mathematics can bring unintended consequences. It is perhaps hardly a surprise that other mathematics educators have called for the bracketing out of the everyday from the mathematics (See Floden, Buchman and Swille, 1987 & Muller and Taylor, 1985). What implications do these observations have for mathematics teachers and the new policy?

CONCLUSION

Inviting the everyday into the mathematics classroom is, like many activities, complex. As children grow up and watch movies or live drama, they learn to appreciate and distinguish between the actor and the character performed (by the actor). With time, they are able to notice that the physical attributes of the actor (e.g. height, colour) somehow resemble that of the character they are performing. They also grow to understand that the events about the character do not necessarily parallel actor’s real life experiences. At least, if the character ‘dies’ a number of young children will not consider it strange to see the same actor appear in another movie or in real life or enquire about funeral arrangements. Thus, with constant exposure, it would seem children do not necessarily take realism to far.
The use of the everyday in the classroom might be a platform to assist learners in negotiating the tough mathematics-everyday boundary. Through these experiences, learners might invite the everyday in or out when it is appropriate to. The everyday embedded in realistic items need not be taken at face value.

REFERENCES


In this paper, the notion of being ‘functional’ (using mathematics in literate ways) is considered with reference to two bodies of research – one finding evidence of mathematical functionality, the other finding widespread evidence of a lack of functionality. It is argued that the key difference is that the first group, located in anthropological and cross-cultural research traditions, has focused on functionality within specific activity settings, whilst the latter group, working within mathematics education, is calling for a much more generalized notion of functionality. The implications of the findings of these two groups for the teaching of Mathematical Literacy are briefly considered, and some learners’ responses from a school where some of these implications have been translated into practice are presented.

**INTRODUCTION**

In the real world we often see mathematical thinking in use in a range of different settings – traders making decisions about the amounts of stock to purchase and on how to price their stock, and nurses calculating medicine dosages for example. These examples illustrate several aspects of the usage of mathematics in real-life – amongst these:

- wide ranging applications of relatively basic mathematical ideas
- situations that are often quite ‘messy’ and complex
- the need to bring in non-mathematical considerations alongside mathematical thinking in order to arrive at decisions for action

All these aspects have been widely documented in a range of studies in mathematics education. Lyn Arthur Steen, a key American advocate of what he terms as ‘quantitative literacy’ has argued that being quantitatively literate involves something different from what is provided in school mathematics:

‘Whereas the mathematics curriculum has historically focused on school-based knowledge, quantitative literacy involves mathematics acting in the world. Typical numeracy challenges involve real data and uncertain procedures but require primarily elementary mathematics’ (Steen, 2001, p6).
If school mathematics has not traditionally developed this kind of literate use of mathematical thinking, we can look at what broader evidence in the field can tell us about approaches that might be more helpful. In this brief report which accompanies the panel discussion, I focus on two areas of research in particular that have produced somewhat contradictory findings – anthropological/ cross-cultural studies of mathematics in use in everyday and/or workplace settings, and school/ employer-based critiques of learners’ poor levels of application skills. I then consider what these two sets of findings might imply for the teaching and learning of Mathematical Literacy in South Africa, before presenting and discussing the responses of learners in one school where some of these implications for practice have been implemented.

FUNCTIONALITY/ LACK OF FUNCTIONALITY?

The first area, drawing upon anthropological and cross-cultural research, consists of studies that have looked at real-life situations in which mathematical thinking is used (e.g. Lave, 1988; Saxe, 1991; Scribner, 1986). These studies have revealed alternative (i.e. not school mathematics-based), situation-specific forms of reasoning to solve problems. For the purposes of this discussion, what is interesting is that these studies give examples of mathematics in use across a range of settings, and show people’s ability to be ‘functional’ or ‘literate’ in these settings using mathematical reasoning and procedures. Saxe’s (1991) study, focused on Brazilian child candy-sellers, showed efficient and accurate calculation and decision-making skills around buying and selling prices from children involved in this practice in a climate of high inflation.

In contrast to this group of studies, critical research within the arena of formal mathematics education points to evidence of lack of sense-making – and sense-making is an essential component of the literate use of mathematics – amongst mathematics learners (Mukhopadhyay & Greer, 2001; Schoenfeld, 1985; Steen, 2001). Higher education and employer reports have also commented on learners’ inability to flexibly apply problem-solving strategies to new or unfamiliar problems (Roberts, 2002; Smith, 2004). A common feature of this group of studies is therefore, the evidence they provide of frequent lack of functionality. This latter group contains many of those who, internationally, have called for the introduction of courses within mathematics education that will incorporate activities, approaches and pedagogies that will better address the need for more people to be mathematically literate. The implementation of Mathematical Literacy in the Further Education and Training phase in South Africa draws from this debate in addition to ongoing concerns to broaden access.

In order to consider how both these positions and findings can be valid, we need to consider the notion of being able to ‘function’ – as viewed by both these groups – more closely.

The first group of studies – which provide evidence of people functioning in everyday settings – has almost exclusively focused on people in specific activity settings. In these settings, novice workers are ‘apprenticed’ into acquiring the specific skills needed to function in these contexts – e.g. the highly context-dependent calculations strategies
used by the Brazilian candy-sellers mentioned above, and the reasoning around capacity used by dairy workers for loading crates in Scribner’s (1986) study. These research studies in which efficient and accurate use of mathematical thinking was apparent provided fertile ground for the development of new theories of learning – situated cognition in particular (Brown, Collins, & Duguid, 1989) – which argued that the (mathematical) thinking brought to bear upon any situation is intrinsically interwoven with the kinds of problems that occur and need to be solved within that context. Thus, the functionality, or literate use of mathematics seen within these studies and the earlier examples is highly situation-specific. This idea of context dependency provides the critical area of contrast with the second group of studies identified above.

This latter group, as stated above, provides evidence of lack of sense-making and/or flexible functionality either within school mathematics learning or following this learning. This body of evidence points to learners who associate mathematics learning with formulaic and unthinking imitation of procedures, and lack of understanding and critique of concepts and methods employed. It is important to note that writers within this group are calling for the development of a general functionality – one that allows people to participate, contribute and critique across a range of problem settings, both mathematical and everyday.

The curriculum statement for Mathematical Literacy in South Africa (DoE, 2003) calls for this kind of generalised sense of literacy within its advocacy of developing the knowledge, skills and attributes that learners will need to function across a range of future life roles:

‘Mathematical Literacy, should enable the learner to become a self-managing person, a contributing worker and a participating citizen in a developing democracy. Mathematical Literacy will ensure a broadening of the education of the learner which is suited to the modern world.’ (DoE, 2003, p10)

The evidence from the two groups of research studies highlighted above points to successes in people being able to learn specific knowledge and skills that are appropriate in particular settings, but widespread failures within mathematics education in terms of the goal of developing a more generalised willingness and capacity to engage with literate mathematical thinking. One feature that links both of these sets of findings is the issue of ‘transfer’ (Carraher & Schliemann, 2002) – being able to translate relevant learning from one context or situation to another – with both groups acknowledging difficulties in achieving this kind of flexible and permeable learning.

So what can these two sets of findings tell us in relation to the aims of the Mathematical Literacy policy in South Africa? Some consequences for the teaching of Mathematical Literacy are considered in the following section. In the concluding section, a selection of Mathematical Literacy learners’ responses (in a school where many of the suggested approaches were used) is presented.
CONSEQUENCES FOR TEACHING MATHEMATICAL LITERACY

The examples of successful use of mathematical thinking in specific situations within the first group of findings above suggest that couching the teaching of concepts, facts and procedures within the context of relevant and realistic problem-solving situations (insofar as these can be made realistic in the context of formal learning situations) may be helpful. Evidence from the application of the Realistic Mathematics Education (RME) approach for example, in which problem-solving begins within a situation that is experientially real for learners, points to these kinds of tasks motivating and supporting the processes of understanding and meaning-making (Barnes, 2005). The Mathematical Literacy policy emphasises the usefulness of real-life situations as starting points for developing literate mathematical thinking, with the Teacher Guide document (DoE, 2006) providing a range of exemplar units for how this might be achieved.

The problem identified in the original cross-cultural and anthropological literature though is that this approach has been successful in developing situation-specific thinking and skills. RME-based research and Boaler’s (1997) study though, point to possible routes out of this highly insulated kind of problem-solving capacity. RME research and Phoenix Park School in Boaler’s study provided learners with ongoing exposure to varied, open-ended and often extended problem-solving tasks, within which mathematical reasoning and tools were developed and taught in response to the need to draw conclusions about the problem situation in focus. As in the everyday examples of functionality, the need to solve a problem motivates and structures the processes of modelling and mathematising. A shift in pedagogic goals underlies this kind of practice. In traditional mathematics classrooms, educators focus on ensuring content delivery. In RME approaches and in Phoenix Park, educators focused on developing learners’ general process abilities - to make sense of problems, to model and mathematise, and to devise strategies to solve problems. The traditional approach tends to emphasise isolated fragments of content; the alternative approaches emphasise the need to make sense of problems, and use mathematical thinking as a flexible and powerful tool to reach conclusions. The literate use of mathematics emphatically involves this latter goal, as the Mathematical Literacy curriculum statement makes clear:

- It is through engaging learners in situations of a mathematical nature experienced in their lives that the teacher will bring home to learners the usefulness and importance of mathematical ways of thought in solving problems in such situations. (DoE, 2003, p42)

Research within the Mathematical Literacy thrust in the Marang Centre has traced the process of policy implementation in a small number of Johannesburg schools. In our main research school, an inner-city school where almost all learners are black, educators (so far, all mathematics teachers) have shifted to using a variety of realistic tasks within Mathematical Literacy lessons – amongst these, water and utilities bills, newspaper reports on dam levels in South Africa, house plan drawings and loan companies’ leaflets on loan rates. Educators and learners in this school also noted the slower pace of work in Mathematical Literacy – enabled by a less densely packed curriculum in comparison to mathematics – that was much more accommodating of learners’ levels of understanding.
Towards the end of last year (2006), Dr. Mellony Graven and I administered a questionnaire to all the Grade 10 Mathematical Literacy learners in this school (n=90, 66 completed questionnaires returned), and interviewed a sample of these learners (n=19) about their experiences in the new subject. Responses which illuminate areas of success (or lack of success) in relation to the development of literate use of mathematics are detailed and discussed in the next section.

LEARNERS’ EXPERIENCES – THE DEVELOPMENT OF MATHEMATICAL LITERACY

Overwhelmingly, learners within this cohort were highly positive about their experiences of learning in Mathematical Literacy. Comments conveying a sense of enjoying the new subject occurred on 55/66 questionnaire open responses, and 57/66 learners reported that they enjoyed Mathematical Literacy all or most of the time. The features that figured within this positive response have been detailed elsewhere (Venkat & Graven, 2007, in these proceedings). In this discussion, I present evidence within our data that related either to the discussion above and/or suggested the literate use of mathematics.

The notion of focusing on problem-solving in context that was noted above as a shift in pedagogic goals away from content delivery was commented on by some learners:

‘Most of the time it makes you active and also it teaches you to focus to what you are doing and come with some solutions for the problem that you are facing. I really enjoyed the subject.’

(Girl, UA’s class, questionnaire, part of open section response)

Connected to this, when learners talked about the work they had covered during grade 10, more comments were received in interviews and questionnaire responses relating to contextual understandings than mathematical content areas or topics.

The ongoing use of tasks focused on real-life situations – a feature that had been observed in school visits across 2006 – was also commented on as a positive aspect by learners:

‘In maths there are certain sections such as trigonometry that I personally don’t consider as substantial for my chosen career. So I really think the DoE has helped empower the young South Africans in believing they can make it by coming up with an alternative subject. Maths literacy views topics from a scenario type of way. It really makes them up to real life situations and I must say it makes it easier to work with.’

(Boy, FD’s class, questionnaire, part of open section response)

Half of the questionnaire responses made some reference to Mathematical Literacy being a useful or practical subject related to everyday life. In many cases, examples of everyday relevance of the subject were detailed:
‘I think I have made a good choice for choosing Mathematical Literacy because it is a challenging subject and people think it is easy but they don’t know anything. The only thing that makes me love Maths Literacy is the fact that it has taught us a lot like knowing how banks work, how do you calculate interest rates; mostly, the fact of how the Reserve Bank operates, (adds mathematical topics covered too)’

(Girl, FD’s class, questionnaire, part of open section response)

Whilst many of these responses referred to ML as potentially useful in the future, some learners spoke (within an interview in the excerpt below) of the subject in terms of active current use:

S: Even like, percentages right. You go to shops like Edgars and you see here they have written 20% whatever, you wouldn't understand but now it's much easier, okay, the price is this much when they're saying 20 percent off. You could calculate that and you know what actually - what money am I going to pay and all this.

MG: So you're actually managing to do that now hey?

S: Yes.

MG: And last year, you maybe weren't doing it?

S: No, not really because in like maths literacy they like to go to more details and like to understand better, you know. It's much nicer.

M: And business contracts. Let's say cell phone contracts and let's say taking a loan from a bank, like R10 000-00, all of sudden you're going to pay R20 000-00. You didn't know where the other R10 000 was coming from. So now whenever like a friend or my mum, she's speaking of getting a phone on contract. She looks at the paper. I always, even if I'm reading the newspaper and then I see this phone you pay R75 for 24 months. Actually take my calculator and calculate how much —

MG: So you're actively looking out.

M: Looking out for my mum and say "ah, you'll end up paying R2000. Just think how many phones you could afford, me and my sister and yours".

S: And like determining like which one is like better to buy - cash or on credit.

(Boys, CN’s class, in paired interview with MG)

The Mathematical Literacy educators in this school continue to stress that levels of mathematical understanding and fluency are still weak. However, they acknowledge that a key achievement of 2006 has been the turning around of highly negative experiences of mathematics learning that the vast majority of their learners came in with to largely positive experiences of learning in Mathematical Literacy. One learner summed up this shift in the following terms:

‘This kind of maths enables us to understand and be able to look at maths from another angle apart from looking at maths as a hard subject.’

(Boy, CN’s class, questionnaire, part of open section response)

A brief summary of concluding remarks that can be made in the face of these patterns of responses from learners is given below.
CONCLUDING REMARKS

I state at the outset of this section that the positive experiences of learners in this school may be ‘atypical’ – some of the educators I work with have described extensive difficulties, and indeed, their inability to transcend the negativity that learners came into grade 10 with at the start of last year. An issue raised by this current lack of generalisability is the lack of broader survey data on whether, and if so, how Mathematical Literacy is being implemented in South Africa. The ‘hows’ of policy implementation are important here for two reasons. Firstly, the discussion of evidence from the two research groupings pointed to the need for shifts in approach if literate use of mathematics was to be achieved. Secondly, analyses stemming from empirical data point to different ‘agendas’ being taken up by Mathematical Literacy educators, with some agendas embodying more change from previous mathematics teaching than others (Graven & Venkat, 2007, in these proceedings)

A further caveat is that the data presented here consists of learner reports of active, or potential future use of mathematical thinking, rather than direct evidence of active usage. This does not imply though, that the data can be dismissed. At the very least, there is evidence here that shifting pedagogic approaches to encompass ongoing work on real-life based tasks and problems, carries potential for the development of literate use of mathematics.

The nature of the shifts in classroom activities in this school indicates a degree of blurring of the boundaries that have traditionally insulated mathematics learning from real-life. This blurring appears to have allowed many learners to think of ways in which Mathematical Literacy learning in class can be borrowed into a range of real-life situations. Current data suggests a very direct translation of in-school learning to out-of-school situations – sale discount prices, compound interest and diesel costs for travel amongst these. This evidence does fall short of the kinds of flexible abstraction, translation and application of learning that are often described as exemplifying ‘transfer’. More research on these aspects would help to build a finer-grained picture of how the features of practice that research suggests are helpful to the development of more literate use of mathematics might be harnessed or adapted to produce learners that are capable and willing to effectively fulfill the range of future life-roles suggested in the South African Mathematical Literacy policy documents.

REFERENCES


INTRODUCTION

The new South African mathematics curriculum through all the grades strongly emphasizes a more relevant, realistic approach focusing a lot on applications of mathematics to the real world. This is in line with curriculum development in most other countries. For example, the influential NCTM Standards sums it up succinctly in the following respective standards for instructional programs in algebra and geometry from prekindergarten through grade 12 (respectively from http://standards.nctm.org/document/chapter3/alg.htm http://standards.nctm.org/document/chapter3/geom.htm):

(Students should be enabled to in ... )

Algebra
- use mathematical models to represent and understand quantitative relationships;

Geometry
- use visualization, spatial reasoning, and geometric modeling to solve problems.

Like the NCTM Standards, the new South African curriculum now also encourages for the first time the use of appropriate technology in modeling and solving real-world problems as follows (Dept. of Education, 2002, p. 3):

- use available technology in calculations and in the development of models.

But how do our learners and students interpret mathematics and its relationship to the real world? What is their proficiency in applying mathematics, modeling real-life problems (and using technology efficiently)?

TWO ILLUSTRATIVE EXAMPLES

I’ve often given the following problem as one of a range of problems to prospective primary mathematics teachers to give to their learners during practice teaching:

*Sally has some dolls and is given 5 more dolls by her grandmother so that she now has 12 dolls in total. How many dolls did she have in the beginning?*

Much to these prospective teachers’ surprise, a substantial number of learners come up
with the seemingly meaningless answer of 17 by simply adding 12 and 5. Firstly, note
that the problem here is clearly NOT one of computational ability, as they
CORRECTLY added 12 and 5. Furthermore, this kind of response was sometimes more
prevalent among older children in Grade 3 than among younger ones in Grades 1 or 2.
How do we explain this?

Without getting in to deeply analyzing the reasons for this error, it would rather seem
that it is a case of interpreting or modeling the situation incorrectly. Moreover, fuelled
perhaps by past experiences of school mathematics being largely divorced from the real
world, learners clearly fail to test or evaluate the meaningfulness of their answers in the
given context. Further research also indicates that some of these learners just picked out
the two numbers and added them because of the word ‘more’ – a dubious strategy
sometimes actually taught by textbooks and teachers for ‘simplifying’ word problems by
identifying certain key words that supposedly suggests the type of arithmetic operation
to be used!

Here is another problem from the 1984 RUMEUS (Research Unit for Mathematics
Education at the University of Stellenbosch) Algebra test I’ve often given to our
prospective high school teachers for use during practice teaching and was also part of
mini investigation of 40 Grade 11 learners by Kowlesar (1992):

A sightseeing trip around the Durban beachfront is given by the following formulae:
Taxi A: C = 80 + 25t
Taxi B: C = 100 + 20t

Draw a graph to show how the cost C (in cents) varies with the time t for Taxi A and Taxi B on the
same graph.

Which of the two taxis is cheaper?

Shockingly in Kowlesar (1992), it was found that NONE of the learners could draw the
correct graphs or determine when one was cheaper than the other! The most popular
response was to plot two “coordinates” namely, (80, 25t) and (100, 20t) on a cost-time
graph. The learners’ explanations were illuminating of some of their underlying
misconceptions, for example:

I thought that 80 was the distance and 25t the time, therefore I plotted these as coordinates.

Did not know what to do. Can’t make certain of what was wanted. Is it cost against time? How do
you plot 25t and 20t?

Dealing with money and time. I won’t be able to plot points like 100t.

I did not have any idea what to do. I’ve never seen a function in that form before.

So what was the problem? Was it because they hadn’t yet been taught linear functions
and graphs or maybe just not remembered the work they’d done earlier on?
However, as became apparent through interviews with some of these Grade 11 learners, they were ALL able to easily draw the graphs of \( y = 80 + 25x \) and \( y = 100 + 20x \). The problem therefore clearly was not one of instrumental proficiency with linear functions and graphs (in standard form with variables \( x \) and \( y \)), but one of recognizing and handling linear functions and variables in REAL WORLD CONTEXTS.

**DECONTEXTUALISED TEACHING AND LEARNING**

The above two examples clearly show that knowing and being able to carry out certain mathematical algorithms and procedures provides NO definite guarantee that learners are able to interpret and solve real world problems meaningfully. Proposals by mathematics educators and professional associations over the past few decades that what needs to be done is to more regularly use real world contexts as STARTING POINTS for developing the mathematics to be learnt, has still to a large degree, fallen on deaf ears and is sometimes even met with fierce resistance by traditionalists.

However, the root of the problem seems quite clear. Is it any wonder that children cannot translate from “pure” mathematics to practical situations, when mathematics is mostly taught in a DECONTEXTUALISED fashion; that is, in a “vacuum”, completely devoid of any relationship to the real world?

For example, in the first problem, just knowing how to add or subtract provides no guarantee that either of these operations will be recognized as the appropriate model for a specific practical situation. Similarly, if linear functions as in the second problem are decontextualised and just taught instrumentally, the chances are that learners would later be unable to apply them to real world situations.

Indeed, one could argue, with substantial support from research evidence, that if mathematical skills and content have been taught completely divorced from any real world interpretations and meanings, attempts to do so at a later stage are mostly futile.

Some years ago as described in De Villiers (1992), I was observing a student teacher dealing with long division in Grade 5. After some preliminary discussion and demonstration, he gave the problem 8.048 kg \( \div 4 \) to the class, upon which they all easily and efficiently solved the problem correctly. A very successful lesson it would certainly seem, not so?

I then asked the class to write a little story describing a real problem situation involving decimals to which this calculation would produce the answer. The results were shocking!

Of the class of 31, not a single child described a relevant, meaningful context, with 30 writing stories not even involving mass at all, for example:

*There are 8.048 cows. We must divide them equally among 4 people. How much does each one get?*

*A farmer has 8048 oranges which he wants to divide into 4 equal groups for needy people.*

49
The only child who described a situation involving mass, wrote the following:

*If John bought a tape recorder of mass 8.048 kg. He has to divide it amongst four people. Each one gets 2.012 kg.*

Although one’s initial response may be one of laughter, further reflection is more likely to move one to tears. Is this what mathematics means to our children? Is this decontextualised, irrelevant and meaningless learning still going on in most of our schools? What is the use if children can do arithmetic operations, but they do not even know what it means nor what it is useful for?

**MODELING AS A TEACHING STRATEGY**

In contrast to traditional decontextualised teaching, modeling of real world contexts can effectively be used as a teaching strategy, i.e. where the mathematical theory is initiated by, and directly developed from a practical situation (to which it can later also be reapplied). For example, in my book *Boolean Algebra at School*, the entire theory is systematically modeled from several electrical switching circuit problems, and only in the last section is it formally axiomatised, and are proofs developed (De Villiers, 1987).

It seems axiomatic that when mathematical concepts, algorithms, theorems, etc. are directly abstracted from practical contexts and problems, the links between such content and the real world must be stronger than when it is attempted to create such links only after the content has been presented in a vacuum. Groundbreaking work by especially the Freudenthal Institute in the Netherlands over several decades, as well as the Problem-Centred Approach of the University of Stellenbosch (Murray, Olivier & Human, 1998) since the mid 1980’s, using precisely such a modeling approach, has shown not only its feasibility, but also substantial gains in children’s ability to relate mathematics to the real world meaningfully (as well as increased conceptual understanding generally).

In traditional decontextualised teaching, children are often appeased when asking the teacher why they need to learn some theory or algorithm by usually being told that they will LATER ON see its practical or theoretical applications – which sometimes takes weeks or months – hardly motivating at all! In contrast, using modeling as a teaching strategy by starting with real world (or theoretical) applications has much higher motivational value as it immediately places the usefulness or value of the content in the foreground.

However, it should also be pointed out that not all mathematical topics in the school curriculum might be equally well suited for development through modeling as a teaching strategy. Historically, we should remember that the source of new mathematics, particularly from the 19th century onwards, was not always the real world, but motivated quite often by needs for symmetry and structure, to solve theoretical problems, or from further reflection, abstraction and generalization of existing mathematics.
UNREALISTIC PROBLEMS AND CONTEXTS

There exist numerous examples in which “reality” is thoroughly degraded in unrealistic, contrived and artificial problems. Consider for example, the following two examples:

A person saves 1c on the first day; 2c on the second day; 4c on the third day; etc. How much does the person save in a year?

Quite clearly no normal person would exhibit the strange saving habit in the first problem; in fact, no-one in the world, including Bill Gates, would have enough money to keep on saving in this manner for a year! The second problem may be amusing and interesting, but clearly it is a “theoretical” problem simply clothed in practical terms – it bears no relationship to any conceivable REAL WORD problem.

In the 2005 Grade 9 Common Task for Assessment for GET (General Education & Training), according to Van Etten & Adendorff (2006), one of the tasks for learners apparently was to predict the total number of elephants in Africa after a decade, only given a total number of 1.3 million elephants in 1979 and a poaching figure of 200 elephants on average per day slaughtered. Clearly, this is highly unrealistic as neither the average birth rate nor the average death rate due to natural disease, aging, etc. is given. Moreover, it is simplistically assuming that a linear model applies. If the purpose of the task was for learners to realize the inadequacy of the given data and use of a linear model, it can still be defended, but it’s not clear whether that was the intention at all.

However, we should also be cautious not to lose genuine mathematics if too much focus is put on the real world context. Jan De Lange (1996: 5) humorously illustrates this with the following (exaggerated) example of the changes in mathematics education over past number of decades:

* in 1960: A woodcutter sells on load of wood for $100. If the production cost is four fifths of the selling price, what is his profit?

* in 1970 (modern mathematics): A woodcutter sells a set of L of wooden blocks for a set M of dollars. The number of elements of M is 100. The subset C of the cost has four fifths of the number of elements of M. Draw C as a subset of M and determine the number of elements of the subset which gives the profit.

* in 1980 (back to basics): A woodcutter sells one load of wood for $100. The profit is $20. Underline the number 2 and discuss its place value.
* in 1990 (socio-politically directed education): By felling beautiful trees, the woodcutter earns $20. Classroom discussion: How would the birds and squirrels feel about the felling of the trees, etc.?

THE MODELING PROCESS
The process of mathematical modeling essentially consists of three steps or stages as illustrated in Figure 1, namely:

1. construction of the mathematical model
2. solution of the model
3. interpretation and evaluation of the solution.

Stage 1
During the construction of the model one or more prerequisites are often necessary, for example:

- the making of appropriate assumptions to simplify the situation
- data collection, tabulation, graphical representation, data transformation, etc.
- identification and symbolization of variables
- the construction of suitable formulae and/or representations like scale drawings or working models

Figure 1
Since the real world is complex and varied, while mathematics deals with ideal, abstract objects, simplifying assumptions always have to come into play when mathematics is applied. In the real world there are no perfectly straight lines, flat planes or spheres nor can measurements be made with absolute precision. Even an elementary arithmetic sum like “2 apples plus 1 apples = 3 apples” is tacitly based on the implicit assumption that these apples are ALL EXACTLY the SAME (in size, shape, and that none of them are rotten!) As shown in Figure 2, the three vertical groups of the “correct answer”, 3 apples, are hardly equivalent at all!

![Figure 2](image)

To what extent are we making learners aware and critical of some of the implicit assumptions in many so-called practical applications of mathematics? Not doing so certainly cannot help learners’ better distinguish between mathematics and the real world, nor help their understanding of the inter-relationship between the two.

Another example is the following typical problem from trigonometry from the FET phase:

* A person on a cliff 100m high sees a ship out at sea. If the angle of declination is 30 degrees, how far out at sea is the ship?

Normally it is expected of learners here to make a little sketch as shown in Figure 3, and then use the tan function to find the distance from the base of the cliff to the ship. A nice simple application of trigonometry, beautifully illustrating the utility of mathematics, isn’t it?

![Figure 3](image)
But what are some of the assumptions that have been made in this problem and its solution? Clearly, there are several over-simplifications of reality, for example:

1. It is assumed that the ocean is flat, i.e. that a straight line can reasonably approximate the distance from the base of the cliff to the ship. In other words, the curvature of the earth is ignored, which on a small scale is not problematic, but on a larger scale may lead to significant deviations.

2. It is assumed that the cliff is perfectly vertical with respect to the line chosen to represent the distance from the base of the cliff to the ship.

3. More over, it is assumed that the person’s height (of say 1.7 m) is negligible compared to the height of 100 m of the cliff.

4. It is also assumed that the ship is far enough away that a single point can reasonably represent its position, which is certainly not the case for a large ship closer to the shore.

Now these are not unreasonable assumptions to make to simplify the problem, and thus to obtain a reasonable ESTIMATE of the distance. However, mathematically educated learners ought to know that the calculated answers we obtain for applied problems such as these are NEVER absolutely precise (leaving out even the issue of how accurately the angle of declination was measured).

Furthermore, by letting learners find a more accurate solution by using a more accurate representation of reality, may help develop understanding that usually the gain in accuracy is offset against increased mathematical complexity. This happens quite often in mathematical modeling in various fields such as physics, biology, engineering, statistics, etc. so that simpler models are quite often chosen because they are easier to work with mathematically, though one should always be aware of the assumptions and therefore the limitations that they may potentially have.

**Stages 2 & 3**

After obtaining some kind of mathematical model to represent reality, we normally proceed with the solution process, which can involve simple arithmetic operations or more advanced ones like factorization, solution of equations, differentiation, transformations, etc. Lastly, in the interpretation and evaluation of the solution we need to check whether it is realistic by critically comparing it with the real world situation.

Let us now briefly consider and discuss Stages 2 and 3 in relation to the following four problems that I’ve also regularly over the years given to prospective primary mathematics teachers to try out with their learners during teaching practice, namely:

1. A group of 100 people have to be transported in minibuses. No more than 12 passengers may be transported in one minibus. How many minibuses are needed?

2. Packages of 12 cookies each have to be made up for a bazaar. How many packages of 12 can be made up if 100 cookies are available?
3. In a famine stricken area, 100 pockets of rice have to be shared equally between 12 families. How many pockets of rice should each family get?

4. 100 people have to be seated at 12 identical tables. How many people are to be seated at each table?

As before, it is quite typical of older children in Grades 5, 6 and 7 who have been taught and drilled extensively in long division, to quickly recognize that all four problems can mathematically be represented by the number sentence $100 \div 12$. Shockingly, however, the majority of them would then usually just offer the answer “8 remainder 4” for all four these problems, showing no appreciation whatsoever that such an “answer” is entirely SENSELESS in each of the four problems. Such learners have effectively become mathematically DISEMPOWERED, contrary to any definition of education!

In contrast, younger children often model and solve these four problems quite sensibly, often using a variety of different ways. For example, Problem 1 could be solved by a young child by repeated addition, by adding up the number of people each taxi can take one after the other, e.g. $12 + 12 \Rightarrow 24 + 12 \Rightarrow 36 + 12 \Rightarrow 48$, etc. until 96 is reached. The child then typically adds up the number of 12’s to find the number of taxis, which is 8, and then adds one more taxi for the remaining 4 people. So the required number of taxis is 9.

(Something one could raise further in a class discussion, is that if additional cost was not an issue, one might as well use 10 taxis to transport 10 people per taxi and which would be more comfortable than having 12 people per taxi. Of course, one could also argue that the remaining 4 people should just be squeezed in, for example by putting one extra person in 4 of the 8 taxis. But then one is raising important issues of road safety and breaking the law by overloading! Even if the class decides that 9 mini-buses will suffice, a decision will still have to be made whether one is going to transport 12 people in each of the 8 taxis, and 4 people in 1 taxi OR whether one is going to transport 11 people in each of 8 taxis plus 12 people in one 1 taxi. From the viewpoint of a little more ‘comfort’ for more people, the latter solution may be favored).

This example clearly shows how a solution in the final stage of interpretation and evaluation can be influenced by several important real world considerations. The other three problems above can be solved similarly with possible respective answers as follows:

2. 8 packages of 12 (here the remainder of 4 cookies could be eaten or put in a smaller packet and maybe sold at 1/3 of the price?)

3. 8 + 1/3 pockets of rice per family (since it is a famine stricken area one would want to divide up the 4 remaining pockets among the 12 families so that each gets 1/3)

4. by seating one extra person at 4 of the tables, we obtain 4 tables with 9 people and 8 tables with 8 people (note how a typical answer of 8 rem 4 is particularly nonsensical for this problem).

Mere computational skill at long division clearly does not guarantee learners obtaining
sensible and meaningful solutions to these four problems. Even using a calculator to carry out the “division” and obtain an answer of 8.3333333 is basically useless without proper interpretation within the given practical contexts. But are our children allowed sufficient opportunities such as these to engage in stage 3 of the modeling process?

Let us also briefly consider again Sally’s doll problem from a modeling perspective:

*Sally has some dolls and is given 5 more dolls by her grandmother so that she now has 12 dolls in total. How many dolls did she have in the beginning?*

As mentioned earlier, many younger children using less sophisticated calculation strategies (e.g. drawing pictures, using physical objects, counting all, or counting on, etc.) are often less likely than older children to simply compute $12 + 5$ as the answer. Closer analysis reveals that one explanation seems to be found in the prescriptive rule-oriented teaching of some teachers.

Where-as the actual number sentence or mathematical model that best represents this problem is $? + 5 = 12$, some teachers apparently INSIST that learners MUST write down a number sentence for this problem in the form $12 – 5 = ?$ These teachers are clearly confusing one possible SOLUTION STRATEGY, namely $12 – 5 = ?$, with that of first accurately MODELING the situation. The number sentence $12 – 5 = ?$ is no reasonable representation of the problem situation at all – the real situation does NOT involve any “subtraction”, i.e. any “taking away” or “decrease”.

In fact, the situation involves an “increase” in the number of dolls, which children readily recognize, but because they are forced to write number sentences with the unknown only on the right side, they now have no alternative but to write $5 + 12 = ?$, and proceed to carry out the calculation. Here is a clear example of INTERFERENCE with children’s correct intuitive perception of a situation by their following a teacher’s prescribed rule as a result of their natural desire to please him/her.

Furthermore, note that a reasonable model for this problem need not be a number sentence, but can also be drawings, counters, fingers, etc. In solving such models, children may also proceed in a number of different ways, for example:

- counting on (6, 7, 8, 9, 10, 11, 12 => 7)
- piecemeal addition (5 + 5 => 10 + 2 => 12; thus 5 + 2 = 7)
- trial & error addition (10 + 5 = 15, too big; 6 + 5 = 11, too small; 7 + 5 = 12)
- counting backwards (11, 10, 9, 8, 7, 6, 5 => 7)
- subtraction (12 – 2 => 10 – 3 => 7)

Unfortunately teachers often think they are HELPING learners by providing them with rules like these, but the sad fact is that learners are being helped into their “mathematical GRAVES”! Prescriptive teaching generally tends to destroy children’s natural creativity and ability to solve real world problems (as well as purely mathematical ones) meaningfully. What happens is that children become so pre-occupied with trying to
comply to the required rules of the teacher, that they no longer think about the actual problem at hand nor try to solve it in any way they can. Another consequence of prescription teaching is that children become more and more teacher-dependent rather than independent, expecting all the time to be first shown an example of how a problem of a certain type should be solved, before dutifully practicing several similar examples.

THE ROLE OF TECHNOLOGY

As discussed and illustrated with several examples in De Villiers (1994), computing technology can assist immensely with particularly the SOLUTION stage of the modeling process. Apart from calculators having the ability to now perform complicated arithmetic calculations with speed, ease and accuracy, symbolic algebra software like Mathematica and Maple that can easily solve equations, factorize expressions, differentiate, etc. are now widely available too. In addition, dynamic geometry software like Sketchpad enables one to easily simulate and solve problems by animation and/or using scale diagrams.

Computing technology therefore strongly challenges the traditional approach, which emphasizes computational and manipulative skills BEFORE applications (and usually at the cost of developing skills in model construction and interpretation). For example, Grade 9 or 10 learners can now solve a traditional calculus problem like the following simply by using a calculator and completing a table of values as in Makae et al (2001) or better still use the graphing capability of Sketchpad as shown in Figure 4:

A water reservoir in the shape of a rectangular prism with a square base has to be built for three villages and needs to contain 80 000 liters. What should its base-length and height be to use the least amount of material?

![Figure 4](image-url)
However, note that in order to use computing technology effectively in this way, it is essential to first obtain an appropriate algebraic model, i.e. the surface area function. Older learners in Grade 12 can even differentiate this surface area function within Sketchpad, draw a graph of the differential function, and then find the minimum where it cuts the x-axis. A heavy emphasis on computation and manipulation BY HAND in mathematics education can therefore no longer be justified with the increased availability of such software, not only on computers, but also on graphic calculators.

![Figure 5](image)

With computing technology, modeling as a teaching approach becomes feasible as illustrated in Figure 5. For example, the following alternative to the traditional “theory-first – applications-later” approach to quadratic functions can now be used:

**Stage 1: Where do quadratic functions come from?**
* Falling Objects, Projectile Motion & other Scientific contexts
* Non-linear revenue & other Economic contexts
* Curve-fitting (e.g. least squares)

**Stage 2: Finding max/min values & solving quadratic equations**
* Guess-and-test
  * By hand: numerically & graphically
  * By calculator: numerically & graphically
  * By numerical methods such as iteration: by hand & calculator
  * By computer: numerically & graphically
* Formal Solutions by Computer
  * Symbolic Processors like Maple, Derive, Mathematica, etc.

**Stage 3: Formal Theory of Quadratic Functions**
* Factorization
* Completion of square
* General solution formula
* Discriminant
* Axis of symmetry, maximum/minimum
* Sum & product of roots
SIMULATION AND ANIMATION

Apart from greatly assisting in the Solution Stage of modeling, computing technology also provides useful modeling tools for simulation and animation. For example, consider the following problem: What is the path of the head of a man's shadow if he is walking past a lamppost AB, in a straight line? As shown in Figure 6 this can be easily modeled, and a solution dynamically found by moving CD along the straight path, and observing the path that E traces out. (Of course, this provides no formal proof that the top of the shadow also moves in a straight line parallel to the straight path the man is walking on, but knowing what one needs to prove is often a necessary first step. In this case, the result can be proved using the properties of similar triangles, and is left to the reader.)

Similarly, as discussed in De Villiers (1999), Sketchpad can very effectively be used to dynamically investigate the best place for kicking to the posts in rugby after the scoring of a try as shown in Figure 7.
Figure 8 shows how the following interesting problem can also be modeled and solved: How large must a mirror be so that you can see your whole body in it?

Through visual animation, Sketchpad can also bring to life the topic of solving simultaneous linear equations. For example, learners could use a sketch as shown in Figure 9 to investigate which of Henri or Emile would win a race of 150 m, if they have respective running speeds of 2.5 m/s and 1 m/s, and Henri has a head start of 45 m. To enhance the connection with the underlying mathematics linear graphs of their respective distances against time can be drawn in the same sketch. Changing the speeds or head start given to Henri further allows learners to dynamically investigate a whole range of different related problems.

**AN EXAMPLE OF MODELING A GEOMETRIC CONCEPT**

Traditionally the concept of perpendicular bisector is just introduced by definition as a “line that perpendicularly bisects a line segment”. This is usually followed by a description of how one can construct a perpendicular bisector of a line segment by compass, and perhaps a proof using congruency that the construction actually works. Next follows the theorem that the perpendicular bisectors of a triangle are always concurrent (and that the point of concurrency is the circumcentre of the circumcircle of the triangle). Apart from the fact that FET learners are no longer required to prove this result, no effort is usually made to connect this result to any real world context.
In De Villiers (2003) the concept of perpendicular bisector is NOT presented directly at first, but learners and prospective teachers are first given the following realistic problem to investigate with a ready-made sketch in Sketchpad:

In a developing country like South Africa, there are many remote rural areas where people do not have access to safe, clean water, and are dependent on nearby rivers or streams for their water supply. Apart from being unreliable due to frequent droughts, these rivers and streams are often muddy and unfit for human consumption. Suppose the government wants to build a water reservoir and purification plant for four villages in such a remote, rural area. Where should they place the water reservoir so that it is the same distance from all four villages?

After first using trial and error and dragging \( P \) until it is EQUIDISTANT from all four villages, learners and prospective teachers are asked to try and find a different, easier method of locating the desired position. In order to do this, they are encouraged to try and find all the equidistant points from two of the villages, and using the drag and TRACE function of Sketchpad, they find that all these equidistant points form a line bisecting the line segment connecting the two villages (and perpendicular to it). So here the concept of perpendicular bisector of a line segment is DEFINED as the “path (or locus) of all the points equidistant from the two endpoints of the segment”.

After constructing the four perpendicular bisectors of the sides of the quadrilateral to note that they all intersect in the desired point of equidistance, learners and prospective teachers are then asked whether one can always find a point equidistant from all four vertices, no matter the shape or size of the quadrilateral. Since most learners and prospective teachers usually expect it to be possible, they then find it SURPRISING when dragging the vertices of the quadrilateral that the perpendicular bisectors are no longer concurrent; i.e. in other words, that not all quadrilaterals have equi-distant points from their vertices!

Having learners and prospective teachers construct a circle with its centre at the circumcentre (the point of concurrency) of the original quadrilateral, and passing through all four villages, the concept of “cyclic quadrilaterals” is now also introduced in a practical, meaningful context. (In contrast to the decontextualised way it is normally introduced in Grade 12).
More-over, when we now move on to next investigate where a water reservoir should be placed so that it is equal distances from three villages, it now comes as a big SURPRISE when DRAGGING, that for a triangle, no matter the shape or size, the perpendicular bisectors always remain concurrent! As explained and discussed in Mudaly & De Villiers (2004), this surprise can now be effectively utilized to introduce proof to learners as a means of “explaining” why this result is always true.

Part of both activities also requires learners and prospective teachers to identify what assumptions have been made to simplify the problems that may not necessarily be true in the real world, and how real world factors such as terrain or size of the villages, may affect the desired solution(s). Consideration is also given to examining the interesting (and challenging) situation of where to put the water reservoir for a quadrilateral that doesn’t have concurrent perpendicular bisectors (i.e. a non-cyclic quadrilateral).

![Figure 10](image_url)

**Figure 10**

**DYNAMIC MODELING OF REAL WORLD OBJECTS**

Lastly, since one can paste pictures directly into Sketchpad, one can easily use it to investigate the geometric properties of real objects from their photographs. For example, Figure 10 shows how the half-circular arcs of the Arc de Triomphe in Paris can be modeled and illustrated easily. Similarly, it is shown in Figure 11 how the arch of the Tollgate Bridge in Durban can be modeled reasonably well using either a parabolic or exponential function.

![Figure 11](image_url)

**Figure 11**
REFERENCES


KEYNOTE LECTURES
This paper provides a theoretical tour I took in a quest to develop a language of description for mathematical tasks which incorporated the everyday. The substance of the argument is that I continuously had to abandon or modify my initial theoretical constructs as dialogue between data and theory ensued. I illustrated this point by retracing how the construct, weak classification, became limited in providing a more accurate description of the type of activities initially categorized as weakly classified. Summoning Dowling’s language of description for texts for this purpose, I also illustrate how I had to modify it in order to provide a better explanation of my study’s purpose. I argue that the ‘lens’ metaphor for theory may be misleading.

THEORY AS ‘LENs’?

In 2001, I embarked on a doctoral study whose focus was the learners’ perspectives on the inclusion of the everyday in mathematics. The empirical data for the study was relatively clear to me. In that regard, I had a better picture of the type of empirical data I needed for the study and how I was going to collect it. Less clear, though, was a theoretical framework I needed and whether I needed one in the first place. This lack of clarity over the role of a theoretical framework was shared by a number of doctoral students. During a South African doctoral students’ session organized by the National Research Foundation (NRF) in 2003, the role and significance of a theoretical framework or theory in research was continuously raised as problematic by the students. Much of the literature, claims Usher (1998:134), views theory as a platform for mapping experience. It helps a researchers not “to go out into the world with completely vacant minds” (Martin, 1997: 24). In other words, ‘theory is like a lens’ through which one views practice (Olivier, 1992:193). However, the metaphor of a lens for theory is problematic and to some extent misleading. Inherent in its use is an acknowledgement that theory enables what experiences a researcher sees and does not see. However, it also backgrounds the role that theory can play in shaping up theory, since it is only through the lens that an object may be viewed and not the other way round. In this paper I outline the way in which data for my study spoke back to my initial theoretical framework based on Bernstein’s concept of classification. Though it provided a focus, it became apparent that the concept of classification was inadequate to help me engage the data I had collected. I will outline this experience by discussing

1. The way in which Bernstein’s concept was appropriated for the study
2. The way in which data forced a modification of my initial theoretical framework
3. The significance of dialogical engagement between theory and data.

SUMMONING BERNSTEIN’S CONSTRUCT - CLASSIFICATION

My study took place within a context of a new South African education system which was in what may be referred to as a boundary-blurring or non-segregationist mode (DoE*, 1997). Not only was there a political intention to blur the boundaries between different races, sexes and classes; there was also an educational intention to blur the boundaries between different school subjects and between each school subject and the everyday. My study hinged on (1) an explication of what the everyday is and (2) what the incorporation of the everyday in mathematics entailed. Bernstein’s constructs helped me conceptualise these two aspects. Firstly, I viewed the everyday as one entity, any common observable phenomena. Common, as Bernstein (2000:157) outlines, “…because all potentially or actually have access to it”. I also recruited Bernstein’s (2000) construct of classification to describe the incorporation of the everyday in mathematics. In particular, Bernstein draws a distinction between weak classification and strong classification. “We can distinguish between strong and weak classifications according to the degree of insulation between categories, be these categories of discourse, categories of gender, etc. Thus in the case of strong classification, we have strong insulation between categories. In the case of strong classification, each category has its unique identity, unique voice, its own specialized rules of internal relations. In the case of weak classification, we have less specialized discourses, less specialized identities, less specialized voices.”

The use of Bernstein’s concepts is quite common amongst mathematics educators (See Ensor, 1999; Dowling, 1998, Cooper and Dunne, 2000). Therefore I also had access to the way in which Bernstein’s constructs were used in relation to empirical data. In particular, Cooper and Dunne (2000: 67) have shown that a child from a middle class background was able to negotiate the everyday/esoteric boundary in mathematics in contrast to a child from a working class background who did not.

Armed with these theoretical constructs, it was clear to me that I was particularly interested in were learners’ perspectives on mathematics lessons which could be categorized as weakly classified. In other words, weakly classified was a notion to describe lessons which incorporated the everyday.

* Department of Education
THE LIMITATION OF CLASSIFICATION CONSTRUCT

I collected data from two schools, Umhlanga and Settlers. I paid attention to lessons in which mathematics teachers used activities which referenced what I regarded as the everyday. The following are two examples of tasks used by the two teachers whose lessons I observed. Task 1 is selected from a set of tasks used by the teacher in one school (Settlers) and task 2 is selected from a set of tasks used in another school (Umhlanga).

**Task 1:** John’s age is $p$ years. Write down Sue’s age in terms of $p$ if Sue is 6 years younger than John.

**Task 2:** If the number of people suffering from AIDS in 2000 is 133.6 million and the world population is 6 000 million; calculate the percentage of people suffering from AIDS in the year 2000.

Both tasks are characterized by an incorporation of the everyday in mathematics, they may thus be grouped together as weakly classified. However, grouping task 1 and task 2 in the same category fails to highlight the different expressions used in presenting the tasks and the different ways in which learners may relate to each context. Firstly, in task 1, knowledge of mathematics symbols is assumed and in the task 2 such an assumption is not made. Secondly, even though John refers to a name of a real person, the claim that John is $p$ years old bears no everyday sense. Categorizing these two tasks as weakly classified obscures these significant differences.

I then summoned Dowling’s framework which provided a more precise language of description enabling a distinction between the two tasks cited above. In describing these tasks, Paul Dowling (1998) uses two categories: mode of expression and the nature of context drawn in. Tasks which have a highly classified mode of expression are those which communicate information in ‘unambiguously mathematical’ terms (Dowling, 1998:135). Such tasks can either draw from the mathematics context or the everyday; in which case they will respectively be labeled ‘esoteric’ and ‘descriptive’. Other tasks employ a weakly classified mode of expression and thus communicate information using non-mathematical expressions. Likewise, these tasks may also either draw from the mathematics or the everyday contexts; they will respectively be labeled “expressive” or ‘public’. The four possible categories emerging from this discussion can be presented in the quadrant below.

<table>
<thead>
<tr>
<th>Strong classification of content</th>
<th>Weak classification of content</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong Classification of mode of expression</td>
<td>Esoteric Domain</td>
</tr>
<tr>
<td>Weak classification of mode of expression</td>
<td>Expressive Domain</td>
</tr>
</tbody>
</table>

Figure 1: Categories produced from an interplay between mode of expression and content
These categories enable a distinction between the two tasks cited above. Task 1 uses a strong classification of mode of expression, characterized by the use of symbols, and it also draws in the everyday context. Thus, it can be categorized as a descriptive domain task. Task 2 can be categorized as a public domain task because it employs a weakly classified presentation mode, characterized by the use of ordinary language and it also draws in the everyday.

By opening up a dialogue between the theoretical notion of ‘classification’ and the empirical data, it was possible to note that my initial theoretical construct of classification needed modification. Dowling’s notions became important for my analysis. However, this model did not take into account the qualitative difference between the different types of the everyday; the point I attend to in the next section.

DIFFERENT TYPES OF THE EVERYDAY

My research interest was much more than what the everyday entailed in mathematics, it was mainly about the way in which learners related to and therefore viewed the role of the everyday. In other words, I was interested in the way a context resonated with the learners’ experiences. Espousing the qualitative difference between contexts, Freudenthal (1970:78) made the following observation; “When speaking about mathematics fraught with relations, I stressed the relations with a lived-through reality rather than with a dead mock reality that has been invented with the only purpose of serving as an example of application”. (my emphasis)

I view these two Freudenthal-based categories as two opposite extremes. On the one extreme, ‘dead mock reality’ references the everyday in a way which is highly unlikely or impossible. An item, for example, which makes reference to an African-American president in the United States of America before 2004 is using a known concept (American president) inauthentically (there was never an African-American one before 2004). I use the term inauthentic for such contexts. On the other extreme, ‘lived-through experience’ makes reference to genuine or not far-fetched use of the everyday. For example, a task which makes reference to ‘John who went fishing with his friends’ is appealing to a context we know little about. However, the possibility of such an event cannot be confidently dismissed. To the extent that such a context is not obviously a make-belief, I will use the term authentic to describe it.

A context, authentic or inauthentic, may reference a scene or event which resonates with learners’ experiences. These would include events that relate to the areas where learners stay and or which take place and are topical during the learners’ lifetime. Such events are ‘near’ to the learners in terms of space (locality) and/or time (period of occurrence). Alternatively, a context may reference a scene or event which does not resonate with the learners’ experiences either because it took place a long time ago or it took place in a place situated physically far from where the learners reside.

I have used the concept of ‘near’ similar to the way in which Royer (cited in Billet: year:8) uses ‘near’ as a qualification for knowledge transfer. He, for example, regards
the ability of a university lecturer to teach with ease in another university as a case of ‘near transfer’ since it permits deployment of skills to a similar context. In a similar way, Royer uses ‘far’ as another qualification for knowledge transfer. Carrying on with an example of a ‘university lecturer’, Royer regards a requirement of a university lecturer to teach at a vocational college or primary school as far knowledge transfer. This is because in this case, there will be a deployment of a skill to a novel situation. In sum, the concept of ‘near’ is related to familiarity or similarity and ‘far’ is related to novelty or unfamiliarity.

Using the concepts of authenticity/inauthenticity and close/far to describe a context, the following four categories emerge

<table>
<thead>
<tr>
<th>AUTHENTIC</th>
<th>INAUTHENTIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLOSE</td>
<td>Authentic and near</td>
</tr>
<tr>
<td>FAR</td>
<td>Authentic and Far</td>
</tr>
</tbody>
</table>

Figure 2: Different categories of the everyday-non mathematical

This model was provoked by the need to describe and differentiate between tasks such as (task 2 and task 3) below which were included in different worksheets and different lessons in Umhlanga high school.

**Task 2:** If the number of people suffering from AIDS in 2000 is 133.6 million and the world population is 6 000 million; calculate the percentage of people suffering from AIDS in the year 2000.

**Task 3:** How did the ancient Egyptians write the number 100?

Using Dowling’s model, the two tasks fit the public domain category since they draw in the everyday and they do not use an unambiguously mathematical mode of expression. However, the contexts drawn in have different appeals to learners. Task 3 references a context which is far (both in terms of time and place) and authentic (there is a particular way in which 100 was written in ancient Egypt). Task 2 draws in a near context of AIDS* even though it references inauthentic data. The use of Dowling’s notions does not highlight the qualitative difference between these two tasks and therefore the different emotions they evoke amongst learners.

* President Thabo Mbeki’s questioning of the transmission of AIDS has provoked heated debates and reactions from AIDS activists at a national level. Umhlanga is located about 20 kilometres from a township in which an AIDS activist Gugu Dlamini was stoned to death for publicly declaring her HIV positive status.
DISCUSSION
In this paper I have reflected on how a dialogical engagement between theoretical constructs and data shapes up these constructs. Whilst the concept of classification helped me locate the type of mathematics lessons I needed to focus on, I could not use it to distinguish between these lessons. Describing the mathematical activities as weakly classified failed to capture the different ways in which the everyday was incorporated in these activities. Dowling’s constructs provided a more accurate language of description for the lessons; yet the different emotions evoked by the contexts in the tasks remained concealed. This led to a development of a new description based on authenticity/inauthenticity and near/far concepts.

The significance of opening up a dialogue between theory and practical settings has been echoed within the mathematics education field. In presenting this argument, Vithal and Valero (1998) and Adler and Lerman (2001) have voiced the need to grant the practical settings of the ‘south’ or ‘developing countries’ in order to better explain the dynamics in those settings. It is this dialogical engagement between theory and practice which is at the centre of Popper’s thesis on conjectures and refutations. Theory divorced from practice becomes no more than a ‘soothsaying practice’ (Popper, 1972:37). In this regard, Popper maintains that “A Marxist could not open a newspaper without finding on every page confirming evidence for his interpretation of history;” (1972:35).

Instead of a lens, theory may be better described as a necessary starting point which becomes shaped up, modified and sometimes abandoned on the basis of data.

REFERENCES


INSIGHTS INTO THE IMPLEMENTATION OF MATHEMATICAL LITERACY

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In this paper, which accompanies our keynote lecture, we present some insights from our research into the implementation of Mathematical Literacy. Two key areas are discussed – learners’ experiences and pedagogic practice. We note that in our main research school, learners’ responses to Mathematical Literacy have been very positive, and are attributed to changes in the nature of tasks used in lessons and the nature of classroom interaction. The notion of changing pedagogic practices leads into our development of a spectrum of agendas guiding educators’ interpretations of Mathematical Literacy. We conclude by summarising the successes and detailing the issues emerging within implementation in our research – key amongst these, ways of thinking about progression in Mathematical Literacy and summative assessment

INTRODUCTION

Mathematical literacy (ML) was introduced in schools in the Further Education and Training (FET) phase in South Africa in January 2006. The subject is structured as an alternative option to mathematics, and all learners entering the FET phase since January 2006 are required to take one or other of these two options. ML is defined in the curriculum statement in the following terms:

Mathematical Literacy provides learners with an awareness and understanding of the role that mathematics plays in the modern world. Mathematical Literacy is a subject driven by life-related applications of mathematics. It enables learners to develop the ability and confidence to think numerically and spatially in order to interpret and critically analyse everyday situations and to solve problems. (DoE, 2003, p9)

The preamble to the curriculum specification in this document and other related policy documentation emphasise the idea of ML involving the development of a ‘mathematical gaze’ on the world. The course aims to develop both a sense of the need to engage with real-world issues and the mathematical tools with which to understand, analyse and critique these issues. Essentially therefore, ML aims to develop the capabilities and willingness to use mathematical thinking to make sense of a wide range of life-related situations.

In this paper, which accompanies our keynote lecture, we share some of our insights into the ways in which ML has been implemented in schools. Our insights are divided into three key sections – firstly, we engage with learners’ experiences in ML classes; secondly, we discuss teachers’ experiences and pedagogic practice in ML; finally, we
bring teaching and learning together and detail some of the successes that need to be highlighted within the implementation of this new subject, and what we see as the issues that need to be addressed. This division is helpful for analytical purposes – we acknowledge the highly interconnected nature of their actual occurrence. Prior to presenting our findings in these areas, we detail the data sources that we have used.

**DATA SOURCES**

Our work within the ML thrust in the Marang Centre at Wits involves a range of strands – research, lecturing and teacher development, and raising public awareness. The data we are using in this paper draws upon feedback from across this work. Our research work centrally involves a longitudinal case study, now in its second year, tracing the experiences of educators and the first cohort of learners taking ML in one inner city Johannesburg school. This work has involved weekly visits to the three ML classes in this cohort across grade 10 and now grade 11 (90 learners in all) with fieldnotes taken, as well as questionnaire data from learners (66 responses received), and interviews with a sample of learners (9 interviews involving 19 learners) and individual interviews with the three ML teachers (all qualified mathematics teachers). Our section on learners’ experiences is drawn from this research.

In relation to our other thrust work, we also draw upon feedback from the teachers we interact with as part of our lecturing at BEd, PGCE, Honours, Masters and doctoral levels, from those who attend our ML teacher support group meetings (four of whom participated in a focus group in November 2006), and the audiences (teachers, researchers and policy-developers – national and international) who have attended our ML seminars. The section on educators’ experiences and pedagogic practices draws from feedback from these teachers as well as the ML educators in our case study school. All names within this paper are pseudonyms.

**LEARNERS’ EXPERIENCES**

Our attention was drawn to learners’ experiences in the early stages of our research. Educators in the main research school and those in our teacher support group commented on the low levels of confidence and self-esteem in relation to mathematical working of the learners in their ML classes, as well as generally weak levels of prior attainment in mathematics. This last aspect reflected an organisational feature that was being used across almost all the schools that we interacted with – strong mathematics learners at the end of grade 9 were strongly advised to take mathematics in grade 10, whilst weak and failing mathematics learners were advised to take mathematical literacy. Given this background, we were somewhat surprised by the overwhelmingly positive responses from learners from the early stages onwards of 2006 when asked within informal classroom interaction about how they felt about doing ML. The questionnaire and interview data provided some insights into the key features noted within the widely reported sense of enjoyment of ML. Learners’ comments describing
their experiences in ML often contained explicit or implicit reference to negative experiences of prior mathematical working. One such comment, reflective of broadly expressed sentiments, taken from one learners’ open response answer in the questionnaire is reprinted in its entirety below because it helps us to open up several of the features figuring within the shift to enjoyment:

I think that mathematical literacy is a subject filled with fun, excitement and generally talked about issues, e.g. how to calculate compound interest and compound decrease. But for me, I would say that this subject is quite okay compared to maths because here it is much easy for you /one to understand what the instruction is / says, because you find us having to research about the national budget which is an issue covered / known by everyone. I would lastly say that I feel it should not only stop with us but continue for other students after us to share in this wonderful experience. Maths literacy is just fun .. fun .. fun. And less complication than having to deal with x’s and y’s, square roots to numbers. It is just easy. So I would advise the current grade 9 and 8 learners to choose this subject because it is one adventure they should not be missing.

(Boy, UA’s class)

A number of features have clearly figured within the positive response from this learner. We summarise these and other aspects raised under three headings – the nature of tasks used in ML, the nature of interaction in ML, and organisational features.

**The nature of tasks used in ML**

The quote above stresses that in the focal school, efforts had had been made to ensure that ML was comprised by investigation and discussion of ‘real’ situations. Learners sometimes described the kinds of tasks they worked with as ‘scenarios’ or ‘story sums’, based on situations that occurred in real-life. It is important to note that neither educators nor learners in this school referred to these tasks in terms of traditional ‘word problems’ – the ‘dressed up’ pure mathematics problems (Blum, 2002) that are a common feature of mathematics curricula. The ‘concreteness’ of tasks in ML was referred to in several ways – that the subject was about things that could be ‘seen’, things that were ‘practical’, and, as in the quote above, things that were more generally talked about. These kinds of tasks appeared therefore to stimulate both visualisation and discursive opportunities in ways that prior mathematics tasks had lacked. Further, these openings for communication led into multiple ways of exploring situations and methods for answering questions, again, in contrast to the singularity that had been experienced in mathematics.

The nature of work *within* a task was also commented upon, with ML tasks seen as avoiding the quickly paced hierarchy of concept-building that was commonly reported in mathematics learning, and which was reported as figuring within their earlier lack of access to understanding:
M: it’s the task and what happens is – maybe there is a section you really find easy, okay, you’re comfortable with it, and oh - then they go to something like algebra, which – Oh God! And it’s confusing because it’s got a lot of concepts in it. There’s an invisible line, there’s this and that, remember there is brackets, remember times, divide this, you know.

HV: So you think this year there has been less need for what you call those ‘invisible concepts’?

Both: Yes.

(Girls, FD’s class, in paired interview)

Implicated within the better understanding in ML that was frequently reported in Grade 10 was the issue of time given to work within a context – classroom observations confirmed that tasks were often covered over a week or two weeks. The structure and pace of work in ML was perceived as being much more responsive to the understandings of learners:

Unlike in maths when they'll just say "aah, just come back after school for like extra lessons, we have to move on". But for maths literacy they just give you time until they see that everyone understands. And in class like lots of activities to ensure that you understand.

(Boy, UA’s class, in paired interview)

Continuous assessment tasks in ML were also reported as being ‘different’ from the kinds of tasks that had been used in mathematics. Research and data collection were reported as common activities within these tasks, and the time and space to find out answers and consult others where appropriate were viewed positively.

The nature of interaction

We noted in the last section that the common use of real situations seemed to open up opportunities for communication. Learners commented that group work and discussion were much more common in ML than they had been in mathematics, and additionally, that a much more collaborative work environment predominated in ML:

And another thing is that maths literacy is unusual, it's full of groups. …. They're like - like many of our teachers they see you work in pairs, so like I think that helped a lot because you really get like other opinions because most of the things in maths literacy, they're like in …things like – we can do – there's the two-way method, it's not like a one-way method so you can get other people’s views, easier methods and stuff like that.

And like, I think like in plain mathematics, you find that there's a lot of individualism. So I find like a person like has to do for himself or something like that. So if you were to ask in a group – you know, it becomes like you're going – you must ask the teacher. If you don't understand like you only like ask your friends but in maths literacy, you like always – most of the time work in pairs. So I think that helps a lot.

(Boy, UA’s class, in paired interview)

Also coming through here are senses of a more diffuse and distributed locus of authority – an environment in which learners are taking on board the role of authoring opinions
and methods rather than leaving this to the teacher. ML educators in turn, were described as being ‘more patient’ than mathematics teachers had been, more willing to take time to explain, and, as noted earlier, to wait for understanding and slow the pace of working when necessary in order to achieve better learner understandings.

Some learners had also been able to extend communication in relation to ML beyond the classroom into their interactions with their families. We received comments from some learners about more active involvement in household accounts and decisions about purchasing – tangible senses of being empowered by the understandings they had acquired:

M: now whenever like a friend or my mum, she's speaking of getting a phone on contract. She looks at the paper. I always, even if I'm reading the newspaper and then I see this phone you pay R75 for 24 months. Actually take my calculator and calculate how much —

MG: So you're actively looking out.

M: Looking out for my mum and say "ah, you'll end up paying R2000. Just think how many phones you could afford, me and my sister and yours".

S: And like determining like which one is like better to buy - cash or on credit.

MG: So you're feeling like you can start to do those calculations now hey?

M: Yes.

MG: And you're having conversations with your family about it.

M: Yes.

MG: Same for you S---?

S: Yes, when like – you know, at home I live with my grandmother. So every time where you like take your money to the bank or something, she always sends me because, you know, I understand and like I'm the one who's changing all those stuff and all that. That's why it's easier than - they ask for me.

MG: Because they see you as somebody who can do the mathematics.

S: Yes.

MG: Do you think that they're doing that more this year than they would have done it last year?

S: This year, ja this year they're doing it more. Like every time, each and every month, I'm the one that goes there and deposit money, draw some cash and all that.

MG: So it's interesting. So they're asking your opinions more in a sense.

M: Yes because now because we understand the maths literacy, we're open with our teacher, we kind-of get overboard and try to do the same thing at home; want to get our mum – our parents to understand. Now they think that we are geniuses and so they want us to do it because they think we're the best.

(Boy and girl, CN’s class, in paired interview)

Whilst this kind of active use of ML learning was expressed by a minority of learners, the notion of ML as a useful subject was widely expressed.
Organisational features

Our final category of learners’ comments is related to aspects about the structuring of ML that learners reflected upon. A key area here related to concerns expressed by a number of learners about what they would be able to proceed onto with a ML pass at matric. Uncertainties around whether ML would be acceptable for entry to University courses in the accounting, business studies, economics and other finance-related areas were the most frequently expressed concerns for learners, many of whom noted that these areas were emphasised in ML and were areas which they had found interesting. Uncertainty remains in relation to these questions and unfortunately, represents a risk to the establishment of ML as an important and useful subject in the FET phase, and, centrally, a subject in this school that has helped to develop some of the concepts, tools and attributes that would be useful for higher level study.

It is important to note here that the shift to overwhelmingly positive experiences of learning in ML and associated shifts in perceptions of the value and power of mathematics in our main research school were intricately connected to changes in classroom experience. In this school, the commonalities of experience across the three classes were linked to the use of the same classroom tasks and some similarities in interactional shifts. The three educators involved did though, view ML in slightly different ways – perceptions that we considered alongside feedback from other teachers that we work with. This leads into our discussion of teachers’ experiences and pedagogic practice in ML.

EDUCATORS’ EXPERIENCES AND PEDAGOGIC PRACTICE IN ML

From our work with teachers we have identified a spectrum of pedagogic agendas which traverse across the purpose of contexts and degree of integration of contexts within pedagogic situations. These are given below:

Table 1: A spectrum of agendas

<table>
<thead>
<tr>
<th>Context driven (by learner needs)</th>
<th>Content &amp; context driven</th>
<th>Mainly content driven</th>
<th>Content driven</th>
</tr>
</thead>
<tbody>
<tr>
<td>Driving agenda:</td>
<td>Driving agenda:</td>
<td>Driving agenda:</td>
<td>Driving agenda:</td>
</tr>
<tr>
<td>To explore contexts that learners need in their lives (current everyday, future work and everyday, and for critical citizenship).</td>
<td>To explore a context so as to deepen math understanding and to learn maths (new or GET) and to deepen understanding of that context.</td>
<td>To learn maths and then to apply it to various contexts.</td>
<td>To give learners a 2nd chance to learn the maths in GET.</td>
</tr>
<tr>
<td>Pedagogic demands</td>
<td>Pedagogic demands</td>
<td>Pedagogic demands</td>
<td>Pedagogic demands</td>
</tr>
<tr>
<td>Involves finding the contexts learners</td>
<td>Involves selecting contexts (can be real,</td>
<td>Involves selecting contexts that GET</td>
<td>Involves revision of GET maths without</td>
</tr>
</tbody>
</table>
currently need and will need in the future as well as contexts the country needs learners to be able to engage with (& critically) (e.g. notions of democracy, national budgets and taxation)

Needs increased discussion of contexts and critical engagement with the underlying function of mathematics embedded in it. (E.g. if one changes the formatting formula of tax rates what happens, who benefits more rich or poor). Might require revisiting or learning new maths but only in so far as it will service the understanding of the context

<table>
<thead>
<tr>
<th>Issues arising</th>
<th>Issues arising</th>
<th>Issues arising</th>
<th>Issues arising</th>
</tr>
</thead>
<tbody>
<tr>
<td>Progression of mathematics usually sacrificed while authenticity of context is maintained in order to meet ‘needs’ of learners. The sacrifice of mathematical progression is not necessarily experienced as a problem</td>
<td>Authenticity of context and progression of math embedded must be balanced. Both authenticity of context and mathematical progression can be experienced as a problem. Summative assessments struggle to align and to deal with</td>
<td>Authenticity of context is often sacrificed so as to meet mathematical goals. Mathematical progression can be developed in the same way as in ‘mathematics’ curriculum. Summative assessments are more familiar and performance is more</td>
<td>Contexts are not a concern as they are not particularly present. Mathematical progression can be developed in the same way as in ‘mathematics’ curriculum. Traditional summative assessments are</td>
</tr>
</tbody>
</table>

Note: the driver is to find contexts that work to unpack this math-context relationship vs finding contexts that are needed by learners for full participation in society.
| Summative assessments struggle to align to this agenda so discrepancies can occur between performance on continuous and summative assessments | issue of progression so gaps can occur between performance on continuous and summative assessments | aligned to continuous assessments | similar to continuous assessments so little discrepancy between performance on these – most likely continued poor performance as in GET |

It should be noted that while the table above shows four distinct categories this should not imply that these categories are strictly bounded. It is precisely because the boundaries are blurred across these categories that we have called it a spectrum. We have not called it a continuum as this might imply that teachers move along it from left to right. Rather it seems that teachers, whilst still reflecting different agendas, foreground certain agendas at different points in time across the FET years.

