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“Mathematical Knowledge for Teaching”

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Editors: JH Meyer & A van Biljon

VOLUME 2
Foreword

“Mathematical Knowledge for Teaching” is, of course, non-negotiable in any reputable education system. How can you teach mathematics without the necessary knowledge?

More important questions are: What kind of mathematical perspective does your knowledge create in your mind? Is your knowledge sufficient enough to react sensibly when you have to deal with the inquisitive minds amongst your learners? Are you prepared to broaden your knowledge on a continual basis, until the day you retire (or die!), or do you just stagnate to the narrow avenues of syllabus-knowledge, sufficient for your learners to obtain a pass mark in the examination? Most importantly: How does your knowledge inspire you to inspire your learners?

It is hoped that this conference will, in the least, create fresh viewpoints on and an awakening of mathematical knowledge - that knowledge exists not only to pass on, but to stimulate the creation of further knowledge.

There are a total of 82 papers to be presented at this congress: 14 long papers, 9 short papers, 21 “How I Teach” papers, 37 workshops (pre-conference workshops included) and one poster. In addition, there will be five plenary sessions, two panel discussions, a keynote address and several interest group discussions. There will also be presentations by exhibitors in the maths market and activity centre. We are also pleased to see a number of presentations on mathematics literacy. This certainly adds to the broadening of the spectrum of mathematical knowledge. Another encouraging fact is the substantial number of presentations by teachers from the maths4stats project, funded by Stats SA.

It is delightful to see an increasing interest in involvement in our mathematics education community. It is hoped that this trend will continue.

The reviewing process certainly had its hiccups. Some of the reviewers had to be reminded several times to send feedback. Some of the authors offered the same kind of difficulty – to send the final versions of their manuscripts in time. Each of the long and the short papers was sent to three reviewers. As a general rule, papers with at least two recommendations for acceptance were selected. Many authors were requested to make modifications to their papers, as requested by the reviewers, before they could be accepted for publication in the proceedings and be included in the congress programme. Each of the “How I teach” papers, workshops and posters was reviewed by at least one person and most of them were accepted (after some rework). We trust that the general standard improved due to this process.

Let me express my sincere thanks to all those reviewers and authors who reacted swiftly and who took the deadlines seriously.

Johan Meyer
Academic Programme Director
Acknowledgement

Each paper submitted to the congress was sent to $n$ reviewers, where $n \in \{1,2,3\}$, for blind reviewing. We hope that the contributors found the comments and suggestions useful and we trust that this process helped to improve the quality of the papers.

Many thanks to our reviewer corps who reviewed the papers in a constructive and helpful spirit:

Hennie Boshoff
Lorraine Botha
Laurie Butgereit
Michael de Villiers
Gawie du Toit
Stephan du Toit
Johann Engelbrecht
Faaiz Gierdien
Nico Govender
Belinda Huntley
Paul Laridon
Caroline Long
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Vimolan Mudaly
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Gerrit Stols
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Nelis Vermeulen
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MATHEMATICAL MODELING – FINDING AND USING T_n

Ian L Atteridge
Dainfern College

Learners with different learning styles and abilities benefit from being able to choose from a variety of techniques. In this presentation I will introduce the use of the regression function on the Casio calculator to derive the T_n (general term) for a number pattern; either linear or quadratic. In my experience learners enjoy the most benefit when relating examples to concrete situations and using technology.

FINDING T_n

Introduction

Mathematical modeling is one of the most exciting additions to our new National Curriculum Statement. It gives the learners examples of where Mathematics is used ‘in the real world’. Using questions and investigations I will present a step-by-step method for deriving a T_n (general formula).

Content

I will demonstrate how to determine if a number pattern is linear, quadratic or neither. I will then show how to input data into the calculator and extract the coefficients of the general equation. I will show how the table function on the calculator could be used to solve questions once the T_n is known.

Conclusion

This approach to determining the T_n as opposed to a more formal algebraic approach, allows weak learners to achieve success. They can then use these techniques to help solve problems.

References

Learning and Skills Improvement Service
http://www.sflqi.org.uk/materials/pd_elements/03_02_01/finaloutput.htm [27 May 2009]
SKETCHING A CUBIC FUNCTION
TRADITIONAL METHODS VS USING TECHNOLOGY

Ian L Atteridge
Dainfern College

With the availability of powerful calculators there are alternate techniques to sketching a cubic function. Learners with different learning styles and abilities benefit from being able to choose from a variety of techniques. In this presentation I will introduce a lesser known technique: Use of the Table function of the calculator. In my experience learners who lack self-confidence find security in using technology to eliminate basic arithmetic errors.

SKECHING THE CUBIC FUNCTION

Introduction
There are many steps to sketching a cubic function. It is often considered as the culmination of algebra in the FET phase. I will show how the important points of a cubic function can be extracted using a variety of methods but placing emphasis on the lesser known method of using a table function on a calculator. The basis of this technique is ‘trial and error’.

Content
Determining the shape will be done intuitively and using the table function. Determining the y-intercept will be done by recognizing the constant term. Determining the x-intercept(s) will be shown using the factor theorem and inspection as well as the table function. X-values of the turning points will be calculated, after differentiation, using the table function. The concavity of these points will be shown intuitively and reinforced using the table function. The point of inflection will be mentioned.

Conclusion
The alternate technique presented aid learners in sketching a cubic function, and allow them to achieve a measure of success without getting bogged down in the accompanying calculations. These methods can also be used by more advanced learners to allow them to visualize other functions.

References
USING TEXT ADVENTURE GAMES TO ENTICE LEARNERS TO PRACTICE ARITHMETIC SKILLS OVER MXIT

Laurie Butgereit
Meraka Institute, CSIR

The matrics of 2009 (those learners turning 18 in 2009) are younger than the internet, younger than cell phones, younger than computer games. Every primary school learner and secondary school learner in school this year has lived his or her entire life in the presence of computers, wireless connectivity, television, the world wide web, cell phones, and computer games. They are a hooked up, wired, connected, multi-tasking, parallel processing generation of learners. Isn't it time the education process catered for these characteristics? This paper describes a facility where learners are enticed or encouraged to practice basic arithmetic skills by playing text adventure games over Mxit on their cell phones. The text adventure game is a traditional “dungeon” type game with rooms and hallways. Unlike traditional “dungeon” games where the weapons of choice are keys, axes, and magic wands, the weapons of choice in our text adventure games are cell phones, calculators, remote digital keys and mathematics textbooks. Whereas in traditional “dungeon” games, magic words would be written on walls, in our text adventure game, numerical calculations are sms’d to the virtual cell phone of the player. The result of these numerical calculations are the secret codes to get through digital safe doors, use keypads and to turn on digital keys.

INTRODUCTION

The learners in primary and secondary school today are a “switched-on” generation. Traditional educational tools such as paper, pencils, and white boards are competing with cell phones, MP3 players, IPODs, PDAs, Wiis and play stations. This paper examines a project where text adventure games with a mathematical twist are deployed over Mxit which participants can play on their cell phones. In order to complete the puzzles laid out in the game, participants must do various arithmetic calculations.

A BRIEF HISTORY OF TECHNOLOGY

By standing on the shoulders of giants, Sir Issac Newton saw farther and could build on what his predecessors had already discovered.

It is difficult for an author to give a brief history of technology because the author never knows how far back that history must go. But let's start in 1969 when four host computers were connected together creating the initial ARPANET configuration [1]. That initial network of four computers grew to be the world wide web. In 1974 one
of the first computer games, *Pong*, was introduced [2]. In 1983 Motorola and AT&T introduced an advanced mobile phone system which grew to over two million subscribers by 1988 [3]. In 1988 Creative Labs also hit the market with their Sound Blaster card for PCs bringing sound to PC gaming [4].

In South Africa, Vodacom (a major cell phone network) was launched in 1993 [5] and MTN (another major cell phone network) was launched in 1994 [6].

And the majority of the matrics for 2009 will be turning 18 in 2009 and were born in 1991. During their entire life there has been internet, cell phones, and noisy computer games.

Marc Prensky [7] maintains that digital technologies have dramatically changed these learners: “Today's students have not just changed *incrementally* from those of the past, nor simply changed their slang, clothes, body adornments, or styles, as has happened between generations previously. A really big *discontinuity* has taken place. One might even call it a 'singularity' – an event which changes things so fundamentally that there is absolutely no going back. This so-called 'singularity' is the arrival and rapid dissemination of digital technology in the last decades of the 20th century.”

This paper will explore a project where computer games with a mathematical twist are accessible to learners on their cell phones using the popular Mxit instant messaging system.

**A BRIEF DESCRIPTION OF MXIT**

Mxit is an instant messaging system (also known as a “chat” system) which runs on cell phones. People who wish to use Mxit must download a small application (known as a “mxit client”) onto their cell phones. This application is then used to send and receive messages and chat with other people. In many ways it is similar to an sms message or a text message. The two major differences between sms messages and Mxit messages are

1. In order to communicate with another person using Mxit both people must be using Mxit at the same time. This is quite different from sms messages or text messages which can be sent at any time regardless of whether the intended recipient has his or her phone switched on.

2. Mxit messages are a fraction of the cost of sms or text messages. Depending on cell phone contracts, sms and text messages may cost up to ZAR1.00 (one South African Rand) per message whereas Mxit messages cost one or two (South African) cents each.

In addition to these features, the designers of Mxit (Mxit Lifestyle (Pty) Ltd based in Stellenbosch, South Africa) keep the Mxit cell phone application vibrant and “kewl” (“kewl” is a common term used in Mxit and other instant messaging systems and is a derivation of the old slang term “cool” meaning fun, funky, exciting, nice, etc) with
frequent updates. The cell phone application is colourful, makes noises, and is easy to use. It is a virtual meeting place for young people.

Mxit currently boasts over 9 million users of their system [8].

A BRIEF DESCRIPTION OF DR MATH

Dr Math is a project which was implemented by Meraka Institute in January, 2007 [9]. Dr Math provides help with mathematics homework using Mxit as a medium. Primary and secondary school learners can use Mxit on their cell phones and reach a tutor in the afternoons after school to help with their math homework problems. The tutors are engineering students from the University of Pretoria. The engineering department has a requirement that students must do 40 hours of community service in order to get their degree. Between twenty and thirty of the engineering students spend their 40 hours as Dr Math tutors. During the first two years of operation (2007 and 2008) over 4000 learners received help from the tutors.

During the second year of operation (2008), Dr Math implemented a number of competitions in which learners could compete against other learners in basic arithmetic and algebra skills during the periods of time when human tutors were not available [10]. The competitions included simple skills such as addition and multiplication and more complicated skills such as factoring a polynomial and finding the intersection point of two straight lines.

These competitions are extremely popular and learners often do hundreds of calculations in order to remain the “top score” of the competition.

However, in reviewing the log files of the competitions, it became obvious that some competitions (such as addition and multiplication) were extremely popular and other competitions (such as division) were not very popular.

Another mechanism was needed to really entice learners to practice skills such as division. The mechanism investigated was the implementation of text adventure games (often also called “interactive fiction”) which can be played over Mxit on cell phones.

A BRIEF DESCRIPTION OF TEXT ADVENTURE GAMES

Text adventure games were some of the first games available on personal computers with Zork being one of the oldest documented text adventure games [11]. Players navigate a world of rooms (or caves as the case may be) by typing commands such as “go east” or “enter door”. Objects in the world can be manipulated with simple commands such as “take ax” or “read book” or more complicated commands such as “throw ax at tree.”
Although this genre of computer games may seem old-fashioned, even “quaint”, in view of the current trend of high end graphics, multi-player, first person shooters, text adventure games are still being developed and deployed. Inform 7 [12] is an open source text adventure game development environment which is freely downloadable. Inform 7 generates a binary file of the adventure game in a format which is an industry standard. The binary file can be played by various other software applications including software on some mobile players.

**DR MATH'S MISSING LAPTOP**

*Dr Math's Missing Laptop* is a text adventure game developed using Inform 7 to be specifically playable on the Dr Math's Mxit contact on cell phones. The game consists of a “world” or “map” of 4 rooms with an interconnecting hallway between them. All of the doors of the rooms are locked with either a digital key pad or are controlled using a digital remote key. In the various rooms, players will find handy objects such as a cell phone, a calculator, an LCD torch, a battery charger, a digital remote key, a mathematics text book, and a rucksack which can be used to carry all of these objects.

Clues on how to get through the digital locks are often written on the walls, on white boards or are “virtually” sms'd to the player. In view of the fact that *Dr Math's Missing Laptop* was written to encourage learners to practice division, all of the clues are division calculations. Here are a few samples of the clues:

- On the desk top, dirty finger prints have written “Sliding door code is 32/4”
- The sms says “The key code for the digital key is 132/12”

The player can then later execute the command

```
Key sliding door to 8
```

and the game will respond with

```
The sliding door is now unlocked
```

and the player will be able to open the door and get through.

In the following sequence of commands, the player needs to get a “virtual” cell phone working. (In the following transcript, the bold print are responses from the game and the normal print are user inputs)

```
Look in desk
In the modern gleaming metal desk are a torch, a cell phone and a digital key
take cell phone
[your score has just gone up by one point]
examine cell phone
The cell phone is currently switched off
```
Turn on cell phone

[your score has just gone up by one point]

Examine cell phone

It is a high-end cell phone with a 5 mega pixel camera, FM radio, HSDPA.

You must really key the cell phone to a pin number.

Key cell phone to 10

Invalid pin for cell phone

look

You can see a sliding door, a modern gleaming metal desk, a broken executive chair and a white board here.

Examine white board

On the white board in large black writing are the words “cell phone pin is 24/12”

key cell phone to 2

the cell phone is now useable

[your score has just gone up by one point]

Players must navigate through the four rooms collecting all the objects in order to eventually find Dr Math’s Missing Laptop. Throughout the game, depending on the routes taken by the players, approximately 5 or 6 division calculations must be correctly done by the player. In this particular case, the game ensures that the division calculations are integer calculations with no remainders or fractions. These calculations are generated by random number generators so that the calculations are different every time the game is played.

Because the game is rather involved, there is a mechanism for the player to save the game status and come back to the game at a later time

RESULTS

By looking at the data from the log files, we find that participants often play the text adventure game for a short period of time, go away, and come back later. In this particular log, Rapper (not his or her real Mxit nick name) started playing for the first time on January 7, 2009, around mid day.

2009-01-07 11:35:14,121 Rapper:

2009-01-07 11:35:14,151 Bot -> Rapper: Today’s TopScore is Gumm be@r with 0. It may be easier to play if you turn the dictionary on. Commands must be spelled out. For example .g help or .g look or .g go or .g take.
He played on and off all day until the early hours of January 8, 2009

2009-01-08 03:04:28,126 Rapper: take

2009-01-08 03:04:28,135 Bot -> Rapper: Today's TopScore is Man wida plan with 3. What do you want to take?

He continues playing on and off for days

2009-01-15 18:33:37,365 Rapper: desk


Until finally on January 18, 2009 (over a week later), he successfully finds *Dr Math's Missing Laptop*

2009-01-18 03:42:10,949 Rapper:

2009-01-18 03:42:10,953 Bot -> Rapper: You are still the top score.

*** You have found Dr Math's laptop. Congratulations. Now it's time to hit your math homework. Bye bye. ***

In that game you scored 21 out of a possible 22, in 163 turns.

We have found that participants often play this adventure game over a period of days. Currently, at the time of writing this paper, there are 70 saved games stored on our system. These are games which are in progress. The participants will come back and continue the game at a later time.

**ETHICS AND SAFETY OF MINOR CHILDREN**

Because this project deals with minor children, the entire Dr Math tutoring project has received ethics approval from the Tshwane University of Technology ethics committee. All conversations are logged for research purposes. All participants are informed when they subscribe to the service that their conversations are logged for research purposes. No personal details of participants and no actual telephone numbers are stored in any of the log files.

**CONCLUSION**

It is clear that children and teenagers can concentrate for long periods of time when it comes to playing computer games. We would like to take advantage of that fact and encourage them to concentrate on items of educational benefit. This initial game *Dr Math's Missing Laptop* was just an attempt to see if it would be technologically possible to deploy text adventure games over Mxit and if children and teenagers would, in fact, play them.
In that respect, the project has succeeded.

From a gaming point of view, however, the introduction of mathematical formulae into the game was a bit “forced” and did not allow the game to flow smoothly. By this we mean that it is a completely unrealistic scenario that a formula for a pin for a cell phone is written on a white board. However, when looking at commercial games, that scenario is no more unrealistic than soldiers having more than one “life” in a first-person-shooter war game.

THE WAY FORWARD

We plan to develop a few more games and deploy them over Mxit. We would also be very interested in venturing into writing science games. Any science or math teacher who has interesting ideas on this is urged to contact the writer of this paper.

In addition, it would be an interesting research project to see if deploying a MUD (multi-user dungeon) game where participants actually played against or with other participants could succeed.

REFERENCES


[2] Prensky, M, Digital Game-Based Learning, ACM Computers in Entertainment, Vol 1, No 1, October 2003


HOW TO INTERPRET THE CUMULATIVE FREQUENCY DIAGRAM (THE OGIVE)

J FEBRUARY
GARIEP HIGH SCHOOL, PRIESKA, NORTHERN CAPE
FET BAND (GRADE 11)

Introduction

This talk is about the interpretation of data using a cumulative frequency diagram. Assessment Standard 11.4.1 (a) states that the learner must be able to calculate and represent measures of central tendency and dispersion in univariate numerical data by using an ogive (i.e. a cumulative frequency diagram).

I do this exercise in class by first recapping what an ogive or cumulative frequency diagram is, making sure the learners know how to draw an ogive, getting them to use the ogive to find measures such as the median and quartiles, and then giving them an activity which requires them to use the cumulative frequency diagram to interpret the measures mentioned and to demonstrate what conclusions they can thus draw about the original set of data.

ACTIVITY 1

The following cumulative frequency table shows the heights of the learners in a Grade One class:

1) Complete the table.

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<thead>
<tr>
<th>Height, h cm</th>
<th>Frequency</th>
<th>CUMULATIVE FREQUENCY</th>
<th>POINTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>85 &lt; h ≤ 90</td>
<td>0</td>
<td>0</td>
<td>(90 ; 0)</td>
</tr>
<tr>
<td>90 &lt; h ≤ 95</td>
<td>5</td>
<td>5</td>
<td>(95 ; 5)</td>
</tr>
<tr>
<td>95 &lt; h ≤ 100</td>
<td>9</td>
<td>14</td>
<td>(100 ; 14)</td>
</tr>
<tr>
<td>100 &lt; h ≤ 105</td>
<td>17</td>
<td>31</td>
<td></td>
</tr>
<tr>
<td>105 &lt; h ≤ 110</td>
<td>28</td>
<td>59</td>
<td></td>
</tr>
<tr>
<td>110 &lt; h ≤ 115</td>
<td>21</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>115 &lt; h ≤ 120</td>
<td>10</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>Total = 90</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2) Use the information given in the table to draw an ogive (cumulative frequency graph or diagram).

**HEIGTHS OF LEARNERS IN GRADE 1**

![Ogive Graph](image)

**NOTE:**

i. The ogive or cumulative frequency graph starts at the point where the cumulative frequency is zero.

ii. For a cumulative frequency curve of continuous data,
   - the **first element** of the ordered pair is the **upper limit of the interval**.
   - the **second element** is the value of the **cumulative frequency**.

iii. When you join the points on the graph you are assuming that the items are evenly spread throughout the groups. However, data items are not necessarily evenly spread on a graph so any reading from a frequency curve is an **estimate**, not an exact reading.

iv. The ogive or cumulative frequency curve allows you to read off the numbers of learners who are less than a certain height. If you wanted to know how many learners were 102 cm or were shorter than 102 cm, you can read up from 102 cm on the horizontal axis and find that there are about 20 learners who are shorter than or equal to 102 cm.

v. Always draw lines on the graph to show where you took the readings

3) Use the ogive to find the approximate values of the following:

   a) Median ..........................
b) Lower quartile (Q₁) .................................................................
c) Upper quartile (Q₃) .................................................................

**ACTIVITY 2**
The following ogive or cumulative frequency diagram shows the test marks out of 100 obtained by learners in a test.

**MARKS OUT OF 100 OBTAINED BY LEARNERS ON A TEST**

1) Use the ogive to find approximate values of
   a) The median .................................................................
   b) The upper quartile (Q₁) ....................................................
   c) The lower quartile (Q₃) ....................................................

2) Complete the following sentences:
   a) Half of the learners achieved a mark lower than ...............  
   b) A quarter of the learners achieved a mark lower than ...........
   c) 75% of the learners achieved a mark lower than ...............  

3) Using the cumulative frequency diagram, what other facts can you conclude about your original data set? .................................................................
Conclusion

Since these terms were familiar to most of the learners, they tackled this activity with enthusiasm and, although a few came unstuck, most of them could complete the activity with very little assistance from me.

Interpretation of any graph still remains a difficult activity for most learners but, once mastered, it helps them to understand other data handling topics such as the median more clearly.

References

Material developed by Meg Dickson and Jackie Scheiber for the maths4stats programme

INTRODUCTION:
My reason for choosing this topic is that I find it very interesting and the learners enjoy constructing box-and-whisker plots.
Box-and-whisker plots are used to identify trends and to summarise information. The quartiles divide the distribution of data into four equal parts. The ‘box’ represents the middle 50% of the data. My presentation involves the calculation of the five-number summary and depicting this as a box-and-whisker plot (box-plot).

CURRICULUM REFERENCE:
11.4.1 (a): Calculate and represent measures of central tendency and dispersion in univariate numerical data by determining the five number summary (maximum, minimum and quartiles) and drawing a box and whisker diagram.

CONTENT:
Before I teach box-and-whisker plots (box-plots), I ensure that my learners know how to calculate the median of a set of data. I prefer to collect data from my learners, e.g. the number of letters in their names, their age, height etc.

I teach my learners to use the following steps to create a basic box-plot.
1) List the data values in order from smallest to largest.
2) Find the five-number summary: minimum, Q₁, median, Q₃, and maximum.
3) Locate the values for Q₁, median and Q₃ on the scale. The quartiles determine the ends of the box and a vertical line is drawn inside the box to mark the value of the median.

4) Draw lines (called whiskers) from the midpoints of the end of the box out to the minimum and maximum values.
**EXAMPLE:** The table below represents data that I have collected from my learners. I used this data to teach the drawing of the box-and-whisker plot. It is good to use their personal data, they love it!

<table>
<thead>
<tr>
<th>Full Name</th>
<th>Letter spaces</th>
<th>Full Name</th>
<th>Letter spaces</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Ashir Singh</td>
<td>11</td>
<td>11. Lungile Shabalala</td>
<td>17</td>
</tr>
<tr>
<td>5. Ntombikazi Buthelezi</td>
<td>20</td>
<td>15. Soobramoney Naidoo</td>
<td>18</td>
</tr>
<tr>
<td>7. Jenisha Ronnie</td>
<td>12</td>
<td>17. Sipho Zwane</td>
<td>11</td>
</tr>
<tr>
<td>8. Tanisha Govindsamy</td>
<td>18</td>
<td>18. Nishaira Gangaparsad</td>
<td>20</td>
</tr>
</tbody>
</table>

1) Arrange the data in numerical order
   11, 11, 12, 12, 12, 14, 14, 15, 15, 15, 17, 17, 17, 17, 17, 18, 18, 19, 20, 20

2) Write down the minimum and maximum values:
   minimum = 11  
   maximum = 20

Since there is an even number of values, the median is the average of the two middle values of the data.

\[
\text{Median (Q}_2\text{)} = \frac{15+17}{2} = \frac{32}{2} = 16
\]

The middle values are now included in the quartile calculations.

To divide the data into quarters, you have to find the medians of the two halves of the data. (i.e. quartiles).

The first half has 10 values: 11, 11, 12, 12, 12, 14, 14, 15, 15, 15.

The lower quartile is the average of the middle two values.

\[
\text{The lower quartile, Q}_1 = \frac{12+14}{2} = \frac{26}{2} = 13
\]

The second half also has 10 values: 17, 17, 17, 17, 17, 18, 18, 19, 20, 20.
The upper quartile is the average of the middle two values.

The upper quartile, \( Q_3 = \frac{17 + 18}{2} = \frac{35}{2} = 17.5 \)

3) Draw the box-plot.
   - An appropriate scale must be chosen.
   - Locate the three quartiles on the scale

   ![Box-plot scale with quartiles marked]

   10 11 12 13 14 15 16 17 18 19 20

   - Mark off the minimum and maximum values. The ‘whiskers’ are drawn from the midpoints of the box to the endpoints. The ‘box’ part goes from \( Q_1 \) to \( Q_3 \).

   ![Completed box-plot with whiskers drawn]

Notice:

- 50% of the data lies between 13 and 17.5 i.e. within the box.
- 25% of the data is less than \( Q_1 \) and lies along the lower whisker.
- 25% of the data is more than \( Q_3 \) and lies along the upper whisker.

EXAMPLE: Reading box-and-whisker plots

When reading box plots you have to remember the following:

![Box-plot diagram with labels]

Minimum \( Q_1 \) \( Q_2 \) \( Q_3 \) maximum

For each of the box-plots, report the corresponding five-number summary.

1)
CONCLUSION:

As you can see, you only need the five values (the minimum, Q₁, median Q₃ and the maximum) in order to draw your box-and-whisker plot. The five-number summary and corresponding box-plot provide a nice summary of a set of data. The median provides a measure of central tendency. The length of the box, the IQR (interquartile range), provides a measure of the spread (dispersion) and the distance of Q₁ and Q₃ from the median, provides an indication of the skewness. However, when examining box-plots, you should be aware that they can hide gaps and multiple peaks (i.e. box-plots don’t fully show the shape of the distribution)

I get the learners work in groups and gather the data themselves. Learners find it is easier to work with data that are familiar to them. I provide learners with squared paper and they use colour pens to draw the box-plots. The advantage is that learners have fun while learning but the disadvantage is that it is time consuming.

REFERENCES:

1) Interactive Statistics(Third edition) by Martha Aliaga and Brenda Gunderson.
2) http://www.purplemath.com/modules/boxwhisk.htm
SCATTER PLOTS AND LINES OF BEST FIT

Raymond Jonklass
George High School
FET Band

Introduction:
Assessment Standard 11.4.1 (b) states that the learner must be able to represent bivariate numerical data as a scatter plot and suggest intuitively whether a linear, quadratic or exponential function would best fit the data.

I find that though this is an advanced topic, learners do not generally have difficulty in grasping these concepts as it is visual and thus easier for them to grasp.

Basics: There are many situations in our world where some kind of relationship between two variables. We often collect information on items where each item has two readings, one for each variable (e.g. for each person we might have their mass and their height).

This kind of data, where each data piece has two readings, is referred to as bivariate data. If one variable is named \( x \) and the other as \( y \), then one is able to plot the data point \((x; y)\) on a Cartesian plane and can then investigate any possible relationship between the two variables. The resulting set of points is called a scatter plot.

For my earlier example thus, each person would be represented by a dot on an \( xy\)-plane where the position would represent the mass \( x \) and height \( y \) of that person.

Other Examples of Bivariate Data:
Learners in a science class will be familiar with the following bivariate data:
- Boyle's law – \((p; V)\) \(\rightarrow\) a hyperbola
- Exponential growth or decay \(\rightarrow\) exponential function
- Displacement(s) against time \((t)\) \(\rightarrow\) a parabola

Once they are familiar with scatter plots they are ready to progress to more advanced topics such as finding lines of best fit and interpreting their findings. This is best done as an activity as described below:
**ACTIVITY**

In an experiment to measure the height that a tennis ball bounces after being dropped from different heights, the following measurements were recorded:

<table>
<thead>
<tr>
<th>Height from which dropped (cm): x</th>
<th>50</th>
<th>70</th>
<th>100</th>
<th>125</th>
<th>150</th>
<th>180</th>
<th>190</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of bounce (cm): y</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>58</td>
<td>68</td>
<td>90</td>
<td>95</td>
</tr>
</tbody>
</table>

Make use of the data in the table to:

1) Draw a scatter plot of the data with the height from which the ball was dropped on the horizontal axis. Label your axes.

2) Write a brief comment about the relationship between the two variables.

3) Draw a line of best fit. (This is done by trying to get the straight line as close to as many of the points as possible.)

4) Determine the equation of your line of best fit i.e. find the values of \( m \) and \( c \) in the equation \( y = mx + c \) (which is equivalent to \( y = a + bx \) in statistics).

5) Use your line of best fit to estimate the bounce if the same ball is dropped from a height of 1,6 m.

6) Use your line of best of best fit to estimate from what height the ball was dropped if the bounce was 0,5m.
Discussion: From this exercise

- I discovered that there were learners who knew the "theory" of straight lines, but who could not find the gradient of their own lines (which should have been covered when dealing with LO 2).
- Learners could compare their lines of best fit to see how their different equations influenced their predictions. It led to discussions about extrapolations (the process of constructing new data points outside a discrete set of known data points) and interpolations (the method of constructing new data points within the range of discrete known data points).
- In the end learners were able to see that a line of best is an estimate and I told them that one could use a scientific calculator to determine the equation of the line of best fit.
- I could ask learners to come up with examples of bivariate data from real life to explore the relationship (correlation) between the quantities during the next lesson.
Conclusion:
I hope that learners would realise that statistics can be integrated with other learning outcomes and other learning areas.

References:
- Maths4Stats – 2006
- National Curriculum Statement.
- Wikipedia
THE BIASED INTERPRETATION OF DATA USING MEASURES OF CENTRAL TENDENCY

Mamoshibudi Mabale
Capricorn District: Limpopo

The presentation is one that I have used when I workshop FET Mathematics and Mathematics Literacy educators on the concepts of measures of central tendency. The presentation does not focus so much on how to calculate the measures of central tendency but on how people interpret a calculated measure of central tendency. It also addresses bias when presenting or interpreting data by using Circuit results.

ASSESSMENT STANDARDS

The following Assessment Standards for Grades 10 – 12 mathematics and mathematical literacy are covered by this presentation:

MATHEMATICS:

10.4.1 (a): Collect, organise and interpret univariate numerical data in order to determine measures of central tendency (mean, median and mode) of grouped and ungrouped data and know which is the most appropriate under given conditions.

10.4.3 (a), 11.4.3 (a) and 12.4.3 (a): Identify potential sources of bias, errors in measurement, and potential uses and misuses of statistics and chars and their effects.

