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PROBABILITY USING TREE DIAGRAMS

Ian L Atteridge

Dainfern College

A gambler's dispute in 1654 led to the creation the theory of probability by two French mathematicians, Blaise Pascal and Pierre de Fermat. Probability allows humans to model random events that happen in the real world. It allows us to make sound business decision as well as the decision to hang out washing to dry.

USING TREE DIAGRAMS TO DETERMINE PROBABILITY

Introduction

Tree diagrams are a simple, visual method in recording and calculating the probability of both dependent and independent events. I will, using a graded series of relevant examples, show how tree diagrams can be used to determine probability.

Content

Introduction to tree diagrams as a summary of possible outcomes. Sketching tree diagrams for independent events and determining resultant probability. Sketching tree diagrams for dependent events and determining resultant probability. Introduction of techniques to make complex situations and the resultant diagrams, simpler.

Conclusion

Tree diagrams are a simple, yet powerful technique in determining probability. A thorough grasp of the technique is essential; especially if the learner whishes to do Paper 3 in the FET phase.

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SKETCHING A CUBIC FUNCTION

TRADITIONAL METHODS VS USING TECHNOLOGY

Ian L Atteridge, Dainfern College

With the availability of powerful calculators there are alternate techniques to sketching a cubic function. Learners with different learning styles and abilities benefit from being able to choose from a variety of techniques. In this presentation I will introduce a lesser known technique: Use of the Table function of the calculator. In my experience learners who lack self-confidence find security in using technology to eliminate basic arithmetic errors.

SKETCHING THE CUBIC FUNCTION

Introduction

There are many steps to sketching a cubic function. It is often considered as the culmination of algebra in the FET phase. I will show how the important points of a cubic function can be extracted using a variety of methods but placing emphasis on the lesser known method of using a table function on a calculator. The basis of this technique is ‘trial and error’.

Content

Determining the shape will be done intuitively and using the table function. Determining the y-intercept will be done by recognizing the constant term. Determining the x-intercept(s) will be shown using the factor theorem and inspection as well as the table function. X-values of the turning points will be calculated, after differentiation, using the table function. The concavity of these points will be shown intuitively and reinforced using the table function. The point of inflection will be mentioned.

Conclusion

The alternate technique presented aid learners in sketching a cubic function, and allow them to achieve a measure of success without getting bogged down in the accompanying calculations. These methods can also be used by more advanced learners to allow them to visualize other functions.

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HOW I TEACH NUMBER PATTERN: Linear and Quadratic patterns

Mr Bossii, S.J (Matsogella Secondary School**)**

Mrs Malapile, C (Nkgoru Secondary School**)**

Mathematics Educators

Limpopo Province (Waterberg District)

INTRODUCTION

A pattern is a repeated decorative design on a surface such as wall papers, cloth or carpet. A regular arrangement of objects to represent shapes is called linear and quadratic number patterns. An example of these patterns may be the arrangement of triangles, dots, squares, etc. we work a lot with number patterns in mathematics from lower grades, i.e. natural numbers, whole numbers, odd numbers, even numbers, etc. In Grade 10 we learned about linear pattern while in Grade 11 we will learn more about quadratic pattern.

DESCRIPTION OF CONTENT

Learning Outcome 1: Number and Number Relationships

When solving problems, the learner is able to recognize, describe, represent and work confidently with numbers and their relationships to estimate, calculate and check solutions.

Assessment standard: 11.1.3

Investigate number patterns (including but not limited to those where there is a constant second difference between consecutive terms in a number pattern, and the general term is therefore quadratic) and hence:

(a) make conjectures and generalizations

(b) Provide explanations and justifications and attempt to prove conjectures.

PRESENTATION

Number patter is an arrangement of numbers according to some rule which generate the patterns in the number pattern. The arrangement of numbers is called sequence. The numbers in the sequence are called terms of the sequence.

Term

$$T_2$$

Term number

The deciphering of a Quadratic pattern

Before we can decipher the quadratic pattern, let's make sure we can work out the rule of a linear pattern.

Example 1: (Linear Pattern)

3; 5; 7; 9; ……….The constant difference is *2*.

To work out how this gives us the formula, we use the general term for a linear pattern which is $mx + c$. In this case we'll use $mn + c$.

If $n = 1$, then $m(1) + c = 3\,(T_1) \longrightarrow$ (i)

If $n = 2$, then $m(2) + c = 5\,(T_2) \longrightarrow$ (ii)

We can solve simultaneously to find *m*.

(ii) – (i) $m = 2$ This means that our formula will start with *2n*. If the constant difference is *3* then the formula will start with *3n* etc.

We then subtract *2n* from each of the terms of the pattern and get *1* as the answer giving us the rule or formula for the pattern as *2n + 1*.

n	Pattern	1st difference	*2n*	Pattern minus *2n*
1	3	2	2	3 – 2 = 1
2	5	2	4	5 – 4 = 1
3	7	2	6	7 – 6 = 1
4	9		8	9 – 8 = 1
	Formula		*2n*	+1

Example 2:

2; 7; 12; 17; ……….The constant difference is *5*.

n	Pattern	1st difference	*5n*	Pattern minus *2n*
1	2	5	5	2 – 5 = – 3

2	7	5	10	7 – 10 = – 3
3	12	5	15	13 – 15 = – 3
4	17		20	17– 20 = – 3
	Formula		$5n$ – 3	

Example 3: (Quadratic Pattern)

Given the pattern 5; 11; 19; 29; ……………, how to work out the general term or formula.

We begin the checking for differences to see if it's linear or quadratic and we find that it has a constant difference of 2. We can use the same idea as we did for the linear pattern to work out the general term.

n	Pattern	1st Diff	2nd Diff	n^2	Pattern minus n^2 (New pattern)	1st Diff	$3n$	New pattern – $0n$
1	5	6	2	1	5 – 1 = 4	3	3	4 – 3 = 1
2	11	8	2	4	11 – 4 = 7	3	6	7 – 6 = 1
3	19	10	2	9	19 – 9 = 10	3	9	10 – 9 = 1
4	29	12		16	29 – 16 = 13		12	13 – 12 = 1
	Formula			n^2			$+3n$	$+1$

Number pattern with second difference constant

This type of sequence is generally represented by $T_n = an^2 + bn + c$ which is equivalent to the quadratic equation or the equation of the parabola.

The following can be used to find the formula (or general term) of the sequence.

First find out whether the sequence is linear or quadratic
Determine the difference (which is either first or second difference)
If the first difference is constant then the pattern is linear

If the first second difference is constant, then the terms of the sequence has the general representation of $T_n = an^2 + bx + c$.

Determine the value of *a, b* and *c*.

Substitute back into the general equation. That will give you the formula (or general term) of the sequence.

To find: *a, b* and *c*:

$T_n = an^2 + bn + c$

If $n = 1$	If $n = 2$	If $n = 3$	If $n = 4$
$T_1 = a(1)^2 + b(1) + c$	$T_2 = a(2)^2 + b(2) + c$	$T_3 = a(3)^2 + b(3) + c$	$T_4 = a(4)^2 + b(4) + c$
$= a + b + c$	$= 4a + 2b + c$	$= 9a + 3b + c$	$= 16a + 4b + c$

T_1; T_2; T_3; T_4; …………….

$a + b + c$; $4a + 2b + c$; $9a + 3b + c$; $16a + 4b + c$; …………………….

First

$3a + b$ $\quad 5a + b$ $\quad 7a + b$

Second difference

$2a$ $\quad 2a$

$2nd.Diff = 2a$

$$a = \frac{2nd.Diff}{2}$$

$$1st..Diff = 3a + b \qquad\qquad 1st..term = a + b + c$$

$$b = 1st.Diff - 3a \qquad\qquad c = 1st..term - a - b$$

Example: Determine the formula or the general term of the sequence:

5; 11; 19; 29; 41; . . . .

Solution: 5 11 19 29 41

6 8 10 12

2 2 2

$$a = \frac{2}{2} \qquad b = 6 - 3(1) \qquad c = 5 - 1 - 3$$

$$a = 1 \qquad b = 3 \qquad c = 1$$

$$T_n = an^2 + bn + c$$

$$T_n = (1)n^2 + (3)n + (1)$$

$T_n = n^2 + 3n + 1$ ⟵ **(General term)**

It is not always easy to determine the rule that defines the pattern. When looking for a rule, we can take the following into consideration:

If the numbers in the pattern are getting bigger, then try to see if something is added to each term, or maybe each term is multiplied by something.

If the numbers in the pattern are getting smaller, then try to see if something is subtracted from each term, or maybe each term is divided by something.

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How I Teach –

Ratios: The POWERTOOL of Mathematical Literacy

Andrew Gilfillan

St. Anne's Diocesan College

Outline

Do your learners struggle to deal with the simplest of problems involving rates and conversions? Dividing when they should multiply and not really knowing the difference? Well fret no further, Ratios are here! The true POWERTOOL of Mathematical Literacy (and your everyday life!!!)

Target Audience

FET – Grade 10, 11 & 12 Mathematical Literacy Teachers

GET – Grade 9 Senior Phase Mathematics

Anyone struggling to find an easy way to work with rates & ratios.