Our analysis of the definition, purpose and post-amble on context and the teacher guide for ML suggests that the second agenda is the one that should be predominantly pursued over the FET band. The post-amble headed ‘context’ following the learning outcomes and assessment standards in the curriculum statement furthermore emphasizes that a literacy approach should be taken in the teaching of mathematical literacy:

> The approach that needs to be adopted in developing Mathematical Literacy is to engage with contexts rather than applying Mathematics already learned to the context. (DoE, 2003, p42)

This statement is important as it distinguishes ML from the application of mathematics to ‘word problems’ encountered in the mathematics curriculum across the bands in the curriculum under apartheid. This point follows up the distinction made in the last section between the nature of the ‘scenario’- based tasks used and traditional word problems in that contexts are not introduced merely to pursue mathematical goals but that engagement with contexts is itself a central goal. This is re-emphasised again in the ML teacher guide published in 2006:

> the challenge for you as the teacher is to use situations or contexts to reveal the underlying mathematics while simultaneously using the mathematics to make sense of the situations or contexts, and in so doing develop in your students the habits or attributes of a mathematically literate person (DoE, 2006, p4)

However, this should not imply that there are not inconsistencies that work against this agenda present in the curriculum and various supporting documents. For example, analysis of the learning outcomes and their assessment standards indicates that this vision is not clearly supported in the details of the curriculum document. For example the inclusion of trigonometric ratios and the sine and cosine rule would seem to suggest that mathematical agendas are sometimes pursued at the expense of a literacy agenda. In the early stages in the process of implementation of Mathematical Literacy it is not
surprising that mixed messages (Venkat, 2007) and ongoing movement in relation to the above spectrum are likely to arise.

Our table highlights various issues that are experienced by teachers when working with a particular agenda. In particular we point to the issues of authenticity of context, development of mathematical progression and discrepancies of continuous and summative assessments. These issues have been experienced by teachers that we are working with in different ways depending on their primary driving agenda. Below we summarise these issues and some of the ways in which they link with the learners’ experiences outlined in the last section. It seems that for the agendas on the left:

- authenticity is less of a problem than for content driven agendas and it would seem results in increased learner participation and a positive attitude toward mathematical literacy
- discrepancies in performance between continuous and summative assessments arise which can have a negative impact on learners’ confidence and attitudes towards mathematical literacy. Such discrepancies seem to be the result of the absence of exemplars of new forms of summative assessment which cohere with more context driven agendas thus resulting in the continued use of more traditional summative assessments

While for the agendas on the right:

- authenticity of contexts is a larger problem when mathematics is the driving agenda but this can result in decreased learner participation as mathematical literacy is experienced as ‘more of the same’. Thus there is less space for learners to develop new ways of being (including confidence and interest) in relation to mathematical literacy
- discrepancies in performance between continuous assessment and summative assessments are not a key issue as these assessments are not dissimilar however learner performance remains low as in the GET since their experience of mathematical literacy is largely ‘more of the same’

A final note on our proposed spectrum - the spectrum is based on our current experiences of working with a range of teachers. It is of course likely that as we continue to work with a wider range of teachers and into grades 11 and 12 our spectrum will continue to be revised and refined.
DISCUSSION – SUCCESSES AND ISSUES

Learners’ responses in our main research school have been largely positive to date, but we are well aware of the fact that this may well not be the case more broadly. Some teachers on our ACE and postgraduate courses have reported low levels of motivation and lack of interest amongst learners in their ML classes (e.g. see Hechter, 2007). Anecdotal evidence from educators suggests that this kind of ongoing negativity is associated with a lack of substantive change in pedagogic practice. In some cases this is due to an interpretation by the teacher of ML as involving ‘basic maths’ (towards the right of the spectrum), and consequentially, teaching that incorporates the kinds of tasks and pedagogic practice that have predominated within learners’ earlier experiences with mathematics. In other cases, educators have told us about being unable to establish pedagogic shifts because of the degree of resentment and de-motivation amongst some learners in their ML classes:

I’m not sure if I’ve actually made any impact on a lot of the class. On some of them, I think that I have, and they have got a lot more positivity towards the subject and maths in general. But some of them just don’t want to be there, don’t want to even try. I don’t have that ‘We want to try and let’s see what we can do’. They still want to – ‘how do we do it?’ Ja, it’s been a very frustrating year.

(Educator, Participant in our ML Teacher support group focus group)

In both cases here (positive and negative learner responses), there is empirical support for the linking of learners’ experiences with pedagogic practice – this is what we have tried to address in this keynote paper. Positive responses in our case-study school and in the schools of some of the other teachers we interact with were related to shifts in the nature of tasks, and in the nature of interaction in ML classes as outlined in our earlier section. A key area of success in these schools has been the turning around of low self-esteem and lack of confidence in relation to mathematical thinking and working for learners, towards a view of mathematics as a potentially powerful and useful tool with which to understand situations. An important feature of this shift has been a willingness amongst learners to work in more active and independent ways when presented with problems:

Francine: you can put something in front of them, and automatically they will get into their twos or whatever. You don’t even have to say anything, and they would sit and read and start making sense, and doodling and trying to find a solution. So they have become far more independent. They don’t need me to tell them how and what to do.

Frank: They are self-discovering, it makes sense to them now.

Francine: (Excitedly agrees) It does! Well you know, the teacher is not going to explain. They’ve got to now make sense of it.

(Two educators, one from the case-study school, another working in an independent school, commenting during the focus group)

In relation to the pedagogic spectrum of agendas, it is interesting to note that we felt that the educators from schools with more positive learner responses foregrounded aspects of
agendas 1-3, whilst those expressing more negative learner responses seemed to have remained within agendas 3 and 4. Opening up access to sense-making using mathematics and more positive responses appear to be key pay-offs of moving leftwards across our spectrum. We need to reiterate at this stage though, that we do not see agenda 1 as the ideal position from a ML perspective. Following the sense that ML should involve tasks and pedagogies that allow learners to understand and deal with real situations using mathematical thinking and problem solving as appropriate, our view is that ML educators need to be encouraged and supported to foreground the second agenda – content and context driven practice. This position has been advocated in several provincial and other ML courses (e.g. Gauteng 2006 Mathematical Literacy training handbook) in the 3-circle ‘content + context = competence’ diagram.

Whilst access has been a key success in the schools mentioned above, teachers here are already asking questions about how best to build on this momentum. Two interlinked issues have been raised as key concerns here – progression and summative assessment in ML. We highlighted different notions of progression that permeated the agendas of our pedagogic spectrum, following feedback from educators about how they perceive development in terms of ML. One of the problems highlighted is that the ML curriculum and many accompanying textbook resources introduce several interesting, but quite complex scenarios for learners to engage with in grade 10 which firstly assume that GET phase content knowledge is secure, and, which secondly, exemplify progression in largely content based terms. It is hard within this formulation therefore to visualise becoming more mathematically literate in terms other than learning more mathematics, and exemplars of increasing ‘complexity of the situation’ (DoE, 2003, p38) remain thin on the ground. This situation plays into the hands of those educators who persist in viewing ML in terms of basic mathematics, and often in turn, into negative learners’ ongoing sense that mathematical thinking and sense making lie beyond their scope.

Concerns about how to think about development and progression in ML lead into questions about how to assess ML. In previous writing (Graven & Venkatakrishnan, 2006), we have noted that whilst the lack of precedents in terms of the format of ML examinations has helped to open up spaces for educators to tailor their interaction and use of tasks to learners’ needs, summative assessment results in ML in our case study school remain relatively low, compounded by uncertainties for teachers in terms of the content, level and format of what to include. Learners complained frequently to us about being unable to complete the set papers in the time allotted. At this stage, with the first cohort of ML learners fast approaching grade 12, more detail on these issues would be helpful. Certainly, the spaces that have been opened up for engaging with teaching and learning in different ways in ML have not been matched currently in the area of assessment, where traditional examination forms have been retained. More broadly therefore, a debate on the ways in which assessment of ML can be tailored to support the aims of the course and build on the potential that is clearly present for changing learners’ engagement with mathematical thinking, would also be useful. Addressing both of these issues can contribute to better understandings of what being, and becoming, mathematically literate can mean for educators, and through this, more
openings for changing the highly negative prior experiences of so many learners coming into Mathematical Literacy.

REFERENCES


This paper will firstly explore some of the symmetries and transformations involved in border patterns and tessellations. This will be followed by some examples of problem solving with transformations. Lastly, we’ll explore some transformations of graphs of functions of the form $y = f(x)$.

Introduction

The new national curriculum statement (Dept. of Ed., 2002a) specifically describes the following outcomes for the Senior Phase (Grades 7-9):

- Use national flags to demonstrate transformations and symmetry in designs.
- Investigate and appreciate the geometrical properties and patterns in traditional and modern architecture (e.g. construction and painting of Ndebele homes).
- Use maps in geography as specific forms of grids.
- Investigate geometric patterns in art (e.g. African and Islamic art).

For the FET phase (Grades 10-12) the following outcomes are specified in the Guidelines for Learning Programmes (Dept. of Ed., 2002b):

- develops conjectures and generalisations, related to triangles, quadrilaterals and other polygons, and attempts to validate, justify, explain or prove them, using any logical method (Euclidean, co-ordinate and transformation)
- generates as many graphs as necessary by means of point by point plotting to test conjectures and hence generalise the effects of parameters on the graphs of functions

Border patterns

Border or frieze patterns are one dimensional, repeating patterns that are often used as decoration on the EDGES of garments, books, buildings, plates, rugs, etc. There are seven types of border patterns based on their symmetry properties of reflection, translation and rotation.

The classification system normally used assigns to each border pattern a two letter/number code to label its type. By definition it is assumed that when a border patterns is classified that it is viewed horizontally. The first letter or number signifies if there is vertical line symmetry or not, while the second letter or number signifies if the pattern has these additional symmetries horizontal line, horizontal glide reflection or half-turn symmetry or not. This table summarizes the two letter/number code scheme.
First Code Letter | Second Code Letter
---|---
m = vertical symmetry | m = horizontal symmetry
1 = no vertical symmetry | g = glide reflection sym, if no horizontal m
2 = half-turn symmetry, if no horizontal m or g
1 = no additional symmetry

**Examples** (Zulu beadwork examples below are from Durban beachfront)

**11-pattern (called a ‘hop’ by John Conway)**

A 11-pattern only has translation symmetry. All border patterns have at least translation symmetry, which means that if it is translated some horizontal distance it will map onto itself. (Note that if we completely ignore the colours in the second example, it is classified as a 12-pattern.)

**m1-pattern (called a ‘sidle’ by John Conway)**

A m1-pattern has vertical line symmetries in addition to its translation symmetry. (Note that if we completely ignore the colours in the second example, it is classified as a mg-pattern.)
**mm-pattern (called a ‘spinjump’ by John Conway)**

A mm-pattern has vertical and horizontal line symmetries in addition to its translation symmetry. (Note that in classifying the second example as mm above, the colours of the vertical lines were ignored, which gives the same classification as if all the colours are ignored. But with the colours of the vertical lines as given, it would actually be a 1m-pattern, because the colouring of the vertical lines does not have vertical line symmetry.)

**mg-pattern (called a ‘spinsidle’ by John Conway)**

A mg-pattern has vertical and horizontal glide reflection symmetries in addition to its translation symmetry.
**1m-pattern (called a ‘jump’ by John Conway)**

A 1m-pattern has horizontal reflection symmetry in addition to its translation symmetry.

**1g-pattern (called a ‘step’ by John Conway)**

A 1g-pattern has horizontal glide reflection symmetry in addition to its translation symmetry. This is not a very common pattern in traditional African art.

**12-pattern (called a ‘spinhop’ by John Conway)**

A 12-pattern has half-turn symmetry in addition to its translation symmetry. (Note that if we completely ignore the colours in the second example, it is classified as a mg-pattern.)
TESSELLATIONS

A tessellation is created when a shape is repeated over and over again covering a plane without any gaps or overlaps.

Another word for a tessellation is a tiling.

A dictionary will tell you that the word "tessellate" means to form or arrange small squares in a checkered or mosaic pattern. The word "tessellate" is derived from the Ionic version of the Greek word "tesseres," which in English means "four." The first tilings were made from square tiles.

A regular polygon has 3 or 4 or 5 or more sides and angles, all equal. A regular tessellation means a tessellation made up of congruent regular polygons. [Remember: Regular means that the sides of the polygon are all the same length. Congruent means that the polygons that you put together are all the same size and shape.]

Only three regular polygons tessellate in the Euclidean plane: triangles, squares or hexagons. We can't show the entire plane, but imagine that these are pieces taken from planes that have been tiled. Here are examples of

- a tessellation of triangles
- a tessellation of squares
- a tessellation of hexagons

When you look at these three samples you can easily notice that the squares are lined up with each other while the triangles and hexagons are not. Also, if you look at 6 triangles at a time, they form a hexagon, so the tiling of triangles and the tiling of hexagons are similar and they cannot be formed by directly lining shapes up under each other - a slide (or a glide!) is involved.

You can work out the interior measure of the angles for each of these polygons:

<table>
<thead>
<tr>
<th>Shape</th>
<th>Angle measure in degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>triangle</td>
<td>60</td>
</tr>
<tr>
<td>square</td>
<td>90</td>
</tr>
<tr>
<td>pentagon</td>
<td>108</td>
</tr>
<tr>
<td>hexagon</td>
<td>120</td>
</tr>
<tr>
<td>more than six sides</td>
<td>more than 120 degrees</td>
</tr>
</tbody>
</table>

88
Since the regular polygons in a tessellation must fill the plane at each vertex, the interior angle must be an exact divisor of 360 degrees. This works for the triangle, square, and hexagon, and you can show working tessellations for these figures. For all the others, the interior angles are not exact divisors of 360 degrees, and therefore those figures cannot tile the plane.

**Naming Conventions**

A tessellation of squares is named "4.4.4.4". Here's how: choose a vertex, and then look at one of the polygons that touches that vertex. How many sides does it have? Since it's a square, it has four sides, and that's where the first "4" comes from. Now keep going around the vertex in either direction, finding the number of sides of the polygons until you get back to the polygon you started with. How many polygons did you count? There are four polygons, and each has four sides.

![4.4.4.4](Image)

For a tessellation of regular congruent hexagons, if you choose a vertex and count the sides of the polygons that touch it, you'll see that there are three polygons and each has six sides, so this tessellation is called "6.6.6":

![6.6.6](Image)

A tessellation of triangles has six polygons surrounding a vertex, and each of them has three sides: "3.3.3.3.3.3".

![3.3.3.3.3.3](Image)

**Semi-regular Tessellations**

You can also use a variety of regular polygons to make *semi-regular tessellations*. A semiregular tessellation has two properties, which are:

1. It is formed by regular polygons.

2. The arrangement of polygons at every vertex point is identical.
Here are the **eight** semi-regular tessellations:

![Tessellations](image)

Interestingly there are other combinations that seem like they should tile the plane because the arrangements of the regular polygons fill the space around a point. For example:

![Tessellations](image)

**Task 1**: If you try tiling the plane with these units of tessellation you will find that they cannot be extended infinitely. Fun is to try this yourself.

1. Hold down on one of the images and **copy** it to the clipboard.
2. Open a paint program.
3. Paste the image.
4. Now continue to paste and position and see if you can tessellate it.

**Task 2**: Explore the symmetries (translation, reflection, glide reflection & rotation) of the regular and semi-regular tessellations. How are they the same or different?

*Note*: There are an infinite number of tessellations that can be made of patterns that do not have the same combination of angles at every vertex point. There are also tessellations made of polygons that do not share common edges and vertices.
Problem Solving with Transformations

Many real world problems can be easily modeled with Sketchpad and elegantly solved by using transformations such as reflections, translations and rotations. Below are some examples that will be briefly discussed.

The Burning House Problem
A man is walking in an open field some distance from his house. It’s a beautiful day and he is carrying an empty bucket with him to collect berries. Before long, he turns around and, to his horror, sees that his house on fire. Without wasting a moment, he runs to a nearby river (which runs in a straight line from east to west) to fill the bucket with water so that he can run to his house to throw water on the fire. Naturally, he wants to do this as quickly as possible. Describe how to construct the point on the river bank to which he should run in order to minimize his total running distance (and time).

Horse Riding
A rider is traveling from point D to point E between a river and a pasture. Before she gets to E, she wants to stop at the pasture to feed her horse, and again at the river to water him. If angle BAC = 45°, EJ = 2 km, AJ = 5 km, DK = 7 km and JK = 10 km, what path should she take to travel the shortest possible distance?
Building Bridges

The following two problems are from Makae et al (2001):

1. At what point should a bridge MN be built across a river separating two towns A and B so that the path AMNB is as short as possible? (It is assumed the river consists of two parallel lines with the bridge perpendicular to it.

2. Solve the same problem as in Question 1 if the towns A and B are separated by two rivers across which bridges PQ and RS have to be constructed (see above).
Building an airport

The following problem is from De Villiers (2003):

Suppose an airport is planned to service three cities of more or less equal size. The planners decide to locate the airport so that the sum of the distances to the three cities is a minimum. Where should the airport be located?

Transformations of $y = f(x)$

As discussed in De Villiers (1991), the study of transformations forms a golden thread linking together many diverse parts of the mathematics curriculum, and beautifully connects geometry with algebra. Particularly interesting and relevant is to investigate different transformations of functions of the type $y = f(x)$ with a dynamic tool like Sketchpad.

Essentially, there are two different, though related questions one could explore:

1. How can we transform a function $y = f(x)$ in a particular way? (For example, how can we reflect it around the $y$-axis?)

2. What happens if we transform $y = f(x)$ in a particular way? (For example, what happens to $y = f(x)$ with the transformation $g(x) = f(x) + b$?)

We shall use the Sketchpad sketch below to find transformations of $y = f(x)$ for each of the following transformations:
Reflection in $x$- and $y$-axes
Reflection in $y = x$
Half-turn around origin
Enlargement from origin
Stretch in $x$- and $y$-directions

It should be noted that in exploring and finding such transformations that one should not (just) use straight-line graphs as these can easily lead to incorrect generalizations, but use sufficiently general or a variety of functions.

REFERENCES
LONG PAPERS
GENERATE GRAPHS AND GENERALISE THE EFFECTS OF PARAMETERS ON GRAPHS BY MEANS OF AUTOGRAPH AND SKETCHPAD
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INTRODUCTION
Dynamic mathematics computer programs like Geometer Sketchpad (The Geometer’s Sketchpad Resource Centre) and Autograph (Autograph, for the Dynamic Classroom) are immensely powerful tools when it comes to the onscreen generation of graphs. In addition, most Mathematical software packages atones for interactive investigations and the generalization of the effects of parameters on graphs. For the purpose of this paper some of the graphs listed in NCS (2007:17) will be generated and the effects of parameter changes be investigated by means of the Sketchpad and Autograph packages.

DESIGN OF A GEOMETER SKETCHPAD SLIDER
A slider is a useful Geometer Sketchpad custom tool which can be utilised to investigate the effects of function parameters on given graphs. To construct a slider the following procedure can be followed (Steketee et al., 2001:197-198).

- Construct a line through two points \( A \) and \( B \).
- Construct a point \( C \) on the line.
- Select \( A, B \) and \( C \), in that order, and choose ratio from the Measure Menu.
- Select and hide the line and the point \( B \).
- Construct \( \overline{AC} \).
- Change the label of measured ratio to the parameter under investigation.
- Slider can be improved by hiding label \( A \) and changing the label of \( C \) to parameter under investigation. (Some screen shots of slider parameter measurements are indicated in the next figure)
INVESTIGATE THE EFFECT OF PARAMETERS ON THE GRAPH OF
\( y = mx + c \)

This equation defines a straight line with gradient (slope) \( m \) and \( y \) – intercept \( c \). To investigate the effect of these two parameters we will consider two possible investigations, namely, (1) Sketch any line, drag points and note the changes or (2) Plot the line defined by \( y = mx + c \) and investigate the effect of the changes to the two parameters on the graph.

Investigation (1)

Follow the following procedure as suggested by Chanan et al. (2002: 40).

- Open a new sketch.
- Draw a line.
- Measure the slope of the line.
- Measure the coordinates of the two points defining the line.
- Measure the equation of the line.
- Snap the points (points will now move to integer coordinates when dragged).
- Drag two points to different locations, observe and discuss the changes in the slope measurements of the line.

**Challenge:** Describe lines that have the following slopes:

- 1, –1, 0, undefined, positive, negative.

Indicate and measure the vertical intercept for the given line

(Hint: Determine intersection between the line and the vertical axis).

- Drag two points to different locations, observe and discuss the changes in the vertical intercept measurements of the line.

**Challenge:** Describe lines that have the following vertical intercepts:

- 0, positive, negative, none.
Investigation 2
Investigate the effect of the gradient and vertical intercept parameters on the graph of a straight line. This can be done by means of a Sketchpad or Autograph investigation.

In the case of a Sketchpad investigation the following procedure is suggested:

- Make two copies of the custom slider tool and label them as \( m \) and \( c \).
- Select both the sliders.
- Create a new function defined by \( y = mx + c \).
- Plot this function.
- Drag and/or animate the slider \( c \) and observe and discuss the effect of this parameter on the graph.
- Drag and/or animate the slider \( m \) and observe and discuss the effect of this parameter on the graph.

In the case of an Autograph investigation the following procedure is suggested:

- Type in the equation \( y = mx + c \) (Note Autograph will automatically assign initial values of 1 to both these parameters).
- Select the line and utilise the Constant Controller.
- Use the Manual option in order to investigate the effect of any one of the parameters on the graph for particular values.
- Use the Animation option to investigate the effect of any one of the parameters on the graph in a dynamic way.
- Use the Family Plots option to investigate the simultaneous effect of any one of the parameters on the graph for a range of values.

INVESTIGATE THE EFFECT OF THE PARAMETERS ON THE GRAPH OF THE PARABOLA IN THE VERTEX FORM \( y = a(x - h)^2 + k \)

In NCS (2007: 17) it is suggested that investigations should be restricted to \( y = ax^2 + k \) in grade 10 and thus limit the parameters to two instead of three. (Note that in this case investigations are restricted to parabolas where the axis of symmetry is always the vertical axis). For the grade 11 level it is suggested in NCS (2007: 17) that investigations should focus on the vertex form of a parabola \( y = a(x - h)^2 + k \). A general way to investigate the effect of a particular parameter is to keep all of the other parameters constant as is normally done in scientific experiments. To investigate the effect of the parameters \( a \) and \( k \) on the graph of \( y = ax^2 + k \) a Sketchpad investigation by means of two sliders or a Autograph investigation by means of the constant controller can be carried out. Chanan et al. (2002: 79) suggest an approach
where the number of parameters are progressively increased in a sequence of investigations. These types of investigations can be carried out by means of Sketchpad or Autograph. The following steps are recommended:

- **Step 1:** Investigate \( y = x^2 \) (The basic parabola or base model). They suggest that if you understand this parabola then you are well on your way to understand all parabolas.

- **Step 2:** Investigate \( y = ax^2 \) by means of slider control. Here we ask the question “What happens if you multiply \( x^2 \) by a constant?”.

- **Step 3:** Investigate \( y = ax^2 + k \) by means of slider direct control. The key question here is “What happens when you add a constant \( k \) to \( ax^2 \).”

- **Step 4:** Investigate \( y = a(x - h)^2 + k \) by means slider control. The key question here is “What happens when you subtract a constant \( h \) from \( x \) in \( y = ax^2 + k \).”

In steps 2 to 4 the parameters are controlled by a slider (Sketchpad) or the constant controller (Autograph).

According to Chanan et al. (2002: 79-80) the following are questions to reflect upon, discuss or report about:

**Question 1:** Describe all the points on \( y = x^2 \) that are visible in the current window and that have integers for both \( x \) - and \( y \) -coordinates. Without scrolling, name four other points with integer coordinates.

**Question 2:** What can you say about the equation of a parabola if its vertex is at the origin?

**Question 3:** What kind of symmetry do all parabolas in the family \( y = ax^2 \) exhibit? Why do they have this symmetry?

**Question 4:** If the point \((a, b)\) is on the parabola, name one other point that must be on the parabola.

**Question 5:** How does the sign of \( a \) affect the parabolas investigated here?

**Question 6:** What does the various graphs look like when \( a = 0 \)?

**Question 7:** How is the coefficient \( a \) in these equations similar to the coefficient \( m \) in \( y = mx \), \( y = mx + b \) and \( y = m(x - h) + k \)? How is it different?

**Question 8:** The axis of symmetry is the line a parabola can be reflected about and still look the same. The axis of symmetry for \( y = x^2 \) is \( x = 0 \) (the \( y \)-axis). What is the equation of the axis of symmetry for parabolas in the families \( y = ax^2 \), \( y = ax^2 + k \) and \( y = a(x - h)^2 + k \)?

**Question 9:** The four fundamental transformations in geometry are translation, rotation, dilation and reflection. Describe which of these transformations apply to the parabola when dragging the sliders \( a \), \( h \) and \( k \)?
INVESTIGATE THE EFFECT OF THE PARAMETERS ON THE GRAPH OF
THE PARABOLA IN THE FACTORED FORM \( y = a(x - p)(x - q) \)

Chanan et al. (2002: 81) suggest that investigations on the parabola in the factored form
\( y = a(x - p)(x - q) \) should also be carried out. This investigation can be carried out
by means of a Sketchpad slider or an Autograph constant controller. According to
Chanan et al. (2002: 82) the following are key areas to focus on during this
investigation:

- Adjust slider \( a \) and summarize the role of this slider in the equation. Remember to
discuss the effect of its sign and the magnitude thereof.

- Dragging slider \( a \) appears to change all points on the parabola but two: the roots of
the parabola.

- Adjust slider \( p \). What happens to the parabola if the values of \( p \) changes? How
does this compare to adjusting the \( q \) slider?

- Adjust sliders so that \( p = q \). How would you describe the graph of a parabola with
two equal roots.

INVESTIGATE THE EFFECT OF THE PARAMETERS ON THE GRAPH OF
THE PARABOLA IN THE STANDARD FORM \( y = ax^2 + bx + c \)

The effect of parameter changes can be investigated by means of a Sketchpad slider
investigation or an Autograph constant controller investigation. By exploring families of
parabolas the effect of parameter changes on the parabola can be investigated and
described. Special attention should be given to the following:

- Adjust slider \( a \) and observe the effect on the parabola. Summarize the role of this
parameter in the equation \( y = ax^2 + bx + c \). Be sure to discuss the effect of the sign
and magnitude of this parameter.

- Dragging \( a \) appears to change all the points on the parabola but one (which one?).
Adjust all three sliders and observe the effects they each have on the \( y \) – intercept.
How does the location of the \( y \) – intercept relate to values of the three sliders?

- Adjust slider \( c \) and describe how the parabola transforms as the value of this slider
changes.

- Adjust slider \( b \) and describe what happens to the parabola as the values of this slider
changes.
INVESTIGATE THE EFFECT OF PARAMETERS ON GRAPHS OF THE TRIGONOMETRIC FUNCTIONS BY MEANS OF AUTOGRAPH AND SKETCHPAD

In the NCS (2007: 17) it is suggested that the graph of \( y = a \sin(x) + q \) should be investigated in Grade 10 while the graphs of \( y = \sin(kx) \) and \( y = \sin(x - p) \) should be investigated in Grade 11 (Similar graphs for \( \cos \) and \( \tan \) are suggested). In this paper and during the presentation thereof the effect of the four parameters on the graph with defining equation \( y = a \sin(k(x - p)) + q \) will be investigated by means of Autograph and the basics steps of similar Sketchpad investigations will be highlighted. As angle-measurements at school level are done in degrees and not in radians great care should be taken in changing the preference as well as the relevant scales. As suggested earlier, a transformation approach to this type of investigation may be the most rewarding. It is therefore suggested that the following sequence if sine-functions be investigated:

Single parameter investigations

\[ y = \sin x \rightarrow y = a \sin x \rightarrow y = \sin kx \rightarrow y = \sin x + q \rightarrow y = \sin(x - p) \]

Two parameter investigations

\begin{align*}
\begin{array}{ccc}
y = a \sin kx & y = a \sin x + q & y = a \sin(x - p) \\
y = \sin kx + q & y = \sin k(x - p) & \\
y = \sin(x - p) + q & & \\
\end{array}
\end{align*}

Three parameter investigations

\begin{align*}
\begin{array}{ccc}
y = a \sin k(x - p) & y = a \sin(kx) + q & y = a \sin(x - p) + q \\
y = \sin k(x - p) + q & & \\
\end{array}
\end{align*}

Finally a four parameter investigation on \( y = a \sin k(x - p) + q \) should be carried out.

PAPER-AND-PENCIL APPLICATIONS

It is suggested that applications to the sine-function should include a selection of the \( 1+4+6+4+1=16 \) (Compare with Pascal’s Triangle) possibilities. In this paper a four-parameter example with be investigated. The design of suitable graph paper is crucial in such application. The design of required graph paper by means of Autograph will be demonstrated during the presentation of this paper. For example let us focus on the transformation approach to sketch the function defined by \( y = 3 \sin 2(x - 15^\circ) + 1 \).
In designing graph paper in this example the educator should consider the following:

- The range for this sketch should be \([-4,4]\) while the domain could be \([-360^\circ,360^\circ]\).
- Which axis numbers should be on display? For example every unit measurement could be displayed on the vertical axis while intervals of \(90^\circ\) could be labelled on the horizontal axis.
- Which distance between grid points should be displayed on the axes. On the vertical axis pips of 1 unit will be suitable for this example while pips of \(15^\circ\) on the horizontal axis may be the best option in this example.

The graph paper below may thus be the best option for this particular example.

**Step 1:** Sketch the base model of \(y = \sin x\)

**Step 2:** Sketch \(y = 3 \sin x\) (Note: \(y = \sin 2x\), \(y = \sin(x - 15^\circ)\) or \(y = \sin x + 1\) are also suitable step 2 options). These alternative options could be included as additional exercises. Important to take note that learners can arrive at the final sketch via many different transformation routes.
Note: If the graph of $y = \sin x$ is stretch vertically by the factor 3 the graph of $y = 3\sin x$ is obtained.

**Step 3:** Sketch $y = 3\sin 2x$ by compressing the graph of $y = 3\sin x$ horizontally by the factor 2. (Note: $y = 3\sin x + 1$ as well as $y = 3\sin (x - 15^\circ)$ are additional step 3 options)

Note: The period of the graph of $y = 3\sin 2x$ is $180^\circ$ and is obtained by dividing the period of the graph $y = 3\sin x$ in step 2 by 2.
Steps 4 and 5: To obtain the final graph two more transformations need to be carried out on the graph of \( y = 3 \sin 2x \). There are two possible ways in which these two final transformations can be carried out.

**1st Possibility:**

\[
y = 3 \sin 2x \quad \xrightarrow{\text{Step 4}} \quad y = 3 \sin 2(x - 15°)
\]

\[
y = 3 \sin 2(x - 15°) \quad \xrightarrow{\text{Step 5}} \quad y = 3 \sin 2(x - 15°) + 1
\]
Interchanging the two transformations in the 1\textsuperscript{st} possibility will result in the following 2\textsuperscript{nd} possibility, but immaterial of the order of the two transformations the final result will be the same.

\textbf{2\textsuperscript{nd} Possibility:}

\[ y = 3 \sin 2x \quad \xrightarrow{\text{Step 4}} \quad y = 3 \sin 2x + 1 \quad \xrightarrow{\text{Step 5}} \quad y = 3 \sin 2(x - 15^\circ) + 1 \]
CONCLUSION

All other graphs listed in NCS (2007: 17) can be investigated by means of the methods described in earlier investigations and are left to the reader as an exercise.

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Homework is a necessary evil in the path of learning mathematics at school. Mathematics homework is traditionally seen as difficult and boring. In the case of difficult homework, “math clubs” and “math extra lessons” are often perceived as even more difficult and more boring. This paper describes a project where learners could get help with their mathematics homework using MXit on their cell phones in the afternoons after school. MXit is a popular instant messaging system where text messages are sent immediately between participants' cell phones and is proprietary software of MXit Lifestyle (Pty) Ltd in Stellenbosch. At the time of this writing, there were over 3 million MXit users in South Africa and nearly 45% of them were teenagers between the ages of 12 and 18. In view of the fact that all high school learners now have to take mathematics or mathematical literacy, MXit offers a fun and exciting medium in which to help learners with mathematics homework. Teachers are welcome to refer their learners to this project or, alternatively, teachers could easily follow the steps taken in this project in order to set up similar systems at their own schools.

INTRODUCTION

Southern Africa needs competent mathematicians for economic development of the region. Unfortunately, by many performance indicators, South Africa is doing poorly in mathematics [1]. There are a number of reasons advanced for this including (but not limited to) not having enough competent educators to teach mathematics [2], out-of-school interventions being normally only available in urban areas for additional cost [3], and mathematics homework being perceived as difficult and boring by the learners [4].

Instant Messaging is a system of communication that is available on computers and cell phones. In many ways, instant messaging is similar to email except that the messages are delivered and received immediately. Instant messaging systems have already successfully been used at university level in distance learning situations [5]. Instant messaging is considered a state-of-the-art communication method.

A North West province high school originally contacted the author of this paper asking for assistance with setting up some sort of “math club” which would make mathematics a fun and exciting topic for teenage learners and would encourage learners to actually do their mathematics homework. This paper describes the resultant project where learners could use MXit instant messaging on their cell phones to get help with their mathematics homework in the afternoons after school.
Although this project originated at one school, “word of mouth” advertising (also known as “viral” advertising) where learners told their friends about the MXit contact ensured that the number of learners grew and the geographical location of the learners spread throughout the country.

Teachers who are interested in this method of helping learners are free to refer their learners to this project. However, teachers who wish to engage learners in a medium which the learners themselves use for hours a day can follow the steps described in this paper and easily set up a similar service for their own schools.

MXIT

MXit is proprietary software of MXit Lifestyle (Pty) Ltd which is based in Stellenbosch, South Africa [6]. The MXit software runs on cell phones and enables participants to instantly send text messages to each other. It is often compared to SMSs; however, one of the major differences between SMSs and MXit is the cost. Messages sent using MXit cost approximately one or two cents. SMSs, however, vary in price depending on cell phone contract type and can cost up to 50 cents each.

The above mentioned cost factor makes MXit very popular with teenagers. Current statistics on the number of people using MXit indicate that there are more than 3 million users in South Africa. Of those 3 million, nearly 45% are between the ages of 12 and 18 [7].

“THE MEDIUM IS THE MESSAGE”

Marshall McLuhan gave the world his famous quote “The medium is the message” [8]. History has proved his statement to be true: publishing a message in a newspaper is a lot different to delivering it by flying Jumbo jets into buildings. The message changes depending on the medium through which it is delivered.

MXit is the latest medium to hit South Africa. It's fast. It's “kewl”. It has its own language or “lingo”. It's popular with teenagers. These reasons (and many more) make it a fun place to discuss mathematics despite some difficulties with geometry and graphing.

It also provides for a private conversation between two people. So for learners who are too embarrassed to ask for help or who are too embarrassed to go to extra math lessons, MXit can give them access to a tutor in the privacy of their own home. And for learners in rural areas, MXit can give them access to tutors in urban areas.
Math on MXit takes advantage of the fact that teenagers are already using MXit to communicate with their friends. Teachers and parents are often perceived as being “anti-MXit” by teenagers. Plus very few adults over the age of 25 can match the speed with which a teenager can type messages on a cell phone keypad.

The MXit software, however, allows participants to talk with other instant messaging systems. The other instant messaging systems are traditionally used on computers connected to the internet and have traditionally sized keyboards. We set up a Jabber account with the name dr.math.help.me@jabber.org. Learners would add our account name and address to their list of contacts on their cell phone indicating that it was a Jabber account and not a normal MXit account. They could then use MXit to contact us and discuss their mathematics homework with a tutor.

From an educational point of view, our tutors did not do the learners' homework. The tutors were there to guide the learners into working out the homework problems for themselves. The tutors did not do calculations. The tutors explained how the homework problems were to be done.

EXAMPLES OF CONVERSATIONS

After the initial novelty of Math on MXit had worn off, the learners asked quality questions and the conversations between the tutors and learners were similar to questions that a learner would ask of a more traditional tutor. (In view of the fact that people usually keep similar aliases or screen names across numerous websites, in the following conversations, all aliases have been changed).

Some of the conversations were short and to the point:

(15:37:58) Smartie: Hey u work out the area of a triangle by using 'half base x perpendicular h'
(15:39:30) dr.math: right, nice seeing u again.
(15:41:42) Smartie: TnX.. And how do u work out the perimeter
(15:42:33) dr.math: side + side + side
(15:43:06) Smartie: Oh ya.. TnX
(15:43:57) dr.math: bye bye
(13:53:40) Angel: hey!wud da circumference of a circle with a radius 2 b pie2?or if nt wt is da answer nd y
(13:53:57) dr.math: r u in class?
(13:54:05) Angel: yes
(13:54:09) Angel: r u?
(13:54:32) dr.math: no, but we only help after class :-( ( you might be writing an exame :-)

109
And some conversations could be quite long and involved:

(13:43:50) dr.math: any math questions
(13:46:21) Cutie: can u pls explain trinomials 2 me
(13:47:10) dr.math: give me an example
(13:48:28) Cutie: xsquare+7x+6
(13:49:25) dr.math: I will use ^ for power x^2 +7x +6   so you need to find two numbers that if you multiply them you get 6 and if you add them u get 7.  what are those two numbers
(13:50:09) Cutie:
(13:50:41) Cutie: 6 1
(13:51:32) dr.math: right so the factors are (x+6)(x+1)
(13:52:58) Cutie: nw wat bout x^2-5x-6
(13:53:36) Cutie: i dnt get da '-' part
(13:54:20) dr.math: so you need to find two numbers that multiplied give you -6 and added give you -5.  that means one of the numbers must be negative and the other positive
(13:56:27) dr.math: does +6 -1 give you -5?
(13:57:44) Cutie: o i c! -6 +1
(13:58:26) dr.math: very good
(13:59:06) Cutie: thanx! u helpd me stax!
(13:59:57) dr.math: good i'm glad to hear that.  come back if you have another problem
(14:00:26) Cutie: i wil!

VOLUNTEER TUTORS

Our Math on Mxit project started originally with one tutor. As the project grew, however, we recruited tutors from the University of Pretoria. The Faculty of Engineering, Built-Environment and Information Technology have a compulsory module for all undergraduates which requires “volunteer” service on a community based project. Math on Mxit met the requirements for the community based project. At the time of writing, we had 9 tutors from the university assisting us in the afternoons after school.

110
HINTS FOR TUTORS

Being a tutor for *Math on MXit* could be stressful at times. We originally geared it towards high school learners but eventually we also had learners as young as Grade 4. So our tutors needed to know everything from how to describe long division up through matric (grade 12) trigonometry.

We did come up with a couple of tricks. In view of the fact that tutor workstations were full sized computers connected to the internet, prior to coming on duty, tutors would point a few internet browsers to http://www.wikipedia.org and http://www.google.com in order to be able to quickly look up any identities or formulas they might have forgotten.

Another thing that occurred was that if one learner had a problem with some aspect of mathematics, there would probably be another learner from the same class who had the same problem. So tutors kept their notes from one session to the next to make it easier to help subsequent participants with the same question.

SOME DIFFICULTIES

Many instant messaging systems (including MXit) do allow for transmitting of pictures and drawings. However, this facility is highly dependent upon the cell phone that the participant is using, and, we did not use that facility.

This did create some challenges when helping people with geometry problems and graphing problems. But with enough time to chat back and forth, we normally succeeded.

(15:08:54) mechanic: It is geometry
(15:09:05) dr.math: Geometry is often difficult over MXit but let's try.
(15:12:36) mechanic: I just need 2 no wat dey r asking... Given: triangleABC with AB=AC. EDT is the perpendicular bisector of AB. BC produced meets EDT in T. Prove dat BT is th third proportional to BC & AB
(15:13:35) dr.math: what is EDT
(15:17:22) mechanic: A line going thru the triangle perpendicular 2 AB ending at 2 points outside the triangle: E on the side of AB & T on da side of AC wher it intersects AB it is cald D wher it intersects AC it is cald F

----- later -----
(15:28:24) dr.math: again, I'm just talking.. cos(b)= BE/BT right?
(15:30:11) mechanic: Bt B & E rtjoiind
(15:30:53) dr.math: what did you call the midpoint of AB?
(15:31:27) mechanic: F
(15:31:33) mechanic: Sory
(15:31:49) dr.math: Ok so \( \cos(b) = \frac{BF}{BT} \) are we together?

(15:33:24) mechanic: Ja

(15:34:33) dr.math: now drop a perpendicular from A down to BC

(15:34:55) mechanic: Ok

(15:35:21) dr.math: so \( \cos(b) = \frac{(BC/2)}{AB} \) right?

(15:36:53) mechanic: Ok ja

(15:37:15) dr.math: So now \( \frac{BF}{BT} = \frac{(BC/2)}{AB} \) you can work it out from there quite easily.

---- later -----

(15:40:11) dr.math: Did you get "So now \( \frac{BF}{BT} = \frac{(BC/2)}{AB} \) you can work it out from there quite easily." You can work it from there

(15:42:48) mechanic: Thanx man u r brillant!!!

(15:42:59) dr.math: but you are solving it yourself. well done.

**LANGUAGE ISSUES**

Our *Math on MXit* was primarily in English because the original tutors did not speak any of the South African languages other than English. We found that Afrikaans speaking learners jumped right in and asked questions interspersing Afrikaans when necessary.

14:13:00) dr.math: so you never told me what you moved onto in math after logs

(14:14:05) Captain Kirk: I dont know what it's called in english:o

(14:14:36) dr.math: OIC. try it in afrikaans maybe I can understand.

(14:15:26) Captain Kirk: Ok its called "rye en reekse":D but that's easy:D

(14:16:03) dr.math: That's beyond my limited Afrikaans vocab. But I'll look it up this weekend

(14:17:42) Captain Kirk: Its like numbers following one another in a pattern:D the first number is usually called T1 and the second one is T2 and so forth:D

(14:18:10) dr.math: series and sequences. Like Fibonacci 1, 2, 3, 5, 8, 13, 21 etc

(14:24:36) Captain Kirk: Exactly:P my maths teacher says its the easiest part of Gr12 maths:D

We eventually started an Afrikaans equivalent service which was, at the time of writing this paper, operating one afternoon per week. The quality of the conversations between the learners and the tutors was also very good.

(15:28:48) Dr Wiskunde: Hallo cupcake, het jy vir 'n wiskunde vraag?

(15:30:32) Cupcake: Ons is op die oomblik besig met breuke mar my eintlike vraag gaan oor Faktorisering!Ek kan dit glad nie doen nie

(15:31:19) Dr Wiskunde: Ok, gee vir my 'n voorbeeld dan help ek jou om hom uit te werk
We also saw from the types of screen names that participants used to identify themselves that there was a growing portion of ethnic African learners taking part. We do have plans to expand the Afrikaans service to four afternoons per week (the same as the English service) to start a Zulu service and Tswana service. Hopefully at the time of presenting this paper, we will have more to report on this aspect of our project.

ETHICAL CONCERNS

We had a number of ethical concerns with Math on MXit – and still do. These concerns revolve around the fact that the people using this facility were minor children and they were freely giving their cell phone numbers out to total strangers. MXit keeps track of participants by their cell phone numbers. When learners contacted us, we would automatically receive their cell phone numbers.

As soon as a learner contacted us, we would ask for a nick name, an alias, or a screen name of some sort. This nick name would hide or override the cell phone number. The cell phone number would still be stored on our computer, but it would not be immediately visible.
We also informed the learners that we were recording or logging all the conversations. We logged the conversations for four reasons:

- Research purposes: we wanted to know if the learners were asking good questions of the tutors
- Quality purposes: we wanted to ensure that the tutors were answering the questions properly
- Safety of the learners: we needed to ensure that the learners were not being enticed into any illegal or unsafe activities by tutors
- Safety of the tutors: we needed to be able to protect the tutors from any false allegations that might arise

We wrote up a code of conduct which tutors needed to sign before being allowed to work on *Math on MXit*. This code of conduct included the following rules:

1. I will not contact any learner who joins the *Math on MXit* program outside of the *Math on MXit* program.
2. I will not give any of the cell phone numbers to which I have access to anybody outside of the *Math on MXit* program.
3. I will not ask any personal questions of any of the participants of *Math on MXit*. The one acceptable exception to this rule is "What grade are you in?" in order to judge the level of help that you can give to the participant.
4. I will not answer any personal questions from any of the participants of *Math on MXit*.
5. I will maintain the log files of all conversations and will not tamper or edit them in anyway.
6. I will upload the log files to the appropriate server after each session.
7. I will limit my conversations to topics in mathematics, science, and school work.
8. I will not discuss sex, drugs, or any illegal activities with any of the participants of *Math on MXit*.
9. I will encourage participants in further study of any subjects in which mathematics is important including science, geography, accounting, and computer studies.
10. I will encourage participants to use their cell phone as a research tool (and not just as a convenience) by also informing them about cell phone browsers and cell phone based calculators.

These rules were primarily designed to prevent any actual real physical contact between tutors and learners. The secondary goal of the rules was to encourage the study of mathematics.
UNEXPECTED SOCIAL ASPECTS OF MATH ON MXIT

One of the many unexpected features of Math on MXit was that many learners developed a virtual social relationship with us and would often just check in after school to say hello:

(13:48:17) dr.math: Ok, what can I help you with today?
(13:51:10) Snake: im in a hostel. i first have 2 eat. after that. im going 2 my room nd then ill need ur help. ply
(13:51:33) dr.math: I'm here until 4, chat l8t
(14:26:50) Snake: ok. thx. in done whith evrything nw.

(14:33:36) pilot: Hey werk j leka
(14:33:53) dr.math: ja
(14:34:15) pilot: Dis kul
(14:34:26) dr.math: any math questions for me?
(14:35:14) pilot: Lots bt I dnt hav the motivation nor energy 2 ask rite nw

Participants enjoyed the enigma of Math on MXit. Our code of conduct did not allow the tutors to give out any personal details about themselves. Many of the participants even wondered whether the tutors were computers or humans. (In the conversation below, the term “aslr” is short hand notation for “age, sex, location, race”.)

(14:45:33) Gander: R U A DUDE OR DUDET

(13:44:50) Carbon Dioxide: how old are u
(13:44:59) Carbon Dioxide: are u a man
(13:45:22) dr.math: no but we don't do aslr stuff ;-)  
(13:45:26) Carbon Dioxide: u could be a sex offender for all i know
(13:46:07) Carbon Dioxide: i dont think i can trust u
(13:46:29) dr.math: that's why we don't ask or answer personal quetsions. but pleease deleete me if you don't trust me
(13:47:41) Carbon Dioxide: wat is pie plus teetha

(20:13:45) Ocean Sprite: Btw how old r u?
(20:13:53) dr.math: Dr math doesn't answer personal quetions
Another unexpected development was the number of learners who specifically asked for counseling over MXit.

LEARNERS' EVALUATION OF MATH ON MXIT
Learners shared with us the fact that they thought Math on MXit was a success:

THE DARK SIDE OF MXIT
There is, unfortunately, a dark side of Math on MXit. It is an extremely sad comment about society when a paper on mathematics education needs to cover sexual activities of minor children. Our tutors received numerous sexual propositions. Our rule was that if
sex or drugs were spoken about by a learner, we would give them a warning that the language or topic was not appropriate. If it continued, we would remove them as a contact on our system.

Some of the conversations were initially innocuous:

(15:42:26) Hot Shot: r u sexy?
(15:45:00) dr.math: Hot Shot, that is not appropriate language and I will delete u as a contact if you talk to me again that way
(15:47:01) Hot Shot: im sorry,my english is bad...
(15:49:26) dr.math: well, i don't think you were asking about the weather ;-) so anyway do you have any math homework
(15:50:45) Hot Shot: hehe... well i do... but i dont knw how 2 tell u in english... really... im afrikaans

(13:28:44) Baby Girl: Are you sexy?
(13:28:47) dr.math: hello Baby Girl howzit
(13:29:56) Baby Girl: Im gud nd u
(13:30:29) dr.math: fine. Baby Girl, remember that I record these conversations. some words aren't really acceptable for dr.math ;-) 
(13:32:13) Baby Girl: Lol okay

(13:25:02) dr.math: hi. what's your nick name
(13:25:23) unknown_3@mxit.co.za: sexy
(13:25:52) dr.math: no, I want a more appropriate nick name please ;-) 
(13:26:16) unknown_3@mxit.co.za: creamy
(13:26:29) dr.math: still pushing your luck :-(( one more try
(13:27:43) unknown_3@mxit.co.za: Beauty
(13:28:22) dr.math: OK, Beauty, that's better. I need to tell you that I record these conversations, is that ok with you.
(13:28:42) Beauty: yes
(13:29:26) dr.math: so, Beauty, how's math class going?
(13:29:41) Beauty: nt gud

Some participants tried to cover up their language or propositions by saying that somebody else had been using their cell phone:

(11:25:40) dr.math: just on MXit, afternoons 2-3 and whenever else I may log in. So u said you get 96% in math. well done.
(11:26:04) Giraffe: yeah. do u want a blowjob
----- contact deleted by Dr. Math -----
----- contact reconnected later in the day -----  
(14:31:35) dr.math: Sorry, I record these conversations. I deleted u. I don't appreciate your vocabulary. keep it clean or I will delete u again.  
(14:31:46) Giraffe: how can i study triangles?  
(14:32:04) dr.math: Just a sec. I'm recording these conversations, is that ok with you?  
(14:32:51) Giraffe: yeah, why u record it?  
(14:33:18) dr.math: For research and quality purposes. If this dr.math is successful we may start dr.science, etc.  
(14:34:10) Giraffe: What did my friend ask u ths mrning- she had m fne?  
(14:34:17) dr.math: so my coworkers and boss, etc, read through the log files just to see how everything is going  
(14:34:38) dr.math: so we keep the conversations clean. And I don't know who yur friend is  
(14:35:09) Giraffe: she chatted on my phne. what did she ask u?  
(14:36:20) dr.math: well, not that I necessarily believe everything I read from MXit, but whatever she or you said was not appropriate.  
(14:36:29) dr.math: Anyway let's drop it. what do u need to know about triangles.  

RESULTS  
At the time of writing this paper, we do not have any hard numerical data on whether any of the participants in Math on MXit actually increased their marks in mathematics or not. We do, however, have conversations with numerous learners which indicated that we had, indeed, helped the learners:  

(15:08:32) daisy: do u have any tips 4 me on (veelterme en polinome)?  
(15:09:07) dr.math: sure. what's the problem that you seem to be having?  
(15:10:26) daisy: no problems jus were writing a test on wednesday jus wanet 2 knw if u hav tips?  
(15:10:57) dr.math: OIC. well, not really....  
(15:11:43) daisy: nt but ure a computer and do u have any help of sme kine 4 me on that?  
(15:12:19) dr.math: will it be covering the roots of the formula, or plotting the formula, we can do some exercises and test questions...  
(15:13:38) daisy: ok that would be nice!  
------- the next day -------  
(14:22:09) dr.math: how was your math test?  
(14:22:55) daisy: it was kind of easy 4 me just hope i get gud marks!  
(14:23:06) dr.math: that's great. what did it cover?  
(14:23:48) daisy: veelterme and smeting like that!  
(14:18:41) Lock: I passed math!
THE WAY FORWARD

Learners have asked us to set up more MXit based “help lines” for help in other subjects. This technique of using MXit to help learners is, obviously, not limited to mathematics. This technique would work equally well for help with science or accounting. However, the technique would not work for any language based courses such as English, Afrikaans, or Zulu. This is because of the abbreviated language used on MXit and the terrible spelling employed.

Another task we have on our “to do” list is to implement a computer networking solution so that we can allow tutors to log in from remote sites without exposing the minor children's cell phone numbers on remote computers. Currently, all tutors must come into our offices (in Pretoria, South Africa) and the log files of the conversations with the children are secured on our computers. This, obviously, limits the pool of volunteer tutors which we can use. We have had numerous offers of volunteers – even as far away as the UK – and we have plans to take advantage of those offers as soon as the security of the minor children's cell phone numbers can be secured.

CONCLUSIONS

Our Math on MXit project was, and still is, successful in helping learners with their mathematics homework in the afternoons. The cost to the participants was minimal. From a learners' points of view, the entire conversation may cost, at most, R1.00. From the tutors' point of view, the cost was the tutors' time and the internet connectivity. Only free open source software was used on the tutors' workstations. MXit software on the cell phones is free of charge.

Learners showed a real eagerness to engage in conversation with an adult. Despite the fact that we made a concerted effort not to reveal any personal data about the tutors, learners rightly assumed that the tutors were adults and treated us as such. Once we made it clear on the ground rules that we would not tolerate foul language and sexual content, most participants were extremely polite when dealing with us.

Besides discussing mathematics, we ended up also discussion other academic topics such as university entrance requirements, tests in other classes, and wishes and dreams of future education. Many participants asked for counseling. We were not in a position to help such learners and we tried referring them to favourite teachers, scout leaders, or Sunday School teachers.

We can highly recommend this medium of communication with teenagers over a wide scope of topics. Besides schools, other organisations such as church groups, youth
clubs, and counseling organisations could be able to use this medium. The one topic which would not really be appropriate would be language education because of the abbreviated form of communication and the creative spelling.

Teachers who do not wish to go to the effort of setting up a service like this are free to refer their learners to our service.

The medium really is the message.

REFERENCES


We analyse our observations of teaching and learning of mathematics at five secondary schools, populated by working class students, with the goal of systematically defining a research problem and the production of research hypotheses. Our initial observations reveal that the pace of teaching and learning is slow but that students fail to learn the content adequately. We draw on research that enables us to more productively discuss the relation between pacing and student learning and find that the nub of the problem centres on the absence of the explicit operation of mathematical grounds to support the elaboration of standard procedures. We generate a series of hypotheses on the negative effects of the absence of mathematical grounds on the constitution of mathematics in the five schools and the slowing of pacing. The hypotheses open up a series of lines of investigation that are being pursued currently.

INTRODUCTION

In this paper we reflect on a series of a initial observations of the teaching and learning of grade 10, 11 and 12 mathematics in five working class schools in the Western Cape. Three of the schools are ex-DET schools situated in townships in greater Cape Town, the fourth is an ex-DET school situated in the Cape winelands, and the fifth is an ex-Model C school with a student population made up entirely of children from “African” and “coloured” working class families.

One purpose of the observations, which took place early in 2007, was to generate a general description of mathematics teaching in the five schools in order to inform the design of intervention strategies aimed at improving the quality of mathematics teaching in the schools as well as the abilities of students to perform successfully on examinations. A second purpose of the observations centred on informing the production of research strategies that could be used to direct a more sustained and rigorous investigation of two interrelated questions: (1) What is constituted as mathematics in the five schools? And (2) how is it constituted?
The focus of this paper is to elaborate aspects of the second of the two purposes of the observations which, of course, necessarily draws on the first. Nineteen lessons were observed across the five schools: 6 grade 10 lessons, 6 grade 11 lessons, and 7 grade 12 lessons. We shall refer to the schools as School 1 to School 5. Table 1 shows a breakdown of the grades and number of mathematics lessons observed at the five schools.

For the series of observations, two researchers observed each lesson and made observation notes which they compared and discussed shortly after leaving the particular school they visited on each day of observation. The purpose of the post-observation research meetings held on each day of observation was for the researchers to arrive at agreement on what had been observed, to consider the questions they had raised in their observation notes, and to begin to generate analytic descriptions of the teaching of mathematics in each school. Prior to the observations the researchers developed a methodological framework to guide their observations as well as the production of their observation notes. The observation notes took the form of a record of the events as they unfolded in each of the classrooms, including records of the time spent on distinct events, along with analytical commentaries and questions.

<table>
<thead>
<tr>
<th>School</th>
<th>Grade 10</th>
<th>Grade 11</th>
<th>Grade 12</th>
<th>Total</th>
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<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
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<td>4</td>
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<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>6</strong></td>
<td><strong>6</strong></td>
<td><strong>7</strong></td>
<td><strong>19</strong></td>
</tr>
</tbody>
</table>

Table 1: Number of mathematics lessons observed by grade

**THE PROBLEM IN ITS IMMEDIACY**

Our initial impression of what we observed in the five schools was: (1) mathematics teaching and learning was happening almost exclusively through the elaboration of worked examples to demonstrate the application of standard procedures; (2) students struggled to reproduce the application of the standard procedures they were expected to learn; and (3) both teachers and students generally worked at a slow pace.

We decided to construct an initial description of the lessons in terms of the manner in which lesson time was used because the research literature that describes and analyses teaching and learning in terms of the construct opportunity-to-learn indicates emphatically that the time spent on the content to be tested is the single most important
determinant of the extent of student success on standard tests and examinations (see, for example, McDonnell, 1995; Reeves, 2005; Reeves & Muller, 2005; Smith, 1998; Smith, Smith & Bryk, 1998; Stevens, 1996; Wang, 1998).

We noted that while teaching in the five schools took place exclusively through exposition, little explicit attention was given to the exposition of the mathematical ideas, principles and definitions that served to ground the procedures that were being rehearsed. Mostly, teachers briefly referred to definitions but without discussing or explicating the mathematical reasons for the production of the definitions. We also noted that worked examples were used in three ways: (1) in exposition by teachers demonstrating the application of standard procedures through worked examples; (2) by students working through exercises to practice and demonstrate their acquisition of procedures; and (3) the routine marking of worked examples by teachers in which privileged solutions were demonstrated. Of course, occasionally classroom time is also spent on activity not related to the topic of a lesson. We therefore have five primary categories of classroom activity that enable us to describe lessons in terms of the use of time: (i) exposition of ideas, principles and definitions, (ii) exposition by worked examples; (iii) marking of worked examples; (iv) working through exercises; and (v) activity unrelated to the lesson topic. The total time spent on the categories of activity numbered (ii), (iii) and (iv) give the total time spent on worked examples in each lesson. Once lesson time is described in terms of the five categories it is a simple matter to calculate the proportion of lesson time consumed by each category of activity for any given mathematics lesson. Rather than use the lesson time as indicated on school timetables, we measured total lesson time as starting from the time students entered a classroom to the time they started packing up their books.
Table 2: The use of time in the five schools

Table 2 presents a summary description of all 19 lessons across the 5 schools in terms of time, from which it is immediately apparent that most time is spent on the elaboration of mathematics through worked examples.

Given that we were able to observe most classes only once, the information summarised here tentatively suggests global trends per school and across schools rather than firm trends for individual classes. Further investigation should enable the production of more finely grained distinctions between individual classes as well as between individual teachers.

Figure 1 shows a graphical comparison of the time spent on worked examples and time spent explicitly on the elaboration of ideas, principles and definitions, from which it is apparent that the way in which is used in the different grades across the schools is very similar.
Time spent on exposition of ideas, principles and definitions compared with time on teaching and learning through worked examples, per grade across Schools 1 - 5

![Graph](Image)

**Figure 1:** Comparison of the use of time on idea, principles and definitions and time spent on worked examples per grade across all schools

Time spent on exposition of ideas, principles and definitions compared with teaching and learning through worked examples, per school across grades 10 - 12

![Graph](Image)

**Figure 2:** Comparison of the use of time on ideas, principles and definitions and time spent on worked examples per school across grades

Figure 2 shows comparison of the use of time on explicitly discussing ideas, principles and definitions and time spent on worked examples for each school across all grades in each school. The results are, once again fairly similar. Two schools stand out as spending more time on ideas, principles and definitions: Schools 1 and 5.

In the case of School 5 the larger proportion of time spent on ideas, principles and definitions is due to a single grade 10 lesson in which the teacher spent most of the time writing definitions of operations on numbers on the chalkboard for students to copy into their notebooks. The teacher, however, did not spend much time discussing the definitions. Of all the schools, it was only at School 1 where it seemed that brief discussions of ideas, principles and definitions happened routinely as part of lessons, and always as an introduction to the discussion of worked examples.
Table 3 summarises the data on the average time spent per worked example per grade per school. Figure 3 shows a graphical comparison of the average times per grade for each school. The last bar in each cluster indicates that the average time spent on worked examples across all grades for each school are very similar: around 10 minutes per worked example.

While it is a very rough estimate of the amount of time spent per worked example, we believe that it is reasonable to expect that mathematics classes will get through 3 to 5 problems per lesson on average, which suggests that mathematics is taught at an extremely slow pace in the five schools.³

The immediate question that arose in response to our observation of the use of time in the five schools was: Why should it be the case that students struggle⁴ with mathematics

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³ Table 2 shows a disturbing trend in average time spent per problem in Schools 2 to 5, where the average time increases from grade 10 to grade 12. This might be indicating that the deficiencies in the students’ knowledge of mathematics becomes more apparent at grade 12 level where they start working on typical problems found in school leaving examination papers.

⁴ All the of the grade 10, 11 and 12 mathematics students at the five schools participated in grade-specific baseline tests designed to get a measure of their mathematics competence on content they should have learnt in previous grades. The results of those tests show that the competence of the students across grades 10 to 12 is very weak. In Grade 10 there were 22,1% correct responses, 66,1% incorrect responses, and 11,8% null responses. The topic for which the most correct responses (28%) were produced was Euclidian Geometry, and most incorrect responses, for Algebra (72,8%). There were 33,5% null responses in Probability. In Grade 11 there were 9,7% correct responses, 68,7% incorrect responses, and 21,6% null responses. The most correct responses was for Number Relations (20,9%) and the most incorrect responses was, again,
when the pace is slow and students apparently have sufficient time to master the knowledge they are required to learn?

![Average time per worked example per Grade across Schools 1 - 5](image)

Figure 3: Comparison of the average time per worked example per grade for each school

Our description of the lessons we observed in terms of the use of time does not enable us to understand what is happening in the teaching and learning of mathematics or explain why students struggle even though they apparently have sufficient time to learn the content. What we now need is move to a reflection on the question thrown up at the moment of immediacy by drawing principled resources into the discussion. Specifically, we need to consider research that has enabled commentaries on the relation between teaching and learning time and student failure in the context of the education of working class children.

**THE PROBLEM UPON REFLECTION**

A number of research studies that attempt to reveal the pedagogic conditions contributing to successful learning outcomes for working class students indicate that a slowing of pacing is a commonly occurring theme.\(^5\) We will restrict our discussion of the pertinent research to two bodies of well-documented studies of school teaching and learning, spanning extended periods of time: the research conducted by the ESSA group in Portugal who use sociological theory to investigate the teaching of science to working class children.

for Algebra (73,4%). The null responses were concentrated in responses to Probability (45,8%). In Grade 12, there were 26% correct responses, 51,6% incorrect responses, and 21,6% null responses. The most correct responses was for Number Relations (39,9%), as for Grade 11, and the most incorrect responses was again for Algebra (58,9%). The null responses were concentrated in responses to Graphs (32,8%).

\(^5\) There is an extensive body of research on the education of working class students showing how the language use of such students as well their general semantic orientation disadvantages them with respect to schooling. We cannot detail that research here, but see Holland (1981) for a discussion of the pertinent issues.
class students, and the Consortium on Chicago School Research, who use the construct opportunity-to-learn to examine education in Chicago public schools and who comment extensively on the failure of education in the city’s “high-poverty schools”.

One conclusion derived from the extensive body of research produced by the ESSA group is that the control of pacing should be weakened so that students can slow pacing when they need to do so.

Our studies have shown that while weak classifications and framings are an essential condition for learning at the level of pacing, for hierarchical rules, for knowledge relations (interdisciplinary, intradisciplinary, academic-non-academic), and for relations between spaces, they are less so at the level of selection (at least at the macro level) and certainly at the level of evaluation criteria. (Morais, 2002: 560)

Paraphrasing Morais’ Bernsteinian language, the first part of her statement asserts that it is essential that students are able to vary the pace when they need to, approach teachers when they need assistance, shift between academic and non-academic (everyday and metaphorical) expressions of knowledge, and be able to interact with others, including the teacher, freely. In other words, control of classroom activity needs to be flexible, so that the resources needed by a student for successful learning are available to the student as and when s/he needs them. The second part of the statement, however, marks as equally essential a strengthening of teacher control (so-called “strong framing”) over the selection of content for study and over the criteria that distinguishes appropriate from inappropriate knowledge statements. The ESSA group conclude that the latter is necessary because it is necessary that evaluation criteria be made explicit to working class students.

Framing is strong when evaluation criteria are made explicit to the acquirer and is weak when evaluation criteria are implicit. Strong framing at this level may lead children to acquire the recognition and realization rules of the school context. This needs time, a weak framing of pacing. But time without explicit criteria may be useless. (ibid.)

What Morais refers to as weak framing of pacing is not exactly the same as slowing the pace of teaching and learning. Strictly speaking, weakly framed pacing allows pace to speed up and slow down as dictated by students’ needs. The penultimate statement in the last extract, however, indicates that a slowing of pace is essential—“this needs time”—for working class students. The final statement in the extract is a caveat, warning that slowing the pace without at the same time making explicit evaluative
criteria⁶ (recognition and realisation rules) is unproductive and therefore does little to assist working class students to meet the learning demands of schooling.

Shifting now from Portugal and across the Atlantic, to the Consortium on Chicago School Research, who report in research they conducted that they found “no connection between slow pacing and improved student learning”, and conclude that all they can say “with the evidence at hand is that steady exposure to slow pacing leaves Chicago’s students farther and farther behind” (Smith, Smith & Bryk, 1998: 2). Their findings on pacing apparently concur with those of the Third International Mathematics and Science Study (Schmidt et al., 1998), and they further report that:

> Across all our analyses, we find that the weak pacing and coherence that limits students’ opportunities to learn persistently characterize many of the system’s high-poverty schools (Smith, Smith & Bryk, 1998: 27; our italics).

Here it should be noted that the Chicago researchers indicate the simultaneous occurrence of two phenomena implicated in the reduction of opportunities to learn: weak pacing and “weak curricular coherence”. The term curricular coherence refers to “the degree to which domain-specific or disciplinary content is systematically presented to learners in terms of the conceptual coherence of its organization” (Reeves & Muller, 2005: 107). Curricular coherence can be predicated in our language as the extent of teachers and students operation on grounding ideas, principles and definitions of the contents of a field of knowledge when teaching and learning.

The Consortium on Chicago School Research appears to arrive at a conclusion similar to that of the ESSA group as regards pacing, but off a very different methodological base. Both research groups argue that working class students are further disadvantaged by the simultaneous occurrence of a slowing of pace and an absence of explicit evaluative criteria (“weak curricular coherence” in the Consortium’s language).

The research thus suggests that where we encounter a pedagogic situation (in the context of the education of working class students) that exhibits a slowing of pace and unsuccessful student learning, that it is likely to be the case that the evaluative criteria are not being made explicit to the students (weak curricular coherence).