10.4.3 (b), 11.4.3 (b) and 12.4.3 (b): Effectively communicate conclusions and predictions that can be made from the analysis of data

MATHEMATICAL LITERACY:

10.4.3; 11.4.3; 12.4.3: Understand that data can be summarised and compared in different ways by calculating and using measures of central tendency for more than one set of data, inclusive of mean, median and mode

10.4.4; 11.4.4; and 12.4.4: Critically interpret data in order to draw conclusions on problems investigated to predict trends and to critique other interpretations

INTRODUCTION

While the purpose of the first part of the workshop was to help teachers to calculate measures of central tendencies, the rest of the workshop was used to help teachers to interpret the calculated values.

Exam results were presented to the teachers. What was important was that they should use the same data to calculate the median, mean and mode so that they could see how the same data can be interpreted differently. I also wanted to show them how their interpretations of the results and their conclusions could lead to misconceptions depending on which measure of central tendency was used.
ACTIVITY

Below is a summary of a circuit’s 2008 final results. (Different schools’ names are represented by different letters of the alphabet.)

In groups answer the following and choose one person to report on your solutions.

<table>
<thead>
<tr>
<th>SCHOOL</th>
<th>% OF GR 12’S WHO PASSED IN 2008</th>
<th>NUMBER OF LEARNERS AT SCHOOL</th>
<th>NUMBER OF LEARNERS WHO PASSED</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCHOOL A</td>
<td>63%</td>
<td>105</td>
<td>66</td>
</tr>
<tr>
<td>SCHOOL B</td>
<td>25%</td>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td>SCHOOL C</td>
<td>57%</td>
<td>384</td>
<td>219</td>
</tr>
<tr>
<td>SCHOOL D</td>
<td>31%</td>
<td>488</td>
<td>150</td>
</tr>
<tr>
<td>SCHOOL E</td>
<td>63%</td>
<td>27</td>
<td>17</td>
</tr>
<tr>
<td>SCHOOL F</td>
<td>36%</td>
<td>11</td>
<td>4</td>
</tr>
<tr>
<td>SCHOOL G</td>
<td>78%</td>
<td>36</td>
<td>28</td>
</tr>
<tr>
<td>SCHOOL H</td>
<td>60%</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>PERFORMANCE OF WHOLE CIRCUIT</td>
<td>46%</td>
<td>1 068</td>
<td>490</td>
</tr>
</tbody>
</table>

TABLE 1: Circuit Results- 2008 Grade 12

Calculate the mean, median and mode of this data..................

Discuss the impression that each one presents about the circuit’s performance. .............................................................

Why is there a difference between the mean and the percentage pass of the circuit? ..........................................................

If you were the circuit manager, which of the marks would you prefer for your circuit and why? ..........................................

Does the mode give a true reflection of the circuit’s performance?
Compare the effects of using the different measures of central tendency on the results of the circuit. …………………………………

Do you think that School E performed better than School C?

Which school would deserve a circuit award and why? ……………

Which type of average do you think a circuit motivator would use?

The DSM looks at the averages and is angry about the district’s general performance. Which average do you think he/she was using?
…………………………………………………………………………

You are a monitoring team leader and the district monitoring plan is to visit underperforming schools first. Which schools should be visited first by your monitoring team and why?
…………………………………………………………………………

Using the mean, mode and median, the circuit did not under-perform. Explain what the 46% is then. …………………

**DISCOVERIES FROM THE WORKSHOP**

It was interesting to see how educators would try to defend their interpretations by putting forward other factors which were not presented in the table. As a result we were able to discuss how a biased interpretation can be created by a person trying to do more with the data other than just presenting it.

Teachers argued within their groups as well as when another group was presenting their solution as they thought that the others were biased.

**HOW THE ACTIVITY WAS USED TO ENHANCE LEARNING**

The teachers were able to see that different values can be obtained for each different measure of central tendency or average, and could see how the different numbers could be used to interpret the data in different ways.

They were also able to see how it was possible to present a biased interpretation of data.

**THEM WERE ABLE TO REALISE THAT NEGATIVE VIEWS AND POSITIVE VIEWS ABOUT THE DATA INFLUENCES THE CHOICE OF THE TYPE OF MEASURE OF CENTRAL TENDENCY.**
REFERENCES

Scheiber, J. 2003. Data handling: Short course for teachers, GET and FET phases. UNESCO and IASE.

Department Of Education. 2007. Assessment Guideline In Mathematics. South Africa

Department Of Education. 2007: Assessment Guideline In Mathematical Literacy. South Africa
HOW I TEACH: FINDING EQUATION OF A REGRESSION LINE USING AVAILABLE TECHNOLOGY

Tlou Robert Mabotja

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Curriculum Advisor “Mathematics”
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Target Audience: FET Band Educators
Duration: 1 hour
Minimum number of participants : 30

INTRODUCTION

Educators are encouraged to be pro-active in developing learner’s mathematical knowledge. They are encouraged also to apply different teaching techniques and methods as long as they can be able to achieve the required aims and objectives as outlined by the principle of NCS. If different kinds of techniques are applied, giving the same outcomes, then they are considered relevant in the implementation of new curriculum.

INTEGRATION OF LEARNING OUTCOMES

Learning Outcome 2:
The learner is able to investigate, analyze, describe and represent a wide range of functions and solve related problems.
Assessment Standard: (10.2.2):
Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures and hence to generate the effects of the parameters on the graphs of the functions.

Learning Outcome 4:
The learner is able to collect, organize, analyze and interpret data to establish statistical and probability models to solve related problems.
Assessment Standard: (10.4.1. b):
Represent data effectively, choosing appropriately from line and broken line graphs.
**MOTIVATION FOR RUNNING WORKSHOP**

This workshop will definitely address the problems our educators and learners are encountering in the teaching and learning environment concerning the functions and how to find equation of graphs using a calculator and a computer. Learners will see the importance of technology and how it will develop their mathematical knowledge and thinking.

**PRESENTATION**

To find equation of a line, one can manually apply mathematical knowledge by first find the gradient there-after substitute the coordinates of a point with the \( m \) - value into the formula or **or** Use a scientific calculator (Casio: Fx – 82ES) **or** Use a computer (Microsoft Excel)

**ACTIVITY**

Find equation of a straight line using above mentioned techniques

First Technique: Mathematical

<table>
<thead>
<tr>
<th>WORKOUT</th>
<th>STEPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-2; 0)) and ((0; 4))</td>
<td>Write the coordinates</td>
</tr>
<tr>
<td>( m = \frac{y_2 - y_1}{x_2 - x_1} )</td>
<td>Find ( m ) (the gradient)</td>
</tr>
<tr>
<td>( m = \frac{4 - 0}{0 - (-2)} )</td>
<td></td>
</tr>
<tr>
<td>( m = 2 )</td>
<td></td>
</tr>
<tr>
<td>( y - y_1 = m(x - x_1) )</td>
<td>Equation of a line</td>
</tr>
<tr>
<td>( y - 4 = 2(x - 0) )</td>
<td>Substitution</td>
</tr>
<tr>
<td>( y = 2x + 4 )</td>
<td></td>
</tr>
</tbody>
</table>
Second Technique: Scientific Calculator

The following key sequences are used with the Casio: Fx-82ES when working out equation of a line if points are given.

<table>
<thead>
<tr>
<th>WORKOUT</th>
<th>STEPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>First get the calculator in the correct mode</td>
<td>[Mode] [ 2: STAT] [2: A + B x]</td>
</tr>
<tr>
<td>Enter the data</td>
<td>for x: 1 [=] - 1 [=]</td>
</tr>
<tr>
<td></td>
<td>[Replay]</td>
</tr>
<tr>
<td></td>
<td>for y: 6 [=] 2 [=]</td>
</tr>
<tr>
<td>To find A</td>
<td>Press: [AC][Shift][1][7;Reg][1:A][=]</td>
</tr>
<tr>
<td>To find B</td>
<td>Press: [AC][Shift][1][7;Reg][2:B][=]</td>
</tr>
</tbody>
</table>

Third Technique: Computer (Microsoft Excel)

<table>
<thead>
<tr>
<th>WORKOUT</th>
<th>STEPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opening Excel window</td>
<td>[start][All programmes]</td>
</tr>
<tr>
<td></td>
<td>[Microsoft office][office Excel]</td>
</tr>
<tr>
<td>Enter data</td>
<td>A1: x</td>
</tr>
<tr>
<td></td>
<td>B1: y</td>
</tr>
<tr>
<td></td>
<td>A2: 1</td>
</tr>
<tr>
<td></td>
<td>B2: 6</td>
</tr>
<tr>
<td></td>
<td>A3: -1</td>
</tr>
<tr>
<td></td>
<td>B3: 2</td>
</tr>
<tr>
<td>Highlight the whole data in Excel window</td>
<td>Click left side of the mouse and move it to the whole data (Data will be colored)</td>
</tr>
<tr>
<td>Follow the steps</td>
<td>• Click: chard wizard</td>
</tr>
<tr>
<td></td>
<td>• Click: x y (scatter)</td>
</tr>
<tr>
<td></td>
<td>• Click: next</td>
</tr>
<tr>
<td></td>
<td>• Click: column</td>
</tr>
<tr>
<td></td>
<td>• Click: finish</td>
</tr>
<tr>
<td>Next steps</td>
<td>• Click: on a dot (right click)</td>
</tr>
<tr>
<td></td>
<td>• Click: Add Trent line</td>
</tr>
<tr>
<td></td>
<td>• Click: linear</td>
</tr>
<tr>
<td></td>
<td>• Click: option</td>
</tr>
<tr>
<td></td>
<td>• Click: display equation on chart</td>
</tr>
<tr>
<td></td>
<td>• Click: Ok</td>
</tr>
</tbody>
</table>
CONCLUSION

These are not the only techniques to find equation of regression lines, educators are encouraged to maneuver different avenues like other software’s; e.g. sketchpad, Autograph etc in order to broaden their skills and that of their learners.

REFERENCE


Using Microsoft Excel: Activity “Handout” The Radmaste Centre, University of the Witwatersrand, Johannesburg, South Africa
TEACHING PROBABILITY THROUGH PLAY

MAKOPO K M

KWAZAMOKUHLE SECONDARY
Senior Phase, FET Maths and FET Maths Literacy

Introduction:
LO5 AS 6 in the GET curriculum states: *Considers situations with equally probable outcomes and determines probabilities for compound events using two-way tables and tree diagrams*

AS 10.4.2 in the FET curriculum states: *Solves probability problems correctly identifying the sample space of a random experiment.*

AS 12.4.5 in the Maths Literacy curriculum states: *Critically engage with the use of probability values in making predications in the context of games and real-life situations.*

I use this activity to get learners to understand how to find the sample space of a statistical experiment and then go onto finding probabilities of certain activities.

Some Basics:
A statistics experiment generally has more than one possible outcome. For example, if a die is thrown once, the possible outcomes of the trial are any of the scores 1, 2, 3, 4, 5 or 6 and the full set is referred to as the sample space of the experiment of throwing a die.

We may find the probability of any collection of possible outcomes (also referred to as an event) by using the classical definition of probability:

- The probability of P of an event E is a measure of how likely the outcome of that event is, and is calculated as:
  \[ P(E) = \frac{n(E)}{n(s)} , \]
  where \( n(E) \) = number of possible outcomes in event E.
  and \( n(S) \) = number of possible outcomes in the sample space S.

- \( P(E) = 0 \) if it is impossible for event E to occur, because \( n(E) \) in \( \frac{n(E)}{n(s)} \) is equal to zero if event E contains none of the possible outcomes in the sample space S.
• \( P(E) = 1 \) if event E is certain to occur because \( \frac{n(E)}{n(S)} \) is equal to \( n(S) \) if event E contains all the possible outcomes in sample space S.

**Note that the classical definition of probability may only be applied if all outcomes of the statistical experiment are EQUALLY LIKELY, i.e. in our example the die has the same chance of falling on a 1 as it has of falling on a 2 or a 3…etc!**

Applying the classical definition of probability is best explained by going through the following activity that I do with my class:

**Activity**

I brought blue and red marbles to my class. I then divided my class into groups and gave each group two blue marbles and one red marble. These marbles are placed in a cup, and from the cup then, a marble was selected at random but not returned to the rest before a second marble was selected.

Learners were then asked what the probability is of selecting

1. two blue marbles
2. a red then a blue marble
3. a blue then a red marble

They know how to use tree diagrams from earlier lessons, so they are given the hint to answer the questions by first drawing a tree diagram.

**Representing the probabilities on the tree diagram**

\[
\begin{align*}
P(E) &= \frac{n(E)}{n(S)} = \frac{2}{3} \\
P(E) &= \frac{n(E)}{n(S)} = \frac{1}{3}
\end{align*}
\]
A group is then selected to demonstrate to the class how they answered the questions and the rest of the class is given the opportunity of responding.

**Conclusion**

This works very well in class because learners can visualize and conceptualize mathematics. Mathematics is not viewed as just a set of rules to be memorized and learners actually enjoy doing something practical like playing with marbles in the mathematics class! It makes them think of mathematics as being more practical and not just theoretical.

**References**

1. Oxford Mathematics Plus Grade 11 Learners Book ,
2. NCS Teacher Training Manual 2006
DETERMINING PROBABILITIES OF COMPOUND EVENTS

Ntebaleng Hendrina Mamabolo
DoE Subject Advisor: District Tshwane South (Gauteng)
Senior Phase

Introduction

I chose this topic since being able to list the outcomes of a random experiment, define a compound event and determine the probability of the compound event forms the basis of probability. I will demonstrate the basic principles using a two-way table and a tree diagram.

Content

From the NCS, LO5: AS 9.5.10 (a) states that a learner has to be able to consider a situation with equally probable outcomes, and:

a) determine probabilities for compound events using two-way tables and tree diagrams
b) determine the probabilities for outcomes of events and predict their relative frequency in simple experiments
c) discuss the differences between the probability of outcomes and their relative frequency

I start by defining the basics – usually revision of an earlier data lesson.

- A two-way table is a table that records two events happening at the same time, i.e. is the grid with all possible outcomes in the rows and columns.
- A contingency table is a two-way table which has the recorded frequencies of all the compound events inserted.

This is best explained by the next activities.

Activity 1

Toss a coin ten times and record the outcomes in the table on the next page, then count the number of heads and tails and complete the question below the table.
The relative frequency for heads = .......... 

The relative frequency for tails = .......... 

**Activity 2**
Toss a coin and roll a die ten times each. Record the outcomes of each trial in the following table:
Calculate the relative frequency of:

1) Obtaining a Head on the coin and an even number on the die ……
2) Obtaining a Tail on the coin and an odd number on the die ……..
3) Obtaining a Head on the coin and a 6 on the die …………………
4) Obtaining a 6 on the die …………………………………………..

**A TWO-WAY TABLE**

We can draw a two-way table to find all the possible outcomes when rolling a die and tossing a coin. The following table shows all the possibilities clearly:

<table>
<thead>
<tr>
<th>COIN</th>
<th>HEADS</th>
<th>TAILS</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1 H</td>
<td>1 T</td>
</tr>
<tr>
<td>2</td>
<td>2 H</td>
<td>2 T</td>
</tr>
<tr>
<td>3</td>
<td>3 H</td>
<td>3 T</td>
</tr>
<tr>
<td>4</td>
<td>4 H</td>
<td>4 T</td>
</tr>
<tr>
<td>5</td>
<td>5 H</td>
<td>5 T</td>
</tr>
<tr>
<td>6</td>
<td>6 H</td>
<td>6 T</td>
</tr>
</tbody>
</table>

From the two-way table, we can list the possible outcomes:

1 H, 2 H, 3 H, 4 H, 5 H, 6 H, 1 T, 2 T, 3 T, 4 T, 5 T, 6 T

Use the two-way table to calculate the probability of:

1) Obtaining a Head on the coin and an even number on the die ………
2) Obtaining a Tail on the coin and an odd number on the die ………
3) Obtaining a Head on the coin and a 6 on the die …………………
4) Obtaining a 6 on the die …………………………………………..

Another way to find the set of all the possible outcomes is by drawing a **TREE DIAGRAM**.
**Activity 3:**

Use the tree-diagram to list all the possible outcomes from throwing a die and tossing a coin.

![Tree Diagram](image)

Use the tree diagram to calculate the probability of:

1) Obtaining a Head on the coin and an even number on the die ……
2) Obtaining a Tail on the coin and an odd number on the die ………
3) Obtaining a Head on the coin and a 6 on the die …………………
4) Obtaining a 6 on the die …………………………………………

**Conclusion**

We can use either a two-way table or a tree diagram to list outcomes and to determining relative frequency or probability of a compound event occurring.

**References**

1. Prof North D (2006), Probability Component of Data Handling in the Senior Phase, University of KwaZulu Natal.
3. Laridon et al (2005), Mathematics for the Classroom Grade 9, Heinemann
INTRODUCTION

This presentation is based on Grade 11 Learning Outcome 4, Assessment 11.4.1(b) – Represent bivariate numerical data as a scatter plot and suggest intuitively whether a linear, quadratic or exponential function would best fit the data. (problems should include issues related to health, social, economic, cultural, political and environmental issues).

In this presentation I am going to focus on the lines of best fit. The reason I chose this topic is to highlight the fact that the interpretation of scatter plots can help us to make predictions about events happening in our daily lives, and to indicate the difference between univariate and bivariate data. I start by first introducing the terminology involved in this section. Then I proceed to what is a scatter plot, examples of data that can be plotted on a scatter plot, lines of best fit and describe the trends in a scatter plot.

CONTENT

We first need to establish the basic terminology:

Univariate data: Data concerned with a single quantity. It is represented by bar graphs, pie charts, histograms, frequency polygons etc.

Bivariate data: Data that consists of two numerical variables. It may be represented on a scatter plot.

Line of best fit: A line on a scatter plot which can be drawn to indicate the trend between the two numeric variables.

Correlation: It is the measure of the strength of the linear relationship between the two numeric variables.

Correlation coefficient: It is a number that expresses the degree of correlation of a bivariate data. It is a number between – 1 and 1.
WHAT IS A SCATTER PLOT?

- It is a graph in which bivariate data is plotted as points on a coordinate grid. A plot represents two readings, one for each variable.
- It is a useful summary of a set of bivariate data (two variables), drawn before working out a linear correlation coefficient or fitting a regression line.
- It gives a good visual picture of the linear relationship between two variables, and aids in the interpretation of the correlation coefficient or regression model.
- It is often employed to identify the potential association between two variables, where one may be considered to be an explanatory variable (e.g. years of education) and another may be considered a response variable (e.g. annual income)
- **Examples of data that can be plotted on a scatter plot:**
  1. Shoe size and length of a person’s forearm.
  2. Height of a mother and height of her daughter.
  3. Size of house and monetary value of house.

LINES OF BEST FIT

- From the definition we note that the line of best fit can help us to predict future values of a particular set of bivariate data.
- Now let us consider the following scatter plot of information obtained by a company, which recorded the years of education of its employees as compared to their annual income.
• It is clear from the above graph that the points tend to form a pattern which resembles a straight line. However the points do not lie on a perfect straight line.

• We can draw a line through some of the points with the aim of getting the straight line as close to as many of the points as possible. A line of best fit for the previous graph is as follows:

![Annual Income vs Years of Education](image)

• A line of best fit does not represent data perfectly but gives you an idea of the trend.

• The equation of the line of best fit that has been drawn above can be obtained as follows:
Select two points on this line, calculate the gradient and hence the equation:

We can choose the points (1; 10 000) and (4; 40 000)

\[(y - y_1) = m (x - x_1)\]

\[(y - 10 000) = 10 000(x - 10 000)\]

Therefore the equation is \(y = 10 000x\)

• We can use the equation of best fit to predict the annual income of employees with 10 years education:
For \(x = 10\); \(y = 10 000(10) = R100 000\) (estimation)

**DESCRIBING TRENDS IN A SCATTER PLOT**

• **Positive trend:** If a line of best fit has a positive gradient, we say that the data has a positive trend. As the variable on the x-axis increases, the variable on the y-axis also increases, e.g. from the graph above where higher incomes correspond to higher education levels and lower incomes correspond to a fewer years of education.
• **Negative trend:** If a line of best fit has a negative gradient, we say that the data has a negative trend. As the variable on the x-axis increases the variable on the y-axis decreases.

• **Correlation:** If the points on the scatter plot are close to the line of best fit we have a strong correlation between the variables and if they are not close the line of best fit we have a weak correlation between the variables. We say that there is no correlation when the scatter plot has a random pattern.

ACTIVITY

The table below represents the number of people infected with TB in a certain region for the year 2002 to 2007:

<table>
<thead>
<tr>
<th>Year</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of people infected with TB.</td>
<td>118</td>
<td>123</td>
<td>131</td>
<td>134</td>
<td>136</td>
<td>138</td>
</tr>
</tbody>
</table>

• Draw a scatter plot to represent this data.
• Determine the equation of the line of best fit and draw it.
• Predict how many people will be infected by the year 2010 if the same trend continued.

REFERENCES:

• Notes by Jackie Scheiber and Meg Dickson RADMASTE Centre, University of the Witwatersrand.
• Exam Success Mathematics: S.Botha et.al(2007 edition)
• Maths 911 notes
• Internet: www.yourteacher.com
This presentation was inspired by the desire to give learners an opportunity to be actively involved in their learning, as well as the belief that learners learn better in a relaxed atmosphere where they interact freely with each other. Appropriate data handling activities can give learners this opportunity, as it will be illustrated in the presentation. Learners will collect, organise and interpret data using practical activities that involve parts of their bodies.

INTRODUCTION

The articulation between mathematics and everyday life is a phenomenon that needs to be investigated and recognized (Ensor & Galant, 2005). Data Handling, as one of the Learning Outcomes in the National Curriculum Statement, has a major role to play in achieving this articulation. Here, learners develop the skills to collect, organise, display, analyse and interpret information, thus enabling them to participate meaningfully in, among other things, social activities. A statistical investigation into human beings’ everyday activities and behaviour would give learners and teachers an opportunity to explain this articulation within a familiar and interesting context…and they will have fun too!

In this presentation, participants will collect information on how they fold their arms, clasp their hands, open bottles and peep through holes. Although the sample would be too small to generalize findings, and chances of possible biases are never ruled out, the following investigations will give learners an opportunity to collect, organise, analyse and interpret data that relates to their own daily life. This presentation addresses Learning Outcome 5, Assessment Standard 5 in Grade 9 from the Revised National Curriculum Statement, which states that “the learner should be able to critically read and interpret data with awareness for sources of error, and manipulation in order to draw conclusions and predictions about characteristics of target groups (e.g. age, gender, race, etc.)”.

ACTIVITIES

The investigations that follow were done for the purpose of trying to answer the question: Is there a relationship between the handedness of a person and the way he or she does most of his or her daily physical activities?
Learners were asked to summarise the data, display it in tables and analyse it by looking at the numbers (frequency) or percentages of observations in each category. Assessment Standard 10.4.2 in the Mathematical Literacy subject statement requires learners to be able to use a variety of methods to summarise and display data in, for example, tables. Assessment Standard 10.4.3 (b) requires learners to be able to effectively communicate conclusions and predictions that can be made from the analysis of data.

INVESTIGATION 1

1) Participants observe each other while they fold arms. Specifically, they need to note if the right or left arm is on top.
2) Next they should clasp their hands and observe whether the right or left thumb is on top.

The observations should be recorded in a table as follows:

<table>
<thead>
<tr>
<th>Participants’ name</th>
<th>Gender</th>
<th>Handedness (left/right/both)</th>
<th>Arm on top (left/right)</th>
<th>Thumb on top (left/right)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Possible questions that could be asked:
- Are the above activities (arm on top and thumb on top) related?
- Do they relate to handedness?
- Do they relate to gender?

INVESTIGATION 2

Small water bottles are supplied to each group and the participants observe and record which hand is used by each of them to open the bottle.

<table>
<thead>
<tr>
<th>Participants’ names</th>
<th>Handedness (left/right/both)</th>
<th>Hand opening the bottle (left/right)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Possible questions that could be asked:
• Does the activity relate to handedness?
• Would it make a difference if the bottle was put on the table or held in the air while it is opened?
• Are there any other interesting questions that relate to this activity?

INVESTIGATION 3

Sheets of paper with a small opening are given to participants to peep through and look at a particular object in the classroom. Observations are made and recorded, to check which eye is used to peep through the paper.

<table>
<thead>
<tr>
<th>Participants’ names</th>
<th>Handedness (left/right/both)</th>
<th>Peeping eye (left/right)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Possible questions that could be asked:
• Does the activity relate to handedness?
• Does the position of the object to be observed matter, if it is straight up in front, to the left or to the right of the observer?
• Are there any other interesting questions that relate to this activity?

DISCUSSION

1. Learners need to be aware of possible biases in the results of investigations that involve human behaviour. Some of the causes of bias involving the above investigations could be:
   • Technological innovations which may have an influence on the natural behaviour of people. For example, the position of gears in motor vehicles; the position of door handles or hinges; the design of taps and so on.
   • Because of the fast life that people are living nowadays, there could be a tendency towards ambidexterity (that is, the ability to use both hands equally well for most activities). For example, talking on the phone while driving; drinking from a cup while busy writing and so on.
   • A particular person’s job description may have an influence on the use of a particular hand. For example, bar tenders and waiters may be able to use both hands equally well.

2. The above investigations could lead to the drawing of contingency tables for the FET learners, where they would consider one variable (for example, handedness) against another (for example hand used to open a bottle).
3. Learners need to think about instances where the results of such investigations as the ones above could be applied. For example, in the police department, sports, film and so on.

CONCLUSION

Learners are more likely to enjoy and gain more statistical insight from the investigations, while they learn more about themselves in the process. By using the above activities, even though they are simple and common, it was hoped that this would give teachers an example of what could be done to demonstrate statistical principles. Teachers should, however, be aware of the possible biases and distractions that could affect the results of the investigations.

REFERENCES


When I was introducing this topic in class, I asked the learners if they had any idea what we mean by measures of central tendency or averages. The word average trigged a positive response from some learners who understood it from the average that is usually calculated after they had written a test in class.

However when they defined it, they only referred to an arithmetic mean. I then used the opportunity to clarify that there are several different types of averages that are used in statistics. The most common ones are the mean, median and the mode. They need to know all these measures of central tendency and also need to know when which measure is more appropriate to use, as mentioned in the following NCS Assessment Standards:

GET Maths – LO5 Grade 9:
Organise numerical data in different ways in order to summarise by determining measures of central tendency

FET Maths – LO4
10.4.1 (a) Collect, organise and interpret univariate numerical data in order to determine measures of central tendency (mean, median and mode) of grouped and ungrouped data and know which is the most appropriate under given conditions

FET Maths Literacy – LO4
10.4.3; 11.4.3 and 12.4.3 Understand that data can be summarised in different ways by calculating and using appropriate measures of central tendency to make comparisons and draw conclusions, inclusive of mean, median and mode.

I start off by defining the different measures of central tendency and then go through some activities that demonstrate these concepts.

Definitions of Measures of Central tendency:

MODE – value that occurs most often

MEDIAN – middle value

MEAN – equal share averages
A measure of central tendency gives an impression of the data without you having to look at all the data items. It shows what is typical in the set of data, i.e. it is a single value that represents the whole data set or shows where it is located on the real number line.

The choice of measure of central tendency depends on what you want to look at in your data. Their use depends on the aspect of location that you need your data to show.

I gave my learners the following example to help them understand the terms. I ask them to calculate all three averages and to then decide which average is most suitable for each example/activity.

Example:
Thandi’s test results at the end of grade 10 are as follows:
English: 63%  Geography: 63%  Technology: 37%
Maths: 57%  Biology: 31%  History: 25%
IsiZulu: 77%
To calculate the mean you use the formula:

\[
\text{Mean} = \frac{\text{Total or sum of all items}}{\text{Number of items}}
\]

\[
= \frac{63 + 63 + 37 + 57 + 31 + 25 + 77}{7} = \frac{353}{7} = 50.4\%
\]

Arranging the data in order we get: 25; 31; 37; 57; 63; 63; 77

The mode is the score that occurs most often. In this case, the mode = 63%. However the mode is not representative of this set of marks as it is the second highest score.

The median is the middle score of items when the set of data is arranged in order from smallest to biggest. The median is clearly 57%. The median is not affected by extremes. Therefore, in this situation, the median describes the data best.

ACTIVITY 1
The following table gives the number of hours that different light bulbs worked before burning out:

<table>
<thead>
<tr>
<th>Make of light bulbs</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of hours</td>
<td>2</td>
<td>23</td>
<td>11</td>
<td>23</td>
<td>87</td>
<td>23</td>
<td>91</td>
<td>23</td>
<td>23</td>
<td>22</td>
<td>5</td>
<td>112</td>
</tr>
</tbody>
</table>
1) Determine the mean, mode and median
2) Which of these three averages would best describe the data?

**Solution:**

- The arithmetic mean of this data is 37. Examining the table we see that this value does not accurately reflect the data.

Arranging the data in order we get:

2; 5; 11; 22; 23; 23; 23; 23; 87; 91; 112

- Both the mode and the median are 23. So we take these values to reflect the data. The mode and the median represent the data better than the arithmetic mean.

**ACTIVITY 2:**

The following data indicate the number of pupils absent from a particular grade over a period of ten days:

<table>
<thead>
<tr>
<th>Day</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of pupils absent</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>1</td>
<td>5</td>
<td>0</td>
<td>5</td>
<td>28</td>
<td>30</td>
</tr>
</tbody>
</table>

1) Determine the measures of central tendency of the above data
2) Which of these three averages would best describe the data?

**Solution:**

Arranging the data in ascending order of magnitude we have:

0; 0; 0; 1; 2; 4; 5; 5; 28; 30

- The arithmetic mean is 7.5 and the mode is 0. Neither of these values accurately reflects the data.
- The two mid-values are 2 and 4. We find the arithmetic mean of the two mid-values if there is an even number of values. The arithmetic mean of these two numbers is 3 which means that the **median** = 3. Thus the data is best represented by the median.
CONCLUSION:
There are situations when the mean is the best measure of central tendency, other when the median is best and even times when the mode is the ideal measure of central tendency. Learners need to know all these measures so that they are aware when they are being given misleading information, as the wrong choice of measure of central tendency could give a distorted message!

REFERENCES:
As a result of innovations in technology, the prevalence of the Internet, and the increasing availability of computers in the classrooms and homes, an enhanced approach for teaching and learning mathematics using manipulative and computers is emerging. This new approach essentially creates a new class of manipulative, called virtual manipulative, as well as a new capabilities, or toolkits, for computer programs that use visual representations. These new virtual manipulative have all the useful properties of existing computer manipulative while overcoming many of their disadvantages, yet very little is known or written about them. The purpose of this presentation is to establish a working definition of virtual manipulative, highlight examples of virtual manipulative on the Internet, and discuss their current and potential classroom use to improve the instruction of mathematics.