Duration

30 Mins

Motivation

A common challenge that learners in Mathematical Literacy face is how to work effectively with rates, unit conversions and other forms of direct mathematical relationship between two quantities. By applying the common, garden-variety ratio to this type of problem, I have found learners are able to confidently work with these concepts and are more empowered in their everyday lives.

Content Description

Tested in the varied environments of both government and private school classrooms, I have developed a unique method of utilizing ratios called "The Pirate Method" that has significantly empowered my learners. I look forward to demonstrating this method and its application.

Session Structure

20 mins: Introduction & demonstration with application to a wide range of problem types

10 mins: Discussion

HOW I TEACH BOX AND WHISKER AND DISTRIBUTION CURVES

PRESENTER: Mr M.J. Hlongoane (Nkgoru Secondary School)

Mr.J.M. Mogashoa (Matopa Secondary School)

Limpopo Province (Waterberg District) "Mathematics Educators"

INTRODUCTION

In statistics we usually find out the results by analyzing, organizing and representing data with the aim of finding patterns, or rule to help us to organize data so that we can deal with it better or remember it better. Data is organized in different ways and conclusions are drawn from the results.

INTEGRATION OF CONTENT

Learning Outcome 4: Data Handling and Probability

The learner is able to collect, organise, analyse and interpret data to establish statistical and probability modes to solve related problems.

Assessment Standard 10.4.1 (a)

Collect, organise and interpret univariate numerical data in order to determine:

Measures of central tendency (mean, median and mode) of ungrouped data, and know which is the most appropriate under given conditions

Measures of dispersion: range, quartile, inter-quartile range

Assessment Standard 11.4.1 (a)

Calculate and represent measures of central tendency and dispersion in univariate numerical data using:

The five number summary (maximum, minimum and quartiles)

Box and whisker diagrams

DESCRIPTION OF CONTENT

In many experimental situations we tend to distrust extreme measures as they may have resulted from poor measurement or behaviour that is not usual. As a result, extreme measures are often discarded. For this reason the middle section of a data set, where most of the data lies, gives you the best description of the data.

The median divides the distribution of a data set into two halves.

Each half can them be divided in half again;

The lower quartile (Q_1) is the median of the first half of the data set

The upper quartile (Q_3) is the median of the second half of the data set.

Quartiles therefore divide the distribution into four equal parts.

Set of data items divided into 4 equal parts:

Lower Quartile

Median

Upper Quartile Q_3

1, **5**, 7, **8**, 8, **14**, 17

The lower quartile (Q_1) is a quarter of the way through the distribution,

The middle quartile which is the same as the median (M) is midway through the distribution.

The upper quartile (Q_3) is three quarters of the way through the distribution.

The median and the upper and lower quartiles tell you about the middle of a distribution but not about the extremes. If you include the minimum and maximum values, along with the median and quartiles, you get the five-number summary of a set of data.

The five-number summary for a set of values consists of:

Minimum: the smallest value in the set of data

Lower quartile (Q_1): the median of the lowest half of the values

Median (M): the value that divides the data into halves

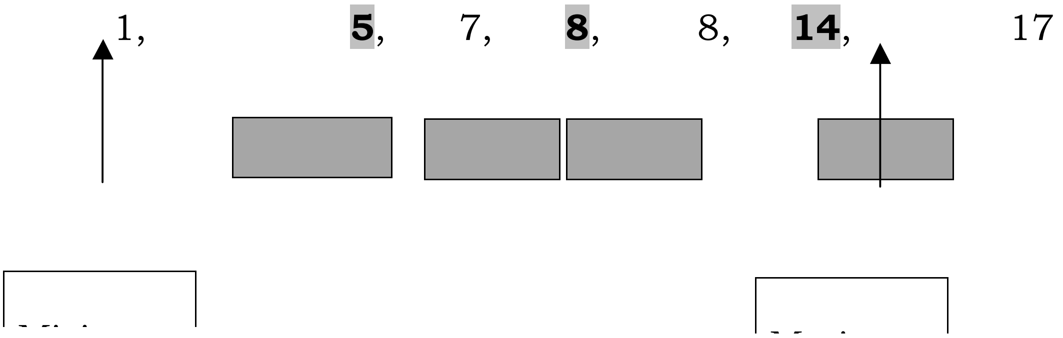
Upper quartile (Q_3): the median of the upper half of the values

Maximum: the largest value in the set of data

Lower Quartile

Median

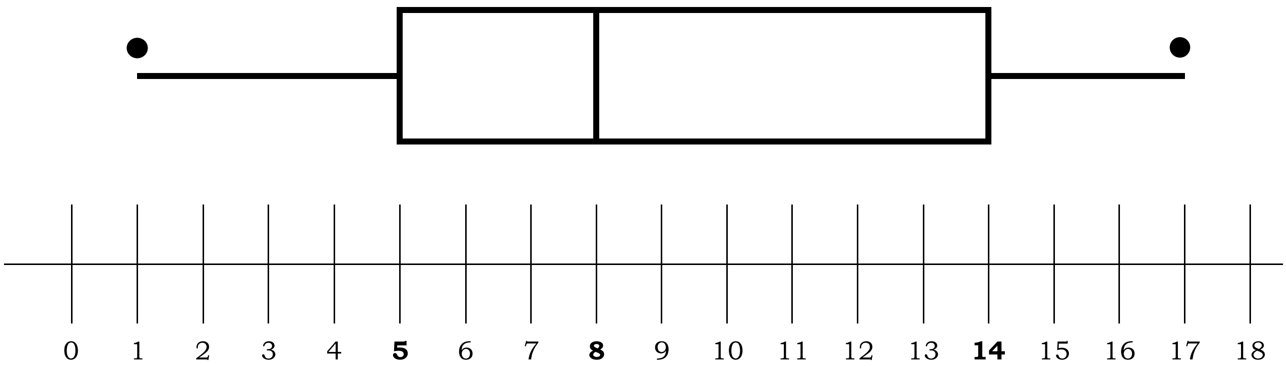
Upper Quartile



A box-and-whisker diagram is a way of representing data that shows the median, the quartiles and the maximum and minimum values of the data set.

To draw the diagram:

Draw a number line, starting at the minimum value of your data and ending at the maximum value of your data.
Draw a line at the median (M).
Draw lines at the lower and upper quartiles (Q_1 and Q_3).
Connect the lower and the upper quartiles to form a box. The median will fall somewhere within the box.
Plot a point at the minimum value and join this to the box – the left hand whisker.
Plot a point at the maximum value and join this to the box – the right hand whisker.



Note:

The lines for the first and third quartiles are joined to make a box corresponding to the central 50% of the data items,

The whiskers are taken out from the box to the maximum and minimum values.

Remember that the quartiles and the median divide the data items into four equal groups. Each section has the **same number of data** items in it. However the boxes and the whiskers may be of varying lengths as these are influenced by the **actual values of the data** items.

Some box and whisker diagrams are symmetrical (showing a very even spread of data) others might be more squashed up (showing skewed data).

SYMMETRICAL AND SKEWED DATA

If the data values are spread out equally or nearly equally or either side of the mean, the data set is said to be **symmetrical**.

The difference between the mean and median ≈ 0

$\therefore$ mean – median $= 0$

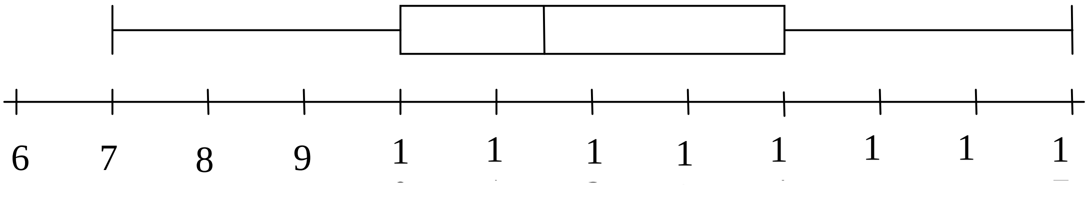
Example : **7; 8; 10; 11; 11; 12; 13; 14; 14; 17**

Mean = $\bar{x} = 11.7$

Median = 11.5

$\therefore$ mean – median $= 11.7 - 11.5$

$= 0.2$



If the data values are spread out more on one side of the median than the other, the data is **skewed**.