So our reflection arrives at a point indicating that the further pursuit of our study demands a focus on investigating the relations between evaluative criteria and pacing. In order to do so, however, we need to develop our use of the notion of evaluative criteria. It is at this point that we diverge somewhat from ESSA and the Chicago consortium on a point of general methodology while nevertheless sharing common

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⁶ For methodological reasons that will become clearer later, we use the notion of evaluative criteria in a manner that diverges from ESSA’s usage. For now we merely note that what ESSA mean by “making explicit evaluative criteria” is for us the enabling of teacher and student operation of the grounding ideas, principles and definitions of a field of knowledge.
theoretical resources\textsuperscript{7} with ESSA.

\section*{THE PROBLEM IN ITS NECESSITY}

For us mathematics is \textit{constituted} through the operation of evaluative criteria, as is the particular specialisation of consciousness that might be vaguely referred to as mathematical thinking. In other words, evaluative criteria are productive of what comes to be mathematics in a pedagogic context as well as productive of the particular manner of thinking mathematics in that context.\textsuperscript{8} We believe that Bernstein’s (1996) predication of the operation of evaluative criteria—as productive of \textit{recognition} and \textit{realisation rules} for the production of legitimate texts—should be treated quite literally, and not as Morais and the ESSA group tend to emphasise. When ESSA researchers speak of the necessity to make explicit evaluative criteria they have in mind only very specific criteria, which may or may not be circulating in any empirically given pedagogic context. For them the criteria to be made explicit are criteria that enable the legitimate reproduction of science in a principled fashion. That is, of course, highly desirable, but such criteria need not be present in a given empirical pedagogic context and are, therefore, externally defined by ESSA as necessary. In other words, for ESSA productive teaching and learning \textit{should} explicitly exhibit the operation of particular evaluative criteria—a prescription for the realisation of good pedagogic practice.

It is possible that evaluative criteria can be very explicit but that the texts generated through the use of the criteria might well be considered non-legitimate from outside of the particular pedagogic context. For example, consider an instance at School 1, where a grade 12 teacher detailed her solution to a problem that required students to find the equation of the line through the points A(-3,2) and B(-3,5):

\[
M_{AB} = \frac{5 - 2}{-3 + 3} = \frac{3}{0} \text{ (1 mark)}
\]

Undefined (1 mark)

Line is undefined (1 mark)

The teacher explained her procedure clearly, even showing the allocation of marks that could be expected for such a problem on tests and examinations. The students accepted the “solution” and marked their work accordingly. The “solution” is clearly incorrect. If we consider the points, A(-3, 2) and B(-3, 5), it is obvious that, \textit{if those points are elements of a line}, then the line in question can only be \(x = -3\) since the \(x\)-coordinate of

\textsuperscript{7} Bernstein’s sociological theory of pedagogic discourse. See Bernstein (1996).

\textsuperscript{8} Here we tend in the direction of Parmenides (and Plato): “The same thing is there for thinking of and for being”.

130
both points has a value of -3. The teacher appeared to be following a standard procedure that started with first finding the gradient of the supposed line, as is indicated by the calculation for $M_{AB}$: the change in $y$-coordinates divided by the change in $x$-coordinates. Presumably, if the points had different $x$-coordinates, she would have calculated the $y$-intercept of the line once the value of the gradient was found and would then have written down an equation of the form $y = ax + b$, or its equivalent.

The evaluative criteria circulating in the pedagogic context are clear: a very specific procedure is to be used to solve the problem of finding the equation of a line through two points: first calculate the value of the gradient, then the value of the $y$-intercept and, finally, write down the equation of the line in the form $y = ax + b$, or its equivalent (like, $Ax + By + C = 0$); additionally, if the gradient cannot be calculated because of division by zero, then it is undefined and, consequently, the line is undefined. So what’s missing here? Why do things go awry?

Let us note that the procedure can be fixed quite easily by adding another rule: one can always draw a line through two points, so if we find that the gradient comes out as undefined by our method, then we are dealing with a line of the form $x = a$ where $a$ is the common $x$-coordinate of the two points (or, if we told that the $x$-coordinates of two points of a line are the same, then the line is of the form $x = a$ and so forth). It is almost always possible to add additional rules to procedures to deal with special cases. So, having shown how to enable the procedure to deal with the specific problem we might be tempted dismiss the teacher’s “solution” as a simple error, an oversight, and then move on. However, things go wrong precisely because a general understanding of the notion of a line does not function as an evaluative resource regulating the teacher’s and students’ use of their procedure.

This reveals the absence of a principle that we suspect is generally neglected by remaining implicit in the teaching and learning of mathematics in schooling: the mathematical operations that inhere in procedures are valid only insofar as (1) they can legitimately be used to operate on the domain of objects implied (or announced) by a mathematical string, and (2) that they do not alter the implicit value of strings, where “value” is usually a particular element of the domain of objects operated upon. In more general terms, solutions to problems are a coordination of two levels of transformations on a mathematical string. First, a mathematical string has an implicit value with respect to and in terms of the domain of objects it operates upon; second, operations can alter the particularities of the expression of implicit value in a string as needed, but not the implicit value. That is, identity is to be strictly preserved at the level of value while difference is permitted, even necessary, at the level of expression. In short, a legitimate solution procedure entails transformations at the level of expression while preserving the values implicit to the original expression. It follows that the mathematical validity of mathematical operations is assured by making sure that transformations of strings preserve value as expression changes, and is regulated by knowledge of the domain of objects being operated upon.
If we return to the empirical instance of an incorrect “solution” to the problem of finding the equation of the line between the points A(-3, 2) and B(-3, 5) we see that the series of operations entailed in the procedure are, in general, legitimate in the sense that those operations can indeed be applied to the specific objects (elements of a line expressed as Cartesian coordinates), but that the series of transformations alters the object being operated upon: the line is judged to be undefined. The “value” which needed to be preserved in this instance was the mathematical existence of a particular line which is an element of the domain of objects that is the set of all lines in the plane. Given that at the point at which the problem is stated all we have are two elements of the line, and that we seek a more general description of that line, it is really the general notion of line-as-such that stands in as a regulative “value”. In other words, the transformations that make up the procedure should not violate the extant notion of line-as-such.

What is of special interest here is the implication, still tentative at present, that the standard procedure for finding the equation of a line through two points comes to assume constitutive precedence over the idea of a line. That is, it is the local procedure that comes to constitute the meaning of the object—a straight line—rather than the other way round, where it would be the general understanding of the nature of the line being used to generate, regulate and evaluate procedures employed to reveal and describe its specific features in local contexts (specific problems).

The teacher’s error and her students’ acceptance of her “solution” tentatively suggest the possibility that the ideas of mathematics and its objects may well be constituted as effects of standard procedures in the class we observed, and possibly so for both the teacher and her students. If such is the case the ideas that are constituted are likely to be inconsistent and unstable. For instance, the teacher’s reasoning displayed earlier implies that even though she and her students routinely draw the line $x=0$ in the Cartesian plane ($y$-axis) when drawing curves, the line $x=0$ cannot be defined because its gradient will come out as undefined if approached in the same manner as she did the line through A(-3, 2) and B(-3, 5). In other words, the line $x=0$ is used as a well-defined mathematical object on the one hand, but must also be encountered as an undefined mathematical object when considered in terms of any pair of its elements, all of which are of the form: $(0, a), a \in \mathbb{R}$.

More generally, the production of a solution to a standard problem requires that one knows both what to do next and how to do what needs to be done. Worked examples clearly reveal much of what needs to be done to solve a problem: for most standard problems the student has merely to perform the same operations in more or less the same order as they appear in a worked example. Unfortunately knowing what to do does not necessarily imply knowing how to do what needs doing, so that having access to worked examples does not necessarily enable the solution of problems. In other words, the how of solving even standard problems demands more than the recognition of what to do.\(^9\)

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\(^9\) For example, knowing that one has to add fractions at a particular point in a problem still requires that one knows how to do so. If one does not, then one cannot proceed any further.
Therefore, if teachers appear to be providing students with a great deal of information on what to do to solve problems, and students still fail, then it is likely that they are not providing sufficient information on how to do so. Minimally, the resources for knowing how to solve problems are the various rules governing the operations that are to be performed in the course of producing a solution. The application of the rules, however, requires something more than knowledge of the rules themselves. First, we need to know which rules to apply (recognition) as well as knowing whether or not our use of the rules is correct. The resource that enables us to monitor our correct use of rules is our knowledge of the mathematical ideas, principles and definitions that function as grounds for those rules.10

We therefore arrive at the conclusion that a fundamental element of appropriate evaluative criteria for school mathematics is knowledge of the supporting ground for the mathematical operations that need to be performed when solving problems. We suspect that it is the operation of evaluative criteria of types excluding the mathematical ground of operations that will effect a reduction in the pace of teaching and learning and also result in a reduction of opportunities-to-learn mathematics for students.

Our problem, finally, becomes one of investigating the extent of the absence, or implicit functioning, of mathematical grounds as elements of evaluative criteria for solution procedures offered to working class students in the five schools, and to reveal the effects of such criteria on the constitution of school mathematics and the thinking of mathematics.

CONCLUDING REMARKS

We suspect that the absence of explicit attention to mathematical grounds in evaluative criteria will produce a series of negative effects that slow pacing because the “mathematics” constituted will be generally inconsistent and hence unstable. We now present the effects we expect to find as hypotheses:

(1) procedures will tend to constitute mathematical notions as effects rather than being grounded by such notions, resulting in inconsistent notions that, therefore, cannot be generalised successfully;

(2) the image of the various solution procedures students are exposed to will function as the chief supporting ground for student work on problems because what “step” to do next according to worked examples will take precedence over the mathematical meaning of expressions if the supporting mathematical ground is absent or merely implicit;

10 For example, if we learn that the so-called “cancellation of terms” is a desirable operation to perform when attempting to simplify algebraic expressions, we must also have some idea of when it is legitimate to do so. That is, we must have some idea of its supporting ground: is it possible operate on two terms legitimately so that the identity element for the operation with respect to the set of elements is produced?
(3) the predominance of the use of the image of a solution of a procedure will predispose some students, possibly many, to treat solutions as iconic resources, in which solutions are produced by attempting to make them look like the worked examples that have been encountered rather than attempting to understand the mathematics;

(4) the lack of stability of the notions constituted as mathematical notions in combination with the use of worked examples as the chief ground supporting the production of solutions, will produce mathematics as highly context dependent, making it difficult for students to transfer knowledge gained in the context of the elaboration of one class of solution procedures as resources when solving problems encountered when considering a different class of procedures.

(5) the overall effect of the absence of explicit attention to mathematical grounds in evaluative criteria result in weak pacing because the resources students use to monitor their work will be inconsistent, unstable and often, non-mathematical.

In the space provided here we could present an analysis of only one instance in sufficient detail, and this is perhaps the appropriate point at which to briefly draw attention to a few more methodological issues.

It may appear strange that we attempt to generate a research problem and hypotheses by way of an analytic discussion that focuses on a single empirical instance. Four methodological points need to be made in that regard.

First, teaching across the five schools observed is very similar. The analysis of the use of time alone indicates that the predominant general object of attention is the worked example without explicit elaboration of the mathematical grounds supporting the procedures applied in worked examples.

Second, the primary object of analysis in our discussion of the single empirical instance is the manner in which mathematics is thought, as indicated in its expression, in the elaboration of worked examples and the effects thereof on the constitution of mathematics.

Third, the particular instance we discussed above was helpful because it might be treated as offering up a parapraxis for analysis, in Freud’s (2001) sense, where parapraxes (slips of the tongue, pen and so forth) are used to reveal the nature of the operation of the unconscious. Our recontextualising of that methodological device from psychoanalysis is motivated by a need to access the principles of pedagogic operation that are not all that visible when things are running along smoothly. Points of breakdown of pedagogy, made to function like Freud’s parapraxes, are exploited to reveal the principles structuring the way mathematics is thought, which provides some insight into the constitution of mathematics in pedagogic contexts. We would, for example, not have been able to gauge much about the way in which mathematics is constituted from the teacher successfully solving a problem like, say, “Find the equation of the line through
the points A(-3, 2) and B(-1, 5)”. The teacher’s procedure would have worked perfectly, but that would have told us little about the way in which mathematics is thought in that particular pedagogic context. At best we would be able to describe teaching and learning in terms of weak descriptive categories, like “teacher-centred” and “learner-centred”, that tell us something about the surface features of pedagogic relations but little of any depth about what is being constituted as mathematics and how it is being constituted.

Fourth, once we are able to generate a robust account of how mathematics is thought in the pedagogic contexts of the five schools we will be able to describe and analyse teaching and learning where things are apparently running smoothly in ways that would not have been possible without an account of how mathematics is thought.

REFERENCES


In this article I intend to share my teaching experience of a particular concept related to Analytic geometry at Pedagogical University in Beira, Mozambique. In 2005, I faced a real classroom situation when I was teaching the concept of lines’ relative position in space to my first year mathematics students pursuing a degree in secondary mathematics teacher education. I was surprised when in the first test the majority of the students were unable to respond correctly to the “easiest” task of the test. Before the correction of the test we worked together on the same task again, as I noticed that there were many conceptual misunderstandings on the above mentioned concept. This situation was very interesting to me, particularly as I had previously considered the concept of lines relative position as one of the easiest tasks of the test.

INTRODUCTION

Geometry is one of the topics widely discussed in Mozambican schools. Discussions around the teaching of secondary school mathematics in Mozambique are common (Revista de Educação Matemática, 2006). These discussions often involve school teachers during their lesson plans or between teachers and students within mathematics lessons. In Mozambique, students start learning simple geometrical concepts at a very early stage, that is, at primary schooling (Grade 1-7), but it is only at Grade 10 (16 – year - olds) that spatial geometry is introduced (Nheze & João, 1998). In Grade 10, many difficulties in understanding and handling spatial geometry concepts among teachers and students emerge, as it was acknowledged by the Project named Support to Teacher Training Programme (STTP, 2005). An inquiry undertaken by STTP on secondary mathematics teachers throughout the country revealed that teachers believed that Spatial Geometry was one of the most difficult topics to teach among others, such as probability and trigonometry. The inquiry was led by university lecturers and had the purpose to help in producing didactical materials to support secondary school mathematics teachers to ensure quality teaching and learning in Mozambican schools.

Difficulties in understanding geometrical concepts are not confined only to secondary school level but they can also be found in several university courses in Mozambique. For instance, university students have many difficulties in interpreting geometrical situations when these are represented graphically in a three-dimensional system. I observed this from my own experience in the geometrical teaching in the Pedagogical University. And from this experience, I learned that the visualization using concrete
material in the geometrical lesson is something very important and helps the student to understand the contents. Many researchers, such as Konyalioglu and others cited by him, refer to the advantages of the visualization:

Graph, diagram, pictures and geometrical shape or models are a tool for visualization of the abstract concept in mathematics (Konyalioglu 2003). It can be considered concepts such as geometric structures and mathematical-physical models for meaningful teaching mathematics. Also, mathematical concepts are abstract that one needs cognitive achievements to assimilate them (Baki 2000). By using visualization approach many mathematical concepts can become concrete and clear for students to understand. The term visualization is used in different meaning between mathematics educators. It is used in this paper as it was defined by Zazkis, Dubinsky and Dautermann (1996), that is, as an act in which an individual establishes a strong connection between an internal construct and something to which access is gained through the sense. Such a connection can be made in two directions. An act of visualization may consist of any mental construction of object or processes that have an external source. Alternatively, it may consist of the construction, on some external medium such as paper, of objects or events. Consequently, the act of visualization is translation from external to mental. Visualization can be alternative method and powerful resource for students doing mathematics (Konyalioglu et al. 2003). The use of the visualization approach enables students to think of the mathematics course, which is usually seen as an accumulation of abstract structures and concepts from a different perspective.

The problem

In the fall of the second university semester in 2005, I gave to my first-year mathematics students the following mathematical task:

<table>
<thead>
<tr>
<th>Let $r$ and $s$ be the equations of two lines in a three dimensional space:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$: $x = 2 + t \land y = -3 \land z = 4$</td>
</tr>
<tr>
<td>$s$: $x = 2 \land y = -3 + t \land z = 4 + t$</td>
</tr>
</tbody>
</table>

a) Represent $r$ and $s$ graphically.

(i) First represent each line in a separate Cartesian system.

(ii) Second represent both lines in a single Cartesian system.

b) What is the relative position of $r$ and $s$?

(This exercise was a part of other tasks included in the first test, 2005)
DESCRIPTION OF THE TEACHING EXPERIENCE

To begin the lesson, I asked the students to resolve the task again. After some a while, the students started to solve the task. They actually solved the same way they did during the test. For the first part of the test, all students presented their graphs. There were small differences in the graphic displays due to differences in the axes angle positioning within the Cartesian system. It was not a problem to me as I was more concerned with the interpretation of the graphs. The second part of the task dealt with the relative position of the two lines, $r$ and $s$, and the table below presents the results.

<table>
<thead>
<tr>
<th>Response</th>
<th>r and s are Coincident</th>
<th>r and s are Oblique</th>
<th>r and s are Crossed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of students</td>
<td>14</td>
<td>16</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 1: Responses to relative position between lines $r$ and $s$ (N = 35)

The students themselves explained the meaning of the responses as follow: By “coincident” they meant that $r$ and $s$ had an infinite number of common points. By “oblique” they meant that $r$ and $s$ had only one common point and the angle between them was not 90 degrees. By “crossed” they meant that $r$ and $s$ were located in different planes and were definitely not parallel lines.

Here are some examples of the students’ responses which are representative of the entire cohort.

(i) $r$ and $s$ are coincident
(ii) r and s are oblique lines

(iii) r and s are crossed lines

When I asked the students to justify their answers, they replied: “r and s are coincident because it is visible from the graph” or “r and s are oblique because the graph shows that”. Other group of students said “r and s are crossed because we can see from the graph that they cross each other though they are in the space”. This kind of responses illustrate that the students based their responses solely on the graphs and did not consider that the lines were projections. This kind of representation can be cognitively troublesome because a projection of a line on the plane can result on a single point or a projection of two perpendicular lines on the plane can result on a single line.
In what follows, I discussed two more tasks related to the previous activity not included in the test.

**Given t = 0 and t = 1 determine the points R₁ and R₂ on the line r.**

**Given t = 0 and t = 1 determine the points S₁ and S₂ on the line s.**

**Based on the points R₁ and R₂ find a vector of the same direction as the line r.**

**Based on the points S₁ and S₂ find a vector of the same direction as the line s.**

This activity was successfully solved by all students, that is, they were able to find a vector of the same direction as the lines r and s using the given points. The common strategy of solving the problem was the following:

Taking t = 0, the students worked out the coordinates of the points: \( R₁(2; -3; 4) \) and \( S₁(2; -3; 4) \). Taking t = 1, they worked out the coordinates of the other two points: \( R₂(3; -3; 4) \) and \( S₂(2; -2; 5) \). Then, I asked the students what they were observing and they immediately replied: “The points \( R₁ \) and \( S₁ \) are coincident because they have common coordinates”. Some of them added: “And \( R₂ \) is not on the line s”. From this observation, the students concluded that r and s were not running on the same pathway, so the lines were definitely distinct. The students also observed that, because \( R₁(2; -3; 4) \) and \( S₁(2; -3; 4) \) are equals, then, the lines r and s intersect each other. This observation led to the conclusion that the two lines were not crossing in the space.

Analyzing the vectors of the same direction as the lines r and s, the students observed that r and s were, in fact, running in different directions. I asked the students to try to draw more conclusions about the lines based on the vector calculations made before but they could not go further.

The following was the last task I gave to students:

**Find the scalar product between r and s \((r \cdot s)\). What is your conclusion?**

All students calculated the scalar product correctly. They did this:

\[ r \cdot s = (1; 0; 0) \cdot (0; 0; 1) = 0! \]

At the same time they started to whisper in small groups “…the lines are perpendicular because the scalar product is zero”, but they were still seeing some kind of contradiction between the calculations and the graphical display. What then should they believe?

From this point the discussing between me (Teacher) and the students was as follow:

Teacher: What comments do you make based on these results? (Scalar product equal to zero).

Student A: It seems that there is a contradiction between what I see on the graph and the calculations.
Teacher: What causes the contradiction?

Student B: Perhaps there is a problem with the axes position. For example, when I draw the Cartesian system only two axes are perpendicular and the third axis is not perpendicular to the other two.

Student C: This is so, because we are attempting to draw lines on a 2D plane while they are actually on a 3D space. It would be much better if we could illustrate this phenomenon using our pens. It is like on a photograph!

The classroom discussion was intense, some students were moving around trying to show what was going on using pens, pencils, books, pieces of chalk and every long object they found inside the classroom. Towards the end of the lesson, I called for five students to the blackboard. One was holding two pens crossed under an angle of approximately 90 degrees. I positioned the other four students in different positions, all of them looking at the two pens. Afterwards I asked them to draw on the blackboard what they had pictured. There were four different drawings reflecting different views on the blackboard. After this, I located the position were the two pens could be seen under an angle of 90 degrees, and asked the students to stand up in that same position and have a look on the pens. After that, I asked them to produce their drawings corresponding to this new view (perpendicular lines). At that time, the students realized that the projection of lines on the plane sometimes could produce misleading results.

**HOW CAN TECHNOLOGY HELP IN UNDERSTANDING THIS PROBLEM?**

Technology has an important application in mathematics teaching. According to the National Council of Teachers of Mathematics (NCTM), technology offer options for students with special needs. Some students may benefit from the more constrained and engaging tasks situations possible with computers. Students with physical challenges can become much more engaged in mathematics using special technologies. Technology cannot replace mathematics teachers, nor can it be used as a replacement for basic understandings and intuitions. The teacher must make prudent decisions about when and how to use technology and should ensure that technology is enhancing students' mathematical thinking.

Certainly, in the issue of this article, using specific computer software is possible to show to students what happens when we change the position from where we see an axis system. But I do not mean that the use of computer software could replace the traditional use of manipulative materials, because even if software is used, a bit of abstraction is still required in order to understand the problem effectively. Although in Mozambican schools computers are few, at university level there are quite a number of them. Despite the abstraction mentioned above, the use of computers motivates the students to learn Geometry.

Valente (1999 in Dullius, Eidelwein, Fick, Haetinger and Quartieri, 2000) state that the use of computers in education allows the integration in the learning process of curricular
concepts in all modalities and levels of teaching, acting as facilitators between the student and the construction of his knowledge. Valente suggests the necessity of the teachers to consider the potential of computers and attempt to adjust the traditional teaching and learning activities to the reality of the computers.

What I showed to my students is well included in the thought of Valente. For example, I have showed the students that what we did together using pens inside the classroom could be seen in a computer screen. I showed them some very interesting results in a computer programme (MATLAB) by changing the position of the axes system (doing rotation – one of the possibilities that the MATLAB offers) as the following:

I drew the two lines in the three-dimensional system using MATLAB software and with the option “rotation” of axes I obtained the different configurations below (figure 1, figure 2 and figure 3). In this way, I introduced the equations of the two lines like this:

```matlab
t=linspace(0,1);x=t+2;y=0*t-3;z=0*t+4;h=plot3(x,y,z);axis('square');hold on;
set(gca,'XLim',[0 5]);set(gca,'YLim',[-5 0]);set(gca,'ZLim',[0 5]);grid on;
x2=0*t+2;y2=t-3;z2=t+4;h(2)=plot3(x2,y2,z2);
```

Figure 1 below shows the two lines in the same coordinate system. So, it is visible that from where we stand and observe the system, the projection of the lines results on a single line. Figure 2 shows the two lines making an angle different from zero and 90 degrees. Figure 3 shows that the lines are making an angle of 90 degrees. In this case, we can imagine that we are seeing the two lines from the top to the bottom.

Figure 1
CONCLUSIONS AND RECOMMENDATIONS

The teaching experience reported in this paper, illustrates that, while learning spatial geometry, the students tend not to confront their analytical calculations with the graphical display that they produce. Even in case that a correct calculation has been performed, decision making is sometimes problematic. The students have difficulties in understanding that lines drawn on the plane represent lines in the space. It is important that the students perceive that the position of the lines in the space cannot be easily grasped from the simple projection on the plane and thus, some analytical calculations are called for to support or reject visual representations.

For the teachers it is paramount to associate the manipulative materials with computer software while teaching spatial geometry concepts. That would, perhaps, increase the level of understanding and motivation of students in mathematics classroom.

REFERENCES


Support to Teacher Training Programme documents (2005). Universidade Pedagógica, Moçambique
This paper focuses on a didactical model that uses base ten decomposition on naturals as cognitive architecture across the divide that exists between arithmetic and algebraic long division. The base ten decomposition as an algorithm for performing division emulates the structure of algebraic expressions and will assist in making the transition to variable seamless. Due to page restrictions the authors will discuss the didactical model, and reflect on key areas of response from the grade 8 learners.

Decomposing naturals into a power relationship in base ten has benefits when teaching long division to learners at all levels. This type of long division has not been sufficiently explored before as an algorithm and it has valuable potential for teaching transition mathematics where the main aim is bridging the cognitive gap (Herscovics and Linchevski, 1994) that exists between arithmetic and algebra, especially as far as division is concerned. It forms the genesis of algebraic division by using the properties of arithmetic, with the algebra not residing in the problem but rather in the thinking. Learning long division in base ten decomposition raises the bar for division on naturals, well above a highly procedural method, and ensures that new knowledge is built from a soundly established former knowledge base.

Firstly, base ten decomposition ensures that place value is fully preserved and in fact enhanced in the division procedure. The critics of the long division procedure argue for its exclusion from the curriculum on the grounds that it un-teaches place value and, amongst other objections ‘encourage(s) learners to give up their own thinking’ (Kamii and Dominick argue that un-teaching place value inhibits the development of number sense. Secondly, base ten decomposition also provides a strong rationale for performing algebraic long division synthetically or by using the method of detached coefficients whilst not devaluing the important conceptual issues with which learners have to engage when performing long division. Base ten decomposition successfully emulates the algebraic division procedure without the variable being present. Instead, the expression in base ten shows to learners that the ‘ten’ is only an indication of the place value of the digit in the number. The ten can thus be ignored once the basic skill of reducing its power every time the division shifts to the next digit is learnt. As this ten can be replaced later by any positive integer, its evolution toward the variable becomes seamless.
Lastly, base ten decomposition changes the cognitive architecture of division, and specifically long division, by creating a clear and yet challenging increase in the cognitive load involved in division on naturals. It maps the division on naturals more clearly and directly onto algebraic long division by offering teachers a new method that can be used in the transition through the cognitive gap (Herscovics and Linchevski, 1994) that exists between arithmetic and algebra. The process of increasing the cognitive load should begin at the intermediate phase, and should gradually increase as learners are introduced to the structure in arithmetic expressions by exploring basic mathematical principles in a more general manner.

De-emphasising the long division procedure in mathematics classroom interaction is not a wise decision as it has multitudinous and diverse effects on mathematics for high school learners, as well as for tertiary learning across various fields of study. It remains a powerful conceptual tool that unpacks meaning and builds conceptual frames without which areas within mathematics cannot be explored concisely and on a deeply conceptual level. I stand in agreement with Wu (1999) when he says “...why not consider the alternative approach of teaching these algorithms properly before advocating their banishment from classrooms...” (p4)? If we advocate the de-emphasis of the long division procedure from our classrooms, it will surely result in superficial understanding of highly conceptual tools in the mathematics curriculum. In the United States fierce opposition was expressed to the decision to de-emphasise long division in the school curriculum, and I strongly believe that we should listen to these petitions and take heed of what is being argued in order to avoid a repetition in the South African context.

This relationship between basic skills and conceptual proficiency is important to establish when teaching for understanding. Hiebert and Lefevre (1986) offer a more comprehensive discussion and analysis of the two types of knowledges. They view conceptual knowledge and procedural knowledge as embedded in the following characteristics:

<table>
<thead>
<tr>
<th>Conceptual Knowledge</th>
<th>Procedural Knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Knowledge that is rich in relationships</td>
<td>Made up of two distinct parts:</td>
</tr>
<tr>
<td>b. Connected web of knowledge</td>
<td>a) Formal language or symbol representation system of mathematics. This is often called the <em>form</em> of maths. Knowledge of the form of math includes knowledge of the syntactic configurations of formal proofs. This implies knowledge of the surface features, and not knowledge of meaning.</td>
</tr>
<tr>
<td>c. A network in which the linking relationships are as prominent as the discrete pieces of information</td>
<td>b) Algorithms, or rules, or procedures for completing mathematical tasks. They are step by step instructions that prescribe how to complete the task. A key feature is that this procedure is executed in a predetermined linear sequence.</td>
</tr>
<tr>
<td>d. Knowledge is conceptual if the holder recognises its relationships to other pieces of information</td>
<td></td>
</tr>
</tbody>
</table>
According to Hiebert and Lefevre an important feature of the procedural system is that it is structured. Procedures are hierarchically arranged so that some are embedded in others as sub-procedures. An entire sequence of step by step prescriptions or subprocedures can be characterized as a superprocedure. In the domain of mathematics, the division procedure takes on both these procedural characteristics.

THE CURRENT DEBATE - A SYMBIOSIS

The work done by Bodanskii (1991) strongly suggests that it is time to seriously consider significant changes in the primary mathematics curriculum. Bodanskii proved that algebraic thinking, embedded within the usual activities of arithmetic, is possible to teach at the beginning of the basic course of education. Carraher, Brizuela and Schliemann (2000) hold the view that the instructional sequence of arithmetic first and then algebra, lends itself to discontinuities between arithmetic and algebra. They point out that there is currently a large gap between arithmetic and algebra, but then ask the question as to the legitimacy of the existence of such a gap. Herscovics and Linchevski (1994) refer to a cognitive gap that exists between arithmetic and algebra as a reason for learners’ difficulties in mathematics. They specifically refer to the learners’ “inability to operate spontaneously with or on the unknown” (p.59).

Warren (2004) adds to this debate by pointing out that early algebra relies heavily on arithmetic. She advocates that the teaching of algebraic thinking with arithmetic thinking can enhance further learning in formal algebra. Warren bases her argument on the fact that learners experience difficulties moving from the arithmetic world to the algebraic world which seem to stem from a lack of appropriate foundation in arithmetic. This view is in line with Slavit (1999) who places the lack of true understanding of operations on the rough transition from arithmetic to algebra. One thing which is certain and is reflected in the work by Kaput and Blanton (2001) is that the most pressing factor for arithmetic algebraic reform is the ability of elementary teachers to ‘algebrafy’ arithmetic. This would imply that teachers need to develop in their learners the arithmetic underpinnings of algebra (Warren & Cooper, 2001). This journey will reach full circle when these underpinnings extend to the beginnings of algebraic reasoning (Carpenter & Franke, 2001).

DEFINING THE COGNITIVE GAP: THE FOCI OF RESEARCHERS

In the endeavour to bridge the divide between arithmetic and algebra, various researchers have described this journey in a variety of ways. A large number see the crossing of this great divide between algebra and arithmetic as a transition process which they have termed as ‘transition mathematics’. It would appear that there are two main foci in this debate: an integrated arithmetic curriculum which aims at extracting algebraic thought and reasoning in the early grades by the algebrafication of arithmetic and a second focus which emphasizes transitional processes.
The role of operation sense has been investigated by Slavit (1999) in the transition from arithmetic to algebraic thought. The role of arithmetic structure in this transition period is the focus of the work of Elizabeth Warren (2003, 2004). This focus is shared by Subramaniam and Banerjee (2004, 2005) as they argue for the development of a procedure and structure sense of arithmetic expressions to assist learners’ growing understandings of arithmetic expressions and beginner algebra. Warren (2004) has particularly pointed to the difficulties that learners experience in moving from the arithmetic world to an algebraic world and she explicitly attributes this difficulty to a lack of appropriate foundation in arithmetic. This claim is supported by the work done by Sfard (1991) who pointed out that the core of learners’ difficulties with algebra and algebraic symbols lie at their inability to conceive the operational-structural duality of algebraic symbols. Gray and Tall (1994) focused on the role of mathematical symbolism within this process and introduced the idea of procept and proceptual thinking as part of this transition between the operational (arithmetic) and the structural (algebra). Here symbolism represents either a process to do or a concept to know.

**COGNITIVE ARCHITECTURE: VARIOUS APPROACHES**

Blanton and Kaput (2005) proceeded to distinguish different categories of algebraic reasoning in a grade 3 classroom as generalized arithmetic, functional relationship, properties of numbers and operations and the algebraic treatment of number. Throughout the research that has been conducted over many years, two main themes emerge very strongly: that of structural thinking and, secondly, the focus on functional thinking.

**a) Structural thinking:**

The transformation to algebra thus involves not only the mastery of algebraic rules, but relies heavily on analyzing structures within expressions (Yerushalmy, 1992). For Bell (1995 in Subramaniam and Banerjee, 2004) an appreciation of the structure of arithmetic expressions is useful for understanding algebraic expressions. Bell argues that arithmetic expressions stand for number and thus focus on the value of an expression. Subramaniam and Banerjee (2004, 2005) make it very clear that algebra requires understanding of the threefold meaning of arithmetic expressions (process, product and relation) and that equality of expressions is the core notation around which structure of expressions can be grasped.

**b) Functional thinking:**

Carraher, Schliemann and Brizuela (2000, 2001) believe that learners’ understanding of additive structures and functional notation provides the fruitful point of departure for algebraic arithmetic. Carraher, Schliemann, Brizuela and Earnest (2006) propose that giving functions a major role in the primary school mathematics curriculum will help facilitate the integration of algebra into the existing curriculum (p.2). Warren and
Cooper (2005) argue that fundamental to relation and transformation is the concept of the function, an exploration into how the value of certain quantities relates to the value of other quantities and how values are changed or mapped to other quantities.

**BASE TEN DECOMPOSITION AS A DIDACTICAL MODEL FOR DIVISION**

Research has increasingly shown that utilizing the potentially algebraic nature of arithmetic is one way of building a stronger bridge between early arithmetical experiences and the concept of a variable (Kirshner, 1989; Kaput, 1985; Kieran, 1990; Subramaniam and Banerjee, 2004; Carraher et al, 2006; Sfard, 1991). There is a huge gap between where primary school stops in the instruction in division and where secondary school takes off. Finding a cognitive root that builds toward algebraic long division is important to lay the foundation on which exploration and new meaningful learning will be solidly rooted in the procedures and concepts that the learners are familiar with. This is followed by a building in difficulty and cognition from this base line. It is also the introductory step towards abstraction, which later prepares for the introduction of the variable into the division procedure - aiming towards a better conceptualization of what polynomial division is all about. A more fitting description would be to refer to this procedure as an emulation of algebraic long division. Lady (2000) warns that no matter how useful the language of mathematics may be, it does not represent the way in which learners develop understanding of concepts. For this reason, procedures with which learners may be familiar, such as arithmetic long division on naturals, may not be as familiar a process when described in an exact mathematical way, especially when abstraction is involved. Lady states that “the mathematics of the twentieth century tends to place emphasis on the rules by which entities behave, rather than on the way they were constructed”(p.4). Fujii (2003) maintains that by “utilizing the potentially algebraic nature of arithmetic, one forms a way of building a stronger bridge between early arithmetical experiences and the concept of a variable” (p.1). By breaking up the numbers (dividends) as well as the divisors into powers of ten, I will attempt to make the seemingly easy procedure of arithmetic long division more complicated for the learners, which will then hopefully transpire into a view that polynomial long division is actually easy. Here my idea is to separate what Skemp (1976) refers to as “instrumental understanding” and “relational understanding”. Learners have a sound instrumental understanding about the process of arithmetic long division, but they do not necessarily understand why they are doing the same when performing algebraic long division. The understanding is thus shallow as Fujii (2003) and Wu (2000) argue. Aiming at relational understanding, I will challenge the learners with higher cognitive division exercises through base ten decomposition division, in order to create this smooth transition toward algebraic long division through successful cognition.

Alongside is a graphical interpretation of what will be done to fill the gap between primary school division and secondary school needs.
Step 1 – Focusing on multiple division procedures that enhance the concept of place value:

Multiple methods of division will develop more understandable alternative ways of exploring the concept of long division on naturals, based on the learners’ division experience in arithmetic long division since primary school. This will offer teachers an alternative to introducing a seemingly difficult area of study by using the principles of metacognition that propose that one must build on existing knowledge in order to acquire new knowledge. Multiple division procedures that enhance place value differently within each method will ensure that the cognitive schema is built on a more solid foundation than just the procedure itself.

Division in base ten decomposition
Expressing divisors as
$10 \pm p$
Expressing dividends as a multiple in a power relation of ten.
Performing the long division

The Division Algorithm
$a \equiv b \cdot q + r$
where $0 \leq r < b$

This focus happens in step 1 and forms the conclusion of each division procedure.

Division that enhances the concept of place value
Multiple representations:
a) Removing multiples of the divisor from the dividend
b) Multiple division with remainder referred to as the 'reset method'

Preparatory School division stops here:

\[
\begin{align*}
5 \text{rem} 3 \\
7) & \quad 38  \\
& \quad 35  \\
& \quad 3
\end{align*}
\]

Secondary School division level for polynomials
Polynomial division and the division algorithm for polynomials
Division using the knowns (coefficients) only.

Polynomial Algebra
Considering the division of 256 by 7:

A) By Arithmetic long division:

\[
\begin{array}{l}
\begin{array}{c|cccc} \\
7 & 256 & \cdot & 7 & = \\
 \hline \\
 0 & 25 & \cdot & 7 & = \\
 21 & 46 & \cdot & 7 & = \\
 42 & & & & \\
 4 & & & & \\
\end{array}
\end{array}
\]

\[
\therefore 256 = 7(36) + 4
\]

B) By removing multiples of the divisor:

\[
\begin{align*}
256 &= 140 + 70 + 42 + 4 \\
&= 7 \cdot 20 + 7 \cdot 10 + 7 \cdot 6 + 4 \\
&= 7(20 + 10 + 6) + 4 \\
&= 7(36) + 4
\end{align*}
\]

C) Multiple divisions with remainder (the reset method):

Procedurally the reset method could be compacted as follows:

\[
\begin{align*}
2 &= 0 \cdot 7 + 2 \rightarrow \text{remainder } 2 \cdot 10 + 5 = 25 \\
25 &= 3 \cdot 7 + 4 \rightarrow \text{remainder } 4 \cdot 10 + 6 = 46 \\
46 &= 6 \cdot 7 + 4
\end{align*}
\]

For the division algorithm:

As Kieran (1990, 1992) and Sfard and Linchevski (1994) warn, that learners find procedural interpretations more accessible than an approach which emphasises the structural character of the operation, this more procedural approach to the division will be adopted to enhance learners’ cognition of the reset method. The reset method, although explained from a sound conceptual frame that enhances place value, would eventually evolve into a procedure with which the learners should be comfortable,
within a very short span of time. The underlying conceptual structure of this method focuses on the remainder and its value within the multiple division with remainder procedure.

**Step 2 – The division algorithm**

It is of utmost importance that the division algorithm be integrated in the first step of the division ladder when summarising each of the division procedures with which the learners will engage. The division algorithm provides the equalising factor to the different division procedures and learners will soon realise that it provides a check point for the correctness of their division procedures. When dividing arithmetically it will only be necessary to compare remainders against one another to determine whether the division procedures have been applied flawlessly. The division algorithm focus of the division ladder is aimed at emphasising and exploring the conditions set on the remainder and the divisor within the algorithm. This will be done in preparation for the base ten decomposition division when the remainders will at times be different to those obtained during arithmetic long division. This will later result in a merger of the two division methods which were dealt with in step 1 and step 3. Without placing such an emphasis on the division algorithm and the conditions on the divisor and the remainder, learners would not experience this application explicitly.

**Step 3 – Base ten decomposition of the divisor and the dividend**

The first two steps of the division ladder can be thought of as preparatory in laying the foundation for abstraction. The third step is building the bridge that will cross the cognitive gap between arithmetic long division on naturals and algebraic long division on polynomials. This step of the division ladder will ensure that algebraic long division is not just presented to the learners as a ready-made object where the role of the variable and the steps are fixed within a rigid framework of conventions. Base ten decomposition is a conceptual tool which will evolve to algebraic division as a product of the sequence of division activities, and can be effectively used on two levels. The first level is imitating the algebraic procedure and, therefore, directing learner thinking to build conceptual discourses that will order their thinking away from number, to the structure in the numbers. This structural aspect of arithmetic and its usefulness in the division procedures of arithmetic has not been explored sufficiently in the past.

This process is to be explored in a similar manner as the following example points out:
Considering the division \(138 \div 11\):

\[
\begin{array}{c|c}
12 \\
\hline
11 & 138 \\
\hline
\quad 11 \\
\quad 28 \\
\quad \underline{22} \\
\quad 6 \\
\end{array}
\]

By the division algorithm we conclude that \(138 \equiv 11 \cdot 12 + 6\).

Decomposing the dividend 138 in base ten: \(138 = 100 + 30 + 8 = 1 \cdot 10^2 + 3 \cdot 10^1 + 8 \cdot 10^0\)

and the divisor in terms of 10 will be expressed as \(11 = 10 + 1\).

So we can write:

\[
\begin{array}{c|c}
10 + 1 & 1 \cdot 10^2 + 3 \cdot 10^1 + 8 \\
\hline
\quad 1 \cdot 10^2 + 1 \cdot 10 \\
\quad 2 \cdot 10 + 8 \\
\quad 2 \cdot 10 + 2 \\
\quad 6 \\
\end{array}
\]

By the division algorithm: \(138 \equiv (10 + 1)(10 + 2) + 6 \equiv 11 \cdot 12 + 6\).

In the final step the 10 will be replaced by the variable \(x\) and the division will be repeated alongside the division in base ten decomposition to finally reach algebraic long division.

So the process will conclude with:

\[
138 = 1 \cdot 10^2 + 3 \cdot 10^1 + 8 \cdot 10^0 \rightarrow x^2 + 3x + 8
\]

\[
11 = 10 + 1 \rightarrow x + 1
\]

\[
\begin{array}{c|c}
x + 1 & x^2 + 3x + 8 \\
\hline
x + x^2 + x + 8 \\
\underline{2x + 8} \\
\underline{2x + 2} \\
6 \\
\end{array}
\]

\[\therefore x^2 + 3x + 8 \equiv (x + 1)(x + 2) + 6\]

These steps that are suggested in the division ladder are by no means claiming to be the solution to the division problems that learners experience when they engage with long division on polynomials, but it does provide a sound didactical model which could be followed to bridge the cognitive gap that exists between division on naturals and polynomial division. This gap could well create the cognitive chaos that some learners experience when attempting to unravel the conceptual building blocks of algebraic division. For these learners algebraic division is a replication of what the teacher said and did on the board, and often they cannot apply this highly procedural approach to division to more complicated problems.
Once the algebraic long division on naturals has been explored by the learners, the cognitive shift will be made horizontally to introduce the division by detached coefficients.

The instruction should then take the form of simply erasing both the tens and the \( x \) from each procedure, to look as follows:

\[
\begin{align*}
329 &= 3 \cdot 10^2 + 2 \cdot 10 + 9 \\
5 &= 1 \cdot 10 - 5 \\
(1 - 5)3 &= 2 + 9 \\
3 &= 15 \\
17 &= 9 \\
17 &= 85 \\
94 &= \\

329 &= 3x^2 + 2x + 9 \\
5 &= x - 5 \\
(1 - 5)3 &= 2 + 9 \\
3 &= 15 \\
17 &= 9 \\
17 &= 85 \\
94 &=
\end{align*}
\]

This will clearly point to the learners that the ten and the \( x \) take on an irrelevant role during the long division procedure, and that the roles of these two place holders only become relevant when formulating the division algorithm. This method will enhance the conditions embodied in the algebraic division algorithm for polynomial division where the emphasis shifts to the degree of the remainder and the divisor.

To conclude this chapter, the full division procedure will be given from step one to step three using the problem \( 329 \div 5 \)

**Step 1 and 2:**

---

**Arithmetic division:**

\[
\begin{align*}
065 \\
5 &\overline{)329} \\
0 & \\
32 \\
30 & \\
29 & \\
25 & \\
4 & \\
\therefore 329 &= 5(65) + 4
\end{align*}
\]

---

**Removing multiples of the divisor:**

\[
\begin{align*}
329 &= 300 + 20 + 9 = 60 \cdot 5 + 4 \cdot 5 + 5 + 4 \\
&= 5(60 + 4 + 1) + 4 \\
&= 5(65) + 4
\end{align*}
\]

---

**Reset Method:**

\[
\begin{align*}
3 &= 0 \cdot 5 + 3 \rightarrow \text{rem} \, 30 + 2 = 32 \\
32 &= 6 \cdot 5 + 2 \rightarrow \text{rem} \, 20 + 9 = 29 \\
29 &= 5 \cdot 5 + 4 \\
\therefore 329 &= 5(65) + 4
\end{align*}
\]

---

155
Step 3: Base ten decomposition

\[ 329 = 3 \cdot 10^2 + 2 \cdot 10 + 9 \]
\[ 5 = 1 \cdot 10 - 5 \]
\[
\begin{align*}
3 \cdot 10 + 17 \\
(1 \cdot 10 - 5)3 \cdot 10^2 + 2 \cdot 10 + 9 \\
3 \cdot 10^2 - 15 \cdot 10 \\
17 \cdot 10 + 9 \\
17 \cdot 10 - 85 \\
94
\end{align*}
\]
\[
\therefore 3 \cdot 10^2 + 2 \cdot 10 + 9 = (3 \cdot 10 + 17) + 94 \quad \text{remainder differs}
\]
\[
\therefore 3 \cdot 10^2 + 2 \cdot 10 + 9 = (5)(47) + 50 + 40 + 4 \quad \text{division continues}
\]
\[
\therefore 3 \cdot 10^2 + 2 \cdot 10 + 9 = (5)(47) + 10 \cdot 5 + 8 \cdot 5 + 4
\]
\[
\therefore 3 \cdot 10^2 + 2 \cdot 10 + 9 = (5)(47) + 5(10 + 8) + 4
\]
\[
\therefore 3 \cdot 10^2 + 2 \cdot 10 + 9 = (5)(47 + 18) + 4
\]
\[
\therefore 3 \cdot 10^2 + 2 \cdot 10 + 9 = (5)(65) + 4
\]

Algebraic long division

\[
329 \rightarrow 3x^2 + 2x + 9
\]
\[
5 \rightarrow x - 5
\]
\[
\begin{align*}
3x + 17 \\
(x - 5)3x^2 + 2x + 9 \\
3x^2 - 15x \\
17x + 9 \\
17x - 85 \\
94
\end{align*}
\]
\[
\therefore 3x^2 + 2x + 9 = (x - 5)(3x + 17) + 94
\]

Division by detached coefficients:

\[
329 \rightarrow 3x^2 + 2x + 9
\]
\[
5 \rightarrow x - 5
\]
\[
\begin{align*}
3 + 17 \\
(1 - 5)3 + 2 + 9 \\
3 - 15 \\
17 + 9 \\
17 - 85 \\
94
\end{align*}
\]
\[
\therefore 3x^2 + 2x + 9 = (x - 5)(3x + 17) + 94
\]

REFLECTIONS AND CONCLUDING REMARKS

Investigating base ten division as an algorithm contributes in preparing learners for the transition across the cognitive gap that exists between arithmetic long division on naturals and algebraic long division on polynomials. In this study the learners reflected on this experience in a positive manner pointing out that base ten division was ‘full of numbers’ that made it appear more difficult. Their response to algebraic division on polynomials was positive and they repeatedly pointed out that ‘this is much easier’ (referring to algebraic division on polynomials) as it does not contain ‘so many numbers’. The reason for this was that the power relationships in base ten were substituted with the variable \(x\) and learners now had to focus mainly on the coefficients of the \(x\)’s to perform the division. Previously the focus was on two aspects of the decomposed number: i) the power relation of ten which emphasised place value in the
division procedure and ii) the coefficients of these powers which were used to perform the actual division. Raising the cognitive load for arithmetic long division by considering long division in base ten decomposition appeared to have had the desired effect on their conceptual basis for division on naturals. The effect of increasing the cognitive load was twofold. Firstly it challenged learners to deal with more than just the digits within the number (dividend) by shifting their focus to the specific place value of each digit and secondly it emulated the algebraic process with which they later divided polynomials into one another. Traditionally this introduction to algebraic long division was not rooted in the cognitive grounds of arithmetic long division, but merely glanced at across the cognitive divide in terms of similarity of procedure. This superficial reference to arithmetic long division could be creating some of the problems experienced by learners when engaging with algebraic long division for the first time as the two processes are currently vastly different in their structure (variable) and their focus (coefficients). The cognitive architecture that base ten decomposition provides as a conceptual building block, structures the two procedures closer by a process of emulation.

One of the most challenging situations arose when learners were required to complete the traditional long division on naturals alongside the base ten decomposition division procedure. Throughout the different stages of the division lessons, it was emphasised that when learners summarise their findings, the division algorithm provides a balancing tool to which all different representations converge. Different remainders were arrived at when learners performed the division, and this created conflicting outcomes for the division procedure. It provided me with an unexpected opportunity to highlight to the learners yet again the special conditions set on the remainder and the divisor within the division algorithm. The outcome of these discussions was measured in the transition to symbol, where the learners soon realised that the division does not continue beyond the remainder as the variable which was introduced can now take on any value and is unknown to them. It was clear to me that, for this group of learners, the concept of the remainder across this transition from number to symbol was preserved and enriched by the conflict that arose when different remainders were obtained. This conflict also served to emphasise the transition from number to variable – thus from known to unknown. Accompanied by the manner in which this transition affects the division procedure in the two domains. This break in conceptual continuity and flow will not be possible to negotiate constructively without base ten decomposition division on naturals, as the cognitive leap is too vast without the proper didactical sequencing of division. It is more probable that the issues of the different remainders that are obtained by arithmetic- and algebraic long division will not emerge naturally, let alone be discussed on a deeply conceptual level, without forcing such a discussion on a superficial level. Throughout the didactical model, the division algorithm takes central position within the division procedures that were presented to the learners as alternatives to arithmetic long division on naturals. The division algorithm was also used as a tool that can be operated upon within the clearly defined parameters as encapsulated in its definition. This continuously reminded the learners that these parameters must be taken seriously. These precisely
defined parameters formed the bedrock on which their division attempts rested. They experienced the remainder as more than just “what is left over” and could judge the division process purely on the value of such a remainder.

REFERENCES


BETWEEN UNDERSTANDING THE LANGUAGE AND UNDERSTANDING THE MATHS? TRANSLATING FROM WRITTEN TO SYMBOLIC FORM IN A MULTILINGUAL ALGEBRA CLASSROOM

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This paper investigates how a teacher in a multilingual classroom supports learners who are struggling to translate written/verbal mathematics into symbolic form. 36 multilingual grade 10 learners in one school (in South Africa) were given a written test involving one question and then a discussion on the solution ensued. The results of the written test by learners, analysis of class discussion and the interview with the teacher reveal the complexity of discerning or situating learners’ difficulty either as due to language limitation or due to lack of understanding of the “math” or both.

INTRODUCTION

After decades of relative under-emphasis of the importance of language in mathematics teaching and learning, the past three decades has witnessed the mathematical education community across several countries grappling to redefine the role of language in school mathematics (Pimm & Keynes, 1994). Various mathematics education research into the interplay between language and mathematics point to the intricate link between language proficiency and mathematical aptitude. Given the complexity of language structures and the confusion that might arise from the verbal-written duality of language and the translation from verbal/written mathematics into symbolic form, the language in the mathematics is not without its difficulties. The study described in this paper explores this intricate relationship between conceptual understanding and linguistic limitation, and the difficulty in translating written (or verbal) mathematics into symbolic form in algebra. It focuses specifically on how a teacher mediates this difficulty. The key question the study seeks to address is the following: in a multilingual classroom, how can a teacher ascertain whether learner difficulty in algebra is due to language difficulties and not due to conceptual limitation of the learners?
THEORETICAL CONSIDERATIONS

What does it mean to know mathematics? Is it context dependent or language dependent? What counts as mathematics is still a subject of debate amongst philosophers and to a lesser degree, amongst mathematicians (Noss, 1997). Douady (1997) contends that to know mathematics involves a double aspect. It involves firstly the acquisition, “at a functional level, certain concepts and theorems that can be used to solve problems and interpret information, and also be able to pose new questions” (p. 374). Secondly, to know mathematics is to be “able to identify concepts and theorems as elements of a scientifically and socially recognised corpus of knowledge. It is also to be able to formulate definitions, and to state theorems belonging to this corpus and to prove them” (p. 375). What role does language play in the knowing of mathematics? Pirie (1998) and Driscoll (1983) contend that mathematics symbolism is the mathematics itself and language serves to interpret the mathematics symbol. In the relationship between language and mathematics, language serves as a medium through which mathematical ideas are expressed and shared (Brown, 1997; Setati, 2005).

It can be argued, as Rotman, (1993, in Ernest, 1994: 38) does, that mathematics is an activity which uses “written inscription and language to create, record and justify its knowledge”. Language, thus, plays an important role in the genesis, acquisition, communication, formulation and justification of mathematical knowledge – and indeed, knowledge in general (Ernest, 1994; Lerman, 2001). Mathematics textbooks, pedagogical practices, and behaviourist view of learning have, in past and present, led learners and educators alike to portray mathematics as the acquisition of ready-made algorithms and proofs through memorisation and proofs (Siegel & Borasi, 1994) and as an activity done in isolation. More and more, new approaches in the teaching and learning of mathematics are supplanting this traditional approach to mathematics and mathematics learning. And with the new approaches, the role of language is increasingly foregrounded.

It is with the above in mind that this study is informed by the socio-cultural theory of learning. The socio-cultural perspective proposes that learning is a social process and happens through participation in cultural practices (Doolittle, 1997). Learning, thus, involves becoming enculturated and enculturation into a community of practice in which a learner finds him/herself and it (learning) is marked by the use of conceptual tools like language. Since the production of mathematical knowledge, for example, involves participation and negotiation of meaning within a community of practice, it then means that the use of language as a communicative tool is integral to the process of mathematical enquiry (Siegel & Borasi, 1994). For the socio-cultural view of learning, therefore, language is essential for participation in a community of practice. Language allows meanings to be constantly negotiated and renegotiated by members of a

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11 They (Pirie and Driscoll) both argue that even though brevity is a strength in symbolism, this symbolism in itself can be the root cause of misunderstanding as many research have revealed.
mathematics community – except for the mathematics register which has a fixed meaning across contexts (Brown et al; Cole and Engeström, in Chernobilsky et al, 2004).

RESEARCH METHOD

Sample

36 grade 10 learners were involved in the research. 25 of these have either Zulu, Xhosa or Setswana as their first language. Their spoken English was however fluent. 11 of the learners have English as their first language. The teacher, Miss Bonga, is fluent in Zulu, Sesotho and English. Bonga believes that the greater part of the difficulty experienced by learners is due to language difficulties. She therefore tries in her class to get learners to understand the language in the mathematics.

Data Collection

Data was collected through observation involving the implementation of the research instrument below:

➢ There are 8 times as many boys as there are girls in a school. Represent the learners in the school in an algebraic equation.

The question is a word problem requiring learners’ correct interpretation and understanding of the language to adequately engage with it. Learners were given 10 minutes to write down the algebraic equation. They then were asked to discuss their solution in groups of four. This was followed by a whole class discussion for twenty minutes where the teacher tried to probe learners’ thinking and requested justification of solution process. The teacher was also interviewed.

ANALYSIS AND FINDINGS

All the learners represented the number of boys with a variable and the number of girls with another variable. For the sake of this paper, I will represent the number of boys with $b$ and the number of girls with $g$. The table below shows how learners engaged with the question in their individual test.

<table>
<thead>
<tr>
<th>SOLUTIONS GIVEN BY LEARNERS</th>
<th>NO. OF LEARNERS WITH THIS SOLUTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8(b) + g$</td>
<td>11</td>
</tr>
<tr>
<td>$8b = g$</td>
<td>24</td>
</tr>
<tr>
<td>$8g = b$ OR $b = 8g$</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1: Learner responses
The above results show that more than half of the learners interpreted the question literally – as it is written in English. 31% of them interpreted the question as asking how many learners were in the school. In the class discussion that ensued, the teacher tried to get learners to justify their solutions.

Teacher: Mabel, what was your answer to the question?
Mabel: $8b + g$, ma’am.
Teacher: How did you get that?
Mabel: I said, if $b$ equals boys and $g$ equals girls, then there would be $8b + g$ learners in the school.
Teacher: read the question again. What does the question require of you?
Mabel: There are 8 times as many boys…(reads the question).
Teacher: So what do you think? What is the question asking of you?
Mabel: It is asking that…that I…I
Teacher: Can someone else help her? Yes, Tsiki
Tsiki: it is asking that represent the above statement as an algebraic equation.
Teacher: Can you say it in your own words?
Tsiki: It wants us to write an equation for 8 times as many boys as there are girls in a school
Teacher: And what do you think?
Tsiki: I think it would be eight times boys equals girls. So, if $b$ stands for boys and $g$ for girls, it would be $8b = g$.
Teacher: What do others think? Temi, what do you think?
Temi: eh…eh…I…
Teacher: What did you write?
Temi: I…I…I (she was one of those who wrote $8b = g$).

Adler (2001) advocates that in a participatory-enquiry approach such as the one used by Bonga, a high demand is placed on learners’ communicative competence. The learners in Bonga’s class have been tackling non-routine problems in diverse areas of the curriculum. As part of their participation in such a learning environment, learners are usually urged to express/justify their thinking and critically examine one another’s solutions. Bonga had noticed that some learners were usually quiet in class and had attributed their lack of participation to their English language competence level or timidity. She made it a point of duty to call on these learners to speak in class. On watching the video-tape of her lesson, Bonga explained that

Even though I was not surprised that learners wrote $8b = g$, I was struck that only one person was able to write the correct equation, and he is a Zulu first language speaker. I then thought to myself that they may be something more than language at stake here. But before trying to figure out what actually could be the problem, I tried to get learners to understand the key words in the question to see if that would help them understand the question better.
After noticing that learners did not understand the question well, Bonga decided that dissecting the question would help:

Teacher: I still need someone to explain to me his/her answer. Yes Dave

Dave: The equation, wouldn’t it be $8b = g$? It is there in the question and it’s easy to see: $8$ times the number of boys equals girls.

Teacher: let us look at the key word or phrase in the question. What is it or what are they? Think. Yes, Luandile

Luandile: Isn’t it “as many”?

Teacher: What do others think?

Mabel: I think it is “8 times as many boys as there are girls”.

Teacher: let’s take it from there: what does that mean? 8 times as many boys as they are girls

Dave: 8 times the number of boys equals girls.

Teacher: Which is greater? Number of boys or number of girls?

Learners: (Chorus) boys

Teacher: How? Who can give me a reason?

Thandi: 8 times as many boys as there are girls. There are more boys.

Even the attempt to analyse the key words or phrases did not help much in the understanding of the question. As Driscoll (1983) argues, the difficulties that learners experience in symbolic formulation of the mathematical written/verbal problem is not due solely to the confusion about words or vocabulary in the question. During the interview, I asked the teacher why she thought the English first language speakers did not answer the question correctly:

Bonga: You know, any interpretative activity such as the question above requires a sort of understanding of language in question. There is understanding the language, there is also understanding the mathematics,

Researcher: In the question, would it be possible to engage with it if you don’t understand the language? And would it be possible to engage with it if you do not understand the math?

Bonga: I believe the two are very related to each other. Learners need to understand both the language and the math to correctly solve the problem. They need the language to interpret the words into algebraic equation, and the math to check if their answer is correct. In short, they need to understanding the language in the mathematics (Bonga’s emphasis). You see, that was why when I noticed that grasping the key words and phrase in the question didn’t work well, I switch to numerical representations hoping this would help (in the excerpt below):

Teacher: Consider the question again, assuming there is only one girl in the class, how many boys would there be?
Thandi: 8 boys.
Teacher: If there are two girls, how many boys would we have?
Thandi: 16 boys
Teacher: Now, if there are g girls in the class, how many boys are there? Yes, Koko.
Koko: 8g
Teacher: Now, if b represents number of boys and g the number of girls, write and equation relating number of boys and number of girls in the class.

Even after this tactics of using numbers, the learners till did not fully grasp the explanation. The confusion inherent in translating the question above to symbolic form as Clement et al (in Driscoll, 1983) points out, is due to the direct translation or mapping of words in English into algebraic symbol. Bonga seemed to believe that learners’ confusion of believing that “8b” represented the larger group and “g” indicated the smaller group (Driscoll, 1983) was due solely to language difficulty. As Pirie (1998) points out, mathematical symbolism in a way, is the mathematics and is interpreted through the medium of oral/written language, but within the mathematics register, verbal/written expressions do not always match symbolic forms. From the above excerpts, it can be observed that learners used the word “times” in the expression “as many times as” to mean “times” as in a times b equal ab.

DISCUSSION

Any teaching or learning of mathematics involves activities of reading, writing, listening and discussing (Pimm, 1991). Language serves as a medium through which these mathematical activities are made possible. By asserting that mathematical activities are essentially interpretative in nature, we give primacy to the interpretative ability of learners and by consequence, their linguistic ability in the language in question. But to what extent can it be deduced that the difficulty experienced by learner is primarily due to the language competency level? In other words, when can we know that the difficulty is due to the English and not to the mathematics itself – not the difficulty of translating from written to symbolic form? Bonga struggled with this dilemma. She could not understand why none of the learners with English as first language could correctly engage with the question. She then surmised that the difficulty in the concept of equation could be at stake here. Still not convinced of her conjecture, she then decided to use numerical values to gradually help her learners to understand the question. Using numerical values to facilitate mathematical algebraic thinking in learners can help mediate language and conceptual problems. But to what extent can this method be successful? Can letters in all algebraic word problems be substituted for numbers? Are there cases where the use of numbers can aggravate rather than alleviate difficulties in the translation from written mathematics to symbolic form? Could code-switching have helped in easing the language problem? If code switching had been used, would learners
have still recognized that $8b = g$ was wrong? These are the questions that should plague the minds of any teacher desirous of mediating the translation from written/verbal mathematics to symbolic form.

**CONCLUSION**

Language is a key component in the construction of mathematical knowledge in the classroom (Gorgorio & Planas, 2001). In algebra as in some other aspects of mathematics, this relationship is a very complicated one and can pose problems to learners in the interpretation of questions. At times, understanding the mathematics and understanding the language in the mathematics are not a straightforward matter, and the teacher’s role of mediation when it comes to symbolic formulation from written expression (where it is not clear that what is at stake is the understanding of the English or the understanding of the math) in a multilingual classroom is a daisy one.

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This paper focuses on a study which I conducted with selected (mostly working class) parents of grade 8 learners of mathematics in an urban school setting. Initially, a survey was conducted with all the grade 8 parents at this school. Most of the parents indicated that they were keen on participating in a parent-assistance programme for mathematics. Using key points from this survey and an extensive literature review, I designed a parent-assistance programme for mathematics. This programme was conducted with three exclusive groups of parents. Each group was considered to be a case study.

An in-depth interrogation of the data in this study revealed that parents’ and children’s perceptions of mathematics were likely to be positively influenced. The data also suggest that children were likely to become more confident and to improve in mathematics. A key factor in influencing parents and children’s perceptions of mathematics was exposure to a wide-range of mathematical applications. This may have also helped to develop children’s confidence in mathematics and their improvement in the subject.

1. INTRODUCTION

The learning of mathematics is a national and international problem (de Lange 1981; Howie 2001, CDE report 2004) and a number of approaches have been taken to remedy the problem, including in-service training conducted by higher education institutions and education departments (Taylor and Vinjevold 1999). Alternatively, in countries such as the USA and the UK, parental involvement has also been used as a strategy to meet the challenges presented by mathematics education.

Parental involvement programmes that have aided mathematics learning in the UK include the “IMPACT programme” (Merttens and Vass 1990) and the “Maths Year 2000 Scotland” programme (Ritchie 2000). Some programmes in the USA include the “Back-to-School nights” (Huetinck and Munshin 2000), “Mathematics Month” in Cape May County in New Jersey (Szemcsak and West 2002) and the Family Maths Programme (Weisbaum 1990). The latter programme was introduced to South Africa in 1996 but was confined to the primary school (Austin and Webb 1998).
It would appear that there are no parental involvement programmes for parents of high school learners in South Africa. Thus, parents of grade 8 children were chosen to participate in this study as it was the first year in high school for their children and it was assumed that at high school level they form the group most likely to become involved in their children’s education.

However, parents’ views of mathematics are shaped by their own experiences of mathematics and their attitudes towards mathematics have an impact on their children’s attitudes. Children whose parents show an interest in and enthusiasm for mathematics around the home will be more likely to develop that enthusiasm themselves (Hartog and Brosnan 2000). At the same time, it is possible that parents with negative experiences of mathematics may pass these attitudes on to their children (de Kock 1999).

Thus, it would appear that parents and children, with negative perceptions of mathematics, need support. It was envisaged that the parent-assistance programme used in this study would attempt to provide this support. The effect of this programme on parents’ and children’s perceptions of mathematics would also be analysed.

2. RESEARCH METHODS AND PROCEDURES

This study took into account both ontological and epistemological perspectives as it is important to examine what is already known about parental involvement in education and how this study fits in with the current knowledge base of this involvement.

The ontological basis of this study primarily focused on the involvement of parents in the education of their children. Parents play a very significant role in their children’s early learning of mathematics and there is no reason why this should not continue throughout their children’s schooling careers.

The epistemology focused on where this study fit in with the current knowledge base about parental involvement in education. Much has been written about parental involvement in education (Wallen and Wallen 1978; Oakes and Lipton 1990; Unger 1991; Cline 2002). The overriding comment of these authors is that parental involvement in education is bound to have a positive impact on the children’s confidence and their results at school.

This study was a qualitative study. Hatch (2002) reports that for qualitative researchers the lived experiences of real people in real settings are the objects of study. This type of inquiry is concerned with understanding how individuals make sense of their everyday lives. Qualitative research also seeks to understand the world from the perspectives of those living in it. In this regard, individuals act on the world based on their perceptions of the realities that surround them. This approach is in line with the interpretive research paradigm, as described by Bassey (1999).

The same procedures were followed with each group of parents in this study, making the study a multiple case study (Yin 1984). In a case study, the researcher typically observes the characteristics of an individual unit: a child, a class, a school or a community. The
purpose of such observation is to probe deeply and analyse the various phenomena that constitute the life cycle of the unit with a view to establishing trends and patterns of coherence (Cohen and Manion 1994:106).

In this study, the interaction between parent and child was observed and documented by the parent in a journal, a process known as participant journaling (Hatch 2002). I then analysed and interpreted the journals. A total of 18 parents participated in the programme. These parents were volunteers, thus, making this sample of parents a convenience sample (Cohen and Manion 1994:88).

Each case study consisted of a parent workshop, the completion of participant journals and telephonic interviews.

2.1 The workshops
The workshops at the beginning of each case study served as preparation for the journaling period. The format for each workshop was the same.

The following matters were discussed in the workshop:

- Welcome and introduction to researcher
- Overview of outcomes-based education
- Outcomes-based education in Mathematics and assessment
- Grade 8 Lesson
- Discussion of parental involvement document
- Discussion of participant journals

The grade 8 lesson
I taught the parents a lesson from the grade 8 learning programme. The lesson was on “the product of two negative numbers”. This lesson was chosen as it was usually a very abstract concept to explain to grade 8 learners. In this regard, “2 x 3 could be easily explained as 2 rows of 3”; “3 x 2 could be explained as 3 rows of 2”. However, this method could not be used to find (-2) x (-3). It is also my experience that grade 8 teachers have difficulty explaining this concept to their learners and usually just give them the answer.

In this lesson, parents were shown two different but easy ways, that (-2) x (-3) = +6. They followed the lesson closely and all of them demonstrated a very good understanding of the work. They were then asked to work out a few examples in class and they attempted this with a great deal of enthusiasm. The objective of teaching this lesson to parents was to build their confidence in mathematics and show them that they could also participate in a mathematics lesson. It would appear from the responses and
the actions of parents in the lesson, that this objective was achieved. After the lesson had been completed, parents were asked to give their views on the lesson.