WHAT ARE MATHEMATICAL MANIPULATIVE?

Defining Mathematical Manipulative:

Mathematical manipulative are objects which are designed so that the student can learn some mathematical concept by manipulating it (Wikipedia, 2006).

Students can use objects as counters, play with geometrical shapes to form other shapes, can build models to understand three dimensional figures, etc. In the manipulative of these materials, students are able to learn abstract concepts in concrete, hands-on ways. These materials make even the most difficult mathematical concepts easier to understand for the student (Uttal, Scudder & Deloache, 1997).
Examples of Manipulative:

- Base Ten Blocks
- Counters
- Fraction Kit
- Geo Strips
- Geo Boards
- Geometric Blocks
- Manipulative Kit
- Tangrams
**The Balloon Model**

Adding with the Balloon Model: addition is associated with the action of "putting on." Thus an addition problem with integers has three parts.

1. The first number represents the initial position of the balloon on the number line.
2. The addition operation following that number indicates that something is being "put on" the balloon.
3. The second number determines how many and what type bags are put on the balloon.

**MATH**

<table>
<thead>
<tr>
<th>Initial Position</th>
<th>Final Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

**Is this game fair?**

- This is your spinner.  
- This is my spinner.  

If we spin both spinners, the person whose spinner lands on the higher number wins. Is this game fair?

**Chip Model**

1) \[ 4 + (-3) = 1 \]

2) \[ 4 - (-3) = 7 \]

**MULTIPLICATION**

<table>
<thead>
<tr>
<th>Initial Position</th>
<th>Final Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>12</td>
</tr>
</tbody>
</table>

This multiplication expression translates in the following way: put on three groups with 4 sand bags in each group.

But consider this: \[ -2 \times 6 \] This one translates as follows: take off two groups with 6 gas bags in each group from the balloon.

References:

Virtual Manipulatives

In the every present growth of technology in our world, nothing goes unaffected. This is also true regarding manipulatives, as virtual manipulatives are now available. Virtual manipulatives can be defined as “interactive, web-based representations of a dynamic object that presents opportunities for constructing mathematical knowledge” (Moyer, Bolyard & Spikell, 2002, p. 373). These manipulatives are still concrete, though they are not “physical” (Clements and McMillen, 1996). Though students are not able to physically pick up virtual manipulatives they can still move the objects on the computer screen and interact with them. Teachers can integrate these representations into their classroom because they can “be more manageable, “clean”, flexible, and extensible” (Clements and McMillen, 1996, p. 271). The biggest advantage of virtual manipulatives is their interactive capabilities. These manipulatives allow students to see mean and relationships based on the results of their actions (Moyer, Bolyard & Spikell, 2002).
Virtual manipulatives can be both static and dynamic. Static manipulatives, or those which are simply “virtual”, offer pictorial or other forms of representation, however no interaction is permitted for students (Moyer, Bolyard & Spikell, 2002). Dynamic, or true virtual manipulatives, however, allow the students to move, count, change and work with objects on the screen to build an understanding of the concept presented (Moyer, Bolyard & Spikell, 2002).

Virtual manipulatives can also be defined as being static images or representations, computer-manipulated images or representations, or virtual manipulative websites. Those which are static images show a pictorial representations which may change, but the user is unable to change or manipulate the image (Moyer, Bolyard & Spikell, 2002). Computer-manipulated images allow the image or object to be manipulated in response to a student’s answer or action (Moyer, Bolyard & Spikell, 2002). Finally, a true virtual manipulative offers the physical movement of objects in virtual form, allowing for the greatest amount of manipulation and interaction for the user (Moyer, Bolyard & Spikell, 2002).

According to Clements and McMillen (1996), computer based manipulatives, or virtual manipulatives, offer many advantages. These are:

- Computer manipulatives allow for changing the arrangement or representation.
- Computers store and later retrieve configurations.
- Computers record and replay students’ actions.
- Computers change the very nature of the manipulative; Students can do things that they cannot do with physical manipulatives.
- Computer manipulatives build scaffolding for problem solving.
- Computer manipulatives may also build scaffolding by assisting students in getting started on a solution.
- Computer manipulatives focus attention and increase motivation.

(p. 272-273)

Another great value for virtual manipulatives is their availability and free expense. Often manipulatives can be expensive and many teachers may not be able to afford access to many varieties of manipulatives. By simply searching online teachers can be provided with many manipulatives to use at their choosing (Hodge and Brumbaugh, 2003). Also, these manipulatives can be made available at home, helping both the struggling student and parents who would normally not have access to such
tools. Teachers will be able to send home homework with manipulatives that may be of assistance for students if they know they will have access to the materials at home (Moyer, Bolyard & Spikell, 2002). This may also help older students who look at using physical manipulatives as “playing with blocks”. Older students can view virtual manipulatives much like a computer game, aiding the student while retaining confidence in the learner (Moyer, Bolyard & Spikell, 2002).

Virtual Manipulative Web sites.

- http://nlvm.usu.edu
- http://illuminations.nctm.org
- http://matti.usu.edu/nlvm/nav/
- http://www.learner.org/channel
- http://www.gomath.com

“The problem of not having enough blocks is controlled by the click of a mouse”
References

- Koontz, T. and Mellilo J. (2009): *(Probability), (Basic Geometry levels), and (Integers and Rational numbers).* Kent State University, USA.
- The National Council of Teachers of Mathematics: (2009) *(Teaching and Learning Strategies).* USA.
- Gutschmidt L. (2009): *(Classroom Teaching Techniques)*, Hudson Middle School, USA.

SAMPLE ACTIVITIES: COMPUTING PROBABILITY FOR SIMPLE EVENTS.

**Activity 1: Coin Tossing:**
Each person in the group should toss a coin 20 times. Individually, keep track of the number of heads that occur during each toss. Display your data and your group’s data on the single graph. Explain in your own words what your graph means. What would you predict if this experiment were repeated?

**Activity 2: Spinner:**
Each person in the group spins the given spinners 10 times and keep track of the number of times they land on a number less than 3 for each of the spinners. Graph the data for each person in the group and totals for the individual spinners. Explain in your own words what your graph means. What would you predict if this experiment were to be repeated?

**Activity 3: Chip drawing:**
Given a bag with 5 blue chips and 5 red chips keep track of the number of red chips, draw per every 10 times you draw a chip. (the chips are replaced after each drawing). Repeat this 2 times for each person in the group. Graph the data for each individual and the group totals. Based on this data answer the following questions:
1. What would happen if you add 5 white chips to the bag and repeated the experiment? Explain.
2. What would happen if you had 50 blue chips and 50 red chips in the bag (instead of the 5 red and 5 blue)?
MISLEADING BAR GRAPHS

Tshililo Mukhaninga

Limpopo maths4stats Provincial Coordinator

FET MATHS AND MATHS LITERACY

INTRODUCTION

Graphs are used for many different reasons and can be found all over. We see them in magazines, newspapers and on television. They help us to communicate information visually. Many different types of graphs exist and each one has certain characteristics that make them useful in a different way.

Interpreting statistical graphs is a very important skill in statistics as it gives a quick glance at the main features of the data set. However, they can also be grossly misinterpreted. We are going to look at how bar graphs can be used to mislead people.

The following Assessment Standards focus on misleading graphs:

MATHMATICS LO 4, AS 10.4.3:

a) Identify potential sources of bias, errors in measurement, and potential uses and misuses of statistics and charts and their effects (a critical analysis of misleading graphs and claims made by persons or groups trying to influence the public is implied here).

b) Effectively communicate conclusions and can be made from the analysis of data.

MATHS LITERACY LO 4:

AS 11.4.6: Demonstrate an awareness of how it is possible to use data in different ways to justify opposing conclusions.

AS 12.4.6: Critique statistically-based arguments, describe the use and misuse of statistics in society, and make well-justified recommendations.

PRIOR KNOWLEDGE

Before doing these activities, learners must be made aware that a graph must have a title as well as labels for the y-axis and the x-axis in order to make it complete – as is shown in the following compound bar graph:
WHAT ARE MISLEADING BAR GRAPHS?
If any of the essential elements or rules for drawing graphs are ignored or modified, there is a possibility that there will be misrepresentation of data.

ACTIVITY 1
A number of managers were sent on a management course. After the course all the managers were tested on their skills. The two graphs compare the test scores.

Graphical Representation of Trained vs. Untrained Managers

Use the graphs to answer the following questions:

Teacher: Are the two graphs the same? Give reasons for your answer.
Learner: .................................................................
Teacher: What is the difference between the number of trained and untrained managers in both graphs?

Learner: …………………………………………………………………………………

Teacher: When would it be better to use Graph 1 and when would it be better to use Graph 2?

Learner: …………………………………………………………………………………

Teacher: Is it permissible to draw a graph where the values on the y-axis do not start at zero?

Learner: …………………………………………………………………………………

ACTIVITY 2

Use the graph to answer the following questions:

Teacher: Approximately how many times taller does the July 1995 bar appear to be than the July 1996 bar?

Learner: …………………………………………………………………………………

Teacher: Approximately how many jobs were there in July 1995? …………… b) in July 1996? ……………

Teacher: How many more jobs were there in July 1995 than in July 1997?

Learner: ……………………………………………………………

Teacher: Compare your answers to questions (1) and (3). What do you notice?

Learner: ……………………………………………………………

Teacher: Could the bar graph be misleading? If so, how would you correct the graph?

Learner: …………………………………………………………………………………
CONCLUSION:
It is a well known fact that statistics can be misleading. They are often used to prove a point, and can easily be manipulated in favour of that point. Learners need to be aware of this, and it is the educator’s responsibility to expose their learners to misleading graphs and statistics.

Reference:
http://www.ms.uky.edu/Graphs
Jackie Scheiber, Maths4stats & RADMASTE Centre (Wits)
Classroom Mathematics (Laridon et al.)
MISLEADING AVERAGES
(MEASURES OF CENTRAL TENDENCY)

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FET Maths and Maths Literacy

INTRODUCTION

Working with data is a very important skill in mathematics. We need some knowledge of statistics to understand what we hear or see on the television and radio. We will only be able to understand matters relating to taxes, welfare, education, etc. if we know statistics. As you gain more knowledge of statistics it will be easier to check the validity of claims made by product advertisers and not be taken in by misrepresentation of data.

The following NCS Assessment standards list what learners need to know about different measures of central tendency:

**FET Maths 10.4.1:** Collect, organise and interpret univariate numerical data in order to determine measures of central tendency (mean, median and mode) of grouped and ungrouped data and know which is the most appropriate under given conditions.

**FET Maths 10.4.3 (a):** Identify potential sources of bias, errors in measurement, and potential uses and misuses of statistics and charts and their effects. (A critical analysis of misleading graphs and claims made by persons or groups trying to influence the public is implied here.)

**FET Maths Literacy 10.4.3, 11.4.3 and 12.4.3:** Understand that data can be summarised in different ways by calculating and using appropriate measures of central tendency to make comparisons and draw conclusions inclusive of the mean, median and mode.

**FET Maths Literacy 12.4.6:** Critique statistically-based arguments, describe the use and misuse of statistics in society, and make well-justified recommendations.
MISLEADING AVERAGES

The mean, median and the mode are all called measures of central tendency and are commonly referred to as averages of a data set. They may be very different; and they can also be very misleading and learners need to know when to use which “average”. The next activity illustrates the difference between these different averages.

DEFINITIONS:

- **Mean** – the equal shares average. We calculate it by finding the total of all the values and then dividing by the number of values.
- **Median** – is either the middle value or the average of the two middle numbers. This value divides the numeric data into two equal parts.
- **Mode** – data values which appear most often, i.e. the observation with the highest frequency.

ACTIVITY

Look at the statements below. All three are correct as they come from the same set of data about Funanani’s Furniture Factory.

**DAILY NEWS**
The average salary at Funanani Furniture Factory is R7 500.

I work as a secretary at the factory. I calculated that the average wage is R6

I am the managing director of Funanani. I calculated the average wage at the factory is R10 500 per month.
<table>
<thead>
<tr>
<th>Position</th>
<th>Number</th>
<th>Salary R</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MANAGEMENT</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chairman</td>
<td>1</td>
<td>150 000</td>
</tr>
<tr>
<td>Managing Director</td>
<td>1</td>
<td>120 000</td>
</tr>
<tr>
<td>Directors</td>
<td>4</td>
<td>100 000</td>
</tr>
<tr>
<td>Executives</td>
<td>4</td>
<td>75 000</td>
</tr>
<tr>
<td><strong>FACTORY WORKERS</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Foreman</td>
<td>10</td>
<td>15 000</td>
</tr>
<tr>
<td>Skilled workers</td>
<td>60</td>
<td>10 500</td>
</tr>
<tr>
<td>Semi-skilled workers</td>
<td>80</td>
<td>7 500</td>
</tr>
<tr>
<td>Unskilled workers</td>
<td>130</td>
<td>6 000</td>
</tr>
<tr>
<td><strong>ADMIN</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Secretaries</td>
<td>10</td>
<td>5 000</td>
</tr>
</tbody>
</table>

1) What is the total amount of money that is spent each month on management salaries (Chairman, Managing Director, Directors and Executives)?

2) What is the total amount of money that is spent each month on the rest of the workers?

3) Work out the mean wage, median wage, and the wage that is the mode of all the employees working for Funanani.

4) Study the 3 averages given on the previous page:
   a) Which average did the secretary use? .....................
   b) Which average did the managing director use? .............
   c) Why do you think each one used the average they did?

   d) What does this tell you about the differences between the 3 averages?
e) Do you think any of the averages used were meant to mislead?

CONCLUSION

- It is very common for mathematics teachers to focus on formulae in data handling when the most important thing is actually the interpretation of what the answers tell us.
- Data is only meaningful if the situation which gave rise to the need for data is known to the learner. If they truly understand the data set, they will have more chance of being able to interpret the answer from statistical formulate such as a measure of central tendency.

REFERENCES

Dickson M, and Scheiber J. 2003. Data handling in the Senior Phase and in the Further Education and Training Band, RADMASTE Centre
DRAWING AND INTERPRETING CUMULATIVE FREQUENCY CURVES (OGIVES)

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INTRODUCTION

According to Assessment Standard 11.4.1 (a) of the NCS, learners are expected to calculate and represent measures of central tendency and dispersion in univariate numerical data by drawing ogives. However, there is confusion about how to accurately draw a cumulative frequency curve (Ogive). Some textbooks, study guides and even the National Memorandum for marking Question 10.2 of Paper 2 of November 2008 bear testimony to this confusion.

This presentation will be focusing on two things:
1. Analysis of November 2008 question paper
2. Drawing a cumulative frequency curve,
   thus illuminating the confusion that exists.

INFORMATION ABOUT OGIVE

- An ogive is a specialised, S shaped curve which is used to determine the number of observations that lie below (or above) a particular value.
- The cumulative frequency is found by adding each frequency to the sum of its predecessor.
- The cumulative frequency is always plotted on the vertical axis of a graph and any other variable is plotted on the horizontal axis.
- The Median is the middle item when the set of data is arranged in order from smallest to biggest, i.e. is the value below which 50% of the data lies.
- The Lower quartile is the median of the first half of the data set or, stated differently, is the value below which 25% of the data lies.
- The Upper quartile is the median of the second half of the data set, or stated differently, is the value below which 75% of the data lies.

NOTE:
The median, lower quartile and upper quartile positions are read off the vertical axis of the ogive and the estimate of the median and the quartiles is read on the horizontal axis of the ogive.
ANALYSIS OF QUESTION 10.2 OF THE NOVEMBER 2008 QUESTION PAPER

Learners were given the following histogram and had to answer questions based on it.

QUESTIONS:

1. Complete the cumulative frequency table on the next page for the sales over November and December.

<table>
<thead>
<tr>
<th>Daily Sales</th>
<th>Frequency</th>
<th>Cumulative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
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</tbody>
</table>
2. Draw an ogive for the sales over November and December.
3. Use your ogive to determine the median value for the daily sales.

**DISCUSSION**

1. Which values from the cumulative frequency table did you plot against the cumulative frequency values?

2. Which one of the two methods (mid-point or upper limit of intervals) is 100% accurate and which one is 100% wrong?

3. What about NCS requirements and marking options in the national memorandum?

4. Why do you think that people use the wrong method?

5. Should the curve be grounded (start at zero on the horizontal axis)? Why?

**FINDING INFORMATION FROM THE CURVE**

In most cases learners struggle to draw meaningful information from the Ogive.

Learners need to follow the following steps when determining the median, lower quartile, upper quartile and interquartile range values from the ogive.

**STEP 1:** Determine the median position and indicate this on the vertical axis.

**STEP 2:** Draw a dotted horizontal line (from left to right) towards the curve.

**STEP 3:** Now draw a vertical line towards the horizontal axis.

**STEP 4:** Read off the estimate of the median at this point on the horizontal axis.

The estimates of the quartiles can be done in the same way by following the steps above.
CONCLUSION

The midpoint curve is TRANSLATED HORIZONTALLY by 5 units to the left to give the correct cumulative frequency curve.

The Mid-Point method is 100% wrong and should not be taught. Learners will lose marks for using midpoints when drawing ogives. Possible reasons for learners using the incorrect method are:

1. Many textbooks are drawing the ogive in this way.
2. Educators and learners are used to using mid-points when drawing other statistical curves such as frequency polygons and histograms.

I hope that by doing this presentation helps to clarify the misconception around drawing the ogive and that the confusion is eliminated.

REFERENCES

3. Data Handling workshop material by Scheiber, J and Dickson, M. RADMASTE Centre, University of the Witwatersrand.
SAMPLING
Nina Scheepers
Diamantveld High School, Kimberley
FET Band

INTRODUCTION
According to the NCS, ASS 12.4.1(a), learners are required to demonstrate the ability to draw a suitable sample from a population and understand the importance of sample size in estimating the mean and standard deviation of a population.

I am going to share with you the information I give my learners about sampling, discuss the terminology associated with sampling, talk about different ways of drawing a sample and then show how the sample can be used to estimate certain aspects of a population.

DO YOU KNOW WHAT A SAMPLE IS?
When asked this question, learners often respond as follows: "A little bit", "A part of the whole thing", "A small portion of something larger" etc. I have found, however, that learners are not sure what sampling is used for.

POPULATIONS AND SAMPLES
In statistics, when we talk about a population, we mean the whole group of people or items about which you want to gather information. Sometimes we can’t get information from the whole population because it is too expensive or time consuming, so we draw a sample, or investigate a subset of the whole population. When doing so, we have to be careful to select the sample in a way that will ensure that the sample has all the characteristics of the population. We do this so that the sample can be used to draw conclusions about the population. The larger the sample, the more reliable these conclusions are as there is more chance that the sample will be similar to the population. We very often want to estimate the population mean by using the sample mean.

Example 1: Suppose a toothpaste manufacturer wants to know people's opinions about their toothpaste. The population in this case is all the people in the country who use toothpaste. The manufacturer will not have the time or money to collect and analyse the opinions about his toothpaste from all people in the country or even in one province in the country. Most probably the manufacturer will collect information from only a part of the total population. This smaller portion of the population is
called a sample. A sample is any subset of the total population that is representative of the whole population.

**Example 2:** If you conducted a survey and found that 2 out of every 3 people like chocolate, the reliability of your conclusion would depend on how many people you asked. If you only asked 3 people then your statement would have little meaning. If you asked 3,000 people and 2,000 said they like chocolate, then your conclusion is of more significance as you investigated a larger group.

**Example 3:** Suppose you wanted to find out how many teenagers in South Africa smoked cigarettes. The population you are interested in would be all the teenagers in South Africa. Who would you ask? It would be impossible for you to ask all the teenagers in the whole country, as there are too many of them. You would therefore need to ask a group of teenagers that would represent all teenagers.

**A FAIR SAMPLE**

When you collect data it is important that you choose a realistic sample to work with. A fair sample:

- is a portion of the population;
- must be representative of the population;
- must reflect the characteristics of the population;
- is used to make conclusions about the population.

**BIAS**

If a sample is biased it means that the sample is not representative of the population, i.e. the sample does not have the same characteristics as the population. Some parts or members of the population may be under-represented or some may be over-represented (e.g. in the teenage smoking investigation above, it would be biased to only interview teenagers in private schools!).

Factors that can influence the bias of a sample are:

- **Size:** If the sample is too small it may not represent the population.
- **Voluntary responses:** Usually only those people who are concerned about the problem will bother to respond. You then do not get another viewpoint.
- **Convenience:** Suppose your sample was made up only from friends. Most probably your friends are fairly similar and are therefore not representative of a larger population. Surveying at only certain times or places can also introduce bias.
SAMPLE SIZE

Sample size is an important aspect in ensuring there is minimal bias. It is important that a sample is big enough (for example 10% of population) to be able to make meaningful inferences from it. A larger sample will be more representative than one that is too small. One always takes the largest possible sample that time and finances will allow!

ADVANTAGES OF SAMPLING

- Cost: It is much cheaper to collect information from 2 000 people than from 2 million people.
- Time: Information is often required within a specific time. A random sample requires less work in the field and less organisation of the data.
- Reliability: Because there are fewer units of information to be collected, greater reliability can be achieved as the people collecting the information can be better trained.

CHOOSING A SIMPLE RANDOM SAMPLE:

A random sample is one in which every member of the population has an equal chance of being selected.

You can use chance (probability) to get a random sample:

If you could put all the members of the population into a hat, mix them thoroughly and then draw them one at a time you would get a random sample.

We are familiar with the Lotto programme on TV. All the lotto tickets bought during the week represent the population. It is impossible to give prizes to all owners of these tickets. The numbers one to forty nine are equally likely to be drawn. The lotto machine draws a simple random sample. Only those people whose tickets show the chosen numbers then get prize money.

Let us look at another example. Eighty nine Grade 10 learners take Mathematics in my school. I found that the mean of their March test results was 49%. I took a simple random sample of 5 learners, and found that their mean was 55.4%. I increased the simple random sample to 19 learners and the mean was 54%. When I took 45 learners’ results the mean was 51.1%.

Population: Grade 10 learners (89) with mean test mark = 49%
Sample 1: Grade 10 learners (5) with mean test mark = 55.4%
Sample 2: Grade 10 learners (19) with mean test mark = 54%
Sample 3: Grade 10 learners (45) with mean test mark = 51.1%
This demonstrates that the larger the sample, the more the sample mean tends to the population mean.

Other methods of selecting random samples include:

- Selecting every tenth person off a voting list
- Selecting people randomly on a street
- Choosing names randomly from a telephone book

Any of these methods might, however, introduce bias, e.g. choosing names out of a telephone book immediately excludes people who do not own a telephone.

**CONCLUSION**

Andrew A. Marino said: "To the scientist, however, representative sampling is the only justified procedure for choosing individual objects for use as the basis of generalisation, and is therefore usually the only acceptable basis for ascertaining truth."

**REFERENCE:**

Material written by Meg Dickson and Jackie Scheiber, RADMASTE Centre, University of the Witwatersrand.
I decided to start off by talking about the stem-and-leaf diagram as this is often the first time that learners appreciate how powerful a representation of data can be! I then follow this up with a demonstration of how this display can be used to calculate measures of central tendency.

ASSESSMENT STANDARDS BEING FOCUSED ON:

A S 7.8 and 8.8: Organise (including grouping where appropriate) and record data, using tallies, tables and stem and leaf displays
AS 7.9: Summarise ungrouped numerical data by determining the mean, median and mode as measures of central tendency and distinguish between them.
A S 8.9: Summarise grouped and ungrouped data by determining the mean, median and mode as measures of central tendency, and distinguish between them.

INTRODUCTION

In the 1960s John Tukey, an American mathematician and statistician, devised a new way of organising data. He called this method a stem-and-leaf diagram or stem-and-leaf plot.

A stem and leaf plot is an efficient way of getting a picture of the way in which the data set is distributed, and gives an easy way of seeing trends. We can use it to find the measures of central tendency (the mean, median and mode).

CONSTRUCTING A STEM-AND-LEAF PLOT

The following results were obtained by 15 learners in class when they wrote a maths test (scores out of 100):
32; 56; 45; 78; 77; 59; 65; 54; 54; 39; 45; 44; 52; 47; 50.
Consider the marks 32; 56;

With 32, the stem is 3 and the leaf is 2

With 56, the stem is 5 and the leaf is 6

<table>
<thead>
<tr>
<th>stem</th>
<th>leaves</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

**STEP 1:** Write the lowest and the highest data value.
- The lowest value in the data set = 32, therefore smallest stem = 3
- The highest value in the data set = 78, therefore the largest stem = 7

**STEP 2:** Separate each number into its stem and leaf.
Since these are all two digit numbers, the tens digit is the stem and the units digit is the leaf.

**STEP 3:** Fill in the numbers on the table

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaves</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2 9</td>
</tr>
<tr>
<td>4</td>
<td>4 5 5 7</td>
</tr>
<tr>
<td>5</td>
<td>0 2 4 4 6 9</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>7 8</td>
</tr>
</tbody>
</table>

**KEY:** 3/2 = 32

**NOTE:**
- In a data set made up of two digit items, the leaf is the digit furthest to the right in the number, and the stem is the digit in the number that remains when the leaf is dropped.
- To show a one-digit number (like 5) in a stem-and-leaf plot, use a stem of 0 and leaf of 5.
- To find the median in a stem-and-leaf plot, count off half the total number of leaves.
- To find the mode, find the leaf that occurs most often.
• The stem and leaf does not really help you find the mean. You calculate the mean in the usual way (i.e. add each number together and divide the total by the number of terms).

**ACTIVITY 1:**
The following stem-and-leaf plot was drawn:

<table>
<thead>
<tr>
<th>stem</th>
<th>leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>11</td>
<td>0 2 3</td>
</tr>
<tr>
<td>12</td>
<td>0 5 5</td>
</tr>
<tr>
<td>13</td>
<td>0 2 5</td>
</tr>
</tbody>
</table>

Key 10/5 = 105

Find:

1. the mean of the data ..............................................................
2. the mode of the data ..............................................................
3. the median of the data ............................................................

**ACTIVITY 2:**
Thirty learners were asked in a survey to say how many hours in a week they spent watching TV. Their answers (correct to the nearest hour) are as follows:

12 20 13 15 22 3 6 24 20 15 9 12 5 6 8
30 7 12 14 25 2 6 12 20 20 18 3 18 8 9

1) Draw a stem-and-leaf display to organise this data.
2) Find:
   a) the mode of the data
   b) the median of the data
   c) the mean of the data
3) Which one of the mode, median and mean would be the best single value to represent the data, and why?
SOLUTION

Step 1: Collect the data in a stem and leaf diagram:

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaves</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

KEY: 2/5 = 25

Step 2: Arrange the leaves in ascending order

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaves</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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<td></td>
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<tr>
<td></td>
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</tr>
</tbody>
</table>

KEY: 2/5 = 25

Step 3: Find the mode and the median of the data

Mode = 

Median = 

Mean = 

Step 4: Which measure of central tendency would be the best to use in this situation? Why?

-------------------------------------------------------------------------------------

-------------------------------------------------------------------------------------

-------------------------------------------------------------------------------------

-------------------------------------------------------------------------------------
CONCLUSION
In a stem-and-leaf diagram the data is listed in intervals that depend on the place value of the digits of each data item. I had the impression that learners enjoyed the activities because they were within their grasp.

REFERENCE
Dickson M, and Scheiber J. 2003. Data Handling in the Senior Phase and in the Further Education and Training Band. RADMASTE Centre
INTRODUCTION

In order for us to be able to make sense of data and be able to use it effectively, the data should be ordered in a way that makes logical sense and is easy to process. One way of sorting data so that it is easy to read is by using TALLY TABLES.

Tally Tables are found in Learning Outcome 5: 4.5.5, 5.5.5 and 6.5.5: Organise and record data using tallies and tables

Where does one start?
It is important for the educator to know the following terminology:

• RAW DATA – data that is a jumble of numbers and not arranged in any particular way
• CLASSIFIED DATA – data that is ordered in such a way that it is easier to read and make sense of.
• FREQUENCY – the number of times each particular item occurs in a dataset

I start off by getting the learners to do the following activity:

ACTIVITY NO 1:
• Learners work in pairs. Each pair is given a box of matches.
• Each pair counts the number of matches in each box and, as each learner finishes, this number is written on the blackboard, in no specific order – Raw data.
• The learners are then asked to order the numbers on the blackboard from the smallest to the largest – Classifying data.
• I then ask them to answer the following questions:
  a) How many boxes of matches were there in total?
  b) Were there any of the boxes that had the same number of matches?
  c) Which number occurs most often?

ACTIVITY NO 2:
I then ask the learners to fill out a tally table (which I give them on a handout) using the data from the previous activity
<table>
<thead>
<tr>
<th>List of all possibilities</th>
<th>Tallies</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**NOTE:**
- The first column is the list of all the possibilities.
- The second column shows the tallies *another way of writing the same number more than once is by using a tally (/)*. The learners must make a tally for each data item that they count.
- The third column shows the frequency of each possible data item. In this column add up the tallies for each possibility.
- The total of all the frequencies should be equal to the total number of data items.

The learners are then asked to do the following activity:
In a survey, 50 people were asked to give the number of children in their families. Here are the numbers they gave in their replies:

1 3 2 2 5 4 1 1 2 3
4 1 1 1 2 1 3 2 4 1
1 3 1 2 2 3 3 5 3 2
2 1 4 6 3 5 6 4 1 1
4 3 2 1 2 7 1 3 3 4

1) Complete the tally table to organize these results:

<table>
<thead>
<tr>
<th>Number of Children</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2) Which number of children was the mode (the most common) of the people surveyed?

CONCLUSION:
Learners had a great time discovering how to do a tally table practically

REFERENCES:
Manual – Data Handling in the GET phase – Jackie Scheiber
A PRACTICAL APPROACH TO TEACHING TREE DIAGRAMS

DESIREE TIMMET
Western Cape maths4stats Provincial Coordinator
SENIOR PHASE – GRADE 9

Introduction

This presentation is based on the Grade 9 Assessment Standard: Considers situations with equally probably outcomes and determines probabilities for compound events using two - way tables and tree diagrams

Statistics is one of the most interesting and practical Learning Outcomes in the Mathematics curriculum. Probability is the fundamental cornerstone of statistics and familiarity with tree diagrams is essential for tackling probability arguments, and needs to be well developed in the earlier grades. Whether a Grade 10 learner chooses maths or maths literacy, the foundations of probability need to be in place in order to be built on up to Grade 12. I am thus going to do some activities that will demonstrate how to determine probabilities of compound events using tree diagrams.