Skewed to the left (**Negatively skewed**)

The difference between the mean and median < 0

∴ mean – median < *0*

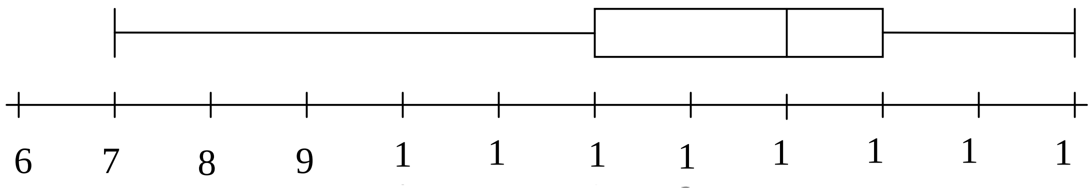
Example : **7; 8; 12; 12; 13; 14; 14; 14; 15; 16; 17**

Mean = $\bar{x} = 12.9$

Median = 14

∴ mean – median = 12.9 – 14

= – 1.1



Skewed to the right (**Positively skewed**)

The difference between the mean and median > *0*

∴ mean – median > *0*

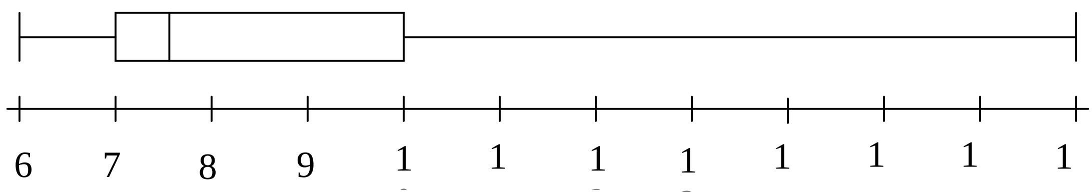
Example: **6; 6; 7; 7; 7; 8; 9; 10; 11; 17**

Mean = $\bar{x} = 8.8$

Median = 7.5

∴ mean – median = 8.8 – 7.5

= 1.3



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HOW I INTRODUCE APPLICATIONS OF CALCULUS

JJ Khumalo

Kwantatshana Secondary School

and the Vula Mathematics Project, Hilton college

INTRODUCTION

When my learners have finished the 'skills' part of calculus, I introduce the applications section with this investigation. They enjoy doing things in mathematics much more than sitting and listening. I also enjoy teaching this way.

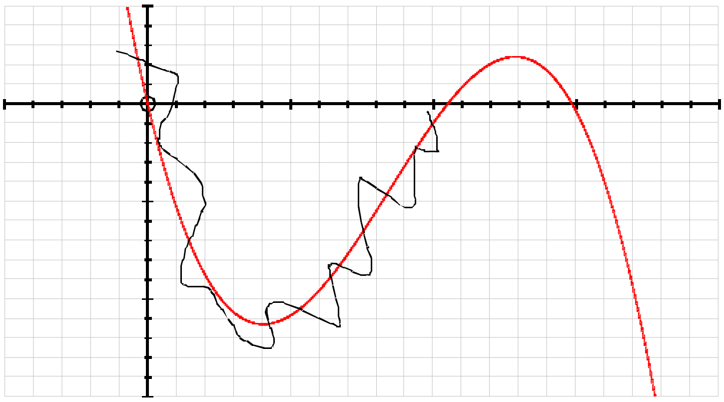
The aim of the investigation is to find the maximum area that can be enclosed by an open-topped made from A4 paper. We start with lots of A4 paper, make the boxes, fill in tables, draw a class graph from it and end with a cubic graph – which they have already learned. This is a good example of modeling. It moves from the concrete (cutting up paper) to the abstract (algebraic graph)

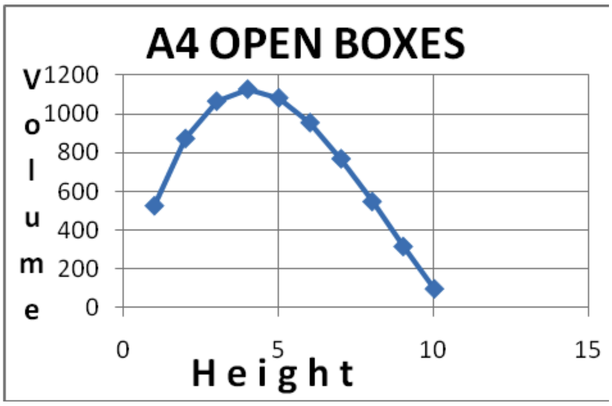
Investigation

We start with a discussion.

We measure the A4 paper and agree that its dimensions are 29,7cm by 21cm. I explain how to make an open-topped box by cutting out identical squares for the corners. The class is divided into ten groups. Each group is given its own box to make and measure. They enter their box dimensions and area into a table. We draw a graph from this table. The graph shows that there is a definite size which will give a maximum volume. Next we develop a mathematical model which is the cubic.

HEIGHT	LENGTH	BREADTH	VOLUME
1	27.7	19	526.3
2	25.7	17	873.8
3	23.7	15	1066.5
4	21.7	13	1128.4
5	19.7	11	1083.5
6	17.7	9	955.8
7	15.7	7	769.3
8	13.7	5	548
9	11.7	3	315.9
10	9.7	1	97





CONCLUSION

I use my laptop with Excel for the table and Autograph for the graph. This is interesting and motivating for my learners. However I also did this activity before I had a laptop and it worked very well. Your learners do lots of talking and discussion which is good.

Mrs SM Southwood helped me prepare this document.

HOW I TEACH HISTOGRAM AND FREQUENCY POLYGON

Presenter: Tlou Robert Mabotja
Institution: Waterberg District (Limpopo Province)
Curriculum Advisor "Mathematics"
Robetlou@yahoo.com

INTRODUCTION

Learners need to be told about the importance of statistics in today's world; surely science is misunderstood and misused by today's people. Thus the reason why some people say statistics is very important, where as others don't trust statistics. This cannot be disputed that learning about statistics and how it affects our lives is an important study in the education of any citizen living in this world.

DESCRIPTION OF CONTENT

Learning Outcome 4: Data Handling and Probability

Assessment Standard: 10.4.1 (b)

Represent data effectively, choosing appropriately from:

- Histogram (grouped data)
- Frequency polygon

Participants should recall that the individual pieces of information used in statistics are called data and for these pieces of information to give meaning they need to be organised, analysed and represented. Therefore the collected data can either be grouped or ungrouped.

Grouped data can be categorised in two ways:

- Discrete data
  - Is data that has exact value (often whole numbers) and it often collected by counting.
  - An example of discrete data can be learner's marks or shoe sizes.
  - Data in this case is used to draw bar graph.

Continuous data

Can be any value within a certain range (not exact values).

Data can be collected by measurement, i.e. heights, kilograms, weight etc.

Example:

The mass in grams of 60 seedlings ready for planting were:

21	29	25	18	22	26	33	28	29	42
26	36	32	34	14	41	31	27	33	31
15	38	42	31	36	18	34	31	25	19
40	29	36	20	29	27	19	40	37	38
45	42	24	37	27	40	40	24	19	16
33	47	45	32	15	25	35	28	28	20

Frequency table

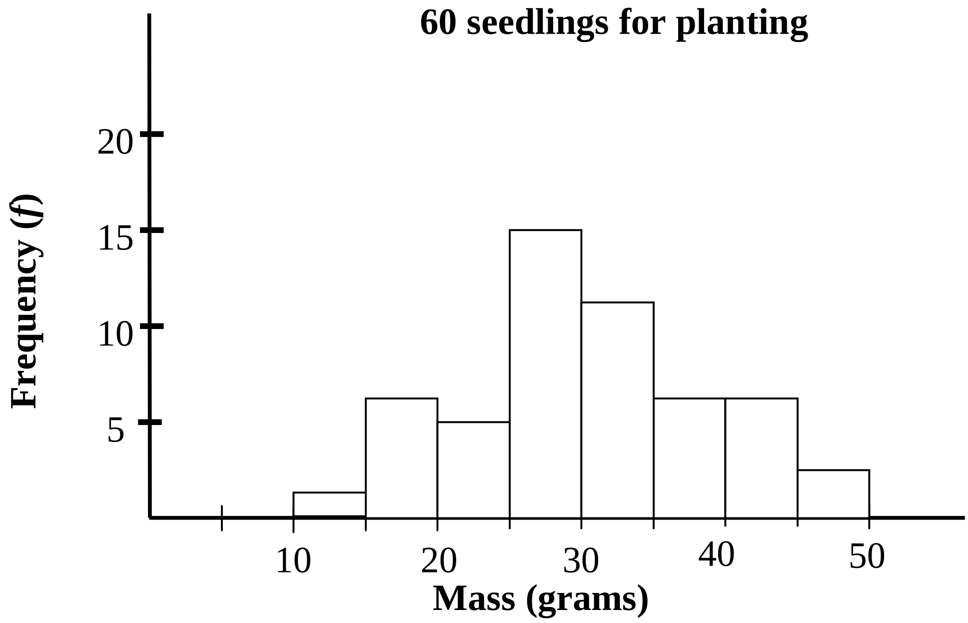
It helps to keep count the number of times a data item occurs by keeping a tally; the number of times the item occurs is called the frequency of that item.

Complete the following frequency table using the above data:

Mass in grams (W)	**Frequency (*f*)**
$10 \le W < 15$	1
$15 \le W < 20$	8
$20 \le W < 25$	6
$25 \le W < 30$	15
$30 \le W < 35$	11
$35 \le W < 40$	8
$40 \le W < 45$	8
$45 \le W < 50$	3
Total	**60**

Data that is continuous can be used to draw a histogram where the horizontal axis shows the class interval and the vertical axis shows the number of frequencies. One should also remember unlike bar graph a histogram has no gaps between the bars.

<u>Histogram</u>



Frequency polygon

Both Histogram and frequency polygon are frequency graphs. The difference between them is that a histogram is made up of bars, where as a frequency polygon is a line graph. The polygon is made up from the lines of the bars and the horizontal axis.