All parents were very appreciative that I had taken the time to teach them a lesson. Some of the comments from parents such as “I can still do mathematics”; “I didn’t realise that mathematics could be so easy”; “The lesson was easy to follow”; “I got all my answers right”; “I enjoyed doing the exercise” appear to show their enjoyment of the lesson and their positive attitude on successfully working out the exercises. At the end of the lesson it was hoped that parents were motivated and that they would be more positive about the rest of the parent-assistance programme.

The discussion document
Some of the matters discussed were: (Note: the last three lines were highlighted in accordance with our congress theme):

* Tips for parents to show that they are interested in their children’s work; why homework is important and how parents can help their children with mathematics homework; what parents can do if the child does not do homework or does not bring homework home; how shopping can become an enriching mathematical experience; the mathematics that we come across in the newspapers; mathematics on television; mathematics in the home; and geometric shapes in the home or environment.*

The following extracts from the document show the kinds of activities that were discussed:

Activities:

(1) **Temperature.**

Ask your child to observe the expected maximum and minimum temperature for Port Elizabeth for 5 consecutive days (from the newspaper or TV):

For example:

<table>
<thead>
<tr>
<th>DAY</th>
<th>Max. temp. (°C)</th>
<th>Min. temp. (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>31</td>
<td>20</td>
</tr>
<tr>
<td>Tuesday</td>
<td>28</td>
<td>18</td>
</tr>
</tbody>
</table>
(a) What was the average maximum temperature for the 5 days?

Add the temperatures: $31 + 28 + 27 + 25 + 27 = 138$ and divide by 5:

Average maximum temperature is $138/5 = 27.6 \, ^\circ C$

(b) What was the average minimum temperature for the 5 days?

Add the temperatures: $20 + 18 + 21 + 18 + 19 = 96$ and divide by 5:

Average minimum temperature is $96/5 = 19.2 \, ^\circ C$.

(c) What was the median maximum temperature? Arrange from highest to lowest: 31; 28; 27; 27; 25: The median maximum temperature is $27 \, ^\circ C$.

(d) What was the mode of the maximum temperature? $27 \, ^\circ C$ appears more than once; so it is the mode.

(2) Rand – Dollar exchange

Explain to your child about the Rand-Dollar exchange and its significance. On __________, the Rand-Dollar exchange was (per dollar):

<table>
<thead>
<tr>
<th>Buying</th>
<th>Selling</th>
</tr>
</thead>
<tbody>
<tr>
<td>R 6,80</td>
<td>R 6,60</td>
</tr>
</tbody>
</table>

(a) You have R10000 to use for an overseas holiday. How many dollars will you get for this amount on this particular day?

You are buying so it is $10000 \div 6.80 = $1470.58$
(b) Your uncle has gone for a three week holiday and returns with $350. How many rands will he get for this dollar amount on this day?

You are selling so it is 350 x 6.60 = R2310

(3) Mathematics on television (A word or two)

EXAMPLE

Suppose a contestant selects 2 high numbers and 4 low numbers. The high numbers are 100 and 25 and the low numbers are 3, 5, 7, 6. The target is 819. How would you get this number?

Solution

100 x 7 = 700
25 x 5 = 125 add: 825 and subtract 6 = 819.

Now you try the following: The two high numbers are 50 and 75. The four low numbers are 2, 4, 7 and 8. The target is 793. Do the necessary calculations. Get your child to practice these calculations.

4. The tile formation in your bathroom or kitchen:

Example: The diagram below represents a 4x4 section of a tiled bathroom wall.
You may ask your child to work out the answers to the following questions:

(a) Calculate the total number of tiles on specific walls/floors in your house

(b) On a section of a wall with tiles in the above formation, how many tiles appear on the diagonal of a

1. 3 x 3 section of tiles
2. 5 x 5 section of tiles
3. 7 x 7 section of tiles

What do you notice?

(c) How many tiles will appear on the diagonal on wall or floor tiles that is 50 x 50?

(d) Suppose each tile in the above diagram is a square. How many squares are there altogether?
In all three case studies, parents expressed their gratitude that an “outsider” (myself) had come to them with a programme to assist them. They were very keen and enthusiastic. Overall, the discussions proceeded very smoothly, with parents making notes where necessary. They just stated that they were happy to have some guidelines on how to support their children’s mathematics learning. They were very keen participants and were very appreciative being informed about the applications of mathematics.

At the conclusion of each workshop, parents were each given a journal to complete over a 7-week period. I went through the journal with the parents and gave them guidance on its completion. There were instructions on the front cover of the journal stating which parts parents and children should, respectively, complete. However, the full responsibility for the completion of the journal lay with the parent. The parent acted as the initiator and the parts that were to be filled in by children had to be completed under parent supervision.

2.2 The journals

Below are examples of activities that were to be done at home:

To ensure that there was regular communication between parents and children about the mathematics that is being learnt at school, the child also had to do the following, under parental supervision, on a weekly basis:

Write down two mathematical terms or words, as well as their meanings or descriptions, that the child learnt during that week; this would be discussed with the parent; and write down two mathematical experiences or encounters that the child had come across at home or in the environment; this would also be discussed with the parent.

The above two activities were intended to communicate to the parent the mathematics that the child was learning at school and the applied mathematical knowledge in the home and environment.

Another activity in the journal was grocery shopping. This had to be completed, once, during the programme. The child had to draw up a shopping list of 10 to 12 items. The child had to estimate the price of the items with parental input. Once the shopping had been completed, a comparison was made between the estimated price and the actual price. The parent had to answer some questions about the shopping experience. This activity meant that the parent had to work closely with the child. It was expected that the child could learn about estimating and comparing prices of items as well as budgeting. The activity also encouraged close parent-child cooperation and socialising.

Parents can play a major role in helping their children notice and use mathematics in an
everyday context (Puddick 2002). It is with this in mind that the next activities in the journal were planned. The first one involved children reading the predicted maximum and minimum temperatures for their city for a specific week. This could be obtained from the newspaper or television. They also had to answer some questions on the temperature readings. This activity showed parents and children that mathematics is not done only at school and that the newspaper provided opportunities for mathematical development. In this activity, children learnt about maximum and minimum and applying the concept of average to a set of real data.

The next one, which parents had to supervise, was the reading of the rand-dollar exchange in the newspaper or on television. The children then had to work out problems based on the rand–dollar exchange. It was expected that, by working through these calculations, children would better understand currency exchange and its effects on our economy.

**2.3 Telephonic interviews**

Telephonic interviews were conducted with parents during each case study of the programme. This was done to motivate and advise them as well as support their efforts in the programme.

**2.4 Follow-up questionnaires**

After each case study, parents and children were given questionnaires to complete in which they were able to relate their experiences in the parent-assistance programme. Besides asking for a range of information from parents and children; the key element of the questionnaire was parents and children stating “their highlights of the programme”

**2.5 Interviews**

A number of questions were put to parents. One of these was:

*What did you and your child learn during the programme that is still helping you?*

After the interviews with parents were completed, the mathematics teachers were interviewed.

**2.6 Focus group discussions**

Immediately after the interviews with parents and teachers of case study 3, parents and children from all three case studies were invited to a focus group discussion (Hatch
2002:24), where their experiences in the parent-assistance programme were discussed and shared with others. There were four focus groups, three comprising parents and one comprising children. The focus group discussion would also help to establish patterns of coherence (if any) and any challenges experienced in the implementation of the programme.

3. THE DATA (RELATED TO THE THEME)\(^{12}\)

**Case study 1**

Five parents participated in case study 1. These parents were called A, B, C, D and E. There was a great deal of evidence in the journals to suggest that the children were discussing the mathematics that they learnt at school with their parents.

**Mathematics studied at school**

All children (with the exception of parent C’s child) were able to list mathematics topics or sections that they had done in their mathematics classes. The following topics or sections were listed.

- The meaning of algebra, and substitution where numbers are substituted for letters;
- The power of a power and the power of a product;
- multiplying the terms according to the given power; an example of this was \((2x^2)^2 = (2x^2) \times (2x^2) = 4x^4\);
- prime factors are factors that can only be divided by itself and 1;
- square roots;
- the meaning of terms and expressions in mathematics;
- the meaning of monomial, binomial, trinomial and polynomial;
- complementary adjacent angles;
- parallel lines, and parallel lines intersected by a transversal;
- describing three types of algebraic patterns;
- explain and describe how flow charts are used in translating flow diagrams into algebraic sentences, e.g. \(y \rightarrow -7 \rightarrow (y-7) \times 2 = 2(y - 7)\);
- word problems;

\(^{12}\) To avoid repetition, only the data from case study 1 is discussed
• algebraic addition where only like terms may be added:
  
  \[
  \begin{align*}
  a + b + 2c \\
  2b + 3c \\
  a + 3b + 5c;
  \end{align*}
  \]

• simplification of expressions such as:
  \[4x + 3x + 7x + 2y + 5y = 14x + 7y;\]

• complementary and supplementary angles;

• drawing and describing parallel lines; and

• alternate angles and corresponding angles.

This section is significant as it would appear that the children found the mathematics they did in class interesting and successfully completed their class exercises. This may have enabled them to recall what they did in class and write about it, thereby highlighting the beauty of mathematics to them.

**Mathematics experiences at home and in the environment**

The inclusion of this item in the journal was for children to see that mathematics could be linked to real-life experiences. In this regard they were able to come up with novel and interesting ways in which they experienced mathematics at home and in their environment:

Some of these were:

• Learning a mathematical card game (Maths 24) which helped in the development of calculating skills;

• counting the laps covered by different cars in motor racing and determining which car was leading and which cars did not complete the race;

• measuring ingredients correctly to get the right consistency when baking a cake;

• watching mathematics lessons on the learning channel on a Saturday morning;

• choosing items and calculating costs during grocery shopping;

• the use of mathematical concepts, such as angles, perimeter and area when building houses;

• sleeping at an angle at night;

• observing that dishes were usually round, had an angle of 360°; and

• the angles between the walls in one’s home were usually 90°.
Mathematics in the home

The grocery shopping activity was included in the journal as it was one way in which children could practise their mathematical skills of estimating, budgeting and calculating. Four of the five children completed this section. The grocery items were listed with the quantity of items and the appropriate weights and measures indicated.

The parents used the following words to describe their children's reactions on being asked to draw up the grocery lists: eager and excited, enjoyed, interested. Parents stated that the shopping experience was an important social outing for them and their children. They claimed that the experience was also educational and informative.

In responding to what they learnt from the shopping experience, the children stated that:

- They became aware of the prices of goods;
- they learnt how to select quality products at "good prices" or bargains;
- they were able to count, measure or weigh articles;
- they became aware of what the weights and measures of the different goods were;
- they were able to compare prices in respect of brands; and
- they learnt about financial responsibility by keeping to a budget.

The section on mathematics in the newspaper was included in the journal because the newspaper was a rich source of mathematical information which parents could use in promoting the mathematical development of their children. However, this section was attempted with limited success. Only two journals had any information about the temperature readings and only one journal had information about the rand-dollar exchange.

(NB: Currency exchange is covered in the NCS and feature strongly in Mathematics and Mathematical Literacy textbooks)

Parents appeared to give favourable views about the programme. They enjoyed interacting with their children and were able to follow the guidelines given and support their children’s mathematics learning. They found out what their children were doing in mathematics and learnt from them. The children appreciated this interest in their work and were pleased to show their parents their work. Parents also commented that they learnt more about mathematics in the home and outside environment. This stimulated their interest in the programme.

The five parents of case study 1 participated in the programme with different levels of interest. At the workshop for parents of this case study, all parents appeared keen and
interested in the programme. However, based on the data received from parents during the implementation of the programme, two of the parents (parents B and E) had difficulty in implementing the parent-assistance programme. They were not able to complete all sections of the journals and also failed to hand in the follow-up questionnaires. This was not the case with the other parents (parents A, C and D). These parents satisfactorily implemented the programme. This ensured that there was sufficient and meaningful data to work with in case study 1.

The structured nature of the journals made it easy for the parents to follow and complete. Parents recorded the discussions they had with their children on what they did at school and it appeared that the children were very involved in school activities. These discussions focused on both academic and extra-curricular matters. The parents were able to check the children’s work and assist them where necessary. This gave them an idea of what mathematics their children were studying.

A further opportunity for parents to know what their children were doing in mathematics came in the part of the journal where the children had to describe to their parents what they were learning in mathematics. This task was done very well; children were able to recall, in a very clear and descriptive manner, what they learnt in their mathematics classroom. The children were able to extend these learning experiences to outside the classroom. They came up with some novel ways in which they used mathematics at home or in their environment.

Parents A, C and D and their children completed the follow-up questionnaires. The follow-up questionnaires revealed information which triangulated with the journal data. The highlights described by these parents, such as; “enjoying being involved in my son’s education”; “finding out easier and simplified ways of studying mathematics”; “being involved in my son’s work” appeared to confirm their interaction with their children and a meaningful involvement in their mathematics learning.

A further confirmation of this involvement could be drawn from the highlights described by the children. Some of the activities they were involved in and what they learnt from these activities were described. These highlights, which showed how the programme affected the children, were:

“the shopping experience”; “liked working with my mother and father and enjoyed their closeness and support”; “drawing up the shopping list and selecting the items in the supermarket”; "how to pay and save money when you want to buy something”

These highlights indicate that the parent-child interaction in this programme, whether at home or outside the home, contributed to the children’s mathematical learning experiences. This also reinforces the remarks of de Kock (1999) who stated that parents have the opportunity of using the everyday contact between them and their children as a means of communication as well for teaching mathematics.

The parents who completed the follow-up questionnaires revealed that they had
difficulty keeping up with the completion of the journals during the week. Parent A reported that administration was not one of her strong points and there were occasions when she did not make any notes during her interaction with her son. She, thus, had to review what had transpired with her son. At times there were other commitments that had to take precedence over the completion of items in the journal. Her son stated that he enjoyed the activities in the programme but had difficulty with the rand-dollar exchange.

Although parent C reportedly had no difficulties, he did explain that his daughter had difficulty keeping up with recording the daily temperatures and the rand-dollar exchange rates. As a result, only the temperature readings were recorded. These readings were backdated from the newspaper. Parent D experienced similar difficulties to those of parent A. However, the difficulties experienced by these parents did not appear to have a negative impact on the way they completed the journals. These difficulties also showed that the context of currency exchange was beyond the comprehension of most children.

Parents A, C and D claimed in the follow-up questionnaires that the programme provided them with opportunities for learning. Parent A indicated that she learnt how to interact with her son; she learnt that problem solving could be made easy to follow and understand. She had, previously, perceived mathematics as a "difficult subject" which she would not be able to understand. After her involvement in the parent-assistance programme, she had come to realise that mathematics was not as difficult as she thought. Parent C reported that he learnt that it is important to have a good understanding of mathematics if you are to be successful in the subject. He claimed that he had not liked mathematics in the past but this had changed and he had developed an appreciation for the subject. Parent D said that she had done mathematics at school until grade 12 but found it to be a boring subject. After being involved in the programme she began to realise that mathematics, in the form of problem solving, was "exciting". These comments by the parents also appeared to be in line with the views they expressed in the journals.

Parents A and C’s suggestions in the journals, that the programme be introduced in the earlier grades, was also repeated in the follow-up questionnaires. This seems to indicate that the programme impacted positively on them and that they felt that more parents, especially parents of junior learners, could benefit from the programme. Parent C’s enthusiasm towards the programme was taken further when he spoke about the programme to family members from another part of the country.

The interviews with the parents reflected a positive disposition toward the assistance programme and shed further light on their involvement in their children’s mathematics learning. All parents, including parents B and E, were interviewed and all stated that the programme had helped change the way their children did mathematics. In most cases there appeared to be an improvement in their children’s attitudes to mathematics and this was corroborated by the mathematics teachers. All parents were keen for their children to continue with mathematics in grade 10. However, parent B was not sure how her
daughter would do.

Parents B and E had difficulty with the implementation of the programme. Their journals were incomplete and they did not hand in their follow-up questionnaires. The interviews with parents B and E shed light on this situation.

The data from the follow-up questionnaires and interviews tended to support and clarify the data from the journal. The highlights described by parents A, C, D and their children in the follow-up questionnaires confirmed that there was meaningful interaction between parents and children and the activities, especially the grocery shopping, was enjoyed by the children and enhanced their budgeting, estimation and calculation skills.

The interviews with parents also revealed that there was meaningful interaction between the parents and the children during the programme, even though for parents B and E, this interaction was limited. The fact that all parents, with the exception of parent B, were able to recall specific incidents from the programme showed the impact of the programme on them. These incidents were linked mainly to the activities prescribed in the programme and included the following:

“concepts such as the rand-dollar exchange and temperature readings”; “compiling a shopping list and budgeting for the items on the list”; “examining the mathematics in the newspaper and on television”, “applications of mathematics in the home and environment”.

An analysis of these incidents suggests that the activities prescribed in the programme left an indelible mark on parents and their children. This does not suggest that those tasks, such as checking and signing their children’s work, are less important. What it does suggest, however, is that these routine tasks should be interlinked with interesting activities in the attempt to promote mathematics learning in children.

With the exception of parent B’s daughter, all children had shown improvement in their work; they were active in class and eager to do their work, although parent D’s son tended to be easily distracted. Despite parent E’s problems in supporting her daughter, the teacher noted that she tried very hard in mathematics.

In examining the various events during this case study, it is noticeable that although parents participated with different levels of interest and commitment, certain trends and features emerge, leading to the following conclusions:

- one of the requirements of the journal was for parents to have discussions with their children about what they did at school; this forced parents to talk to their children about what they did at school; these discussions took place regularly, were constructive and focused on general and mathematical issues,

- parents were able to check their children’s work and assist them when necessary; they became aware of the mathematics their children were studying when examining their children’s mathematics books and their children’s journal
writings on their mathematics experiences at school; these mathematics experiences were extended to outside the school; the children came up with novel ways in which mathematics could be used at home and in their environment;

- although the activities involving real-life experiences had mixed success, the children were able to use a wide range of mathematics skills during the activities; the success of the grocery shopping experience was due to the active role played by the children where they drew up the list, estimated prices and selected the items in the supermarket, thereby using both their estimation and calculation skills and learning to stay within a budget; the activity on temperature readings was attempted with some success, but the one on the rand-dollar exchange was poorly executed; although all these activities incorporated real-life experiences, the lack of success for the latter activity could be attributed to the fact that the concept of currency exchange may not be part of their daily experiences; and

- despite the mixed success with the mathematical activities, nearly all parents were able to recall these activities during the interviews, thus, highlighting the impact of these activities during the programme; this suggests that any programme that promotes mathematics learning should incorporate interesting activities that would enhance this learning:
4. FOCUS GROUP DISCUSSIONS

The parents were placed into groups that reflected the case study in which they were involved. In this section only aspects related to the theme of this congress are discussed:

The parents of case study 1 were very keen to share their experiences with each other. All parents in the group claimed they learned a lot from the programme. The programme helped them understand how mathematics is used in their daily lives. The programme gave them a focus on how they could assist and support their children's mathematics learning.

In case study 2 it was noted that the programme emphasised the importance of mathematics to the parents. They got involved and learnt at the same time.

Again in case study 3 parents said they “learned a lot from the programme”. The shopping experience was an important learning experience for all of them; they learned to shop wisely and work with a proper budget.

The fourth group consisted of the children present. They found that before being in the programme, they would have conflict with their parents since their parents' way of doing mathematics was different from what the teacher did in class. They were happy to have their parents take an interest in their work but sometimes they became too "in your face". Parents were "doing fine" in their interactions with them. They also felt there was a need for parents to be continually informed of what the content was. They were fine then but the children were concerned about what would happen when the work changed. They understood that mathematics was important.

Through the focus group discussions more light was shed on the experiences of parents and children in the parent-assistance programme. In general, the programme was seen as a learning experience by the parents. Communication between parents and children improved and parents enjoyed interacting with their children. Because the parents now understood some of the new ways of doing mathematics there was less conflict between parents and children. As a result the children enjoyed having their parents interested in their work. The children became keener in the subject and there was improvement in their results.

Parents were complimentary of the programme but expressed the need to be continually supported. They believed that teachers should play an active role in this process by forming partnerships with parents. In this regard, a long-term strategic plan for parent involvement needed to be put in place at the school.
5. FINDINGS

The data emerging from the three case studies suggest that the programme used as a basis for this research affected both the parents’ and children’s perceptions of mathematics. Those with a positive disposition to mathematics were likely to continue in this way as they saw mathematics in a new light which helped reinforce these feelings toward mathematics while those with a negative disposition were likely to change.

It is, thus, likely that parents’ perceptions were positively affected for the following reasons.

- They were *keen to participate* in the programme where they were given assistance;
- they *enjoyed* their interaction with their children;
- the parents wanted their children *to do well* in mathematics; and
- they *appreciated* the importance of mathematics and the role it was likely to play in the *future careers* of their children.

The children’s perceptions of mathematics also appeared to be affected in a positive manner. The following reasons may have caused this to happen.

- They *enjoyed* their parents’ interest in their work;
- they *appreciated the support* given by their parents;
- parent-child discussion took place in a *private home setting* without any outside distraction; this discussion focused mainly on mathematics; it gave children an opportunity to *tell their parents* about the mathematics they were learning at school;
- their children enjoyed looking for *applications* of mathematics in their homes and environment and *shared* these with their parents; this made them see mathematics in a *new light*; and
- it would appear that while the *top achievers continued* with their good performances in mathematics, many of the other children also appeared to *improve*.

The data collected in this study and the interpretation thereof provide strong evidence to suggest that the parent assistance programme, used in this study, affected both children’s and parents’ perceptions of mathematics in a positive and constructive manner.
6. CONCLUSION

This study provided very detailed and rich data which resulted in a number of findings. It is now possible to draw certain conclusions from these findings.

6.1 The effect on children’s perceptions of mathematics

In analysing the data emerging from the three cases of this study in terms of the effect on children’s perceptions of mathematics, certain trends emerge and these were explained in the summary of the findings. From these findings, the following conclusions are drawn on how this study affected children’s perceptions of mathematics.

The interest that parents showed in their children’s mathematics and the support they gave their children resulted in most of the children, including those with perceived negative attitudes to mathematics at the outset, developing positive attitudes to mathematics. By becoming aware of the various applications of mathematics at home and in the environment, children’s perceptions of mathematics as a difficult and abstract subject began to change. The grocery shopping activity, in which the children played a key role, also fostered positive attitudes to mathematics. The change in children’s perceptions and the fostering of positive attitudes to mathematics predicted further study in mathematics. Most parents stated in their interviews that their children would proceed with mathematics in grade 10 as the subject would be required for their intended careers. Even those parents who were not sure, stated that their children would, most probably, also opt for mathematics as they believed the subject would later offer them a wide range of careers to choose from.

It would be reasonable to conclude that the fostering of positive attitudes to mathematics in children was a key feature of this study and that the perceived importance of mathematics as a school subject was enhanced in the process. This was possible because of the close interaction between the parents and the children in this study. The relationship that developed between parents and children also strengthened as a result.

6.2 The effect on parents’ perceptions of mathematics

In analysing the data generated in this study in terms of the effect on parents’ perceptions of mathematics there were certain findings and these were stated earlier. From these findings, the following conclusions may be drawn on how the parents’ perceptions of mathematics were affected as a result of their participation in the programme.

The parents’ workshops made them aware of the role they could play in their children’s mathematics learning and they played this role with effectively. Their interaction with their children made them aware of the mathematics their children were doing. They found the outcomes-based approach to mathematics to be different from what they were used to; but they began to understand this approach and their role in this new
approach. They became aware of the **wide-ranging applications** of mathematics; something that was absent when they were at school. Notwithstanding what their perceptions of mathematics were, most parents viewed mathematics as a **key subject** in the curriculum and their participation in this study made them **more appreciative** of the subject. This appreciation for mathematics surfaced in the parents’ interviews, when most parents indicated that their children would continue with mathematics in grade 10.

One may, thus, conclude that, as a result of their participation in this study, **parents’ perceptions** of mathematics were **affected**. Those parents with **positive** attitudes to mathematics had these attitudes **reinforced** while those with **negative** attitudes to mathematics had a **rethink** and **changed** these perceptions once they **understood** what their children were doing in mathematics.

### 6.3 The perceived effect on children’s achievement in mathematics

Jones’s (2001) suggestion, that a well-designed parent involvement programme could boost academic achievement and raise students’ test scores, has relevance for this study. Although it is not possible to make such claims for the assistance programme used in this study as the cases were held in different school terms and may or may not have included summative assessment, it may be useful to examine what the perceived effect of this study on the children’s achievement in mathematics was.

A summary of the parents’ and teachers’ perceptions of the effect of the parent-assistance programme on the children’s achievement in mathematics appeared to show that the majority of the children became more confident in mathematics and developed positive attitudes to the subject. For most of the children, this meant an improvement in their mathematics results and overall averages.

It is likely the children’s **increased confidence** and development of **positive attitudes** to mathematics came about as a result of **parental involvement**. It is also likely that this involvement resulted in an **improvement** in the children’s mathematics.

### 7. CONCLUDING REMARKS

It is vitally important that parents become more involved in their children’s mathematics learning as there is widespread concern about the success rates of mathematics learners. There are many reasons why children do badly in mathematics with two possible ones being related to this study. These are the lack of interest by parents and the failure by teachers to involve parents. The data in this study supported the view that parental involvement in education and support of their children’s mathematics learning was likely to result in improved mathematics results and raised enthusiasm and confidence levels in mathematics. Parents themselves, stated that they benefited from the programme. It would be reasonable to claim that awareness of “the beauty, applicability and utility of mathematics” impacted positively on both parents and children.
If a comprehensive parent-assistance programme for mathematics is implemented at a school, this study showed that it is possible that confidence in mathematics will increase and the mathematics results of children could improve. This may have a positive effect on the school and ensure that children proceed to the next grade with the full confidence and support of their parents. A real challenge for schools is to get all parents involved. Although this is not likely in the foreseeable future, it is important that a start be made to getting an increasing number of parents involved.

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CRITIQUE OF MATHEMATICAL MODELS AND APPLICATIONS: A NECESSARY COMPONENT OF MATHEMATICAL LITERACY

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A case is presented for the incorporation of the critical engagement with mathematical modelling and applications in the teaching of Mathematical Literacy. The different purposes of mathematical modelling and applications are discussed and it is demonstrated how the interrogation of mathematical models and applications can be realised with the different kinds models. Examples are drawn from contemporary South African issues such as Black Economic Empowerment (BEE) and funding for educational institutions.

INTRODUCTION

In most countries it is emphasised that school mathematics curricula must, amongst other goals, have the development of democratic competence as a goal. Niss (1989) states that this goal is one of the five basic arguments posited for the incorporation of mathematical applications and modelling in the school mathematics curriculum. He views democratic competence as critical potential and states the goal as “to generate, develop and qualify a critical potential in students towards the use (and misuse) of mathematics in extra-mathematical contexts.” (Niss, 1989: 23).

Within the South African school mathematics curricula similar sentiments, although not as clearly as Niss does, are expressed. The National Curriculum Statement (Grades 10–12, General): Mathematical Literacy (Department of Education, 2003) manifests this intention. A few random statements from this document clearly demonstrate this desire (All quotations are from The National Curriculum Statement (Grades 10–12, General): Mathematical Literacy (Department of Education, 2003)).

The kind of learner that is envisaged is one who will accordingly be imbued with the values and act in the interests of a society based on respect for democracy, equality, human dignity, and social justice. (p 5)

Mathematical Literacy will develop the use of basic mathematical skills in critically analysing situations… (p 9)

To be a participating citizen in a developing democracy, it is essential that the adolescent and adult have acquired a critical stance with regard to mathematical arguments presented in the media and other platforms. (p 10)

The above statements clearly point in the direction of a competence to critique mathematical models and applications since “Mathematical Literacy is embedded in
mathematical modeling and applications.” Julie (2006: 36). In essence the envisioned critical competence can only be realized in the mathematical applications and modelling component of the Mathematical Literacy curriculum. An important facet then of the teaching of Mathematical Literacy in school mathematics is dealing with the critique of mathematical applications and modelling as evinced by Niss in the quotation above.

REFLECTIVE KNOWLEDGE AS THE NEEDED KNOWLEDGE TO CRITIQUE MATHEMATICAL MODELS AND APPLICATIONS

Skovmose (1990) argues convincingly that to realise the intention of critical engagement with mathematical models and applications, more than mere mathematical knowledge or technological knowledge—“knowledge about how to build and how to use a mathematical model” (p 0)—is needed. He defines the needed knowledge to engage in the critique of a model as “reflective knowledge, to be interpreted as…metaknowledge for discussing the nature of models and the criteria used in their constructions, applications and evaluations.” (p 0). Reflective knowledge does not develop automatically. Learners have to be engaged in specific activities of which the specific goal is the development of the ability to critique models.

In order to critically engage with mathematical applications and modelling consideration must be given to the reality situation of which the mathematical model is a representation. As Julie (2003) points out the reality situation is transformed through consensus where interests and purposes are settled and the resulting mathematization relates to this consensus-generated reality rather than to the “real-world reality.” The resulting mathematical model or application is thus a model of this consensus-generated reality. To critically engage with mathematical applications and modelling it is important to question the origin of the consensus-generated domain. “Which issues are included? Excluded? How considered are the decisions made to assign values to certain variables and parameters of a model?” are but some of the reflective questions that need to be considered in interrogating a mathematical model. In general this implies the consideration of the underlying assumptions of the mathematical application and model.

PURPOSES OF MATHEMATICAL MODELS AND APPLICATIONS AND THEIR CRITIQUE

In this section the different purposes which mathematical models and applications serve are discussed. Examples of existing mathematical models are used for illustrative purposes. It is not the intention to delve into the construction and the underlying mathematics of the models that are presented. This leads to another kind of critique which primarily has to do with increasing the fidelity of the model. Nor is the intention to provide an exhaustive critique of the presented models. The idea is rather to give indications or pathways that can be followed to dissect the models from the perspective of mathematically literate citizen perspective. It is thus so that some will agree with the
proposed direction of critique and others will go in some other direction based on their interests and experience. The contention is that this is how should be in a society which strives for robust democratic participation on issues of social. Furthermore, the models used as examples are more dealing with social issues and not natural phenomena since, again from a mathematical literacy perspective, these are the kind of models with the general population can, by and large, relate to.

According to (Davis and Hersh, 1986: 115) mathematical applications and models “can serve to describe, to predict, or to prescribe. These modes are interrelated, but they are not identical.”

A mathematical model can be used to describe a phenomenon. So for example, when a stone is thrown its trajectory can be described as approximating that of a parabola given by \( ax^2 + bx + c \). More esoterically there are also attempts to describe something like a person’s state of happiness by using what is known as “the happiness quotient” as described in the following excerpt:

\[
Happiness = P + (5 \times E) + (3 \times H) \ldots P \text{ stands for Personal Characteristics, including outlook on life, adaptability and resilience. E stands for Existence, and relates to health, financial stability and friendships. H stands for Higher Order Needs, including self-esteem, expectations, ambitions and sense of humour.}
\]

[To calculate your happiness quotient you do the following test:]

Are you outgoing, energetic, flexible and open to change?

Do you have a positive outlook, bounce back quickly from setbacks and feel that you are in control of your life?

Are your basic life needs met, in relation to personal health, finance, safety, freedom of choice and sense of community?

Can you call on the support of people close to you, immerse yourself in what you are doing, meet your expectations and engage in activities that give you a sense of purpose?

Answer the questions on a scale of one to 10, where one is “not at all” and 10 is ‘to a large extent’. Add your scores for question 1 and 2 to find your P value. The score for question 3 is your value for E, and question 4 for H. Apply your scores to the \( P + (5 \times E) + (3 \times H) \) formula to determine your Happiness Quotient out of 100.

*From “Happiness is No Laughing Matter”, by Pete Cohen and Carol Rothwell (Oakes, 2003: 13, 14)

This procedure is to be used solely by the individual him/herself to determine his/her state of happiness and is thus of the descriptive variety. Critique of this model can be via an interrogation of the 4 questions which forms the heart the model.

Some mathematical models which have a descriptive purpose can also be used to predict. In some sense most descriptive models are constructed for predictive purposes. But it should be borne in mind that not all descriptive models could be used for predictive purposes. The happiness quotient above is a case in point. However, the quadratic model to describe the trajectory of a projectile can be used given the values for
some of the parameters to determine the position of projectile at a particular time—say, the formula is of the form \( \text{height} = a \times t^2 + b \times t + c \), where \( t \) is the time.

The third use that mathematical models are put is to prescribe which “directly leads human actions or technological actions.” (Skovmose, 1990: 9). The prescriptive use of mathematical models is many times seamless and we are not aware of it. Take, for example, camera traps for speeding, or drawing money from an autoteller. Behind the scenes of these very everyday actions are complex mathematical models which prescribe when pictures should be taken for speeding or when a transaction at the autobank should proceed or not dependent on a variety of factors: whether the PIN number is correct, whether we have money in the bank, and so forth.

Nearer to present day South Africa there is the issue of equity with respect to the award of government tenders and Black Economic Empowerment. Some government departments such the Department of Safety and Security (Western Cape) use a point system prescribing how tenders will be ranked. They give the following procedure:

3.1 Responsive tenders will be adjudicated by the Province using a system which awards points on the basis of

- The tendered price
- Local manufacturing and Local Content
- Equity ownership

4.1 A maximum of 80 points is allocated [for price (NP)] on the following basis:

\[
Ps = 80 \left( 1 - \frac{(Pt - P\text{ min})}{P\text{ min}} \right)
\]

where

- \( Ps \) = the number of tender adjudication awarded for price
- \( P\text{ min} \) = the price of the lowest tender on a comparative basis
- \( Pt \) = in each case the comparative price of the relevant tender

6.5 Preference points for equity ownership by historically disadvantaged individuals who had no franchise in national elections prior to the introduction of the Constitution of the RSA, 1983 (Act 110 of 1983) or the Constitution of the RSA, 1993 (Act 200 of 1993)

\[
EPC = 10 \times \frac{EP}{100}
\]

where \( EPC \) is the number of points scored for equity ownership by people of colour, and \( EP \) is the percentage of people of Equity Ownership of people of colour within the enterprise…

6.6 Preference points for equity ownership by historically disadvantaged individuals who are women

\[
EOW = 3 \times \frac{EW}{100}
\]

where \( EW \) is the percentage Equity Ownership of Women within the enterprise…
Preference points for equity ownership by historically disadvantaged individuals who are disabled.

\[ E O D = 2 \times \frac{E D}{100} \]

where ED is the percentage Equity Ownership of Disabled within the enterprise…

TOTAL TENDER ADJUDICATION POINTS

The total number of tender adjudication points awarded (N) is, is the sum of:

\[ EPC + EOW + ECD + NP + \text{Local Manufacture and Local Content} \]

where local manufacture and local content are awarded a maximum of 5 points if the local content of the product/service is 60% or more of the total tender price excluding VAT.

This model for points adjudication is a prescriptive model. It does not describe nor does it predict. It merely is a calculation procedure for coming to some ranking on which to base decisions for the award of tenders. However, the given ranking can have consequences to whom the tender will be awarded. For example, whom does the 80 allocated for price favour? Can those with more economic power and resources tender a lower price than those with lesser power and resources? [What has to be borne in mind that with models of this nature, it makes economic sense to allocate tenders to those tenders which will be the cheapest.]

With respect to the critiques related to purpose it is important not to confuse the purpose and interest—descriptive, predictive or prescriptive—of the model constructor. Skovsmose (1990: 9) refers to this confusion as “confusion of guiding interest” and warns that “it is a serious misunderstanding if we identify a prescriptive [as] a descriptive use.”

CONCLUSION

From the above narrative to critique a mathematical models requires that their assumptions, guiding interests and purposes be interrogated. Developing critiques of models does not have the aim of finding fault with the model. It entails the scrutiny, dissection, extension and adaptation of existing models with the view to come to grips with the underlying mechanisms of mathematical model construction and the assessment and evaluation of constructed mathematical models. One of the aims of critique is thus to develop alternative models and in situations of social stratification—intended or otherwise—to take into account the “voices of the other” which are many times excluded even in present-day South Africa. This is particularly the case with normative models—those that have to do with value judgements—in the terrain of the social. In many of these cases there is a tendency “to preserve a static and homogeneous notion of
whiteness.” (Bloom, 1994: 16). A case in point here is the prescriptive model to allocate parliamentary and other elected positions to address gender inequality. They prescribed that one-third of such positions should go to females on the ranked list of candidates provided by their provinces and the application procedure is that every third position should go to a female. The procedure is implemented as follows: There is a list of possible candidates which is ranked—highest to lowest—according to the number of votes they obtained in their provinces. A possible configuration for the ranked list of 7 persons is: 1-M; 2-M; 3-M; 4-F; 5-M; 6-M and 7-F (M – Male; F – Female). Say now a political party is qualifies for 5 seats then the allocation will be four males (Numbers 1, 2, 3 and 5) and one female (Number 4). This results from the prescription “every third position should go to a female.” This prescribes where the counting procedure should start and result in the distortion. Had the procedure been “of every 3 candidates the last two should be male” then the allocation counting procedure would have started with the females and the five representatives would then have been: 4-F; 1-M; 2-M; 7-F and 3-M giving 2 females and 3 males—a much better dispensation, I think. But illustrative of this example is how in apparently progressive situations whiteness is preserved and results in unintended consequences.

The importance of allowing for the critical engagement with mathematical models and applications can not be underestimated. In part it lies at the heart of democracy and equity since “broad ideas of mathematical science…lie behind the ways in which they [decision-makers] reach their decisions and explain them [and if] the public were more familiar with these concepts they would be able to question organisations more effectively.” (Hunt, 2007: 2 – 24)

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INVESTIGATING THE USE OF LEARNERS’ HOME LANGUAGES TO SUPPORT MATHEMATICS LEARNING

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The paper presents an investigation into how learners’ home language can be used as a support for learning mathematics. This qualitative case study was conducted in primary school where learners were taking mathematics in English, which is not their home language. The school worked in collaboration with the Home Language Project to facilitate the learning of mathematics. The study revealed that when learners use their home languages they interacted better and freely with their peers and their teachers. The home language served as a reference point for words that were ambiguous and unfamiliar to learners. Mathematical practices such as conceptual understanding, procedural fluency, adaptive reasoning and strategic competence were furthermore facilitated by the use of learners’ home language.

INTRODUCTION

This was a qualitative case study, conducted in a grade three class in Johannesburg. The main purpose of this study was to investigate how the learners’ home language(s) can be used to support the teaching and learning of mathematics in a class of multilingual learners. The study was guided by the following questions:

a) How do learners use their home languages when they interact with mathematical tasks?

b) What mathematical practices are carried out by the use of the learner’s home language?

The paper will furthermore highlight how the research was conducted and this will be followed by a brief review of the literature and the debates on how language relates to mathematics. The analysis and findings of the data collected will be described later in the paper.

THE RESEARCH

The school where the study was conducted was working in collaboration with the Home Language Project (HLP), which started in 2001 as an initiative by parents and governing bodies from six Johannesburg schools. The aim of the HLP was to assist learners whose
home language is not English, to use their home language as a resource for learning, and to achieve bilingualism. The project provides home language support in a group of schools using English as the LoLT. The overarching goal of the HLP was to promote the ongoing learning of African home languages alongside the formal language of learning and teaching in schools where the LoLT is English or Afrikaans.

The lessons in this study were conducted by three teachers i.e. two teachers from the HLP and a mathematics teacher. The two HLP teachers assisted learners with the reading of home language tasks because learners were not yet proficient in reading their home languages. The mathematics teacher conducted the lessons in English.

THE DEBATES

It is widely accepted that language is important for learning and thinking. Researchers such as Pimm (1987), Pirie (1998), Torbe and Shuard (1982) have long recognized the relationship between mathematics and language. They argued that mathematics is like a language and learning it is much more abstract. They furthermore highlighted learning ways of speaking, reading and writing that are appropriate in mathematics.

What is still under constant debate and investigation in both the public domain and in research is which language is most appropriate for learning subjects such as mathematics, especially in a multilingual context such as South Africa.

Adler (1998, 2001) and Setati & Adler (2001) argued that multilingualism per se does not impede the learning of mathematics. They maintain that home language can be used as a useful resource for learning mathematics. Setati & Adler (2001) recommended code switching as a valuable resource for learning mathematics. Setati (2003) maintained that code switching is also one of the ways in which a teacher can encourage conceptual discourse, by allowing learners to speak informally about their mathematics: explaining, exploring and arguing about their interpretations and ideas. This talk is an important technique for learners to develop ideas in a comfortable environment.

Dawei (1983, cited in Yushau, 2004) also investigated the effect of teaching mathematics (in English) to students that have English as their second language. He focused on Punjabi, Impure, Italian and Jamaican learners who grew up in England. The results showed that first language competence was an important factor in the children’s ability to do mathematical reasoning in English as a second language.

Chan (1982, cited in Yushau, 2004), in his investigation on the difference in discourse patterns between bilingual and monolingual Mexican-American mathematics students he observed that where English was the only language used for teaching and learning, students were unable to engage in both procedural and conceptual discourse. Setati (2003) found the same in South African classrooms, while Rakgokong (1994) whose study also involved primary school children in multilingual classrooms in South Africa maintains that using English only as a LoLT has negative effects on learners’ meaning making and problem solving ability.
Moschkovich (1996, 1999, and 2002) whose work focuses on Latino bilingual mathematics learners argues that learners bring into the classroom different ways of making meaning in the mathematics classroom. In her analysis of the discourse that took place in the classroom, she noted the importance of supporting learners by “revoicing or modeling” their utterances. She argues that “revoicing” learners’ incorrect English utterances and modeling learner contributions enabled the teacher to get a deeper understanding of the mathematical discourses that students bring into the classroom. Moschkovich maintains that if we focus on students’ failure to use technical terms, we might miss how students construct meaning for mathematical terms or uses multiple resources such as gestures, objects or everyday experiences to communicate mathematically. She emphasizes the importance of valuing learners’ first language and mathematical discourses.

Gorgorio and Planas (2001) explored the role of language as a social tool that is crucial for the construction of mathematical knowledge in a multilingual classroom. They considered language as a wide notion where social, cultural, linguistic, emotional and cognitive tools are intertwined. They argue that teaching and learning should be a continuity between home and school or else new meaning or new words learnt in the school can turn into a wide variety of cultural conflicts and disruptions of the learning process.

On the other hand, there are other researchers such as Lim (1998 cited in Yushau, 2004) who do not view the use of home language as a resource for learning mathematics. They argue for the continued use of English as the LoLT in multilingual contexts and maintain that efforts should be made to improve English language proficiency of the learners. Lim studied the relationship between language and mathematics among Korean–American students and found that bilingual students’ success in problem solving is inextricably intertwined with their level of proficiency in English and other factors that relate to English proficiency. He recommended greater exposure to the language of the classroom (English) and the language of mathematics. Lim’s findings and recommendations resonate with those by Howie (2001), who argues that the solution to improving South African second language learners’ performance in mathematics is to develop their English language proficiency. Like Lim, Howie (2001) maintains that proficiency in the English language is related to performance in mathematics.

THEORETICAL FRAMEWORK.

I have used Kilpatrick, Swafford and Findell’s (2001) five strands for mathematical proficiency to analyze my data. Kilpatrick et al (2001) argue that these five interwoven and interdependent are essential for developing proficiency in mathematics. They furthermore indicate that these five strands would enable learners to cope with the mathematical challenges of daily life.

The five strands are as follows:
• **Conceptual Understanding**: Conceptual understanding refers to an integrated and functional grasp of mathematical ideas. Learners with a conceptual understanding know more than isolated facts and methods (Kilpatrick et al 2001).

• **Procedural fluency**: Procedural fluency refers to knowledge of procedure, knowledge of when and how to use them appropriately and the skill of performing them flexibly, accurately and efficiently (Kilpatrick et al 2001).

• **Strategic competence**: Strategic competence refers to the ability to formulate mathematical problems, represent them and solve them. This strand is also known as problem solving and problem formulation in the literature of mathematics education and cognitive science (Kilpatrick et al 2001).

• **Adaptive reasoning**: Adaptive reasoning refers to the capacity to think logically about relations among concepts and situations. Learners use adaptive reasoning to navigate through many facts, procedures, concepts and solutions methods to see that they all fit together in some way, that they make sense (Kilpatrick et al 2001). Adaptive reasoning does not only include informal explanation and justification but also intuitive and inductive reasoning based on pattern, analogy and metaphor.

• **Productive Disposition**: Productive disposition refers to the tendency to see mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics (Kilpatrick et al 2001). In short, a good attitude towards mathematics is imperative for acquiring the five mathematical strands.

**HOW LEARNERS USED THEIR HOME LANGUAGE AS A SUPPORT FOR LEARNING MATHEMATICS**

In order to illustrate how learners used their home language as a support for learning, I will use the following vignette to highlight how learners used their home languages to demonstrate the four strands. In the vignette the learners are discussing the attached task. The task was titled “the rainy days”. The task was prepared in a variety of languages: English, IsiZulu, Sesotho, Setswana, Sepedi, Xhosa, Xitsonga and Tshivenda. The learners were given the task in two languages: English and their home language. I give the version that was given to the Sepedi group. This version has the pictograph presented in Sepedi and is followed by questions in Sepedi and then in English. The translations of the months of the year were written as indicated in the task below.
THE ENGLISH TRANSLATION

Questions

Look at the calendar. Answer the questions.

1. Which month has the most rain?.................................................................
2. Which month has the least rain?............................................................... 4. How many more days does it rain in February March?..............................
3. Which months have the same amount of rain?........................................... 5. How many days does it rain in September and October altogether? .............
6. Which month has 20 days of rain?............................................................ 6. Which month has 20 days of rain?.............................................................
**English only questions:**

7. How many more days does it rain in May than September? .............................................

8. Which three months together have the fewest rain? .................................................................

9. Are there more or fewer days in May than in October? .........................................................

10. Are there more or fewer rainy days in January than in December? What is the difference between them? .........................................................................................................................

11. Which 3 consecutive months have the least rain? .................................................................

12. Which 2 consecutive month have the least? ..............................................................................

**THE MONTHS OF THE YEAR IN ENGLISH**

1. JANUARY       2. FEBRUARY        3. MARCH       4. APRIL
5. MAY    6. JUNE    7. JULY          8. AUGUST
9. SEPTEMBER   10. OCTOBER   11. NOVEMBER    12. DECEMBER

**The vignette**

In this vignette, the three learners were working on question two of the task described earlier, which involved the use of the concept “least”. The three learners are Sifiso, Nhlanhla and Sipho.

[ ] represent action taken in the lesson

( ) represents a translation to English

HLPT: Home language project teacher

MT: mathematics teacher

L: Any learner in the class, (name not allocated)

R: for researcher

*the three learners are reading the question loudly in Zulu, then in English and then in Zulu again*

**The group:** Yiphi Inynaga enemvula encane? [ they turned to the English version] (Which month has the least rain?) [They turned to home language version and read.]

**Sipho**: June, July....... agunethi ngo June no July ku ya bhanda (its June and July it does not rain in Winter). … [The other group members shook their heads]

**Nhlanhla**: masi zi counteni futhi.(lets count again) [They refer to the pictograph to see what has been represented in each, month.]

**Nhlanhla**: Hu August, there is nothing in August
Sifiso: but bathe imvula encane, a kuna mvula ngo August, hu July. (but they said the month with the least rain, there is no rain in August)

Nhlanhla: encane (the least) a kuna niks ……..ho “naught” (its zero).

Sifiso: Sifiso pointing to compare at the August and the July pictograph. He eee hu July one mvula encane ……..(No…… but it is July that has the least rain) [They all agreed on that July is the month with the least rain, Sifiso was insisting on July after the argument with Nhlanhla. Now the whole group writes July on their worksheet].

The three learners in the vignette, began by reading and deciphering the meaning of the words used in their home languages and then moved to the English version. The home language version acted as a source of reference in instances where they did not understand the meaning of words in English. They further used their home language version to ensure that they understood what was required in the question.

Sipho’s response to the question was “agunethi ngo June no July ku ya bhanda”'(its’ June and July, it does not rain in Winter). Sipho did not refer to the graph to figure out the answer, he used his everyday experience. His answer may not be correct but he understood what was required in the question. The meaning of the word “least” which meant “encane” in Zulu was understood. Sipho could relate the concepts least with “nothing”. However, his interpretation of the concept “least” was interpreted differently by Sifiso. The different interpretation of the concept “least” heated up the discussion between Sifiso and Nhlanhla. Nhlanhla argued that the month with the least rain is August. “Hu August, there is nothing in August “(its August there is nothing in Augusts) but Sifiso insisted that “but bathe imvula encane, a kuna mvula ngo August, hu July . (But, they said the month with the least rain ). Sifiso was implying that there should be a certain amount of rain for the month to be declared the month with least rain. He maintains that the question was stated as: “Hi yiphi inyanga enemvula encane (the month with least rain) and not “hi yiphi inyanga engenamvula” (the month that does not have any rain). He interpreted the concept “least” as something that can be quantified. It can be argued that the meaning he attached to the concepts “least” is influenced by his linguistic background. In ordinary language (both English and Zulu languages) “encane” or “least” cannot be related to nothing. We talk about “encane” when there is presence of something. Mathematically, least means the lowest quantity and zero or nothing. Zero is the smallest value in this case; as a result, the absence of rain should be the least amount. Numerically zero is less than one (Kaplan, 1999).

The discussion between the two learners demonstrated how learners used their home languages to argue, justify their answers, talk about the ideas and interpret mathematical concept. Sifiso who interpreted the question as “Hi yiphi inyanga enemvula encane (the month with least rain) and not “hi yiphi inyanga engenamvula”(the month that does have any rain) justified his answer. He displayed an excellent and logical adaptive reasoning for a grade three learner. He might not have understood that zero is also a number for quantifying things but he understood that the concept least meant “encane” which meant the lowest value. The interaction between the two learners displayed a high
level of adaptive reasoning, conceptual understanding and procedural fluency. They used their home languages to put forth what they believed to be valid explanations.

CONCLUSION

The use of home language for reading, translating English words, talking, sharing ideas, disputing ideas, challenging each other’s ideas facilitated the four strands of mathematical proficiency. Learners displayed skills of procedural fluency when they were solving tasks. Procedural fluency also played an important role in developing learners’ strategic competence. Students develop procedural fluency as they use strategic competence to choose among effective procedures. They also learn that solving challenging mathematics problems depends on the ability to carry out procedures readily and, conversely those problems solving experience helps then to acquire new concepts and skills (Kilpatrick et al 2001). Learners in this study have displayed a sound skill in procedural fluency, which is crucial for developing strategic competence.

They could connect their understanding of the concepts least with zero. This is a significant indicator of conceptual understanding; i.e. being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes. The degree of student’s conceptual understanding is related to the richness and extent of the connections they have made. Connections are useful when they link related concepts and methods in appropriate ways (Kilpatrick et al 2001).

The other important strand that learners have displayed in their discussion is adaptive reasoning. They could justify their work and explain ideas in order to make their reasoning clear. They could use evidence to critically reason, generate explanations with their data, and defend them orally or in writing. This interaction enabled them to reach a higher level of understanding, which developed their capacity to solve mathematical problems collectively. All mathematical ideas, interpretations, reasoning and thoughts were filtered through communication in the classroom. As they communicated their ideas, they learned to clarify, explain, refine and consolidate thinking (Sfard, Nesher, Streetland, Cobb & Mason, 1998).

In short, the mathematical conversations they had were good for mathematical thinking, reasoning, conceptualizing and solving problems (Sfard et al, 1998).

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LEARNING MATHEMATICS THROUGH INTEGRATION WITH CONTEXTS THAT DRAW ON LEARNERS’ EVERYDAY EXPERIENCES BOTH IN MATHEMATICS AND MATHEMATICAL LITERACY CLASSES: A PILOT STUDY

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In this paper I draw on a pilot study to describe and explain the learning of mathematics through integration with contexts that draw on learners’ everyday experiences both in Mathematics and Mathematical Literacy classes, using Bernstein codes of recognition and realization rules and Bourdieu’s notion of habitus. This pilot is part of a wider study for my doctoral research which is still in progress. The study is located within a qualitative approach, adopting a multiple case study which will focus on two secondary schools from contrasting backgrounds in South Africa. For piloting and for this paper I chose only one township school to explain and discuss the results. In this school two classes of approximately 30 learners in each were chosen, one Mathematical Literacy class and one Mathematics class. The 5 learners in each class (both Mathematics and Mathematical Literacy) were selected based on what they have written in their learners’ background questionnaires scripts. Thus a total of 10 learners were chosen for working on the task and for oral presentation. Data was collected using task instruments and interviews. The focus is on two issues that arose from this pilot – the problems some learners have when negotiating the boundary between esoteric mathematical knowledge and their everyday knowledge and learners’ background differences in relation to their views with mathematics and mathematical Literacy.

INTRODUCTION

This study investigates how the learning of mathematics by grade 11 learners is enabled or hindered through integration with contexts that draw on learners’ everyday experiences. The study is guided by the following critical questions: How do learners’ everyday experiences and habitus interact with their mathematical experiences to solve ‘real world’ problems that require mathematical modelling?; How does the learning of mathematics through integration with learners’ everyday experiences affect the recognition and realisation rules within the classroom? Are there any differences in relation to the first and second questions in the chosen Grade 10 Mathematics classes versus chosen grade 10 Mathematical Literacy classes? If so what are the sources and the nature of such difference and what are the implications of the above for: the design of context based materials, role of the teacher in the mediation of context based activities and current curriculum implementation of both Mathematics and Mathematical Literacy in the FET band.
LEARNING MATHEMATICS THROUGH INTEGRATION WITH LEARNERS EVERYDAY EXPERIENCES

It is of grave concern that even when students have procedural fluency within specific concepts in mathematics, they often struggle with solutions to problems that are posed in a contextualized form. The original Curriculum 2005 exhibited very weak classification between school subjects (weak internal classification). Thus several subjects were amalgamated into three “learning areas” for the first three school grades. Curriculum 2005, from grade 0-9 was characterized by weak classification between school and everyday knowledge. A heavy emphasize was also placed on “integration, which determined that all knowledge was to be studied under the rubric of so-called programme organizer— which were to provide topics or themes through which school knowledge was to be integrated with the everyday experiences of learners (weak external classification). The motivation behind weak classification was that it would facilitate the acquisition of school mathematical knowledge by relating it to the everyday; and on the other hand, facilitate the application of formal knowledge to problems of the real world (Taylor, Muller and Vinjevold, 2003).

Researchers like Cooper and Dunne (1998; 2000); De Lange (1996); Nyabanyaba (1999; 1998; 2002); Lubieski (2000); Sethole (2004); Chisholm et al (2000) highlight the difficulties of learning mathematics through everyday experiences. In particular, the reports quote that the body of knowledge that defined mathematics is obscured or dominated by non-mathematical considerations. In view of this problem, the review committee recommended that the programme organizers be scrapped and the learning programmes be separated into distinct learning areas. Adler, Pournara and Graven (2000: 2), however, argue that despite the fact that some of the design features of Curriculum 2005 such as phase and program organizer have been dropped, integration remains one of the driving principles in outcomes based curriculum. The Curriculum Review Committee, then, proposes that a revised, streamline National Curriculum Statement be produced for Early Child Development, General Education and Training, Further Education and Training and Adult Basic Education and Training.

The National Curriculum Statement FET (2005) asserts that all grade 10, 11, and 12 learners must choose either Mathematics or Mathematical Literacy as one of their subjects from 2006. Both in Mathematics and Mathematical Literacy documents, the learning of mathematics through integration with learners’ everyday life experiences is still a focus. This is evident both in their definitions and their Learning Outcome 2 where mathematical modeling becomes more prominent. That is, using the context to learn the mathematics and using the content to make sense of the context using mathematical models such as graphs, tables and formulae. Verhage, Adendorff, Cooper, Kasana, Le roux, Smith and Williams (2000); Blum and Niss (1989; 1991); Dunne and Galbraith (2003); Galbraith (2004); De Lange (1996); Galbraith and Clatworthy (1990); Julie, Cooper, Daniels, Fray, Fortune, Kasana, Le Roux, Smith, and Williams (1998) purport that tables, graphs and formulae are also presented as mathematical tools that are useful in the process of modeling a context situation. Verhage et al (2000), in particular
argue that in the Realistic Mathematics Education (RME) tables, graphs and formulae are used at all levels of teaching, and employed as powerful tools to facilitate learning, and to master the said concepts related to the understanding of functions. Hence, I am interested in both Mathematics and Mathematical Literacy class in terms of how learners’ everyday experiences and habitus interact with their mathematical experiences to solve ‘real world’ problems that require mathematical modelling. It is explicit that both Mathematics and Mathematical Literacy highlights differences with respect to the emphasis on integration on these two statements. Thus part of this study is to explore if there are any differences in learners experiences on teaching approaches, working on the task in terms of their everyday experiences and habitus interacting with their mathematical experiences when solving ‘real world’ problems that require mathematical modelling. If so what are the sources and the nature of such differences? Are those differences in learners’ responses on the task congruent or incompatible with those outlined in the two curriculum statements?

THEORETICAL ORIENTATIONS

In this study I use the work of Bernstein and Bourdieu as a lens through which to analyze the data. The work of Bernstein is widely noted for its usefulness in providing tools for analysis especially of changes in education curriculum. Bourdieu’s notion of habitus and field is particularly useful in theorizing how social differences are manifested and legitimated through school mathematics. For Bourdieu (1997), habitus is the embodiment of culture, and it provides the lens through which the world is interpreted. Habitus predisposes (but does not determine) thoughts, actions, and behaviors (Zevenbergen 2000: 202). Through this it will be possible to describe how different individuals use their agency to act as a go-between the social unfairness as propagated by the everyday knowledge and school knowledge. Both Bernstein and Bourdieu’s works bear relevance to my concern in a sense that I am interested in exploring how learners’ everyday experiences and habitus interact with their mathematical experiences to solve ‘real world’ problems that require mathematical modelling. There are also two codes mentioned by Bernstein that are the core of this study. That is, the Recognition and Realization rules.

Recognition rules “at the level of the acquirer”, are the means by “which individuals are able to recognize the specialty of the context they are in” (Bernstein, 1996: 31). The recognition rule, essentially, enables appropriate realization of putting things together. The acquirer must first recognize the type of response demanded by a particular context. At the lowest level, this requires choosing between a community or school codes. Then the acquirer must correctly identify the specialized language of the particular discourse, whether this is art, Mathematics or Sunday dinner with the family (Taylor, Mullar and Vinjevold, 2003).

According to Bernstein (1996: 32), the realization rule determines how we put meanings together and how we make them public”. Realization rule mean that the acquirer
(learner) is able to produce a legitimate text in the required discourse. “Text” refers to anything which attracts evaluation: is something as simple as the way one solves an algebraic problem. Brodie (2000) defines learner text as learner’s contributions or productions. Brodie argues that correct answers indicate that the learner has acquired the realization rules. Possession of the realization is reflected in the ability to produce (act, speak or write) the expected (legitimate) text. Thus the study tries to explain how the two codes, recognition and realization rules affected when learners are drawing from their everyday experiences in the processes of learning mathematics in Mathematics and Mathematical Literacy classroom.

RESEARCH DESIGN AND METHODOLOGY

For piloting, the study focuses on one township secondary school in Soshangoeve. The research is located within a qualitative approach, adopting a multiple case study approach. In this school, two classes of approximately 30 learners in each the structured questionnaire about their backgrounds and their views about Maths Literacy and mathematics was administered to them, five in each class were chosen – five from Mathematical Literacy class and five from Mathematics class. The five learners in each class were chosen based on their responses on the questionnaire and also based on the help of the teacher to select learners who are communicative. The teacher also help me to select five learners from mathematics class who are weaker mathematically, so that they could be in the same footing with five from mathematics class in terms of their mathematical knowledge. These 10 learners were interviewed. The interview was a follow up from their written responses on the questionnaire. The purpose of interview and structured questionnaire was to establish if there are any differences between learners’ views on mathematics and mathematical Literacy before the project could be administered to them. Four days mathematics lessons were observed, and later transcribed noting the responses of learners. Multiple tape-recorders were used to cover what was happening in each group.

Knowledge acquisition is situated in contexts. Knowledge acquisition goes hand in hand with participation in a community in which the target knowledge is shared (Lave and Wenger, 1991). Wood, Cobb and Yackel (1992) support that mathematical knowledge that can be used in a variety of situation as an intellectual tool is acquired by construction, and social interaction, and meanings are constructed through a process of negotiations of meanings forms the source for the analysis of the role of interaction in learning.

It was therefore ideal to have a classroom which provides settings for students to interact with others while solving open-ended contextualized mathematical problems. In these two groups of learners, learners could work together in groups when solving activities. My role as a researcher was two fold: to make sense of how learners might be thinking through probing their responses, and intervene in ways that do not impose my ideas on learners. Thus, my task was to use questions appropriately as some kind of scaffolding.
through which I would try to elicit learner’s meanings, listen to learners interpretively rather than evaluatively when working with them.

Furthermore, the activities that were used in this study were categorized as doing mathematics task or activities—that are task that require an explanation, has real-world context, and uses diagrams, uses manipulative. There was no predictable pathway suggested by this task. The focus was on looking for the underlying mathematical structure which required complex thinking.

After learners had worked on the project for four days, they then discuss their strategies to the class through presentation on the fifth day wherein the video record was taken. Thus learners were justifying, explaining their thinking to others as well as trying to understand and questioning the reasoning of others. The lessons found in these classrooms and the nature of discourse that evolves during the interactions helped me to explain and discuss learners’ recognition and realization rules and the notion of habitus by which learners learn mathematics from a social constructivist view using multiple case study methodology. During presentations, learners were explaining their methods for solving mathematical problems and others would listen and decide if the explanation provided did not make sense, then others would ask questions or provide information to help clarify meaning.

KEY FINDINGS AND ISSUES EMANATING FROM THE TEACHER AND LEARNERS INTERVIEWS

Teachers and Learners views on the teaching and the learning of mathematics

The data used here is drawn from a wider study in progress which involves secondary school learners. I analyze individual interviews with 10 learners from Mathematics and Mathematical Literacy class.

R: Researcher
T: Teacher

R: What would you say is different about your teaching of Mathematical Literacy over the last year compared to your teaching of Mathematics with respect to the teaching approaches?

T: Basically… there are no differences because if you look, the textbooks that we are using are the same especially in the first chapters in terms of the content but ML has more word problems than in Mathematics textbook, that could contribute on using different teaching approaches like in ML I often use group work than in Mathematics class that demand individual approach, so in ML class learners often discuss, debate, analyze and interpret word problems than in math class which is so straightforward.

The teacher’s response seem to suggest that there is no much difference in content, particularly in the first chapters of both maths and ML textbook that she is using. However she appear to be contradicting herself by pointing some differences in terms of maths literacy been contextualized than mathematics. She seems to suggest that the differences have an impact on the teaching approaches. ML literacy because of its nature
of been contextualized tends to demand group work wherein learners often discuss, debate, analyze and interpret those words problems and Mathematics tends to demand individual approach because it is just straight forward. This is also confirmed by what the teacher said on the extract below:

R: What about the activities?

T: As I have already indicated, activities are the same especially in the first chapters of both M and ML textbook, but ML has more of word problems than in M which has only one which is interest and compound.

The teacher’s response is congruent to the majority of learners’ responses who responded thus:

L1: the activities were ratio, decimal places, percentage and equation

R: Are these activities connected to your everyday life

L1: ML is about maths in everyday life.

R: What are your experiences with respect to teaching?

L1: I can say teachers in this school (the name of the school) are very active, energetic and communicative so I find them very good at teaching us.

R: for ML teachers which approaches do they normally use in the class when teaching ML

L: What do you mean by approaches, can you please explain?

R: Do they use group work or do you work alone in class.

L1: No teachers give us case study and scenarios to discuss in group so I find ML teachers more hyperactive than those of math.

R: (Laugh), what do you mean by hyperactive?

L1: I mean ML teachers are flexible than Math teachers.

R: What are your experiences about Maths teachers?

L1: Maths teachers are very strict and stereotype, I mean they are not communicative like ML teachers who let us to discus and debate in class.

One of the findings that emanate from this pilot study is the issue of language in mathematical literacy and mathematics. The teacher highlighted the fact that English language is a problem. She raised an issue of learners’ difficulties with the English Language and comprehension required in mathematical Literacy due to its more applied contextualized and real life problem nature in contrast to mathematics which does not demand too much of English comprehension due to its symbolic language or mathematical register. This seems to suggest that English as a language could hinder the learning of mathematics. This finding is confirmed by the following extract:

R: what about learner’s progress in learning of these two subjects?

T: The problem with ML learners is English, ML is more of worded problems in the form of scenario and case studies that need to be analyzed and interpreted so our learners fail ML because they cannot understand English, so English is a problem to them. But, in Math class is simple and straight forward learners need not to analyses and interpret it is just straight forwards solve for x, Simplify, factories, proof without too much of words.
Maths learners seem to dislike ML because it is full of words sums and paragraphs that demand too much of reading and comprehension.

L2: I love maths than ML because maths is straight forward and simple, if it solve for $x$ is solve for $x$ and proof is proof, but ML is of more of paragraph, words and I don’t like reading. And ML is for small kids.

L3: I don’t like ML because it is full of sentences and demands too much of reading. In maths there is no too much sentenses, it is just straight forward solve for $x$ is solve for $x$ and nothing else.

L4: ML is full of paragraph, we maths people don’t like to read…mmm too much paragraphs no…

The other finding that was raised by the teacher is the learning pace in these two groups of learners. The teacher highlighted the fact that ML learners understand slower than mathematics learners, due to their different mathematical knowledge. She suggests that mathematics learners have strong mathematical knowledge than mathematical literacy learners. She further suggests that their different mathematical knowledge could be attributed to that fact that some of ML learners did not have mathematics as a subject in grade 9.

R: What is different in these two classes with respect to lesson structure/ pace?

T: Math class learners understand faster than the ML learners?

R: why?

T: It is because in the ML class, some of the ML learners were not doing mathematics in grade 9, so when they come to high school they had to choose between two streams of commercial subjects and sciences and these subjects are linked to ML because ML can be integrated easily with accounting so learners who did not do maths in grade 9 lack mathematical background that is why they are little slower than maths learners. The other thing is some of the learners who failed mathematics in grade 10 and pass other subjects we force them to take ML in grade 11.

From the above extract, it appears as if in these school learners who are taking mathematical Literacy are advised by their teachers to do so base on their performance of mathematics in grade 10. Thus, although in some learners some level of free choice is allowed, in this school learners who failed maths in grade 9 are strongly advised to take mathematical literacy in grade 10. And learners who pass mathematics in grade 9 are strongly advised to take mathematics in grade 10.

This teacher also believe that mathematical literacy is a basic watered down maths

R: wow? Why are you doing so?

T: It is because ML is the basic of pure math.

The same believe is also strongly shared by mathematics learners who believe that ML literacy is so easy and as result it is for small kids.
R: if you had to advise Gr 9 learner considering whether to take maths or ML what would you say?
L2: Sir, I would advise him to take Maths, because ML is for small kids.
R: between maths and ML which one do you love most?
L2: I love maths than ML because maths is straightforward and simple, if it solve for x is solve for x and proof is proof, but ML is of more of paragraph, words and I don’t like reading. And ML is for small kids.
R: Why are you saying ML is for small kids?
L3: No, I don’t like ML because it is so easy and it is for small kids.

The pilot study also reveals that learners in both maths and ML class do learn mathematical concepts. Learners increased access to mathematical thinking. The difference is that mathematical literacy is contextualized while mathematics is decontextualised.

T: Both subjects have “solve for x”, surds, ratio, decimal places, percentages and proportion, but in ML are in worded form.
L1: The activities were ratio, decimals places, percentage and equation.
R: Can you tell me more about the topics covered thus far?
L2: We did ratio; percentage; proportion.