What is a tree diagram and why do we use it?

A tree diagram is a graphical method for obtaining the outcomes of a random experiment. A random experiment has various possible outcomes and tree diagrams are thus used in order to get a listing of all possible outcomes.

We use the following formula to find the probability of an event E occurring:

\[ P(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Total number of outcomes}} \]

Note that this classical definition of probability may only be used where all outcomes of the random experiment are equally likely (e.g. rolling a die)

EXAMPLE WITH A DIE

Suppose a die is rolled twice. Let \( O \) denote an odd number was obtained and \( E \) denote an even number was obtained. This can be represented in the following tree diagram:
At the end of the first “branches” the possible outcomes of the first roll is given, which is repeated on the second level of branches. The possible outcomes are read by going to the end of each second level branch and reading the outcomes along the way.

For this example, then

```
possible outcomes
  E ← EE
```

```
  E
  O ← EO
  E ← OE

  O
  O ← OO
```
The set of all possible outcomes for this experiment

\[ = \{EE; EO; OE; OO\} \]

**ACTIVITY (Group work)**

Each group will receive an envelope (suitcase) of clothes consisting of blue jeans, khaki jeans, brown shoes, black shoes, red t-shirt, white t-shirt, and pink t-shirt. They must physically pack out the different outfits for each day, for example:

- Day 1: red t-shirt, blue jeans and black shoes.
- Day 2: white t-shirt, blue jeans and black shoes

1) For how many days can you wear a different outfit?

........................................

2) Draw a tree diagram to illustrate the number of possible outcomes

3) Use the tree diagram to determine the following:
   a) What is the probability of wearing a red t-shirt, blue jeans and brown shoes?
      ........................................
   b) What is the probability of wearing a white t-shirt? .........................

Refer back to the tree diagrams showing all the possible outcomes when rolling a die twice

1) Use the tree diagram to find the probability of obtaining an odd and even number on the two rolls of the die.

   **Solution:**
Let \( A \) denote an odd and even number. \( P(A) = \frac{2}{4} \) using the classical definition of probability as any of \((EO, OE)\) are favourable to the outcomes of getting an even and an odd number while there are 4 possible outcomes.

2) Use the tree diagram to find the probability of obtaining even numbers on the two rolls of the die.

Solution:

\[ \text{Solution:} \]

\[ \text{Conclusion: Using this practical, visual approach, learners are easily able to grasp the concept of using a tree diagram to determine probabilities of compound events.} \]

References

Prof D North, School of Statistics, University of KZN: Probability notes

White t-shirt

Red t-shirt

Brown shoes
Pink t-shirt
Blue jeans
Black shoes
Khaki jeans
ADD THE ANGLE TO THE 2- and 3- DIMENSIONS

SONIA ABRAHAMS

KLIPHEUWEL PRIMARY

Max no of participants: 30
Audience: Intermediate phase
Duration: 1 hour

DESCRIPTION OF THE CONTENT OF WORKSHOP

Display of wall charts and other equipment etc.
Equipment - pencils, rubbers, rulers, geo-boards, mirrors, elastics, geo-solids
Overhead projector needed

Handing out activity sheets which will be used during the course of the workshop. Equipment and Instruments will be displayed on working areas and made use of. Lessons will be given as been done in classroom.

Only practical work (hands-on activities) which educators will experience while doing it, themselves. Equipment and instruments which will be used to make lesson interesting and successful. The use of an overhead projector to complete the continuation of the follow-up work.

Educators will have a better understanding of 2- and 3 dimensions. Different type of worksheets will be given to the educator to understand the concept of the work. Every participant that attend my workshop have to be physically involved.
About 80% will be practical and 20% will be given to discussion and questions

MOTIVATION

From my point of view, the educator get the chance to experience the different methods to introduce 2D & 3D to the learner. That if you work hard and through perseverance you will achieve what you set out to do, is to educate the learner.

To gain knowledge of the subject presenting in a simple and fun manner. To make use or explore the advantages of how to use simple equipment of the immediate environment as well as your cultural background make it so much easier in difficult circumstances.
**Mathematics**

Grade: 6

Learning Outcomes: 3 Space and Shape (Geometry)

ASS. STD. 6.3.1 Recognises, visualises and names two-dimensional shapes

---

**IDENTIFY THESE SHAPES AND NAME THEM BY CHOOSING FROM THE LIST:**

<table>
<thead>
<tr>
<th>pentagon</th>
<th>rectangle</th>
<th>octagon</th>
<th>circle</th>
</tr>
</thead>
<tbody>
<tr>
<td>oval</td>
<td>rhombus</td>
<td>trapezium</td>
<td>hexagon</td>
</tr>
<tr>
<td>square</td>
<td>triangle</td>
<td>parallelogram</td>
<td></td>
</tr>
</tbody>
</table>

---

![Shapes](image)
Mathematics

Grade: 6

Learning Outcome: 3 Space and Shape (Geometry)

ASS. STD. 6.3.2.2 Describes and classifies two-dimensional shapes and three-dimensional objects in terms of properties including:

- number of sides/length of sides
STUDY THE FOLLOWING SHAPES AND COMPLETE THE TABLE THAT FOLLOWS:

<table>
<thead>
<tr>
<th>NR.</th>
<th>NUMBER OF CORNERS</th>
<th>NUMBER OF LENGTHS</th>
<th>NAME OF SHAPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>5.</td>
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<tr>
<td>6.</td>
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<tr>
<td>7.</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Enlarge the following objects
Reduce the following drawings

Remember: A reduction is enlargement, but the scale factor is a proper fraction.
Mathematics

Grade: 6

Learning Outcome: 3 Space and Shape (Geometry)

ASS. STD. 6.3.1 Recognises, visualises and names two-dimensional shapes and three-dimensional objects in natural and cultural forms and geometric settings including those previously dealt with and focusing on:

- Similarities and differences between tetrahedrons and pyramids:

------------------------------------------------------------------------------

<table>
<thead>
<tr>
<th>Shape</th>
<th>Name of faces</th>
<th>Number of edges</th>
<th>Number of vertex</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cube</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Triangular Prism</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rectangular Prism</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Square Pyramid</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hexagonal Prism</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Triangular Pyramid</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Describe the similarities and differences between triangular and square pyramids
Mathematics

Grade: 6

Learning Outcome: 3 Space and Shape (Geometry)

ASS. STD. 6.3.1 Recognises, visualises and names two-dimensional shapes and three-dimensional objects in natural and cultural forms and geometric settings including those previously dealt with and focusing on:

NAME THE FOLLOWING THREE-DIMENSIONAL OBJECTS. WRITE YOUR ANSWERS ON THE TABLE BELOW:
<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>E</td>
<td>F</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Mathematics**

Grade: 6

Learning Outcomes: 3

Ass Std. 6.3.3 Investigates and compares three-dimensional shapes

◊ nets provided by teacher

COMPARE THE THREE-DIMENSIONAL SHAPES TO THE NETS

1 2 3 4

5 6 7

---
This paper proposes a contextual and practical approach to the teaching and learning of platonic and plane figures in order to increase understanding, and by making activities more meaningful and relevant. This investigation involves a wide variety of practical activities, including the introduction and development of geometrical concepts and terminology. By means of guided re-invention pupils are afforded the opportunity to explore, investigate, discover patterns, and construct meaning for themselves. The activity starts with everyday contexts, involving the identification of geometrical shapes (polyhedra) in terms of usage, external features, sorting according to common and distinctive features, and culminates in expressing relationships among faces, edges and vertices algebraically.

INTRODUCTION

Based on my 12 years of in-service mathematics training (in the Senior and FET Phases), involving lesson observation and lesson critique, I have become agonizingly aware of the lack of or absence of real 3-D (everyday household) objects in the mathematics classrooms when this mathematical content was taught. Dealing with Space and Shape was almost always only a pen / pencil and paper exercise, far removed from pupils’ daily experiences. Pupils are not always afforded opportunities to classify and explore the properties of platonic and plane figures as a basis for the systematic study of these figures within a specific axiomatic system.

Mathematics pupils almost never experience the transition from ‘the 3-D object to the 2-D sketch on paper’ in a practical manner. Consequently, many pupils never arrive at the higher Van Hiele levels of geometric development since they are not exposed to certain geometrical activities facilitating progression from one Van Hiele level to another. This might be the reason why so many grade 12 students encounter difficulty visualizing or interpreting 3-D figures when having to solve angles of elevation or depression in trigonometry on paper. Visualisation and spatial reasoning to solve problems both within and outside of mathematics need to be developed. Thus the focus is on activities that should arouse pupils’ interest, allow pupils to make
meaningful connections between familiar everyday and mathematical shapes, and investigate relationships.

**THE ACTIVITIES**

These activities expose pupils to practical teaching methodologies based on constructivist and realistic mathematics approaches: working from everyday 3-D objects and contexts through four levels of mathematization, namely from the situational to the formal. Diagrammatically the relationship between the four levels (reflecting vertical mathematization) is represented as follows (Drijvers, 2003:54; Gravemeijer, 1994:102):

![Diagram of four levels of mathematization](image)

They will thus be exposed to mathematics material that facilitates self-exploration and self-activity, thus occupying pupils constructively - in a way that enhances learning and understanding. This material is by nature quite flexible and allows mathematics teachers to arrange chunks of content in such a way so as to compile tutorials, investigations, projects, formative as well as summative tests, etc. At the end of the workshop teachers would have developed particular skills and knowledge to help teach this content effectively.

The following questions are taken from module 13 (Space and Shape). The question numbers have been kept the same as they appear in the module (Van Etten, 2003: 3–14):

1. Try and obtain as wide a variety of packaging as possible, and bring them to class.

   a. Bring all the packaging you obtained to your class and mix them up. Arrange the packaging according to the same contents of each package for example those containing liquids, cleaning agents, soap, etc.
When we arrange objects according to one or more characteristics, we call the process **sorting**.

b Sort your packaging according to a different characteristic than in a. Write down the characteristic you used to help sort your packaging and indicate which ones belong together.

c Now choose a particular property or characteristic and sort the objects in the pictures below according to the chosen characteristic.

The three Heading Styles should be adequate to structure your paper. Please avoid numbering sections (as opposed to lists and footnotes).
SORTING ACCORDING TO SHAPE

Some shapes have names. The names appear in the list below.

**Prism:**

Pyramid:

**Cone:**

**Cylinder:**
Prisms, pyramids, cylinders, cones, spheres and cubes are called spatial figures.

A spatial figure also has surfaces/faces. Some of the figures have flat faces/surfaces or curved surfaces.

3 Read the important information above, look at the list of shapes and answer the following questions:

a Which spatial figures only have flat surfaces?

b Which spatial figures only have curved surfaces?

c Which spatial figures have no flat surfaces?

4 The flat surfaces can also be classified according to shape. Write down which shape of surface occurs in which spatial figure:

<table>
<thead>
<tr>
<th>shape of surface:</th>
<th>triangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>can occur in prism</td>
<td>yes</td>
</tr>
<tr>
<td>can occur in pyramid</td>
<td></td>
</tr>
<tr>
<td>can occur in cylinder</td>
<td>no</td>
</tr>
<tr>
<td>can occur in cone</td>
<td></td>
</tr>
<tr>
<td>can occur in sphere</td>
<td></td>
</tr>
<tr>
<td>can occur in cube</td>
<td></td>
</tr>
</tbody>
</table>
shape of surface:

- can occur in *prism*
- can occur in *pyramid*
- can occur in *cylinder*
- can occur in *cone*
- can occur in *sphere*
- can occur in *cube*

5 For each of the spatial figures below, write down the name of the shape of each surface. Also write down the number of surfaces for each spatial figure. In some of the drawings, not all surfaces can be seen.

25 In the sketches of the prisms below, not all the detail is shown. Complete the sketches by including the faces, vertices and edges you cannot see.
a Now complete the table below:

<table>
<thead>
<tr>
<th>prisms</th>
<th>No. of edges</th>
<th>No. of faces</th>
<th>No. of vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>E</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b The prisms listed in the table below are not drawn. You can, however, complete the table since certain information is given:

<table>
<thead>
<tr>
<th>prisms</th>
<th>No. of edges</th>
<th>No. of faces</th>
<th>No. of vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td></td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>K</td>
<td></td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>M</td>
<td></td>
<td>1002</td>
<td></td>
</tr>
</tbody>
</table>

c If you look at the information in both tables regarding the number of faces, vertices and edges, what can you deduce about prisms?
In the sketches of the pyramids below, once again count the number of faces, vertices and edges.

a Now complete the table below:

<table>
<thead>
<tr>
<th>pyramids</th>
<th>No. of edges</th>
<th>No. of faces</th>
<th>No. of vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
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<td>D</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b The pyramids listed in the table below are not drawn. You can, however, complete the table since certain information is given:

<table>
<thead>
<tr>
<th>pyramids</th>
<th>No. of edges</th>
<th>No. of faces</th>
<th>No. of vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td></td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td></td>
<td></td>
<td>11</td>
</tr>
<tr>
<td>K</td>
<td></td>
<td></td>
<td>1001</td>
</tr>
</tbody>
</table>

c If you look at the information in both tables regarding the number of faces, vertices and edges, what can you deduce about pyramids?
Now look at the different spatial figures below and complete the table:

<table>
<thead>
<tr>
<th>figure</th>
<th>No. of edges</th>
<th>No. of faces</th>
<th>No. of vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
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<td></td>
<td></td>
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<td>D</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Even for these spatial figures, there exists a relationship between the number of faces, vertices and edges. This relationship is represented by a rule called Euler’s Rule, named after a mathematician called Leonard Euler. The rule reads as follows:

For spatial figures with flat faces/surfaces and no holes; the following is valid: \( \text{number of edges} + 2 = \text{number of faces} + \text{number of vertices} \)

28 Determine if the Euler’s rule is valid for the spatial figures above.

29 Is this rule also valid for rectangular prisms, prisms and pyramids?
Take any object/spatial figure in your class and see if Euler’s rule is valid for that object.

INSTANCES OF INTEGRATION

This investigation activity is an integrated exercise. Integration by nature occurs within this mathematics activity since it addresses a variety of learning outcomes simultaneously. The pertinent mathematics learning outcomes in this instance involve:

- **Number:** The use of number to find the number of edges; vertices, and faces; decomposing numbers into factors in an effort to find relationships between variables.
- **Functions:** Patterns are developed in terms of the number of edges, vertices or faces (identifying number patterns, such as multiples of 3 and multiples of 2); tables (completing number tables); equations (once a formula has been established to change the subject of the formula) and graphs (using graphical representations depicting linear relationships existing between edges and faces for example).
- **Space and Shape:** Handling and investigating the properties of a variety of spatial figures (identifying 2-dimensional forms such as the triangle, square, rectangle, etc. by examining household objects and packaging.

Simultaneously, it also integrates other Learning Areas such as:

- Technology (designing spatial figures) and,
- Life Orientation. (geometrical figures in everyday household containers and packaging are investigated).

CONCLUDING REMARKS

This investigation involves a wide variety of practical activities, including the introduction and development of geometrical concepts and terminology. Also in this case, the idea of guided re-invention (Gravemeijer, 1994; Drijvers, 2003; Van Etten and Smit, 2005) by means of which pupils are afforded the opportunity to explore, investigate, discover patterns, and construct meaning for themselves, is evident. The activity starts with everyday contexts, involving the identification of geometrical shapes (polyhedra) in terms of usage, external features, sorting according to common and distinctive features.

The concern with the context situation gradually shifts to focus increasingly on differences, identifying 2-dimensional shapes, counting edges, faces, vertices, and completing tables, by supplying missing values and studying emerging patterns; analysing patterns; exploring numerical and algebraic relationships; expressing these
relationships algebraically; and representing these relationships graphically (Verhage, Adendorff, ea, 2000: 38).

REFERENCES


THIS IS HOW I TEACH THE ENLARGEMENT OF A THREE-DIMENSIONAL OBJECT BY A FACTOR K AND ITS INFLUENCE ON THE SURFACE AREA AND VOLUME OF THE OBJECT

Lorraine F Botha
Centre For Education Development (CED)
University of the Free State
FET Band Grade 10

The Centre for Education Development (CED) is a site for the “EQUALS Institute for the Professional Development of Mathematics Teachers” (Lawrence Hall of Science, University of California at Berkeley). The following activity is an adaptation of an EQUALS investigation.

According to Assessment Standard 10.3.1 of the NCS, learners are expected to “Understand and determine the effect on the volume and surface area on right prisms and cylinders, of multiplying any dimension by a constant factor k”. A constructivistic approach will be used in this activity and provision is made for the accommodation of learners with different learning styles by making use of concrete material to discover the effect of enlargement on the surface area and volume of a three-dimensional object. The material allows for the determining of the surface area and volume of the object without making use of formulae. During the session participants will work in pairs to construct an enlarged three-dimensional object and by determining the surface area and volume of the object, discover the new concepts for themselves.

INTRODUCTION

For effective learning to take place it is essential that the needs of analytical as well as global thinking and learning styles should be provided for, especially during the discovering and fixing of new concepts. By making use of concrete material that are colourful and easy to use and by creating a fun-filled atmosphere, more learners might be able to master concepts successfully, before moving on to more abstract work and applications.

CONTENT

This session will focus on the following aspects:

- The six elements of effective learning according to De Corte and Weinert (mention only).
- The steps of a constructivist lesson which will be followed during the session (mention only).
• How do analytical and global thinkers differ? How should teachers provide for both types of thinkers and resulting learning styles? (Very briefly).

• Participants will be working in pairs and each pair will be supplied with a set of Cuisenaire rods (to be returned after the workshop), a worksheet and graph paper.

• Participants will construct an enlargement of a three-dimensional structure provided and determine the surface area and volume of the structure.

• By comparing the original structure with its enlargement, certain conclusions will be made and discussed.

This activity has been used in EQUALS teacher training workshops and was enthusiastically received and enjoyed by the teachers attending.

However, some of the teachers mentioned that they will not be able to do the activity at school as they do not have the necessary resources. Several alternative ideas for inexpensive and readily available material suitable for this activity will be suggested at the session.

Another remark from teachers was: “It is too time-consuming”. The author is of the opinion that, if there was not enough time to do it properly at first, where will the time be found to do it over again? Is it not perhaps time well-spent? Is the aim not to have more learners to be successful? These questions will be reflected on after the session.

REQUIREMENTS FOR THE SESSION

A maximum number of 30 teachers can be accommodated. Late-comers can not be accommodated because of practical work to be done in a very limited time.

• A minimum time of 1-1½ hours.

• Facilities to enable the use of PowerPoint.

• Separate small tables which will allow teachers to work in pairs.

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THINGS TO DO WITH CELL PHONES IN A MATHEMATICS CLASSROOM

Laurie Butgereit
Meraka Institute, CSIR

The days of being able to ban cell phones from a school are over. The cell phone has become a necessary part of life: parents use cell phones to arrange the pick up time of their children; children use cell phones to get missed homework assignments from their friends; teachers use cell phones to plan their much deserved holidays. This paper describes various things which teachers can encourage pupils to do with a cell phone to assist in their learning mathematics. If you are planning to attend this workshop, to maximise the benefit to you, it is important that you visit your cell service provider and have them configure “GPRS” on your cell phone for you before attending the workshop.

INTRODUCTION

Africa has the highest cell phone usage in the developing world [1]. Cell phones have gone far beyond being merely a way to talk to a friend or family members. Cell phones are used for banking purposes [2]. Cell phones are used in medical applications [3]. Cell phones are used in crime prevention [4]. This paper discusses some things which can be done with a cell phone to help pupils enjoy learning mathematics. Practical examples, demonstrations and handouts will be given.

WORKSHOP PREPARATION

If you are planning to attend this workshop, it would be ideal if you have already contacted your cell service provider had them configure “GPRS” for you on your phone. Some service providers require 24 hours notice in order to do such configuration. In addition, to actively benefit from this workshop, you will need approximately R5.00 airtime for this workshop.

TEENAGERS AND ADULTS ATTITUDES TOWARDS TECHNOLOGY

Adults are often intimidated by children's and teenagers' abilities to use technology. Give a teenager a new electronic device (digital camera, cell phone, IPOD, or MP3 player) and he or she will “fiddle” with it until it is configured and working to his or her satisfaction.

Adults, on the other hand, often hear terms such as “WAP”, “GPRS”, and “3G” and shy away from the conversation because they are embarrassed to admit they don't understand the terms. The author of this paper is the first to admit that her teenage children know much more about configuring cell phones than she does.
Adults must not be shy about asking teenagers “Could you please help me get GPRS configured on my cell phone?” From experience, teenagers will not think less of an adult because the adult asked for help configuring a cell phone. Just the opposite. The teenager will probably think that the adult is “kewl.”

**CALCULATORS**

Most cell phones come with calculators of varying capabilities ranging from simple calculations such as addition and multiplication up to more complicated scientific and/or financial calculators. Encourage your pupils to find the calculator on their cell phone and experiment with its capabilities. Have the pupils answer the following questions:

1. How many decimal places does your calculator have?
2. What happens when you try to divide by zero? Why?
3. Does your calculator have trigonometric functions?
4. If your calculator can do trigonometric functions, does it operate in degrees or radians? (This is extremely important even if the pupils do not know what radians are because some calculators may only operate in radians.)
5. If your calculator can do trigonometric functions, what happens when you try to calculate tan(90)? Why?
6. Does your calculator have financial functions?

**CAMERAS**

Most modern cell phones have built in cameras. Some sample projects include:

3. Pupils studying geometry can be asked to look around the classroom and take photos of triangles, rectangles, circles, right angles, acute angles, obtuse angles, or the intersections of lines.
4. Pupils could be given homework assignments to take photos of specific examples of mathematical concepts around the school campus or at home.
5. Math Literacy students could take photos of signs in shopping centres advertising finance arrangements.
6. Math Literacy students could visit stores and take photos of the same product being offered at different prices in different stores.

One problem is lighting (or lack of lighting) in the classroom. This often works best outdoors.

**VIDEOS**

Most modern cell phones also have the facility to take videos.

1. Pupils can make presentations explaining certain mathematical concepts or they can give a lesson on specific topics such as fractions.
2. Pupils could also video the teacher while he or she is presenting a topic in order to revise the presentation at home.

3. Pupils could also video the whiteboard or blackboard while the teacher is explaining algebraic manipulations such as FOIL or factoring a polynomial.

As with still photos, lighting can be a problem. Again, this may work best outdoors.

**BLUE TOOTH**

Most modern cell phones have Bluetooth communication. This is a free communication mechanism for short distances (within a few metres of each other). It is often called “side loading” information (as opposed to “down loading” information from a website or “up loading” information to a website).

During the workshop, you will

1. Turn your Bluetooth on.
2. Set up your Bluetooth name and visibility.
3. Try sharing the photos or videos which you have just taken to another person in the workshop.
4. Turn off Bluetooth.

One of the pitfalls of Bluetooth, however, is that normal phones and computers will only allow one Bluetooth connection at a time. Although it is possible for pupils to Bluetooth to the teacher, only one pupil can do this at a time and you will get virtual queues in the classroom.

**INTERNET**

Most modern cell phones can be configured to access the internet. This facility, however, is often not configured when you initially purchase a cell phone, get a new sim card or cell phone contract. If you are too shy to ask a teenager to help you, go to your service provider (MTN, Vodacom, or Cell C) and ask them to configure it for you (Hopefully you have done this before attending this workshop).

Using your phone to access the internet is a relatively inexpensive way to access a world wide library of information. Current costs are approximately R2.00 per megabyte. A megabyte is the equivalent of more than 6000 sms's!

In fact, if a pupil asks a research type question in class, that pupil could easily find the answer using the internet on their own phone.

Projects for pupils could include:

1. Math Literacy pupils can look up the foreign exchange rates for today by accessing any of the South African banking websites. An example website is http://www.scmb.co.za/servlets/za.co.scmb.datafeeds.forex.GetforexServlet
2. Math Literacy pupils could find the interest rate on government retail bonds by accessing http://www.rsaretailbonds.gov.za
3. Math Literacy pupils could access any number of South African bank websites and determine which offer the “best deal” for a chequing account or a home loan.
4. All pupils could access websites for South African universities to find out what type of mark they need in mathematics or MathLit in order to enter their preferred field of study.

DIGITAL CITIZENSHIP

Once pupils are connected to the internet (and, remember that the pupils are probably already connected even if the teachers are not connected), they need information on digital citizenship or digital safety. This is not specifically a mathematics topic, but if teachers are encouraging young people to access information on the internet via their cell phones, there should also be some instruction on how to act safely and responsibly.

MXIT AND DR MATH

“Dr Math” is a facility where pupils can get help with their mathematics homework using the popular Mxit chat facility on their cell phones. Besides linking pupils with tutors, “Dr Math” provides games and competitions to encourage pupils to hone their math skills. More specific information about the “Dr Math” project can be found at [5][6].

In order to access Mxit and “Dr Math”, you must have a “GPRS” enabled cell phone. You must point your cell phone browser to the website http://www.mxit.co.za and follow the instructions for downloading Mxit onto your cell phone. Once Mxit is installed on your cell phone, “Dr Math” can be accessed by adding the contact 27-79-992-3960

UFUDU

Ufudu is a formula graphing utility which is designed to work on a wide range of cell phones as long as the phone is WAP enabled. The formulae (along with any optional domain and range restrictions) is sent up to a central server. The graph is generated on the central server and the image is sent back to the cell phone. At current cell
phone costs (approximately R2.00 per megabyte), each graph costs less than 1 cent to
download.

Ufudu can be downloaded free of charge from http://146.64.81.63/ufudu/wap.

CONCLUSION

A modern cell phone can be used as a tool in education. Pupils can use their cell
phones to research projects for school. Pupils can use their cell phones to get help
with homework. Teachers can use cell phones to brush up on any topics with which
they may not feel 100% comfortable explaining to children.

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INVESTIGATING BALANCING POINTS OF POLYGONS

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University of KwaZulu-Natal. Edgewood
http://mysite.mweb.co.za/residents/profmd/homepage.html

Target Audience:  FET & Further Education
Duration:  2 hours
Required:  Computer Lab with Sketchpad 4 plus data projector & OHP
Max. Participants:  Dependent on no. of computers - max. 2 per computer

In this workshop, participants will firstly be given hands-on computer experience of dynamically finding the balancing point (centroid) of a triangle based on an activity from De Villiers (2003). Using worksheets, participants will in the first hour be guided through a standard synthetic proof of the concurrency of the medians. It will then be shown that the respective balancing points of a ‘cardboard’ triangle and a ‘point mass’ triangle coincide.

However, participants will then be led to discover that for a quadrilateral and other higher polygons, the “cardboard” centroid and ‘point mass’ centroid do not in general coincide. Subsequent related problems that will be mentioned and discussed further in the remaining time are the location of balancing points:

1. for triangles with unequal ‘point masses’ at the vertices (leading to Ceva’s theorem),
2. for other polygons with unequal ‘point masses’ at the vertices,
3. for a ‘wire’ triangle (to discover that it is not the same as the other two)
4. for concave polygons, and an application in high jumping.

Use will also be made of cardboard cut-outs and a gravity simulator in Cinderella to visually illustrate some results. Depending on time, alternative proofs of the concurrency of the medians using advanced geometry results from homothetic polygons or Desargues’ theorem might be given. Overall, the workshop plans to provide an opportunity for engaging participants in conjecturing, logical explanation, generalising, refutation, and verification. During the activity some of the roles played by proof in mathematics such as verification, explanation, discovery and intellectual challenge may also be briefly discussed.

Reference

One of the biggest paradigm shifts that has faced teachers, old and new, in the NSC has been adjusting to setting assessments according to cognitive levels. Experienced teachers will say that they always did this automatically but the practice of analysing papers in this way is improving the style of question papers and making them fairer and more accessible to a broader cross-section of students. This brief workshop is targeting the setting of the higher-order problems and motivating the need to include these questions in order to challenge our thinking students. The aim is that teachers go away more equipped with ideas and resources in order to improve their questioning across the taxonomical scale.

DESCRIPTION OF THE CONTENT OF THE WORKSHOP

- The aim of the workshop will be to give the delegates some practice in solving and setting/choosing problem solving question for NSC Mathematics examinations
- A brief outline will be given of how to interpret a Level 4 or Problem solving question and how much of these questions we can expect in NSC examinations
- The idea of 'Problem Solving questions' will be broken down into various types and examples will be given of each type and worked through with the delegates
- Can we prepare students for problem solving - a dilemma?
- The delegates, in teams, will work through some examples and there will be some ensuing discussion over the level of difficulty of the problems and their suitability in the context of examinations. What are we assessing through these problems and what is the purpose of 'problem solving' questions?
- A problem solving or 'unseen' problem - fair or unfair? Are we levelling the playing fields or setting up some students for failure?
- To what extent can we give realistic problems in an NSC Mathematics examination? Are all 'realistic' problems contrived at this level?
DATA HANDLING IN THE FOUNDATION PHASE

MS SHANBA GOVENDER

AUSTERVILLE PRIMARY/AMESA KZN – DURBAN SOUTH BRANCH

Duration of Workshop: one hour
Maximum Number of Participants: 60
Type of Audience: Foundation Phase

1. **THE PURPOSE OF THE WORKSHOP**:  
   To create an understanding of Learning Outcome 5 – Data Handling.  
   To enhance the teaching of Data Handling in the Foundation Phase.  
   To familiarize educators the various terminology to be used.  
   To stress the importance of using concrete apparatus.  
   We are learning how to solve a problem by collecting data and organising it in a way which is easy to read and understand.

**WHAT IS DATA and DATA HANDLING**

Data is information collected, recorded and then examined for various reasons. We collect data in the following ways

- By observing something.
- By experimenting - count, measure or try out.
- Survey - questionnaires
- Census
- Use of somebody else’s data from various resources e.g. internet.

Raw Data: this is information that is not sorted or arranged in any specific order.

**SORTING OF DATA**: Data is sorted according to criteria or attributes – a single attribute e.g. colour.

- more than one attribute e.g. shape and colour or shape, colour and size.
REPRESENTING DATA

Tally Chart: used to organize frequency distribution. A tally shows how often something is happening.

\[ \begin{align*}
11 & \quad 8 \\
1111 & \quad 12
\end{align*} \]

We generally present collected data in a form of a visual representation - tables and graphs.

**A Pictograph** also called a picture graph or pictogram - pictures are used to stand for quantities. A picture can stand for one or many things.

* ONE TO ONE REPRESENTATION IN THE FOUNDATION PHASE.