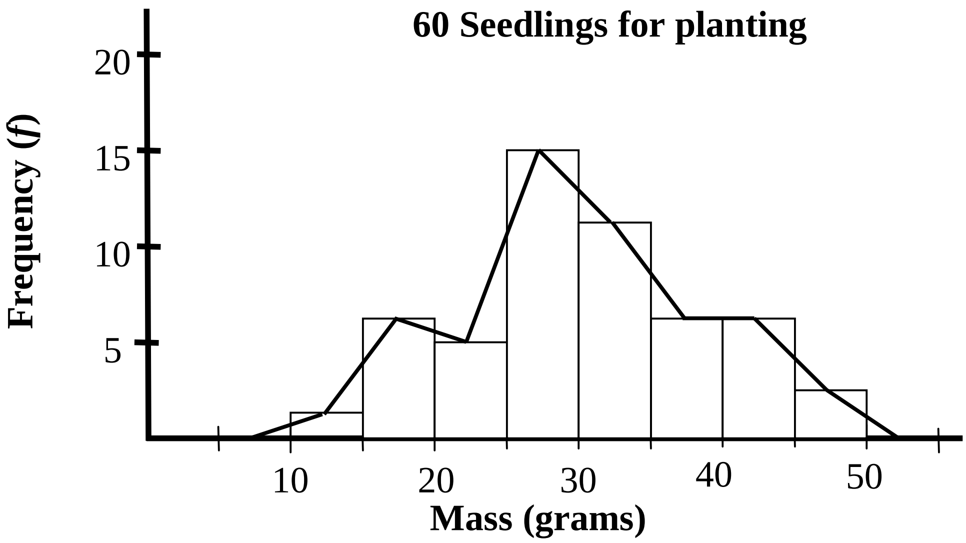
NB: Polygon is a closed 2 – Dimensional shape made up of line segments.

Drawing a frequency polygon:

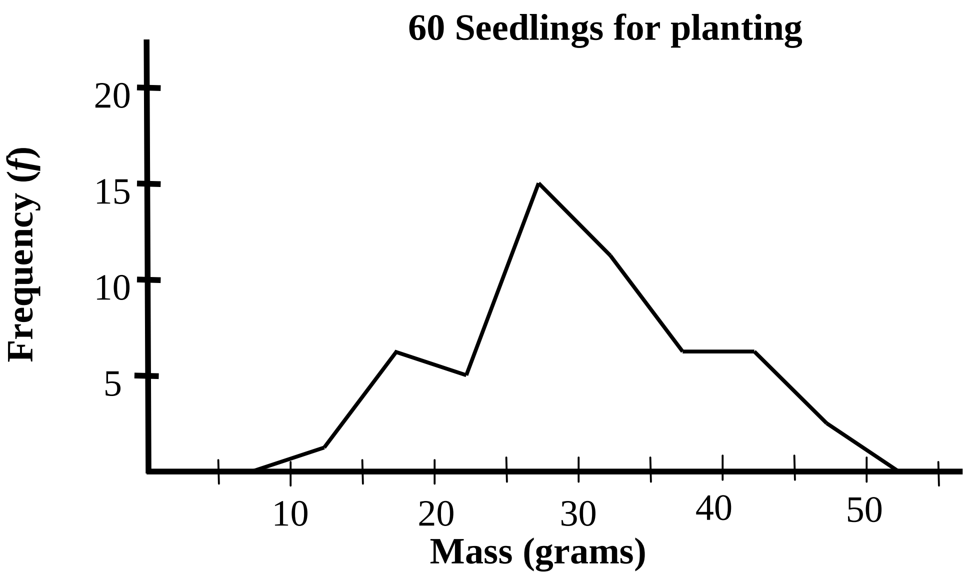
- Finding the midpoints of the class intervals as ½ (lower class boundry + upper class boundry).
- Mark the midpoints of the top line of each bar.
- Join points, from left to right, with straight lines to make the lines into a polygon, join the ends to the horizontal axis.

To find the points on the horizontal axis, one have to imagine that there is an extra class interval at each end of the "real classes". These classes have zero (0) frequency, so the tops of their "bars" are on the horizontal axis. Mark the midpoints of these imaginary bars join these two midpoints to the line graphs.

This same information from Histogram can be represented in a **frequency polygon** by joining the midpoints of the bars of the histogram like this:



and then erasing the bars like this:



A **frequency polygon** is a line graph of the data. It is a useful way of representing the data as it gives another way of reading frequency. Frequency polygons are often used to compare different frequency distributions. You may have seen line graphs in the newspaper that are actually frequency polygons.

CONCLUSION

Histogram and frequency polygon graphs are very important since they give a picture of grouped data they represent. However taken by the data themselves, they often give a limited view of the whole picture. Through this graphs one can detect how the rest of the data are grouped, whether closely grouped or scattered more widely.

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USING TECHNOLOGY TO TRANSFORM THE BASIC TRIG GRAPHS

Sicelo Majola

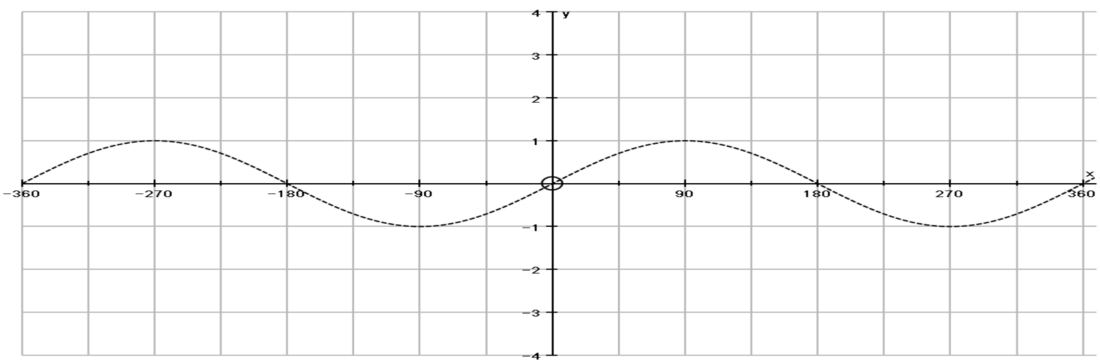
Tshanibeswe High School and the Vula Laptop Project at Hilton College

INTRODUCTION

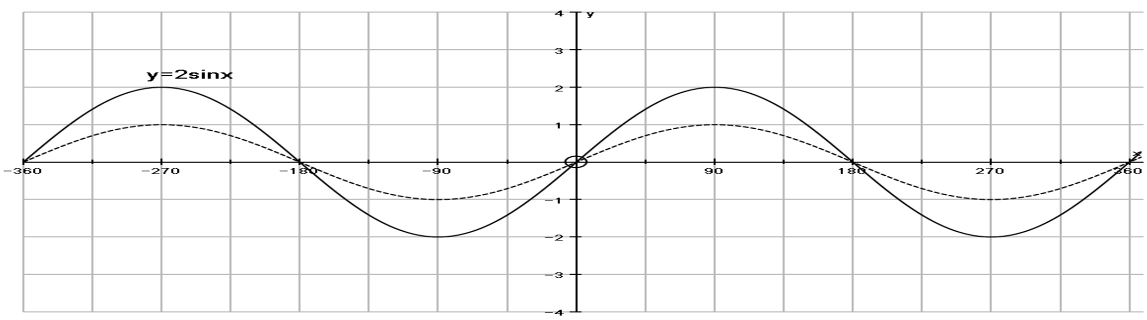
My presentation will focus on types of transformation that the sine , cosine and tangent graphs can undergo. We will draw a number of graphs to establish the effect of ‘a’ ; ‘q’ ‘k’ and ‘p’ from y = a sin k (x + p) + q . Most learners find it very challenging to draw graphs of this kind specially where there is vertically and horizontally translation.

Since I started to use computer to help me to make my presentation more meaningful and exciting in the classroom, I have noticed my learners have good attitude and are being extrinsically motivated in doing Mathematics. I will use Maths software (Autograph) to demonstrate how I teach transformation in trigonometric graphs. We will use the sine graph and then generalize the information discovered to the cosine graph and tangent graph.

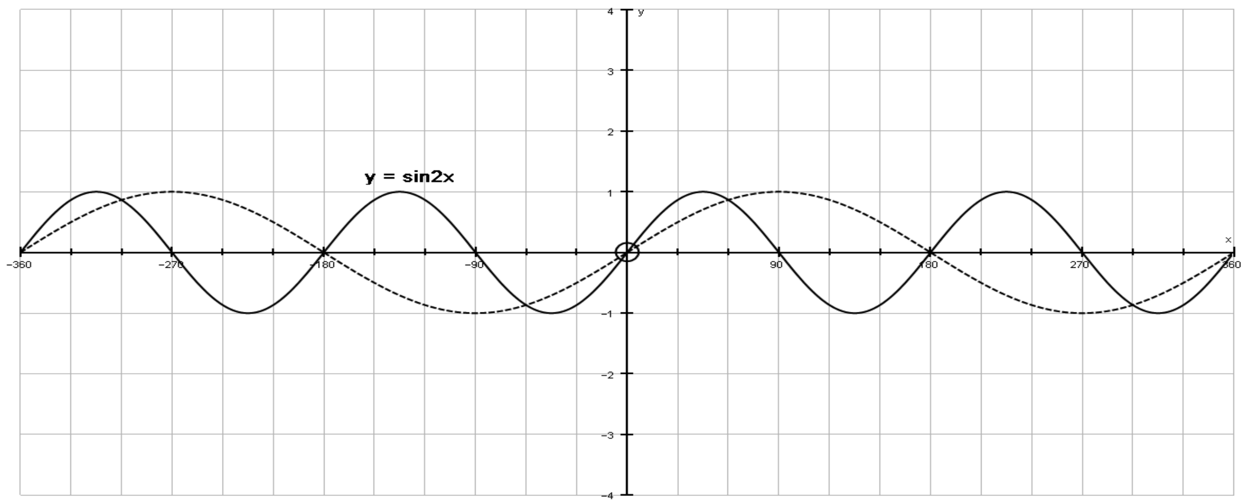
The dotted graph below is : y = sin x



The effect of ‘a’ in y = a sinx – changes the amplitude of the graph.