Findings from the project
In this project learners were to explore and critically analyse the context of phone charges- Cell phone as against fixed phone using mathematics to determine the best “value for money” telephone option. Two tables of tariffs for the different cell phone options as well as TELKOM rates for using a fixed phone were included. Learners were to thoroughly investigate the financial implications of using a fixed phone versus using a cell phone. Learners had to think of all factors that influence the decision, for example, the cost of calls at different times or over different distances. They also had to decide which information is not necessary when solving the problems. As these activities are modeling tasks, learners must had to make use of assumptions, investigations, justifications generalizations and mathematics, such as calculating the cost of the calls made during peak time and during off peak time. Their investigation and decisions must be based on mathematics and personal preference. The project had 3 activities. The first activity was about finding the formulae when given the two tables of tariffs of vodacom and MTN. The second activity was about finding the formulae when given telkom tariffs over different time and distance. The third activity was about the comparison and decision making of the three telephone providers based on the calculation from first and second activities. It was very much interesting to see some of the learners ignoring the
mathematical calculations they have done in the first two activities and answering the third activity based on their personal preferences when they were prohibited from doing so. Activity 3 was structured as below:

WHICH TELEPHONE OPTION IS THE BEST VALUE FOR MONEY BASED ON YOUR MATHS ANALYSIS IN ACTIVITY 1 AND 2 YOUR INDIVIDUAL NEEDS?

a) Consider your own need for a telephone. Take into account your current use and your available budget for a telephone. If you don’t have a phone, plan and budget as if you want to get one. Decide if you should get a VODACOM phone, an MTN phone or a TELKOM phone.

- State your assumptions. That is, give some evidence of when you usually need to make calls, how many calls you make during a month, where you call to and so on. This information can be sourced from old accounts or by careful estimation.
- Make a decision for your situation and justify your decision mathematically.

Below are the responses of maths and maths literacy learners taken from both mathematical literacy and mathematics learners’ scripts.

Lerato; ML: Vodacom, because is the leading network and they charge their services at low rates than other networking providers. Like when you want to make a call on off peak time it charges R0, 68 and weekend and holiday.

MTN = R0, 68 only on off peak time per minutes.

Telkom = R0. 86 per second.

Sibongile Maths Literacy

Sibongile; ML: I think I will go for telkom, because when I compare for example,

<table>
<thead>
<tr>
<th></th>
<th>Vodacom</th>
<th>MTN</th>
<th>Telkom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subscription</td>
<td>129</td>
<td>129</td>
<td>57, 26</td>
</tr>
<tr>
<td>Call rates at peak time</td>
<td>2.51</td>
<td>2.51</td>
<td>0.16</td>
</tr>
<tr>
<td>(To landline)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Call rates at off peak time</td>
<td>0.68</td>
<td>0.68</td>
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<tr>
<td>(To landline)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>To cell at peak</td>
<td>1.80</td>
<td>1.87</td>
<td>1.60</td>
</tr>
<tr>
<td>To cell at off peak</td>
<td>0.68</td>
<td>0.68</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Therefore Telkom rates are cheaper.
Unice; M: I would choose MTN because it has a personalized callback which can be changed once daily. If you are using CPS (call per second) you are charged at 2 cents per second and 35c a sms on off peak time. I usually make calls during off-peak time and I make 2 to 4 calls daily and calling people here in Soshanguve and some in Mpumalanga so in a month. I probably make about plus minus 120 calls. During the day on a peak time a sms is 10c or R1.50 depending on the time of day. I sms more than calling because it is lesser and it allows you say more. I send 2 to 5 sms daily and I usually speak for more than 3 minutes on the phone.

MTN off peak sms

2c x 120 calls + 35 cents x 5 sms.

Thami; M: Vodacom is a network I choose because first of all when coming to advertisement is a network that is leading even if you don’t like to look at advertisement but vodacom will make you to enjoy, not only in advertisement, but also in free air time. I never heard that MTN is giving free cell phone or call discount. But with vodacom you can get free cell phone and again is a South African leading network in terms of supporters. It also sponsors African soccer challenge.

Margaret; M: I would prefer MTN network because you only pay R 95, 26 for connection fee and that is less than the R190.00 you pay for the vodacom connection fee. And it is less than the vodacom connection fee. In MTN you also get free goodies, for example T-shirt, bags, caps, juice bottle and lanyard etc. And when you make call during off peak time your monthly account will be monthly subscription + connection fee+ the amount charged for off peak time. But I think MTN is better excluding the free goods you get, MTN is also affordable.

Nthabiseng; M: According to the project that we have done and our different findings on contract phones with all the tariffs, i.e. Monthly subscription, connection fee, call rates etc, I happen to find Telkom as the network provider so cheap but depending on the different distances, for instance Telkom charges 16c for 0-50 km per minute (standard time). With telkom I will make as many calls as possible, But of course depending on my distance and on a call more time and the monthly rental of 57,26. But the disadvantage of this is with the land line phone you will have to wait for the call. So I prefer vodacom because in our system of axes where by we compare MTN and VODACOM ‘s monthly costs option for peak time calls to another mobile we happen to find that the vodacom costs are LOWER than the MTN according to our sequence of numbers, for example in

MTN = the x line and the y line

10 50

Vodacom= the x line and the y line

10 47

One again Vodacom is again the cheapest comparing MTN mobile to mobile calls, with VODACOM =180 but MTN 1, 87

It is evident on the above learners’ responses on the task that some of the children, such as Thami and Unice (in the mathematics class) have explicitly imported their everyday knowledge when it is inappropriate or “illegitimate” to do so. Their responses have a
direct relation to a specific material base. Their responses are embedded in a local context, in local experience. Their incorrect responses reflected their general tendency to respond to these test items within an everyday frame of reference. As a result, one could say they lack the recognition and realization rules. Lerato (in the ML class) and Margaret (in the mathematics class) partially responded correctly. However, Nthabiseng for mathematics and Sibongile for maths Literacy, by referring to their position in the transmission of knowledge, indirectly relation to a specific material base. This enables them (Nthabiseng and Sibongile) to engage in an appropriate or legitimate manner with the task. They know, in Bernstein’s terms, the recognition and realization of this context. They seem to have access to an appropriate realization rule and they required “mathematical “operation. It was also evident that, Unice and Thami did not seem to have posses both the recognition and realization rules, because both their responses are based to their personal preferences.

RESULTS AND CONCLUSION

In conclusion, it is very difficult to establish the difference between these two groups of learners from their written responses. Hence, the one from maths and the other from ML seemed to posses the realization and recognition rules partially. Again, one from M and one from ML seemed to posses the recognition and realization rules on the written task. However, Thami and Eunice both for maths seem to lack the recognition and the realization rules on the written tasks. However, from the interview it is evident that ML learners enjoy the subject due to its simplicity with respect to mathematics and it is the subject which is linked to their everyday life situation. Even if there is some little contradiction with the latter statement in their responses that ML is very challenging due to its form of case study and scenarios. It is also evident from maths learners that they do not like ML because it demands a lot of reading and also, it is considered as watered down or standard mathematics which therefore is considered as maths for “small kids”. The same sentiment has been slightly echoed by their teacher when she responded by saying “: It is because ML is the basic of pure math”. And learners who fail maths are sometimes forced to take ML. What actually emerges as a concern from the teacher is the English language as one of the factor that could inhabit the learning of mathematics in ML. What is also evident in the interview with both learners from the two groups and the teacher is the common mathematical content which is pursued.
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AN EXPLORATORY STUDY INTO THE INTRODUCTION OF MATHEMATICAL LITERACY IN SELECTED CAPE PENINSULA HIGH SCHOOLS

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This is an exploratory study to investigate the challenges of implementing the new mathematical literacy curriculum in high schools in the Western Cape. Eight schools from previously disadvantaged areas in the Cape Peninsula of the Western Cape Province of South Africa were studied through class visits, interviews with teachers of mathematical literacy, video taping of lessons and an analysis of students’ results in the June and November examinations in 2006. Initial results of the study show that in these schools, the majority of teachers teaching mathematical literacy have professional qualifications in mathematics. There are also some teachers with other subject specialisations who also teach the subject. Teachers report that learners have a negative attitude to mathematical literacy and are struggling to understand the new subject as can be attested to by the dismal showing in the June examinations.

INTRODUCTION

The South African National Department of Education began to implement a new curriculum in all high schools in 2006 with a specific focus in the Further Education and Training (FET) phase. This new curriculum is a logical progression Curriculum 2005, which was introduced by the new South African government after 1994 as a replacement for apartheid education. The new curriculum, which was introduced in the FET phase in South African schools in 2006, has seen every student doing seven subjects instead of six, with four of these being compulsory subjects. Mathematical literacy would be compulsory for all those students not taking mathematics.

At the beginning of 2006, in response to the proclamation by the Minister of Education in 2005, all schools in the country began to implement this new policy. In anticipation of this new policy implementation, the National Department of Education began, in 2004, to register teachers in an Advanced Certification in Education specializing in mathematical literacy, at Higher Education Institutions throughout the country.

A strategic decision was taken to consider the retraining of teachers who were not qualified mathematics teachers to cope with the large numbers of learners who would be doing mathematical literacy. Many of these teachers would only graduate in 2006 whilst the implementation of the new policy was in progress.
The state realized that it could not formally qualify enough teachers to cope with the huge number of students who would be doing mathematical literacy at the beginning of 2006. Hence, during 2005 and 2006, a number of short courses were conducted in all provinces to train teachers in the implementation of mathematical literacy in Grade 10.

This training focused on training teachers who did not necessarily have mathematics professional qualifications and some of them had undergone a course in mathematical literacy as a qualification through higher education institutions.

This is the context in which this study was undertaken. It is a preliminary study whose aim was to find out how teachers and learners in a few high schools have experienced the teaching and learning of mathematical literacy in 2006. The study also aimed at finding out what content was been taught and learnt at these schools, the contexts within which mathematical literacy was been taught and also to look at learner achievement in the subject in 2006.

RESEARCH QUESTIONS

The study sought to understand teachers’ perceptions about the training they received in particular from the Western Cape Education Department (WCED) and how they applied their training in teaching mathematical literacy in grade 10 classes during the first half of 2006. It sought an understanding of their challenges and their joys in teaching this new subject and the feelings either of inadequacy or achievement in their teaching and in learners’ achievement. It also sought to understand the content of the mathematics taught and the contexts chosen for the subject in class and how learners have achieved in semester assessments. The following are the relevant research questions that were pursued:

- What training did teachers receive on the teaching and learning of mathematical literacy?
- What strategies did schools adopt in teaching mathematical literacy?
- What challenges and experiences have teachers encountered in the teaching of mathematical literacy?
- What was the learners’ academic achievement in mathematical literacy in the first year of implementation?

LITERATURE REVIEW

To pursue an understanding of mathematical literacy, it is perhaps, prudent to start with the notion of literacy from a linguistic perspective. Whilst the Concise Oxford Dictionary defines literacy as “the ability to read and write”, the idea of literacy is very complex and dynamic and is dependent on the development of society. Literacy can be regarded as:
a complex set of abilities needed to use and understand the dominant stems of a culture-alphabets, numbers, visual icons -for personal and community development. The nature of these abilities and the demand for them vary from one context to another. (http://www.centreforliteracy.qc.ca)

From this definition, one can understand the notion of mathematical literacy as deriving from the need of societies to understand their complexity and the role that mathematics plays in society. Because much of human activity is described mathematically, it has become more important than ever for citizens to be mathematically literate. Understanding this imperative, the South African National Department of Education saw the wisdom of introducing mathematical literacy in all high schools as an alternative to “pure mathematics” - pure mathematics in this case being the non-contextual mathematics.

Mathematical literacy, the new addition to the FET curriculum in South African schools, is viewed by the Programme for International Student Assessment (PISA) as:

…the capacity to identify, understand and engage in mathematics, and to make well-founded judgements about the role that mathematics plays in an individual’s current and future private life, occupational life, social life with peers and relatives, and life as a constructive, concerned and reflective citizen (OECD, 2001:22).

What this definition implies is that society should see mathematics and people’s daily existence as indivisible because mathematics permeates all that they do in their daily lives. To understand what mathematical literacy is, one should start with the notion of literacy, which refers to one’s ability to function in society because one is able to understand the written word. Hence, from this conception, mathematical literacy would refer to an individual’s ability to function in society because the individual understands the language of mathematics, which describes aspects of society and how it creates a participating citizen.

The International Programme Committee for ICMI Study 14 (2002) sees mathematical literacy as belonging to the realm of mathematical applications and mathematical modelling (International Programme Committee for ICMI Study 14, 2002). Applications of mathematics deals with the use of mathematics to solve real life problems, whilst modelling takes a real life problem, and then provides a mathematical model which can then be used to solve this problem and other related problems.

The South African conception of mathematical literacy focuses on applications of mathematics to solve real-life problems in contrast to conceptions that focus on modelling. In emphasizing the applications dimension of mathematical literacy, the National Curriculum Statement (NCS) of the South African National Department of Education (NDE) conceptualizes mathematical literacy as being: “…driven by real-life applications of mathematics. It enables learners to develop the ability and confidence to think numerically and spatially in order to interpret and critically analyse everyday situations and solve problems (NCS, 2003:10)”
The NCS further notes the situations that confront society on a daily basis like financial transactions, hire purchase, mortgage bonds, investments, the ability to read maps and time tables, using medication and so on. It states that a self managing person must be able to understand these situations and be able to solve problems related to these. The tool to use in understanding and coping with these situations is mathematical literacy (NCS, 2003:9).

Hence mathematical literacy was introduced in the South African education system to facilitate the growth of mathematically literate learners who are able to do mathematics informed by contexts in real life. This will lead to citizens who are mathematically literate.

One would have hoped that students who do mathematical literacy as a subject would not do it as a substitute for mathematics nor regard it as a junior cousin to mathematics.

THE SAMPLE AND DATA COLLECTION

At the beginning of September 2006, eight schools in established townships in the Cape Peninsula were approached with the aim of gaining access to observe mathematical literacy classes in session and to interview teachers who were teaching mathematical literacy. All the schools agreed to participate in the study. This happened after permission had been obtained from the Western Cape Education Department (WCED) to conduct the study.

Data for this study was obtained through observation of five classrooms in which mathematical literacy was taught and interviews with teachers who teach mathematical literacy in the eight schools mentioned above. A total of 13 teachers were interviewed. It was not possible to observe classes in all schools because the researcher was only able to reach the schools towards the closing dates for the third term and sometimes schools were involved in revision programmes or end-of-term examinations. The classroom observation was recorded through field notes with two classes observed video taped. The interviews were structured from an interview schedule which contained nine questions ranging from the teachers’ understanding of mathematical literacy, whether they received any training in the teaching the subject, adequacy of the training, the challenges and joys of teaching mathematical literacy, interaction with learners and learner performance in the first semester examinations. The teachers’ responses were audio-taped. The table below shows the profiles of teachers who were interviewed for the study:
Table 1: Profiles of teachers who participated in the study

As can be gauged from this table, the average teaching experience of these teachers is 13 years with the majority, 62%, being male. Of these teachers the majority, 77%, are qualified mathematics teachers who also teach mathematics as part of their overall teaching responsibilities. The rest are teachers with subject specialization of Xhosa and History.

DATA ANALYSIS AND FINDINGS

As indicated above, the interviews held with teachers were audio-taped. Transcripts of the audio-tapes were made and then the responses were analysed to obtain those responses relevant to the research questions. For the classroom observations, observation notes were taken and two lessons were video taped and it is from these notes and videos that findings on classroom teaching of mathematical literacy were obtained.

The following findings are derived from teachers’ responses to the research questions.
TRAINING IN MATHEMATICAL LITERACY

With regard to training in the teaching of the new subject, the study has found out that the majority of teachers, 62%, did not receive training in the implementation of mathematical literacy. It is interesting to note that those who did not receive specific training in mathematical literacy are teachers who are qualified in mathematics and are offering mathematics in their schools. These teachers stated that they did not see the need to go for training because as far as they were concerned, they would not encounter any conceptual problems in Mathematical Literacy. Interestingly they seemed to assume that the same methodology of teaching would be employed in the teaching of Mathematical Literacy as is the case in the teaching of mathematics. The rest either attended WCED-organized courses or attended a two-year Advanced Certificate in Education (ACE) course in Mathematical Literacy run by either the University of Cape Town (UCT) or The Cape Peninsula University of Technology (CPUT). These institutions are two of four institutions which won a tender from the WCED to teach the ACE course in Mathematical Literacy to 140 teachers in the Western Cape (Mbekwa, 2006) From the table one can observe that the teachers who attended courses organised by the WCED or the ACE in Mathematical Literacy are not qualified mathematics teachers but are teachers whom the WCED has identified for retraining.

All the teachers interviewed stated that the training they received from the WCED course was not enough. Some indicated that the two weeks set aside for the course was not enough. One teacher put it: “The course was too compressed. For non-maths teachers it was not adequate. Lecturers had advanced assumptions that teachers had understanding of maths concepts”. One teacher stated that many of the teachers attending the course found it difficult: “If teachers could find it so difficult, imagine what it would be like for learners.”

Teaching mathematical literacy at school

Of the five classes observed, one class dealt with measurement, and two classes dealt with functions and two classes dealt with space and shape. The two lessons on functions were similar in the sense that both dealt with the derivation of an algebraic formula from a table of values with a functional relationship. One class, for instance, was given a table showing the relationship between the number of packets of biscuits and the selling price of these packets of biscuits. Students were asked by the teacher to complete the table and then derive from the table the formula that indicates the linear relationship existing between the number of packets of biscuits and the selling price. Students worked in groups on the following table:
No of packets (n) | 1 | 2 | 3 | 4 | 5 | 6
---|---|---|---|---|---|---
Cost of biscuits (C) | 4,50 | 9,00 | 13,50 | ……… | 22,50 | ………

Table 2: Showing functional relationship between packets of biscuits and their cost.

Students were then asked a number of questions relating to the nature of the function and also to sketch a graph of the function. The teacher then went around the class to see if students were deriving the correct formula. It is interesting to note that the teacher described the number of packets of biscuits as “conservative numbers” instead of consecutive numbers. This was brought by the researcher to the notice of the teacher after class so that she could rectify this terminological error. The students were able to deduce the relationship between the number of packets and the cost in rand as defined by C=4,5n. I did not get a sense of how the students came to this equation. A question was asked and one student came up with the answer. No student queried how this one student came up with the answer - the answer was simply accepted without interrogation. They also identified the dependent and the independent variables in the table without unpacking the concepts. The teacher did not probe whether learners understood the meaning of the concepts of dependence and independence.

Two schools used computer technology enabling learners to utilise the power of technology to reinforce their learning of mathematical literacy. This implied using computer laboratories with Master Maths programmes with mathematical literacy problems. One could see the enthusiasm and animation of the students while they were doing mathematical literacy etched on their faces. One teacher described this thus: “I have never seen so many kids occupied with mathematics at one time as I saw in this class.” One of these schools, in addition to using computer technology, engaged learners in doing practical research tasks. An example of such a research task involved learners investigating the price of the same model of car from two different car dealers. Learners found out that the prices differed although the car was the same model and year. Learners had to motivate why the marked prices differed and what dealers had to take into consideration when pricing a car. They also had to work out instalments on the different cars given a 10% deposit or on the same priced car if different deposits were paid. They had to look at instalment options for longer term and short term payments. They also had to look at payment on the same car if, for instance, one paid a deposit on the car to a dealer who then arranged bank finance or when the buyer bought the car through a personal loan from a bank and paid the loan from the bank back. Learners then had to make a decision as to the best option of the two.

The two other lessons which were observed through video tape dealt with topics on space and shape. One dealt with the area concept from a tangram activity. Learners were required to find out for instance the ratio of the area of a triangle to the area of a...
parallelogram or to the area of a square. The last lesson dealt with calculations related to the theorem of Pythagoras.

CHALLENGES OF TEACHING MATHEMATICAL LITERACY

All the teachers interviewed concurred that the greatest challenge to the teaching of mathematical literacy was the lack of understanding of learners and their lack of motivation. Teachers indicated that students do not really want to do mathematical literacy but do it because it is compulsory if they do not do mathematics. As one teacher put it: “These learners are negative towards mathematical literacy. They do it because they have no choice.” They also stated that students’ performance in mathematical literacy is not good. According to some of the teachers, what contributes to poor performance is the poor preparation of learners in lower grades. What also contributes to this poor performance is the English language in which Mathematical Literacy problems are couched. Most of the problems are verbal in nature and students find it difficult to make sense of what is required. Teachers have to spend a lot of time translating the language of the problem into Xhosa, their mother tongue. It is only then that students can start developing problem solving strategies.

Two of the teachers interviewed stated that whilst the majority of students struggle with mathematical literacy, one finds one or two students who are very brilliant and obtain distinctions in all tests. In one school, one such student was placed in a mathematical literacy class because he came from another province and registered later than other students after the commencement of the academic year. The teacher of this student was contemplating placing this student in a mathematics class. This thinking is a reflection of the idea that mathematical literacy is a dumping ground for mathematics underperformers and hence this brilliant student is misplaced in a mathematical literacy class.

Amongst other challenges mentioned, one challenge mentioned by teachers who were interviewed refers to the difficulty of finding teachers willing to teach mathematical literacy. They state that this is because mathematics teachers do not want to teach mathematical literacy as they see it as an inferior kind of mathematics. Teaching mathematics at schools is seen as a high status role. Hence the department of education has wisely decided to utilise teachers who have been redirected from other subject streams and re-skilled, to teach the subject. One such teacher, whose specialisation subject is History, was retrained by the department of education and states that she is quite “comfortable to teach mathematical literacy to grade 10 students. I approach Ms…to assist me when I encounter a problem.” This teacher uses the mathematics teacher, that she mentions, as a consultant to overcome mathematical conceptual problems. Most schools pair mathematical literacy teachers with mathematics specialists so that the mathematics specialists perform a mentoring role to the non-mathematics specialists.
STUDENTS’ PERFORMANCE IN EXAMINATIONS

I requested the teachers to supply me with the statistics of mathematical literacy students in their schools and how these students performed in the first semester and final examinations in 2006. Of the eight schools participating in the study, seven provided pass statistics for mathematical literacy for the June and November examinations. The following table represents these figures:

<table>
<thead>
<tr>
<th>Name of school</th>
<th>No. of Candidates</th>
<th>Passes</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>June: 152</td>
<td>21</td>
<td>13,8</td>
</tr>
<tr>
<td></td>
<td>Nov: 163</td>
<td>107</td>
<td>65,6</td>
</tr>
<tr>
<td>B</td>
<td>June: 143</td>
<td>14</td>
<td>9,8</td>
</tr>
<tr>
<td></td>
<td>Nov: 134</td>
<td>100</td>
<td>74,6</td>
</tr>
<tr>
<td>C</td>
<td>June: 358</td>
<td>47</td>
<td>13,1</td>
</tr>
<tr>
<td></td>
<td>Nov: 424</td>
<td>211</td>
<td>49,7</td>
</tr>
<tr>
<td>D</td>
<td>June: 71</td>
<td>4</td>
<td>0,05</td>
</tr>
<tr>
<td></td>
<td>Nov: 77</td>
<td>30</td>
<td>38,9</td>
</tr>
<tr>
<td>E</td>
<td>June: 154</td>
<td>82</td>
<td>53</td>
</tr>
<tr>
<td></td>
<td>Nov: 156</td>
<td>116</td>
<td>74</td>
</tr>
<tr>
<td>F</td>
<td>June: 215</td>
<td>56</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>Nov: 217</td>
<td>185</td>
<td>84,9</td>
</tr>
<tr>
<td>G</td>
<td>June: 79</td>
<td>8</td>
<td>10,1</td>
</tr>
<tr>
<td></td>
<td>Nov: 74</td>
<td>39</td>
<td>52,7</td>
</tr>
</tbody>
</table>

Table 3: Statistics of students taking mathematical literacy and the pass rate per school

The pass rates shown in the table above are represented diagrammatically in the following bar chart:
Training in mathematical literacy

In reflecting on the findings of this study, it is instructive to note that these findings are derived from the observation and interaction with a few schools and thus these findings cannot be projected to all high schools in the province or the country as a whole. In this study it can be seen from the teacher profile that the majority of these teachers who teach mathematical literacy are Mathematics teachers. One can thus recommend that the education department should utilise Mathematics teachers in schools to teach mathematical literacy and also to serve as mentors to those teachers who have received a basic training in teaching the subject. It is also interesting to note that almost all the teachers interviewed did not have conceptual problems with Mathematical Literacy. Perhaps it is due to the nature of mathematical literacy being mostly numeric or arithmetic and less algebraic.

The researcher was informed during contact with teachers that courses and workshops on mathematical literacy are continually being conducted by the education department to sharpen teacher skills and to be kept abreast of developments in mathematical literacy.
It is recommended that the department of education should use its data base on its teaching staff to target teachers who passed mathematics as one of their grade 12 subjects for training in the teaching of mathematical literacy.

**Teaching mathematical literacy**

It was found that in three of the five lessons observed, the lessons focused on context, which conforms to the notion of Mathematical Literacy as dealing with contexts especially as it relates to day to day living. The case of the school, for instance, that gave learners a task to look at pricing practices of a car dealership definitely made the subject more real to its learners. If more schools could subject learners to this kind of activity, it would go a long way in making Mathematical Literacy relevant and promote sense making. It is also recommended that, whenever possible, schools make use of technology and thus achieve one of the critical outcomes of the NCS. Research also shows that computer technology promotes concept formation and improved student attitudes to learning (Baker, Gearhart, & Herrman (1994); Harel (Ed). (1990); Harel & Papert. (1991); Kulik (1994); Sivin-Kachala (1998); Scardamalia & Bereister. (1996); Wenglinsky (1998).

**Challenges of teaching Mathematical Literacy**

Looking at the challenges of teaching mathematical literacy it is instructive to note the negative attitudes of both the teachers and students towards mathematical literacy. This might arise from the fact that these teachers have not been educated about what mathematical literacy is and what the motivation is for introducing it into the school curriculum. This might go a long way to eliminating the perception of mathematical literacy being a poor cousin to mathematics or a dumping ground for learners who are performing poorly. The issue of code switching to assist the learners in comprehension concurs with recommendations deriving from research in the use of the mother tongue in the teaching and learning especially in mathematics and science. Research shows that using the first language of learners in explicating mathematical concepts helps in conceptual understanding (Allen, 1988; Chapman, 1993; Desai, 2006; Langenhoven, 2006; Nomlomo, 2006; Moschovich, 2007; Setati, 2002; Setati, Adler, Reed & Bapoo, 2002; Setati; 2005). It becomes important therefore for the education department to have teaching and learning support materials translated into the mother tongue of learners. That has not yet happened. It is hoped that teachers will volunteer and be involved in the translation of teaching and learning support materials to assist learners in understanding the subject matter and so facilitate problem solving in mathematical literacy.
Students’ performance in examinations

Looking at students’ results in the June examinations one can observe the poor performance of students, which confirms the teachers’ observation that students generally do not do well in mathematical literacy. The pass rates range from a minimum of 10.1% to a maximum of 53%, which gives a range of 42.9%. It is quite interesting to note that the November examination results range between 38.9% and 84.9%. The pass rates in June are all below 40% whilst in November only one school has a pass rate below 40%. One observes the increases between the June and November results ranging from a minimum of 11% to a maximum of 64.8%. It seems quite striking that all schools did well in the final examinations whilst in the June examinations they did so badly.

One suspects that there might have been an upward adjustment of marks. What formula was used to adjust the marks and whether this was consistent in all schools in the province one cannot say at this stage. This needs some more probing and probably an expansion of the study to include schools outside of the targeted sample. Only one of these schools has a low difference (11%) between the June and the November results and it is only this school which had a pass rate above 50% in June.

CONCLUSION

This study looked at the challenges of teaching and mathematical literacy at some high schools in the Cape Peninsula. The target group of the study was mathematical literacy teachers who highlighted the fact that learners and mathematics teachers are not motivated to become involved with the subject and learners who initially did not do well in the subject. Results of examinations corroborate the teachers’ observation of a lack of motivation on the part of learners. It is the intention of the researcher to further probe the radical improvement in the performance of learners in the November examinations and also to solicit learners’ opinions on their experience of mathematical literacy.

REFERENCES


231


Ways of presenting and formulating concepts in Mathematics that makes the subject comprehensible to others is at the core of effective teaching. Current research warns that the Mathematics that teachers need for teaching Mathematics is not of the kind that their learners do, neither is it just being a step ahead in content of the students learn and neither is it some kind of a watered-down version of formal Mathematics. It is in fact a more serious and demanding area of Mathematical work and therefore should be seen as tacit and as a distinct branch of Mathematics. Because achievement in mathematics has always been seen as symptomatic of the quality of education and training in a country, developing this special kind of mathematical knowledge for teaching within a teacher has been one of the most critical issues on the agenda for teacher professional development endeavors the world over. However, while the issue of teachers’ knowledge of mathematics has been a prominent one for several decades, little progress has been made towards a consensus on the question of ‘what mathematics teachers need to know.’ It is like trying to describe the air, we know it is there, we can feel it is there but it is a very elusive concept to define or describe!

This paper gives an analysis of the current discourses on the dilemmas that policy formulators and implementers face with regards the conceptualization of this mathematics-for-teaching and the debates for and against subject matter knowledge for Mathematics and the Mathematics-for-teaching. The paper will end with some suggestions for the way forward especially for developing countries as they try to grapple with ways of improving the teaching and learning of Mathematics for their citizens.

BACKGROUND:

As an external examiner for Mathematics for the University of Zimbabwe, I traveled to Teacher Training Colleges, which are associates of the university, to examine pre-service teachers’ work in Mathematics. In one of the colleges that I visited in November 2005 in that role of an external examiner, I noticed that one of the pre-service teachers who was being presented by the college as having failed the theory/content in Mathematics had actually passed teaching practice in Mathematics with a distinction. The assessment criteria in this college require a pre-service teacher to pass both the theory/content and the teaching practice separately before proceeding to teach as a qualified teacher. This candidate was therefore not allowed to proceed to teach because
of the theory/content component which was not well done and so had to remain repeating that section of the course. This triggered the interest in me to focus on issues to do with matters of effective teaching of mathematics and what teachers need to know.

There are a number of questions that came to the fore from this observation but all of them could just be captured in one question: “What implication therefore does this have on the relationship between subject content knowledge and pedagogical content knowledge or simply the ability to teach Mathematics?”

POLICY SHIFTS BASED ON THE ASSUMPTION OF SUBJECT MATTER KNOWLEDGE FOR MATHEMATICS

Most curriculum reform in Mathematics the world over and teacher professional development efforts thereof have always been designed on the assumption that teachers had weak subject matter knowledge for mathematics. Efforts to improve learners’ performances in Mathematics have therefore focused on increasing teachers’ content knowledge for Mathematics which, it was hoped, would translate into better understanding by the learners. Very little if any desirable change has however been witnessed through such approaches.

In Zimbabwe for example, there was a recent policy shift in Mathematics curriculum which came about as a result of recommendations from a study which was carried out by the Nziramasanga Commission of 1998. From this commission’s findings and those of other researches prior to the commission’s research, there was overwhelming evidence that a greater number of pupils did not like Mathematics and were not doing so well in the subject (Kilborn, Dhlwayo, Gudza, & Nguru, 1996). This in all the researchers’ view was mainly because of a teaching force, especially at Primary Level, with inadequate skills to cope with the demands of the subject. Specifically they pointed to weak subject knowledge within the teachers and in their view, learners in the schools ‘were not possibly benefiting from being taught by such teachers.’ Recommendations thereof all pointed to the same direction that more content knowledge for teachers would translate into better performance by learners.

But Zimbabwe is not alone in this belief. Kerslake (1986), as cited in Kilborn et.al. (1996), confirm that this notion of linking weak subject matter knowledge with poor teaching in Mathematics was a well-known pattern from other research works about teachers’ performances the world over. According to Taylor and Vinjevold (1999) and following a similar argument, concerns about the state of educators in South Africa have been raised consistently in the past decade or so, in such documents as the NEPI Report (1992), the National Teacher Audit (1996) and the EDS Report (1999). The Presidential Education Initiative (PEI) Report as cited by Taylor and Vinjevold (1999) was particularly critical of teachers. It suggested that South African educators’ conceptions of their activity were superficial and their subject knowledge was weak: “teachers’ knowledge of key mathematics and science topics at the Grade 5 to 7 levels is little better than that of their pupils…” (Taylor & Vinjevold, 1999, p. 141). This bleak picture
regarding teachers’ knowledge was also confirmed by the Review Committee for Curriculum 2005 which reported that many teachers had a shallow understanding of the curriculum and that there was a wide gap between “what teachers say they know and what they actually do in their classrooms” (Department-of-Education, 2000, p. 78)

However, according to Jaworski and Gellert (2003), that mathematics teachers need to know mathematics is uncontested; but the claim researchers are making is that mathematical knowledge alone is not sufficient. As Wilson et.al (1987) wrote, “While personal understanding of the subject matter may be necessary, it is not a sufficient condition for being able to teach” (p.115) Quite a number of other researchers are of the same view that, as has been thoroughly developed, no causal link can be established between subject knowledge for mathematics and the ability to teach mathematics (Askew, Brown, Rhodes, Johnson, & William, 1997; B. Davis & Simmt, 2003, 2006; B. Davis & Sumara, 1997; B Davis & Sumara, 2000). In fact, a number of researches have also shown that even where teachers had rich understandings of Mathematics, they were not always able to help children construct similar understandings.

So on one hand there is the policy formulator who argues that in order to improve performance in school mathematics, professional development activities must focus on improving teachers’ subject content knowledge. On the other hand researchers have demonstrated that emphasis on more mathematics may be inappropriate for mathematics teachers because there was a weak relationship between those two when it comes to teaching of mathematics. All what this points to is that we possibly need to understand more of this mathematical knowledge for teaching (MKT), for until we get the conceptual view of this knowledge, efforts to improve effectiveness in the teaching of mathematics may never bear fruit.

Highlighting the importance of finding out what teachers need to learn, the National Academy of Education report (NAE 1999 p 66) as cited in the (Department-of-Education, 2000) commented thus:

> We are politically and economically poised to invest in teacher development in ways unlike previous reform. But despite the recent interest, our understanding of professional development and factors that affect what teachers learn and do is uneven and underdeveloped. Therefore our knowledge of the processes whereby it can improve education is weak.

In support of this view of gaining an understanding of what mathematics teachers need to know, Fiske and Ladd (2002) note that within the South African research arena, there is now a move away from the illustrations that teachers fail to teach and learners fail to perform, towards a better understanding of what it is teachers should do and why. They do this by way of a quotation as follows:

> While the pathology is widespread, and no doubt in many ways justified, we are concerned that research has thus far failed to ascribe to teachers and learners a positive subjectivity. We know what they don’t do, but we have not adequately grasped what they should do and why they do what they do (Ensor & Galant, 2005).
This statement indeed seems to confirm the challenges researchers are facing in trying to conceptualize this mathematical knowledge for teaching (MKT).

**ATTEMPTS TO CONCEPTUALIZE THE MATHEMATICS-FOR-TEACHING**

According to Davis and Sumara (1997) relying on some of the popular “how-to” manuals that have been prepared for teacher education programmes, one might come to the mistaken conclusion that what it means to teach and how one learns to teach are largely settled matters. Because of its dynamic and nested character, mathematics-for-teaching cannot be considered a domain of knowledge to be mastered by individuals. It always occurs in contexts that involve others and hence an awareness of how others might be engaged in productive collectivity is an important aspect. In fact according to Cobb (1999) there is a current conceptual shift away from Mathematics as content and toward emergent terms. As he explains the content metaphor entails the notion that Mathematics is placed in the container of curriculum, which then serves as the primary vehicle for making it accessible to students. By contrast, in his emergent terms, mathematical ideas were seen to emerge as the collective practices of the classroom community evolved. He further highlights that this perspective is compatible with the view of mathematics as a socially and culturally situated activity.

In fact drawing on their complexity science to theorize about mathematics-for-teaching, Davis and Simmt (2006) concur with that notion and go on to suggest that this knowledge for teaching mathematics is tacit and should be seen as a distinct branch of Mathematics which is qualitatively different from the knowledge expected of students that teachers teach. In their quest for conceptualizing the mathematics-for-teaching, researchers have pointed to the need to take heed that an exclusive concern with the components of teaching, has always been and continues to be inadequate for preparing mathematics teachers for the complex situations within which they will be working. In trying to emphasize this bridging of the gulf between theory and practice the National Academy of Education report (NAE 1999 p 76) as cited in (Department-of-Education, 2000) reported:

No matter what gets emphasized in teacher education or what kind of institutional structure exists for professional development, skills, knowledge and dispositions are taught in isolation from the setting in which they must be used, thereby setting up the problem of transfer.

Teaching and learning cannot be studied as though they occur in isolated and closed systems. Dichotomies have been maintained between studies of knowledge and studies of practice without yielding positive results especially in the teaching/learning of mathematics. As Hopkins (2001) cited in Dembele (2005) puts it, teaching mathematics is more than just presenting material, it is about infusing the curriculum content with appropriate instructional strategies that are selected in order to achieve the learning goals the teacher has for the learners. Effective teachers for mathematics therefore try to make their subject more comprehensible to learners by building bridges between their sophisticated understanding of subject matter and their students’ developing...
understanding and adapting their instruction to the variation in ability and background presented by their learners.

So in trying to understand the mathematics that teachers need to know in order to improve effectiveness in their teaching, there is also need to understand simultaneously the context in which this competence is to be applied. Only then does competence become responsive and developmental. In trying to define competence Julien (1997) argues that competent action is not grounded in individual accumulations of knowledge but instead, generated in the web of social relations and human artifacts that define the context of our action. This focus then shifts our explanations of the competence we observe from knowledge held by an individual to the social context of the activity we are examining. In Bellis (1999) view, learning or performance that does not know the context in which it is expressed and of the impact it makes on the ‘system’, cannot truly stretch capability and neither can it have an impact on the level of national potential. Boaler (1996) points to recent evidence which suggests that the most important opportunities for professional development are provided by teachers’ everyday experiences in school.

“DESCRIBING THE AIR” AS A METAPHOR FOR CONCEPTUALISING MKT

It should be clear from the preceding paragraphs that it is quite some challenge trying to conceptualize the mathematical knowledge for teaching hence the use of the metaphor “trying to describe the air.” Firstly researchers using complexity science to understand the mathematics-for-teaching agree that boundaries of complex systems cannot usually be unambiguously determined. However Davis and Simmt (2006) have attempted to trace out some of the defining features of MKT and these include mathematical objects, curriculum structures, classroom collectivity and subjective understanding.

Researchers also argue that an isolated focus on either questions of mathematics as content and questions of learning the subject, is inadequate, as efforts are made to understand teachers’ mathematics for-teaching (B. Davis & Simmt, 2006). In fact such emphases may be counterproductive as there exists an inextricability of teachers’ knowledge of established mathematics and their knowledge of how mathematics is established. So what this points to is that for teachers the knowledge for established mathematics is inseparable from knowledge of how mathematics is established, in other words, the mathematics-for-teaching cannot be distinguished from the subject matter knowledge for mathematics but what is certain is that it is situated in practice and also needs to be learned in practice.

This is actually where the mathematics-for-teaching becomes an elusive concept just like “the air we cannot see but can feel it is there”. This is so because mathematics-for-teaching is seen as dynamic in nature and nested in character and so cannot be considered a domain of knowledge to be mastered by individuals. It always occurs in contexts that involve others and hence an awareness of how others might be engaged in productive collectivity is an important aspect. In other words it is a difficult concept to
get to grips with, and so how can teacher development efforts be targeted at something we cannot get to grips with especially considering that it is dynamic, occurs in contexts, and cannot be mastered by an individual and that it can only be learned/mastered in the classroom situation.

**IMPLICATIONS FOR POLICY FORMULATORS AND IMPLEMENTORS**

The first observation that this paper draws our attention to is that effectiveness of mathematics teachers cannot possibly be measured accurately through content tests for teachers yet most commissions of inquiry into mathematics education would like to adopt this as an approach for doing so. Admittedly it is a user friendly approach in terms of costs but certainly it has its own limitation in terms of accuracy when measuring teacher competence in the classrooms. Mathematics-for-teaching is situated in practice and needs to be learned in practice and therefore needs to also be measured in practice.

The other observation for both formulators and implementers of policy is that while personal understanding of the subject matter may be necessary, it is not a sufficient condition for being able to teach. Different practices in both teacher training and professional development efforts have in the past been held in place by an assumption that formal mathematical knowledge by an individual was sufficient for effective teaching of mathematics. “He, who knows mathematics, knows how to teach it.” However research seems to disprove this and this raises questions such as where pre-service teachers for mathematics should spend more of their training time, in the classrooms on teaching practice or in the colleges learning theory. It would appear that efforts to improve teacher effectiveness should be grounded not in theory or content but more in practice.

This paper also points to a critical issue in mathematics teacher development that the mathematical knowledge for teaching is qualitatively different from the knowledge expected of students that teachers teach. The assumption that teacher training institutions have always held in place when developing a curriculum for their pre-service teachers is that the mathematics a teacher needs in order to teach effectively is that which is a step or steps higher than the learner he/she is going to teach. It is not surprising therefore that literature is replete with the censure of teacher education programmes as being inadequate in preparing the “new” teacher for mathematics. Institutions are therefore being encouraged to look for something qualitatively different to put in the curriculum for their mathematics pre-service teachers.

The last but perhaps the most critical issue raised in this paper is that, because of its dynamic and nested character, mathematics-for-teaching cannot be considered a domain of knowledge to be mastered by individuals in a contrived environment like a lecture theatre. Mathematics-for-teaching is situated in practice and needs to be learned in practice and so developmental efforts which remove teachers from their classes of practice may not be doing justice to the time and resources ‘wasted’ in such endeavors.
Professional development efforts which focus on enhancing capacity for mathematics teachers should therefore be carried out as much as possible within their classrooms.

REFERENCES


239

This paper provides an overview of my Master’s Research report. The study explores the deliberate use of multiple languages to support the development of grade 11 learners’ mathematics proficiency in a multilingual classroom. The study is an action research aimed at transforming my teaching. A well-selected mathematical task set in multiple languages was used for teaching and multiple languages were used in a planned and proactive manner.

INTRODUCTION

I teach mathematics to grades 10 and 11 learners, who are multilingual learners and have limited fluency in English, which is the language of learning and teaching in the school. The participation of these learners is usually very limited during classroom discussions, especially when they have to interact with me. When the discussions are between learners during group activities, they usually use their home language(s) in addition to English, and most of them do take part in such discussions. Sometimes they ask questions or make inputs in the public domain in their home language(s), and when I insist that they use English, most of them refrain from asking questions or making inputs. This makes learning and teaching difficult.

From my conversations with other teachers I have come to realize that what my learners are doing is not unique. Most mathematics teachers are faced with these challenges in their multilingual classrooms. Learners on the other hand, are faced with a challenge of understanding mathematical terminology, concepts and meanings explained in the language that they are still learning. According to Pretorius (2002), the use of more than one language has become a strategy that teachers in multilingual classrooms rely on to explain concepts.

The South African language-in-education policy encourages multilingualism. Schools have the right to choose their LoLT, and this right must be exercised with the aim of promoting multilingualism (Education Labour Relations Council, 1996). One way of promoting multilingualism in mathematics classrooms is by using multiple languages during teaching. The debates around language and learning in South Africa tend to create a dichotomy between learning in English and learning in the home language(s). These debates create an impression that the use of the learners’ home languages for teaching and learning must necessarily exclude English, and the use of English must necessarily exclude the learners’ home languages. The aim of the study reported here
was to challenge this dichotomy between the use of home languages and the use of English for teaching and learning of mathematics.

The study explored the deliberate use of multiple languages to support the development of Grade 11 learners’ mathematical proficiency in a multilingual classroom. The study was guided by the following research questions:

- How can the learners’ home language(s) be used to support the development of learners’ proficiency in mathematics?
- To what extent can tasks set in multiple languages support the development of learners’ proficiency in mathematics?
- To what extent can learners’ interactions with the tasks in multiple languages support the development of learners’ proficiency in mathematics?

This was an action research aimed at transforming my teaching. I refer to this as transformation because I taught in a way that was different from my usual way of teaching.

**What Was Transformed?**

While I sometimes used learners’ home languages in my teaching, I used them in an unplanned manner. The tasks that I previously used in my class, which were mostly from textbooks, were not set in multiple languages, but in English only. Due to learners’ limited proficiency in English, the tasks I selected were comparatively easy and the mathematics thereof was generally of a lower cognitive demand. I used them generally for assessment purposes with the aim of assessing learners’ understanding of mathematical procedures, algorithms and rules. While these kinds of tasks are in general worth doing with learners, in the study, I gave a mathematics task of a higher cognitive demand set in multiple languages and used it for learning and teaching. However, the mathematics of the task was not compromised. The cognitive-level demands of the tasks, as suggested by Stein, Smith, Henningsen and Silver (2000), were taken into consideration. Furthermore, multiple languages were deliberately used for teaching and learning. This implies that learners’ home languages were used in a planned and proactive manner.

**THEORETICAL FRAMEWORK AND LITERATURE REVIEW**

This study was broadly informed by Vygotsky’s theory of socio-cultural development. According to this theory, the child’s cognitive development depends on external factors, i.e. language, cultural and social processes. This development, from the Vygotskian perspective, occurs in and through socially mediated activity, and language plays a key role in mediation (Vygotsky, 1978). This theory is helpful because it provides a framework within which one can describe the role that the learners’ home languages as
used by the teacher, the learners and in the tasks can play in a multilingual classroom. Central to Vygotsky’s theory is the fact that the higher mental functions are formed through social interaction (Vygotsky, 1978). This resonates with Hatano’s work who argues that humans construct their own knowledge, and the process of knowledge reconstruction “is not a purely individual enterprise but is constrained socioculturally” (Hatano, 1996: 199). In other words, learners do not construct mathematical knowledge on their own but they do so in interaction with resources around them, both human and non-human. Hence this study highlights the importance of well thought-out tasks and an environment where learners can interact with each other and with the teacher. According to Vygotsky’s general genetic law of cultural development,

Any function in the child’s cultural development appears twice, or on two planes. First it appears on the social plane and then on the psychological plane. First it appears between people as an interpsychological category, and then within the child as an intrapsychological category (Wertsch, 1979; Vygotsky, 1978 in Tharp and Gallimore, 1991; Vygotsky, 1966, 1978).

This law suggests that in order to explain the psychological, we must look at the external world or the social plane in which the child’s life develops and not only at the child (Tharp and Gallimore, 1991). The interpsychological or the social plane is the classroom in which learning and teaching occurs through the use of English and the learners’ home language(s), and through tasks set in multiple languages. On the interpsychological plane there are learner-learner and teacher-learner interactions whereby the learners and the teacher communicate mathematical ideas and concepts through language.

The Role of Language

Language is used by teachers to lead the discussions, give direction and guide learners’ constructions (Jaworski, 1997; Wertsch, 1984). Berger (1998) indicates that language serves as a cognitive re-organiser that aims at changing the quality of teaching and therefore helps in developing thinking. This suggests that when a learner exchanges mathematical ideas with his/her peers through language, he/she moves from one state of knowing to another state of knowing, since learning is an additive process (adding to what is already there). Through language, the teacher’s assistance and effects of peers (Ginsberg, 1985:9), and the use of mathematical tasks, learners become active participants in the process of knowledge construction. In my view, the use of multiple languages may play a vital role in this regard. The learners’ language background should also be considered when tasks are selected or created because if a task is in the language which learners have limited fluency in, they may have difficulty in understanding and interpreting it. Even if the task matches learning goals, such goals may not be realised.
Mathematical Tasks

One of the important aspects of learning mathematics is developing mathematical understanding and the learners’ ability to think mathematically. However, developing mathematical thinking is not easy. Learning mathematics is not only about application of rules and procedures. According to Ball and Bass (2003), knowing procedures and mathematical ideas as just routine or mere fact without mathematical reasoning is not enough. Through tasks, learners can learn how to reason mathematically.

Well-selected tasks can provide learners with an opportunity to gain mathematical power, that is, to be mathematically competent, which Kilpatrick, Swafford and Findell (2001) refer to as mathematical proficiency. Kilpatrick et al. (2001) suggest five interwoven and interdependent components/strands that are significant in developing learners’ proficiency in mathematics. These strands are conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. The use of tasks, which are classified into lower-level and higher-level demands tasks, during the learning and teaching of mathematics is important in developing learners’ abilities in the five strands of mathematical proficiency because they (tasks) form the basis of learners’ opportunities to learn mathematics (Stein et al., 2000). Therefore, cognitive demands of the tasks, must be taken into consideration because “it is the level and kind of thinking in which students engage that determines what they will learn” (p.11). In other words, what the learners will learn is dependent on the opportunities that the tasks they are working on provide.

Lower-level demand tasks are classified into ‘memorization’ tasks and ‘procedures without connections’ tasks. Since memorization tasks and procedures without connection tasks do not have the capability to engage learners in complex forms of thinking, they can be used to develop learners’ procedural fluency, which “refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently” (Stein et al., 2000:121).

Strategic competence, which is “the ability to formulate mathematical problems, represent them, and solve them” (Kilpatrick et al., 2001:124), can be developed through the use of ‘procedures with connections’ tasks and ‘doing mathematics’ tasks, which are tasks of higher-level demand. Such tasks assist in the development of meaning as they can be represented in various ways, and a certain amount of cognitive effort is needed when working on them. Therefore, the procedures used cannot be followed without understanding. Since ‘doing mathematics’ tasks “demand self-monitoring or self-regulation of one’s own cognitive processes” (Stein et al., 2000:16), they may require learners to explain and justify how the solution can be obtained, developing the proficiency in adaptive reasoning. As learners’ reasoning skills improve and are able to justify and explain mathematical ideas, their conceptual understanding also improves. Conceptual understanding, which refers to “comprehension of mathematical concepts, operations, and relations” (p.116), enables learners to know the importance of certain mathematical ideas and the contexts in which they are relevant. As Kilpatrick et al. (2001) suggests, learners must “see sense in mathematics, perceive it as both useful and
worthwhile, believe that steady effort in learning mathematics pays off, and see oneself as an effective learner and doer of mathematics” (p.131), which is a competence referred to as productive disposition. The development of this strand, which occurs over time, depends on the development of the other strands and also helps the other strands to develop.

The instructional task used in the study was selected from the Malati Draft Materials (30-04-2005). It consists of 10 questions. As Stein et al. (2000) suggest, the task features determine its level of cognitive demand. The task I selected had the following features: a ‘real world’ context is used, it is a problem situation, some questions require an explanation, some questions may require the use of a calculator, learners must use prior knowledge and experiences when working on the task, and an approach to follow is not suggested. The task was used to teach ‘Linear Functions’ because it is one of the topics included in the Grade 11 syllabus. Since the aim of this study was to use a mathematical task to support the development of learners’ proficiency in mathematics, the type of task selected had to provide learners with such opportunities. The English version of the task used in the study is shown below.

THE MATHEMATICS TASK

COST OF ELECTRICITY:

*The Brahm Park electricity department charges R40 – 00 monthly service fee then an additional 20c per kilowatt-hour (kwh). A kilowatt-hour is the amount of electricity used in one hour at a constant power of one kilowatt.*

1. The estimated monthly electricity consumption of a family home is 560 kwh. Predict what the monthly account would be for electricity.

2. Three people live in a townhouse. Their monthly electricity account is approximately R180 – 00. How many kilowatt-hours per month do they usually use?

3. In winter the average electricity consumption increases by 20%, what would the monthly bills be for the family home in (1) above and for the townhouse?

4. In your opinion, what may be the reason for the increase in the average electricity consumption in (3) above?

5. Determine a formula to assist the electricity department to calculate the monthly electricity bill for any household. State clearly what your variables represent and the units used.
6. a). Complete the following table showing the cost of electricity in Rand for differing amounts of electricity used:

<table>
<thead>
<tr>
<th>Consumption (kwh)</th>
<th>0</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>700</th>
<th>800</th>
<th>900</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost (in Rand)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b). Draw a graph on the set of axes below to illustrate the cost of different units of electricity at the rate charged by the Brahm Park electricity department.

![ELECTRICITY COSTS](image)

After careful consideration, the electricity department decide to alter their costing structure. They decide that there will no longer be a monthly service fee of R40 – 00 but now each kilowatt-hour will cost 25c.

7. What would be the new monthly electricity accounts for the family home and the townhouse?

8. a). Complete the following table showing the cost of electricity in Rand for differing amounts of electricity used using the new costing structure:

<table>
<thead>
<tr>
<th>Consumption (kwh)</th>
<th>0</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>700</th>
<th>800</th>
<th>900</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost (in Rand)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b). Draw a graph on the same set of axes in question 6.b. to illustrate the cost of electricity for different units of electricity using the new costing structure.
9. Do both the family home and the townhouse benefit from this new costing structure? Explain.

10. If people using the electricity had the option of choosing either of the two costing structures, which would you recommend? Clearly explain your answer using tables you have completed and graphs drawn in questions (6a) and (6b) and (8a) and (8b) above.

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**RESEARCH DESIGN AND METHODOLOGY**

**The Research Context**

The study was undertaken at a school where I teach. It is a multilingual high school located in Soweto, a township southwest of Johannesburg. It caters for learners from Grade 8 to Grade 12. I selected to do the study in a Grade 11 class I teach because I also taught the same group of learners in 2004 while they were in grade 10. It was convenient for me to do the study in that class, as I did not have to interfere with the daily school timetable or arrange visits to another school. There were 36 learners of varying abilities, ethnic groups and gender in the class. The class was multilingual, and the home languages of learners were SeTswana, XiTsonga, IsiZulu and TshiVenda. All the learners learnt English and their respective home language(s) as subjects. As indicated before, learners’ fluency in English was limited. As the teacher who was teaching the class I was a critical participant in the study. I am multilingual and able to communicate in the following languages, Setswana, Sesotho, Sepedi, Xitsonga, IsiZulu, Tshivenda, English and Afrikaans. My home language is Setswana. I have been teaching mathematics at secondary school level for 15 years.

**Methodological Approach**

I have chosen an action research approach because I was researching my own practice so as to transform and improve it. I referred to this as transformation because as explained earlier, I did things differently from how I usually used to do them. Unlike other research approaches like Case Study, Survey, Experiments and many others, action research was an approach that gave me an opportunity to research my own teaching so as to change or transform it. As Davidoff and van den Berg (1990) argue,

> Action Research is a way of taking a systematic, close, critical look at the way in which we teach, with a view to changing it so that the classroom experience becomes a more meaningful one for all those involved in it (p.28).
The quotation suggest that through action research, an opportunity was created where I systematically reflected on my classroom activities so that effective and efficient changes that were beneficial to both learners and I during learning and teaching could be made. Reflecting critically on what takes place in one's classroom is key to the beginning of a process of classroom transformation. Action research allowed for modifications of the intended intervention after analysing, evaluating and reflecting upon the first cycle of data collected (Opie, 2004).

**Data Collection**

During data collection, four lessons were presented over a period of four successive days. Since the action research process is cyclic, each lesson was taken as a cycle. Each cycle included planning, implementation, observation, reflection, and planning a revised action (Davidoff and van den Berg, 1990). Each lesson was 80 minutes long. In all lessons, the main instrument, a mathematics task set in multiple languages was used for teaching and learning. Learners were divided into home language groups and given both the English version and the home language version of the mathematics task. Since the task consisted of ten questions, the questions were divided across the four lessons. Two observers were present during lessons, one taking notes and the other one video-recording lessons. At the end of the fourth lesson, four learners, one from a different home language group, were selected for individual learner interviews which were conducted by a fellow researcher, who was once a senior teacher of mathematics in the same school. The interviewer was not present during lessons. Figure 1 below shows the cyclic process of the planned action research:
How Data Was Analysed

When analyzing data, research questions were not dealt with separately but holistically. Kilpatrick et al.’s (2001) five interwoven and interconnected strands of mathematical proficiency were used as a framework to analyse data. Video recordings of data collected during the four lessons in this study were transcribed and then utterances in English and the learners’ home languages were counted to investigate which languages were used. Using Kilpratrick’s et al.’s (2001) strands of mathematical proficiency, the transcripts were then categorized to explore if and how the deliberate, proactive and planned use of multiple languages contributed to the development of learners’ proficiency in mathematics. The main categories were then categorised further into sub-categories which I refer to as the ‘action’ that either the teacher or learners displayed in each strand. The five strands used for the categorisation of the transcript were procedural fluency (PF), strategic competence (SC), adaptive reasoning (AR), conceptual understanding (CU), and productive disposition (PD). Below is the description of sub-categories:
• **Displaying procedural fluency (DPF):** Evidence of knowledge of procedures to be used, when and how to use them suitably, and evidence of skills to perform them flexibly, accurately and efficiently.

• **Displaying strategic competence (DSC):** Learners indicating the knowledge of number of strategies and also know which strategy might be useful to find a solution for a particular mathematical problem. This simply means evidence of mathematical problem formulation, representation and solving skills by learners.

• **Displaying adaptive reasoning (DAR):** Learners showing the ability to explain and justify mathematical ideas and how the solution to a specific mathematical problem was obtained. This is displaying a skill of explaining, justifying, and thinking logically about and reflecting upon the relationships among concepts and situations.

• **Displaying conceptual understanding (DCU):** Learners showing the ability to comprehend mathematical concepts, ideas, operations and relations, and knowing the importance of certain mathematical ideas and the contexts in which they are relevant.

• **Displaying productive disposition (DPD):** Evidence of some indication by learner(s) of seeing sense in mathematics, regarding it as both useful and practicable, believing that constant effort in learning mathematics pays off, and seeing oneself as a capable learner and doer of mathematics.

**RESULTS AND FINDINGS**

Due to limited space in this paper, I only present a summary of results and findings. Table 1 below shows the prevalence and the frequency of the use of the learners’ home language(s) and English during each cycle.

<table>
<thead>
<tr>
<th></th>
<th>CYCLE 1</th>
<th></th>
<th>CYCLE 2</th>
<th></th>
<th>CYCLE 3</th>
<th></th>
<th>CYCLE 4</th>
<th></th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lang.</td>
<td>Teacher</td>
<td>Learner</td>
<td>Teacher</td>
<td>Learner</td>
<td>Teacher</td>
<td>Learner</td>
<td>Teacher</td>
<td>Learner</td>
<td></td>
</tr>
<tr>
<td>HL</td>
<td>70</td>
<td>52</td>
<td>88</td>
<td>39</td>
<td>66</td>
<td>29</td>
<td>36</td>
<td>34</td>
<td>414</td>
</tr>
<tr>
<td>ENG</td>
<td>98</td>
<td>98</td>
<td>149</td>
<td>69</td>
<td>138</td>
<td>88</td>
<td>73</td>
<td>45</td>
<td>758</td>
</tr>
</tbody>
</table>

Table 1. Frequency of the use of multiple languages by learners and the teacher in each cycle.

Even though the learners’ home languages were encouraged and used deliberately in each cycle, Table 1 shows the dominance of the use of English by both the teacher and learners across the four cycles. This is not surprising because English is the official LoLT of the school and the aim was not to exclude its use during teaching and learning
but to create a platform for learners to use their home language(s) as a resource. Furthermore, the dominance of English does not in and of itself necessarily imply lack of understanding by learners. What is not visible in Table 1 is whether and how using multiple languages contributed to the development of the learners’ proficiency in mathematics. This is reflected in Table 2 below.

<table>
<thead>
<tr>
<th>SUB-CAT</th>
<th>LANG.</th>
<th>CYCLE 1</th>
<th>CYCLE 2</th>
<th>CYCLE 3</th>
<th>CYCLE 4</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCU</td>
<td>HL</td>
<td>14</td>
<td>10</td>
<td>04</td>
<td>05</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>ENG</td>
<td>27</td>
<td>19</td>
<td>37</td>
<td>17</td>
<td>100</td>
</tr>
<tr>
<td>DPF</td>
<td>HL</td>
<td>05</td>
<td>05</td>
<td>00</td>
<td>01</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>ENG</td>
<td>10</td>
<td>10</td>
<td>02</td>
<td>02</td>
<td>24</td>
</tr>
<tr>
<td>DSC</td>
<td>HL</td>
<td>04</td>
<td>03</td>
<td>00</td>
<td>01</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>ENG</td>
<td>07</td>
<td>04</td>
<td>06</td>
<td>07</td>
<td>24</td>
</tr>
<tr>
<td>DAR</td>
<td>HL</td>
<td>02</td>
<td>06</td>
<td>00</td>
<td>03</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>ENG</td>
<td>03</td>
<td>02</td>
<td>03</td>
<td>09</td>
<td>17</td>
</tr>
<tr>
<td>DPD</td>
<td>HL</td>
<td>00</td>
<td>00</td>
<td>00</td>
<td>00</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>ENG</td>
<td>00</td>
<td>00</td>
<td>00</td>
<td>01</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4. Prevalence of strands of mathematical proficiency.

In Table 2 the prevalence and the frequency thereof, of the strands of mathematical proficiency and the language associated with each strand in each cycle, as mentioned earlier on, are shown. Though the prevalence of each strand is shown separately in the table below, this does not mean that each strand was developed independently from others, since the strands of mathematical proficiency are interdependent and interwoven (Kilpatrick et al, 2001). The above table present an overview of the extent at which each strand of mathematical proficiency was developed in each cycle. Table 2 shows that all the five strands of mathematical proficiency were displayed, but generally conceptual understanding was dominant across all cycles as compared to other strands and was most frequently displayed (DCU) in English. This suggests that most of the interactions were conceptual, unlike in the primary multilingual classrooms that Setati (2005) observed, where procedural discourse dominated. In this study, procedural discourse was second in terms of dominance and was displayed (DPF) mostly during cycle 1 and cycle 2. Since procedures to solve Questions 1, 2, and 3 were not suggested in the task, it was also significant that learners knew, as Kilpatrick et al (2001) suggest, a number of strategies and identify a strategy (DSC) which might be of use to each problem. The understanding of mathematical concepts and their relationships between them, enabled learners to identify a useful strategy and to remember the appropriate
procedures to solve problems in the mathematics task (Kilpatrick et al, 2001). Table 2 further shows that adaptive reasoning was most frequently displayed (DAR) during the second and fourth cycles. This was due to working on question 4 and questions 8 and 9 from the task sheet, which were done during the second cycle and the fourth cycle respectively. These type of questions may “demand self-monitoring or self-regulation of one’s own cognitive processes” (Stein et al., 2000:16) as they required some explanations and justifications, and therefore supported the development of learners’ proficiency in adaptive reasoning. What is interesting is that even though the use of English dominated the interactions, Table 2 shows that during Cycle 2 the display of adaptive reasoning was mostly in the learners’ home language(s), suggesting that learners’ used their home language(s) as a resource to give an ‘opinion’, that is responding to Question 4.

As table 2 shows, productive disposition, a strand that develops when the other four strands have developed, was only displayed in the fourth cycle. It was not surprising that productive disposition was the least frequent because, according to Kilpatrick et al (2001:131), “seeing sense in mathematics, regarding it as both useful and practicable, believing that constant effort in learning mathematics pays off, and seeing oneself as a capable learner and doer of mathematics”, is not a once-off thing, but is developed over time. Below is an extract showing the display of conceptual understanding (DCU), procedural fluency (DPF) and adaptive reasoning (DAR) by a learner.

In the extract, Given uses Tshivenda as a resource to display conceptual understanding, procedural fluency and adaptive reasoning as she explains how the solution to a mathematical problem was obtained.

Given: Hei, nayo … ar … (Giggles) … So forty rhanda hi monthly cost ne, then ba yieda nga twenty cents kha kilowatt for one hour. Then after that, angado shumisa …, baibidza mini? Heyi … ndoshumisa one kilowatt nga twenty cents kha one hour /...So forty rand is the monthly cost, then they add twenty cents per kilowatt-hour. ... they use..., what do they call it? Heyi ... they use one kilowatt-hour for twenty cents/. Sipho: Eya [Yes].

Given: Boyieda, maybe boshumisa twenty cents nga one hour [They add it, maybe they use twenty cents per hour].

Sipho: Eya, yantha [Yes, one hour].

Given: Iba […It becomes…].

Given and Sipho: Forty rand twenty cents.

Sipho: Yes, vhoibadela monthly, ngangwedzi ya hona. Yo fhelela, yes. Sesiyaqubheka […, the pay it monthly, each month. It is complete, yes. We continue].

The extract shows Given displaying conceptual understanding by explaining that “forty rands is the monthly cost” and that “they use one kilowatt-hour for twenty cents” which
is added to forty rands. This indicated that Given eventually understood the concepts involved, i.e. ‘the R40-00 service fee and the 20c per kilowatt-hour, and how the concepts relate to each other. By explaining, she was displaying adaptive reasoning as she was also justifying the obtained solution, R40-20c.

Learners’ Reflections and Views on the New Approach

The objective of analysing the learner interviews was to identify learners’ views, focusing mainly on two things, the nature of the mathematical task used in this study, and the deliberate use of multiple languages. All learners interviewed indicated that the four lessons conducted during this study were different from their normal (daily) mathematics lessons and they generally gave an indication that they liked the new approach.

The Nature of the Task

When responding to the interview question “What was different about the lesson?”, all but one learner began by referring to the nature of the mathematical task used and not the use of home language. While the two learners, Nhlanhla and Colbert pointed to the context used as a change in the way of teaching, Sindiswa pointed only to the mathematics of the task.

Interviewer: What … what was so special about the lesson?

Sindiswa: It does not include those maths … maths. It is not different, but those words used in maths didn’t occur, didn’t occur but we weren’t using them. Er … ‘simplifying’, ‘finding the formulas’, ‘similarities’, …

Colbert: Iya, basenzele [Yes, they did it] in order to … ukuthi ibe [so that it should be] simple and easy to us, because most of people, uyabona, aba-understend like i … like i-card ne meter [you see, they do not understand like a… like a card and a meter]. Abanye bathi i-meter is … i-price yakhona i-much uyabona, i-card iless i-price yakhona, that’s why uyabona [Some say a meter is … costs more you see, the cost of a card is less, that’s why you see]. So, abantu abana-knowledge, uyabona, bakhuluma [So people do not have knowledge, you see, they talk…] just for the sake of it. So, I think for us, because we have learnt something, both are the same.

Nhlanhla: Okokuqala mem, ilokhuza, la sidila ngama-calculations awemali, manje ku-maths asisebenzi ngemali [Firstly mem, the.., here we work with calculations that involve money, now in maths we do not work with money].

One interesting point that Sindiswa makes about the task is the language of the task, which is different from the type of language used in mathematics textbooks they normally use. She mentions terms like ‘simplify’, finding the formula’, ‘similarities’, terminology that are usually found in mathematics textbooks. This suggests that even
with the English version of the task used, the language was made more accessible and the task enjoyable as it was less threatening for learners. Colbert goes beyond the lessons as he sees the nature of the task as having clarified the real-life situation since they now know how the two costing structures work. For Nhlanhla, what was different with the approach was that they dealt with calculation involving money, which is what they do not usually do in mathematics.

**Deliberate Use of Multiple Languages:**

While using either English or home languages did not make any difference for Sindiswa, the three learners, Sipho, Nhlanhla and Colbert liked the use of home language and saw both languages as a resource. These learners indicated that the use of home languages assisted in understanding of the task and made most learners to participate.

Interviewer: uMr Molefe ungitshele ukuthi kule vike beka fundisa completely different, ubona ukuthi izo …iyasebenza le ndlela ayisebenza manje [Mr Molefe told me that this week he was teaching completely different, do you think it will... does the approach he uses now work]?

Nhlanhla: Yes mem, I think iyasebenza, ngoba ama-learners amaningi, maybe like, uma ungasebenzise ama-home language wabo, abaphathisipheiti kakhulu. Mabanikezwana ama-home language abo, I think bazokhona ukuphathisipheita [Yes mem, I think it works , because most learners, maybe like, if you do not use their home languages, they do not participate that much. If they are given their home languages, I think they will be able to participate].