**A Bar Graph** show information by using bars. The bars are all the same thickness and can be horizontal or vertical.

**A Block Graph** is made up of blocks. Blocks are usually square. Blocks can be horizontal or vertical.

Other common graphs and tables are the line graph, pie graph and Carroll Diagram.

**** Graphs should have a Title saying what is being shown.

**LEARNING OUTCOME FIVE:**

The learner is able to collect, summarise, display and critically analyse data in order to draw conclusions and make predictions, and to interpret and determine chance variation. The learner is able to COLLECT, SUMMARISE, DISPLAY AND CRITICALLY ANALYSE. (will discuss and refer to NCS).
# NUMERACY ASSESSMENT STANDARDS AND MILESTONES

<table>
<thead>
<tr>
<th>GRADE ONE</th>
<th>GRADE TWO</th>
<th>GRADE THREE</th>
</tr>
</thead>
<tbody>
<tr>
<td>✤ Collect data according to given criteria or category</td>
<td>✤ Collect data according to given criteria or category</td>
<td>✤ Collect data according to given criteria or category</td>
</tr>
<tr>
<td>✤ Sort physical objects according to one attribute.</td>
<td>✤ Sort physical objects according to one attribute.</td>
<td>✤ Sort physical objects according to one attribute.</td>
</tr>
<tr>
<td>✤ Give reasons for collection being grouped in a particular way.</td>
<td>✤ Give reasons for collection being grouped in a particular way.</td>
<td>✤ Give reasons for collection being grouped in a particular way.</td>
</tr>
<tr>
<td>✤ Draw a picture as a record of collected objects.</td>
<td>✤ Draw a picture as a record of collected objects.</td>
<td>✤ Draw a picture as a record of collected objects.</td>
</tr>
<tr>
<td>✤ Construct pictographs where stickers or stamps represent individual elements in a collection of objects.</td>
<td>✤ Construct pictographs where stickers or stamps represent individual elements in a collection of objects.</td>
<td>✤ Construct pictographs that have a 1-1 correspondence.</td>
</tr>
<tr>
<td>✤ Explain how it was sorted.</td>
<td>✤ Explain how it was sorted.</td>
<td>✤ Explain how it was sorted.</td>
</tr>
<tr>
<td>✤ Answer questions about it.</td>
<td>✤ Answer questions about it.</td>
<td>✤ Answer questions about it.</td>
</tr>
</tbody>
</table>

## MILESTONES ASSESSMENT TASKS

### TERM ONE

- collect and sort objects according to one attribute
- introduction to graphs

### TERM TWO

- Collect and display data
- collect and sort data according to given criteria.

- sorts, orders and organises own data according to 2 attributes.
TERM THREE

- Collects, sorts explains and draws a collection of objects according to one attribute
- Collects, sorts, describes and constructs pictographs according to one attribute chosen by the teacher
- Is able to collect, sort and organise supplied data and then draw a bar graph using the data.

TERM FOUR

- Collects, sorts, explains and constructs pictographs
- Analyses data to draw a conclusion.
- Is able to read and interpret data in a simple table.

ACTIVITY:  Step One - Educators sort data.
          Step Two - Tabulate this on a frequency / tally chart.
          Step Three : Using information data is displayed on graphs.
          Step Four : Draw conclusions from the representation.

CONCLUSION:  Success Criteria.

On completion of the workshop delegates would have learnt how to organise data in different ways, which make the data easy to read and will help solve problems and answer questions quickly and easily.

We should also gained

► greater knowledge and understanding.
► strategies in making teaching and learning more enjoyable.

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FUNCTIONS AND INVERSES IN CONTEXT

Marina Van Heyningin and Ramesh Jeram

Institute for Mathematics and Science Teaching,
University of Stellenbosch

Target audience: FET Mathematics teachers, grades 10-12

Maximum no. of participants per workshop: 30 participants, broken up into groups 2/3.

Duration: 2 hours

Many mathematics textbooks approach inverse functions from a formal mathematics perspective. This activity puts inverse function in the context of everyday life and highlights its importance. The contextual approach will help teachers move from the informal to the formal. This activity will also highlight the prior knowledge required by learners to do inverse functions, viz. transformations (reflections), number patterns and graphs. This activity will also introduce practical teaching methodologies based on constructivism and realistic mathematics.

FRUITS AND MONEY

Mr. Kagee sells fruit and vegetables. He currently has a special on peaches. He sells his peaches at R1,50 each.

(a) Complete the table:

<table>
<thead>
<tr>
<th>Number of peaches</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Write out a rule/formula which will represent the cost of any amount of peaches.

(c) There are 24 peaches in a tray. How much will it cost to purchase two trays? Is the cost of 48 peaches twice the price of one tray?

(d) You will notice that with each amount of peaches there is an associated cost. We can write these numbers as an ordered pair, viz. (amount of peaches; cost). Use the values in the table and write the values as a set of ordered pairs.

(e) In the above relationship, which is the dependant and which is the independent variable? Why?
These ordered pairs show an association of pairs of numbers. For every amount of peaches purchased, there is an associated cost with it. This is an example of a function; the quantities of peaches make up our domain and the cost make up the range.

(f) In your own words, explain what you understand by the terms **function, domain and range**. Refer to the set of values represented in the table.

Not everyone who comes to Mr. Kagee’s shop stipulates that they want a certain quantity of peaches. In most cases, people walk in and say I want R30 worth of peaches. In other words, the “cost” is known and we want to find out what the associated number of peaches is.

(g) Complete the table:

<table>
<thead>
<tr>
<th>Cost</th>
<th>R3</th>
<th>R4,50</th>
<th>R6,00</th>
<th>R15,00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of peaches</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(h) Write out a rule/formula which will represent any amount of peaches, if the cost is known.

(i) How did you do this?

(j) Would you consider this new relationship to be a function?

(k) What is the domain and range of this new relationship?

(l) In the above relationship, which is the dependent and independent variable? Why?

(m) Write out the ordered pairs for this relationship.

(n) If you compare these ordered pairs with the ordered pairs in (d), do you notice the pattern?

(o) Describe this pattern in your own words.

(p) Draw graphs, on the graph paper provided, of the two relationships (in different colors) on the same system of axes. Use the tables to help you.

(q) Where do these graphs intersect?

(r) Draw the line \( y = x \) (use a dotted line). Mark off the points \((5; 7.50)\) and \((7.50; 5)\) on your line graphs. Measure the perpendicular distance of each point to the line \( y=x \). What do you notice and what conclusion can you draw about the line \( y=x \)?
ARRIVE ALIVE!!!

When a motorist brakes, the car does not come to an immediate stop. While he brakes, the car still moves a little further. This distance covered by the car is called the braking distance. The braking distance covered depends on the speed of the car. The braking distance for a speed of 120 km/h is greater than for a speed of 60 km/h. There is a formula which helps us to calculate the braking distance.

The formula is:

\[
braking\ distance = \frac{3}{4} \times \frac{\text{speed}^2}{100}
\]

The speed must be in kilometre per hour, then the braking distance will be in metres.

(a) Complete the table:

<table>
<thead>
<tr>
<th>speed in km/h</th>
<th>0</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>140</th>
</tr>
</thead>
<tbody>
<tr>
<td>braking distance in m</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Use the formula to calculate the braking distance if someone drives at 60 km/h.

(c) Use the formula to calculate the braking distance if someone drives at 120 km/h.

(d) Is the braking distance for 120 km/h twice as much as 60 km/h?

(e) Use the values in the table and write the values as a set of ordered pairs.

(f) In the above relationship, which is the dependant and which is the independent variable? Why?

(g) In your own words, explain what you understand by the terms function, domain and range. Refer to the set of values represented in the table.

After an accident, a car had a braking distance of 48 metres. The question is now: “How fast was the motorist driving?”
(h) Complete the table:

<table>
<thead>
<tr>
<th>braking distance in m</th>
<th>0</th>
<th>3</th>
<th>12</th>
<th>27</th>
<th>48</th>
<th>75</th>
<th>108</th>
<th>147</th>
</tr>
</thead>
<tbody>
<tr>
<td>speed in km/h</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(i) Write out a rule/formula which will represent the speed if the braking distance is known.

(j) Would you consider this new relationship to be a function?

(k) What is the domain and range of this new relationship?

(l) In the above relationship, which is the dependent and independent variable? Why?

(m) Write out the ordered pairs for this relationship, in accordance with the rules stated above.

(n) If you compare these ordered pairs with the ordered pairs in (e), do you notice the pattern?

(o) Describe this pattern in your own words.

(p) Draw graphs, on the graph paper provided, of the two relationships (in different colors) on the same system of axes. Use the tables to help you.

(q) Where do these graphs intersect?

(r) Draw the line \( y=x \) on your graph (use a dotted line). Mark off the points \((40; 12)\) and \((12; 40)\) on your graphs. Measure the perpendicular distance of each point to the line \( y=x \). What do you notice and what conclusion can you draw about the line \( y=x \)?

**Discussion points:**

- inverse: the dependent variable becomes the independent one if you write number of peaches as function of cost or speed as function of braking distance.

- therefore names on axis must be different

- in formal maths we associate \( y \) with the word “dependent variable” and that is why we “swop \( y \) and \( x \)”

- The graphs are continuous; does this make sense in reality?

- The independent variables are all positive – is this the case if we draw graphs of these relationships without context?

- To be or not to be?? When is a relationship a function??
BACTERIA!!!

Food goes rotten when there is too much bacteria present. The number of bacteria increases very quickly at room temperature. If the food is kept in a fridge, the bacteria does not increase that much. The health services visit an old age home at 10 am to inspect a prepared meal in the fridge. They confirm that at that stage, there were approximately 1000 bacteria per gram in the meal. For calculating the number of bacteria, the following formula is quite reliable:

\[ \text{number of bacteria} = 1000 \times 2^{\text{time}} \]

- \( \text{number of bacteria} \) is the number of bacteria per gram food.
- \( \text{time} \) is the number of hours after 10 o’clock in the morning.

(a) At 11:00 am there are already approximately 2000 bacteria per gram in the food. On what can one base this statement?

(b) Complete the table:

<table>
<thead>
<tr>
<th>time of day</th>
<th>10 o’clock</th>
<th>11 o’clock</th>
<th>12 o’clock</th>
<th>1 o’clock</th>
<th>2 o’clock</th>
<th>3 o’clock</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of bacteria</td>
<td>1000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(c) By how much did the amount of bacteria per gram food increase between 11 o’clock and 12 o’clock.

(d) By how much did the amount of bacteria per gram food increase between 12 o’clock and 1 o’clock.

(e) What do you notice about the answers in (c) and (d)?

(f) Is the increase during two hours double the increase during one hour?

(g) Use the values in the table and write the values as a set of ordered pairs.

(h) In the above relationship, which is the dependant and which is the independent variable? Why?

(i) In your own words, explain what you understand by the terms function, domain and range. Refer to the set of values represented in the table.

Suppose we know the number of bacteria per gram food, can we figure out what time it is?

(j) Complete the table:

<table>
<thead>
<tr>
<th>number of bacteria</th>
<th>1000</th>
<th>2000</th>
<th>4000</th>
<th>8000</th>
<th>16000</th>
<th>32000</th>
</tr>
</thead>
<tbody>
<tr>
<td>time of day</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(k) Write out a rule/formula which will represent the time (the number of hours after 10 o’clock in the morning) if the number of bacteria per gram food is known.
As you can see we need to introduce a new concept... called logarithm.

**Definition of a logarithm:**

If $a^x = y$, then $y = \log_a x$.

- **(l)** Now try again to answer no (k).
- **(m)** Would you consider this new relationship to be a function?
- **(n)** What is the domain and range of this new relationship?
- **(o)** In the above relationship, which is the dependent and independent variable? Why?
- **(p)** Write out the ordered pairs for this relationship, in accordance with the rules stated above.
- **(q)** If you compare these ordered pairs with the ordered pairs in (g), do you notice the pattern?
- **(r)** Describe this pattern in your own words.
- **(s)** Draw graphs, on the graph paper provided, of the two relationships (in **different colors**) on the **same** system of axes. Use the tables to help you.
- **(t)** Where do these graphs intersect?
- **(u)** Draw the line $y=x$ on your graph (use a dotted line). Mark off the points (12 o’clock; 4000) and (4000; 12 o’clock) on your graphs. Measure the perpendicular distance of each point to the line $y=x$. What do you notice and what conclusion can you draw about the line $y=x$?

**NOW WE GO THE FORMAL MATHS WAY!!**

1. **Consider the following equation:**

   $2^x = y_1$

   - **(a)** What type of equation is this? Is it a function?
   - **(b)** Complete the table:

     | $x$-values | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
     |------------|----|----|----|---|---|---|---|
     | $y_1$-values |    |    |    |   |   |   |   |

   - **(c)** Use the values in the table and write the values as a set of ordered pairs.
   - **(d)** In the above relationship, which is the dependant and which is the independent variable? Why?
   - **(e)** In your own words, explain what you understand by the terms function, domain and range. Refer to the set of values represented in the table.
Suppose we want to make $x$ the subject of the equation/formula…

(f) Complete the table:

<table>
<thead>
<tr>
<th>$y_1$-values</th>
<th>0.125</th>
<th>0.25</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$-values</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(g) Write out a rule/formula which will represent the $x$–value when the $y_1$–value is known

(h) Would you consider this new relationship to be a function?

(i) What is the domain and range of this new relationship?

(j) In the above relationship, which is the dependent and independent variable? Why?

(k) Write out the ordered pairs for this relationship, in accordance with the rules stated above.

(l) If you compare these ordered pairs with the ordered pairs in (c), do you notice the pattern?

(m) Describe this pattern in your own words.

(n) Draw graphs, on the graph paper provided, of the two relationships (in different colors) on the same system of axes. Use the tables to help you.

(o) Do these graphs intersect?

(p) Draw the line $y=x$ on your graph (use a dotted line). Mark off the points $(1 ; 2)$ and $(2 ; 1)$ on your graphs. Measure the perpendicular distance of each point to the line $y=x$. What do you notice and what conclusion can you draw about the line $y=x$?

2. Consider the following equation:

\[ \left( \frac{1}{2} \right)^x = y_2 \]

(a) What type of equation is this? Is it a function?

(b) Complete the table:

<table>
<thead>
<tr>
<th>$x$-values</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_2$-values</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(c) Use the values in the table and write the values as a set of ordered pairs.

(d) In the above relationship, which is the dependant and which is the independent variable? Why?

(e) In your own words, explain what you understand by the terms function, domain and range. Refer to the set of values represented in the table.

Suppose we want to make $x$ the subject of the equation/formula…

(f) Complete the table:

<table>
<thead>
<tr>
<th>$y_2$-values</th>
<th>8</th>
<th>4</th>
<th>2</th>
<th>1</th>
<th>0.5</th>
<th>0.25</th>
<th>0.125</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$-values</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(g) Write out a rule/formula which will represent the $x$ value when the $y_2$ value is known

(h) Would you consider this new relationship to be a function?

(i) What is the domain and range of this new relationship?

(j) In the above relationship, which is the dependent and independent variable? Why?

(k) Write out the ordered pairs for this relationship, in accordance with the rules stated above.

(l) If you compare these ordered pairs with the ordered pairs in (c), do you notice the pattern?

(m) Describe this pattern in your own words.

(n) Draw graphs, on the graph paper provided, of the two relationships (in different colors) on the same system of axes. Use the tables to help you.

(o) Do these graphs intersect? If they do, where?

(p) Draw the line $y=x$ on your graph (use a dotted line). Mark off the points $(1;0.5)$ and $(0.5;1)$ on your graphs. Measure the perpendicular distance of each point to the line $y=x$. What do you notice and what conclusion can you draw about the line $y=x$?
REGRESSION MODELLING

Paul Laridon
RADMASTE Centre Wits University

Further Education and Training Band

2 Hours in duration

As an introduction, participants will make a scatter plot of a small set of data and then draw a line of best fit through the points. The essentials of the manner in which a least squares regression can lead to the best fit line will be explained using the scatter plot. Calculators capable of providing the parameters for the line will then be used. Further sets of bivariate data arising out of real contexts will then be explored so as to provide regression models of various types such as linear, quadratic or exponential for these data. Decisions will be made as to which model best suits a particular set of data. This will lead to predictions based on the model adopted. The development of mathematical models arising out of data is a very important aspect of the use of mathematics in the modern world. The inclusion of regression modelling in the new curriculum provides the opportunity of developing insight into how such models are used and so enhancing the mathematical literacy of our FET learners.

Maximum number of participants: 25

Work in pairs.

You will be provided with graph paper.

Please hand calculators back after the workshop.
1. DETERMINING THE EQUATION OF A LINE OF BEST-FIT ON A SCATTER PLOT

1.1 Plot the following data on the graph page provided in order to get a scatter plot.

<table>
<thead>
<tr>
<th>Reading</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>t (sec.)</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>d (cm)</td>
<td>44</td>
<td>80</td>
<td>110</td>
<td>140</td>
<td>170</td>
</tr>
</tbody>
</table>

Where \(d\) (cm) is the distance that a ball rolls along a straight track from the start of the track, and \(t\) (s) is the corresponding time of that reading.

1.2 Draw a line through the scatter plot so that the line is “closest to as many of the points as possible.” This is the line of best-fit.

1.3 Determine the equation of the line you have drawn by finding the gradient and \(d\)-intercept of the line.

2. USING A CALCULATOR TO DETERMINE THE EQUATION OF THE LINE OF BEST-FIT

In this workshop we will be working with the CASIO \(fx-82ES\) PLUS/NATURAL DISPLAY.

2.1 Entering the data

Switch on the calculator. Press [MODE] then select STAT by pressing [2]. The following screen will appear:

```
1 1-VAR  2 A + BX
3 _ + CX^2  4 ln X
5 e ^ X  6 A . B ^ X
7 A . X ^ B  8 1/X
```

Now press [2] for linear regression. Your screen should look something like this:

```
x | y
---|---
1  |   
2  |   
3  |
```

Enter the data given in 1.1 into this
table. Then press [AC]. The screen clears but the data remains stored.

2.2 Finding the constants (parameters)

Now press [SHIFT] [1] to get the stats computations screen which is shown on the right. Choose Regression by pressing [7].

From the screen that now appears press [1] and [=] to get the value of the \(d\)-intercept, \(A\).

To get the slope, \(B\), use the following key sequence:

\[
\text{[SHIFT]} [1] [7] [2] [=]
\]

Write down the equation of the line of regression: \(\hat{d} = A + Bt\).

Compare this value with that found for the line of best-fit in 1.3.

2.3 Predicting values

Press [AC] [60] [SHIFT] [1] [7] [5] [=].

This will give a (predicted) value for \(d\) at \(t = 60\). (By extrapolation since \(t = 60\) is outside the given set of \(t\)-values.)

2.4 Finding the correlation value

Press [SHIFT] [1] [7] [3] [=]

This gives the value of \(r\) which is an indication of how well the line fits the data values. The closer \(r\) is to 1 the better the fit.
3. NON-LINEAR REGRESSION MODELS

3.1 Breaking distances.

By “breaking distance” is meant the distance a car will travel until it comes to a stop from the instant the foot is taken off the accelerator and jammed onto the brake pedal.

The table below gives the breaking distances recorded against speeds at which it was travelling for a motor car on test.

<table>
<thead>
<tr>
<th>Speed $v$ (m/s)</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Breaking distance $d$ (m)</td>
<td>2</td>
<td>10</td>
<td>25</td>
<td>40</td>
<td>70</td>
</tr>
</tbody>
</table>

3.1.1 Draw a scatter plot of $d$ on the vertical axis against $v$ on the horizontal. Do you think it appropriate to adopt a linear model for the situation? What type of function would you think is better suited?

3.1.2 Fitting a function to the data.

Get to the regression screen as described in 2.1.

First choose a linear regression. Enter the $v$ and $d$ data into the table. Proceed as in 2.2 and 2.4 to find the parameters and the $r$ value.

Again get to the regression screen but this time choose the quadratic function $d = a + bx + cx^2$. Enter the data and determine the parameters as before.

Which function would you choose to best model the data?

3.1.3 Predicting values.

Use the function which you decide on to predict the breaking distance for the car if it were travelling at

(i) 60 km/h

(ii) 100 km/h
3.2 The Pendulum

The diagram represents a pendulum consisting of an object A which swings at the end of a piece of string held up by a support.

The length of the piece of string, \( \ell \), is varied and the corresponding time for one complete swing \( T \) (the period) is determined experimentally.

The table below gives a set of data found in this way.

<table>
<thead>
<tr>
<th>( \ell ) cm</th>
<th>150</th>
<th>120</th>
<th>100</th>
<th>80</th>
<th>60</th>
<th>40</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T ) s</td>
<td>2,487</td>
<td>2,214</td>
<td>2,001</td>
<td>1,785</td>
<td>1,559</td>
<td>1,325</td>
<td>0,920</td>
</tr>
</tbody>
</table>

3.2.1 Draw a scatter plot of \( T \) on the vertical axis against \( \ell \) on the horizontal axis. Does the plot indicate that a function other than linear could best fit the data?

3.2.2 Fitting a function to the data.

Get to the regression screen as described in 2.1.

First choose a linear regression. Enter the \( \ell \) and \( T \) data into the table. Proceed as in 2.2 and 2.4 to find the parameters and the \( r \) value.

Again get to the regression screen but this time choose the power function \( y = Ax^a \). Enter the data and determine the parameters and \( r \) values as before.

Which function would you choose to best model the data?
3.3 Times for the women’s 100 m race

The table provides world record times for the women’s 100 m race in athletics since 1922.

<table>
<thead>
<tr>
<th>Name</th>
<th>Time (s)</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marie Mejzlikova</td>
<td>13.6</td>
<td>1922</td>
</tr>
<tr>
<td>Mary Lines</td>
<td>12.8</td>
<td>1922</td>
</tr>
<tr>
<td>Gundel Wittman</td>
<td>12.4</td>
<td>1926</td>
</tr>
<tr>
<td>Kinue Hitomi</td>
<td>12.2</td>
<td>1928</td>
</tr>
<tr>
<td>Myrtle Cook</td>
<td>12.0</td>
<td>1928</td>
</tr>
<tr>
<td>Tollien Schuurman</td>
<td>11.9</td>
<td>1932</td>
</tr>
<tr>
<td>Stanislawa Walasiewicz</td>
<td>11.8</td>
<td>1933</td>
</tr>
<tr>
<td>Stanislawa Walasiewicz</td>
<td>11.7</td>
<td>1934</td>
</tr>
<tr>
<td>Stanislawa Walasiewicz</td>
<td>11.6</td>
<td>1937</td>
</tr>
<tr>
<td>Fanny Blakers-Koen</td>
<td>11.5</td>
<td>1948</td>
</tr>
<tr>
<td>Marjorie Jackson</td>
<td>11.4</td>
<td>1952</td>
</tr>
<tr>
<td>Shirley Strickland</td>
<td>11.3</td>
<td>1955</td>
</tr>
<tr>
<td>Wilma Rudolph</td>
<td>11.2</td>
<td>1961</td>
</tr>
<tr>
<td>Irena Kirszenstein</td>
<td>11.1</td>
<td>1965</td>
</tr>
<tr>
<td>Wyomia Tyus</td>
<td>11.08</td>
<td>1968</td>
</tr>
<tr>
<td>Renate Stecher</td>
<td>11.07</td>
<td>1972</td>
</tr>
<tr>
<td>Inge Helten</td>
<td>11.04</td>
<td>1976</td>
</tr>
<tr>
<td>Annagret Richter</td>
<td>11.01</td>
<td>1976</td>
</tr>
<tr>
<td>Marlies Oelsner</td>
<td>10.88</td>
<td>1977</td>
</tr>
<tr>
<td>Marlies Gohr</td>
<td>10.81</td>
<td>1983</td>
</tr>
<tr>
<td>Evelyn Ashford</td>
<td>10.79</td>
<td>1983</td>
</tr>
<tr>
<td>Evelyn Ashford</td>
<td>10.76</td>
<td>1984</td>
</tr>
<tr>
<td>Florence Griffith Joyner</td>
<td>10.49</td>
<td>1988</td>
</tr>
</tbody>
</table>

(Source: Infostrada Sports) (Times since 1968 recorded electronically.)

3.3.1 Draw a scatter plot of these data with the year on the horizontal axis.

3.3.2 By hand draw in a graph that you think best fits the data.

3.3.3 Use your calculator to find the relevant constants for the following regression models of the data:
   (i) linear,    (ii) quadratic,    (iii) power.

3.3.4 Argue for which of the models you consider to be the best.

3.3.5 Use the model you preferred to predict what the world record might be in 2009.

3.3.6 Shelly-Ann Fraser’s time for winning the final of the 100 m at the Beijing Olympics in 2008 was 10.78 s. Compare this time with the time you got in...
3.2.5 and comment on how good or otherwise the prediction turned out to be.

3.3.7 Argue for a point of view as to the manner in which the record times will behave into the future.
Problem Solving using Heuristics

John Luis
Advtech Schools

Target Audience: GET Intermediate and Senior Phase Teachers
Duration: 2 Hours
Number Of participants: Venue capacity

Problem solving strategies given to students are often too general and students find the application of these strategies to specific problems difficult. A heuristic is a good guide to solving a problem. The workshop looks at solving problems from Grade 3 to Grade 9 using a specific type of heuristic, namely “drawing a diagram”.

Using a Diagram to Solve Problems
Teachers will be guided through the “part-part model” used for simple addition and subtraction problems in Grades 3 and 4, as well as the “comparative model” used for solving problems from Grades 5 to 9.

The “Part-Part” Model
This model is used mainly for addition and subtraction problems in Grades 3 and 4. Here is an example of a problem which can be solved using the “part-part” model:
John has 5 Marbles. His mother gives him 8 more marbles. How many does he have altogether?

The “Comparative” Model
This model is used to solve more difficult problems asked of students in Grades 5 to 9.
Here is an example of a problem which can be solved using the “comparative” model:
In a school, there are 18 fewer male teachers than female teachers. If 45% of the teachers are male, how many teachers are there in the school?

References:
Zalina A. Jalil(2007). My Pals are Here!
THE VERSATILE USE OF PATTERN BLOCKS IN THE CLASSROOM

Marionette Maart
Northway Primary School
Intermediate Phase Educator

“I never knew that one can use pattern blocks in so many ways. Previously I gave it to my learners only to play. I will now be able to use the pattern blocks with great effect. I had fun, and I know my learners will have fun also.” Those were the words of an educator who attended a previous workshop. During this session the educators will be given ideas as to how using this equipment can stimulate the learners’ thinking while they are having fun at the same time.

INTRODUCTION

A fun-filled atmosphere will be created, where educators will have hands-on experience of all the activities that can be done in the classroom. These activities will not only stretch the mind of the learners, but also the mind of the educators who will be attending this workshop. With the hands-on activities educators will get a better understanding of how to use the equipment during the different activities, and how to start creating their own activities.

CONTENT

This session will focus on the following aspects:

1) how to build different 2D shapes by combining the different pattern blocks, e.g. using 2 triangles, 1 rhombus, 1 trapezium to build a pentagon

2) using the pattern blocks to teach algebra in the intermediate phase

3) using the pattern blocks to teach angles by discovery

4) an easy way to use the pattern blocks to represent money and using it during a group activity

5) demonstrating a fun way to teach symmetry and rotational symmetry through individual and group work

6) teaching transformation, reflection and rotation in a fun way
During previous workshops educators found the activities of great value, because of the lack of knowledge on how to use the MST kit. This workshop will give educators an idea of how valuable the pattern blocks can be, and how educators can develop their own activities. Learners love using the pattern blocks, and will always, when given a chance, do some of the activities on their own.

**REQUIREMENTS FOR THE SESSION**

A maximum number of 30 teachers can be accommodated. Practical work will be done all the time, so educators will be busy with activities, although learners work and reaction will also be shared with the educators.

- A minimum time of 1½ hours will be used to complete all the activities.
- Facilities to enable the use of a PowerPoint presentation.
- Separate small tables which will allow teachers to work in pairs.

**REFERENCES**

Supedi Trust Maths Program
AN INTRODUCTION TO FUNCTIONAL RELATIONSHIPS

Makgoshi Manyatshe
RADMASTE Centre, University of the Witwatersrand
makgoshi.manyatshe@wits.ac.za

Target Audience: Senior Phase Educators

Duration: 2 Hours

Maximum number of participants: 30

Description of the content of workshop:

- Workshop participants will get hand on experience with activities in the worksheets.
- They will also be given the opportunity to share with one another on their personal experiences in their classrooms regarding the teaching of functional relationship.

Motivation:

Learners struggle to see the relevance of functions in their daily lives, and see it purely in terms of variables and graphs with little meaning. It is hoped that by introducing learners to functional relationships in the real world this will have more meaning to them and assist in problem solving and graph interpretation activities.
BEADWORK AND MATHEMATICS:
 PATTERNS, FUNCTIONS AND ALGEBRA

Mary Morokolo Mei
Radmaste Centre, Wits University

Tel.: 011 717 6070 (work)
Cell: 082 298 5155
e-mail: mary.mei@wits.ac.za
A 2-hour workshop aimed at Senior Phase (Grades 7 –9)

MAXIMUM NUMBER OF PARTICIPANTS: 35 – 40

What will be done in the workshop:

Step 1:
Participants will work in groups of 4 or 5. They will each receive a copy of LO 2 and will start off by looking for the Assessment Standards that deal with Patterns, Functions and Algebra. Having found the ASs they will then look for what each Grade has to know, and will also look for the progression from Grade 7 to Grade 9. When they are done there will be a discussion around that activity. This will take about 30 minutes.

Step 2:
Participants will be given four worksheets which focus on growing patterns and finding formulae and will have the opportunity to engage with them.

Step 3:
The last 15 minutes time will be spent discussing the activities and their link to LO 2.

MOTIVATION FOR RUNNING WORKSHOP:
Having gone through the material on Zulu beadwork developed by Helene Smuts under the project “Africa meets Africa”, I decided that I wanted to share the knowledge with fellow educators as I know that they struggle with the topic in the Senior Phase and also have problems with integration between learning areas.
USE OF MATHEMATICAL GAMES AS AN ALTERNATIVE, INFORMAL TEACHING METHOD

Dr Annari Milne
Xhariep District, Free State Education

Target Audience: Grade 9 and 10 Teachers (Senior Phase and Fet Phase)
Duration: 2 hours
Max. participants: 30

By using games a mathematics lesson can be made excited, interesting and enjoyable. Mathematical games give learners opportunities to be actively taking part in the learning and teaching process. Because games allow learners to experience success and satisfaction, their enthusiasm and self confidence can be boosted. Furthermore can their writing, reading, speaking and listening in/of mathematics be enhanced. The experience of success and satisfaction will lead to the fact that better understanding will happen and thus better achievement can be achieved. (Hildebrandt, 1998:192). More learners can possibly choose mathematics as a subject and also continue with the subject in further studies in such a way that our country can get more mathematicians and mathematical literate people.