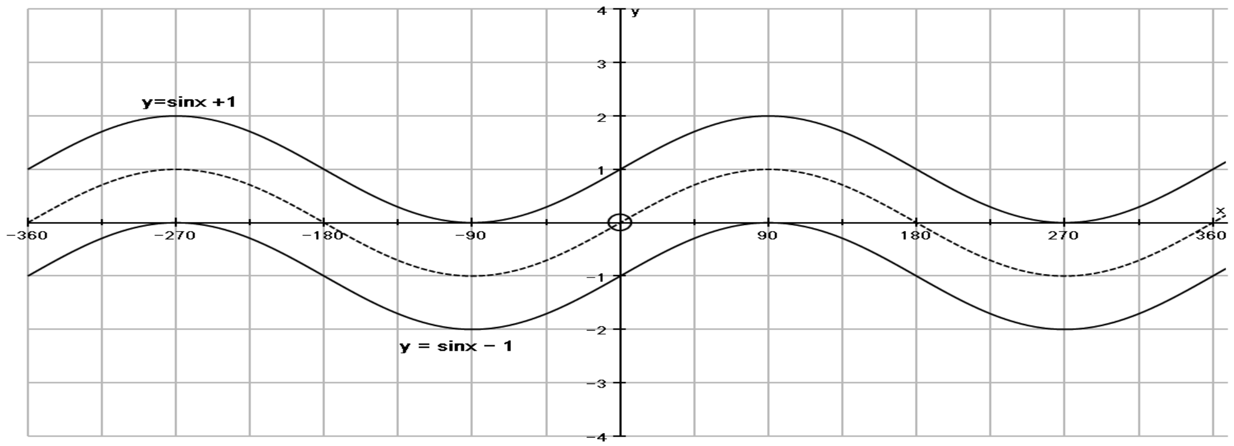


The effect of 'k' in y = sin kx :



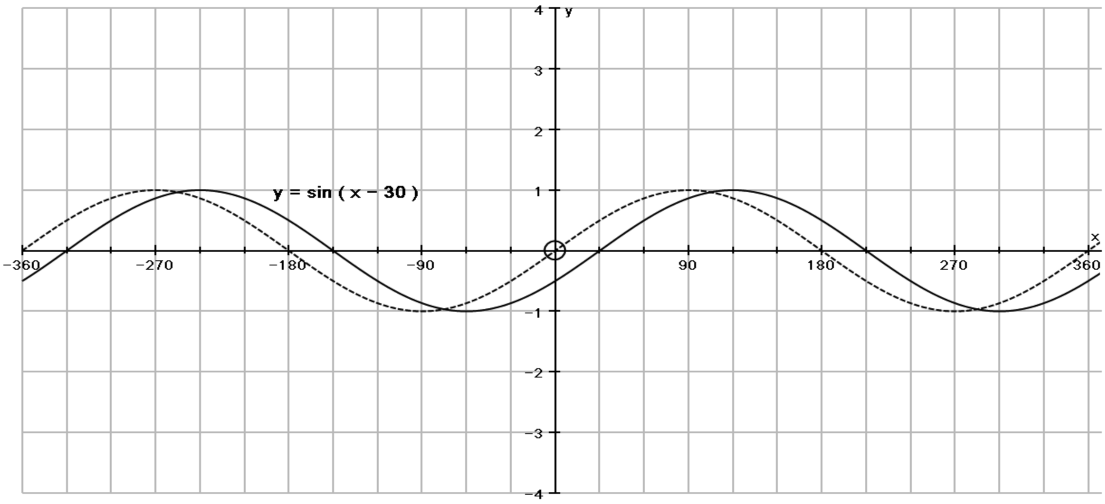
k has the effect of changing the number of cycles that are shown for the given domain by changing the period of the graph , making it $\frac{360°}{k}$

Vertical translation: y = sin x + q



'q' has the effect of moving the whole graph up q units , if q is positive and
'q' has the effect of moving the whole graph down q units , if q is negative.

Horizontal translation : y= sin (x + p)



The graph of $y = \sin(x - 30°)$ is shown as a solid line. The effect of $-30°$ is to shift each point on the graph of y= sinx by 60° to the right along the x – axis.

CONCLUSION

When I use Maths software, my learners understand better the transformation of trigonometric graph**,** hence they are always keen to draw their own graphs. Using a laptop with Maths programs has challenged and changed me and my methodology of teaching.

Page setup

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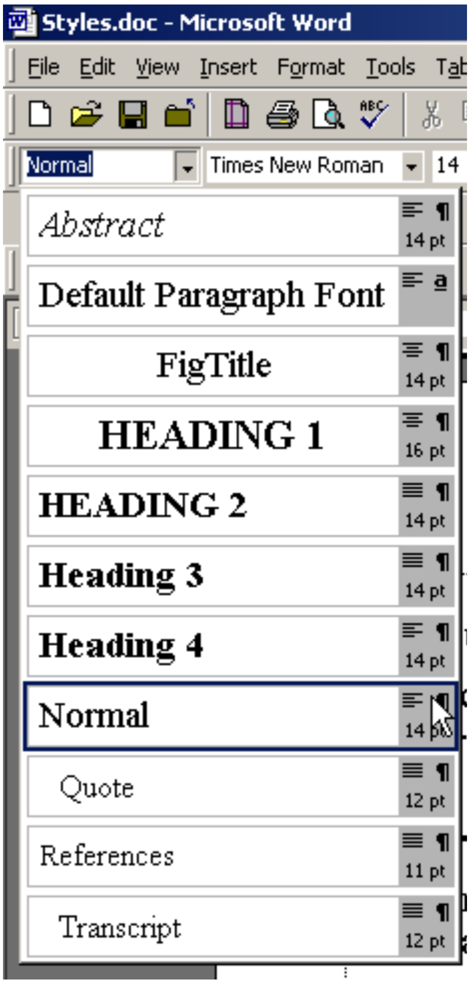
How to use this document

You can work directly in this document – the page setup is correct and you can apply the defined styles to your paragraphs.

Alternatively you can copy the styles in this document to your Normal template (in Word 2003 go Tools, Templates, Organizer, … ; in Word 2007 go Styles window, Manage Styles, Import/Export, …) and copy the styles from Styles.doc to Normal.dot. Then the styles are available in any new document.

You can see see a drop-down menu of available styles in the Styles window:



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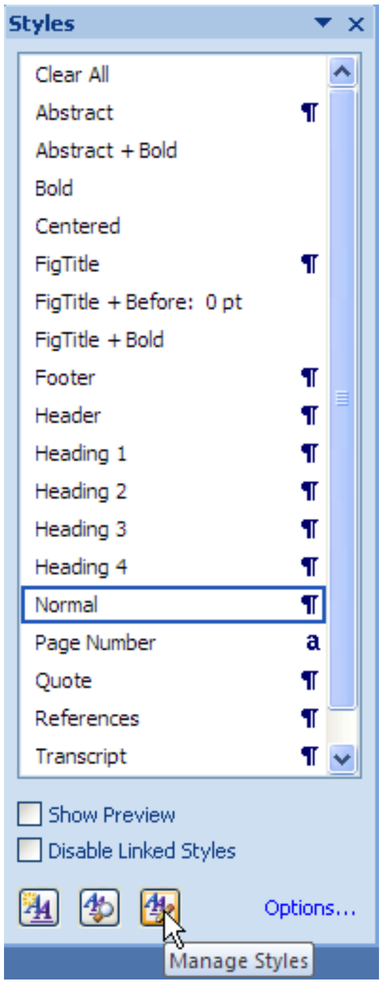


In Word 2007

TITLE: HOW I TEACH CONDITIONAL PROBABILITY

NAME: MAPONYANE DUNGILE

ORGANISATION: BATHOKWA SECONDARY SCHOOL

(MATHS4STATS)

(BAKENBERG CLUSTER, WATERBERG DISTRICT, LIMPOPO PROVINCE)

PHASE: FET

ABSTRACT: *Being part of the mathematics curricula for both primary and secondary schools, probability has been a difficult part to teach for mathematics educators. As such there has been an outcry nation wide to develop educators in this regard.*

Introduction

Teaching this lesson was an absolute blast, and to me a great success. In this lesson learners did not only understand how to determine conditional probability and conditional probability using the formula but they managed to give their own interpretation and approaches to conditional probability, and they understood the difference between probability and conditional probability. My reason for choosing this topic is that I find it interesting and challenging at the same time, as such I would like to share my experience with other mathematics educators. Since well it involves the section which is new and demanding to educators.