Colbert: Iya yes, I think is a good idea, uyabona, ngoba iyenza ukuthi … iyenze izinto zibe simple, ngoba if singa-understendi ngeEnglish, sicheka ku … our languages, aba simple bese siyakhomphera [...you see, because what it does ... it makes things to be simple, because if we do not understand in English, we check in...our languages, they become simple and then we compare].

Sipho: Because kaofela digroup they were participating, wa utlwisisa mem. Le bane ba sa phathisipheiti ko klaseng, ne setse ba phathisipheita. Nna ke maketsa gore ‘he banna, mothaka o kajeko ke ena oe arabant so maths’ [Because all the groups were participating, do you understand mem. Even those who used not to participate in class, were now participating. I was surprised that ‘oh man, is this guy the one who responds so much in maths today’](Clicks fingers).

In Nhlanhla’s and Sipho’s views, using learners’ home language(s) encouraged learners to participate actively during lessons. While they both refer to learner participation as an indication that the new approach can be a success, how they say it is interesting. Nhlanhla says that “uma ungasebenzise ama-home language wabo, abaphathisipheiti kakhulu [if you do not use their home languages, they do not participate that much]”, whereas Sipho says “Le bane ba sa phathisipheiti ko klaseng, ne setse ba phathisipheita [Even those who used not to participate in class, were now participating]”. For
Nhlanhla, most learners do participate during mathematics lessons, but the absence of their home language(s) minimises active participation. On the other hand, for Sipho using home language(s) has encouraged all learners to participate. Nhlanhla refers to the ‘level of participation’ while Sipho refers to the number of learners who were participating. For Colbert, using learners’ home language(s) assists with comprehension of the task if the English version is incomprehensible for learners as they may simply switch to their home language version. These learners’ responses suggest that they view their home language(s) as both visible and invisible resources (Lave and Wenger, 1991, cited in Setati, Molefe, Duma, Nkambule, Mpalami and Langa, 2007) in the sense that they had an opportunity to draw from it and were neither a distracter nor an obstacle (switching to home language version of the task and having access to more clearer information).

IMPLICATIONS AND CONCLUSION

While research in mathematics and language shows that code-switching is used as one of the strategies by mathematics teachers to mediate learning in multilingual classrooms (Adler, 2001; Setati, 1998, 2002; Moschkovich, 1999), the findings from this study show that the deliberate use of learners’ home languages in addition to English can be resourceful in facilitating the development of learners’ proficiency in mathematics. As Zevenbergen (2000) argues, the learner’s home language and the way in which it is used can be a form of capital. In this paper, the findings from the study have indicated that learners had two languages to draw from in dealing with the task. Even though the results of the study show that the use of English dominated, learners interviewed mentioned that the advantages of using their home languages in the manner that the teacher used them were that it helped in understanding of the mathematical task, and encouraged learners to participate during lessons. Since literature suggests that well-selected tasks can provide learners with an opportunity to gain mathematical power, that is, to be mathematically competent (Stein et al., 2000), the analysis in this paper indicates that the nature of the mathematical task used plays an important role in supporting the development of learners’ proficiency in mathematics.

While this was a small-scale study done in one classroom, of which one may argue that its findings may not be generalised to other multilingual mathematical classrooms, it provides one with some ideas of transformation that one can make in ones’ classroom in order to assist learners to gain mathematical power. Other practitioners and researchers can learn from this study as it provides an important contribution for those who teach in multilingual classrooms and may encourage practitioners to research their own practice. However, teachers who would want to use the new teaching approach in their multilingual classroom as in this study are cautioned that using learners’ home languages in addition to the LoLT on its own is not enough. What is key in using the approach is that one has to be multilingual and mathematically strong to enable one to support the development of learners’ proficiency in mathematics in multilingual classrooms.
REFERENCES


The Sunday Times (29 February 2002). *English versus the rest in battle for the classroom*


This report is based on a doctoral thesis in research psychology, titled "Behavioural Correlates and Specific Attitudes in Students Exposed to Mathematical Programmes at Interfaculty Levels", in which the effects of short- and long-term mathematics anxiety was probed with reference to both short and long-term study behaviour and ultimate performance in both algebra and geometrical examinations. Differences in the dependent variables were not always accompanied by corresponding differences in performance and the procedures and results are discussed as below.

MATHEMATICAL UNDERPERFORMANCE IN TERTIARY STUDENTS

This proposed or imposed mathematics subcourse is notoriously the undoing of many a student in a largely noncommercial or nonscientific curriculum (U.C.L.A. Academic Advancement Program Manual, n.d.), sometimes to the extent of precluding otherwise capable students from completing their degrees. A quick perusal of any university’s prospectus will reveal that most university degrees either presuppose or actively impose at least one sub-course in mathematics or statistics. According to Cohen (1995), the overwhelming majority of postsecondary mathematical students are taking either remedial or a compulsory introductory course in mathematics, as opposed to already being reasonably number literate on matriculation. In fact, according to the Mathematical Association of America, 1988, as quoted in a local mathematics literacy project, the Malati Project:

- Colleges and universities should treat quantitative literacy as a thoroughly legitimate and even necessary goal for baccalaureate graduates;
- Colleges and universities should expect every college graduate to be able to apply simple mathematical methods to the solution of real-world problems;
- Colleges and universities should devise and establish quantitative literacy programs each consisting of foundation experience and a continuation experience,
and mathematics departments should provide leadership in the development of such programs; and

- Colleges and universities should accept responsibility for overseeing their quantitative literacy programs through regular assessments.

This research report will attempt to isolate and intercorrelate factors that directly and indirectly affect mathematical performance, in and out of the test situation, and suggest how the manipulation of these factors could greatly improve eventual grades.

**DEFINITION AND DIAGNOSIS OF MATHS ANXIETY**

For the purposes of this study and precisely because we were looking at both long and short term factors, maths anxiety was seen as the sum of both generalised anxiety and test anxiety.

- Generalised anxiety is seen as having both a state (short term) and a trait (long term personality) component (Spielberger, Anton & Bedell, 1976).
- Test anxiety is the anxiety at the prospect of a test situation looming and the possibility of failing it. Sarason (1961, in Anastasi, 1990) made the interpretation that both school achievement and intelligence test scores correlated negatively with test anxiety.
- Maths anxiety is separate from dyscalculia or acalculia, which may, strictly speaking and especially in the case of brain injury, be entirely free of anxiety of any sort.

Research by D’Ailly *et al.* (1992) into mathematics anxiety, in which they defined maths anxiety as the sum of mathematics test anxiety and general numerical anxiety in everyday life, has identified mathematics test anxiety as a separate phenomenon that correlated less with general numerical anxiety and more with gender (female), (also Dwinell & Higbee, 1991) and generalized test anxiety levels.

The Richardson and Suinn Maths Anxiety Rating Scale (MARS) is the definitive diagnostic tool in the USA and generally considered the most reliable and valid measurement available (D’Ailly & Bergering, 1992). Both the MARS and the Mathematics Avoidant Behaviour Test (Anton & Klish, 1995) are highly predictive of mathematics avoidance behaviour, the latter with particular reference to gender-related mathematics avoidance in that females frequently choose to not take courses in advanced mathematics.

Professor J G Maree of the University of Pretoria has also locally developed a study orientation questionnaire in Mathematics (SOM), (Maree, 1997), which has been extrapolated to several versions suitable for different age groups. However this instrument also does not focus on the effect of crippling anxiety *in situ*, but rather on a cluster of generalised cognitions and emotions around mathematics, in terms of which a
student may never choose to take mathematics as a subject, or may otherwise opt out as soon as reasonably possible.

The Incidence of Maths Anxiety

Very few authorities would care to deny that maths anxiety exists, even if they chose to lament the supposed lack of backbone in those who suffer most from it. Although diagnostic tests have been developed to identify maths anxiety in a given population, there are no statistics available directly measuring the incidence of mathematics anxiety in the student population at large, either here or abroad. The most apparent reason is that students do not care to admit to a dread of mathematics because they are afraid that they will be ridiculed, either by the teacher or their fellow students. This can be taken to the extreme of the finding of Rounds and Hendel (1980, in D’Ailly & Bergering, 1992) that, when given the option to do so, mathematics-anxious students avoided answering even questions related to mathematics test anxiety or concrete everyday number situations.

THE CONSEQUENCES OF MATHS ANXIETY

Richardson & Suinn (1972, in Anton & Klish, 1995 and in Betz & Hackett, 1983) state that: "Mathematics anxiety may prevent a student from passing fundamental mathematics courses or prevent his pursuing advanced courses in mathematics or the sciences”. Moreover, according to Ashcraft (2002), mathematics avoidance is the ubiquitous and iniquitous consequence of mathematics anxiety.

Anyone could point out, that the more one plays truant, the more reason one will have to fear maths and the testing situation and that the avoidance behaviour will very soon become a form of self-sabotage. It is therefore clear that an adequate understanding of mathematics fear needs to take cognizance of not only what happens in the test situation, but of the antecedents leading up to the student’s personal choice of whether or not to address that fear.

RESEARCH DESIGN

In this experiment the experimental variables of this study are divided into long-term and short-term components, which relate to the examination situation (in situ psychobiological measurement of state anxiety and distractibility), as well as off-site

13 “Situational variable (state)”: According to Thorndike (1913, in Dictionary of Psychology, 1972) “there are no broad, general traits of personality, no general and consistent forms of conduct which, if they existed, would make for the consistency of behavior and personality, but only of independent and specific stimulus-response bonds of habit.” Therefore, a situational (state) variable would appear to be a variable caused by an occurrence firmly located outside of the individual, as opposed to a trait variable caused by an internal experience and therefore existing in the individual even before the trigger event.
measurements of trait anxiety and distractibility behaviour (conceptualized as trait anxiety and trait avoidance and generally typical of entrenched behaviours that may not require extreme short-term stress to manifest themselves).

The participants were thirty-four students at the then University of the Witwatersrand, all of whom were doing Applied Mathematics II in the year 2001. Their participation was in return for a fee as, due to a last-minute cancellation by the initially-selected sample from another tertiary institution, they had to be recruited at very short notice.

<table>
<thead>
<tr>
<th></th>
<th>FREQUENCY</th>
<th>PERCENTAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>MALE</td>
<td>23</td>
<td>67.65</td>
</tr>
<tr>
<td>FEMALE</td>
<td>11</td>
<td>32.35</td>
</tr>
<tr>
<td>BLACK</td>
<td>20</td>
<td>58.82</td>
</tr>
<tr>
<td>WHITE</td>
<td>14</td>
<td>41.18</td>
</tr>
</tbody>
</table>

Table 1: Percentages by Gender and Ethnicity.

Procedur

The first two testing occasions were in the examination hall when the participants were actually writing their June examinations. For the first measurement of responses under moderate stress, the sample was concurrently engaged in an examination on algebra. The second measurement was of one response under extreme stress, as the examination being written (vector calculus), was characterized by Dr Mormoniat, their academic coordinator, as the most feared subsection of the course. Participants were required to produce a urine sample while writing their examination or immediately afterward, and the Symbol Digit Modalities Test tests were individually performed after each student handed in his or her completed examination papers.

The baseline measurements of Symbol Digit Modalities Test and cortisol/creatinin ratios were taken after the students had returned from their June vacation, as were the long-term assessments of personality variables.

14 “Pre-existing variable (trait)”: Usually used in the context of trait anxiety, and according to Allport (1937, in the Dictionary of Psychology, 1972) “Traits are not directly observable; they are inferred. ... Without such an inference the stability and consistency of personal behavior could not possibly be explained. Any specific action is a product of innumerable determinants, not only of traits but of momentary pressures and specialized influences. But it is the repeated occurrence of actions having the same significance (equivalent of response) following upon a definable range of stimuli having the same personal significance (equivalent of stimulus), that makes necessary the postulation of traits as states of being)”.
DISCUSSION OF THE ASSESSMENT INSTRUMENTS

Biographical Questionnaire
This was administered to all students a week before the first examination. This included basic demographics, an explanation of the purpose of the research (oral presentation) as well as a disclaimer regarding the uses to which their information would be put. All students had to sign this form before the trials proper started.

Cortisol/ Creatinin Measurement (CCR)
Cortisol has shown itself to be highly responsive to stress induced by exposure to mathematical tasks (Al’Absi, Lovallo, M’Key et al., 1994). Granted, his participants were borderline hypertensives, but the similarity to the participant pool at hand is that the students had been under considerable examination stress for quite some time. Previous research has not specifically compared anxiety levels between mathematics and lower-stress examinations, yet a comparison between psychology and physiology examinations did yield cortisol reading differentials (Armario, Marti, Molina et al., 1996). This study was aimed at replicating the tendency using the difference in fear levels between algebra and vector calculus.

Symbol Digit Modality Test (SDMT)
According to Barr, (2001), the Symbol Digit Modalities Test was developed as a measure of sustained attention and concentration by researchers at the University of Michigan in the late 1960’s, and comprises a matrix of digits up to nine, with corresponding symbols. A participant is given ninety seconds in which to scan the “key” matrix, and then to commence filling in every alternate row of a second matrix, each empty block with the digit corresponding to the symbol given. This test is known to be particularly sensitive to inattention, since three of the symbols are inverse images of the other. Moreover, Bledsoe & Perkins (1976) reported that the ability to manipulate novel stimuli was in fact a substantial predictor of academic performance in mathematics.

The Symbol Digit Modalities Test was administered either individually, or in small groups of two to three, as each participating student completed their examination. A single group test or consistent individual testing would have been preferable but, despite heavy bribing, once the students had written their examination they simply wanted to get out of the hall as quickly as they could. Raw scores were used, obtained by counting the number of correct responses obtained in the ninety seconds of testing.

Grades
Three sets of grades were used: the grade obtained in the actual examinations in which the cortisol/ creatinin and Symbol Digit Modalities testing was done, which were algebra and spatial mathematics (not the formal course designation). The third set of grades were the composite grade obtained by the students the previous year.
Rotter Internal/ External Locus of Control Scale

The very quintessence of the Rotter Locus of Control Scale is whether or not study effort, which is under one’s personal control, will improve one’s grades - or whether one’s own efforts are entirely irrelevant due to external circumstances entirely beyond one’s control. According to Brenenstuhl & Badgett (1977) the Rotter Locus of Control scale has been one of the most frequently used predictors of academic success in the past.

The locus of control questionnaire is also known for being little influenced by social desirability factors yet remains an accurate measure of attributions over a wide variety of social spectrums. The 23 items were marked according to an internal rating with some questions having been inverted to ensure an accurate response, as some participants have a tendency to answer all questions positively.

The questionnaire was completed in the students’ own time in the same manner as the Millon Clinical Multiaxial Inventory . The questionnaire was marked according to the internal scale.

Millon Clinical Multiaxial Inventory II Scale (MCMII)

The Millon Clinical Multiaxial Inventory II is a self-report personality inventory, meaning that the participant is required to report on internal states or beliefs about themselves without relying on feedback from others. The advantage of self-report is that, as opposed to qualitative research, since all respondents complete the same questionnaire, their results are comparable. Since in this case they answered the questionnaire at their own convenience (again a matter of student cooperation), there would be little control over variables such as a quiet undisturbed environment or mood at the time. However, it is added that the ideal candidate would be at least reasonably intelligent, fluent in English and have some insight into their own personality and willingness to divulge (Choca, 1992).

The subscales relevant to this study were defined as follows:

- “Avoidant = social detachment, approach-avoidance conflict, socially apprehensive so avoid interpersonal contact to reduce anxiety. [also mistrust of others, feelings of worthlessness, desire to isolate self, low self-esteem, suppression of own feelings, sexual inhibition]”
  and

- “Anxiety = apprehension, phobic reactions, indecision, tension, restlessness, associated physical discomfort, poor confidence in abilities, low self-esteem, [also feel unwanted, unappreciated, sudden tears/ anger, dependent, is very sensitive indicator of psychological distress and disturbance]”.

263
STATISTICAL PROCEDURES AND FINDINGS

Statistical manipulations were performed by Professor H. S. Schoeman of Clinstat, using the statistical package SAS Version 8.

<table>
<thead>
<tr>
<th></th>
<th>LONG-TERM</th>
<th>SHORT-TERM (2 X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANXIETY</td>
<td>Trait Anxiety (Baseline cortisol/creatinin Readings, Millon Clinical Multiaxial Inventory Anxiety Score)</td>
<td>State Anxiety (Test cortisol/creatinin Readings) Symbol Digit Modalities Test</td>
</tr>
<tr>
<td>MOTIVATION</td>
<td>Intrinsic Versus Extrinsic Motivation, Locus of Control (Rotter)</td>
<td>Attention (Symbol Digit Modalities Test) at test site Cognitive Coping Strategies</td>
</tr>
<tr>
<td>AVOIDANCE</td>
<td>Procrastination (Millon Clinical Multiaxial Inventory Avoidance)</td>
<td>Distractibility (Symbol Digit Modalities Test)</td>
</tr>
<tr>
<td>ULTIMATE MATHEMATICAL PERFORMANCE</td>
<td>Procedural Memory During Mathematical Calculations</td>
<td>Short-term Memory During Mathematical Calculations</td>
</tr>
</tbody>
</table>

| GENDER                |                                                                            |                                                                                |
| ETHNICITY             |                                                                            |                                                                                |

Table 1: schematic representation of variables

DISCUSSION OF THE MOST SIGNIFICANT RESULTS
CORTISOL/CREATININ RATIO’S

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>TOTAL NUMBER</th>
<th>MEAN</th>
<th>STANDARD DEVIATION</th>
<th>P-VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCCR 01 (all)</td>
<td>31</td>
<td>.55</td>
<td>.39</td>
<td>&lt; .0001</td>
</tr>
<tr>
<td>LNCCR 02 (all)</td>
<td>33</td>
<td>.22</td>
<td>.38</td>
<td>.0024</td>
</tr>
<tr>
<td>LCCR 01 (male)</td>
<td>9</td>
<td>.65</td>
<td>.37</td>
<td>.0007</td>
</tr>
<tr>
<td>LCCR 02 (male)</td>
<td>11</td>
<td>.08</td>
<td>.33</td>
<td>.4141</td>
</tr>
<tr>
<td>LCCR 01 (female)</td>
<td>22</td>
<td>.51</td>
<td>.40</td>
<td>&lt; .0001</td>
</tr>
<tr>
<td>LCCR 02 (female)</td>
<td>22</td>
<td>.29</td>
<td>.39</td>
<td>.0027</td>
</tr>
<tr>
<td>LCCR 01 (black)</td>
<td>18</td>
<td>.39</td>
<td>.34</td>
<td>.0002</td>
</tr>
<tr>
<td>LCCR 02 (black)</td>
<td>18</td>
<td>.11</td>
<td>.39</td>
<td>.2412</td>
</tr>
<tr>
<td>LCCR 01 (white)</td>
<td>13</td>
<td>.78</td>
<td>.34</td>
<td>&lt; .0001</td>
</tr>
<tr>
<td>LCCR 02 (white)</td>
<td>14</td>
<td>.37</td>
<td>.32</td>
<td>.0009</td>
</tr>
</tbody>
</table>

Table 3: Basic Statistics for Cortisol creatinin ratio’s
LCCR is log-transformed cortisol creatinin measurement (the log transformation was performed to ensure a normal distribution).

T-tests were performed to compare the fear reaction of males and females to the two testing procedures (algebra and vector calculus) in comparison to the measure taken after their vacation, i.e. when they were relaxed and rested. In other words, this is a comparison not of individual students, but of groups, to see whether the means between the groups were truly significantly different (Trochim, 1997). No significant gender-related differences occurred on either the algebra or vector calculus cortisol/creatinin ratio’s when examinations were looked at separately, and it can be inferred that although both males and females experienced fear when writing their exams, the differences in individual fear levels were not due to gender.

Both black and white students showed increased cortisol/creatinin ratio’s when writing their algebra examinations, as opposed to when they wrote their vector calculus exams several days later.

Yet only white students showed an increased cortisol/creatinin ratio during the vector calculus. This could perhaps be put down to exhaustion, as it is well known that eventually the fear reaction will subside, which is the body’s natural mechanism to protect especially the kidneys and parts of the brain from undue wear and tear. There is no explanation to hand for why black students might have been more exhausted than white ones, unless one puts it down to a difference in living conditions, and general stress factors such as longer transport hours, money worries and perhaps inadequate access to good lighting or a quiet place to study.

Symbol Digit Modalities Test

On both testing occasions the female students scored markedly better on attention levels than their male counterparts. At the first examination (algebra) only males showed increased distractibility but at the second examination (vector calculus), both males and females showed increased distractibility.

When one compared attention levels for the algebra and the vector calculus examinations to the same attention measurement taken when the students were at rest, it was clear that both black and white students were less focused, more markedly the black students at the algebra examination, with the inverse result at the vector calculus examination. On the other hand, only black students showed significant differences in attention levels at both tests when compared to the baseline score of the Symbol Digit Modalities Test, with white students showing a decrease in attention that came close to significance in the vector calculus examination, but not in the algebra examination.

This tallies with the observation in fear levels above where with the algebra examination both black and white, male and female students did show increased fear, but where white students showed more fear in the vector calculus examination, even though the loss of focus (attention) shown by the white students was not statistically significant.
Grades

<table>
<thead>
<tr>
<th>Grade Level</th>
<th>Total Number</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 0 (all)</td>
<td>34</td>
<td>57.88</td>
<td>12.96</td>
</tr>
<tr>
<td>Grade 1 (all)</td>
<td>31</td>
<td>62.35</td>
<td>11.67</td>
</tr>
<tr>
<td>Grade 2 (all)</td>
<td>31</td>
<td>53.06</td>
<td>19.74</td>
</tr>
<tr>
<td>Grade 0 (male)</td>
<td>23</td>
<td>58.30</td>
<td>11.36</td>
</tr>
<tr>
<td>Grade 1 (male)</td>
<td>22</td>
<td>63.05</td>
<td>11.81</td>
</tr>
<tr>
<td>Grade 2 (male)</td>
<td>22</td>
<td>54.36</td>
<td>20.58</td>
</tr>
<tr>
<td>Grade 0 (female)</td>
<td>11</td>
<td>57.00</td>
<td>16.40</td>
</tr>
<tr>
<td>Grade 1 (female)</td>
<td>9</td>
<td>60.87</td>
<td>11.83</td>
</tr>
<tr>
<td>Grade 2 (female)</td>
<td>9</td>
<td>49.89</td>
<td>18.26</td>
</tr>
<tr>
<td>Grade 0 (black)</td>
<td>20</td>
<td>56.40</td>
<td>12.27</td>
</tr>
<tr>
<td>Grade 1 (black)</td>
<td>18</td>
<td>61.72</td>
<td>10.21</td>
</tr>
<tr>
<td>Grade 2 (black)</td>
<td>18</td>
<td>53.44</td>
<td>21.19</td>
</tr>
<tr>
<td>Grade 0 (white)</td>
<td>14</td>
<td>60.00</td>
<td>14.08</td>
</tr>
<tr>
<td>Grade 1 (white)</td>
<td>13</td>
<td>63.23</td>
<td>13.83</td>
</tr>
<tr>
<td>Grade 2 (white)</td>
<td>13</td>
<td>54.54</td>
<td>18.36</td>
</tr>
</tbody>
</table>

Table 3: Grade Means by Course (Measurements for each course, as opposed to comparisons to baseline.)

At a glance, the male students usually appear to have scored very slightly higher than their female counterparts, but the t-tests to follow showed that this was not significant. White students usually appear to be approximately a percentage point above their black counterparts, but the t-tests to follow showed that this was not significant.

Vector calculus grades were noticeably lower than those for either algebra, or the previous year’s composite mark. This agrees with the prior observation that vector calculus tends to be feared much more than algebra (enough of the students were white males – greater fear, poorer attention levels - to bring down the average grade for vector calculus)

No significant gender or racial grade differences were found in any of the testing occasions.
INTERACTION BETWEEN LONG-TERM VARIABLES (ALL) AND SHORT-TERM VARIABLES (ALL)

As exploratory statistics had revealed no significant relationships, gender and ethnicity was not brought to bear upon the Millon Clinical Multiaxial Inventory (avoidance and long term anxiety) and the Rotter Locus of Control Scale (the extent to which one believes that one’s efforts determine one’s rewards).

The most plausible general explanation for the lack of findings is that long-term variables could have been expected to impact on general study habits. However, since initial results revealed no statistical relationships, no further analyses were performed.

Schematic representation of results obtained.
CONCLUSION AND RECOMMENDATIONS
The Yerkes-Dodgson law (Smith, 1990) states that there is an optimal level of arousal for efficient performance, with stress seen as impairing performance simply because it resulted in either the overstimulation or understimulation of the participant. During the course of this study, the author encountered an abundance of research into the effect of various levels of anxiety on the various components of performance. It may well be argued that the participants were experienced mathematicians who had freely chosen their mathematical careers, and that even if they did experience some anxiety, they had learned to deal with it without letting it affect their performance.

However an entirely different result might have been obtained for students who had not chosen to do mathematics, which in the here and now equates to the vast majority. Given that anxiety consumes energy as well as generating it, it would appear that mathematics educators of the future will need to identify an optimum level of arousal that facilitates attention and effort rather than hampering it and learners will need to reciprocate by focussing their efforts on getting to grips with the mathematics.

This study was not able to focus deeply on issues of gender and race, as it was designed to be primarily quantitative and the analysis by gender and race was done after the fact. However the hallmark of good research is that it provokes more questions than it answers. It is to be hoped that further research may follow that will probe into factors affecting issues of gender and race, and above all into factors that can be manipulated to the advantage of student and teacher alike.

Arrows denote where a relationship emerged between the variables; variables not connected with arrows did not show significant any relationship with any of the other variables. Variables have also been grouped into long-term, demographic and short-term variables (the gray area).

BIBLIOGRAPHY


In this paper I reflect on three experiences in my Grade 10 Mathematical Literacy classroom last year in 2006 that opened my eyes to the complexities and difficulty involved in teaching and learning Mathematical Literacy.

Have you ever experienced one of those “Light Bulb” moments in your teaching career, when something happens in class that opens your eyes to the wonder and complexity of teaching Mathematics? In 2006 while teaching Mathematical Literacy, I experienced several of these moments, each showing in a different way the complexities involved in using mathematics to solve problems in the physical world.

In this article I share three “Light Bulb” moments that I experienced and attempt to explain how these moments have shaped my teaching. I also use these three scenarios to explain why I believe Mathematical Literacy is a difficult subject to teach and learn.

LIGHT BULB MOMENT #1: THE MIDMAR MILE

The Midmar Mile is a mile (≈ 1.6 km) long swimming race that takes place at Midmar Dam near Pietermaritzburg in KwaZulu Natal at the beginning of February each year. The race is the biggest open water swimming race in the country and attracts thousands of swimmers.

Given that many of the girls at my school swim the Midmar Mile every year, I gave my class a mathematical literacy activity relating to this race. This activity formed part of a sequence of activities on ratios, and its purpose was to provide students with practice in using a conversion table and ratios to convert units of measurement from meters to yards to miles.
Activity: Rachael’s Midmar Mile Training

Rachael has decided to swim the *Midmar Mile*. On Monday and Wednesday afternoons she trains in the small pool and on Tuesdays and Thursdays she trains in the heated pool. The small pool is 25 yards long and the heated pool is 25 m long.

The table below is a conversion table for converting yards to meters.

<table>
<thead>
<tr>
<th>meters</th>
<th>←</th>
<th>Yards</th>
<th>→</th>
<th>miles</th>
</tr>
</thead>
<tbody>
<tr>
<td>9,1</td>
<td></td>
<td>10</td>
<td></td>
<td>0,006</td>
</tr>
<tr>
<td>18,3</td>
<td></td>
<td>20</td>
<td></td>
<td>0,01</td>
</tr>
<tr>
<td>45,7</td>
<td></td>
<td>50</td>
<td></td>
<td>0,03</td>
</tr>
<tr>
<td>91,4</td>
<td></td>
<td>100</td>
<td></td>
<td>0,06</td>
</tr>
<tr>
<td>457,2</td>
<td></td>
<td>500</td>
<td></td>
<td>0,3</td>
</tr>
</tbody>
</table>

Use the conversion table to determine how many lengths of the small pool and how many lengths of the heated pool Rachael must swim to swim a mile.

The fascinating thing about this activity is that two students answered the question in completely different ways and came up with vastly different answers.

One student found that 67 lengths of the small pool and 61 lengths of the big pool each equal a mile. The other student found that 80 lengths of the small pool and 73 lengths of the big pool equal a mile!

If both students’ methods are correct and there are no mistakes in their calculations, how is it possible that their answers differ so much? And if you are training for the *Midmar Mile*, whose answer should you believe? After all, if you are as unfit as I am, there is a big, big difference between swimming 67 lengths or 80 lengths.

To answer these questions, let us have a look at how each student arrived at her answer.
**Teegan’s Solution**

*Small pool (25 yards):*

0,006 miles = 10 yards

\[ \Rightarrow 1 \text{ mile} = \frac{10 \text{ yards}}{0,006} = 1,666,67 \text{ yards} \]

(rounded off to two decimal places)

\[ \therefore \text{ No. of lengths in a 25 yard pool} \]

\[ = \frac{1,666,67 \text{ yards}}{25 \text{ yards}} \]

\[ = 66,67 \]

\[ \approx 67 \text{ full lengths} \]

*Big pool (25 meters):*

9,1 m = 0,006 miles

\[ \Rightarrow 1 \text{ mile} = \frac{9,1 \text{ m}}{0,006} = 1,516,67 \text{ meters} \]

(corrected to two decimal places)

\[ \therefore \text{ No. of lengths in a 25 m pool} \]

\[ = \frac{1,516,67 \text{ m}}{25 \text{ m}} \]

\[ = 60,67 \]

\[ \approx 61 \text{ lengths} \]

---

**Penny’s Solution**

0,01 miles = 20 yards = 18,3 m

0,01 miles × 100 = 1 mile

20 yards × 100 = 2,000 yards

18,3 m × 100 = 1,830 meters

*Small pool:*

2,000 yards ÷ 25 yards = 80 lengths

*Big pool:*

1,830 m ÷ 25 m = 73,2 lengths

\[ \approx 73 \text{ full lengths} \]

Teegan has answered the question using the method that I taught them in class. Penny, on the other hand, has answered the question using a far less structured approach.

So why the vast difference in their answers?
The problem, in fact, lies not with either of the student’s calculations. The problem lies with the way that I set up the conversion table.

Consider the first two rows of the conversion table:

<table>
<thead>
<tr>
<th>meters</th>
<th>←</th>
<th>yards</th>
<th>→</th>
<th>miles</th>
</tr>
</thead>
<tbody>
<tr>
<td>9,1</td>
<td></td>
<td>10</td>
<td></td>
<td>0,006</td>
</tr>
<tr>
<td>18,3</td>
<td></td>
<td>20</td>
<td></td>
<td>0,01</td>
</tr>
</tbody>
</table>

If 10 yards = 0,006 miles, then:
20 yards = 0,006 miles × 2 = 0,012 miles

The conversion table, though, lists 20 yards as being equal to 0,01 miles. This is because I rounded off the values in the miles column of the conversion table to two decimal places to make the table more user-friendly.

If the conversion table listed 20 yards equal to 0,012 miles then Penny’s calculations may have looked something like:

\[
0,012 \text{ miles} = 20 \text{ yards} = 18,3 \text{ m}
\]

\[
0,012 \text{ miles} \times 83,333 = 1 \text{ mile (corrected to three decimal places)}
\]

\[
20 \text{ yards} \times 83,333 = 1,666,66 \text{ yards}
\]

\[
18,3 \text{ m} \times 83,333 = 1,524,994 \text{ meters (corrected to three decimal places)}
\]

**Small pool:**

\[
1,666,66 \text{ yards} \div 25 \text{ yards} = 66,67 \text{ lengths}
\]

= 67 full lengths

**Big pool:**

\[
1,524,994 \text{ m} \div 25 \text{ m} = 61 \text{ full lengths}
\]
Why was this a “light bulb” moment?

In Grades 8 and 9 and in Core Maths in Grades 10, 11 and 12, we teach our students to round off to three decimal places within a calculation and to one or two decimal places for a final answer. How many of our students, though, actually understand the impact that rounding off too soon can have on an answer and the validity of that answer? How many of our students actually understand that in real-world calculations rounding off can have serious implications for whether a project will succeed or fail – whether, for example, a cake will rise or flop, concrete will set, your books will balance or you will be arrested for fraud for rounding off financial figures?

The Midmar Mile activity was a “light bulb” moment for me because it highlighted in a real and practical way the significant impact that rounding off can have on a solution. It was also a “light bulb” moment for me because I now realise that I have not been swimming far enough in my training for the Midmar Mile. Oops! 😊

Light Bulb Moment #2: Mixing Concrete

For much of this year there has been a great deal of construction at my school. First, a new sports pavilion with change rooms was built and then extensions to the school chapel were started. Given that there was so much building activity around the school, I decided to use the opportunity to teach my students about some of the different aspects involved in construction – for example, working with plans, mixing concrete, estimating brick quantities, and so on.

The question below is an extract from a classroom activity on mixing concrete. This activity was part of a larger sequence of activities on ratios. The purpose of the activity was to provide students with practice in reading information from a table and using ratios to calculate various quantities.

274
Activity: Mixing Concrete

The table below shows the mix quantities of cement, sand and stone for making the type of concrete needed for the foundations of the chapel.

**Medium Strength Concrete**

<table>
<thead>
<tr>
<th>Mix quantity (m³)</th>
<th>Bags of Cement</th>
<th>Sand (m³)</th>
<th>Stone (m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,1</td>
<td>1</td>
<td>0,1</td>
<td>0,1</td>
</tr>
<tr>
<td>0,3</td>
<td>2</td>
<td>0,2</td>
<td>0,2</td>
</tr>
<tr>
<td>0,6</td>
<td>4</td>
<td>0,4</td>
<td>0,4</td>
</tr>
<tr>
<td>1,5</td>
<td>10</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>15</td>
<td>100</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

If the foundations of the chapel has a total volume of 28,6 m³, use the table to calculate the number of full bags of cement needed to make the concrete for the foundations.

My students’ answers to this question were fascinating, not because they came up with different answers, but because of the number of different ways in which they answered the question. Out of a class of 19 students, seven different methods were used to answer the question. These different methods follow.
### Method 1 – Chelsea’s Method
(The method I taught the class)

Mix quantity: 3 m³ = 20 bags

\[ \Rightarrow 1 \text{ m}^3 = \frac{20 \text{ bags}}{3} \]

= 6,667 bags

(rounded off to 3 decimal places)

∴ For 28,6 m³ the builders will need:

6,667 bags \times 28,6 = 190,7 bags

= 191 full bags

### Method 2 – Gina’s Method

You need 28,6 m³ to be covered.

\[ \therefore 15 \text{ m}^3 + (3 \text{ m}^3 \times 4) + 1,5 \text{ m}^3 + 0,1 \text{ m}^3 = 28,6 \text{ m}^3 \]

15 m³ = 100 bags

3 m³ \times 4 = 20 bags \times 4 = 80 bags

1,5 m³ = 10 bags

0,1 m³ = 1 bag

∴ Total amount needed is:

100 bags + 80 bags + 10 bags + 1 bag

= 191 bags

### Method 3 – Angela’s Method

Total volume of concrete = 28,6 m³

15 m³ = 100 bags

∴ 28,6 m³ \div 15 m³ = 1,91

1,91 \times 100 bags = 191 bags of cement

### Method 4 – Anna’s Method

28,6 m³ of concrete needed.

Round this off to 30 m³ of concrete.

Then:

Bags of cement for 15 m³ is 100 bags

∴ Bags of cement for 30 m³ = 100 bags \times 2

= 200 bags.

∴ 200 bags of cements were needed.

### Method 5 – Shannon’s Method

28,6 m³ (amount of concrete used) \div 3 (mix ratio) = 9,543

∴ 20 bags cement (ratio to 3 m³ of mixed quantity) \times 9 543 = 190,86 bags

∴ 191 bags of cement are needed.
Method 6 – Sinead’s Method

Total volume = 28,6 m³
∴ 0,1 + 0,3 + 2(0,6) + 2(1,5) + 3(3) + 15 = 28,6 m³
0,1 m³ = 1 bag
0,3 m³ = 2 bags
0,6 m³ = 4 bags
1,5 m³ = 10 bags
3 m³ = 20 bags
15 m³ = 100 bags

∴ 28,6 m³ = 1 bag + 2 bags + 2(4 bags) + 2(10 bags) + 3(20 bags) + 100 bags
= 191 bags of cement

Method 7 – Megan’s Method

28,6 m³ of concrete needed.
15 m³ [28,6 m³ − 15 m³ = 13,6 m³]
(3 m³ × 4 = 12 m³) [13,6 m³ − 12 m³ = 1,6 m³]
(0,6 m³ × 2 = 1,2 m³) [1,6 m³ − 1,2 m³ = 0,4 m³]
(0,3 m³) [0,4 m³ − 0,3 m³ = 0,1 m³]
0,1 m³

100 bags + 20 bags × 4 + 4 bags × 2 + 2 bags + 1 bag
= 100 bags + 80 bags + 8 bags + 2 bags + 1 bag = 191 bags

Why was this a “light bulb” moment?

In Grade 8, 9 and the current Grade 12 year, most questions can be answered using only one or two different methods. This is because much of the work that we teach is “compartmentalised” into sections of content and many of the problems that students face relate to a single section of mathematics. To solve these problems, students are expected to draw on knowledge and skills relating to a specific section of work and are not required to link different sections of content or make use of a variety of different skills.
In a Mathematical Literacy classroom, the situation is very different. This is because real-life problems often cannot be compartmentalised into sections of content, and solving problems posed in real-life scenarios usually requires linking together various sections of mathematical content and a variety of different skills. It is for this reason that most problems posed in real-life scenarios can be solved using a variety of different methods.

The *Concrete Activity* was a “light bulb” moment for me because it highlighted the fact that Mathematical Literacy is a very difficult subject to learn and to teach. It is difficult to learn because often the problems that students have to solve require them to make links between different sections of content and use a variety of different skills. It is difficult to teach because every student in the class may interpret the problem differently and may use different methods to solve the problem. It is also difficult to teach because of the time it takes to make a marking memo if there are several different ways to solve every problem! 😊

**LIGHT BULB MOMENT #3: MARKING THE ATHLETICS TRACK**

My third light bulb moment last year actually didn’t arise from a lesson in my Mathematical Literacy classroom. Rather, it arose in a discussion with a Geography teacher about how to mark the lanes and staggered starts on the athletics track for athletics day.
**Staggered Starts Explained:**

On an athletics track, the distance around the inside lane is shorter than the distance around the second lane, the distance around the second lane is shorter than the distance around the third lane, and so on. For this reason, the starting positions of the athletes in each lane need to be *staggered* so that every athlete runs the same distance around the track.

![Diagram of an athletics track with staggered starts](image)

In the third term of last year I was asked by my school’s groundskeeper to help him work out where to mark the staggered starts for the 200 m, 400 m and 800 m races on the athletics track. Working this out for the 200 m race was the most important job since the 400 m race would just be double the number of laps of the 200 m race and the 800 m race four times the number of laps of the 200 m race.

Being the keen mathematician that I am, I immediately gathered all the information on the dimensions of the school’s athletics track and set about drawing a replica of the athletics track. I then used the formula for the circumference of a circle to calculate the distance around the inside lane of the track and, using the dimensions of the width of each lane, calculated the distance around the remaining five lanes. After an hour and a half of drawing and calculating, I was able to calculate how far along each lane the staggered starts for the 200 m race would need to be positioned to ensure that every athlete would run the same distance.

By this time I was feeling quite pleased with myself, not only because I had been able to solve the problem, but because here was evidence of the usefulness of mathematics in daily life. Here was an example that I would be able to use to convince my students of the importance and applicability of mathematics to their lives.

With this in mind, I arrived at the lunch table boasting about my mathematical achievements and wasted no time and energy explaining to those unfortunate enough to be there about how I had approached and solved the problem. I even went so far as to
comment that this example is clear proof that mathematics is far more useful than any other of the school subjects.

It was at this point that the Geography teacher, who had been listening quietly but intently at the end of the table, completely and utterly destroyed my bubble. He simply stated:

“If you wanted to know where the start of the 200 m race should be in each lane, why did you not just use a measuring wheel and, starting at the finish line, walk around each lane in the opposite direction to which the athletes would be running until the measuring wheel recorded 200 m? This would show where the athletes must start from in each lane to run 200m”.

If you are feeling as confused as I was at the time, then perhaps the picture below will help to explain what he was suggesting.

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**Why was this a light bulb moment?**

As a mathematician, I am always tempted to try to solve problems that I encounter in the world around me using a mathematical approach. Unfortunately, real-life is often more complicated than the classroom because there are variables that come into play in real-
life problems that never even appear in the textbooks. Sometimes, real-life problems that involve mathematics can be solved more easily and quickly without using any mathematics at all.

Consider my story about marking the staggered starts on the athletics track. I approached the problem from a purely mathematical perspective and solved the problem using calculations and formulas. The geography teacher, on the other hand, approached the problem from a far more practical perspective and solved the problem using logic and measurement rather than mathematical calculation. What enabled him to do this? He is a runner with a great deal of life experience relating to athletics tracks and running distances.

In this scenario, although my mathematical approach to the problem yielded the correct answer, the geography teacher was able to solve the problem more easily and more logically by looking at the problem through non-mathematical eyes. He was able to solve the problem by drawing on his life experience and by using logic, rather than by trying to see the problem from a scientific perspective using scientific methods.

My discussion with the geography teacher was a “light bulb” moment for me because it opened my eyes to the dangers involved in trying to approach real-life problems from a purely mathematical perspective. It brought about the realisation that a mathematical approach to a real-life problem does not always provide the best solution and of the importance of logic and life experience in solving problems based in real-life scenarios. It also, once again, reinforced the difficulty involved in both teaching and learning Mathematical Literacy: the difficulty of teaching because teachers are no longer only teaching content but also logic and life experience as well; and the difficulty of learning because students now have to rely on more than just mathematical knowledge and textbook explanations to solve problems.
In this paper, I describe and discuss ways in which connections are described in the South African Mathematics National Curriculum Statement and related documents. The centrality of connections in the conceptualization of mathematics and the learning outcomes and assessment standards is identified and discussed. The notions of representation and integration as key aspects in understanding connections in mathematics are analysed with respect to the National Curriculum statement. Finally, theoretical and practical implications of connections in the curriculum are identified.

INTRODUCTION

The new curriculum allows us to see mathematics in new ways. It also allows for the development of teaching strategies that help us see teaching in new perspectives. The new vision in the curriculum also helps us to conceptualise assessment in ways that help educators to recognize that all learners can do and succeed in mathematics. This paper sees the new curriculum as presenting an opportunity for engaging and understanding curriculum itself. The paper grapples with the broad question: what is our current understanding of the FET mathematics curriculum that is presently being implemented in South Africa? There have been some recent and highly informed attempts to understand the context (Cross, Mungadi & Rouhani, 2002) and practice (Graven, 2004; Naidoo & Parker, 2005) of curriculum and its implementation in South Africa. Although the concept of connections lies at the heart of key deliberations concerned with our new curriculum (see for example, Forgasz, Jones, Leder, Lynch, Maguire & Pearn, 1996), none of these discussions have put connections as an object of study and understanding. This paper contributes to this gap in our understanding of the FET mathematics curriculum. It identifies and analyses the range and nature of connections that are in this curriculum. The issues and questions that are posed in this paper with respect to connections are intended to open possibilities for thinking about connections that allow for and acknowledge complexities in curriculum and practice.

In the sections that follow, I present an analysis of the way connections are presented in the curriculum. The analysis commences by examining connections in definitions of mathematics. It then describes connections in the way the learning outcomes and assessment standards are presented. The paper concludes with a presentation of an emerging picture of connections in the curriculum.
CONNECTIONS IN THE DEFINITION OF MATHEMATICS

Connections are an underlying principle of the constitution of mathematics. According to the National Curriculum Statement (NCS), mathematics as a discipline is viewed as follows:

Mathematics enables creative and logical reasoning about problems in the physical and social world and in the context of mathematics itself. It is a distinctly human activity practised by all cultures. Knowledge in the mathematical sciences is constructed through the establishment of descriptive, numerical and symbolic relationships. Mathematics is based on observing patterns; with rigorous logical thinking, this leads to theories of abstract relations. Mathematical problem solving enables us to understand the world and make use of that understanding in our daily lives. Mathematics is developed and contested over time through both language and symbols by social interaction and is thus open to change (Department of Education (DoE), 2003, p. 9)

Relationships, hence connections, are at the heart of the definition of mathematics, at least in the way mathematics is conceptualized in the NCS. These connections are concerned with what mathematics is: where it comes from – human activity, a construction, a development and contestation that is time-and-socially dependent), and what it does: problem-solve and understand the world and daily living. Mathematics is not about reasoning for its own sake. It is concerned with reasoning, symbolizing and thinking, processes that are connected to activities and problems of the social, physical and mathematical worlds involving human practices in all cultures.

In the foregoing, it is clear that there is a conceptual and social dimension to mathematics, its essence and use. Mathematics has both social and conceptual connections. Mathematics is a highly conceptual field, a field of knowledge consisting of concepts that are structured in specialised ways. This entails that the processes of coming to know and understand mathematics are also specialized. Ability to do mathematics well: to represent and communicate mathematics effectively hinges on learners having achieved a conceptual understanding of mathematical concepts and procedures and relations between concepts and procedures (Kilpatrick, Swafford, & Findell, 2001). The “powerful conceptual tools” that are made available by mathematics enable learners to “analyse situations and arguments; make and justify critical decisions; and take transformative action” (DoE, 2007, p. 7).

WHAT KIND OF CONNECTIONS APPEARS TO BE MORE PRIVILEGED?

According to the National Curriculum Statement, “opportunities for making connections” across various mathematical contents involved within and across learning outcomes “should be sought in requiring the solution to standard as well as non-routine unseen problems” (DoE, 2003, p. 54). However, it appears that the kind of connections that are expected to be demonstrated by learners are potentially exclusive. This concern can be seen in the following statement regarding the “purpose” of mathematics:

An important purpose of Mathematics in the Further Education and Training band is the establishment of proper connections between Mathematics as a discipline and the application of Mathematics in real-world contexts (DoE, 2003, p. 10, emphasis added).
What is meant by “proper connections” in the foregoing statement? Does this suggest there are certain realms of connections between mathematics and its applications that may be “improper”? And according to whom are these connections considered to be proper? Who authenticates these connections? I suggest that the naming of certain connections being considered as “proper” cannot be considered without considering who is making sense of those connections. In this respect, Presmeg (2006) has argued that “for the purposes of connecting knowledge in the teaching and learning of mathematics, it is essential to take the meaning-maker into account” (p. 172).

The second concern regards the privileging of connections that are more mathematically focused. For example, the NCS specifies the following competence descriptions in connection with the kind of learner that this curriculum is expected to produce:

- By the end of Grade 10 the learner with meritorious achievement can:
  - make connections among basic mathematical concepts (p. 74);
- By the end of Grade 11 the learner with meritorious achievement can:
  - make connections between important mathematical ideas from this and lower grades (p. 75);
- By the end of Grade 12 the learner satisfactory achievement can:
  - make connections across important mathematical ideas and provide arguments for inferences (p. 77);
- By the end of Grade 12 the learner with outstanding achievement can:
  - synthesise across different outcomes and make connections with other subjects (DoE, 2003, p. 73);

In the above, there is a potential privileging of mathematical content – basic and important mathematical ideas and mathematical argumentation – in the type of connections that are envisaged to be produced by learners with satisfactory/meritorious achievement. We also see that there is an expectation that Grade 12 learners with an outstanding achievement need to “synthesise across different outcomes and make connections with other subjects”. However, we note here that making “mathematical” connections does not seem to be a key feature of the competences anticipated for learners with adequate, partial or inadequate achievement. Is this an indication that the kind of connections being anticipated and emphasized at this level are of such a conceptual nature that they cannot be demonstrated by under-achieving learners? It might also be that the kind of connections being valued here are those of a more “proper” and “correct” nature. Connections or relationships that may be attempted but which may be regarded as “not correct and not explained” (DoE, 2006, p. 64) may be unrecognised.
CONNECTIONS IN THE LEARNING OUTCOMES AND ASSESSMENT STANDARDS

In the National Curriculum Statement, there are connections in the key elements (knowledge, skills and values) of learning outcomes and experiences to be gained by learners. According to the NCS, “it is important not to think of Learning Outcomes as independent of each other” given, for example, that it is “impossible to study measurement without having an understanding of numbers and operations involving numbers”. Every Learning Outcome is associated with a number of Assessment Standards which “describe the minimum level at which learners should demonstrate their achievement of the Learning Outcome(s) and the ways of demonstrating their achievement”. The assessment standards are intended to be specific to a grade and show “how conceptual progression will occur in a learning area” (DoE, 2006, p. 16, emphasis added).

According to Learning Outcome 5, the learner is expected to “deal with data in significant, social, political, economic and environmental contexts with opportunities to explore relevant issues (e.g. HIV/Aids, crime, abuse, environmental issues)”. The associated Assessment Standard 8.5.1 indicates that this outcome will be established if the learner “poses questions relating Human Rights, social, economic, environmental, political issues” in his own environment. It is expected that the learner “should be critical and aware of the use, and especially abuse, of data representation and statistics” in the analysis and interpretation of data. So the engagement of the learner in data handling not only provides entry into the mathematical concepts involved but also allows the possibility for learners to understand and learn about the social and everyday contexts with which they may not have been familiar. This is accomplished through Mathematical modeling. According to DoE (2003, p. 10), mathematical modeling “provides learners with the means to analyse and describe their world mathematically, and so allows learners to deepen their understanding of Mathematics while adding to their mathematical tools for solving real-world problems”. This understanding is also accomplished by “posing questions” related to issues relating to the particular context (DoE, 2006, p. 18).

There are also connections in the critical outcomes and assessment standards. One of the critical outcomes expects learners to “demonstrate an understanding of the world as a set of related systems by recognising that problem-solving contexts do not exist in isolation”. The associated Assessment Standard 8.2.1 indicates that this outcome will be established if a learner is able to “investigate and extend numeric and geometric patterns looking for relationship or rules, including patterns found in natural and cultural contexts” (DoE, 2006, p. 20).

It is evident in the above analysis that there are connections that emerge in learners’ engagement in solving problems and investigating contextualized situations that involve the real world. What is at issue here is the extent to which learners will be able to pose problems given that they are more attuned to traditional classroom cultures which present them with already formulated problems.
REPRESENTATIONS AS AN ASPECT OF CONNECTIONS

A key aspect of critical outcomes for the NCS is learners’ abilities to make “representations”. For example, within the domain of geometry, while ensuring that learners are mathematically literate, they are required to work towards being able to:

- describe, represent and analyse shape and space in two and three dimensions using various approaches in geometry (synthetic, analytic transformation) and trigonometry in an interrelated or connected manner” (DoE, 2003, p. 10).

The use of representations is also a key aspect across all learning outcomes. In Learning Outcome 1, learners are required to “recognise, describe, represent and work confidently with numbers and their relationships to estimate, calculate and check solutions” when solving problems. Within this outcome, learners are expected to “expand the capacity to represent numbers in a variety of ways and move flexibly between representations” (DoE, 2003, p. 12). In Learning outcome 2, learners are required to be able to “investigate, analyse, describe and represent a wide range of functions and solve related problems”. Within the FET band, learners “should… use symbolic forms to represent and analyse mathematical situations and structures” (DoE, 2003, p. 13). The foregoing statement captures the power of the algebraic component associated with this learning outcome. It signifies a connection between symbolic forms and the analysis of mathematical situations and structures. According to the DoE (2003),

A fundamental aspect of this outcome is that it provides learners with versatile and powerful tools for understanding their world while giving them access to the strength and beauty of mathematical structure. The language of algebra will be used as a tool to study the nature of the relationship between specific variables in a situation. The power of algebra is that it provides learners with models to describe and analyse such situations (p. 12, emphasis added).

The concept and language of algebra is so critical that learners need to have access to mathematical knowledge and its structures if they are to understand their world and decipher “unknown information” about situations. It does not seem like learners have a choice. They must develop symbolic knowledge. There is very little sense from the above that there could be other means available for learners to understand and analyse their worlds and situations apart from the use of mathematics and algebra in particular. What is being suggested here is for learners to be make other connections that might be different from those suggested by the NCS. This is important as it recognises the fact that the curriculum is one but not the only resource from which learners can draw in order to make sense of the world they are living in.

The powerful nature of the concepts of algebra and function is also evident from the fact that these concepts constitute a learning outcome of their own (LO2). There is therefore a requirement that learning programmes be structured in ways that provide for appropriate learning experiences and situations that develop these key concepts and enable learners to “experience the power of algebra as a tool to solve problems” (DoE, 2003, p. 13). There is a requirement to represent mathematical models of situations in different ways: “in words, as a table of values, as a graph, or as a computational
procedure (formula or expression)” (p. 12). Learners need to work fluently and flexibly with conversion between numerical, graphical, verbal and symbolic representations.

In Learning Outcome 3, learners are required to “describe, represent, analyse and explain properties of shapes in 2-dimensional and 3-dimensional space with justification” (DoE, 2003, p. 13). The study of space, shape and measurement enables learners to: “link algebraic and geometric concepts through analytic geometry” and to “analyse natural forms, cultural products and processes as representations of shape and space” (p. 14). According to the DoE, the proposed content for Learning Outcome 3 “really only becomes meaningful and alive when used to address issues of importance to the learner and to society” (p. 60).

With respect to Learning outcome 4, engaging learners in collecting, organising, analysing and interpreting data to establish statistical and probability models to solve related problems enables them to “become critically aware of the deliberate abuse in the way data can be represented to support a particular viewpoint” (p. 14).

The ability to produce and work with representations is a key competence particularly for learners that are categorized as “outstanding achievers”. According to the DoE (2003),

By the end of Grade 10 the learner with outstanding achievement can:
make use of appropriate mathematical symbols and representations (graphs, sketches, tables, equations) to communicate ideas (p. 72);

By the end of Grade 11 the learner with outstanding achievement can:
use appropriate mathematical symbols and representations (graphs, sketches, tables, equations) to communicate ideas clearly and creatively, linking across Learning Outcomes (p. 73);

By the end of Grade 12 the learner with outstanding achievement can:
communicate solutions effectively, thoroughly and concisely, making use of appropriate symbols, equations, graphs and diagrams (p. 73).

Across the three grade levels, there is an emphasis on learners being able to “use appropriate mathematical symbols and representations” and to communicate mathematical ideas and solutions effectively using symbols and representations. However, it appears that this kind of competence (making “proper” representations) does not seem to be anticipated for learners with adequate, partial or inadequate achievement.

**INTEGRATION AS AN ASPECT OF MAKING CONNECTIONS**

According to the DoE (2003, p. 3),

Integration is achieved within and across subjects and fields of learning. The integration of knowledge and skills across subjects and terrains of practice is crucial for achieving applied
competence as defined in the National Qualifications Framework. Applied competence aims at integrating three discrete competences – namely, practical, foundational and reflective competences. In adopting integration and applied competence, the National Curriculum Statement Grades 10 – 12 (General) seeks to promote an integrated learning of theory, practice and reflection.

The DoE recognizes integration within mathematics by acknowledging that “Learners need to be able to see the interrelatedness of the Mathematics they are learning” (DoE, 2006, p. 49). It is also observed that “integration within a learning area is automatic in the sense that you cannot work with measurement without integrating with number” (p. 76). This means that integration within learning areas is an inevitable activity.

Adler, Pournara and Graven (2000) have identified various levels of integration: “integration of the various components of mathematics, between mathematics and everyday real world knowledge; and where appropriate, across learning areas” (p. 3). They have argued that while integration is desirable, the extent of the demands placed upon teaching makes integration less feasible. However, integration across learning areas seems to be more feasible at the lower grades than at the higher. While proposing that contexts\(^{15}\) are a useful way in which to “integrate” learning areas, the DoE (2006) have noted the following:

> Up to Grade 6, it is customary to select a couple of contexts, for example four per year, and deal with all the Mathematics content under each one of those contexts. We could, for example, decide to choose Our School as context for the first term and relate all the Mathematics to the school. Then for the second term you could decide to choose Building a bridge as your context... However, note that the further we progress from grade to grade, i.e. from primary school to secondary school, it becomes increasingly difficult to define one context that is relevant for all core knowledge and concepts.

It appears that ways of proceeding with integration determines what kinds of integration can be made possible. According to the DoE,

> In Mathematics It is more sensible to identify the different core knowledge and concepts (these then become the headings for the different Lesson Plans), and then choose a context for each activity to make the learning material more enriching and more meaningful to the learner (DoE, 2003, pp. 27-28).

There seems to be a claim here that integration becomes “more sensible” if mathematics is the starting point. What does “more sensible” mean here? Arguing from the perspective of “transfer”, Parker (2006) has pointed out that “the idea of transferability of everyday knowledge into mathematics is absent” in the National Curriculum Statement. The focus appears to be “on the establishment of proper connections between mathematics as a discipline and the application of mathematics in the real world”

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\(^{15}\) According to DoE (2006), “contexts refer to the situations or conditions in which content is taught, learnt and assessed. These are derived from “different sources” such as: the nature of the Learning Area being taught, the socio-economic environment of learners, national and other events, interests, nature and needs of learners, and the integration of appropriate Assessment Standards from other Learning Outcomes and other Learning Areas (p. 27).
Does that mean that connections that involve moving from the everyday into mathematics are not “proper”? It appears that making connections that involve moving from mathematics into the everyday has a more privileged status than the making of connections that employs moves from the everyday world into mathematics.

**CONNECTIONS IN THE CURRICULUM: AN EMERGING COMPLEX PICTURE?**

In the above analysis, I have presented and discussed ways in which connections are described in the NCS curriculum. This discussion has captured the following: the centrality of connections in definitions of mathematics (as evidenced in the NCS); the privileging of “proper” connections in mathematics; connections in learning outcomes and assessment standards; representation and integration as key components of the curriculum statement and learning outcomes. The range of connections that are described in the NCS that gives spaces for the curriculum to develop and emerge as a “conversation”. Two key forms that this conversation is taking are: conversations within mathematics and conversations between mathematics and/or the intended educator/learner audience. These conversations are important given the nature of the broader curriculum that is informing teaching and learning in South African schools in the current society.

How do we need to understand these connections? There are a number of issues and questions that need to be posed in order to understand these connections. The first concerns the kind of theories that need to be invoked in attempting to understand and make sense of the nature of connections that are apparent in the NCS curriculum for mathematics. We need to ask the question about the kind of assumptions about curriculum that need to inform our analysis of connections in the curriculum. I suggest here that there is an opportunity, in such an analysis, for taking a perspective of “curriculum as conversation”, as elaborated elsewhere (see for example, Reeder, 2005)?

The second is an issue about the kinds of implications that these connections have for practice. This is particularly important in relation to the requirement that teaching needs to integrate across learning areas. It needs to be acknowledged that working in integrated ways in the school curriculum makes available new visions and realities for schooling. However, bringing about this vision into reality is a complex activity that is likely to meet with challenges both within and beyond specific curriculum disciplines. There is a need to understand the identities of educators and learners (their beliefs about pedagogy and schooling) that need to support the enactment of this vision. There is a critical challenge here that concerns the mathematics, and in general, the knowledge that is needed for teaching and learning. For example, at present, we do not know much

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16 There is a complexity that is inherent in all curricula here. This concerns the point that there are bound to be contradictions and biases in the statements of outcomes and intensions of curriculum. Translating into practice a curriculum with its inherent contradictions and biases is therefore a complex phenomenon.
about what happens when the mathematics needed in order to make learners understand a science concept that requires mathematics is not well understood. The training that is currently being provided to enable educators to implement the curriculum needs to be acknowledged. However, there is an important gap that has not been recognized both in education policy and in teacher education practice that concerns the preparation of learners to work in a reformed curriculum that demands making connections within mathematics and integrating across disciplines. Many approaches to implementing reform concentrate on preparing teachers to implement such reforms. The workshopping of teachers for the South African NCS/FET mathematics curriculum is a sound example. There have been many inadequacies with this way of proceeding in implementing curriculum reform. Some of this inadequacy has to do with the attendant conceptions of curriculum that are informing (or need to inform) curriculum implementation. There is a need to conceptualise approaches that acknowledge that just as new curriculum proposals place heavy demands on teachers, they also place demands on learners. The conceptual as well as practical questions linked to this position add to the complexity of implementing new curricula. Does it mean that when institutions prepare teachers for the new curriculum they are also preparing learners to anticipate and plan for the respective demands being placed upon them? Is there a place for “workshopping” learners for the new curriculum? Are there any approaches to curriculum implementation that consciously prepare learners to respond to the demands of the new curriculum?

The fact that there is an emphasis on connections in the National Curriculum Statement in mathematics education in South Africa is consistent with developments in mathematics education globally. In the context of mathematics education in the USA, the National Council of Teachers of Mathematics (NCTM) has a curriculum standard dedicated to connections. In the NCTM curriculum standards for grades 9-12 (a phase similar to the South African FET), it is proposed that:

- The mathematics curriculum should include investigation of the connections and interplay among various mathematical topics and their applications so that all students can:
  - recognize equivalent representations of the same concept;
  - relate procedures in one representation to procedures in an equivalent representation;
  - use and value the connections among mathematical topics;
  - use and value the connections between mathematics and other disciplines (NCTM, 1989, p. 148, emphasis added).

In the above statement, there is an emphasis on recognizing and valuing of connections and representations by all students. The requirement for students to be engaged in “investigation of mathematical connections” so that students can “use a mathematical idea to further their understanding of other mathematical ideas” (NCTM, 1989, p. 84) underlines a key element of mathematical activity that has potential to make connections within mathematics possible. In the context of mathematics education in South Africa,
we need to deepen our understanding of not only what these connections are but also what purposes these connections are intended to serve.

There is also a further critical issue in understanding connections in mathematics which concerns the matter of what needs to be appropriate and practical starting point for recognizing and understanding connections. In her discussion of the “connections” standard in the context of mathematics education in the USA, Presmeg (2006) has noted two ways of making connections between mathematics and everyday life.

Start with an everyday practice that is meaningful to the participants, and then see what mathematical notions grow out of the chaining as it is developed. Secondly, one might focus on a mathematical concept that is to be taught, and then search for a starting point in the everyday practices of students that can lead to this concept in several links of the chaining process (p. 167).

In relation to the South African curriculum and curriculum textbooks, we need to ask the question: what appears to be the starting point for making connections? Does the making of connections start with mathematical content/concept or with everyday/non-mathematical contexts? What kinds of learning are made possible by these different starting points? What kinds of starting points do teachers recognize and/or prefer in their teaching? What kinds of learning opportunities about mathematics do these make possible? There is also the general question about when and why might it be more appropriate to start from a particular practice (within or outside mathematics) rather than another when making connections.

Extended engagement with the issues and questions identified here would contribute to the deepening of our understanding of the new NCS curriculum which is replete with demands upon educators and learners for making connections, producing representations and working in integrated modes within mathematics and across curriculum disciplines.

REFERENCES


TEACHER’S SELECTION OF CONTEXTS IN MATHEMATICS
LEARNING SUPPORT MATERIALS FOR TEACHING THE
CONCEPT VARIABLE

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INTRODUCTION

Over the years mathematics has been perceived by many people as a subject that is generally difficult to understand and to learn. Barbeau (1990, as cited in Picker and Berry 2001: 65) puts it more aptly by saying: ‘Probably no area of human activity is afflicted as mathematics with a gap between the public perception of its nature and what its practitioners believe it to be.’ Efforts to close this gap have resulted in several explorations and researches on how to make the subject accessible to the broader public (Boaler, 1999; Cobb, 1986; Dossey, 1992; Furinghetti, 1992; Hammond, 1978).

The South African mathematics curriculum has opted for the adoption of the selection and use of contexts as one of the approaches in the teaching and learning of the subject. The first four Learning Outcomes (LOs) of mathematics in the Revised National Curriculum Statement (RNCS) (Grades 7 – 9) which are (1) number, operations and relationships; (2) patterns, functions and algebra; (3) space and shape (geometry) and (4) measurement are characterised by a statement that leads with the words: ‘Contexts should be selected ...’ (Department of Education, 2002: 62 – 65). Although the words are conspicuous by their absence in Learning Outcome 5 (LO5): Data Handling, the very data that has to be handled is contextual.

The variability of contexts precludes them from what is considered in the mainstream as characteristic of mathematics. Accuracy and exactness posits mathematics as strongly classified. Bernstein (1982: 159) defines classification as the degree of boundary maintenance between contents and as the ‘division of labour of educational knowledge’. The stronger the classification the more sharply is the boundary between the contents drawn. This strong classification also reduces the power of the teacher over the knowledge that is to be constructed.

It is apparent that the intention of the new curriculum in advocating for the use of context is meant to reduce these boundaries. It has to be recognised however that it is not only improbable to obtain objects in the everyday context that are equal to one another but quantifying or qualifying them in terms of accuracy and exactness is also highly unlikely.

The use of contexts in the curriculum, especially relating to their use in mathematics, is a fairly recent practice in South Africa and is one of the significant features of outcomes-based education (OBE). It is important to investigate how teachers deal with the issue in practice. This paper explores how a teacher, faced with the expectation of using contexts in the teaching and learning of mathematics, selects contexts in the
learning support materials in order to promote the understanding of the concept variable amongst learners. In the Grade 7 assessment standards of Learning Outcome 2 (LO2) the learners are expected to represent and use relationships with variables (DoE, 2002: 74).

The following critical questions guided the exploration:

- What contexts are selected by a Grade 7 teacher to promote understanding of the concept variable and why?
- How are the selected contexts intended to be used by the teacher in order to promote the understanding of the concept variable?
- To what extent do the contexts selected for use have the potential to promote the understanding of the concept variable?

Mathematics practitioners would be greatly assisted if they could be informed about what constitutes the selection and use of context in the teaching and learning of the concept variable in a manner that will promote its understanding.

**CONTEXT**

The research was conducted at Siyabonga\(^{17}\) Primary School, a former Model C school in Gauteng, which is situated near a township with the result that almost the entire learner population is Black. The teacher population includes Blacks, Whites, Coloureds and Indians. The facilities in the school are way above those of a typical township school with the school fees charged per learner being more than five times what is averagely charged in the neighbouring township schools.

The study involved Bongi\(^{18}\), an enthusiastic and vibrant grade 7 mathematics teacher. The teacher had done mathematics up to Grade 12 and proceeded to obtain a diploma in teaching, specializing in Mathematics and Physical Education. She has been teaching mathematics in Grade 6 for the past three years and 2006 was her first year of teaching mathematics in Grade 7. She had attended inservice-training on the RNCS in 2005 but had attended none in 2006.

**THEORETICAL FRAMEWORK AND LITERATURE REVIEW**

The paper draws on the socio-cultural theories that portray understanding as developing through interpersonal activity. It is underpinned by the notion that understanding of a concept presupposes and is presupposed by its formation.

According to Crook (2003) the socio-cultural approach recognises mediation as its central concept in learning and teaching. The emphasis on mediation gives rise to three

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\(^{17}\) pseudonym

\(^{18}\) pseudonym
themes, which are:

- cognition is more than the repertoires of circumscribed and private processes but an activity of integrated functional systems;
- cognitive acquisitions are initially situated and tied to contexts; and
- cognition is a profoundly social phenomenon.

The adoption of a socio-cultural framework is largely informed by the importance of the mediatory roles assumed by teachers and learning support materials in the social interaction that take place in order to facilitate learners’ understanding.

The RNCS states that mathematical ideas and concepts are build on one another to create a coherent structure (DoE, 2002, 4) while White and Gunstone (1992) indicate that in order to understand a concept some information about that concept must be in existence in some way or another in the experience of the participant. The understanding of a concept presupposes and is presupposed by its formation. Skemp (1981) indicates that the initial stages of concept formation is characterised by the bringing of past experiences to the present situation by first classifying previous experiences and then fitting the present experience into one of these classes. He states that the formation of a concept requires a number of experiences which have something in common while at the same time highlighting the importance of contrast and of non-examples which make the similarity between members of a particular classification more noticeable.