MATHEMATICAL GAMES

Play as on of the four didactical base-forms can be use with much effect in the NCS-curriculum (Department of Education, 2004:2-9), but does not really come to realisation in the teaching and learning process. The question about why mathematical games must be played, can now be answered. According to Hildebrandt (2005:1-11), a specialist in the area of using games, it is necessary to play games because:

- group-games give a rich context for mathematical developement in the child’s learning enviroment;
- through repetative play the learners develop new strategies to do mathematical calculations;
- learners become more interested in and motivated for the mathematics.

Tapson (1997:2-4) expands on the above by referring to two basic approaches towards the use of games in the classroom. He groups the link between mathematics and games into two groups of questions/statements. The first five questions indicate a covert approach were the games are played and the mathematics which is intrinsically present. The second group of five statements needs that some extrinsic mathematics must be done and is aksing for some written work.
The links is further explained by Tapson (1997:2) by means of looking at possible questions which the player can ask him/herself before the games is played.

The first five questions can be summarised as follows:

<table>
<thead>
<tr>
<th>QUESTION</th>
<th>Mathematical Link</th>
</tr>
</thead>
<tbody>
<tr>
<td>* How do I play the game?</td>
<td>* Interpretation</td>
</tr>
<tr>
<td>* Which is the best way to play?</td>
<td>* Optimisation</td>
</tr>
<tr>
<td>* How can I make sure that I will be the winner?</td>
<td>* Analysis</td>
</tr>
<tr>
<td>* What will happen if...?</td>
<td>* Variation</td>
</tr>
<tr>
<td>* What is the possibility that...?</td>
<td>* Probability</td>
</tr>
</tbody>
</table>

These questions leads to the following statements:

<table>
<thead>
<tr>
<th>STATEMENT</th>
<th>Mathematical Idea</th>
</tr>
</thead>
<tbody>
<tr>
<td>* This games is the same as ...</td>
<td>* Isomorfisme</td>
</tr>
<tr>
<td>* You can win by ...</td>
<td>* A specific case</td>
</tr>
<tr>
<td>* It works with all these games...</td>
<td>* Generalisation</td>
</tr>
<tr>
<td>* Look, I can show that ...</td>
<td>* Proof</td>
</tr>
<tr>
<td>* I communicate (orally, written) about the game as follows...</td>
<td>* Simbolisation and notation</td>
</tr>
</tbody>
</table>

The first five questions include the implicit mathematics in the games while the second group of statements the possible opportunities indicate which needs a response and thus the mathematics is made explicit.

**VALUE OF MATHEMATICAL GAMES:**

Games with educational value can also be fun and exiting. According to Hildebrandt (2005:1-11) the value of mathematical games lies in the fact that it gives a context in which the learners can (1) develop their sense of numbers; (2) look for different solution strategies; (3) develop their communication skills. Calitz (2005:21) indicates that with the development of the level of play, there also development of abstract thinking is. She clarifies this by saying that mathematics is to a great extent simbolic actions and through play learneras can leran how to tackle problems and also develop dimensional insight. The successfull use of games as a teaching method depends on
the teacher’s ability to choose the correct game, to ask penetrating and directed questions about the outcome of the game as well as the ability to be able to create the correct classroom environment that is productive towards investigation. Shaftel, Dass & Schnabel (2005:25-30) states that the focus with the playing of a game, must rather fall on the cognitive process as on the absolute accuracy of the outcome. The process through which a ‘wrong’ answer is obtained, must be used with the same effect as the process through which a ‘correct’ answer has been obtained. The fact that the process is more important than to just getting the answer, is also strongly emphasized by the NCS (Department of Education, 2004:9) as well as the SAG document (Department of Education, 2005d:7-12).

Skemp (1986:131) and Brown et al. (1989:35-37) has found during their research that learners have the need to be successful in their social environment in the classroom. They must experience success in mathematics in order to want to learn more about mathematics. They must be placed in environments where their learning of mathematics will lead to opportunities of experiencing success. A very important factor indicated by these researchers is the impact of how learners experience their making mistakes. This experience has got a direct impact on their selfconcept, motivation and anxiety about the subject.

Learners recoup from making mistakes during playing games because the circumstances of structured play is not intimidating and they do not feel that they will fall behind. Mistakes must be seen as the route/way to success that will follow later in the game. This can happen because learners can follow their own pace in games and thus can rectify their mistakes with success.

Ernest (1986:35-49) is of the opinion that success in mathematics teaching is greatly depended on the active involvement of learners in the process and games needs active involvement. Mathematical games needs the active participation of the learners and can thus be used very effectively in the teaching and learning process of mathematics.

HANDS-ON EXPERIENCE OF MATHEMATICAL GAMES

In order to incorporate a game successfully in the classroom, the teacher have to play the game him-/herself first to determine the value of the game. The example game that will be used in the workshop to illustrate the effective use of mathematical games is *Top Score D*.

**Source:** Funkey Maths- Cambridge  ([www.keystolearning.co.za](http://www.keystolearning.co.za))

**Learning Outcome:** LO1 – basic calculations; LO2 – manipulation of algebraic expressions, fractions

**Players:** 1 – 6 players

**Needed:** pen, paper, stopwatch
Contents: 51 playing cards and 3 jokers. Four numbers visible on each card (whole numbers). Three numbers are around the sides and one number in a cloud. The number in the cloud is the answer that must be determined.

Rules:

1. Players must build as many as possible number sentences using two or three of the numbers. They can make use of the operations plus, minus, multiplication and division as well as the use of brackets where needed.

2. Each player takes one card without turning it over. The moment the stopwatch starts, they must turn their cards over.

3. Each player has got one minute now (time depending on the development level of the learners) to make as many as possible true number sentences.

4. After the time is finished, each player must in turn explain his/her answers to the other players.

5. Marks are allocated as follows: one mark if only two numbers have been used correctly; three marks if all three numbers have been used correctly; seven marks for a joker.

6. Marks are noted down per round.

7. After each player has had five cards, is the scores totaled in order to determine the winner.

Example of a playing card:

If this card was used in a round, the following possible answers available:

The number in the cloud, a 5, is the answer that must be obtained.

3 – (-2) = 5 only two numbers used, thus getting 1 mark

-1 – (-2 x 3) = 5 all three numbers used thus getting 3 marks

3 + (-2 x -1) = 5 all three numbers used thus getting 3 marks

Total for this round: 7 marks

After playing the games in groups of 3 people for 40 minutes, their will be a question/answer time of 20 minutes where the presenter will answer questions about the implementation of mathematical games as an informal, alternative teaching
method. This will be followed by a discussion of 30 minutes on the use and influence of mathematics games in the classroom.

**INFLUENCE/USE AND CONCLUSION:**

The researcher came to the conclusion at the end of the whole research process that the games that was played, resulted in the fact that learners:

- knew more about basic number-facts;
- skills to build and manipulate number sentences on different difficult levels, has improved;
- worked together much easier with their fellow classmates;
- social skills developed because solutions/answers had to be explained, motivated, supported and justified to the group for acceptance or discussion;
- really learned how to listen to each other;
- learned to differ about answers and not with the person;
- very quickly learned to think out of the box in order to find alternatives to come to an answer/solution;
- levels of logical, mathematical reasoning developed and increased in difficulty;
- confidence were build through the continuous revision process of mathematical knowledge and –skills with the help of the games and also as a result of the exposure to other learners thinking patterns;
- basic knowledge/skills have been enhanced in a very positive, hands-on way (practical application through games);
- everyone, weaker or stronger, can be winners because an element of luck is build into the games in order to make it on the one hand more competitive and on the other hand promoting cooperative work;
- are motivated by playing the games (during the games by means of marks and bonus marks; intrinsically because learners come to the knowledge that they can be successful in mathematics).

According to the researcher the use of mathematical games in the teaching and learning of mathematics can also have the following advantages:

- Because rounds are following each other, there will be no more a problem of bored learners in your class.
- Learners alternate between competing against and working together.
- Learners cannot copy; they have to do their own work.
- Because fellow learners are checking the calculations, accurate and responsible work is encouraged.
• Games are making ordinary repetitive work fun. Learners are learning without thinking consciously about it.
• Because games are played against a time constraint, it helps learners to work faster with accuracy. Thus the learners are supported in being able to finish formal assessments in the allocated time.
• Learners where getting frustrated if they could only find one solution. This resulted in the learners trying over and over to find more than one solution. As a result the elasticity of the learners reasoning was enhanced. Logical reasoning could thus move on to a higher level.
• The learners really enjoyed the games.
• Games are really a wonderful resource to bring mathematics to the learners in a more practical, informal way.

Out of many aspects of the research it became apparent that quality time spent to make learners mathematically more powerful can have many positive results. Rich and diverse problems/situations must be developed because it will result in the development of communication-, reasoning- and presentation skills. Regular discussions of the actions in a problem helped the learners to become more confident with different problem solving strategies and thus they could answer problems with more ease. The process to obtain appreciation for oral and written sharing, flexibility and reflection, plants and feed a powerful mathematics seedling in the learners thoughts, which will grow and flourish in the correct circumstances. Powerful reasoning patterns will develop which will be of great help to the learners in their studies and life after school. Learners are thus much more equipped with mathematical knowledge and skills which are practical and usable, also after school. Mathematical games as alternatives, informal teaching method deserves its rightful place in the OBE-classroom in which the NCS can be taught and learned with success.

MOTIVATION:

The technological environment of the modern child is responsible for the fact that he/she does no longer readily play ‘ordinary’ games just for the fun of it. Many teachers are also of the belief that learners cannot discover and learn if they are not kept 100% under control. Playing games in the classroom is very often seen as a waste of time. Is there then any value in the playing of mathematical games?

Tapson (1997:2) answer this question with three reasons why games must be played with vigour in the classroom: (1) the value of the intrinsic mathematics which is always present in mathematical games; (2) a high level of motivation and interest that can be obtained with games; (3) the deeper understanding and insight of situations that can be obtained when the games are played more than once. Calitz (2005:15-16) agrees with Tapson that games is not only there for fun and the building of selfconcepts, but that the playing of games learner can help to (1) understand
mathematical concepts, (2) develop mathematical skills, (3) learn mathematical facts, (4) learn the subject language and vocabulary and (5) to develop cognitive mathematical skills.

The teachers will thus be able to experience it themselves how powerful the use of mathematical games can be in the classroom. They will realise how effective mathematics games can be in enhancing the achievement of their learners.

Reference

*Funkey Maths*: Verkry van die wêreldwyse netwerk op 10/01/07.  
[www.keystolearning.co.za](http://www.keystolearning.co.za)

Exceptional Children, January /February: 25-30.


Learning Outcome 3: SPACE AND SHAPE (GEOMETRY)

The learner will be able to describe and represent characteristics and relationships between two-dimensional shapes and three-dimensional objects in a variety of orientations and positions.

Reference: Mathematics Education Primary Programme MEPP
E-mail: info@mepp.co.za

3.1

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<thead>
<tr>
<th>Grade R</th>
<th>Grade 1</th>
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<th>Grade 3</th>
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<tr>
<th>R.3.1 RECOGNISES, IDENTIFIES AND NAMES THREE-DIMENSIONAL OBJECTS IN THE CLASSROOM AND IN PICTURES, INCLUDING:</th>
<th>1.3.1 Recognises, identifies and names two-dimensional shapes and three-dimensional objects in the classroom and in pictures, including:</th>
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<tbody>
<tr>
<td>• BOXES (PRISMS)</td>
<td>• boxes (prisms) and balls (spheres):</td>
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<td>• BALLS (SPHERES)</td>
<td>• triangles and rectangles;</td>
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<td>• circles</td>
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<td>2.3.1 Recognises, identifies and names two-dimensional shapes and three-dimensional objects in the school environment and in pictures, including:</td>
<td>3.3.1 Recognises, identifies and names two-dimensional shapes and three-dimensional objects in the environment and in pictures, including:</td>
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<td></td>
<td>• boxes (prisms), balls (spheres) and cylinders;</td>
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<td>• triangles, squares and rectangles;</td>
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<td>• circles.</td>
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<td>• cones and pyramids.</td>
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| • Identifies solid shapes around the school and inside the classroom.  
• Make a display with solid shapes and discuss it.  
• Let the learners play with Lego blocks, plasticine and cake shapes, empty containers and boxes and let them talk about the shapes of the different objects they are handling.  
• Look at the shapes of leaves, clouds, shadows, motorcars, prams, etc. and discuss it.  
• Compare objects; classify and arrange them according to characteristics, size and shape.  
• Make off-prints with different shapes of blocks in the sand, look at the prints and identify the blocks that were used for it.  
• Draw 2-D shapes and put them down on the floor. Let the learners walk around them and say what they experience.  
• Handle 3-D shapes like bricks, boxes, balls and cones.  
• Colour in shapes, cut them | If there are not enough /any of the above mentioned objects in the classroom ‘plant ‘some (hide them). The teacher knows exactly where the objects are hidden and how many of them there are.  
• Conduct a survey: - divide the class in small groups and let them find out and record how many boxes, balls, triangles, rectangles and circles they can identify in the classroom - they draw pictures of the objects and shapes and make ticks under it when they find something;  
• they report back about their findings and say where they found the objects and shapes;  
• the results of the different groups are compared to find the winning group.  
• Make up riddles, e.g. I have 3 corners and 3 sides, I am made out of paper, and I sleep on teacher’s table.  
• Make up riddles about shapes in a picture, e.g. Sometimes I am big, sometimes I am small, Sometimes I am coloured, sometimes I have patterns, I can roll on the ground and fly | Boxes (prisms), balls (spheres), and cylinders:  
• Put empty containers like boxes of different sizes, balls of different sizes and different cylinders (like toilet rolls, kitchen paper rolls, etc) in a big box. Let the learners sort it into the different categories.  
• Make headings like boxes, balls, cylinders and make sure that they know what each heading says.  
• The learners take turns to take an object out of the big box, describe it according to it’s sides, corners and edges and decide whether it belongs in one of the sorting boxes. They must motivate their decision.  
• Hide boxes, balls and cylinders in the classroom and give stickers with the words box, ball and cylinder on it or with a big label (boxes, cylinders) for the whole collection. Add a collection of balls with different sizes and add a label to it.  
• Point to box-, ball- and cylinder shapes in a picture. Triangles, squares, rectangles and circles: Let the learners trace triangles, rectangles, squares and circles:  
• let the learners make circles: with one friend, five together, with all the boys/girls, etc.  
• let the learners find out which letter shapes (or parts of letters) | Boxes (prisms), balls (spheres), and cylinders:  
1. Let the learners bring different empty containers like boxes, toilet rolls, kitchen paper rolls, different shapes of cardboard/plastic-/glass containers, to school. They sort it: all the boxes (prisms) together, and the cylinders together and mark each with a label (box, cylinder) on it or with a big label (boxes, cylinders) for the whole collection. Add a collection of balls with different sizes and add a label to it.  
2. The learners try to find two-dimensional examples of the above mentioned shapes in old magazines, cut them out and paste them on a chart which is put with the sorted objects. Triangles, squares and rectangles: Make sure that the learners know the distinguished features of each of the shapes and do the same as above.  
• Circles: - let the learners make circles: with one friend, five together, with all the boys/girls, etc.  
• let the learners find out which letter shapes (or parts of letters)
out and use them for collages. Many adults and children use me and I am in the middle of the picture you are looking at now. Who am I? (Answer: a ball)

- Let the learners look at the pictures in story books and ask them whether they recognise pictures of 3-D shapes they have worked with.

<table>
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<tr>
<th>Circles of different sizes and colour them in: the triangles red, squares green, rectangles blue and the circles orange. Draw a line from the shapes to their names in the box: ( \Delta ) Triangle circle oval square diamond rhombus rectangle star</th>
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<tbody>
<tr>
<td>Give an A-4 sheet of paper with circles, squares, rectangles and triangles. Let the learners cut out the shapes and use it for making collages, e.g.</td>
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<td>give each learner an A-4 sheet and ask them to put a square in the middle of the sheet, then a triangle above the square, a circle on the left side and the right side of the square and a rectangle at the bottom of the square.</td>
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<td>Make a sketch of your picture.</td>
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<td>Let the learners recognise the triangles, rectangles, squares and circles in pictures.</td>
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<td>The teacher points to different 3-D objects in the classroom while the learners say which shape it reminds them of. Who am I? The teacher describes a 2-D shape according to sides and corners and the learners guess what shape it is.</td>
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<td>Can be linked to a circle;</td>
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<td>- let they make circular movements with their hands and feet;</td>
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<td>- let them make a list of objects with a circular shape they identify outside the classroom and compare it with that of their friends;</td>
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<td>- let them page through magazines, identify circular objects, write down the page numbers and ask a friend to check their survey.</td>
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<tr>
<td>Cones and pyramids:</td>
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<td>- start with a good discussion on the shape of cones and pyramids and where and how they are used in our daily lives;</td>
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<td>- let them search for pictures and real objects with this shape, e.g. cones used for knitting yarn, chocolate boxes, drinking yogurt containers, etc. and put it on display (as mentioned above);</td>
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<td>- have a discussion about the advantages and disadvantages of using containers with this shape for thicker liquids and whether they can think of other liquids that can be put in these containers, e.g. dishwasher.</td>
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<td>Grade R</td>
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<tr>
<td>R.3.2 DESCRIBES, SORTS AND COMPARES PHYSICAL THREE-DIMENSIONAL OBJECTS ACCORDING TO: • SIZE: • OBJECTS THAT ROLL: • OBJECTS THAT SLIDE:</td>
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<td>Grade R</td>
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| • Size:  
- use big tins and smaller tins (cylinders), big blocks and smaller blocks, big beads and smaller beads, big boxes and smaller boxes, etc.  
- let the learners work in groups when they sort the objects, e.g. one group put all the big beads in a container while another group put the smaller beads in another container, etc.  
- afterwards they discuss why certain objects were put together.  
• Objects that roll:  
- learners must know the concept of rolling;  
- they put the objects which can roll together – even if they look different;  
- they test the selected objects to make sure that they can roll;  
- they discuss why some objects can roll and others not.  
• Objects that slide:  
- do the same as with the objects that roll;  
- also try to find out which objects can roll and slide, e.g. coins and tins. | • Let the learners work in small groups. Put examples of three-dimensional objects which can roll and can slide and shapes that have straight and round edges on the floor:  
- give each of the learners an opportunity to describe 3 of the objects according to their size and whether they can roll or slide.  
- do the same with two-dimensional plastic- or paper shapes asking whether they have straight or round edges.  
• Put a variety of three-dimensional objects that can roll, slide and with different shapes in a big box or in different boxes. Ask the learners to sort out the objects in groups that can roll, that can slide and in different sizes.  
• Let the learners look at objects in picture books and say whether they can roll or slide, compare there sizes and say whether they have straight or round edges.  
• Make up riddles like: I am made out of soft metal, I | • Size:  
- Find circular-, triangular- and other shapes in the classroom, and  
- classify them as big or small;  
- discuss the shapes and compare their sizes;  
- use the appropriate vocabulary like bigger than/smaller than.  
• Give a paper with pictures of 2-D shapes (in two different sizes) on it to the learners, let them colour in, e.g. the big triangles with red and the small triangles with green. Use different colours for the different shapes. Let the learners go to the playground and try to find one example of each of the shapes. When they are back in the classroom they tell the other learners what they have seen or what they have found and classify it as, e.g. an example of a big circle he/she has seen (the ring of the netball poles) etc.  
Objects that role or slide:  
- the learners sit in a circle and send around a bag with different objects in it, the learners take one of the objects out when the bag | Two-dimensional shapes in or on the faces of three-dimensional objects:  
- let the learners bring empty containers of different substances to school; they examine the containers to see whether there are any two-dimensional shapes in pictures on the containers, describe it and sort the containers according to these shapes;  
- do the same with other three-dimensional objects like pots for plants, vases, mugs, plates etc.  
Flat/straight and curved/round surfaces and edges: Let the learners try to find examples of these in the classroom and on the playground, make a list of it and discuss it in the classroom. |
|   | have a lid that can roll, and People pack different kinds of things in me.     (Answer: a cake tin) | stops with them, look at it and say whether it can roll or slide or roll and slide– the objects that can roll are put together and those that can only slide are put together;  
- show the learners a big picture with pictures of different objects, let the learners point to the objects that can roll:  
- let the learners page through old magazines and find 3 pictures of objects that can roll and 3 pictures of objects that can only slide. Shapes that have straight or round edges:  
- ask the learners to think about 2 objects with straight edges and 2 objects with round edges while the rest of the learners say whether they agree or not – the class decide together with the teacher what is right and what is wrong, they discuss wrong answers and give reasons why it is wrong. |   |
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<tbody>
<tr>
<td>R.3.3 Builds three-dimensional objects</td>
<td>1.3.3 Observes and builds given three-dimensional objects using</td>
<td>2.3.3 Observe and creates given two-dimensional shapes and three-</td>
<td>3.3.3 Observes and creates given and described two-dimensional shapes</td>
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<td>using concrete materials (e.g.</td>
<td>concrete materials (e.g. building blocks and construction sets).</td>
<td>dimensional objects using concrete materials (e.g. building blocks,</td>
<td>and three-dimensional objects using concrete materials (e.g. building</td>
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<td>building blocks).</td>
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<td>construction sets and cut-out two-dimensional shapes).</td>
<td>blocks, construction sets, cut-out two-dimensional shapes, clay,</td>
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<td>drinking straws).</td>
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| • Identify solid shapes inside and outside the classroom. | • Use empty boxes of different sizes if building blocks and/or construction sets are not available:  
- let the learners build any construction;  
- make a sketch on the board and let them build according to it, e.g.  
- let the learners try to draw a sketch of their own constructions. | • Give the apparatus to the learners and observe while they are working. When they have finished they describe what they have done.  
• Draw a diagram comprise of triangles, circles, squares and rectangles on the board and ask the learners to build it with their paper shapes and draw a diagram of it while the teacher changes the diagram on the board. The learners build the new diagram and draw a picture of it, then compare the two pictures according to differences and similarities. | • Draw diagrams in which different shapes are used (use triangles, squares, rectangles and circles) on the board or on cards and let the learners build it with blocks or with their two-dimensional shapes (each learner should have a set of these shapes).  
• If there are construction sets in the classroom, let them use the notes with instructions for building the models that come with it.  
• Let the learners draw their own diagrams to build.  
• Let the learners build something with the construction blocks and make a sketch/diagram of their model. |

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| • Look out for certain characteristics, of containers, boxes, tins, Lego-blocks, etc.: hollow, round, square, etc. | • Make shapes with clay. Compare 3-D shapes with 2-D shapes. | • Make shapes in sand.  
• Give different solid shapes to the learners to build with. Let them describe their models and tell which shapes they used to build it.  
• Let the learners pack solid blocks in a box with a square shape and explain afterwards what they did and how they did it. | • Give the apparatus to the learners and observe while they are working. When they have finished they describe what they have done.  
• Draw a diagram comprise of triangles, circles, squares and rectangles on the board and ask the learners to build it with their paper shapes and draw a diagram of it while the teacher changes the diagram on the board. The learners build the new diagram and draw a picture of it, then compare the two pictures according to differences and similarities. |
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<tr>
<td>• Do activities in which the front and the back sides of their own bodies and that of their friends are:  - give a piece of paper to each learner and ask them to hold it at the front of their bodies and then at the back of their bodies;  - ask the learners to get a partner and let them work in pairs doing the same by asking each other to hold the paper at the front and at the back of their bodies;  - give each learner a book and let them show the back part and the front part of the book;  - let them describe the front part and the back part of the book.</td>
<td>• Let the learners point to the left side and the right side of their own bodies and to their left and right hand, foot, leg, arm, ear and eye while they say, e.g. “This is my left hand”, etc.  • Do the same with “The back and front of my body.”  • Play games like:  - put your hands at the back of your head/at the font of your head;  - hold up your right/left hand  - stand on your right/left leg, etc..  • Point to the front/back of the class room/and the reader.  • Point to the left-hand/right-hand page of the book.</td>
<td>• The learners get a partner and face each other. They find the middle-line on each other’s body and discuss symmetry by referring to the positioning of our ears (on the left and right of our heads), our eyes, our arms, hands, legs and feet.  • Compare the bodies of known animals to their own bodies as far as symmetry is concerned.  • Give different shapes of paper to the learners and let them do the following:  - fold it in half;  - open it, and  - put a drop of ink/paint on the right hand side of the fold;  - close it and rub it over the top ‘page’;  - open it and discover a ‘picture’ with a definite middle-line.  • Let the learners page through magazines and see whether they can identify the middle-line in pictures.</td>
<td>Fold a paper in half and put the fold on the left-hand side for right handed learners and on the right-hand side for left-handed learners. Only draw the right side (left side for left-handed learners) of the picture on the fold:  Keep the paper folded and cut on the line. Open the cut out piece to see the whole figure. The learners can draw his clothes.  Other figures that can be used for this are: butterflies, trees, stars, etc.</td>
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<td>R.3.5 Describes one three-dimensional object in relation to another (e.g. ‘in front of’ or ‘behind’).</td>
<td>1.3.5 Describe one three-dimensional object in relation to another (e.g. ‘in front of’ or ‘behind’).</td>
<td>2.3.5 Recognises three-dimensional objects from different positions.</td>
<td>3.3.5 Recognises and describes three-dimensional objects from different positions.</td>
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<td>• Start with instructions where they are asked to hold an object (e.g. a crayon) in front of their bodies and a block behind their bodies.</td>
<td>• Give instructions like the following to the learners: - stand in front of/behind your chair; - Peter go and stand behind the door; - put your pencil in the front/at the back of your book; - look at pictures (maybe in their readers) and ask questions like: “Where does … stand?” - ask the learners to draw a house with a tree in front of it; - give exercises to learners in which direction plays a role and ask them to encircle/to draw a circle around the first and the last picture in each row, e.g.</td>
<td>Put a big object in the middle of the class on a table. It can be e.g. a box with lots of print on it. Let the learners look at it from where they are sitting and describe what they see according to it’s shape and the print on it. The learners will experience that they all see different things. Why?</td>
<td>Put a big object (like a reading lamp, or a coffee tin) on a table in the middle of the classroom and ask the learners to draw it – only that part of the object as they see it from where they are. When they are finished they compare their drawings with each others’. Now they move to a different place and draw the object again and compare their own two pictures with each other. This is followed up with the teacher who moves the object to different places in the classroom. The learners draw it and compare their pictures.’</td>
</tr>
<tr>
<td>• Make a row by asking the learners to stand/sit in the front or at the back of the row. Ask the learners who is sitting in front of/behind them and point to them while talking.</td>
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<tr>
<td>• Do the same when they are sitting at their tables/in their desks.</td>
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<tr>
<td>• Give pictures like the following to the learners where ‘in front of’ and ‘behind’ is determined by the direction in which the objects move. Let them point to the first and the last one and say why they are first and last.</td>
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</tbody>
</table>
### Grade R

<table>
<thead>
<tr>
<th>R.3.6 Follows directions (alone and/or as a member of a group or team) to move or place self within the classroom (e.g. ‘At the front’ or ‘At the back’).</th>
</tr>
</thead>
</table>

### Grade 1

1.3.6 Follows directions (alone and/or as a member of a group or team) to move or place self within the classroom or three-dimensional objects in relation to each other.

### Grade 2

2.3.6 Positions self within the classroom or three-dimensional objects in relation to each other.

### Grade 3

3.3.6 Reads, interprets and draws informal maps of the school environment or of an arrangement of three-dimensional objects and locates objects on the map.

---

### Grade R

- Give e.g. instructions like the following to the whole class:
  - Stand behind your chair,
  - Stand in front of your table/desk;
  - Girls go and stand behind another girl and boys behind another boy;
  - Do the same with ‘stand in front of’;
  - Take one step forward;
  - Walk two steps back;
- Give instructions to individual learners, e.g.:
  - Tom go and stand behind the door
  - Susan go and stand in front of the wash-basin,
- Discuss position and direction by asking questions, e.g.:
  - Where are standing John?
  - Susan, where is John standing? Etc.

### Grade 1

- Give the following instructions to the learners:
  - Sit on your chair/under your chair; sit next to, in front of, behind, on, under your table/desk;
  - Take your food box and put it on, under, in front of, behind, on the left-hand side, on the right-hand side of your table/desk;
- Get a partner and decide who will follow the instructions first and start, e.g.:
  - Stand behind, in front of, on the left hand side, on the right-hand side of your partner, change places.

### Grade 2

- Use the box of 2.3.5 in the same way, but the learners move around this time and say where they are in relation to the box, e.g. in front of, behind, next to, at the back/front, far away, etc.

### Grade 3

- Let the learners draw a map of the route from the classroom to the toilet as well as the permanent objects that they pass on their way. The learners compare their maps and walk the route to see whether it is correct.
- Other routes that can be drawn are e.g. from the classroom to the gate, from the classroom to the principal’s office, etc.
- The teacher can draw some known routes and ask the learners to find out what route is shown on each card.
### 3.7

<table>
<thead>
<tr>
<th>Grade R</th>
<th>Grade 1</th>
<th>Grade 2</th>
<th>Grade 3</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>2.3.7 Describes positional relationships (alone and/or as a member of a group or team) between three-dimensional objects or self and a peer.</td>
<td>3.3.7 Describes positional relationships (alone and/or as a member of a group or team) between three-dimensional objects or self and a peer.</td>
</tr>
</tbody>
</table>

- See 2.3.6 – change the object: it can be the black board, the teacher’s table, the door, the lights, the floor or anything else. Look for interesting things in the class to force the learners to use different words to describe their position.

- Put a three-dimensional object in a strategic place in the classroom:
  - each learner gets the opportunity to describe his/her position in relation to the object, e.g. I am sitting right across it, diagonally behind it, etc.
  - each learner also describe the position of another learner in relation to the object, and then
  - describe their own position in relation with another learner, e.g. we sit right across each other and the object is right in the middle of the distance between us.
MATHEMATICS GAMES

VISHNU NAIDOO

BUFFELSDALE SECONDARY

WORKSHOP: TWO HOURS

TARGET GROUP: GRADES 2 - 11

INTRODUCTION

Large number of learners in South Africa perform poorly in Mathematics. One of the challenges that face us is to change these learners attitude towards the subject. Giving them more classroom Mathematics is not going to make a difference.