CONTENT

Before I teach conditional probability I ensure that my learners know the basic terminology of probability:

Probability –The measure of how likely it is that some events will occur.

Sample Space – The set of all possible outcomes in a probability experiment.

Event – A subset of the sample space containing collection of some of the possible outcomes (grouped outcomes with something in common)

Certain Outcome – An outcome that will definitely happen.

Uncertain Outcome - An outcome that will either happen or not.

Impossible Outcome – An outcome that will never happen.

What is conditional probability?

It is the probability of some event B, given that event A has occurred.

Conditional Probability is written as P (B/A), which is read as "the probability that event B, given that event A has occurred.

Example 1:

Suppose a fair die is rolled. Define events as follows:

A: an even number obtained.

B: a 4 appeared on the die.

C: an odd number obtained.

ANALYSIS

P (A) =P (obtaining an even number) =3/6=1/2… (1)

P (B) =P (a 4 is obtained) = 1/6…(2)

P (B/A) = (a 4 obtained given that an even number came up.)

= P (got a 4 if I know one of (2; 4; 6) came up)

= 1/3……… (3)

P (B/C) = P (a 4 is obtained given that an odd came up)

= P (got a 4 on the die if we know one of (1; 3; 5) came up)

= 0

NB: (2) and (3) show that event A and B are not independent (i.e. events A and B are dependent) since the knowledge of the occurrence of event A changed the probability if the occurrence of event B.

Being convinced that the grabbed all the necessary steps of determining conditional probability. I then gave them the formal definition of conditional probability. For any two events A and B, the conditional probability of event A given that B, may be calculated by:

$$\mathbf{P\ (A/B)} = \frac{P(AandB)}{P(B)}$$

$$= \frac{P(A \cap B)}{P(B)}$$

After going through this example with my learners I gave them example 2 to work on as groups

Example 2:

A mathematics teacher gave her class two tests. 25% of the class passed both tests and 42% of the class passed the first test. What percent of those who passed the first also passed the second test?

Before we can analyze the learners' responses let us attempt the following problems first:

ACTIVITY 1

The probability that it is Friday and that a student is absent is 0.03. Since there are 5 school days in a week, the probability that it is Friday is 0.2. What is the probability that a student is absent given that today is Friday?

A jar contains black and white marbles. Two marbles are chosen without replacement. The probability of selecting a black marble and then a white marble is 0.34, and the probability of selecting a white marble on the second draw, given that the first marble drawn was black?

Here follows some of the responses from the learners :

REFERENCES

Data Handling short Course for teachers – Compiled by Jackie Scheiber and Dr Delia North.
Internet – www.mathforum.org

HOW I TEACH GAME OF CHANCE TO GET LEARNERS

Presenter: Ms Maraka, K

Institution: Given Mangolo Primary School (Bakenberg Cluster)

Target group: Grade 6 and 7 Educators

Duration: 30 minutes

NB: EQUIPMENTS NEEDED "Dice, coins and Cards"

INTRODUCTION

The key of being successful in the teaching of Mathematics in your classroom is to have a positive attitude and the will to try new approach. Positive attitude create positive teaching environment. Probability is one topic that allows learners to have fun with Mathematics while on the other hand reinforcing the content at the same time.

DESCRIPTION OF CONTENT

From RNCS Learning Outcome 5: **DATA HANDLING** states that:

The learner will be able to collect, summarise, display and critically analyze data in order to draw conclusions and make predictions, and to interpret and determine chance variation by:

- Predicting the likelihood of events in daily life based on observation, and places them on a scale from 'impossible' to 'certain'.
- Listing possible outcomes for simple experiments (including tossing a coin, rolling a die, and spinning a spinner).
- Counting the frequency of actual outcomes for a series of trials.

MOTIVATION FOR RUNNING WORKSHOP

This workshop aims to introduce the game of chance (probability) to teachers in the lower grades. These games can be used in the intermediate and the senior phase.

PRESENTATION

When you talk about the probability of something happening it is the same as talking about the chance of it happening or the certainty or likelihood of it happening. You might be absolutely certain that an event will happen, or there might be little likelihood it will happen, or you might be unsure of whether it will happen or not.

As you know, some events are absolutely certain to happen, while others are uncertain or less likely and others are impossible. Look at these descriptions of the likelihood of an event occurring:

Impossible - it has no chance of happening.

Unlikely - it has a greater chance of not happening than of happening.

Evens - it has as much chance of happening as of not happening. It is equally likely to happen as to not happen. This is sometimes referred to a 50- 50 chance.

Likely - it has a greater chance of happening as of not happening.

Certain - it is sure or certain to happen.

ACTIVITY 1

The space that you think best describes each of the following events:

EVENT	**Impossible**	**Unlikely**	**Evens**	**Likely**	**Certain**
You will lose all your pocket money through					

negligence.					
Your father will pick up a diamond in your backyard.					
You will grow up to be a successful business person.					
The sun will not rise tomorrow.					
The best player will win.					
You will drive a car when you grow up.					
The first throw of a dice will be six.					
You will go to bed tonight.					
You will win the lotto.					
The frog will turn into a prince.					

PROBABILITY SCALE:

Impossib	Unlikel	Evens	Likely	Certai
↓	↓	↓	↓	↓
0	¼	½	¾	1
↓	↓	↓	↓	↓
0%	25%	50%	75%	100%

The probability scale with ***numbers*** improves the accuracy.

The nearer the value of the proabability is to 1, the greater the chance of the event happening.

The nearer the value of the proabability is to 0, the less likely the even is of happening.

A probability of ¾ means that 75 % of the time the event will happen.

$$\textbf{Probability = P (A) =} \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

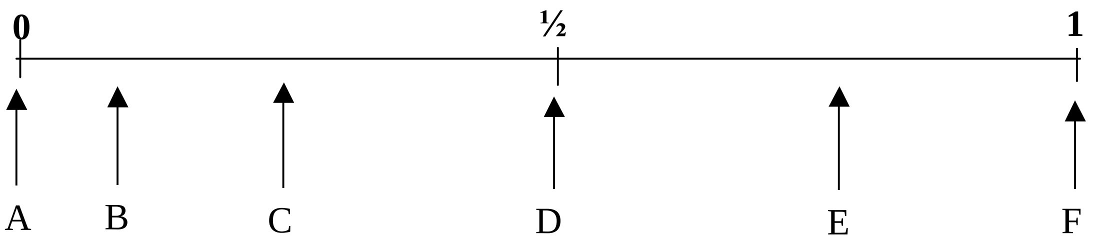
Note:You can only use this formula if the outcomes of the event are all equally likely.

To calculate the probability of an event you need to list all the possible outcomes and the entire successful (or favourable) outcome first. You can then count up the number of outcomes.

Probability can be integrated with fractions, decimals and percentages.

ACTIVITY 2

Match each of the events listed below with the capital letter marked on the following probability scale.



The soccer captain will win the toss of a coin.
You will live to be 400 years old
A card drawn from a pack of cards will not be a ‘diamond’.
A square will have 4 sides.
It will snow in Polokwane in April.
A letter picked from the alphabet will be a vowel.

There is a hot-drinks machine in the school staff room. The probability of teachers choosing each of the different drinks are:

Coffee = $\frac{2}{5}$ Tea = $\frac{3}{10}$ Hot chocolate = $\frac{1}{4}$ Cappuccino =

$\frac{1}{20}$

Which drink is bought most often?
Which drink is bought least often?
Arrange the drinks in order from most popular to least popular.

Weather forecaster estimates that the likelihood of rain over 3 consecutive days to be:

75% on Saturday
an even chance on Sunday
$\frac{1}{20}$ on Monday

Describe these probabilities in words.
Place them on a probability scale.
What is the probability that it will not rain on Monday?

CONCLUSION

At first learners will encounter problems in understanding probability as it will be the first time they come across it. I found it very easy when using practical models like cards, coins and dies. Learners thereafter understand the concepts better than they did before.

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Department of Education (2002) Revised National Curriculum Statement Grades R - 9 (Schools) Mathematics.

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SEARCHING LINES: HOW I INTRODUCE OBJECTIVE FUNCTIONS

Phelelani I Mkhize

Kwamncane High School and the Vula Laptop Project, Hilton College

INTRODUCTION

Since Linear Programming plays such an important role in industry and in the world in decision making. I would like to share with you my experiences on how to introduce language in Linear Programming and how to use technology to illustrate the search line for optimization.