Remillard (2005: 212) provides a framework for characterising and studying teachers’ interactions with curriculum materials. She states that: “…the teacher-curriculum relationship is intertwined with other teaching practices, is dependent on the particular teacher and curriculum, and is situated in a specific context”. She identifies four theoretical perspectives on the conceptions of curriculum use, viz. following or subverting the text, drawing on the text, interpreting the text as well as participating with the text.

The framework is used together with an adaptation of the situated perspective which explores learning as a community of practice. The latter is described in terms of legitimate peripheral participation (LPP) which is the descriptor of engagement in a social practice. Increased participation of newcomers (apprentices) in the ongoing practice shape their gradual transformation into old-timers (masters) (Lave and Wenger, 1991; and Lave, 1993).

For the purposes of this paper the curriculum is placed as part of the socio-cultural context and the teacher is viewed as playing the role of an old-timer. The learners are newcomers to be apprenticed into the envisaged form of participation. The proposed framework therefore takes the form of a “classroom participation” model which is captured in the accompanying Figure 1.
The model places the teacher (old-timer) as one whose role is to ‘apprentice’ the learners (new-comers) in order to ‘participate’ with regard to the concept variable. The ‘participation’ is only viewed in terms of the extent to which learners are expected to understand the concept variable as intended in the RNCS. The participation described here is by no means the same as the one described in Lave and Wenger’s LPP. It should be acknowledged that classrooms cannot automatically be seen as communities of practice. This participation is not viewed in terms that are beyond the selection of materials by the teacher in the endeavour to promote the understanding and use of the concept variable by Grade 7 learners. It was rational to expect that whatever contexts that the teacher had selected in order to ‘apprentice’ the learner for ‘participation’ would have some effect not only on the ‘apprenticeship’ of the learner but on the learners’ manner and level of ‘participation’ as determined by the learners’ understanding of the concept variable.

The definition of a variable has undergone refinement as being amongst others ‘a letter that is used in formulas and mathematical expressions’ (Abdelnoor, 1979), ‘where letters appear in a number sentence (equation) assuming different values which make the number sentence true’ (Dekker and Visser, 1984), and ‘a letter or other symbol that is used to represent any number in a specified set’ (Cobran, 1982). Cangelosi (1996) states, however, that a variable is a quantity, quality or characteristic that can assume more than one value. It is important to note that this definition does not restrict the variable to the use of a letter and in fact highlights the implicit link between the variable with contexts. Quantity, quality and characteristic are all descriptions of everyday experiences. This paper therefore aligns itself with Cangelosi’s perspective of the concept of variable.

Van Etten and Smit (2005) argue that the Realistic Mathematics Education (RME), developed in Netherlands, offers an approach that enhances the realisation of the learning outcomes of the RNCS in the South African education. The RME has its roots in Freudenthal’s (1973, 1991) idea of ‘mathematics as an human activity’ where it is
encouraged that learners be given opportunities to reinvent mathematics by the mathematising of subject matter from reality and the mathematising of mathematics matter (cited in Gravemeijer and Stephan, 2002). Mathematisation is viewed as the translation of contexts into mathematics.

**METHODOLOGY**

A qualitative research approach in the form of a case study was pursued. According to Opie (2004) a case study is an in-depth study of a single instance in an enclosed system where certain features of social behaviour or activities in particular settings, together with other factors, influence the situation. Initially a questionnaire was administered in order to gather data that would be used later to inform questions in the interviews.

Two interviews were conducted with the first one mainly focusing on gathering biographical data of the teacher while the second one was used for eliciting data not only on the teacher’s selection activities but on what informed such activities. Interviews encourage the respondents to develop own ideas and allow the latter to say what they think and this is done in greater richness and spontaneity (Oppenheim, 1992: 82). In order to deal with issues of validity and reliability which are problematic in qualitative research, materials that the teacher had used in class were also collected and analysed.

Although the results of the study cannot be generalised they highlight practical situations that teachers encounter in their working environments.

**PRESENTATION OF THE CONCEPT VARIABLE IN THE MATERIALS AVAILED BY THE TEACHER**

The learning support materials that were made available by the teacher to the researcher presented the concept *variable* in a variety of manners. Number patterns, tables, flow charts and placeholders, for instance, were used as precursors for the eventual presentation of the definition of the concept *variable*. The definition is presented in the textbook in the form: ‘In Grade one we learnt that placeholders, such as shapes, can be used to mark the place of a number. For example \( b + 4 = 5 \). This year we learnt that we can use lower case letters of the alphabets as placeholders for numbers. For example, \( b + 4 = 5 \). We call the placeholder letter or unknown, a variable’ (Yule, Du Preez and Omar, 2005: 178). It is, however, significant to point out the absence of an indication of the use of the same letter to represent a variety of numbers in the definition.
ANALYSIS OF THE MATERIALS USED BY THE TEACHER IN THE PRESENTATION OF THE CONCEPT \textit{VARIABLE}

The teacher provided five assessment activities 3.6, 3.7, 4.1, 4.2 and 4.3 that she had used in her class and these were analysed. The activities were all controlled by the school’s head of department of the senior work (name obscured for ethical reasons). Although it appeared in the interview that the teacher’s pedagogical content knowledge with regard to the concept \textit{variable} was apparently inadequate, the adequacy of the materials to deal with the issue was quite the opposite. It was apparent that the selection of the assessment activities by the teacher relating to the concept \textit{variable} had to do with the fact that they were in the learning support materials. This finding supports the conjecture in Collopy’s (2003) study that curriculum materials may facilitate teacher learning. Another possibility was that of interpreting the teacher’s actions in terms of Remillard’s (2005) conception of curriculum use, namely that of users following the text.

In one of the assessment activities (figure 3) the idea of conveying \textit{variables} as letters that were to be replaced by numbers is pursued. It was significant to note that this was the case in the major parts of the activity, as evident in parts 1, 2, 4 and 5. A table was utilised to show the variability of \textit{a} and \textit{b} in the number sentence \textit{a} + \textit{b} = 6. The heights of indigenous trees were used to generate the two graphs in the figure.

![Assessment activity 3.6](image.png)
The concept of variable is captured in this assessment activity in form of a substitutive function where numbers are to be used in the place of letters or vice versa. It is significant however to highlight that in parts 1 and 2 of the activity, the variability of the variables is captured by the apparent infinite number of solutions for the provided closed sentences.

By the notion of mathematisation which is defined in Van Etten & Smit (2005) as the translation of contexts into mathematics, the variable differences identifiable in the kinds of trees mentioned in the activity are ignored in answering the question 6b “How many trees are shown?” The objectification of trees as equal to one another needs to be employed in order to generate the answer. This is possible through mathematisation of the variable trees. It can be argued that the determination of the difference between the shortest and the longest trees will not be possible considering the inherent inaccuracy associated with measurement, an everyday contextual activity. The determination of difference of the variable heights of the trees—albeit inherently impossible—is however possible through mathematisation, where particular units and numbers will be used to respectively objectify and quantify the heights of the trees, for example, three meters (3 m).

ANALYSIS OF THE INTERVIEW

In terms of the classroom participation model described in this paper it was apparent that the teacher’s role as old-timer was determining what she would select for participation with the learners. In spite of the fact that the materials that the teacher had selected contained letters other than x as representing variables, the teacher opted to use only the x in her teaching. This is captured by her saying: ‘You know I have been working this year only with the x’s, not so with the y. How to work with the x’s. But I was telling my kids that next year, they will learn more of the alphabetic words when they go to high school, more of the letters a, b, c and upwards. But this year I have been working on the value of x. How to “work out x”. I have been working on that.’

Even though it is apparent that the learners may have had access to the same materials that were in the teacher’s possession, where letters other than x were depicted (see figure 3), it was the teacher who was determining the form and the level of ‘participation’ regarding the learner’s ‘apprenticeship’ in the understanding of the concept variable. The following figure captures the form of the ‘participation’ and inadvertently signifies the importance of the role that is played by the teacher in this process.
Using this model for analysis, the teacher selected only the $x$ and communicated its exclusive usage to other letters by indicating to the learners that the other letters will be dealt with later in their schooling. As a result, the ‘participation’ in the class as far as the teacher was concerned regarding the concept \textit{variable} was apparently in terms of ‘working out the $x$’.

The teacher’s understanding of the concept \textit{variable} appeared to be more in terms of the substitution of numbers by letters which are called ‘placeholders’ or ‘unknowns’ as defined in the learning support materials used by the teacher. The teacher seemed to view these unknowns, in turn, as objects to be worked out so that they would come back to be in the form of numbers again. The process takes the following form:

<table>
<thead>
<tr>
<th>variable</th>
<th>number</th>
<th>replaced</th>
<th>‘unknown’</th>
<th>work out</th>
<th>number</th>
</tr>
</thead>
</table>

Responding to the question of how the $x$ that she was talking related to other number in mathematics, she said: ‘Yeah, there is a connection. If I can... – I don’t know how to say $x$ – if I can make maybe a typical example: if you go to shop, maybe there is a certain amount of product is $x$-amount, so you have to work out what how much that product is, it can take back to what that product is costing in the shop’. The notion of viewing the concept \textit{variable} in terms of the substitution or the replacing of numbers by letters, as propagated in the learning support materials is elicited in this response. The number that represented the amount of the product was simply substituted by the letter $x$, which in turn had to be ‘worked out’ back to a number again. This highlights some of the challenges encountered by a teacher in dealing with the concept \textit{variable}. 

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It was shown in the analysis of the assessment activity 3.6 that transition from context to mathematics was possible through the notion of mathematisation. Mathematisation may act as the launch pad for initial stages of the formation of the concept and therefore posits itself as a very important tool in the transition from context to mathematics.

**CONCLUSION**

The contexts that the teacher selected for the teaching and learning of the concept *variable* were largely congruent with what and how they were depicted in the learning support materials in which the obscuring of number is pursued. This is done through the use of a placeholder or letter in which the value has to be determined.

But with variable contexts precluding their use in the highly classified mathematics in terms of accuracy and exactness, it transpired that the teaching of the concept *variable* using context in a way that may enhance learners’ understanding remained elusive for the teacher. This paper suggests that the notion of mathematisation can be used as the means by which the boundaries can be weakened in order to transit from context to mathematics in a far as the concept *variable* is concerned. It further posits the concept *variable* is one of the means by which the link between context and mathematics can be made.

It is apparent from this study that access of mathematics to the broader public is compromised in some way by how it is presented in the materials that are used as well as by the activities of those whose obligations and intentions are to facilitate its access. It seems that the gap of the perceptions of the nature of what mathematics is between the public and practitioners will persist for a foreseeable future unless matters such as the ones highlighted in this paper are addressed.

**REFERENCES**


DoE (2002) see Department of Education


LOOKING AT HOW A GRADE SIX EDUCATOR PROMOTES CONCEPTUAL UNDERSTANDING WHEN INTRODUCING MIXED AND IMPROPER FRACTIONS.
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This research looks at how a grade six teacher promotes conceptual understanding in learners when teaching improper and mixed fractions. This study is informed by notions of conceptual and procedural mathematics. The study is also informed by constructivist theory and looks at how the learners construct and restructure knowledge when learning. The study was conducted in one school. Data was collected through videotaping a lesson and interviewing the teacher with a tape recorder. In doing the analysis the researcher demarcated the work into different categories. The findings indicated that the teacher does promote conceptual understanding in learners, but in conversions of fractions she did not. The recommendations suggest that in order to promote conceptual understanding, teachers should use different representations of fractions.

INTRODUCTION
The purpose of this study is to investigate how one grade six educator deals with the challenges she comes across when introducing mixed and improper fractions. I want to find out how she promotes conceptual understanding and procedural fluency in learners when introducing improper and mixed fractions and when dealing with conversion of fractions. This study specifically explores the following research question and sub-questions.

- HOW DOES A GRADE SIX EDUCATOR INTRODUCE IMPROPER AND MIXED FRACTIONS TO LEARNERS IN GRADE 6?
  - How does she deal with procedural fluency and definitions when introducing and converting improper and mixed fractions?
  - How she enables conceptual understanding amongst learners?
  - HOW DOES SHE EXPLAIN WHEN LEARNERS DON’T UNDERSTAND?

WHY THIS STUDY?
My interest in this study comes as a result of my experience as a learner and also as a mathematics teacher in a primary school. As a learner I first encountered the topic of
improper fractions in grade 7. This topic was introduced to us in the form of rules to follow. Through my experience as a teacher I am well aware that most educators experience problems when trying to introduce improper fractions especially in the intermediate phase. It is difficult to address the ‘why’, as they were following procedures without understanding. Kilpatrick et al (2001) argue that it is important that we promote conceptual understanding in learners and not just depend on procedural fluency. They say it is important that teachers should promote conceptual understanding in learners so that they may be able to explain and justify their answers because these are interrelated (Kilpatrick et al, 2001). There is a concern about the learners’ conceptual understanding because Kilpatrick et al (2001) say that once the learners develop conceptual understanding they know more than isolated facts and methods. Kilpatrick et al (2001) further argue that it is easy to remember facts and methods learned with understanding because they become connected and they can be reconstructed when forgotten. This study will therefore explore if this teacher enables conceptual understanding as well as procedural fluency so that her learners may know when and how to use procedures appropriately, flexibly, accurately and efficiently (Kilpatrick et al, 2001).

THEORETICAL FRAMEWORK

Mathematics

The Mathematics part of the theoretical framework will be based on Kilpatrick et al’s (2001) paper. Kilpatrick et al stress the importance of five strands that are conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. Mathematical reasoning is one of the important aspects of the new curriculum in South Africa (DoE, 2002). Learners are expected to actively question, examine, conjecture, and justify their solutions and present arguments. Educators are thus expected to promote these practices in learners when teaching mathematics. Teachers have to ensure that the learners are exposed to mathematical practices that promote mathematical reasoning. Furthermore, the Revised National Curriculum for Mathematics General Education and Training indicates that the mathematics programme should provide opportunities for learners to develop and employ their reasoning skills and be able to evaluate the arguments of others (DoE, 2002).

The focus of this paper will be on conceptual understanding and procedural fluency because that is where the research questions are based. As much as procedural fluency is important it is very important that the learners’ conceptual understanding is well developed. The learner has to be able to explain procedures in order to prove that he/she understands the given problem. If the learner does not have conceptual understanding of a problem it is usually difficult for him/her to explain the procedures involved.

CONCEPTUAL UNDERSTANDING

In conceptual understanding the learners become able to comprehend mathematical concepts. This implies that the learners have a functional grasp of mathematical ideas.
They learn facts and methods with understanding and they become able to explain those facts and methods. The learners also become able to explain concepts and connections. After understanding the basics the learners then become able to invent their own procedures. In the case of my research question I believe that learners need to develop in this area so that they may be able to convert fractions with understanding.

**Procedural fluency**

This strand refers to the knowledge of procedures. The learner must know when and how to use the procedures flexibly, accurately and efficiently. The learner knows how to perform algorithms because he/she knows all the procedures involved. There is however a danger that the procedures might be known without understanding which might lead to difficulty in solving other mathematical problems. My desire to move from procedural fluency to conceptual understanding will help me to be able to help my learners to have deeper understanding of conversions. This understanding will help my learners to be able to come up with their own procedures.

**CONSTRUCTIVISM**

This study is informed by constructivist theory. Constructivists believe that knowledge of reality is a result of going beyond the information given (Bently et al., 1991). Bently et al. (1991) say that according to Piaget the knowledge that is constructed makes sense in relation to the knowledge that already exists. Piaget says that knowledge is actively constructed because the mind is actively involved. Hatano (2001) alludes to the same sentiment as he says that knowledge is not acquired by transmission only but also by construction. He says that for knowledge to be usable in a variety of problem solving situations it has to be reconstructed first. While learners are in the process of learning they are in the act of reconstructing. The study is broadly informed by the social constructivists’ theory of learning (Von Glasersfeld, 1987; Vygotsky, 1987; Taylor and Campbell-Williams, 1993) cited in Jaworski, (1999) which recognises the importance of a knowledgeable other in the construction of knowledge. Vygotsky’s theory focuses on teaching and conceptual development. He acknowledges that the child develops through the intervention of knowledgeable others, namely peers or adults. Vygotsky (1996) argues that during the process of development the teacher should take cognisance of the development already achieved by the learner. The learners are not in the process of construction, only but they are in the process of restructuring, so that they may be able to move to the advanced version of mathematics. The learners combine their prior knowledge or existing knowledge with the new knowledge and then they try to make sense of this new knowledge. To do this they reconstruct and restructure their prior knowledge, (Hatano, 2001). This shows that learners’ prior knowledge plays a very important role in learning of fractions. It is therefore important that teachers understand the learners’ level of understanding of fractions before introducing new fraction concepts.
Vygotsky (1996) explains the zone of proximal development as the distance between the child’s actual developmental level and the level of potential development. He says “by means of copying the child is able to perform much better when together with and guided by adults than left alone, and can do so with understanding and independently” (Vygotsky, 1996, p 175). He says that through social interaction the child reaches his potential under the guidance of an adult or in collaboration with more capable peers. While the teacher is in a process of working towards mathematical proficiency, learners’ prior knowledge might affect the process of conceptual understanding (Hatano, 2001). Smith et al (1993) argue that flawed content and misconceptions interfere with learning of new concepts. They further argue that some misconceptions are powerful enough to influence what students actually perceive, thereby decreasing the chances of understanding new knowledge successfully. Introducing fraction concepts to learners could then be hindered by the powerful misconceptions rooted from the prior knowledge. That is why working with conceptual understanding of fractions is so important.

TEACHING AND LEARNING OF FRACTIONS

There is a lot of research that has been done focussing on fractions and some of this research focuses on learning and teaching. Lukhele and Murray (1999) conducted a study entitled “Learners’ understanding of addition of fractions” which involved grade 3 and grade 4 learners. The findings show that indeed there is a serious problem about fractions. Lukhele and Murray (1999) suggest that the problem lies with the teachers i.e. one of the problems could be that teachers present fractions in an abstract way or the learners have to do algorithms for the operations on fractions before they have understood the concept. They believe that teachers need to get in service training in order to be able help learners understand fractions conceptually. These findings are similar to those in Jooste’s (1999) study. While Jooste (1999) did not focus only on addition, but more generally on how grades 3 and 4 learners deal with fractions, she argues that learners could not use fractions learned in class in their everyday life because their teachers taught fractions out of context. Murray et al (1999, p306) support this argument that learning fractions out of context may lead to the “adverse effect of rote procedures on students’ attempts to construct meaningful algorithms for operations of fractions.” (Murray, 1999, p 3-306).

Booker (1998) cautions teachers that children’s initial understanding of whole numbers form the basis of the learners’ understanding of fractions in future. In his paper he refers to Streefland (1991) who argues that “indeed prior knowledge of whole numbers is often a hindrance to developing a meaning for initial fraction ideas.” Booker’s research was being done in a predominantly middleclass background. Murray et al (1999) support Booker’s argument that the influence of whole number schemes may encourage students to interpret the fraction symbol as two separate whole numbers if they don’t have sufficient experience there. Lukhele et al (1999) who did their research in township schools are also concerned about learners’ prior knowledge. They argue that some
errors done by children when doing fractions arise from a very common limiting constructions arising from the learners’ experience with whole numbers and the set algorithms which are taught for whole number arithmetic. (1999, p 47).

Lukhele et al (1999) further argue that the learners’ errors may be traced back to two causes: weak or non-existent understanding of the fraction concept; and no understanding of the symbolic representation of a fraction. For example children cannot explain what $\frac{3}{4}$ means. These authors are making us aware that for learners to be able to connect fractions to real life and to have a conceptual understanding of fractions they need to be skilled so that the learners may not encounter problems in future.

Murray et al (1999) in their paper also argue about fraction problems in another form. Murray et al (1999) find themselves in a school and classroom environment with serious practical and organisational problems and they have to remedy the fraction problem that is already there (Murray et al, 1999). Murray et al (1999) however acknowledge that it is not an easy task to rectify the problem that is why they encourage prevention strategies.

Looking at the six papers reviewed above it is clear that the authors are trying to make the readers aware that fraction problems are everywhere in school mathematics learning and teaching. Dickson et al (1984) refer to many other researchers who did their studies in different countries. These studies included schools of different socio economic backgrounds. The underlying argument in all these studies is that indeed prior knowledge of whole numbers is often a hindrance to developing a meaning for initial fraction ideas Klein et al, (1998) worked with a sample of prospective and qualified teachers and the programme focussed on teachers’ understanding of the learners’ problems with rational numbers. While reading the paper one sees that before the teachers were workshopped, even though they were qualified teachers, they still encountered problems with fractions (Klein et al, 1998). They worked with three samples, namely: prospective teachers; qualified teachers who had never been workshopped; and qualified teachers who had undergone in-service training. They showed that even though teachers are qualified, they still need in-service training in order that they may understand learners’ misconceptions in fraction concepts. This is the reason why Klein et al (1998) started a STAR programme to help qualified teachers to understand learners’ problems in rational numbers better.
METHODOLOGY

This is a qualitative study investigating how a grade six educator introduces improper and mixed fractions. The study sought to find out about challenges a grade six teacher comes across concerning conceptual understanding and procedural fluency when teaching improper fractions and mixed fractions. A grade 6 class was selected because it was heterogeneous in terms of ability. It had a range of learners from those who achieve high marks in mathematics to those who were struggling to pass. This was important so that the findings may not be based on just few learners’ responses but the whole class. The research focused on the whole class so that the educator could be observed presenting her lesson to the whole class. The observed lesson focused on improper fractions and mixed numbers.

The lesson was video recorded to ensure that as much as possible of what the teacher and the learners did in class during the lesson presentation was captured. The video camera was able to capture the mathematical conversations that emerged while the teacher interacted with the learners. During the course of the lesson, the teacher was moving towards the learners while they were responding to her questions. The video focused on the interaction between the teacher and the learners and how the teacher communicated with the learners to promote conceptual understanding.

After thoroughly watching the video-cassette different categories in relation to the statement of purpose were identified. In each category there was a discussion whether and how the teacher promoted procedural fluency and/or conceptual understanding.

- Category 1: Drawing on prior knowledge
- Category 2: Examples and definitions.
- Category 3: Different representations.
- Category 4: Mathematical terminology.
- Category 5: Reasons for procedures.

Category one: Drawing on prior knowledge

Introducing new concepts to learners in Mathematics is a challenge. Through my experience as a teacher I have realised that a lot of teachers struggle when introducing fractions. It sometimes takes very long for learners to understand fraction concepts. Hartung argues in Dickson et al’s paper (1984, p 276) that “the fraction concept is complex and cannot be grasped all at once. It must be acquired through a process of sequential development”. Wu (2001, p 5) also supports this argument by saying that “the basic fact in mathematics, (is), that mathematics is cumulative and hierarchical: learning any one topic requires a knowledge of most, if not all of the topics preceding it.” In introducing improper fractions and mixed fractions, Thandi made the assumption that learners have the knowledge of the preceding concepts eg whole numbers and proper
fractions. Drawing from prior knowledge is supported by Piaget who says that the new knowledge makes sense in relation to the knowledge that already exists (Bently et al, 1991). This assumption that the learners already knew whole numbers and proper fractions was clear in the first question when she did not explain what the word fraction means. She started by asking a question that required the learners to give examples of fractions that they already knew.

Thandi: Give me any fraction that you know, any example of fraction that you know
Learners: One over eight, one over two, one over four, four over eight and five over six. (Thandi wrote $\frac{1}{8}, \frac{1}{2}, \frac{1}{4}, \frac{4}{8}, \frac{5}{6}$ on the chalkboard)
Thandi: Why do we say they are proper fractions? Who can tell me?
Learner: Because they have big numbers.
Thandi: Where do you see big numbers?
Learner: Four over eight ($\frac{4}{8}$)

Learners could not give the teacher the correct definition of a proper fraction by just observing examples given.

Thandi: Okay a good explanation is that a proper fraction is the fraction where the value of a numerator is less than the value of a denominator, all these are examples of proper fractions (pointing at the given examples)
Thandi: Now give me the examples of big numbers.
Learners: Fifty over seventy, eighty over hundred (she writes $\frac{50}{70}$ and $\frac{80}{100}$)
(Thandi started explaining the concepts ‘numerator and denominator’ to learners.)
Thandi: (she looked at $\frac{80}{100}$ and pointed at 100 ) This is the denominator, angithi niyayibona inkulu (you can see it is big). Who does not understand the numerator and the denominator.

From the example that the learners gave in the extract above it is clear that these learners have an idea of what a fraction is, but cannot define a proper fraction. So Thandi worked with what they know and added to it. She seems to be using what the learners know to introduce what they don’t know. This is the hierarchy of knowledge in mathematics that Wu (2001) is referring to. A hierarchy where prior knowledge is useful in order to acquire the new knowledge. Also Vygotsky (1996) argues the zone of proximal development requires that during the process of development the teacher should take cognisance of the development already achieved by the learner. Thandi shows the learners 100 as a denominator. When the learners responded that they see that 100 is big, she continued without making a follow up to make sure that the learners understand the difference between the denominator and the numerator. She does not find out if they still remember the definition from the previous lesson.
In this category Thandi tells the learners that for them to be able to recognise the proper fraction; they always see it when the numerator is smaller than the denominator. She could not challenge her learners to show the interwoven ness in the strands mentioned by Kilpatrick et al (2001). The teacher imposed procedures to learners. This resonates with what Schiffer (2001) aptly points out in his study that he realised that many teachers needed help in order to develop the skills necessary to assess the validity of a mathematical argument because they have gone to school, sometimes very successfully, memorising facts and procedures. Learners were not given an opportunity to convince others how and why they came up with particular procedures and solutions; the teacher did not encourage the deeper understanding of concepts. Carpenter et al (2003) argues that justification is central to mathematics and mathematics cannot be learnt with understanding without engaging in justification. They maintain that young children cannot learn mathematics with understanding without engaging in justification (Carpenter et al; 2003).

Ignoring the learners also suggested that Thandi is interested in getting to the correct answer. When the learner in the above utterance gives \(\frac{4}{8}\) as an example of big numbers the teacher does not make a follow up on the answer. Instead, she gives the correct definition and thereafter moves on to get more examples. Schiffer; (2001) argues that listening plays an important role in promoting mathematical thinking in learners. The constructivists argue that the learners combine their prior knowledge with the new knowledge and try to make sense out of it (Hatano; 2001). In this case \(\frac{4}{8}\) could have had a certain meaning to the learner, but the teacher does not work with it, she ignores it. Thandi explained the conceptual understanding in the interview but did not promote it in the lesson while explaining the concept of a proper fraction.

Researcher: You were teaching your learners about improper and mixed fractions but I saw you starting with the proper fractions, can you tell me why you did that? Why didn’t you just start with the improper fractions?

Thandi: Ok mam, firstly it is important for learners to understand proper fractions before they understand improper fractions and mixed numbers because proper fractions are fractions between 0 and 1 and improper fractions are greater than one and from improper fractions we can have mixed numbers so it’s going to be simpler to understand. In proper fractions it’s simpler to understand because we only use one whole, for an example if you divide one cake to learners it’s not going to be difficult because you are using only one whole.

She is talking about dividing a cake in the interview. She talks about wholes in the interview but does not talk about them in the lesson I observed. Talking about the cake makes it possible that maybe the teacher used an example of the cake in the previous lesson that is why she does not go into details with explaining proper fractions to learners.

In this lesson she is focusing on procedures, there is no promotion of conceptual understanding although she spoke about it in the interview.
As is evident in the extract above, Thandi uses examples to exemplify the concepts and then asks learners to come up with the definition based on the observation. In line 2 the learners are giving Thandi examples of proper fractions. Thandi wanted the learners to give her the definition of proper fractions after they had given her the examples. Though the learners could not give her the definition of proper fractions she did not abandon her pattern. After she had given them the definition she demanded more examples of proper fractions. She wanted examples of big numbers, in other words she wanted learners to give examples of two digit numbers instead of one digit numbers. The learners managed to give her the examples. What Thandi is doing is consistent with Wu’s (2001) argument that the teaching of any topic in mathematics has to start with clear definitions because mathematics is a logical unfolding of ideas. Thandi’s teaching suggests that she follows a particular pattern in her introduction of new concepts and there is some consistency in her pattern, i.e. examples and definitions and more examples. She is aiming at letting the learners know the mathematics in the improper and mixed fractions through using her pattern.

As much as Thandi is giving reasons why proper and improper fractions are defined, the definitions are still abstract, and were being followed procedurally. By looking at the definition the learners managed to give her examples of proper fractions. Learners seemed to be able to see that the numbers above the line were smaller than the numbers below the lines. There is however; no guarantee that they gave the correct answer because they fully understood what a proper fraction really represented. Also when Thandi was introducing improper fractions she still focussed more on procedural fluency. The extract below is an example of how Thandi starts with examples of improper fractions before getting into the definition. In this case she gave the examples and then asked for the definition.

Thandi:  Now we look at improper fractions (she then wrote the word ‘improper fractions’ using a number line)
Thandi:  You see from these numbers \(\frac{4}{3}, \frac{5}{3}, \frac{6}{3}, \frac{7}{3}\) they are called improper fractions and we are using a numerator and a denominator, who can try to explain what is an improper fraction?
Learner:  It is when the numerator is bigger than the denominator;
Thandi:  You see the good explanation of improper fraction is when the value of the numerator is greater than the value of the denominator. Can you give me examples of improper fractions?
Learners:  Six over one, five over two, twelve over five, eleven over ten and ten over three.
Thandi:  So do you see the difference between a proper and improper fraction?
Learners:  Yes.
Thandi wanted the learners to look at the examples and come up with the definition of an improper fraction on their own. After each definition she demands more examples of the new concept. This suggests that she wants the learners to come up with the definition by observing the given examples. She focuses on the ability to procedures however there is no where where she ensures understanding of why they have to follow this procedure. The learners did not seem to have forgotten the previously learned rules, definitions, and formulas; however, they were never given an opportunity to explain why they were using those rules. The teacher managed to probe the learners until they managed to grasp the pattern of proper fractions.

In introducing mixed fractions Thandi continued with her pattern of giving examples and then a definition. The teacher is trying to persuade the learners to recall the facts about the numerator and the denominator. Learners seemed to remember the facts about proper fractions but had difficulty in grasping the pattern for mixed fractions. This resonates with what Kilpatrick et al (2001) argue about, that without sufficient procedural fluency, learners have trouble deepening their understanding of mathematical ideas or solving mathematical problems.

In this category learners were not provided with opportunities, encouragement and assistance to engage in thinking, reasoning and sense making in the mathematics classroom (Ball & Bass; 2003). Focus on procedures dominated the entire category. The teacher probed learners to recall, reproduce and use formulas and rules without ensuring understanding.

**Category three: Different representations**

In introducing improper fractions, Thandi used a number line. Representing fractions on a number line shows another way of conception of a fraction as a number. This is one way in which a fraction can be interpreted (Dickson et al, 1984).

From my experience as a teacher I have seen that the use of a number line does promote conceptual understanding depending on how it is used. When a number line is used together with the diagrams, it is found to be promoting conceptual understanding much better than when it is used alone. Kilpatrick et al (2001) argue that the degree of a student’s conceptual understanding is evident when he/she can be able to connect various representations. They say that “a significant indicator of conceptual understanding is being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes” (Kilpatrick et al, 2001; p 119). But with this teacher there was mainly procedural fluency in the use of a number line, to show the difference between improper and mixed fractions.

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312
Having presented the number line Thandi drew the learners’ attention to the improper fractions on it. She used the visual picture of improper fractions on the number line to get the learners to define the improper fractions. The strength of what Thandi is doing is that on the number line the learners can see the continuum through common fractions and to improper fractions. The learners can see the numbers growing in front of them from $0/3$ to $7/3$. They don’t just see $7/3$ in isolation. The gradual growth of fractions in the number line gives a visual picture of the improper fractions. This is the other example that Thandi used in order to explain improper fractions.

![Number Line Diagram]

Thandi uses the number line to make the learners aware of the difference between the proper fractions and improper fractions by observing the numerator and the denominator.

Thandi: You can also say from the number line, you can see that the proper fractions lie between zero and $7/7$.

Thandi: What number comes after $7/7$?

Learners: Eight over seven

Thandi: Is it a proper fraction or improper fraction?

Learners: An improper fraction.

Thandi: So you see the denominator is small (she said that pointing at $8/7$ in the number line) proper fractions lie between zero and 1 which is $7/7$.

She uses the number line to make available the interpretation of an improper fraction as a point on a number line. Dickson et al (1984: 282) argues that the use of number line possesses several advantages, for example it makes improper fractions appear more natural, and probably more important, emphasises that the set of fractions form an extension of the set of natural numbers, helping to ‘fill in the holes’ in between them, the number line model thus leads naturally to the use of fractions in measurements of all types.

At the same time Dickson et al (1984: 282) are also concerned that the number line does not incorporate the notion that the fraction can be thought of as part of the concrete object, or as part of the set objects but reduces it to an abstract number.
In this case what Dickson et al (1984) allude to is that it might be possible in Thandi’s lesson that the learners have only an abstract understanding of fractions. The reason could be that she did not even show the positions of whole numbers in the number line. Dickson’s concern draws us to the different types of fractions (discussed in chapter two) e.g. a fraction as a sub area of a unit area, fractions represented by a point in a number line etc. Dickson et al (1984) argue that the learner might know an improper fraction just as the symbol. The learners’ understanding might be insufficient. They might see $\frac{4}{3}$ without fully understanding what it really represents. For example while $\frac{4}{3}$ is a point on a number line, it can also be represented pictorially as follows:

![Diagram of fractions](image)

When introducing mixed fractions; Thandi did not succeed in using a number line in order to explain a mixed fraction. The learners could not give her the definition of a mixed fraction just by looking at the examples. That led her to use another representation ‘a diagram’. Using a diagram made her lesson promote conceptual understanding in learners. She first wrote the word ‘mixed numbers’ on the chalkboard and then pointed on the number line that she had drawn previously for improper fractions.

Teacher: if we start here (pointing at $\frac{4}{3}$) we say three goes into four how many times?

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After giving the rules: dividing the numerator by the denominator to get a mixed number she got mixed fractions. Thandi is really promoting procedural fluency in the concept of mixed fractions.

Thandi: you see these numbers (she pointed at 1 $\frac{1}{3}$; 1 $\frac{2}{3}$; 2 $\frac{1}{3}$) we call these numbers mixed numbers.

After giving these examples she challenged the learners again to give a definition of a mixed fraction by observing the given examples in the number line.
Thandi: who can try to explain what a mixed number is?

In this case learners could not come up with a definition. Muangnapoe in cited Dickson et al (1984) argues that various difficulties were encountered repeatedly when teaching fractions, one of them being the understanding of fractions greater than one unit. This is evident in Thandi’s class when she required her learners to give her the definition of a mixed fraction after looking at the examples. The learners struggled, but they did not struggle with improper fractions. The learners might have struggled because there was a whole next to the proper fraction. They were still used to a numerator and a denominator in proper and improper fractions. The struggle suggests that that the learners have not developed conceptual understanding in improper fractions otherwise they would have been able to recognise a whole. I am saying this because Thandi’s number line for mixed fractions has indicated whole numbers on top. It seems as if the whole numbers did not mean anything to learners.

When the learners could not define what a mixed fraction was Thandi moved to the use of diagrams. She went to a conceptual understanding by using the example of a diagram below.

She used the above diagram to represent $1\frac{1}{4}$ which is the same as $\frac{4}{4} + \frac{1}{4}$. The extract below shows how Thandi talked to the learners about the diagram

Thandi: What is the answer? Five over four which is $\frac{5}{4}$, your denominator is four not eight.

After writing $1\frac{1}{4}$ on the chalkboard she went back to the focus on the definition of a mixed fraction. This time she did not ask the learners to offer their own definition. She gave them her definition in the extract below.

Thandi: The explanation ya (of) mixed number comes from here (pointing at the diagrams showing $1\frac{1}{4}$) the explanation that I want is the sum of a natural number and a fraction, so do you understand where mixed numbers come from, do you all understand?”

Learners: Yes

In the above extract I think Thandi was referring to conceptual understanding because she was pointing at the diagram. Using two different examples strengthened Thandi’s
approaches. The learners saw two visual pictures, i.e. they saw a mixed number in a number line and also saw a mixed fraction represented in a diagram. I think the diagram eases Dickson’s (1984) concern about the number line limiting the learners’ understanding to the abstract. In this case the learners did not just end up with the abstract knowledge of a mixed fraction. They saw the 1¼ in a concrete form, a diagram which could enhance the learners’ understanding of mixed fractions.

The use of a number line however; does not guarantee that the learners fully understand what improper and improper fractions are. The move from using the number line to using diagrams was not just a change in the visual representation but it was a change in conception. From a conception of a fraction as a point in the number line to a conception of a mixed fraction defined in relation to a given whole.

**Category four: Mathematical terminology**

Thandi’s focus is on using the mathematical terminology. Though she relates the terminology to examples, it still promotes procedural fluency more than conceptual understanding. In the above extracts we find her using the words ‘numerator’ and ‘denominator’. She is also naming fractions mathematically e.g. proper fractions, improper fractions and mixed numbers (mixed fractions). Thandi is using the mathematical terminology appropriately, particularly because her definitions of fraction terms focussed on the numerator in relation to the denominator. She however; allows the learners to name the fraction incorrectly by using the word ‘over’ e.g. five over two instead of five halves. Smith et al (1993) argue that flawed content interfere with learning of new concepts. This could impair learners’ understanding of fraction concepts.

The learners actually see the fractions in the number line and also when the teacher talks she points to the fractions in the number line so that they may see the numerator and the denominator. Looking at the examples of improper and mixed fractions given by the learners it is evident that they understood what a denominator and a numerator was though it cannot be concluded that the whole class understood. However because Thandi used the number line only when introducing proper and improper fractions one would not say for sure that they can definitely explain what a numerator or a denominator represent in a fraction like this one below.

Looking at the terminology ‘mixed fractions’; the teacher explained them well in the number line, diagrams and also in definitions. The diagram of 1¼ that explained the
mixed fractions enhanced the conceptual understanding of mixed fractions in learners. They could see a diagram representing a whole next to a proper fraction. They could see a whole number has been put together with a fraction hence a mixed fraction.

**Category five: Reasons for procedures**

In this category I only see the teacher emphasising procedural fluency. She is promoting the use of rules. When dealing with the conversions of fractions; there is nowhere where she promotes conceptual understanding in learners concerning the conversions of fractions. The teacher tells the learners that for them to be able to move from an improper fraction into a mixed fraction they have to divide the numerator by the denominator. Look at the extract below.

Thandi: if we start here (pointing at \( \frac{4}{3} \)) we say three goes into four how many times?

She also gave them the rule that if they have to move from a mixed fraction to an improper fraction they have to multiply the natural number by a denominator and add the numerator. She taught the use of the rules without understanding. Lukhele and Murray (1999) suggest one of the problems could be that teachers present fractions in an abstract way or the learners have to do algorithms for the operations on fractions before they have understood the concept. There was no moment when the teacher explained to the learners why she was using these rules of conversion. When asked about the use of rules in the interview Thandi blamed the college that trained her to be the teacher.

Researcher: in your lesson you were emphasising that the learners should not forget the rules of converting improper fractions into mixed fraction and mixed numbers into improper fractions. Why these rules?

Thandi: Well this is what I was taught at college. It’s a rule that must be followed.

The learners seemed to understand the teacher’s explanation because after she had explained the rules to the learners some of them managed to do the conversion using her method. When learners were given class work some of them gave correct answers. This however does not guarantee that they had developed conceptual understanding in the conversion of fractions. Murray et al (1999: 306) support this argument that learning fractions out of context may lead to the “adverse effect of rote procedures on students’ attempts to construct meaningful algorithms for operations of fractions.” (Murray,1999: 3; 306). While explaining the mixed fractions and when the learners could not come up with the definition, the teacher resorted to the use of a diagram to explain better. This shows that learners have difficulty in understanding fraction concepts. When looking at the research papers that have been done in different countries (Dickson et al, 1984) it is
evident that the issue of learners struggling to understand fractions is universal.

Researcher: Have you ever bothered yourself by trying to find out why this rule is being used?
Thandi: Mh.., well no I have never thought about it, maybe I will now start to think about it now that you have talked about it.

The teacher’s role here was to explain the rules of converting improper fractions into mixed fractions and mixed fractions into improper fractions.

**Class work given to learners**

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<td>1. (a) In an improper fraction the …………………….is greater than the ……………………..</td>
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<td>(b) A mixed number consists of a ……………………. and a …………………….</td>
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<td>2. Place each of the numerals in the correct column</td>
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<td>( \frac{2}{3}; 1 \frac{1}{3}; \frac{3}{4}; \frac{320}{100}; 8 \frac{17}{100}. )</td>
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After explaining all the instructions to the learners they then started writing their class work. The teacher was moving around the learners’ desks while they were busy writing.
RESEARCH FINDINGS

When introducing proper, improper and mixed fractions Thandi promotes more of procedural fluency than conceptual understanding. Often a number line is seen as promoting conceptual understanding, but in this case it was more of procedural fluency. Too much focus on procedural fluency deprived the learners a chance to conjecture, present their arguments and to prove their conclusions. Carpenter et al (2003) also support the importance of conjecturing by stating that they have “found that it is productive to ask children whether their conjectures are always true and how they know they are true” (p. 102). Thandi seemed to be depending on the number line in order to enhance the learners’ understanding of new concepts. As she wanted the learners to know the fraction as a point in the number line she seemed to have succeeded when dealing with proper and improper fractions, though it cannot be claimed that the whole class understood.

Thandi was not just limited to procedural fluency. When realising that the learners could not explain the mixed fraction she proved that she could also promote conceptual understanding in learners. She was able to quickly move to the use of diagrams so that the learners cannot just see a mixed fraction in a number line but also as a diagram. When the learners did not understand she seemed to be capable of moving from procedural fluency to conceptual understanding. Also; when dealing with definition of mixed fraction Thandi seemed to be able to draw some conceptual understanding but not for conversions. In dealing with the rules of converting improper fractions to mixed fractions and vice versa, Thandi stated clearly to me that she only knows the procedures. In other words she could not promote conceptual understanding when it came to the conversions of fractions. As stated in the analysis, she used the rules and emphasised the rules to the learners. She taught them that moving from a mixed fraction into an improper fraction they must multiply the natural number by a denominator and add the numerator.

To make sure that the learners grasped the rules she gave them class work just after the lesson presentation. When I asked her why she was focussing so much on procedural fluency when it came to conversions, she stated it clearly that this is how she was taught at college. Kilpatrick et al (2001) argue that procedural fluency is not limited to the ability to use procedures, it also includes an understanding of when and how to use them. Thandi was not teaching conceptually enough. Failing to explain why she was using the rules of conversion shows that this kind of procedural fluency was not flexible, accurate, efficient and appropriate. There is no guarantee that the learners will be able to use their knowledge of procedure in different contexts (Kilpatrick et al, 2001).

The teacher’s concern of starting with proper fractions is supported by Hartung in Dickson et al (1984) that the fraction concept is complex and cannot be grasped at once. It must be acquired through a long process of sequential development. The implication in Dickson’s paper is that sometimes it makes understanding simpler if we start from the learners’ experience before moving to the complex aspects of mathematics.
RECOMMENDATIONS
It is recommendable to use different representations when teaching fractions to ensure maximum understanding in learners. Using fraction symbols only may limit the learners’ understanding of fractions. Symbols sometimes look very abstract Dickson et al (1984). Gipps (1994) cited in Dickson et al (1984) argues that focusing on symbols only might lead to memorisation and rote learning. Learners might be able to give correct answers without the conceptual understanding of fractions. They might have just memorised the rules not knowing why they are using those rules.

The NCS document emphasises the importance promoting mathematical reasoning in learners and not just in the learners’ correct responses. Kilpatrick et al (2001) is also encouraging the promotion of mathematical proficiency in learners which is conceptual understanding, procedural fluency, strategic competence, adaptive reasoning as well as productive disposition (see theoretical framework).

It is not sufficient for a teacher to select an appropriate task. Teachers should encourage mathematical communication. Moschkovich (2002, p. 193) indicates that “in many classrooms teachers are incorporating many forms of mathematical communication and students are expected to participate in a variety of oral and written practices such as explaining solution processes, describing conjectures, proving conclusions and presenting arguments”. As Kilpatrick et al (2001) argue, the five strands are interdependent, this means that procedural fluency cannot exist without the presence of conceptual understanding. If the teacher focuses more on developing procedural fluency and ignores the other four strands then the learners would not have the kind of procedural fluency that Kilpatrick et al (2001) talks about. Hence it’s interdependency with the other four strands.

The manner in which the task is implemented is also critical. As Schiffer (2001) argues, before the teacher gives the task to the learners he should consider the following:

- How he can attend to the mathematics in what the learners will be saying and doing?
- How he will assess the mathematical validity of learners’ ideas?
- Have the skill to be able to listen to the sense in learners’ mathematical thinking even when something is amiss?
- Have ability to identify the conceptual issues the learners are working on?

Schiffer (2001) argues that though teachers may have stronger mathematics background they still need the skills in order to be able to attend to learners’ mathematical thinking in order to promote conceptual understanding. Teachers need to be able to listen to learners with sharpened curiosity and interest and even be able to know which questions to ask when learners respond and how to mediate the learning process. (Schiffer, 2001).

The above-mentioned points by (Schiffer, 2001) also require a teacher who will have developed good listening skills mentioned by Davies (1997). Listening is an important aspect of engaging with learners’ thinking. It is important to the teacher to consider how
he listens. Davies (1997) talks about three different kinds of listening i.e. evaluative listening, interpretive listening and hermeneutic listening. Evaluative listening is displayed when the teacher have “correct” answers in mind. Davies (1997) argues that the teacher whose listening is evaluative strives for a structured lesson. Evaluative listening limits the learners’ contributions as either correct or incorrect. Again, as much as interpretive evaluation seeks for information from learners than just responses that is not enough. Elaborate answers with some sort of demonstration or explanation are also not enough when promoting conceptual understanding. Listening for the promotion of conceptual understanding should be more than attending to answers in different ways as this still leads to listening for a particular response. When promoting conceptual understanding the teacher has to develop hermeneutic listening skill. He should be able to listen in order encourage participation and interaction in learners. The teacher’s listening should not make him direct the learners to some pre-given understanding but must show the willingness to interrogate the learners in order to get reasoning behind their responses (Davies: 1997).

CONCLUSION

As much as procedural fluency is one of the most important strands of mathematical proficiency, focus should not be given to it more that the other strands. The teacher should also let the learners give details when using and explaining their strategies as Kilpatrick et al (2001) argue that the more the learners interact about mathematical ideas and concepts, the more their mathematical proficiency will develop. The use of clues should not lead to procedural routes. The teacher also needs to pay attention when learners respond so that he does not end up accepting incorrect answers as he accepted the use of ‘over’ when naming fractions.
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LEARNERS’ EXPERIENCES OF MATHEMATICAL LITERACY IN GRADE 10
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Mathematical Literacy was introduced as a new subject in the Further Education and Training (FET) phase in January 2006. It is structured in the FET curriculum as an alternative option to Mathematics, with all learners in this phase having to take one or the other option. This changes the previous situation in which some 40% of all FET learners took no mathematics at all.

Our focus in this paper is on learners’ experiences of Mathematical Literacy (ML) in Grade 10. The data used was collected from intensive longitudinal research in one inner-city Johannesburg school which had three classes taking ML in Grade 10 (alongside three Mathematics classes).

To date, publications in the area of ML in South Africa have (predictably) tended to focus on more theoretical elements such as the nature of ML as a subject (Christiansen, 2006), policy, or text-based analyses (Bowie & Frith, 2006; Venkatakrishnan & Graven, 2006) and teacher development issues (Brown & Schafer, 2006). Much of this writing, including our own, has drawn attention to potential problems, tensions and contradictions within the policy texts, and the mixed messages conveyed about the nature and processes associated with ML. Set against this backcloth of potential problems, our weekly observations and interactions with learners in the ML classes at the school mentioned above, pointed from the early stages, to positive experiences of learning. In this paper, we begin to examine the nature of learners’ experiences more deeply, framed as they were, in overwhelmingly positive terms in comparison to prior experiences of learning mathematics. Within this notion of reference back to mathematics, our data also provided recurring examples of vividly negative memories of mathematics learning for the majority of Grade 10 ML learners.

Prior to considering the nature of learners’ experiences, we provide some background detail about the broader study, the focal school and the three ML teachers involved, and the data sources drawn upon within this paper.

BACKGROUND TO THE STUDY
Our broader study aims to examine classroom practices and learning within the process of implementation of ML. In order to do this, we are focusing on in-depth longitudinal research in one city school, involving weekly observations across 2006 and now, 2007,
supplemented by one-off visits to a small sample of other schools (to date n=3) and feedback from ML teachers in a range of other schools taking part in our postgraduate courses and/or ML teacher support groups. Almost all learners in the focal school are black; the school contains significant numbers of immigrant children with varying degrees of proficiency in English, the main language of learning and teaching; the teaching body is very mixed, with black, coloured and white staff. The school, which used to be an independent school but is now public, continues currently to work towards the IEB examinations. The three teachers involved in teaching ML in Grade 10 are all qualified mathematics teachers, one with three years’ experience, another with ten years (the head of maths), and the third with almost 25 years. Across 2006, the head of maths designed activities that were used by all three teachers, though not always in the same way. The tasks were drawn from a range of sources – newspaper articles, utilities bills, and ML textbooks amongst these, and usually involved real or realistic situations.

DATA SOURCES
The data used within this paper is drawn from two main sources – a questionnaire administered to all Grade 10 ML learners in late September 2006, and semi-structured interviews carried out with a sample of these learners in October/November 2006. The questions used within both of these instruments were informed by data from our ongoing classroom observations. The questionnaire in particular, made explicit reference to the comparisons with mathematics learning that many learners had alluded to in informal conversations across the year, and which they raised again within the interviews. The questionnaires contained six scaled response items, an open response section, and a section asking for areas of contrast between ML and mathematics. Four of the scaled response questions related to enjoyment and ease of work in Grade 10 ML and in Grade 9 mathematics; the fifth and sixth questions were related to perceptions about progress and test performance in ML. The open question asked for general comments on experiences of ML learning. 66 questionnaire responses were received (from 90 ML learners in Grade 10).

Six learners from each of the three ML classes were interviewed in pairs. Whilst most ML learners in this school had been told that they had to take ML (usually as a result of failing mathematics, or failing across a range of other subjects in Grade 9), some learners in this school had elected to take ML in spite of good prior performance in mathematics, and against the advice of teachers at the school. We therefore selected a girl and a boy from each class with good prior performance in mathematics (based on end of Grade 9 mathematics marks), and conducted paired interviews with them. We then selected four further learners – two girls and two boys, one of each gender who had achieved within the third quartile of performance, and one of each gender who had achieved in the first quartile of performance based on Grade 10 ML June examination marks – and conducted paired interviews. Within the interviews we probed learners’ perceptions of the classroom practices associated with ML, and the opinions we had
heard expressed in informal classroom conversations and within the questionnaire responses.

A combination of quantitative and qualitative methods was employed in data analysis. Questionnaire data was analysed; the interviews were recorded and transcribed. Our presentation of findings now follows.

**FINDINGS**

The questionnaire responses pointed to two clear areas in which ML was viewed in different ways to mathematics – enjoyment and ease. Across the year, when learners had expressed their enjoyment of ML in classroom interaction, this had been related to comments that the new subject was ‘less stress’ than mathematics had often been for them. Questionnaire results confirmed this sense with clear differences in the reporting of enjoyment between the two subjects:

<table>
<thead>
<tr>
<th></th>
<th>ML Gr10</th>
<th>Mathematics Gr9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Do you enjoy ML lessons:</td>
<td></td>
<td>In Gr 9, did you enjoy maths lessons:</td>
</tr>
<tr>
<td>never</td>
<td>3</td>
<td>never</td>
</tr>
<tr>
<td>sometimes</td>
<td>6</td>
<td>sometimes</td>
</tr>
<tr>
<td>mostly</td>
<td>32</td>
<td>mostly</td>
</tr>
<tr>
<td>all the time</td>
<td>25</td>
<td>all the time</td>
</tr>
</tbody>
</table>

This result was backed up by overwhelmingly positive comments in the open sections of the questionnaire in which versions of the word ‘enjoy’ – ‘enjoying’, ‘enjoyable’ and near-synonyms such as ‘fun’, ‘wonderful’ and ‘cool’, as well as ‘interesting’ and comments relating to ‘liking’ or ‘loving’ ML, figured more frequently than comments relating to any other feature (at least one of these words was used on 55/66 open responses). Interview responses also reflected this sense of enjoyment of ML in comparison to mathematics, revealed here in an excerpt from an interview with two high attainers who elected to take ML in grade 10:

MG: So what are your experiences of maths literacy so far?
T: It's very, very interesting. I actually enjoy it; really I do.
B: You won't normally enjoy maths but maths literacy is nicer.
MG: So if you compare it to the maths you were doing last year, you were enjoying this more?
B: Ja.
T: Ja, basically. It's really much more interesting, ja.
MG: So what is it that you think is making it more interesting? What are the types of activities you're doing? What are the kinds of topics that you're covering? You've mentioned statistics.
T: Ja, we mentioned – you know, we deal with basic stuff. It's probably more interesting because you look at things in a perspective of scenarios and how you deal with it in real life; unlike in maths where you do trigonometry and you don't know where you're going to meet something like that.

The notion that ML was ‘easy’ was also frequently mentioned, and the sense that it was easier than mathematics had been was also confirmed:

<table>
<thead>
<tr>
<th>ML Gr10</th>
<th>Mathematics Gr9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Have you found the work in ML this year:</td>
<td>Did you find the work in Gr9 maths lessons:</td>
</tr>
<tr>
<td>very hard</td>
<td>very hard</td>
</tr>
<tr>
<td>hard</td>
<td>hard</td>
</tr>
<tr>
<td>about right</td>
<td>about right</td>
</tr>
<tr>
<td>easy</td>
<td>easy</td>
</tr>
<tr>
<td>very easy</td>
<td>very easy</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>2</th>
<th>35</th>
<th>21</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7</td>
<td>26</td>
<td>25</td>
<td>7</td>
<td>1</td>
</tr>
</tbody>
</table>

However, both the open section of questionnaires and interview data pointed to mixed perceptions about how ‘easy’ ML was as a subject. ML learners often began by saying that the subject was easy – a judgment that they noted was often made by mathematics learners and in some cases, other teachers. However, as they continued to reflect on their experiences, this judgment was often transformed en route, with comments about the ‘different’ nature of the subject, it requiring thinking and hard work, and of it being ‘challenging’, in instances, challenging even to their peers in mathematics who commented on ML being easy. Comments such as the one below reflected a move from an externally given judgment of ML as easy, straightforward, to an experience of working in ML that did not always reflect this judgment:

‘The subject is not ‘easy’ as many would say but it is very challenging and it requires a person who is not lazy to think or work.’

These comments connect also to learners’ perceptions about the nature of ML as a subject, and the ways in which it differed from prior experiences of mathematics. Learners’ responses to their classroom experiences suggested that ML was viewed, as indicated in the earlier quote, as strongly connected to ‘real-life’. Variants on this theme pointed to a subject that was described as more ‘practical’, more applicable, and somehow more ‘visible’ than mathematics had been:

‘Yes, so like – okay, what I can say that – you see mostly interest – ja, I like Maths Literacy and all that because you know, it's talk about like things that I will see everyday; things we use, percentages all that; time and speed. It's talking about something that we know. We know – we usually see and it's much easier to understand.’

Related to the notion of applicability to real-life, many learners’ commented on ML as
being more ‘useful’ than mathematics, with ‘usefulness’ linked to everyday situations, future needs and in particular, careers linked to business and accounting, with learners reflecting on the frequent occurrence of finance-related work across the topics covered in class.

ML was also reported to be comprised by ‘scenarios’ or ‘story sums’; these were contrasted with the ‘x’s and y’s’ which clearly haunted many of their mathematical histories. The story sums were described as being easy to understand and to ‘see’, again contrasting with the lack of any visualisation that they appeared to have experienced, particularly in relation to prior algebraic work:

S: I think when we are using like story sums, we use things which are like, we use in our daily lives such as chocolates, cold drinks. Some people understand, rather than as in using x’s and y’s.

B: Another thing, when you read in this story, you know how to calculate – I take this and I take that and I calculate. You know how to calculate, you know when you are using the subtraction sign.

HV: Okay, so would that have meant, when you were doing it last year and you had fewer story sums – you probably had some story sums but you said not as many – (T: Yes.) Did you read a question and find yourself more often, not knowing what to do?

B: That was the problem because most of the time I used to read the question, and don’t understand the question and then I get the answer wrong – because of the question. So now it’s kind of clear. Although you do get sums wrong, but you do understand the question.

Learning in ML was generally described in terms of this kind of better understanding. Openings for better understanding were associated with having more time available to digest and work with a problem situation, and being in a less paced and pressured environment. ML teachers were described as being much more likely to wait for understanding before moving on, and as being more patient – both features that appeared to be strikingly absent in their prior experiences in mathematics classrooms:

HV: Both of you, just from what I’ve heard already, seem to be happier this year than you were last year – would that be true? (Both: Yes.) So what were your experiences of mathematical learning, maths learning last year like compared to your learning of maths literacy this year?

B: It was confusing at times.

HV: Okay.

B: It was really confusing.

HV: Anything else you want to add to that T--?

T: Yes ma’am, because like, if you get confused, every lesson when you come, you’re not up to the lesson. So you end up like, always being lost and you don’t care if you know or not.

B: And when they ask ‘Who doesn’t understand?’ – you don’t have the guts to just put up your hand, because if the majority of learners in the class understand and you don’t understand, then it becomes a problem for you, because you’re thinking ‘Okay, I’m just stupid alright’ – something. Or they’re going to laugh at you or the teacher will comment.

T: But even when you put up your hand ma’am, some times they just say ‘Ah, move on. He’ll catch on with us’. Whereas you’re still desperate.
Differences in the nature of progression between mathematics and ML were also commented upon, with mathematics seen as difficult because concepts were built upon each other too quickly. This contrasted with the longer time frames given within ML to understand a problem situation – one or two weeks were mentioned frequently.

The pedagogical organisation of learning was also reported as helping to garner understanding, with group work and discussion-based activities reported much more commonly in ML than in mathematics, which was viewed as embodying a much more individualistic style of learning. Within the more collaborative learning environments in ML, there appeared to be more room for sharing ideas and discussing a range of alternative solution strategies:

‘It’s different ma’am, because this year you are allowed the chance to work with a partner or the group and then you share your point of views, your understanding. Maybe one partner might have a different view of doing the sum, the other might have another, and so you combine your ideas and you see, rather than struggling all alone.’

The nature of assessment was also commented upon as different in some ways for ML, with continuous assessment tasks involving research and data collection about real-life situations more often than had been the case previously – a feature that learners viewed positively. However, formal assessment continued to be problematic for both learners and teachers. There was uncertainty for educators around the format, breadth and level at which to set papers, and on how different these papers should be from mathematics papers. Poor performance on formal assessment for many learners was often reported as the key negative element of ML experiences in Grade 10, and discrepancies between performance in continuous assessments and summative assessments (primarily mid and end of year examinations) were also noted. Lack of time to complete the paper was commented on frequently in relation to the June examination.

CONCLUSIONS

Across Grade 10, what appears to have been achieved in ML in this school is a shift in the relationship between a cohort of learners and mathematical working. The vast majority seemed to have moved from viewing themselves as largely unable to make sense of mathematics to seeing mathematics as a potentially useful tool with which to begin making sense of situations. Acquisition of the process skills needed for this is still clearly in its early stages, but low self-esteem issues that learners came in with have been addressed successfully. One learner described her experiences across the year thus:

‘I really enjoyed every lesson this year for Mathematical Literacy because I could understand the work and I understand why I have to do maths. At first I did not see the reason to do maths, mostly the complicated maths that has to do with ‘finding y’s and x’s’. But Maths Literacy is the best and it works for the kind of job I want and starting up my business one day.’

Whilst the absence of a history of Senior Certificate ML papers seems to have opened up spaces for educators to support learners to develop mathematical confidence at their own pace and in their own way, more discussion and guidance on the type, content and
structure of assessment that will support the aims of ML still appears necessary. Overall though, the key achievement of the ML educators and learners in our focal school has been to overturn the highly negative experiences of mathematics learning that so many of them came into ML with, and open their eyes to the possibility of finding mathematical working both useful and enjoyable to engage with.

REFERENCES


PREPARING LEARNERS FOR A MATHEMATICAL LITERACY OLYMPIAD

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In 2006, Mathematical Literacy was introduced as a fundamental subject in the Further Education and Training Curriculum in South Africa. This subject was directed at learners who, in the past, would stop studying mathematics after grade 9. This situation perpetuated high levels of innumeracy in our population. The inclusion of this subject in the curriculum “will ensure that our citizens of the future are highly numerate consumers of mathematics”. The orientation training programme for grades 11/12 teachers was scheduled for July 2006. Due to unforeseen circumstances, the training had to be postponed. The PE District eventually conducted its training in November 2006. The GMSA Foundation was asked to present a 2 hour slot on “The use of Multimedia Mathematical Literacy classroom”. The Mathematical Literacy teachers were enthusiastic and actively involved throughout the training. They also found the Multimedia slot very interesting and informative. This prompted the GMSA Foundation to meet with the writer of this paper and come up with a firm proposal to enhance the status of Mathematical Literacy as a subject in our schools.

This paper traces the progress made thus far:

INTRODUCTION

Mathematics Olympiads or competitions usually target learners with “higher-order” problem solving abilities. Currently, there are a number of Mathematics Olympiads in which learners from various grades participate.

The competitions organized by the NMMU Mathematics Department together with AMESA (PE) set competitions for grades 3; 5; 7; 9 and 11. This is undertaken on a massive scale and comprises 2 rounds. In round 1, learners write at their respective schools and in round 2 selected learners write the papers at central venues. This venture has proved to be very popular. It is exciting to see parents with their children flocking to the prize-giving ceremony in large numbers.

Mathematical Literacy is a new subject in the FET band in South African school curriculum. It is accorded the same “status” as Mathematics. Each learner, in the FET band, must have either Mathematics or Mathematical Literacy as a compulsory subject. However, the feelings of teachers and learners at some schools are that Mathematical Literacy is inferior to Mathematics. Thus, we have learners who should be doing Mathematical Literacy doing Mathematics. Their teachers say that they are not coping with Mathematics. However, they “refuse” to do Mathematical Literacy because it is an
“inferior” subject.

The GMSA Foundation Mathematical Literacy Olympiad is intended to change this view of Mathematical Literacy and raise its profile at our schools. Mathematical Literacy learners are being given an opportunity to participate in their own Olympiad and interest in this project from teachers and learners in the Nelson Mandela Bay area (incorporating Port Elizabeth, Uitenhage and Despatch) has been phenomenal.

The aim of the Mathematical Literacy Olympiad (MLO) is to work hand-in-hand with educators to develop the numeracy skills of Mathematical Literacy learners in the Nelson Mandela Bay area. This project is being undertaken in a partnership with officials from the local education district offices.

A key problem plaguing many learners in Mathematical Literacy (according to reports from teachers) is a lack of basic numeracy skills. The Mathematical Literacy Olympiad (MLO) project aims to motivate educators and learners to focus on mastering the four basic operations in mathematics – addition, subtraction, multiplication and division. This will be done by means of the support materials given to assist learners to prepare for the Olympiad in the 3rd term.

The GM South Africa Foundation (initially called Delta Foundation) was established by the Delta Motor Corporation (now called GM South Africa) in 1994 as an independent development agency. Since inception, the GM South Africa Foundation has pioneered a unique approach to corporate social investment - Corporate Social Action (CSA). This involves the management of innovative pilot projects and the making use of all lessons learnt to produce new developmental models. These models – which are really sets of implementation guidelines - are structured so that the lessons learnt can be shared throughout South Africa and so that findings can also be used to assist the various authorities in the formulation of policies and programmes.

MATHEMATICAL LITERACY VERSUS MATHEMATICS

Mathematical Literacy as a subject is defined in the South African Curriculum as follows:

Mathematical Literacy provides learners with an awareness and understanding of the role that Mathematics plays in the modern world. Mathematical Literacy is a subject driven by life-related applications of mathematics. It enables learners to develop the ability and confidence to think numerically and spatially in order to interpret and critically analyse everyday situations and to solve problems (DoE 2003a: 9).

On the other hand Mathematics is defined as:
Mathematics enables creative and logical reasoning about problems in the physical and social world and in the context of Mathematics itself. It is a distinctly human activity practised by all cultures. Knowledge in the mathematical sciences is constructed through the establishment of descriptive, numerical and symbolic relationships. Mathematics is based on observing patterns; with rigorous logical thinking, this leads theories of abstract relations. Mathematical problem solving enables us to understand the world and make use of that understanding in our daily lives. Mathematics is developed and contested over time through both language and symbols by social interaction and is thus open to change (DoE 2003b:9)

From these definitions, we see that there are both similarities and differences between Mathematics and Mathematical Literacy

**Similarities:**

- Role of mathematics in the world and the applications of mathematics
- Numerical and spatial thinking
- Solve problems

**Differences:**

In Mathematics there is observation of patterns leading to theories of abstract relations; this is not a feature of Mathematical Literacy. This is one of the key differences between Mathematical Literacy and Mathematics.

However, looking at the similarities between Mathematical Literacy and Mathematics, it becomes apparent one of the foci of Mathematics and/or Mathematical Literacy Olympiads should be:

*The solving of problems involving the applications of mathematics in real-life situations; this involves numerical and spatial thinking.*

In Mathematical Literacy this may be the only focus.

**MEETING WITH TEACHERS**

Teachers were invited to a meeting called by GMSA Foundation to explain the purposes of the Mathematical Literacy Olympiad. The writer of this paper discussed the following issues with teachers:

- What an Olympiad is
The purpose of the first ever Mathematical Literacy Olympiad
Procedures involved in the Olympiad: Benchmark test; Learner Support; structure of the Olympiad

The teachers were also free to raise any issues regarding the Olympiad. About 40 teachers attended this initial meeting. However, 65 high schools participated in the Benchmark tests comprising approximately 1755 learners.

5 tutors (2 retired Mathematics Teachers and three University students) were appointed by GMSA Foundation to assist in this project. These tutors were trained and given certain roles: These roles included:

- Delivering material to schools
- Marking benchmark (baseline assessment) tests
- Supporting selected learners over a 10 week period (9 weeks in term 2 and one week in term 3)

**BENCHMARK TEST**
The benchmark test consisted of 15 questions. This comprised the following:

The first 5 questions were on basic arithmetic involving the four operations (no calculator usage for these questions);
Question 6 was converting a decimal to a mixed number;
Question 7 was on number patterns;
Question 8 involved recognizing a 90° angle;
Question 9 was on metric conversions;
Questions 10 – 13 were word problems; and
Questions 14 and 15 were on interpretation of graphs

A quick analysis of the test revealed the following:
Learners had difficulty with the basic operations; they had difficulty with the word problems with language being a major factor; they had difficulty in highlighting key points from a given problem. However, they were able to interpret graphs reasonably well.
An analysis of the performance in the benchmark tests revealed that the majority of learners needed support. Thus, a series of learner support activities were developed by this writer. Learners would complete these tasks throughout the second term and early in the third term. These support activities were developed partly from the benchmark test analysis and partly from selected assessment standards for grades 10/11 Mathematical Literacy subject statements. This was done so that learners would see that their completion of the support activities would help them prepare for the Mathematical Literacy Olympiad as well as assist them with their daily Mathematical Literacy class tasks. Learners were required to use calculators when working through the support activities.