It is for this reason I decided to introduce games in Mathematics.

One of the objectives is to popularize Mathematics through games.

A good game should involve both skill and luck. By introducing the element of luck the game appeals to all learners, including those who perform poorly in Mathematics.

These games are not only played at social but at a competitive level as well.

Educators will be taught the game. They will play each game. In addition they will also be trained in respect of organizing a competition.

THE THREE MATHEMATICS GAMES

• Maths 36 (1)

Challenging number game played with cards- includes rules ;score sheets competition aspects. High level of computational and problem solving skills are required. This is a local game and has been modified after input by the local people.

• Math-a-Move (2)

A stimulating and exciting board game played with dice. This board game covers data handling as well as problem solving and computational skills. Game has been modified in terms of the new curriculum-local game
Maths 20 (3)

A very challenging and enjoyable game played with cards. Advanced problem solving and computational skills. The rules; score sheets and organizational aspects covered. This game has also been modified with local input- a South African game.

All three games have been developed and manufactured in South Africa.

These games are very neutral in terms of language; number sense; computational skills; logic etc.

We have observed that high levels of computational skills promote good Mathematics performances. This body of knowledge has been created in South Africa.
WORKSHOP: 2 hours

INTRODUCTION

The Mathematics Learning Area includes interrelated knowledge and skills.

The knowledge that learners are expected to take ‘ownership’ of are:

- numbers, operations and relations
- patterns, functions and algebra
- space and shape
- data handling
- measurement

The skills that learners need to be ‘empowered’ with are:

- representation
- interpretation
- estimation
- communication
- calculation
- reasoning
- problem solving
- investigation
- describing
- problem posing
- analysing

One of the major challenges that face educators is to ensure that learners are given the opportunity of acquiring the skills mentioned. It is important that the materials are carefully selected to ensure this acquisition. I will attempt to highlight some of these skills by using selected problems.
PROBLEM SOLVING

1. John has 3 plastic digits 3; 5 and 2.
1.1 Write down all 2 digit numbers John could make.
1.2 Find the sum of all such 2 digit numbers made.

2. Sipho was trying to work out the correct 3 digit numbers so that he could open his bag.
   He was given the following clues (If 1 correct is given then it means that 1 digit is correct and in the correct place).
   Work out Sipho’s correct number.
   
   394  --------------------------------------- 1 correct
   379  --------------------------------------- 1 correct
   734  --------------------------------------- 1 correct
   793  --------------------------------------- 1 correct

3. Sandy can make 3 bags in 20 minutes.
   Mandy can make 2 bags in 15 minutes.
   How many bags can both make in 2 hours if they work at the same rate?

4. 4 10 18
   4 matchsticks are used to make the first figure.
   10 matchsticks are used to make the second figure.
   18 matchsticks are used to make the third figure.
   3.1 How many matchsticks are used to make the 6th figure?
   3.2 How many figures are possible from 200 matchsticks?
5. Guess the number I stand for.
- I am a 3 digit number.
- My unit’s digit can be counted in 3’s.
- If you add my digits you will get 22.
- I am greater than 500.
- My ten’s digit is 2 less that my unit’s digit.

6. Red beads ● and green beads 0 were arranged as follows:-

●●●00●●●00●●●00●●●00●●●00….

75 beads were used.
How many red beads were used?

7. Mandy is 2 years older than Betsy.
   Betsy is 3 years younger than Carol.
   If Carol is 9 years old then what is Mandy’s age?
   (A) 8  (B) 10  (C) 12

8. The following is in balance.

Which is the heavier?
(A) □  (B) △  (C) ○

9. Find the mystery number.

X | Y

* This is a 2 digit number.
* X is double Y.
* X + Y = 9
(A) 45  (B) 36  (C) 63
10. There are 24 numbered blocks below:-

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<td>24</td>
</tr>
</tbody>
</table>

J moved 3 blocks to the right, 2 blocks down and 1 block left. On which block is J now on?

(A) 16  (B) 17  (C) 18

11. Sweety moves from one circle to the next (A to B to C to D and so on).

Sweety visited 68 circles starting from A.

Which circle did Sweety end up in?

(A) D  (B) E  (C) F

12. Links and Chains

Double the number and add 1 each time to move from one block to the other

1.                       2.                   3.                      4.

13. After giving $\frac{1}{4}$ of his sweets Les still had 24 sweets.

How many sweets did Les give away?

(A) 32  (B) 24  (C) 8
ASTROS? OR SMARTIES? ASTROS? OR SMARTIES?

Marc North
The VULA Programme
mnorth@telkomsa.net

Target audience: Mathematical Literacy teachers and Mathematics Teachers
Grades 8 - 10

Duration: 2 Hours
Max. Participants: 50
Workshop Length: 2 Hours

TARGET AUDIENCE:
The content covered in this workshop will be relevant to teachers of Mathematical Literacy (all grades) and Core Mathematics (especially Grades 8, 9 and 10).

WORKSHOP OUTLINE:
I recently walked into a shop to buy a chocolate and was faced with the dilemma of whether to buy a box of Astros or a box of Smarties. Although Smarties are more expensive, I suspect that a box of Smarties is bigger than a box of Astros. This is difficult to tell, though, because with Astros the weight of the product is not listed on the box.

In this workshop participants will attempt to answer the question that has been bugging choc-a-holics for years – whether it is more cost effective to buy a box of Astros or a box of Smarties. In doing so, participants will calculate the volumes of rectangular boxes, spheres (Astros) and “almost” elliptical objects (Smarties). Participants will also use basic knowledge of statistics to determine the average number of Astros and Smarties in a box.

Part 1: Determining the volumes of the boxes (10 minutes)
Participants will sample one Astro and one Smartie to alleviate any hunger pains that may have arisen. Thereafter, participants will determine and compare the volumes of both the Astro and Smartie boxes. Participants will then be asked to discuss and answer the question: “Does knowing the cost and volumes of each of the boxes provide us with enough information to be able to make a decision about which type of chocolate it would be more cost effective to buy?”
Part 2: Determining the average number of Astros and Smarties in each box (15 minutes)

After all of their exhausting work in Part 1, participants will now be allowed to eat another Astro and Smartie for sustained energy.

Following this, each participant will need to record data on the number of Astros and Smarties in their box. As a group we will collate this data and then determine the average number of Astros and Smarties that one can expect to find in a box of each type of chocolate. A discussion will ensue of what the most appropriate measure of central tendency is for this situation.

Part 3: Determining the volume of an Astro (30 minutes)

After sampling another Astro and Smartie, participants will be provided with the formula for calculating the volume of a sphere and will be shown how the formula is derived. Participants will then use this formula to determine the volume of one Astro and the consequent total volume of all of the Astros in a box.

Part 4: Determining the volume of a Smartie (30 minutes)

By this stage some participants may be passing out from hunger, and so they will be encouraged to eat two Astros and two Smarties before continuing. After recovering, participants will need to strategise ways in which to estimate the volume of each Smartie. Participants will then use the formula for the volume of an elliptical object to estimate the volume of each Smartie and the consequent total volume of all of the Smarties in a box.

Part 5: Determining which chocolate is more cost effective (15 minutes)

Since we no longer need the actual Astros or Smarties, participants will be able to wolf down the remaining chocolates at this point. With hearty smiles and full tummies, participants will be asked to use ratios and rates to determine which type of chocolate is the most cost effective to buy. A suggested method that promotes the calculation of ratios and rates with understanding will be discussed.

MOTIVATION FOR THE WORKSHOP

The purpose of this workshop is to provide participants with an illustration of the process of problem solving – namely, the application of mathematics in order to solve a problem posed in a real-world context. The workshop is also aimed at providing participants with a fun and interesting way to explore the concepts of volume, averages, and ratios and rates.
It is important to note that volumes of spheres and elliptical objects are not part of the Core Assessment Standards in the Mathematical Literacy curriculum. However, it is the opinion of this facilitator that if the equations for determining the volumes of these objects are provided for the learners, then the focus of the activity becomes more about using unknown or unseen equations and making a comparison (both of which are very much a part of Mathematical Literacy) and less about the volumes of spheres and elliptical objects.

**DISCLAIMER:**
The facilitator of this workshop accepts no responsibility for any weight gain that may arise as a result of attending this workshop, nor for any disputes (verbal, physical or otherwise) that may result over whether Astros or Smarties taste better. 😊
LE TOUR DE MATHS –
USING A BICYCLE TO TEACH MATHEMATICS

Marc North
The VULA Programme
mnorth@telkomza.net

Target audience: Mathematical Literacy teachers and Mathematics Teachers Grades 8 - 10
Duration: 2 Hours
Max. Participants: 50
Workshop Length: 2 Hours

TARGET AUDIENCE:
The content covered in this workshop will be relevant to teachers of Mathematical Literacy (all grades) and Grade 8 and Grade 9 Mathematics teachers.

WORKSHOP OUTLINE:
While driving near Bergville in KwaZulu-Natal recently I was amazed to see numerous learners from one particular school riding home on bright yellow bicycles, all sporting equally dazzling helmets. I stopped to ask them where the bicycles had come from and was told that they had been given to the school by the Department of Education as part of an initiative to provide alternative means of transport to children who live very far away from the school. It was then that I decided that given the importance of these bicycles in these children’s lives both as a source of enjoyment and a means of transport, the context of the bicycle would provide the perfect setting for illustrating the role and application of mathematics in the world.

In this workshop participants will be shown how a bicycle can be used as an effective context for introducing students to various mathematical concepts. In particular, participants will explore the mathematics of “ratios” at play in the gears of the bicycle, the mathematics of “speed” through a speedometer, and the mathematics and calculation involved in determining the heart-rate of an athlete.
Part 1: The Great Bicycle Race (30 minutes)
Participants will have the opportunity to ride a bicycle and to experience how gears and a speedometer work. Participants will also have the opportunity to measure their heart-rates using a heart-rate monitor. For the more energetic, there will be a competition for the longest wheelie and the highest bunny-hop, and a prize for the most spectacular fall!

Part 2: The Mathematics of Gears (30 minutes)
Participants will be shown an animated demonstration of how bicycle gears work. They will have the opportunity to handle some gears and will be asked various questions about the gears – in particular why it becomes harder to pedal as bigger gears are selected in the front of the bicycle but easier to pedal as bigger gears are selected at the back.

Part 3: The Mathematics of the Speedometer (30 minutes)
Participants will explore the concept of speed and how a speedometer calculates “average” speed over a journey. The difference between instantaneous speed and average speed will be discussed, and participants will be asked to develop alternative ways for determining average speed if a speedometer is not available.

Part 4: The Mathematics of Heart-Rate (30 minutes)
Participants will be shown how to measure their heart-rates both with and without the use of a heart-rate monitor. A comparison will be made between the heart-rates of the male and female participants, and the concept of “average heart-rate” will be explored. Participants will also calculate their maximum heart-rate using an appropriate formula and will explore how and why athletes use heart-rate as a training tool.

MOTIVATION FOR THE WORKSHOP:
One of the primary purposes of the subject Mathematical Literacy is help students to develop skills that will enable them to become effective problem solvers. In other words, enable them to be able to identify and apply basic mathematical content in order to solve (sometimes) complex real-world problems.

The purpose of this workshop is to provide teachers with an example of a realistic and practical context in which numerous strands of mathematical content have application. The context of a bicycle has been specifically chosen because of the relevance and accessibility of the bicycle in the lives of many teenagers. It is hoped
that by exposing learners to various mathematical content in the context of the bicycle, they will find meaning in the mathematics and will come to experience firsthand the applicability of mathematics in a real-world situation.
Teaching mathematics and numeracy learners to think. Is this fantasy, magic or possible? Meet the goals of the Foundations for Learning and Quality of Learning and Teaching Campaigns by learning how to develop stimulating numeracy or mathematical lessons that teach creative and critical mathematical thinking. In this workshop you will discover which thinking skills to implement and how to develop them in your learners. Hands-on exciting and practical activities will have you learning how to think creatively and critically and applying these new skills to enrich your classroom practice of teaching and learning!

TARGET AUDIENCE:
This workshop will give educators from grade 1 to 9 creative and innovative ideas to make the teaching of mathematics more creative and fun.

DURATION:
This will be a two hour workshop

NUMBER OF PARTICIPANTS
30

DESCRIPTION OF WORKSHOP

OUTCOMES:
By the end of the workshop participants will have reached the following outcomes:

They:

1. will understand and apply brain gym exercises in the classroom
2. will understand what a dynamic Mathematics teacher is
3. will be able to teach thinking activities in the classroom to enhance learner perception
4. will be able to help and support learners to do problem solving through creative and critical thinking
5. will be able to do energetic and lively activities and put magic back into Mathematics by planning well

Application driven, hands-on and learner-centred activities will be done. They include theory and activities on the following:

**BRAIN GYM (10 MIN)**

The programme is based on the idea that all learning begins with movement.

**BRAIN GYM EXERCISES (15 MIN)**

Whole Brain Activation: PACE – Brain gym

- Brain Buttons – 'switching" on the brain before lessons begin
- Cross Crawl – to help coordinate flow between the left and right brain hemispheres
- Cook's hook-ups – Settling nerves before a test or a speech

**DISCUSSING THE MEANING OF WHAT IT MEANS TO BE A DYNAMIC MATHEMATICS TEACHER (10 MIN)**

- Discuss the meaning of 'A Dynamic Mathematical Teacher'
- Reflecting on own teaching practices

**DYNAMIC INVESTIGATIONS (25 MIN)**

- Examples of and progression in investigations
- Making sure your learners understand what you expect of them

**TEACHING LEARNERS TO THINK (30 MIN)**

Thinking can be taught, and it is possible to raise the level of intelligence of any child through the mediation of the teacher.

- Emotional ‘wisdom’ or intelligence
- Self image
- The working of the brain
- Games helping learners to think
WHAT IS THINKING? (15 MIN)
An important goal of teaching is helping learners learn how to think more productively by combining creative thinking (to generate ideas) and critical thinking (to evaluate ideas)

- Logical Thinking
- Mathematical Thinking, involving the processes of:
  - Creative thinking – creating hypotheses, using insight and inspiration
  - Critical thinking – applying logical chains of reasoning
  - Problem solving
  - Creative and critical thinking within the curriculum
- Thinking activities that will enhance thinking skills

Problem Solving
Explore the concepts of problem solving.

PLANNING TO BECOME AN EFFECTIVE AND DYNAMIC TEACHER (15 MIN)
A teacher must never enter a classroom without knowing clearly and explicitly what he or she will communicate during the class period. How you will make instructional content ‘interesting’ to your learners is a second dimension of lesson planning.

Plan for quality lessons with a 'WOW'
- Energetic and lively activities to start your lesson with

REFLECT ON OWN CLASSROOM PRACTICE IN TERMS OF THIS WORKSHOP

MOTIVATION FOR RUNNING THIS WORKSHOP
Teachers will enhance their Mathematical classroom practice as described in the first four Integrated Quality Management System (IQMS) Performance Standards (Lesson Observation).

The following will be gained
Knowledge:
The process of creative, critical thinking and problem solving

Skills:
Thinking Skills: lateral, parallel, constructive, creative and critical thinking

Values:
Value the learners for their own uniqueness.

Learning Outcomes:
LO 1 and 2 as well as the critical outcome of creative thinking and problem solving.

Reflection:
Reflecting on own teaching practice; teaching learners to think and reflect on your own learning.
EXPLORING GRAPHS THROUGH TABLES

DORY REDDY

CALCULATOR PROMOTIONS AND TRAINING

CASIO

The scientific calculator is no longer just a tool to perform calculations. Modern scientific calculators, such as the CASIO ES series have been designed to serve both as a teaching and a learning aid. The many new features of these calculators, if exploited fully, can be used to develop and reinforce mathematical concepts. This workshop session will be used to demonstrate the use of one of the features of the calculator, namely the Table Mode, to enhance learners understanding and knowledge of functions and their associated graphs.

INTRODUCTION

The workshop is intended mainly for teachers who are unfamiliar with the use of the CASIO ES series scientific calculator. Participants will be shown how to use the Table Mode to generate a table of values for a given function.

Teachers will be given the opportunity to use the scientific calculator to select the Start and End values and an appropriate Step size to determine the intercepts on the axes where applicable and explore other key information used in sketching various types of graphs.

Teachers will also learn how the tabulation facility of the calculator can be used

• to search for solutions to a cubic equation.
• to show the systematic growth in simple and compound interest over a given period of time.

The purpose of the exercises is to give participants an opportunity to develop fluency with the use of the Table Mode on the FX 82 ES calculator.

DRAWING GRAPHS USING THE TABLE MODE

1. Draw the graph of \( y = 2x - 3 \).

   • Using the Table Mode input the function \( f(x) = 2x - 3 \).
   • We use the constant \(-3\) to decide on the start and end values. In this case we choose \(-3\) to start and \(3\) to end.

   • The co-efficient of \( x \) is used to determine the step value. In this case we use \( \frac{1}{2} \), since \( 2x - 3 = 0 \) gives \( x = \frac{3}{2} \) which is a multiple of \( \frac{1}{2} \).

A section of the table is shown below. The highlighted rows show the \( y \) and the \( x \) intercepts respectively.
1. We can choose the x intercept (0;−3) and the y-intercept (1.5;0) to draw the straight line graph.

2. **Draw the graph of** \( f(x) = 3x + 4 \).
   - Input \( f(x) = 3x + 4 \).
   - Use the constant 4 to select −4 and 4 as the start and end values.
   - Choose 1/3 for the step value input. Why?

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
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<tbody>
<tr>
<td>1</td>
<td>−3</td>
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<tr>
<td>2</td>
<td>−2.5</td>
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<tr>
<td>3</td>
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</table>

3. **Draw the graph of** \( f(x) = x^2 − 2x − 3 \)
   - Key in the function as before.
• Use the constant -3 to decide on the start and end value inputs. In this case choose -3 and 3 for the start and end values respectively.
• The coefficient of $x^2$ (ie. 1) is entered as the step value.
• Select the x and y intercepts from the table.

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• Axis of symmetry: $x = \frac{\text{sum of } x\text{-intercepts}}{2} = \frac{-1+3}{2} = 1$.
• Substitute $x = 1$ into the function to obtain $f(x) = -4$.
\[\therefore \text{the turning Point is (1; -4).}\]
\[\text{[NB: Use formula if roots are not rational]}\]

4. **Draw the graph of** $f(x) = -2x^2 + x + 6$.
• Input the given function.
• Select the highest prime factor of the constant for the start and end values, viz. -3 and 3.
• For the step value, choose $\frac{1}{2}$ since the co-efficient of $x^2$ is 2 (one of the zeros of the function will have a denominator of 2.)
• Select the x and y intercept from the table.
• Axis of symmetry: \( x = \frac{-1.5 + 2}{2} = \frac{1}{4} \).

• Substituting \( x = \frac{1}{4} \) into the function we obtain \( f(x) = 6\frac{1}{8} \). The turning point is \( \left(\frac{1}{4}; 6\frac{1}{8}\right) \).

5. Draw the graph of \( f(x) = \frac{4}{x-2} + 1 \). Show clearly the intercepts on the axes and the asymptotes of the function.
Explain the error in line 7.

6. **Draw a sketch graph of the function** $y = \cos 2x$, $x \in [-180^\circ; 180^\circ]$.

   - Key in the given function and the Start and End values of $-180$ and $180$ respectively.
   - For the Step value choose $45$ ($90$ divided by the coefficient of $x$).

   ![Graph of $y = \cos 2x$]

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<tr>
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</tr>
<tr>
<td>9</td>
<td>180</td>
</tr>
</tbody>
</table>

   - Using the table (or the graph) one can read off the period and the maximum and minimum values.

7. **Without drawing the graph of the function** $f(x) = 3\sin 3x$, **write down the period, maximum and minimum values and the amplitude of the function**.

   - Input the given function. For the Start and End values we enter $0$ and $360$ respectively.
   - For the Step value enter $30$ ($90$ divided by the coefficient of $x$).
<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>60</td>
</tr>
<tr>
<td>4</td>
<td>90</td>
</tr>
<tr>
<td>5</td>
<td>120</td>
</tr>
<tr>
<td>6</td>
<td>150</td>
</tr>
<tr>
<td>7</td>
<td>180</td>
</tr>
<tr>
<td>8</td>
<td>210</td>
</tr>
<tr>
<td>9</td>
<td>240</td>
</tr>
</tbody>
</table>

8. Sketch the graph of \( f(x) = x^3 + 3x^2 - 4x - 12 \)

- Key in the function.
- Select the highest prime factor of the constant 12 which is 3. For the Start and End values input –3 and 3 respectively. You may change this for other factors of 12 if one or more of the zeros of the function do not appear in the table.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-3</td>
</tr>
<tr>
<td>2</td>
<td>-2</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
</tr>
</tbody>
</table>
SOLVING EQUATIONS & FACTORISING AN ALGEBRAIC EXPRESSION

1. Solve for x : \( x^3 + 7x^2 + 4x - 12 = 0 \)
   - Use the Table mode and let \( f(x) = x^3 + 7x^2 + 4x - 12 \).
   - As before we select the highest prime factors first. For Start, input –3. For End, input 3. For step input 1.
   - The table reflects two zeros for the function, namely -2 and 1. We can use this to obtain the third root. (How?).

\[
f(x) = x^3 + 7x^2 + 4x - 12 = (x + 2)(x - 1)(x + \ldots )
\]

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-3</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
<td>-10</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>-12</td>
</tr>
<tr>
<td>5</td>
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<td>0</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>32</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>90</td>
</tr>
</tbody>
</table>

NOTE : We can try other factors of 12 for the Start and End values. The numbers -6 and 6 for the Start and end values will give all three roots.

2. Factorise completely : \( 3x^3 - 4x^2 - 17x + 6 \)
   - Input \( f(x) = 3x^3 - 4x^2 - 17x + 6 \).
   - The highest prime factor of 6 is 3. Input -3 for Start and 3 for end.
   - We use the coefficient of \( x^3 \) which is 3 to determine our Step value of \( \frac{1}{3} \). (How? and Why?). Part of the table that your calculator generates is shown below.
Since the roots are \( x = -2 \ ; \ \frac{1}{3} \ ; \ 3 \)

The corresponding factors will be

\[
f(x) = 3x^3 - 4x^2 - 17x + 6
= (x + 2)(3x - 1)(x + 3).
\]

### SUMMARISING:

Use Rational Roots Theorem to determine the Start, End and Step values.

For \( f(x) = ax^3 + bx^2 + cx + d \), Start : -d , End : d , Step : \( \frac{1}{a} \)

### COMPARING SIMPLE AND COMPOUND INTEREST

Formula for Simple Growth : \( A = P + (P \times i \times n) \)

Formula for Compound Growth : \( A = P(1+i)^n \)

1. Calculate the Simple Growth on an investment of R10000 over a period of 10 years at an interest rate of 10% per annum.

Use the Table mode on the fx 82/991 ES Plus. Let A be represented by \( f(x) \) and \( x \) the period in years. Hence \( f(x) = 10000 + (10000 \times 0.1 \times x) \)
2. Calculate the Compound Growth on an investment of R10000 over a period of 10 years at an interest rate of 10% per annum compounded annually. Use the Table mode on the fx 82 ES. Let A be represented by \( f(x) \) and \( x \) the period in years. Hence \( f(x) = 10000(1 + 0.1)^x \)

**Simple Growth**

<table>
<thead>
<tr>
<th>Year</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>R11000</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
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<tr>
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<td></td>
</tr>
<tr>
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<tr>
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<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
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</tbody>
</table>

**Compound Growth**

<table>
<thead>
<tr>
<th>Year</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>R11000</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>R16105</td>
</tr>
<tr>
<td>6</td>
<td></td>
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<td>7</td>
<td></td>
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<tr>
<td>9</td>
<td></td>
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<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>
3. Draw a graph to represent the two forms of investment on the same system of axes.

4. Write formulae that would generate only the interest earned each year on both forms of investments illustrated in examples 1 and 2 above. Use the Table Mode to generate the two types of interest.

**Practice Exercises**:

1. Use the Table Mode on the fx 82 ES to generate tables for the following functions and draw their corresponding graphs:

   1.1 \( y = x^2 - 4, -3 \leq x \leq 3, \text{ step } 1 \)
   1.2 \( f(x) = 2\cos x, [0^\circ ; 360^\circ], \text{ step } 90^\circ \)
   1.3 \( f(x) = \sin 2x, [0^\circ ; 180^\circ], \text{ step } 45^\circ \)
   1.4 \( y = 2^x, [-3; 3], \text{ step } 1 \)
   1.5 \( f(x) = \log x, 0 < x \leq 1, \text{ step } 1 \)
   1.6 \( f(x) = \frac{4}{x}, [-4;4], \text{ step } 1 \)
   1.7 \( f(x) = x^3 - 3x^2 - x + 3, [-3;3], \text{ step } 1 \)
   1.8 \( f(x) = -2(x - 3)^2 + 8 \)
   1.9 \( f(x) = 2x^3 - x^2 - 8x + 4, [-3;3], \text{ step } 0.5 \)

2. Given \( h(x) = 4^x \) and \( f(x) = 2(x - 1)^2 - 8 \).

   Sketch the graphs of \( h \) and \( f \) on the same system of axes. Indicate ALL intercepts with the axes and any turning points.
3. Solve for x: \( 4x^3 - 4x^2 - 7x - 2 = 0 \) \[ \frac{1}{2}; 2 \]

4. Factorise completely:
   4.1 \( x^3 - x^2 - 14x + 24 \) \[ (x-2)(x+4)(x-3) \]
   4.2 \( 2x^3 - 3x^2 - 8x + 12 \) \[ (x+2)(2x-3)(x-2) \]
   4.3 \( 8x^3 - 20x^2 + 14x - 3 \) \[ (2x-1)(2x-1)(2x-3) \]

5. Show that the equation \( 2x^3 + 2x = x^2 + 1 \) has only one real solution.
   What is this value of x?

6. Draw sketch graphs of the following:
   \( f(x) = \cos x - \frac{1}{2} \) and \( g(x) = \sin(x + 30^\circ) \) for \( x \in [-120^\circ ; 60^\circ] \).
THE INTERPRETATION OF QUARTILES AND PERCENTILES IN
MATHS LITERACY

Jackie Scheiber
RADMASTE Centre, Wits University

2 hour workshop for FET Maths and Maths Literacy educators

*It is not easy to know what is meant by the Maths Literacy Assessment Standards which talk about interpreting quartiles and percentiles. In this workshop the participants will start working towards an understanding of what is meant by interpreting quartiles and percentiles.*

1) CURRICULUM REFERENCES

<table>
<thead>
<tr>
<th>Grade 10</th>
<th>Grade 11</th>
<th>Grade 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.4.3 Understand that data can be summarised in different ways by calculating and using appropriate measures of central tendency and spread (distribution) to make comparisons and draw conclusions, inclusive of</td>
<td>11.4.3 Understand that data can be summarised and compared in different ways by calculating and using measures of central tendency and spread (distribution) for more than one set of data inclusive of the</td>
<td>12.4.3 Understand that data can be summarised and compared in different ways by calculating and using measures of central tendency and spread (distribution) inclusive of the</td>
</tr>
<tr>
<td>o Mean</td>
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<tr>
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<td>o Mode</td>
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<tr>
<td>o Range</td>
<td>o Range</td>
<td>o Range</td>
</tr>
<tr>
<td></td>
<td></td>
<td>o Quartiles (INTERPRETATION ONLY)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>o Percentiles (INTERPRETATION ONLY)</td>
</tr>
</tbody>
</table>
2) QUARTILES

A *quartile* is any of three values which divide *a sorted data set* into four equal parts, so that each part represents one quarter (¼) of the sampled population.

- The **lower quartile** (Q₁) is the smallest of these three values.
- The **middle quartile** or **median** (M) is the second of these three values.
- The **upper quartile** (Q₃) is the largest of these three values.

**NOTE:**

The word quartile is derived from the Latin word *quartus* meaning fourth.

Suppose the following diagram represents a set of data:

```
xxxxxxxxxxx Q₁ xxxxxxxxxx M xxxxxxxxxx Q₃ xxxxxxxxxx
```

We can reach the following conclusions about the data:

a) One quarter (25%) of all the data has a value that is less than, or equal to, the value of the lower quartile Q₁.

b) One half (50%) of all the data has a value that is less than, or equal to, the value of the median M.

c) One half (50%) of all the data has a value that is more than, or equal to, the value of the median M.

d) Three quarters (75%) of all the data has a value that is less than, or equal to, the value of the upper quartile Q₃.

e) One half (50%) of all the data lies between the lower quartile Q₁ and the upper quartile Q₃.

f) Three quarters (75%) of all the data has a value that is more than, or equal to, the value of the lower quartile Q₁.

**ACTIVITY:**

The heights of all the girls and all the boys in Grade 11 in a particular school were measured. The heights were ordered, and were then organised in the following way:

<table>
<thead>
<tr>
<th></th>
<th>GIRLS</th>
<th>BOYS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of the shortest boy/girl</td>
<td>120 cm</td>
<td>120 cm</td>
</tr>
<tr>
<td>Lower quartile Q₁</td>
<td>130 cm</td>
<td>145 cm</td>
</tr>
<tr>
<td>Median M</td>
<td>140 cm</td>
<td>148 cm</td>
</tr>
<tr>
<td>Upper quartile Q₃</td>
<td>150 cm</td>
<td>160 cm</td>
</tr>
<tr>
<td>Height of the tallest boy/girl</td>
<td>160 cm</td>
<td>170 cm</td>
</tr>
</tbody>
</table>
1) Comment on the heights of the girls compared to the heights of the boys

2) How many boys are taller than the tallest girl?

3) PERCENTILES

When a set of data is arranged in size order, the $n^{th}$ percentile is the value such that $n\%$ of the data must be less than or equal to that value. $n$ must be a whole number from 1 to 99.

This means that you can find the 1$^{st}$ percentile, the 2$^{nd}$ percentile, the 3$^{rd}$ percentile, up to the 99$^{th}$ percentile.

- Low percentiles always correspond to lower data values
- High percentiles always correspond to higher data values.

For example:

If 70% of the population were shorter than you, then your height would be said to be at the 70$^{th}$ percentile.

NOTE:

- The word percentile comes from the Latin word *per centum* which means “per hundred”.
- Percentiles are generally used with large sets of data so that dividing it up into 100 equal parts seems realistic.
The lower quartile corresponds to the 25\textsuperscript{th} percentile; the median corresponds to the 50\textsuperscript{th} percentile and the upper quartile corresponds to the 75\textsuperscript{th} percentile.