ACTIVITY / CONTENT

Language

x must be **at least** 5	$x \geq 5$
y must be **at most** 7	$y \leq 7$
sum of x and y must be **at least** 30	$x + y \geq 30$
sum of x and y has to be 40	$x + y \geq 40$
sum of x and y has a **minimum** of 50	$x + y \geq 50$
sum of x and y is 45 **maximum**	$x + y \leq 45$
sum of x and y is **more than** 35	$x + y > 35$
x must **exceed** 25	$x + y < 25$
y has does **not exceed** 20	$x + y < 20$

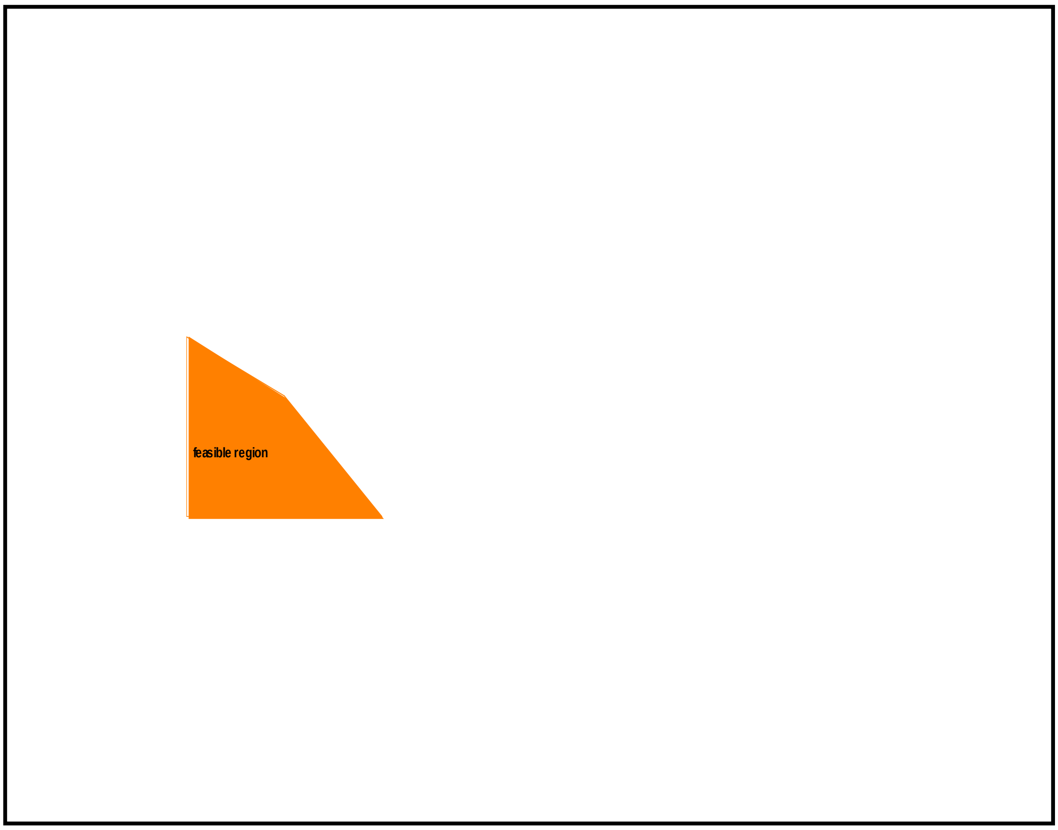
Search line

The animation function of Geometers Sketchpad is very useful when explaining the concept of optimization using a graphical method. It allows you to move the search line up and down and also to pause it at any point.

The example below is the first that I use in my presentation.

> A retailer wishes to buy a maximum of 20 pianos. He can buy either type A for R1500 each or type B for R3000 each. R45 000 has been set aside for the purchase of the pianos. However, at least 6 of each type must be purchased.
>
> Let type A = x and type B = y

Write down the 4 inequalities which satisfy the above conditions.
Represent the inequalities graphically and indicate the feasible region.
If the retailer makes a profit of R400 on each piano of type A and R1000 on each piano of type B. Write down the Profit equation (P).
Hence use the search line to determine the how many of each type he should buy to make a maximum profit.
If he makes a profit of R500 on each type A and R1000 profit on each type B piano, how will this affect the quantity of each type of piano purchased for his stock.



In this example, there are two search lines. The first (which is shown in the diagram) has a gradient of $-\frac{2}{5}$ and allows you to find the single point which gives the optimal solution. The second search line has a gradient of $-\frac{1}{2}$ which is the same as that of one of the boundary lines. The technology shows, very clearly, that there are three possible optimal solutions. They are the points with coordinates (6 ; 12) , (8; 1) and (10; 10)

CONCLUSION

I have made a simple worksheet of suitable Linear Programming questions. Each question has a Sketchpad diagram which goes with it and shows the boundary lines and feasible region, together with a search line which can be animated. This will be available on the Vula website vula.hiltoncollege.com after the Congress.

The Benefits of Using Concept Maps to Teach Mathematics

Urmilla Moodley, Wingen Heights Secondary School

Despite the emphasis of meaningful learning that characterises the reform standards of our National Curriculum statement (2006), methods that enhance this type of learning is clearly lacking in our classrooms. This dichotomy requires that students understand the meaning behind mathematical algorithms in which they demonstrate much fluency. Understanding of basic concepts is essential for students to gain the command of mathematics. A concept is well understood if a learner can link it to past knowledge and other aspects of a particular topic. Concept maps are visual tools that externally portray and describe connections between concepts. The depth of learners understanding is determined by the number, accuracy and quality of these connections between concepts. In this paper I report on the use of concept maps to gauge the connections and hence understanding between concepts for the topic of real numbers for grade eight learners.

What are concept maps?

A concept map is a visual technique that employs a two dimensional, hierarchical, node-link representation (Novak & Gowin, 1984). It is a summarised description of how learners think concepts are associated to one another. Figure 1 below shows the mathematical relations between concepts for quadrilaterals. As seen in the example below, concepts are enclosed in a shape pertaining to a subject domain. The phrase along the line that links concepts, describes the mathematical relationship between the two concepts. Two concepts together with the linking phrase is referred to as a proposition. Examples can also be included in concept maps to indicate a complete understanding of a particular topic.

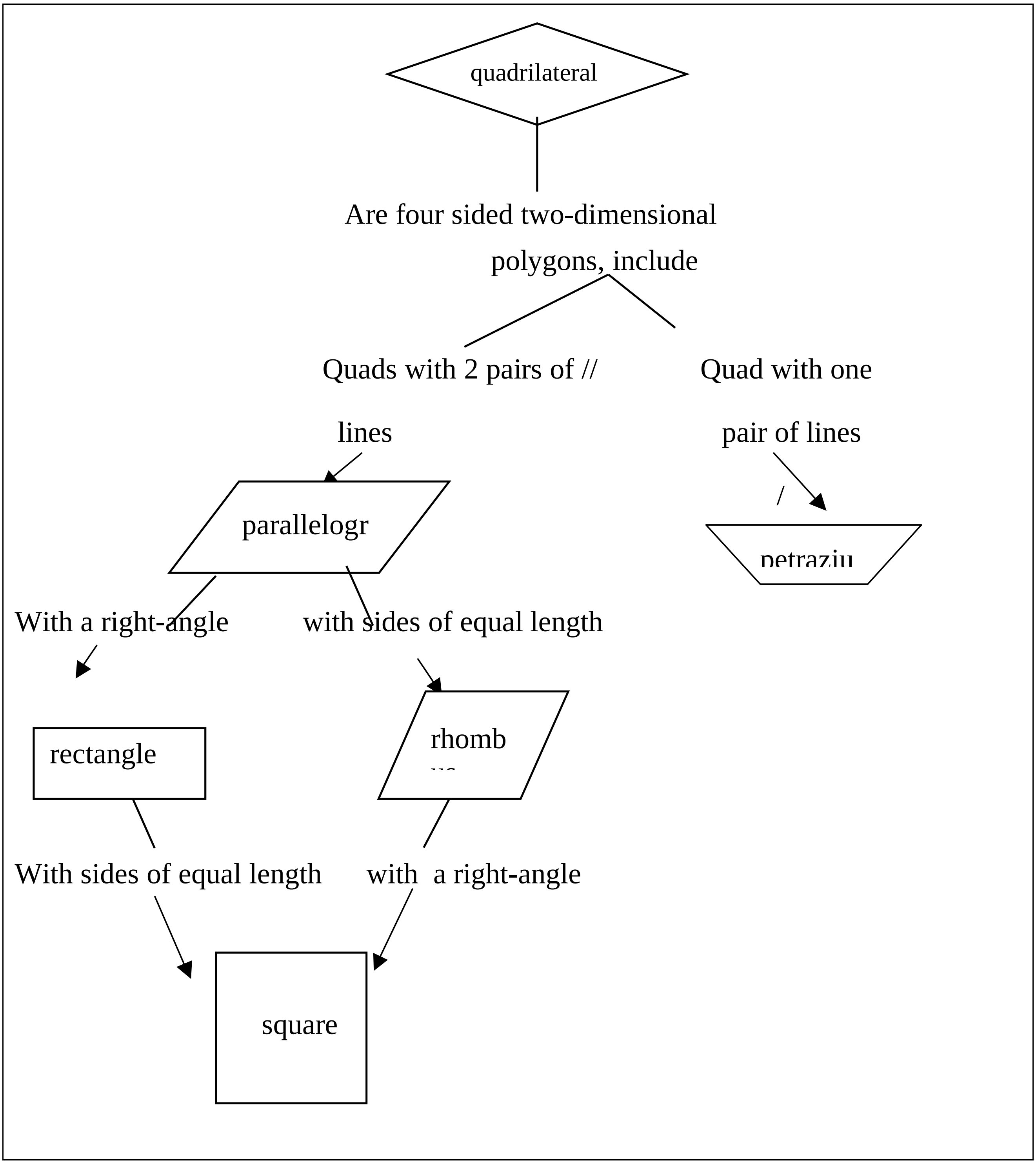


Fig 1 Concept map showing connections between quadrilaterals

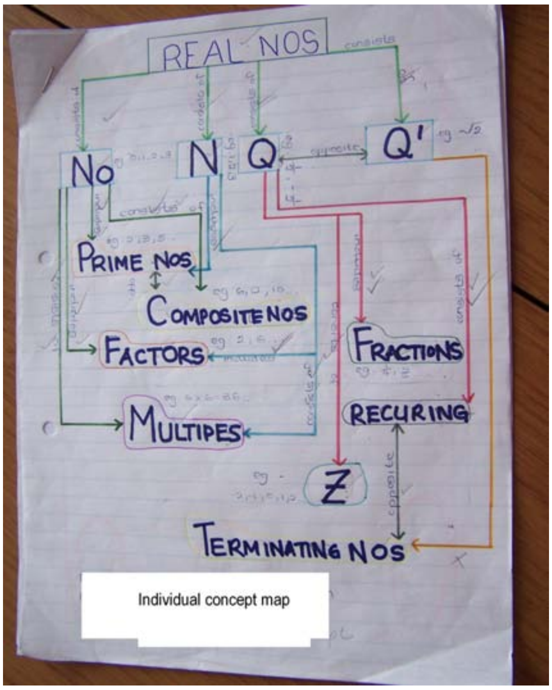
Why use concept maps to teach?