The five tutors were allocated to support 25 schools. (Due to budget constraints only those schools that indicated they needed support were allocated tutors). This support would involve tutors meeting with selected groups of learners in the afternoons and assisting them with the tasks given in the support materials.

THE MATHEMATICAL LITERACY OLYMPIAD

The support activities in the 2nd and early in the 3rd term served as preparation for the Olympiad which will consist of 2 rounds.

Round 1:
To be written at their schools in August 2007: All learners who wrote the benchmark tests will participate in round 1. The round 1 paper will be on basic knowledge and routine applications.

Final Round
From the learners who write round 1, 500 will be invited to participate in the final round. This will be based mainly on performance in round 1. However, to ensure inclusivity a minimum of 2 learners per school will be invited to the final round. The final round paper would include more advanced applications.

The prize-giving ceremony will be in October 2007. A detailed analysis of the learners’ results will be presented at the 2008 AMESA Congress in Port Elizabeth.
REFERENCES


In this paper we share our experiences of working with mathematical literacy over the past two years. From these experiences we have developed a spectrum of agendas which we believe provides a useful tool for thinking about the different nature of mathematical literacy lessons which are occurring as a result of current curriculum implementation in Grade 10 and Grade 11. Our presentation will particularly focus on this spectrum and will invite feedback from participants as to the usefulness and resonance of such a spectrum for teachers of mathematical literacy.

INTRODUCTION

Since South Africa’s first democratic elections in 1994, there has been major educational reform. Implementation of a new curriculum for school in the General Education and Training (GET) band began in 1997 and in the Further Education and Training (FET) band began in 2006. This new curriculum encompasses radical shifts for teachers both in terms of the content covered and the nature of teaching and assessment. Within the curriculum mathematics is acknowledged for its important role of supporting learners to become active participators in the new democracy. The crucial role of mathematics in developing actively participating citizens is further acknowledged by the introduction of a compulsory mathematics learning area in the FET band. Thus all learners in the FET band must take either Mathematics or the newly introduced option of Mathematical Literacy.

Previously Graven (2000a; 2000b) analysed the introduction of Mathematical Literacy, Mathematics and Mathematical Sciences (MLMMS) in Curriculum 2005 for the GET band of schooling from the perspective of four different mathematical orientations. The four orientations identified in MLMMS\textsuperscript{19} (and in supporting documents such as illustrative learning materials, teacher guides and texts) were:

\begin{itemize}
\item Orientation 1: Critical Literacy
\item Orientation 2: Participatory Literacy
\item Orientation 3: Humanistic Literacy
\item Orientation 4: Mathematical Literacy
\end{itemize}

\textsuperscript{19} These orientations continue to be present in the Revised National Curriculum Statement even while the prominence of orientations 1&2 obtained in their presence in the specific outcomes shifted to the rationale of the RNCS when the specific outcomes were abandoned as primary organising features of the curriculum. In line with this shift the name Mathematical Literacy, Mathematics and Mathematical Sciences was replaced with just Mathematics.
1. Mathematics for critical democratic citizenship. It empowers learners to critique mathematical applications in various social, political and economic contexts.

2. Mathematics is relevant and practical. It has utilitarian value and can be applied to many aspects of everyday life.

3. Mathematics inducts learners into what it means to be a mathematician, to think mathematically and to view the world through a mathematical lens.

4. Mathematics involves conventions, skills and algorithms that must be learnt. Many will not be used in everyday life but are important for further studies.

Primary arguments in that research were that, while all four orientations were present in the curriculum, the way in which they were encountered and experienced by teachers, through their interaction with texts, assessments, departmentally organized workshops and other curriculum support materials, was contradictory. Teachers were often confused by pendulum swinging between orientations and messages that seemed to convey different orientations as good and others as bad at different points in time. Furthermore, the research highlighted that the demand that teachers should work with each of these orientations and integrate across them was unrealistic without a great deal of teacher support and intervention. Even for those teachers involved in intensive in-service education aimed at supporting teachers in navigating their teaching among these orientations, integration between these orientations was a challenge.

In this paper we look at how these orientations appear in the FET band of the curriculum. In particular we look at the presence and absence of these orientations in the Mathematical Literacy option of the FET curriculum and at how a spectrum of agendas is emerging for teachers working with this curriculum.

The four orientations above are clearly present in the FET curriculum. However the splitting of the FET curriculum into pursuing either Mathematics or Mathematical Literacy involves a clear splitting of the orientations and roles in terms of both presence and emphasis.

The FET curriculum is designed in such a way that Mathematics and Mathematical Literacy are different ‘in kind and purpose’ (Brombacher, 2006) and thus Mathematical Literacy is not subsumed in Mathematics. This, combined with the fact that one cannot take Mathematical Literacy if one takes Mathematics, means that learners choosing Mathematics, by and large, lose out on learning mathematical ways of being in the world. By this we mean learning to use mathematics to “analyse and interpret their own lived experiences, make connections between these experiences and those of others, and, in the process, extend both consciousness and understanding” (Walsh, 1991, p6).

In Mathematics, as was the case with Mathematics in the previous FET curriculum, a strong mathematical agenda is clear with ‘rigorous logical reasoning’ and ‘theories of

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20 This is different to the introduction of Functional Mathematics (similar to Mathematical Literacy) in England where the intention is that it is taken in addition to mathematics.
abstract relations’ (DoE, 2003b, p9) being emphasized. The curriculum is content driven and the context of learning is primarily ‘in the context of mathematics itself’ (DoE, 2003b, p9). While the definition includes ‘logical reasoning about problems in the physical and social world’ (DoE, 2003b, p9) which ‘enables us to understand the world and make use of that understanding in our daily lives’ an analysis of the vast amount of content to be covered indicates that this is largely based on the hope that the mathematics content learnt in its abstract and generalisable form will be transferred by learners to use in their daily lives. This is in contrast to an approach which provides contexts and processes that enable learners to experience mathematics in their daily lives. Contexts are useful insofar as they provide access to, and/or motivation for learning mathematics and thus in the learning area ‘Mathematics’ contexts can be contrived in order to meet this purpose. Thus we argue that Mathematics in the FET focuses mainly on the last two orientations with little inclusion of the first two orientations. Concerns that Mathematics is too abstract, catering primarily to meet the needs of an elite minority proceeding to further mathematically or scientifically oriented studies, are addressed by providing Mathematical Literacy as an alternative.

The literacy agenda of Mathematical Literacy is captured both in the name itself and in the definition:

“Mathematical Literacy provides learners with an awareness and understanding of the role that mathematics plays in the modern world. Mathematical Literacy is a subject driven by life-related applications of mathematics. It enables learners to develop the ability and confidence to think numerically and spatially in order to interpret and critically analyse everyday situations and to solve problems” (DoE, 2003a, p9).

The post-amble headed ‘context’ following the learning outcomes and assessment standards, furthermore emphasizes a literacy approach:

“The approach that needs to be adopted in developing Mathematical Literacy is to engage with contexts rather than applying Mathematics already learned to the context.” (DoE, 2003a, p42)

This is re-emphasised in the teacher guide published in 2006:

“the challenge for you as the teacher is to use situations or contexts to reveal the underlying mathematics while simultaneously using the mathematics to make sense of the situations or contexts, and in so doing develop in your students the habits or attributes of a mathematically literate person” (DoE, 2006, p4)

The three quotations above provide a range of views on the relationship between mathematical content and contexts. However all three quotations make it clear that
Mathematical Literacy in the FET band, in intention at least\(^2\), has the integration of both a literacy and mathematical agenda as its core business. Thus we argue that the integration of orientations 1, 2 and 4 are required in Mathematical Literacy.

We now turn to focus on how teachers’ navigate their teaching along a whole spectrum of pedagogic agendas in order to work with the demands of Mathematical Literacy and the integration of the various orientations identified above.

The spectrum we propose below is based on our work with teachers engaging with mathematical literacy in:

- PGCE, ACE and B Sc Hons Mathematical Literacy courses
- A Marang co-ordinated Mathematical Literacy teacher support group
- Marang workshops and research seminars on the implementation of mathematical literacy

- as well as through our intensive longitudinal research of policy implementation in one inner city Johannesburg school.

An overarching aim that traverses across the spectrum of agendas for the teachers we are working with is to increase confidence in mathematical thinking and reduce mathematical anxiety. This issue relates both to the structure of mathematical literacy in the curriculum (set up as an alternative to mathematics) and the current uncertainty as to what the qualification can lead to or provide access to. In most cases, rather than allowing learners to choose Mathematics or Mathematical Literacy, schools make this choice for learners based on learners’ prior mathematical performance (established through assessments in the GET band). The result of this is that most learners taking Mathematical Literacy have far weaker mathematical histories (and confidence) than those taking mathematics.

The spectrum of agendas we have identified here traverses across the question of the nature and degree of integration of context within pedagogic situations, and cuts across orientations 1, 2 and 4 in different ways. The agendas and the ways in which they interact with Graven’s earlier identification of orientations are presented below.

\(^2\) We say in intention at least since various aspects of the curriculum document have been criticized for having far too much ‘traditional’ mathematics content which detracts from the literacy agenda (AMESA, 2003). The inclusion of Trigonometric ratios and the sine, cosine and area rule in the learning outcomes is a clear example of a distraction from a literacy agenda.
<table>
<thead>
<tr>
<th>Context driven (by learner needs)</th>
<th>Content &amp; context driven</th>
<th>Mainly content driven</th>
<th>Content driven</th>
</tr>
</thead>
<tbody>
<tr>
<td>Driven agenda:</td>
<td>Driving agenda:</td>
<td>Driving agenda:</td>
<td>Driving agenda:</td>
</tr>
<tr>
<td>To explore contexts that learners need in their lives (current everyday, future work and everyday, and for critical citizenship).</td>
<td>To explore a context so as to deepen math understanding and to learn maths (new or GET) and to deepen understanding of that context.</td>
<td>To learn maths and then to apply it to various contexts.</td>
<td>To give learners a 2nd chance to learn the maths in GET.</td>
</tr>
<tr>
<td>Pedagogic demands</td>
<td>Pedagogic demands</td>
<td>Pedagogic demands</td>
<td>Pedagogic demands</td>
</tr>
<tr>
<td>Involves finding the contexts learners currently need and will need in the future as well as contexts the country needs learners to be able to engage with (&amp; critically) (e.g. notions of democracy, national budgets and taxation)</td>
<td>Involves selecting contexts (can be real, contrived, authentic, edited or messy) that enable the above. Also involves discussion about context and possible revision of GET maths or relearning of this maths in new way.</td>
<td>Involves selecting contexts that GET maths can be applied to (contrived or more real) and editing these to allow ‘unmessy’ application.</td>
<td>Involves revision of GET maths without pedagogic change except to slow down the pace. (orientation 4)</td>
</tr>
<tr>
<td>Needs increased discussion of contexts and critical engagement with the underlying function of mathematics embedded in it. (E.g. if one changes the formatting formula of tax rates what happens, who benefits more rich or poor). Might require revisiting or learning new maths but only in so far as it will service the understanding of the context. Note: the driver is to find contexts that work to unpack this math-context relationship vs finding contexts that are needed by learners for full participation in society.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(orientation 2&amp;4 almost equally balanced with some of 1 depending on the context)</td>
<td>(mainly orientation 4 + some of 2)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(orientation 1&2 are key with 4 only being important where necessary to service 1 & 2)

<table>
<thead>
<tr>
<th>Issues arising</th>
<th>Issues arising</th>
<th>Issues arising</th>
<th>Issues arising</th>
</tr>
</thead>
<tbody>
<tr>
<td>Progression of mathematics usually sacrificed while authenticity of context is maintained in order to meet ‘needs’ of learners. The sacrifice of mathematical progression is not necessarily experienced as a problem. Summative assessments struggle to align to this agenda so discrepancies can occur between performance on continuous and summative assessments.</td>
<td>Authenticity of context and progression of math embedded must be balanced. Both authenticity of context and mathematical progression can be experienced as a problem. Summative assessments struggle to align and to deal with issue of progression so gaps can occur between performance on continuous and summative assessments.</td>
<td>Authenticity of context is often sacrificed so as to meet mathematical goals. Mathematical progression can be developed in the same way as in ‘mathematics’ curriculum. Summative assessments are more familiar and performance is more aligned to continuous assessments.</td>
<td>Contexts are not a concern as they are not particularly present. Mathematical progression can be developed in the same way as in ‘mathematics’ curriculum. Traditional summative assessments are similar to continuous assessments so little discrepancy between performance on these – most likely continued poor performance as in GET.</td>
</tr>
</tbody>
</table>

A spectrum of agendas

Based on our analysis of the definition, purpose and post-amble on context and the teacher guide for mathematical literacy, our sense is that the second agenda is the one that should be predominantly pursued over the FET band. However, this should not imply that there are not inconsistencies that work against this agenda present in the curriculum and various supporting documents. For example, analysis of the learning outcomes and their assessment standards indicates that this vision is not clearly supported in the details of the curriculum document. For example the inclusion of trigonometric ratios and the sine and cosine rule would seem to suggest that mathematical agendas are sometimes pursued at the expense of a literacy agenda. In the early stages in the process of implementation of Mathematical Literacy it is not surprising that mixed messages (Venkat, 2007) and ongoing movement in relation to the above spectrum are likely to arise.
Our table highlights various issues that are experienced by teachers when working with a particular agenda. In particular we point to the issues of authenticity of context, development of mathematical progression and discrepancies of continuous and summative assessments. These issues have been experienced by teachers that we are working with in different ways depending on their primary driving agenda.

In the presentation we will discuss these issues further and share some of our data that has led us to engage with these issues. A final note on our proposed spectrum - the spectrum is based on our current experiences of working with a range of teachers. It is of course likely that as we continue to work with a wider range of teachers and into grades 11 and 12 our spectrum will continue to be revised and refined. Furthermore it is hoped that through presenting this work at the AMESA conference we will receive feedback from participants that will guide us in these revisions.

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THE NEED FOR RESEARCH INTO MATHS FEAR IN SOUTH AFRICA:
A physiologically-based assessment of the legitimacy of the concept of maths fear
and how it affects different aspects of mathematical learning

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Mathematics anxiety is often blamed for poor performance in mathematics and especially when a learner is known to be neither lazy nor lacking in ability. The expression of maths anxiety is rooted in a very well-known biological alarm chain reaction and neurological reasons are posited for the affect of anxiety on mathematical performance. It is also posited that the parts of the brain most affected by strong emotion, are the centres for visuospatial processing (i.e. the number line and any visualization involved in a word problem, as well as geometrical conceptualisation) and that these centres form the foundation of more advanced calculations. It has been well corroborated that students tend to fear geometrical mathematics more than pure algebra, and research is encouraged into the exploration of reasons for this difference.

INTRODUCTION
Mathematics is one of the most elevated of human literacy and coping skills, and fear, one of the most fundamental emotional responses. (Dehaene, 1997; Jourdain, 1919). Their intersection in the classroom and examination hall has until recently never been considered worthy of legitimate recognition - to the extent that overcoming the fear was part and parcel of the test.

However, the observation of weak mathematical performance due to fear has always been nonspecific insofar as unless the teacher knew the learner well, he or she would be unable to determine whether the underperformance was due to any or a mixture of the following factors (or more):

- A simple lack of ability, or at least the learner’s perception that he or she lacks the ability (Ablard & Mills, 1996);
- Fear of failure, manifested as an unwillingness to apply one’s mind to the task at hand;
- An unwillingness to seek help in the above, possibly due to a fear of ridicule.
DEFINITION OF MATHS ANXIETY

All forms of anxiety are expressed through a nonspecific chain of physiological events, (Selye, 1974 Unreferenced at end, in Everly & Rosenfeld, 1981 and in Sourkes & Gagner, 1980), but individuals differ in how they experience that anxiety, when they experience it and how they deal with it. Mathematics anxiety is different from test anxiety in that, whereas very few people have a problem with admitting to test nerves, mathematics anxiety is very much more of a hidden problem.

Whereas Neethling & Rutherford (2001) held forth that poor performance in mathematics may, for a particular student, be attributable to nothing more than a temporary feeling of helplessness and therefore not a barometer of past or future success, the conclusions drawn by earlier research are far less reassuring.

Richardson & Suinn (1972) (in Anton and Klish, 1995 and in Betz & Hackett, 1983, also Ashcraft, 2002) define mathematics anxiety as: ”Feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematics problems in a wide variety of ordinary life and academic situations. Mathematics anxiety may prevent a student from passing fundamental mathematics courses or prevent his pursuing advanced courses in mathematics or the sciences”.

Mathematics anxiety can generalize to the extent that the participant may avoid even talking about it or answering questions about it. Rounds & Hendel (1980, in D’Ailly & Bergering, 1992) stated that, when given the option to do so, mathematics-anxious students avoided answering even questions related to mathematics test anxiety or concrete everyday number situations. For the same reason, statistics on measurable levels are very hard to obtain.

MATHS ANXIETY AGAINST THE SPATIAL NATURE OF MATHS

Notwithstanding the specific factors involved, the problem of underperformance in mathematics cannot be fairly judged without also taking into consideration that there are historical reasons for it in South Africa, and other factors such as biology and the cultural history of mathematics that are certainly not unique to South Africa. In recent years remarkable new research has surfaced to demonstrate that, not only does emotional and physiological arousal affect the efficiency of cognitive processing, such as test anxiety affecting examination performance (Anton & Klish, 1995; Arkin, Calvo & Carreiras, 1993; Liebert & Morris, 1999; Mwamwenda, 1994 as well as Smith,

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23 “Anxiety”: Usually connotes an experience of varying blends of uncertainty agitation and dread (also anticipatory neurosis as defined by Kraepelin [diagnostic and statistical manual – IV] and only too well known to those facing exams). As opposed to the nonspecific arousal responses (Selye, 1974, in Everly and Rosenfeld, 1981 and in Sourkes & Gagner, 1980), anxiety here usually refers to the emotion in awareness and also interacting with other components of consciousness.

24 “Cognitive”: An expression for every process by which a living creature learns about the external environment. It includes perception, discovery, recognition, imagining, judging, memorizing, learning, thinking and often speech.
Snyder & Handelsman, 1982), but also that certain components of mathematical performance (Dehaene, 1997) are processed by portions of the brain previously considered to be primarily emotional.

Mathematically verifiable relationships may or may not exist independently of humanity, but modern-day thinking is to posit an internal representation of external events or states (Damasio, 1994, 2000 and 2004). Even extreme dualism has to admit that mathematics has to be processed by a brain to be any good to humanity;

If it is conceded that numbers are human constructs imposed on potentially verifiable mathematical relationships, then it follows that it has to obey the same rules of neuropsychology, ontogeny, phylogeny, history etc. as any other sphere of human functioning (Damasio, 1994, 2000 and 2004; Dehaene, 1997; Butterworth, 1998 and 2002; and Geary, 2000a and b); and

According to Ansari & Karmiloff-Smith (2002), current cognitive neuroscience has shifted from the culturally dependent and traditional concept of numbers to an examination of its “representational primitives”, by which is generally understood the intuitive numerosity as set forth by Dehaene (1997). By this is meant that subhuman species and very young children simply do not have the neural layers needed to process numbers as such, and that they therefore revert to a very quasi- primitive number line representation. One rather startling example of this is the documented tendency in approximately ten percent of the sampled population (Berlin, 1998) (unreferenced at end) to visualize numbers in color. The author can personally testify to having done this in childhood, with the digits up to five having the strongest contrasts (one white, two yellow, three green, four dark blue, five red, and all subsequent numbers being shades of beige and brown).

According to Carey (1998) and Dehaene (1997), the intuitive numerosity that underlies formalised number theory is the product of how human beings developed from australopithecines to today, and also how different cultures chose to represent number for a given purpose. Even so, a concept of number as a line rather than collections of discrete dots or symbols and digits (Dehaene, 1997) does appear to be established across cultures and time. It is also true that the basic rules of mathematics and mathematics language are highly spatial in exposition.

Not only is the number line integral to highly advanced mathematical theory, but it is also intuitive and almost innate to the extent that it serves as an introduction to mathematical reasoning even at primary school level. It is formally taught in the form of rulers and slide rulers, graphs and trigonometry, as well as an aid in teaching children to add and subtract fractions.

Biologically and maturationally speaking, the outer layers of the brain that handle the more complex mathematics that involve explicit numbers and calculation, are completely dependent on the function of the lower layers, which process emotion as well as intuitive non-numeric reasoning. Fear of any sort is processed mostly in the limbic system, which is paleontologically and developmentally a far older and more primitive
part of the brain than the overlying ‘grey matter’. Emotionally charged memories are stored in the amygdala, which is itself an integral part of the adrenaline and cortisol hormonal response to danger and emotional stress (real or perceived); the hippocampus which is also part of the limbic system, has the task of converting short and medium-term memory (up to ten second span) into longer term memory.

One can take that argument even lower down if need be. Further down in the brain, the reticular formation is responsible for general activation levels (to some extent motivation and the ability to focus) and the medulla oblongata is responsible for gross motor coordination. Baenninger & Newcombe (1989) made the telling finding that children who practiced spatial movements themselves, improved in their ability to manipulate spatial quantities and relationships in their minds (geometry). The significance of this is that basic equilibrium and crude motor control is known to reside in the cerebellum, even below the limbic cortex, which underscores the participation of subcortical cortex even in complex mathematics.

It is established neurological fact that if a lower layer of the brain is incapacitated, then the higher levels would be correspondingly incapable of their normal response. Therefore, if the limbic brain is awash with panic hormones and buzzing with a virtual electrical storm of chaotic impulses, clearly most if not all of its resources are attending to the panic response, which leaves fewer neural tracts available to process even basic mathematical intuition such as being able to generate a very approximate estimation of what the solution to the problem is likely to be. By mathematical intuition Dehaene (1997) means a non numerate but possibly spatial grasp of the problem, and an appreciation of what the answer should be more or less. One wouldn’t expect to know offhand what the square root of 60 was, but even a very few seconds should suffice to tell you that it would be between five and 10.

**CONCLUSION**

So far the evidence for the correspondence between mathematics anxiety and the neural localization of each type of mathematical activity is by inference. An MRI study would be the only conclusive way to establish that intuitive mathematics is in fact processed in the exact same part of the brain as the fear dynamic, and one realises that such a study at present falls outside of the scope of educational studies and outside of the budget of psychological studies.

However, these tantalizing clues do offer many prospects for future research, among which my favorite is a neurally- or hormonally based study into the differences between the amount of fear experienced in both algebraic and geometric mathematics. It was a central tenet of my thesis, as borne out by verbal confirmations from mathematics lecturers, that students tended to fear spatial mathematics far more than algebraic mathematics. The hypothesis would be that since it was spatially based, it was that much more dependent on limbic input and therefore far more vulnerable to interference due to fear. The following issues refer:
• Whether the perceived difference in fear levels is real;
• Whether there are gender differences in fear levels between geometric and algebraic mathematics;
• Whether this fear is related to ultimate performance or not (Kimura (2002) stated that there were definite sex differences in that girls tended to prefer algebra and rely more heavily on memory to perform such tasks, whereas boys showed a definite preference for spatially-based tasks), and;
• Whether the teaching paradigm can be analysed and adapted to correct this imbalance.

Such research could make vast contributions towards an understanding of how students and teachers allow their fear to sabotage their efforts to come to grips with mathematics. The psychobiological element is inescapable, and psychobiology ranks emotional processing before that of higher functions. No mathematical learning will take place until the emotional element has been dealt with, and it is crucial that the mathematics educators of the future be willing to allow emotion into the equation.

BIBLIOGRAPHY


350


This paper is a reflection on the development and implementation of a diagnostic test to a cohort of Grade 10, 11 and 12 learners. The authors aim to discuss the guiding principles informing the selection of items and to show the extent to which the items were able to test learners’ content knowledge as well as their proficiency in using mathematics to perform routine and complex operations and solve problems which required higher-order thinking skills. Some comment on the framework for analysis is also made.

INTRODUCTION
In the South African curriculum, the mathematical content is organised into Learning Outcomes (LOs) and within each LO, into Assessment Standards (ASs). This organisation lends itself to criterion-referenced testing as one is able to assess whether the learners meet the standards for academic achievement. The items in the test were aligned to the ASs from the previous grade for each test. This implies that Grade 10, 11 and 12 learners were tested on the Grade 9, 10 and 11 curricula respectively. Items were loosely arranged according to the LOs, although we yielded to the principle of integration so that sometimes knowledge and skills spanned more than one LO.

FRAMEWORK FOR TEST ITEM SELECTION
The purpose of any test is to assess whether learning has taken place. This we do by determining how much knowledge a learner has acquired. Ideally we should also be able to make some conclusions about the quality of that knowledge. To enable a more “fine-grained analysis of proficiency”, tests should be aligned to content and process delineations (Schoenfeld: 33).

In our opinion, the type of testing most suited to the measurement of the quality of learners’ knowledge is criterion-referenced testing. A characteristic of criterion-referenced assessment is that the analyst is able to measure individual learner performance against specific criteria and standards (Dunn et al, 2002). A criterion, as defined by Dunn et al, is a characteristic by which quality can be judged. A standard is a statement about the degree of quality attained (Dunn et al, 2002). We will use these definitions to unpack the criteria used in the test as well as the standards by which learners’ responses were categorised.
Criterion-referenced testing would facilitate an analysis of the knowledge and skills which students have acquired, which indirectly gives access to levels of learning and teaching (Bond: 1996). It would also allow us to compare learner performance to a defined content domain (Nefec, no date). The literature we consulted supported the development of a criterion-referenced test based on a set of pedagogic principles. The National Council of Teachers of Mathematics (NCTM) Standards (Shannon, 1999:12) influenced the design of the framework for our test. The principles employed in the design of our test were:

- **Content** – all learners should have been exposed to the content in their previous grade. For this purpose, all items were based on the assessment standards of the previous grade.
- **Learning** – the assessment would ultimately enhance learning, as it would be used as a tool for developing intervention strategies.
- **Equity** – no student should be disadvantaged by unfamiliar contexts or complicated language.
- **Openness** – the assessment should inform students of their shortcomings.
- **Inferences** – the assessment should allow the programme convenors to make valid inferences about what the learners know. It would be more difficult to make inferences about what they do not know.
- **Coherence** – the assessment task should reflect the coherence of how learners were taught.

As our criteria we used the assessment standards contained in the relevant National Curriculum Statements. The aim of the project is to increase the number of learners who qualify to enter Mathematics and Science streams at university, therefore we identified mathematical concepts from the assessment standards and which are traceable across the grades in order to be able to trace the progress in conceptual development of individual learners. The concepts (referred to by mathematical topic) which we identified were:

- **Number relations.** Items tested learners’ knowledge and understanding of: Definitions, classification and representation of numbers as well as laws of exponents and surds. They also tested learners’ ability to estimate and perform basic operations with or without the use of the calculator. Finance was included as a context.
- **Algebra.** Items tested learners’ knowledge and understanding of: Multiplication and factorisation, setting up and solving equations and inequalities, finding the rule for number patterns and simplification of expressions involving all basic operations. There are also items which test learners’ understanding of the transformation from table to graph to algebraic rule.
- **Graphs.** Items tested learners’ knowledge and understanding of: Graph sketching, interpretation and analysis.
- **Measurement.** Items tested learners’ knowledge and understanding of: Measurement and calculation of length, area, perimeter and volume.
- Transformational and co-ordinate geometry. Items tested learners’ knowledge and understanding of: Position and transformation of points, lines and polygons on the Cartesian plane as well as the use of relevant formulae to perform calculations.

- Trigonometry. Items tested learners’ ability to interpret and analyse trigonometric graphs, define trigonometric ratios and use trigonometric relationships to solve triangles, equations and prove identities.

- Data Handling. Items tested learners’ ability to recognise, interpret and analyse data in various representations and to perform basic calculations relating to measures of central tendency and spread.

- Probability. Tested learners’ knowledge of formulae to calculate probability.

- Euclidean Geometry. Items tested learners’ knowledge and understanding of: Symmetry and congruence, relationships relating to lines and angles including the use of the theorem of Pythagoras, properties of polygons including visualisation with nets, application of theorems relating to triangles and circles. This only affects the Grade 12 learners who are still following the interim syllabus.

The test items

In this section we discuss the types of test items we included. The grounding principle for item selection was that the assessment should be balanced – that it should allow learners to “show that they have a repertoire of mathematical skills, that they understand mathematical concepts, that they can use mathematics to solve problems ...” (Shannon, 1999: 14). Another guiding principle we used in item selection, was some of the categories contained in the NCS:

- Knowledge - The learner does not need to perform any mathematical procedure to derive the answer. This category of questions tested learners’ knowledge. These items enable one to assess students’ factual and procedural capabilities. Learners have to recall knowledge. Since there is no need for learners to demonstrate how they reach their answers, these questions were mostly multiple-choice.

This question appeared in the Grade 11 test:
Which one of the following statements is TRUE?

A. \( \sqrt{5} < 2 \)
B. \(-\sqrt{5} = \sqrt{-5}\)
C. \(\sqrt{5}\) is irrational
D. \(\sqrt{5} = \sqrt{3} + \sqrt{2}\)

The question tests various aspects of learners’ knowledge of the number system, particularly irrational numbers. Learners should know that \(2 = \sqrt{4}\). Thus they can rule out the first statement. They should know that \(\sqrt{-5}\) is an imaginary number which rules out the second statement. They should know (or be able to deduce) that \(\sqrt{5} \neq \sqrt{3} + \sqrt{2}\).

- Performing routine procedures - The learner uses one algorithm or standard procedure to derive the answer. This category of questions tested routine procedures. Because we wanted to be able to analyse learners’ proficiency in using algorithms and routine procedures, learners needed to show their calculations.

A question requiring routine operations from the Grade 10 test was:

A 25 m tall tree casts a 40 m shadow on the ground.

What is the distance, \(x\), from the top of the tree to the end of its shadow? Round your answer to one decimal place.

The question requires learners to find the length of the hypotenuse of a right-angled triangle, using Pythagoras. This is algorithmic and because the right-angled triangle is sketched for them, there is little conceptual demand.

- Performing complex procedures - The learner uses multiple concepts, or needs to link procedures in order to develop a coherent mathematical argument. Complex procedures relate to problem types which the learner should have encountered regularly.
An example from the Grade 10 test is:

The figure below consists of 5 squares of equal area. The area of the whole figure is 245 cm².

Find the area of one square.
Find the length of one side of one square.
Find the perimeter of the whole figure in cm.

This is a multi-step routine problem. Learners need to apply the relevant formula to calculate the area of one square. From this they can deduce the length of one side, and use that answer to calculate the perimeter. For recording purposes, these three operations were organised as three separate, consecutive questions on the test.

Similarly, in the Grade 12 test:

When \( x^3 + px^2 + 3x - 2 \) is divided by \( (x + 2) \) the remainder is\( -8 \).
Calculate the value of \( p \).

In this question learners need to recognise that -2 is a factor of the polynomial; they also need to recognise that \( f(-2) = 8 \). They should synchronise these two pieces of information to calculate the value of \( p \).

- Problem-solving - The learner has to select or develop a strategy in order to develop a coherent mathematical argument. Problem solving relates to situations which the learner is unfamiliar with. Learners demonstrate their problem-solving abilities. The mathematical argument would also include mathematical procedures and concepts.

This question appears in the Grade 12 test:

A carpenter makes two types of tables, round and square. He has 600 m² of floor space available. Each round table requires 50 m² of floor space and each square table requires 60 m². He can only invest R1 600 in equipment that amounts to R160 per round table and R100 per square table. If he makes a total of \( x \) round tables and a total of \( y \) square tables, write down two inequalities that represents the above-mentioned situation.

Learners should recognise that the round and square tables are variables and that they need to develop a set of equations to account for all the parameters.

Another test item requiring problem-solving strategies (from the Grade 10 test) is:
Which figure would have 98 small triangles?

You could use this table to help you with the answer.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Number of small triangles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Here learners could deduce the answers for figures 3 and 4 easily. The skill lies in detecting a pattern and developing a general equation which describes it.

Whilst the tests were balanced in that each question-type was included, there were fewer items requiring higher-order thinking skills. We took this decision to accommodate for the functional achievement level of the learners. We deemed it more useful to assess learners’ basic knowledge and operational skills as we anticipated a poorer response to questions which demanded higher-order skills. As a result, the majority of questions were of the knowledge and routine operations type: 77% for grade 10, 70% for grade 11 and 67% for grade 12. There were instances where questions could have been posed as problem-solving items, but where the question was scaffolded for learners.

The following discussion is based on an analysis of results of the tests for grades 10, 11, 12.

**THE FRAMEWORK FOR ANALYSIS OF THE TEST RESULTS**

In this section we will comment on the test framework.

In keeping with the concept of criterion-referencing, the following standards were developed for classifying learners’ responses:
Dunn et al (2002) classify mathematics as a hard pure discipline\(^{25}\). For school mathematics this implies that there is usually one (set of) solution(s) to a problem, with little room for inferences. The criteria for the correctness of an answer are generally uncomplicated. Piper et al (1996) note that the more mathematical the discipline, the less likely it is that a professional judgement will be questioned. The marking memorandum which accompanied each test made the standards explicit for each item where learners needed to show their calculations. An example is:

A 25 m tall tree casts a 40 m shadow on the ground.

What is the distance, \(x\), from the top of the tree to the end of its shadow?
Round your answer to one decimal place.

\[
x^2 = (25)^2 + (40)^2
\]
\[
\therefore x = \sqrt{2225} = 47.2 \text{ cm}
\]

**Use all codes**

Careless error: computation error e.g. \((25)^2 + (40)^2 = 625 + 160\)

Conceptual error:

\(^{25}\) Hard pure knowledge is typified as being cumulative and atomistic in structure, concerned with universals, simplification and a quantitative emphasis. Professional judgement relies upon a concrete knowledge base that is shared by the knowledge community.
- Sets up theorem of Pythagoras to solve incorrect side of triangle e.g. \((25)^2 = (40)^2 - x^2\) OR \(x^2 = (40)^2 - (25)^2\)
- Does not solve for \(x\) i.e. does not find \(\sqrt{x}\) as a last step
- Does not round off answer correctly e.g. \(\sqrt{2225} = 47.1\)

We realise, in retrospect, that categories 1 and 0 are not mutually exclusive. Nevertheless, besides being able to ascertain learners’ level of knowledge, the standards facilitate some analysis of why learners get answers wrong. Field workers in the project would be able to isolate test items relating to a particular content strand and use the standards to inform their intervention. For example, if many learners made one (or more) conceptual error(s), the field worker would be able to analyse what these errors are and address the problem(s). Another advantage is the ability to use subsequent tests to measure whether there has been a shift in conceptual development.

A different type of analysis is possible if one looks only at those answers which were correct, wrong or unanswered (categories X, 0 and 4). The graph below refers to the grade 11 test.

From the graph it is immediately evident that Probability, and to a lesser extent, Trigonometry, have not been adequately dealt with in the class. The levels of incorrect answers suggest that field workers should do a deeper analysis which includes categories 1,2 and 3.

The framework for the analysis of the data facilitates that analysis. The graph below shows grade 11 learner performance on 3 related questions:
Grade 12 – a special case

The grade 12 test deviated from the rest because this cohort of grade 12 learners is still following the interim syllabus. This implies a break in the continuum from grade 10 to 12, as the results of the test cannot be used as a comparative measure for future grade 12 cohorts. A new test needs to be designed for the 2008 cohort.

CONCLUSION

With this initial baseline exercise we realised that mathematical topics that are represented by a greater number of questions as well as questions of the complex procedure and problem-solving type, provide a more accurate measure of learner proficiency. We have discussed and justified the fact that test items were delineated in terms of knowledge (content) and process skills. The question we ask is whether that delineation facilitated a valid assessment of the quality of learners’ knowledge and problem-solving abilities. This is a future challenge we set ourselves.

References


Carmody, G., Godfrey, S and Wood L., Diagnostic tests in a first year Mathematics subject, University of Technology, Sydney, Australia

Cornwell R., You Can't Manage What you Can't Measure - Assessment of Learning


Washington, D.C. 1999


Frith, J., Frith, V. and Conradie, J., *The assessment of prospective students’ potential to learn undergraduate mathematics*, University of Cape Town, South Africa.


This entertaining presentation demonstrates how formulae for the sums of the terms of five sequences can be derived using pictures. The first three sums are those of consecutive natural numbers, the squares of consecutive natural numbers and the cubes of consecutive natural numbers. i.e. $1 + 2 + 3 + \ldots$ and $1^2 + 2^2 + 3^2 + \ldots$ and $1^3 + 2^3 + 3^3 + \ldots$. The 4th sum is that of the odd numbers and the 5th could easily be found in a matriculation paper. The question that is asked throughout the presentation is "Is this a proper proof or is it just a pretty picture?"

THE SUM OF CONSECUTIVE NATURAL NUMBERS

A demonstration of a use for the sum of the first 9 consecutive natural numbers.

Did you know that if there are 10 people in a room and they all shake hands with each other once, you can calculate the number of handshakes that take place by adding the first 9 consecutive natural numbers?

\[
\begin{align*}
&1 + 2 + 3 + 4 + \ldots + 8 + 9 \\
&9 + 8 + 7 + 6 + \ldots + 2 + 1
\end{align*}
\]

Adding $10 + 10 + 10 + \ldots + 10 + 10$

\[
\begin{align*}
+9 & \\
+8 & \\
+7 & \\
+6 & \\
+5 & \\
+4 & \\
+3 & \\
+2 & \\
+1 & \\
+9 & \\
+8 & \\
+7 & \\
+6 & \\
+5 & \\
+4 & \\
+3 & \\
+2 & \\
+1 & \\
+9 & \\
+8 & \\
+7 & \\
+6 & \\
+5 & \\
+4 & \\
+3 & \\
+2 & \\
+1 & \\
\end{align*}
\]
This is equal to 10 added 9 times but is doubled the required sum, so

\[ 1 + 2 + 3 + 4 + \ldots + 8 + 9 = \frac{1}{2} \times 10 \times 9 \]

This method is easily extended to \( n \) terms where \( n + 1 \) would be added \( n \) times and the product halved. This is a familiar technique and the one by which we find a formula for the sum of the terms of an arithmetic sequence in Grade 12.

**A justification of a formula for the sum of \( n \) consecutive natural numbers.**

![Diagram showing the sum of the first 9 natural numbers as a rectangle of buttons.](image)

The diagram shows the first 9 natural numbers as an arrangement of dark buttons and a second arrangement of the first 9 natural numbers as an arrangement of lighter buttons. Together they form a \( 9 \times 10 \) rectangle of buttons. This rectangle has twice the number of required buttons indicating that the sum of the first nine natural numbers could be computed as \( \frac{1}{2} \times 9 \times 10 \).

This method could be extended to the sum of the first \( n \) natural numbers with the sum being computed using the formula \( \frac{1}{2}n(n + 1) \)

### THE SUM OF THE SQUARES OF CONSECUTIVE NATURAL NUMBERS

"How many squares are there on a chess board?"
The chess player's answer to this question is 64.

The mathematician's answer to the question is 204, the 204 being the sum of the square numbers $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2$. This is demonstrated by showing that there is one 8 x 8 square, there are four 7 x 7 squares, nine 6 x 6 squares, sixteen 5 x 5 squares, twenty-five 4 x 4 squares, thirty-six 3 x 3 squares, fourty-nine 2 x 2 squares and sixty-four 1 x 1 squares.

"How many squares are there on a 100 x 100 grid?"

Here the sweat-it-out method of adding all the square numbers from 1 to 10 000 is impractical. The solution is to find a formula for the number of squares in an $n \times n$ grid and substitute 100 into it. This formula is the same as the one for the sum of $n$ consecutive square numbers.

The derivation of the formula involves the analysis of a three-dimensional figure built from unit cubes.

The explanation that follows uses the sum of the first four square numbers.

The figure shows three orientations of congruent towers of unit blocks, each tower containing

$$1^2 + 2^2 + 3^2 + 4^2 = 30 \text{ blocks.}$$

In this figure the three towers of unit blocks have been fitted together as the arrows above indicated.

The new solid has three times too many blocks.

In the first figure below, the top layer of unit blocks is about to be sliced in half and fitted next to the remaining half. This has been done in the second figure, making the new solid 4½ units high. The final solid has dimensions 4 x 5 x 4½. Remembering that the original construction used three towers of blocks, the number of blocks in this final solid is still three times too many.
DIVIDING THE NUMBER OF UNIT BLOCKS IN THE FINAL CUBOID BY 3, IT BECOMES EVIDENT THAT $1^2 + 2^2 + 3^2 + 4^2$ CAN BE COMPUTED AS $\frac{1}{3} \times 4 \times (4 + 1) \times (4 + \frac{1}{2})$.

If this explanation were to be repeated with the number 4 being replaced by the words "the number of squares", the explanation would become a generalization:

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{1}{3} n(n+1)\left( \frac{n+1}{2} \right) = \frac{1}{6} n(n+1)(2n+1)$$

THE SUM OF THE CUBES OF CONSECUTIVE NATURAL NUMBERS

The author has failed to find a real-life example that uses the sum of the cubes of consecutive natural numbers. She has found this wonderful proof of the formula for the sum of the cubes of consecutive natural numbers but, like Fermat, finds that this page has not enough room in which to describe it. Perhaps a picture will suffice.
The actual presentation illustrates how this figure is built up, square by square, with the overlapping square pieces compensating for the empty squares. It relies on the fact that every cube can be thought of as \( n \) lots of \( n^2 \). From the figure

\[
1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 = (1 + 2 + 3 + 4 + 5 + 6 + 7)^2
\]

Again the explanation of this sum could be generalized: the sum of the cubes of the natural numbers is equal to the square of the sum of the natural numbers.

\[
1^3 + 2^3 + 3^3 + \ldots + n^3 = (1 + 2 + 3 + \ldots + n)^2
= \left( \frac{1}{2} n(n + 1) \right)^2
\]

**THE SUM OF CONSECUTIVE ODD NUMBERS IS ALWAYS A SQUARE**

\[
1 = 1
\]

\[
1 + 3 = 4 = 2^2
\]

\[
1 + 3 + 5 = 9 = 3^2
\]

\[
1 + 3 + 5 + 7 = 16 = 4^2
\]

\[
1 + 3 + 5 + 7 + 9 = 25 = 5^2
\]

\[
\ldots
\]

\[
1 + 3 + 5 + \ldots + (2n - 1) = n^2
\]

The picture on the right explains the sums on the left. Does it justify the generalization the "The sum of the first \( n \) odd numbers is equal to \( n^2 \)?

**A GEOMETRIC SUM**

If asked to evaluate the series \((\frac{1}{2})^2 + (\frac{1}{2})^4 + (\frac{1}{2})^6 + (\frac{1}{2})^8 + \ldots\) the average Grade 12 learner would classify it as a geometric series, identify the first term \((\frac{1}{2})^2\), find the constant ratio \((\frac{1}{2})^2\) and substitute them both into the formula for the sum of an infinite GP and get an answer of \(\frac{1}{3}\).

He/she would find the following wording of the same question much more difficult:

"Evaluate \((\frac{1}{2})^2 + (\frac{1}{4})^2 + (\frac{1}{8})^2 + \ldots\)"

He/she might find this pictorial explanation interesting but not very convincing:
How would you explain that limit of the sum of the areas of the shaded squares is $\frac{1}{3}$?

A problem to ponder . . .
Should this presentation have been entitled Can Pretty Pictures Provide Proper Proofs? Did you see proofs that merely explained or were they proofs that actually proved?

REFERENCES

MATHEMATICAL LITERACY: TERMINATOR OR PERPETUATOR OF MATHEMATICAL ANXIETY?

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Until 2005 South African learners with mathematical anxiety could avoid mathematical studies in grades 10 to 12. From 2006, however, they are required to take either Mathematics or Mathematical Literacy. It is reasonable to expect that a percentage of learners in any Mathematical Literacy class will be suffering from mathematical anxiety. This paper provides a brief description of mathematical anxiety, its symptoms and possible causes. It also explains why proper teaching and learning of Mathematical Literacy can reduce mathematical anxiety. It then describes a research project that was undertaken during 2006 among Grade 10 learners who took Mathematical Literacy as a subject. During the research, learners’ levels of mathematical anxiety as well as their attitudes towards and beliefs about mathematics were measured at the beginning and the end of 2006, in an attempt to determine whether their exposure to Mathematical Literacy had had any effect on these.

INTRODUCTION

Many people have negative feelings towards mathematics. In a number of cases these negative feelings develop into a real fear of anything mathematical. We have learners suffering from mathematics anxiety in our schools. Many of them try to follow the avoidance route. Until 2005, these learners could avoid mathematics in the “Senior Secondary” phase (Grades 10 to 12). From 2006, however, they cannot: in the new FET band they either have to take Mathematics, or Mathematical Literacy.

How will we deal with learners in our Mathematical Literacy classes who have negative feelings towards mathematics, or who suffer from mathematical anxiety? Will we be able to reduce, even terminate, their levels of mathematical anxiety? Or will their anxiety continue, and even increase?

THE TYPICAL MATHEMATICAL LITERACY LEARNER?

It is reasonable to assume that the majority of Mathematical Literacy learners will be those who are less than enthusiastic about Mathematics and mathematical activities. The Draft Mathematical Literacy Guidelines for the Development of Learning Programmes (LPG) state that “It is expected that the majority of learners who choose Mathematical Literacy may have experienced mathematical learning negatively.” (South Africa, 2003). Vermeulen, et al (2005:12) conclude: “This may result in Mathematical Literacy
learners with a low level of confidence in their mathematical abilities (knowledge and skills), and high levels of anxiety and resistance towards anything mathematical.”

It is therefore very likely that a percentage of Mathematical Literacy learners will suffer from mathematical anxiety.

**WHAT IS MATHEMATICAL ANXIETY?**

Curtain-Phillips (2004) defines mathematical anxiety as “feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a variety of ordinary life and academic situations.” Zaslavsky (1999:6) defines mathematical anxiety as “A state of mind that makes it difficult or even impossible to use the mathematics skills one already has. It may be accompanied by physical symptoms, such as headaches, nausea, heart palpitations, and dizziness. It surfaces in the most extreme form when one has to take a test. Although a certain amount of anxiety about a new or stressful situation is perfectly normal, and may even stimulate one to perform well, the term ‘math anxiety’ is reserved for a condition that is traumatic and debilitating”.

**WHAT ARE THE SYMPTOMS OF MATHEMATICAL ANXIETY?**

Most sufferers of mathematical anxiety feel powerless, out of control, lacking in self-esteem. Many of them manage to conceal their negative feelings about mathematics until they are confronted with a situation that forces them to “come out of the closet” (Zaslavsky, 1999:19). Andrews (2003) suggests that disruptive learners in mathematics classes may exhibit this behaviour because their levels of mathematical anxiety are so high that this is the only way they think they can get by without everyone finding out.

Physically, sufferers from mathematical anxiety may experience nausea, perspire profusely, develop a headache or tight muscles, increased heart rate, rapid but shallow breathing from the upper lungs, tightness in the throat, difficulty swallowing, shortness of breath, dizziness and lightheadedness. Mentally they become confused or disorganized, unable to concentrate. They make lots of careless mistakes, forgot formulas they knew, cannot think clearly, or blank out completely (Arem, 2003:26).

According to Ashcraft and Kirk (2001:236) mathematical anxiety leads to “a reduction in the available working-memory capacity….mathematical anxiety disrupts the ongoing, task-relevant activities of working memory, slowing down performance and degrading its accuracy.”
WHAT CAUSES MATHEMATICAL ANXIETY?

Mathematical anxiety does not appear to have a single cause. It results from negative experiences of mathematics; parents’, teachers’ and peers’ attitudes towards mathematics; poor self-concept; the inability to handle frustration; and teaching methods that place an emphasis on drill and practice rather than on understanding (Friore, 1999).

Many sufferers of mathematical anxiety are able to trace their fears to specific incidents in the primary school. Often the problem begins with the failure to understand some concepts (Zaslavsky, 1999:12), and is exacerbated by a lack of confidence in their mathematical ability (Zaslavsky, 1999:6).

Teacher beliefs

Teachers’ beliefs as to what mathematics is, and how it should be taught and learnt, are often based on common misconceptions or myths, such as:

- Mathematics involves a lot of memorization of facts, rules, formulas and procedures
- You must follow the procedure set down by the teacher and the textbook
- Mathematics must be done fast. If you cannot solve a problem in a few minutes, you might as well give up.
- Every problem has only one correct answer, and it must be exact.
- You must never count on your fingers or use concrete material (manipulatives) to solve a problem
- You must keep at it until you have solved the problem.
- Mathematics is hard. Only a genius or a “mathematics-brain” can understand it.
- Mathematics language is unrelated to ordinary everyday language.
- Mathematics is rigid, uncreative, cut-and-dried, complete. It does not involve imagination, discovery, invention, creativity. There is nothing new in mathematics.
- Mathematics is exact, logical and certain. Intuition does not enter into it.
- Mathematics is abstract. It is unrelated to history or culture.
- Mathematics is value-free. It is the same for everyone all over the world.

(Zaslavsky, 1999: 21)
Teachers’ teaching methods:
Evidence suggests that mathematical anxiety results more from the way the subject matter is presented than from the subject matter itself (Greenwood, 1984).

Teachers’ beliefs about mathematics influence the way they teach. If the misconceptions in the previous section inform a teacher’s belief about mathematics, then he or she will teach mathematics as a set of (unrelated) rules, not to be understood, but to be remembered, promoting instrumental rather than relational understanding. He or she will convey the idea that mathematics is not related to other parts of mathematics, or other subjects, or life-related situations. Furthermore, the teacher is the authority – never to be questioned.

Mismatches between teachers’ instructional style and learners’ learning styles further increase levels of mathematical anxiety (Zaslavsky, 1999:169).

Teacher attitudes:
Many mathematical anxiety sufferers can relate the origin of their condition to unsympathetic teachers: yelling at learners, throwing objects at learners, making learners feel “stupid”, or gender discrimination.

It is believed that teachers’ attitudes towards their learners matter more than their teaching methodology: “They learn far more from our faith in them that they can and will learn mathematics, than from our most lucid explanations or most brilliant innovations” (Hallet, 1983 in Zaslavsky, 1999:20).

Classroom culture:
Incorrect teacher beliefs, poor teaching methods and negative teacher attitudes lead to a poor classroom culture, which does not promote proper learning of mathematics, and which increases levels of mathematical anxiety.

Society:
Society often supports a number of incorrect beliefs, based on stereotyping, such as:
- Males inherently have more mathematical ability than females
- Males have more spatial ability than females, and that makes them superior in mathematics
- Some people can do mathematics; others simply cannot
- You have to have a “mathematics-brain” to do mathematics
- If you don’t have the mathematics gene, you cannot do mathematics
- Race and class groups have different mathematical abilities
Parents:
Parents may be influenced by the various myths and stereotypes, for example that girls are less capable of doing mathematics. Or they may believe that women’s careers do not require knowledge of mathematics (Zaslavsky, 1999:16). Parents admitting that they did not like mathematics at school, or that they (also) could not do mathematics, sometimes in an attempt to console their child, do their children no favour. Parents may have suffered from mathematical anxiety themselves, and convey this overtly or covertly to their children. The culprits are often, but not exclusively, mothers.

Pressure exerted by parents on their children to perform in mathematics (sometimes because the parent was good in mathematics; sometimes because the child needs to make up for the parent’s failure in mathematics; other times merely because a well-meaning parent realises the importance of his or her child being proficient in mathematics) leads to “fear of failure”. This in turn leads to poor test and exam results.

Peer influence:
More recent theories about the influence of adults on children have focused attention on peer group effects. For example, Harris (1995) concludes that peer affiliations become increasingly more influential on shaping attitudes than parents and teachers. If children are greatly influenced by their peers, they may avoid the pursuit of mathematics if the peer group regards it negatively for any reason.

CAN (TEACHING AND LEARNING) MATHEMATICAL LITERACY REDUCE MATHEMATICAL ANXIETY?
A properly planned and implemented Mathematical Literacy learning programme should be able to reduce, and even overcome, mathematical anxiety. This hypothesis is based on the fact that most, if not all of, the factors contributing towards mathematics anxiety (as described earlier), can be avoided and/or positively addressed in the Mathematical Literacy classroom. An analysis of Mathematical Literacy as a subject should support this point of view, for example:

- The very nature of Mathematical Literacy, i.e. that “Mathematical Literacy is a subject driven by life-related applications of mathematics” (South Africa 2005:7), and “Mathematical Literacy’s predominant focus should not be to further learners’ mathematical ‘content’ learning, that the emphasis should be on applications, and that development in Mathematical Literacy should be understood in terms of learners’ ability and willingness to solve problems in increasingly complex contexts.” (Venkatakrishnan & Graven, 2006:19).
- Suggested teaching approaches in Mathematical Literacy, e.g. developing and applying the required knowledge and skills within relevant real-life or simulated contexts, rather than as sets of rules and procedures detached from learners’ real world experiences.
- An emphasis on understanding, original quality thinking and problem solving, rather
  than on rote memorization of rules and procedures, or rote application of formulas
- An equal importance of process and product
- The central role of discussion in analyzing and solving problems
- Collaborative learning/working in groups
- The use of manipulatives

These examples stand in direct contrast to the earlier stated common misconceptions on
which mathematics teaching is too often based, and which may lead to negative attitudes
towards mathematics, and even to mathematical anxiety.

THE RESEARCH

Four schools in the Western Cape took part in the research project. The educators in
these schools who would be teaching Mathematics Literacy to Grade 10 in 2006,
completed an ACE in Mathematical Literacy at CPUT during 2004 and 2005. One
school is an ex-model C school, one is in a coloured community, and two are township
schools. Grade 10 learners from these schools completed two questionnaires: 379
learners completed Questionnaire 1 consisting of 53 questions at the beginning of 2006
before they started with classes in Mathematical Literacy, and 355 Grade 10 learners
completed Questionnaire 2 (similar to Questionnaire 1, but focusing on Math Lit) at the
end of the year after their classes finished, but before the examination started. These
questionnaires were based on the Attitudes Toward Mathematics Inventory (ATMI)
developed by Tapia and Marsh (2004). During the year I visited the schools to observe
teachers’ classroom practice and learners’ learning behaviours.

Teachers also completed a separate questionnaire, reporting their classroom practice and
their experiences.

The primary purposes of this research project were to determine

- the extent and levels of mathematical anxiety among Grade 10 learners starting with
  Mathematical Literacy (as measured by Questionnaire 1).
- whether properly planned and implemented learning programmes for Mathematical
  Literacy can be instrumental in reducing learners’ mathematical anxiety (as measured
  by Questionnaire 2).

Secondary purposes of the research project were to determine

- Learners’ general attitude towards mathematics (and whether these changed during
  one year of studying Mathematical Literacy) (Questionnaires 1 and 2)
- Learners’ general beliefs about mathematics (and whether these changed during one
  year of studying Mathematical Literacy) (Questionnaires 1 and 2)
PRELIMINARY RESULTS

A number of sample results follow below.

A. Mathematical anxiety (Q1 = Questionnaire 1; Q2 = Questionnaire 2)

A1. Question: When the teacher asked the class a question, and I did not know the answer, I would avoid eye contact with the teacher, hoping that he/she would not ask me.

   Results:

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th>Percent</th>
<th>Cumulative Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Math (Q1)</td>
<td>Math Lit (Q2)</td>
<td>Math (Q1)</td>
</tr>
<tr>
<td>Never</td>
<td>79</td>
<td>92</td>
<td>21,5</td>
</tr>
<tr>
<td>Hardly ever</td>
<td>41</td>
<td>53</td>
<td>11,2</td>
</tr>
<tr>
<td>Sometimes</td>
<td>150</td>
<td>126</td>
<td>40,9</td>
</tr>
<tr>
<td>Usually</td>
<td>58</td>
<td>33</td>
<td>15,8</td>
</tr>
<tr>
<td>Always</td>
<td>39</td>
<td>18</td>
<td>10,6</td>
</tr>
<tr>
<td>Total</td>
<td>367</td>
<td>322</td>
<td>100,0</td>
</tr>
</tbody>
</table>

A2. Question: When the teacher asked me a question directly, and I did not know the answer, I would feel like running out of the class.

   Results:

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th>Percent</th>
<th>Cumulative Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Math (Q1)</td>
<td>Math Lit (Q2)</td>
<td>Math (Q1)</td>
</tr>
<tr>
<td>Never</td>
<td>193</td>
<td>171</td>
<td>59,9</td>
</tr>
<tr>
<td>Hardly ever</td>
<td>23</td>
<td>18</td>
<td>7,1</td>
</tr>
<tr>
<td>Sometimes</td>
<td>59</td>
<td>81</td>
<td>18,3</td>
</tr>
<tr>
<td>Usually</td>
<td>25</td>
<td>19</td>
<td>7,8</td>
</tr>
<tr>
<td>Always</td>
<td>22</td>
<td>14</td>
<td>6,8</td>
</tr>
<tr>
<td>Total</td>
<td>322</td>
<td>303</td>
<td>100,0</td>
</tr>
</tbody>
</table>
A3. **Question:** When the teacher asked me a question directly, and I did not know the answer, I tried to make a joke.

**Results:**

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th>Percent</th>
<th>Cumulative Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Math (Q1)</td>
<td>Math Lit (Q2)</td>
<td>Math (Q1)</td>
</tr>
<tr>
<td>Never</td>
<td>227</td>
<td>177</td>
<td>65,0</td>
</tr>
<tr>
<td>Hardly ever</td>
<td>35</td>
<td>32</td>
<td>10,0</td>
</tr>
<tr>
<td>Sometimes</td>
<td>61</td>
<td>65</td>
<td>17,5</td>
</tr>
<tr>
<td>Usually</td>
<td>16</td>
<td>19</td>
<td>4,6</td>
</tr>
<tr>
<td>Always</td>
<td>10</td>
<td>12</td>
<td>2,9</td>
</tr>
<tr>
<td>Total</td>
<td>349</td>
<td>305</td>
<td>100,0</td>
</tr>
</tbody>
</table>

A4 **Question:** During Mathematics/Math Lit tests I “struck a blank” (did not know what to do).

**Results:**

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th>Percent</th>
<th>Cumulative Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Math (Q1)</td>
<td>Math Lit (Q2)</td>
<td>Math (Q1)</td>
</tr>
<tr>
<td>Never</td>
<td>68</td>
<td>69</td>
<td>18,1</td>
</tr>
<tr>
<td>Hardly ever</td>
<td>37</td>
<td>48</td>
<td>9,9</td>
</tr>
<tr>
<td>Sometimes</td>
<td>179</td>
<td>181</td>
<td>47,7</td>
</tr>
<tr>
<td>Usually</td>
<td>55</td>
<td>33</td>
<td>14,7</td>
</tr>
<tr>
<td>Always</td>
<td>36</td>
<td>17</td>
<td>9,6</td>
</tr>
<tr>
<td>Total</td>
<td>375</td>
<td>348</td>
<td>100,0</td>
</tr>
</tbody>
</table>

B. **General attitude towards Mathematics (Questionnaire 1) and Math Lit (Questionnaire 2)**

B1. **Question:** I looked forward to attending Mathematics/Math Lit classes.

**Results:**

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th>Percent</th>
<th>Cumulative Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Math (Q1)</td>
<td>Math Lit (Q2)</td>
<td>Math (Q1)</td>
</tr>
<tr>
<td>Always</td>
<td>137</td>
<td>147</td>
<td>36,4</td>
</tr>
<tr>
<td>Usually</td>
<td>45</td>
<td>54</td>
<td>12,0</td>
</tr>
<tr>
<td>Sometimes</td>
<td>124</td>
<td>118</td>
<td>33,0</td>
</tr>
<tr>
<td>Hardly ever</td>
<td>35</td>
<td>17</td>
<td>9,3</td>
</tr>
<tr>
<td>Never</td>
<td>35</td>
<td>12</td>
<td>9,3</td>
</tr>
<tr>
<td>Total</td>
<td>376</td>
<td>348</td>
<td>100,0</td>
</tr>
</tbody>
</table>
B2. **Question:** I felt relaxed in Mathematics/Math Lit classes.

**Results:**

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th>Percent</th>
<th>Cumulative Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Math (Q1)</td>
<td>Math Lit (Q2)</td>
<td>Math (Q1)</td>
</tr>
<tr>
<td>Always</td>
<td>50</td>
<td>87</td>
<td>13,6</td>
</tr>
<tr>
<td>Usually</td>
<td>46</td>
<td>59</td>
<td>12,5</td>
</tr>
<tr>
<td>Sometimes</td>
<td>177</td>
<td>166</td>
<td>48,0</td>
</tr>
<tr>
<td>Hardly ever</td>
<td>41</td>
<td>21</td>
<td>11,1</td>
</tr>
<tr>
<td>Never</td>
<td>55</td>
<td>14</td>
<td>14,9</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>369</td>
<td>348</td>
<td>100,0</td>
</tr>
</tbody>
</table>

B3. **Question:** I was confident to ask the teacher questions in the Mathematics/Math Lit class.

**Results:**

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th>Percent</th>
<th>Cumulative Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Math (Q1)</td>
<td>Math Lit (Q2)</td>
<td>Math (Q1)</td>
</tr>
<tr>
<td>Always</td>
<td>71</td>
<td>89</td>
<td>18,7</td>
</tr>
<tr>
<td>Usually</td>
<td>61</td>
<td>51</td>
<td>16,1</td>
</tr>
<tr>
<td>Sometimes</td>
<td>158</td>
<td>156</td>
<td>41,7</td>
</tr>
<tr>
<td>Hardly ever</td>
<td>48</td>
<td>23</td>
<td>12,7</td>
</tr>
<tr>
<td>Never</td>
<td>41</td>
<td>29</td>
<td>10,8</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>379</td>
<td>348</td>
<td>100,0</td>
</tr>
</tbody>
</table>

B4. **Question:** I was confident to ask a fellow learner questions in the Mathematics/Math Lit class.

**Results:**

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th>Percent</th>
<th>Cumulative Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Math (Q1)</td>
<td>Math Lit (Q2)</td>
<td>Math (Q1)</td>
</tr>
<tr>
<td>Always</td>
<td>98</td>
<td>87</td>
<td>26,1</td>
</tr>
<tr>
<td>Usually</td>
<td>81</td>
<td>44</td>
<td>21,6</td>
</tr>
<tr>
<td>Sometimes</td>
<td>145</td>
<td>157</td>
<td>38,7</td>
</tr>
<tr>
<td>Hardly ever</td>
<td>13</td>
<td>27</td>
<td>3,5</td>
</tr>
<tr>
<td>Never</td>
<td>38</td>
<td>34</td>
<td>10,1</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>375</td>
<td>349</td>
<td>100,0</td>
</tr>
</tbody>
</table>
C. Beliefs about Mathematics (Questionnaire 1) and Math Lit (Questionnaire 2)

C1. **Question:** Mathematics/Math Lit is a set of rules which one does not have to understand.

**Results:**

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th>Percent</th>
<th>Cumulative Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Math (Q1)</td>
<td>Math Lit (Q2)</td>
<td>Math (Q1)</td>
</tr>
<tr>
<td>Strongly agree</td>
<td>72</td>
<td>62</td>
<td>20,2</td>
</tr>
<tr>
<td>Agree</td>
<td>96</td>
<td>94</td>
<td>27,0</td>
</tr>
<tr>
<td>Do not know</td>
<td>72</td>
<td>87</td>
<td>20,2</td>
</tr>
<tr>
<td>Disagree</td>
<td>78</td>
<td>58</td>
<td>21,9</td>
</tr>
<tr>
<td>Strongly disagree</td>
<td>38</td>
<td>27</td>
<td>10,7</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>356</td>
<td>328</td>
<td>100,0</td>
</tr>
</tbody>
</table>

C2. **Question:** To be successful in Mathematics/Math Lit, one needs to remember a lot of rules, and apply the correct rule to solve a problem.

**Results:**

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th>Percent</th>
<th>Cumulative Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Math (Q1)</td>
<td>Math Lit (Q2)</td>
<td>Math (Q1)</td>
</tr>
<tr>
<td>Strongly agree</td>
<td>21</td>
<td>15</td>
<td>6,0</td>
</tr>
<tr>
<td>Agree</td>
<td>31</td>
<td>31</td>
<td>7,5</td>
</tr>
<tr>
<td>Do not know</td>
<td>48</td>
<td>48</td>
<td>11,5</td>
</tr>
<tr>
<td>Disagree</td>
<td>136</td>
<td>130</td>
<td>32,7</td>
</tr>
<tr>
<td>Strongly disagree</td>
<td>114</td>
<td>107</td>
<td>27,4</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>350</td>
<td>331</td>
<td>100,0</td>
</tr>
</tbody>
</table>
C3. **Question:** In Mathematics/Math Lit the answer is more important than how one arrives at the answer.

**Results:**

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Math (Q1)</td>
<td>Math Lit (Q2)</td>
<td>Math (Q1)</td>
<td>Math Lit (Q2)</td>
</tr>
<tr>
<td>Strongly agree</td>
<td>51</td>
<td>57</td>
<td>14,7</td>
<td>17,1</td>
</tr>
<tr>
<td>Agree</td>
<td>86</td>
<td>85</td>
<td>24,9</td>
<td>25,5</td>
</tr>
<tr>
<td>Do not know</td>
<td>55</td>
<td>71</td>
<td>15,9</td>
<td>21,3</td>
</tr>
<tr>
<td>Disagree</td>
<td>106</td>
<td>93</td>
<td>30,6</td>
<td>27,9</td>
</tr>
<tr>
<td>Strongly disagree</td>
<td>48</td>
<td>27</td>
<td>13,9</td>
<td>8,1</td>
</tr>
<tr>
<td>Total</td>
<td>346</td>
<td>333</td>
<td>100,0</td>
<td>100,0</td>
</tr>
</tbody>
</table>

**DISCUSSION OF RESULTS**

At the time of writing this paper, no statistical analysis had been carried out yet to determine statistical significances of any changes. Therefore, I can only comment on the raw data, as reported by participating learners. A few of these are discussed below.

**A1:** While 26,4% of learners at the beginning of 2006 reported that they would *usually or always* avoid eye contact with the teacher when they did not know the answer to a question, only 15,8% reported this at the end of 2006. This may signify a more confident attitude among learners, which in turn may be an indication of a potential reduction in mathematical anxiety.

**A4:** 24,3% of the learners at the beginning of 2006 reported that they *usually or always* “struck a blank” during Mathematics tests. 14,4% reported this at the end of 2006. This may signify more self-confidence in their abilities, with a concurrent possible reduction in the level of mathematical anxiety.

**B1:** 48,4% *always or usually* looked forward to their Mathematics classes until 2006, while 57,8% reported this for Mathematical Literacy. This may signify a generally positive shift in learners’ attitude.

**B2:** 26,0% reported at the beginning of 2006 that they *always or usually* felt relaxed in Mathematics classes, while 42,1% reported this regarding Mathematical Literacy at the end of 2006. Although one would hope for a larger increase in this value, at least there appears to be a marked improvement.

**B4:** While 47,7% reported at the beginning of 2006 that they *always or usually* was confident to ask a fellow learner a question, this decreased to 37,5% at the end of 2006. This is disconcerting, and requires further investigation.
C1: 47.2% agreed and strongly agreed at the beginning of 2006 that Mathematics is a set of rules which one does not have to understand. Disconcertingly, 47.6% reported this about Mathematical Literacy at the end of 2006. This also requires further investigation.

C3: The 42.6% who agreed and strongly agreed at the end of 2006 that in Math Lit the answer is more important than how one arrives at the answer, should sound the alarm bells for the teachers concerned. Issues such as these need to be discussed with these teachers.

CONCLUSION

Although learners’ responses indicate a number of improvements in their state of mathematical anxiety, as well as in their general attitudes and beliefs regarding mathematics, these do not appear to be dramatic. They also report a number of disconcerting decreases. This investigation will be repeated at the end of 2007 to determine whether two years’ exposure to Mathematical Literacy has had a more significant impact on learners’ mathematical anxiety, attitudes and beliefs regarding mathematics.

REFERENCES