A score at the 84\textsuperscript{th} percentile states that 84\% of people taking the test scored the same or lower and 16\% scored higher.

A score at the 50\textsuperscript{th} percentile is a median score: 50\% score higher and 50\% score the same or lower.

A percentile may or may not correspond to a value judgement about whether it is “good” or “bad”. The interpretation of whether a certain percentile is good or bad depends on the context of the situation to which the data applies. In some situations, a low percentile would be considered “good”; in other contexts a high percentile might be considered “good”.

Example:
If a learner, when given a test back, found that he or she had a mark of 33. Would that be a good mark or a bad mark?

- Based only on the information given, it would be impossible to tell. The 33 could be out of 35 possible marks and be the highest mark in the class, or it could be out of 100 possible marks and be the lowest mark, or anywhere in between. The mark that is given is called a raw score.
- Suppose the raw score of 33 is transformed into a percentile rank of 98. This means the student did better than 98\% of the students who took this test. In that case the student would feel pretty good about the test.
- If, on the other hand, a percentile rank of 3 was obtained, the student might wonder what he or she was doing wrong!

Example 1:
The time taken by a learner to finish a test was 35 minutes. This time was the lower quartile of the time taken to finish the test. Interpret the lower quartile in the context of the situation.

Solution
- 25\% of learners finished the exam in 35 minutes or less
- 75\% of the learners finished the exam in 35 minutes or more.

\textit{NOTE: Here a low quartile or percentile would be considered good, as finishing more quickly on a timed test is desirable. (If you take too long, you might not be able to finish.)}
Example 2:
On a test out of 20, the 70th percentile for the number of correct answers was 16. Interpret the 70th percentile in the context of this situation.

Solution:
• 70% of the learners answered 16 or fewer questions correctly.
• 30% of the learners answered 16 or more questions correctly.

NOTE: Here a high percentile would be considered good, as answering more questions correctly is desirable.
**ACTIVITY:**
Answer the following questions in pairs:

1) For runners in a race, a low time means a faster run. The winners in a race have the shortest running times.
   a) Is it more desirable to have a finish time with a high or a low percentile when running a race?

   b) The 20th percentile of running times in a particular race is 5.2 minutes. Write a sentence interpreting the 20th percentile in the context of the situation.

2) A bicyclist in the 90th percentile of a bicycle race between two towns completed the race in 1 hour and 12 minutes.
   a) Is he amongst the fastest or slowest cyclists in the race?

   b) Write a sentence interpreting the 90th percentile in the context of the situation.

3) For runners in a race, a higher speed means a faster run.
   a) Is it more desirable to have a speed with a high or a low percentile when running a race?

   b) The 40th percentile of speeds in a particular race is 12 km per hour. Write a sentence interpreting the 40th percentile in the context of the situation.
4) On an exam, would it be more desirable to earn a mark with a high or low percentile? Explain.

5) Mina is waiting in line at the Traffic Department to renew her licence. Her waiting time of 32 minutes is the upper quartile of waiting times.
   a) Is that good or bad?
   
   b) Write a sentence interpreting the upper quartile in the context of the situation.

6) In a survey that collected data about the salaries earned by teachers, Mary found that her salary was in the 78th percentile. Should Mary be pleased or upset by this result? Explain.

REFERENCES
• Bloom Roberta: *Descriptive Statistics: Practice 3: Interpreting Percentiles* http://cnx.org/content/m18845/latest/
SETTING A MATHS LITERACY EXAMINATION
Jackie Scheiber
RADMASTE Centre, Wits University

A 2-hour workshop aimed at Maths Literacy teachers

In this workshop I will start off by going through the technical requirements for a Maths Literacy examination as specified by the Maths Literacy Subject Assessment Guidelines (SAG) and the Maths Literacy Exam Guidelines.

We will also briefly go through the content listed in the Core Assessment Standards, and will discuss the four taxonomy levels required in every examination as listed in the SAG and the Exam Guidelines.

The educators will then be given resource material and will have the opportunity to set their own examination questions using this resource material. These questions will be shared with the rest of the group, so that by the end of the session, everyone will have a question bank that they can take back with them to the classroom.
UNEXPECTED RELATIONSHIPS BETWEEN SHAPE, SURFACE AREA AND VOLUME

Jessica Sherman
RADMASTE Centre
University of the Witwatersrand
GET Senior Phase, FET and Maths Literacy

REQUIREMENTS FOR THE SESSION

- 35 teachers can be accommodated.
- The workshop will take 2 hours.
- Suitable table for working in groups of 4.

This investigation can be used in the classroom at most levels in the high school. It is an alternative to the method of giving formulae for volume or surface area of polyhedra followed by endless exercises. It helps to develop learner’s thinking about relationships between space and shape giving meaning to this Learning Outcome. It also helps to train learners to look beyond what we think is obvious and to verify all our assumptions.

This activity is an adaptation of an investigation which is found in the 4th unit of the book “Teaching and Learning Maths in Diverse Classrooms” produced by SAIDE (South African Institute for Distance Education). The whole book is available in pdf format on the internet. The relevant pages will be provided.

It was first presented to the GDE ACE teachers who were teaching at a GET level. They found it challenging but engaged with it in their own ways and came to some conclusions which they hadn’t anticipated.

INTRODUCTION

Why are products on the shelves in supermarkets usually in cylindrical or rectangular containers. Do they look bigger than triangular shaped containers. Why
are soup tins usually cylindrical?................. This activity might help to answer these questions.

Each group will be provided with a couple of sheets of light cardboard out of which they will be expected to build certain different types of containers. They will then investigate relationships between the size and shape of these containers (polyhedra).

**CONTENT**

These are some of the aspects which will form the focus for the workshop:

- Conjectures about the relationships between size and shape. e.g. Does the triangular prism have more volume than the rectangular prism when built by the same size piece of card?
- Predicting relationships between various polyhedra. Enough practical work will be done so that predictions can be made.
- Testing and verifying of conjectures and predictions. These will be done both practically and algebraically. Formulae can be brought in to test conjectures and to make accurate comparisons.
- Participants will be encouraged to look for ways to extend the activity.
- Refining conclusions. This will involve sharing of conclusions with the rest of the groups and finalizing our thoughts.
THE NUMBER SYSTEM BOX

Sue Southwood
VULA Mathematics Project, Hilton College

Target audience: Educators in the Senior and FET Phases who subscribe to the philosophy “Students learn when they are actively engaged.” and are interested in different ways of teaching the concept of numbers.

Audience size: Thirty
Time: One hour

Numbers and their classification seem quite straightforward to an educator, but somewhat boring. Learners find the topic esoteric and obscure. The workshop presents some ideas for making the topic more fun. Some examples are set out below.

Part 1: 5 minutes

**Competition:** The learner who chooses the lowest positive integer that no one else has chosen will win a prize. Write your name and number on a piece of paper. Only one entry per learner.

Part 2: 5 minutes

**Assignment:** Consider the items on the line below. Discuss their differences and similarities.

\[ \frac{22}{7}, 3.14, 3.141592654, \pi, \text{circumference/diameter}, \pi, \text{Einstein's birthday} \]

Part 3 15 minutes

**Activity:** You have been given three pages divided into squares, each of which contains 36 numbers. Cut out the squares and sort the numbers into the following piles:

- Non-real numbers
- Irrational numbers
- Rational numbers
- Integers
- Natural numbers

(These sets are not discrete so this leads to an interesting discussion.)
Part 4: 30 minutes

In groups: Make your own number box as per the demonstration. Place each number from the previous activity in its correct compartment.

(Each workshop participant will be provided with an empty photocopy paper box, a pair of scissors, sticky tape, two nets printed on A4 card – one labeled N and the other Z – and a black marker pen.)

Extension:

Internet search: Find out why the set of integers is called Z and the set of rational numbers is called Q.

Requirements: Data projector and screen.

Tables and space

Boxes, scissors, card nets, sticky-tape, activity sheets will be provided by the presenter
QUESTIONS NOT TO ASK IN A MATHS LITERACY LESSON

Hamsa Venkat and Aarnout Brombacher
Wits University and Brombacher & Associates

In this workshop we share examples of questions we believe to be inappropriate for use in Maths Literacy lessons. We use these examples to share our rationales for why we consider these tasks to be inappropriate and to stimulate discussion around the issues we raise. We then share and discuss evidence from our experiences of research and teacher development in Maths Literacy that point to aspects of the policy messages that appear to be relatively widely accepted in this, the 4th year of implementation, such as the need to bring in both mathematical content and everyday contexts. This is followed by our use of the examples to highlight the reasons why this condition on its own – without an anchoring in a problem that can be made sense of – is insufficient to build mathematical literacy in a holistic and integrated way.

The session is motivated by ongoing evidence of difficulties for teachers in designing tasks that meet the aims of Maths Literacy. It is aimed at Maths Literacy teachers and teacher educators, as well as researchers interested in the area of designing and working with contextualised tasks in mathematics.
DEVELOPING ALGEBRAIC THINKING IN THE PRIMARY SCHOOL

Nelis Vermeulen

Faculty of Education and Social Sciences, Cape Peninsula University of Technology

vermeulenc@cput.ac.za

Target audience: Grades 3 to 8 teachers

Duration: 2 hours

Max. number of participants: 50

Description of workshop content:

Participants will discuss the question: “what is algebra?” (15 min)
- discuss the question: “what is algebraic thinking?” (5 min)
- analyse the NCS (GET) to determine what the curriculum prescribes regarding algebra and algebraic thinking (15 min)
- receive a short presentation on the above activities (10 min)
- do activities that can develop primary school children’s (and participants’) algebraic thinking (50 min)
- design their own activities that can develop primary school children’s algebraic thinking (20 min)
- closure (5 min)

Motivation for running the workshop:

For many years now, mathematics educators have been concerned about the quality of learners’ knowledge and understanding of elementary algebra. In the 1980’s and 1990’s, numerous research projects had been undertaken internationally, and a wealth of knowledge regarding learners’ misconceptions in algebra had been accumulated. Based on these insights, researchers have made a number of recommendations to improve the practice of elementary algebra teaching. These not only included a change in the approach to teaching elementary algebra in the early high school years, but also essential groundwork to be done in the primary school years. Curricula implemented in a number of countries since 2000, notably Australia, New Zealand, the USA and South Africa, reflect this need for the development of algebraic thinking in the lower grades, before learners formally encounter algebra in the high school.
This workshop hopes to assist participants to understand what algebraic thinking is, and how it can be developed in the primary school, by analyzing the NCS (GET) for Mathematics, exposing them to typical activities, and to engage them in designing their own activities.

**WORKSHEET 1**

1. **In groups of about 4, discuss the question: “What is algebra?”**
   
   **Note:** Formulate a final answer for the group, and be prepared to be called upon to share your answer with all the participants.
   
   **Total time: 15 min**

2. **In groups of about 4, discuss the question: “What is algebraic thinking (or algebraic reasoning)?”**
   
   **Note:** Formulate a final answer for the group, and be prepared to be called upon to share your answer with all the participants.
   
   **Total time: 10 min**

3. **Analyse the NCS (GET) to determine what it states regarding:**
   
   (Total time: 15 min)

   3.1 **What is algebra?**

   3.2 **What should be done in the primary school to develop learners’ algebraic thinking (focus on LO1 and LO2)**

4. **Short presentation on algebra, algebraic thinking, and the “Big Ideas of Algebra” (10 min)**
WORKSHEET 2
Activities to develop learners’ algebraic thinking
Work in groups
Number patterns:
1. If you start with 3 and count in fours, you get the following sequence:
   3; 7; 11; 15; 19; …
1.1 Write down the next three numbers.
1.2 Find the 20\textsuperscript{th} and 50\textsuperscript{th} numbers in the sequence.
1.3 Will 487 be a number in the sequence? If yes, what is its position in the sequence?
1.4 What “Big Idea(s)” of algebra is/are emphasized by 1.1 to 1.3?
2.1 Is it true that the sum of two even numbers is always an even number? Investigate.
2.2 Is it true that the product of an even number and an odd number is always an odd number? Investigate.
2.3 Is it true that the sum of any number of consecutive odd numbers is always a square number? Investigate.
2.4 Is it true that the sum of two consecutive numbers (e.g. 1 and 2) is a prime number?
2.5 What “Big Idea(s)” of algebra is/are emphasized by 2.1 to 2.4?
3. Study the following pattern:

<table>
<thead>
<tr>
<th>Row 1</th>
<th>Row 2</th>
<th>Row 3</th>
<th>Row 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>5</td>
<td>10</td>
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<tr>
<td>3</td>
<td>3</td>
<td>6</td>
<td>11</td>
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<tr>
<td>4</td>
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<td>7</td>
<td>12</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>8</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>9</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td></td>
<td>15</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>16</td>
</tr>
</tbody>
</table>

3.1 How many numbers are in Row 50?

3.2 What is the first number in Row 21?

3.3 What “Big Idea(s)” of algebra is/are emphasized by 3.1 and 3.2?

WORKSHEET 3
Activities to develop learners’ algebraic thinking
Work in groups

Geometric patterns:
Each of the following shapes is made from matches. In each case (1 to 4)
• draw the next two pictures
• describe in words how many matches are required to move from one picture to the next picture
• complete the table that follows after the shapes

1.

![Figures 1 to 4](Fig.1, Fig.2, Fig.3, Fig.4)

2.

![Figures 1 to 4](Fig.1, Fig.2, Fig.3, Fig.4)
What “Big Idea(s)” of algebra is/are emphasized by these activities?
WORKSHEET 4
Activities to develop learners’ algebraic thinking
Work in groups

1. Brick wall:

Instructions for your learners:
Study the “growing” brick wall.

1.1 Describe in words how you continue the brick wall.

1.2 How many bricks will be necessary for
(a) Brick wall 4?
(b) Brick wall 5?
(c) Brick wall 10?
(d) Brick wall 50?

1.3 Which brick wall can be built with (a) 55  (b) 101  (c) 200 bricks?

Questions for workshop participants:

1.4 In what grade(s) could this activity be given?

1.5 Can this activity be used to develop learners’ algebraic thinking? If yes, how?

1.6 Which of the “Big Ideas of Algebra” are addressed here?
2. Another brick wall:

Repeat questions 1.1 to 1.6 for this brick wall.
WORKSHEET 5
Activities to develop learners’ algebraic thinking
Work in groups

Activity 1:
Often the opportunity to develop learners’ algebraic thinking arises spontaneously in class.
A Gr 2 learner makes the following comment:
“Miss, I have discovered that \(3 + 5 = 5 + 3\).”

1.1 How will you respond to this learner?

1.2 Is there an opportunity here to develop learners’ algebraic thinking? If yes, how?

1.3 Which of the “Big Ideas of Algebra” are addressed here?

Activity 2:
Grade 3 or 4 learners are given the following problem: “Farmer Jackson plants young orange trees. He plants 18 rows, each containing 32 trees. How many orange trees did he plant?”
You observe that several learners answer the question by “breaking down” one or both of the numbers, e.g.
Learner 1: \(32 = 30 + 2\). \(18 \times 30 = 540\) and \(18 \times 2 = 36\).
Therefore \(18 \times 32 = 540 + 36 = 576\)

Learner 2: \(18 = 20 – 2\). \(32 \times 20 = 640\) and \(32 \times 2 = 64\).
Therefore \(32 \times 18 = 640 – 64 = 576\)

Learner 3: \(32 = 30 + 2\) and \(18 = 20 – 2\). \(30 \times 20 = 600\), \(30 \times 2 = 60\), \(2 \times 20 = 40\) and \(2 \times 2 = 4\). Therefore \(32 \times 18 = 600 – 60 + 40 – 4 = 576\)
2.1 Are the methods of all three learners acceptable?

2.2 Is there an opportunity here to develop learners’ algebraic thinking? If yes, how?

2.3 Which of the “Big Ideas of Algebra” are addressed here?
WORKSHEET 6  
Activities to develop learners’ algebraic thinking  
Work in groups

1. **Perimeter (distance around the perimeter of a shape):**

   We as teachers know that the perimeter of any rectangle is given by the formula $P = 2 \times (\text{length} + \text{breadth})$, or then $2(l + b)$.

1.1 Where does this formula come from?

1.2 How should we teach this formula?

1.3 Can this formula be used to develop learners’ algebraic thinking? If yes, how?

1.4 Which of the “Big Ideas of Algebra” can be addressed in the process?

2. **Area of a rectangle:**

2.1 Do you think learners can calculate the area of the rectangle shown alongside in different ways?

2.2 If yes in 2.1, show the different methods.
2.3 Is there an opportunity here to develop learners’ algebraic thinking? Discuss.

2.4 Which of the “Big Ideas of Algebra” (if any) can be addressed in the process?
WORKSHEET 7  
Activities to develop learners’ algebraic thinking  
Work in groups

Seeing the generality through the specifics, using technology:

1.1 What would you prefer when you buy an item at a 20%-discount sale:  
That the 20% discount is subtracted first from the price, and then 14% VAT is added,  
Or  
That the 14% VAT is first added to the price, and then the 20% discount is subtracted?

Investigate by studying specific cases by hand, or preferably generated by an Excel spreadsheet.

Your conclusion:

1.2 Will your conclusion in 1.1 only apply for 20% discount and 14% VAT (or for any %)
1.3 Is there an opportunity here to develop learners’ algebraic thinking? Discuss.

1.4 Which of the “Big Ideas of Algebra” (if any) can be addressed in the process?
NOTES

1. **What is algebra?**
   - Algebra is the language for investigating and communicating most of Mathematics. Algebra can be seen as generalised arithmetic, and can be extended to the study of functions and other relationships between variables. A central part of this outcome is for the learner to achieve efficient manipulative skills in the use of algebra. (NCS (GET)).
   - Algebra enables us to express and manipulate generalized numerical statements.

2. **What is algebraic thinking (or algebraic reasoning)?**
   Looking for, and identifying regularities/patterns/relationships, generalizing them, formulating conjectures and investigating/justifying/refuting them.

   Algebraic thinking can be developed and supported through the use of “algebrafied” tasks (in arithmetic and geometry/measurement) that help children look for regularities/patterns/relationships.

3. **What are some of the “Big Ideas” of algebra that need to be developed as part of algebraic thinking?**

   **1. GENERALISING**
   Every mathematical technique or procedure, such as addition, extending a pattern, solving a linear equation, etc. is a “general method” for resolving a class of similar problems. Thus, every mathematics lesson affords opportunities for learners to generalize for themselves.

   It is important that learners articulate what they do/did during the generalising tasks in order to become aware of the generality of their actions. Teachers can use “standard” arithmetic and geometric tasks to develop and support learners’ awareness of generalization. In these tasks learners do not only focus on finding the (correct) answer to the problem, but become aware that the same procedure is carried out each time a similar problem is solved, e.g.

<table>
<thead>
<tr>
<th>“Standard” task</th>
<th>Generalising the “standard” task</th>
</tr>
</thead>
<tbody>
<tr>
<td>What is the next number in the row of</td>
<td>What is the next three numbers in the row of numbers: 1; 3; 5; 7; ...?</td>
</tr>
<tr>
<td>numbers: 1; 3; 5; 7; ...?</td>
<td>And the 50th number? And the 100th number?</td>
</tr>
</tbody>
</table>

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Investigate whether the following is true:
3 + 4 + 5 = 3 x 4

Investigate whether the following are true:
6 + 7 + 8 = 3 x 7
11 + 12 + 13 = 3 x 12

Can you make more such statements?
Can you make similar statements with five numbers, e.g. 1 + 2 + 3 + 4 + 5 = 5 x 3?
Will it work for 7 numbers? 4 numbers?

Find the sum of 24 and 42.

Find the sum of 16 and 61, 35 and 53, 27 and 72. What do you see in each case?

2. PATTERNS and RELATIONSHIPS
While developing learners’ notion of relationships, the notions of variables/variability and generality are simultaneously developed.

<table>
<thead>
<tr>
<th>“Standard” task</th>
<th>Generalising the “standard” task</th>
</tr>
</thead>
<tbody>
<tr>
<td>What is the next number in the row of numbers: 1; 3; 5; 7; ...?</td>
<td>What is the next three numbers in the row of numbers: 1; 3; 5; 7; ...? And the 50\textsuperscript{th} number? And the 100\textsuperscript{th} number?</td>
</tr>
<tr>
<td>Blocks are stacked to form a wall, as shown below. How many block are needed to build a wall 5 levels high?</td>
<td>Blocks are stacked to form a wall, as shown. How many block are needed to build a wall 5 levels high? And 10 levels high? And 50 levels high?</td>
</tr>
<tr>
<td>Addition and multiplication tables to find individual answers</td>
<td>Addition and multiplication tables to find relationships between rows and columns.</td>
</tr>
<tr>
<td>Calenders are used to find dates</td>
<td>Calenders are used to find relationships between dates.</td>
</tr>
</tbody>
</table>
3.  **STRUCTURE and EQUIVALENCE**

While developing learners’ notion of structure in arithmetic, the notions of equivalence and generality (and often relationships) are simultaneously developed.

Here learners must become explicitly aware of the properties of operation, in particular the distributive, commutative and associative properties. This can be achieved by allowing learners to become aware of what they often intuitively do when doing arithmetic calculations, such as breaking down (decomposing) numbers and regrouping them. For example, in a relatively simple calculation such as $18 + 27$, learners would often proceed as follows: $18 + 27 = 10 + 8 + 20 + 7 = 10 + 20 + 8 + 7 = 30 + 8 + 2 + 5 = 30 + 10 + 5 = 45$. Implicit in this are both the commutative and associative properties. Learners must articulate what they are doing.

Concurrent to developing an increasing awareness of the properties of operation, the notion of equivalence is developed, i.e. that different (numeric) expressions are equal in value, for example:

\[
3 + 5 = 5 + 3 \quad \text{(commutative property)}
\]

\[
3 \times (5 \times 7) = (3 \times 5) \times 7 \quad \text{(associative property)}
\]

\[
6 \times (20 + 7) = 6 \times 20 + 6 \times 7 \quad \text{(distributive property)}
\]

4.  **LANGUAGE**

The NCS only requires learners for the first time in grade 8 to “describe, explain and justify observed relationships or rules in own words or in algebra”. Therefore, the symbolic language of algebra need not be introduced in the lower grades; rather, learners should express their generalizations in words. However, where learners spontaneously start replacing words or traditional symbols such as □ with algebraic symbols such as $x$ or $a$, they should be allowed to do so. They should however be required to clearly articulate why they do so, and what the meaning of the $x$ or $a$ is, and the teacher should satisfy herself that the learner understands this “new” symbol as a placeholder for any suitable number.

**LIST OF REFERENCES**


WEB 2.0 TOOLS IN THE CLASSROOM
USING SOCIAL NETWORKS, BOOKMARKING, RSS, POD/VIDEO-CASTS, WIKIS, BLOGS AND TWITTERING IN THE MATHS/MATHEMATICAL LITERACY CLASSROOM FOR INTERACTIVE AND COLLABORATIVE LEARNING AND TEACHING

Maggie Verster
ICT4Champions

Target audience:

This workshop is aimed at all phase teachers. Teachers should be at the intermediate level of computer skills

Maximum no. of participants:

As many as can fit into a computer laboratory.

Description of content of workshop:

This workshop will show teachers how to use “FREE and FRIENDLY web 2.0 tools" to engage a new generation of 21st century digital native learners in a more interactive and collaborative way. We will brainstorm some ideas on how to use every tool practically in the classroom.

During the workshop we will:

- Discuss what web 2.0 tools are and why it is important for us it in our classrooms using a social network (www.mathsliteracy.co.za ). (20 minutes)
- Register for a twitter account (www.twitter.com) and use it to give feedback and ask questions during the workshop. Follow fellow teachers! (15 minutes)
- Discuss a given maths/mathsliteracy topic by creating a blog post (http://mathsliteracy.wordpress.com) and using it to comment and keep track of twitter questions. (15 minutes).
- Create a collaborative mindmap (www.bubbl.us) about a classroom topic and embed some of the mindmaps in the blog. (20 minutes)
- Demonstrate what a wiki and RSS are and how to use it. Teachers can then add and change pages. (20 minutes)
- Subscribe to and use a social bookmarking system to track and store collaborative resources (20 minutes)
Motivation for doing the workshop

Our learners grow up in a digitally enabled environment and will be required to use computers and digital media responsibly when they leave our care. We therefore need to learn how to use these tools ourselves in order to support our classroom activities and engage our learners in interactive and meaningful ways. Social media tools encourage

- Group-centred learning
- Project-based learning
- Problem solving
- Inquiry learning
- Collaborative learning
- Experiential learning

These tools can also assist us to stay on top of our subject through learning networks and professional development. We can use it to teach smarter not harder!
This workshop is a feast too good to miss. A Hors d’oeuvres is a starter at the beginning of a meal, and this workshop is titled as such because it looks at exciting ways of starting your Mathematics lessons.

Intermediate and Senior phase

2 hours

Approximately 30 participants

What is a Maths Starter?

- 5 – 10 minute
- Mental exercise
- Beginning of each lesson
- Unrelated topic
- Sets the tone of the lesson.

Disadvantages:

- Time

Advantages:

- Fun!!!!
- Encourages mental arithmetic.
- Revision tool.
- Saves time being wasted.

Examples:

- Guess the Magic Number
- Target Number
- Fizz Buzz
- BINGO
- Loop Game
• Paired cards
• Crack the code
• Building Bricks
• … and for dessert…Your Age by Chocolate Maths

During this workshop participants will look at the examples of Maths starters, and have some fun engaging in them.
They will then be divided into groups according to the grade that they teach, and they will develop some of their own Maths starters.
This poster presents the results of testing Intermediate and Senior Phase learners at Eight Schools in the Overberg Region of the Western Cape in March 2007, November 2007 and November 2008.

The sample size of each class was 40 learners, and these were selected in such a way that the sample included learners from each quartile in the classes. About 1 300 learners took part in the tests. Numbers were assigned to learners, and the same set of learners were tested each time.

1. Introduction

The Enlighten Education Trust is an NGO situated in Hermanus. The core business of the Trust is the training, re-training and upgrading of teacher qualifications from Early Childhood Development to post-graduate university accredited qualifications. However, our focus is on the training and support of teachers in their own classrooms in schools which have been underachieving.

In order to assess progress and to provide quantitative data for teachers we have designed baseline tests for the Intermediate and Senior phases, both for Literacy (language) and Mathematics. Only the results of the mathematics tests are discussed in this presentation, but similar trends were noted for Literacy. One test paper was used for grades 4, 5 and 6, while another paper was used for grades 7, 8 and 9. The time allowed for each test paper is 1 hour.

Most test questions were based on work that teachers had done with learners in class, while some were based on ideas from textbooks. (See references).

The tests were administered in March 2007, November 2007 and November 2008. The same test papers were used for successive tests. The results of the tests were analysed and are presented in graphic form.
Separate tests were developed for the Intermediate and Senior phases. The Intermediate Phase test paper is directed at Grade 5 level and the Senior Phase test at Grade 8 level. Grades 4 and 6 were not expected to do well on the tests, while Grades 7 and 9 were expected to achieve higher marks.

The test questions were set on the different learning outcomes as outlined in the NCS.

Tests were translated into English, Afrikaans and Xhosa so that all learners could be tested in their home language.

The tests were administered in March 2007 November 2007 and November 2008. It is expected that the tests will be administered again in November 2009.

Learners who participate in the tests are selected by the Trust (not by their teachers) from class lists and according to an equal spread of the best, average and slower learners in each grade.

Results of tests are made available to the schools and are discussed at meetings with the principal and staff.

Schools A, C and H cater for grades 4 through to 9, schools D, E and G for grades 4 to 7, while schools B and F for grades 8 to 12.

2. Results of mathematics tests.

The results of the tests for mathematics are presented in a set of bar graphs.

Slides 1 and 2 show the averages of all participating schools for the Intermediate and Senior phase. In general the Senior Phase results were poorer than for the Intermediate phase.

It is interesting to note that the results of the November 2008 grade 4 learners shows a significant “dip”. Although several reasons for this have been suggested, it is still not perfectly clear why this has happened. Similarly, the grade 6 group did not fare well in November 2008. On a more positive note, a small improvement is apparent with the grade 5 group.

The results of the grade 9 learners indicate a slow, steady decline from March 2007 to November 2008.

Slides 3 to 8 give the results for grades 4 through 9 for each school for the March 2007, November 2007 and November 2008 tests. School J was tested in November 2008 to serve as a reference or control school.(School J was a former model C school).

From slide 3 it is clear that only schools D and H did not show the decline in results for November 2008.
In slides 3, 4 and 5 we see that results for the entire Intermediate Phase (grades 4, 5 and 6) have become worse at school C.

Slide 8 shows that only at School A did the grade 9 learners show some improvement. There was a weakening of grade 9 results at all the other schools.

Slide 8 at School C. One of our invigilators was ill during the March 2007 tests, and a teacher at the school supervised the class. However, it was found that the teacher was openly helping the learners and the results obtained were very high. The results for this test have been declared invalid and were excluded.

Slides 9 to 14 show the results for each school in the different learning outcomes. Two problem areas have been identified from this data. One of these is LO5 Measurement. Learners have difficulty in the correct use of measuring instruments, reading and interpolating any scales, such as rulers, thermometers etc with little regard to the correct application of units. There is also a problem with telling time on analog clock faces. The other problem area seems to be with LO1 Numbers, Operations and Relationships. Learners from grade 4 up to grade 9 seem to have great difficulty with fractions. With addition of fractions the learners in some Intermediate phase classes simply add the denominators instead of using a LCM.

References:

Enlighten Education Trust

Comparison of Grade Results (Intermediate Phase)

Comparison of Grade Results (Senior Phase)

Grade Averages by School. Mar 07, Nov 07, Nov 08

Grade 5 Averages by School. Mar 07, Nov 07, Nov 08
Enlighten Education Trust
Grade Averages by School Mar 07, Nov 07, Nov 08

Grade 6 Averages by School.
Mar 07, Nov 07, Nov 08

<table>
<thead>
<tr>
<th>School A</th>
<th>School C</th>
<th>School D</th>
<th>School E</th>
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<th>School H</th>
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Grade 7 Averages by School.
Mar 07, Nov 07, Nov 08

<table>
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Grade 8 Averages by School.
Mar 07, Nov 07, Nov 08

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Grade 9 Averages by School.
Mar 07, Nov 07, Nov 08

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</table>
Enlighten Education Trust
School Averages per Learning Outcome. Mar 07, Nov 07, Nov 08
Enlighten Education Trust
School Averages per Learning Outcome. Mar 07, Nov 07, Nov 08