Cognitive psychologists are of the opinion that within the brain representations of knowledge resembles network of ideas that are organised and structured (Hiebert &

Carpenter, 1992; Royer, Cisero & Carlo, 1993). Completed concept maps provide external representations of how the learner organises and interconnects concepts. Concept maps also encourage the brain to work in a way that is fast, efficient and in a way that it does naturally. Since constructing a concept map requires explicitly defining connections, concept mapping is therefore an invaluable tool for determining the meaningful learning of mathematics.

How can teachers use concept maps to determine learners' understanding?

To assist this discussion I have included a concept map done by two learners on real numbers. They are represented below as figure 2 and figure 3.



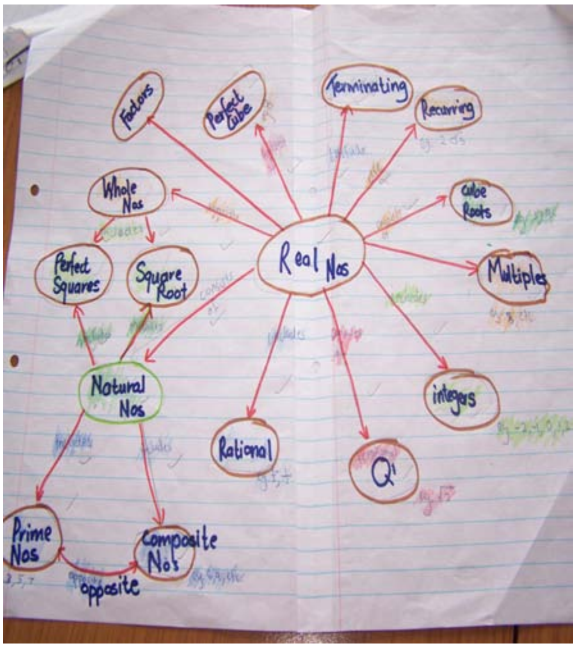


Figure 2

Figure 3

<u>Concepts and linking words</u>

"Concepts' and "valid links" serves as a quantitative estimate of mathematically acceptable knowledge that is typically assessed in conventional evaluation measures (Markham, Mintzes and Jones, 1994). The number of connections and depth of understanding can be determined by the number of correct propositions in a student's concept map. In figure 2 the learner indicates that ‘recurring’ and ‘terminating decimals’ fall within the subset of ‘rational numbers’. The learner includes linking phrases such as ‘consists of’ between the concepts. This indicates a good understanding of these concepts since the learner sees the correct relationship between the concepts. The second concept map, figure 3 reveals several links from real numbers with the non-descriptive linking words of ‘consist of’. There are no links indicated between the various concepts. Although all the concepts on the map fall within real numbers, the learner does not understand how one concept is related to the other. This reflects a superficial understanding of the concepts in this topic.

Cross-links

Cross-links on a concept map indicate how previous concepts could be integrated with new knowledge (Karoline Afamasaga-Fuata'I, 2004). In figure 2 the learner includes a cross-link between ‘recurring’ and ‘terminating’ decimals with the phrase ‘opposites’ between them. This indicates meaningful learning as the learner sees a relationship across the concepts in the map. In figure 3, although a valid cross-link exists between ‘prime’ and ‘composite’ numbers there are gaps in the learners’ knowledge structure. The learner could see the relationship between these two concepts but could not make sense of how other concepts are linked in real numbers.

The benefits of using concept maps

Affective factors

Learners enjoy the “freedom” and “creativity” that concept maps allow. They find it particularly enjoyable since it is different from current exercises found in mathematics classrooms. However these exercises are necessary as they develop procedural knowledge, whilst concept maps helps to determine conceptual knowledge.

Group work

Concept mapping is very effective for group work. Concept maps encourage debate and animated discussions about what concepts should be included or omitted in a concept map (White and Gunstone, 1993). This forces learners to think seriously of how concepts are related to each other.

Information for the teacher

Examining concept maps makes it easier to gauge misconceptions and gaps in learners’ understanding. It helps the teacher to determine whether learner understanding is coherent with educational expectations. Shortfalls in teaching can be easily detected and immediately rectified.

Conclusion

Since the constructing of concept maps requires learners to become actively involved in relating concepts to each other, it is an explicit way to determine learners’ understanding of a particular topic. It is also an interesting way to engage learners in the learning of mathematics since present practices in mathematics classrooms do not promote effective acquisition, retention and transferability of mathematical concepts. Concept maps are a novel and appealing method to promote conceptual development and to make learning of mathematics fun and enjoyable.

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HOW I TEACH MIDPOINT AND DISTANCE FORMULAE

PRESENTER: Sekgana Motimele

INSTITUTION: MASEDIBU HIGH SCHOOL

Phase: FET

INTRODUCTION:

The paper is based on developing the formulae for both the midpoint and distance by using deductive approach. In the paper we will use the worksheet and tracing paper to find out how to develop the formulae.

The reason why I chose the section is that I have discovered through my years of teaching that the section is very simple but yet our learners don't get it right because of the way we present it.

The main objective is to demystify Mathematics so that learners can enjoy it more as it is the way that it should be done. We as educators must move from the level where learners see Mathematics as a set of rules called formulae to the level of making it more practical.

CONTENT:

Presentation:

The learners will be given worksheets in which different points will be indicated. The learners will then be given a task of finding the midpoints and distance by just counting from the given worksheet. This will be used so that learners can be able to develop the concepts before being subjected to working with the formulae.

Activity 1:

Learners will find the distance and then the midpoints of the points which are parallel to the x-axes.

Learners will then be given the other activity wherein they are supposed to find the midpoints and distance in the case where the points are parallel to the y-axes.

The learners must discover on their own that to find the midpoint and distance we consider only the quantity that has changed. e.g.

Distance between A (2; 5) and B (2; 11) will be **11 - 5= 6** while the

Midpoint will **be** **M (2; 8)**, where only the value of y is affected.

Activity 2:

Learners will then be given the activities wherein they have to find the distance and midpoints of the points which are neither parallel to the y-axes nor to the x-axes.

E.g. P (4;5) and Q (7;9)

Finding the **distance by measurement**

Completing the right angled triangle using the given co-ordinates and the lines parallel to the x-axes and y-axes respectively.

Finding the **third side** in the given right angled triangle by using the theorem of Pythagoras.

Midpoints:

The procedure will be the same where learners will be given the two points to find the midpoints by **measurement.**

Where they cannot find the answer by measurement, **tracing paper can be used to** determine the midpoint.

After the concept of midpoint is developed, is then that the formula can be introduced.

CONCLUSION

In concluding the lesson, learners must come to the realization of what the midpoint and distance entails, while they also understand the reasoning behind the two formulae. I have discovered that, by approaching the section in this way, they will understand more and hence I get more input (in the case of learners' understanding.)

REFERENCE:

1. Study and Master (Grade 10)

2. Grade 10 Mathematics (A class Text) – the answer series

TREE DIAGRAMS; MUTUALLY EXCLUSIVE EVENTS AND INDEPENDENT EVENTS

Presenter: Musetha, D.R

Institution: Waterberg District (Limpopo Province)

DCES "Mathematics and Mathematical Literacy" FET Band

musethadr @ webmail.co.za

INTRODUCTION

Counting can be much more than adding up the number of boys and the numbers of girls in your class. You can use counting rules to determine the number of different possible outcomes of an experiment as well as the number of different ways the outcome of interest can occur. If you can count these possibilities, you can determine probability. The probability that something of interest (event) will happen is defined as the number of ways the outcome of interest occurs, divided by the total number of different possible outcomes.

DESCRIPTION OF CONTENT

Learning Outcome 4: Data Handling and Probability

Assessment Standard 11.4.2

(a) Correctly identify dependent and independent events (e.g. from two-way contingency tables or Venn diagrams) and therefore appreciate when it is appropriate to calculate the probability of two independent events occurring by applying the product rule for independent events: *P (A and B) = P (A).P (B).*

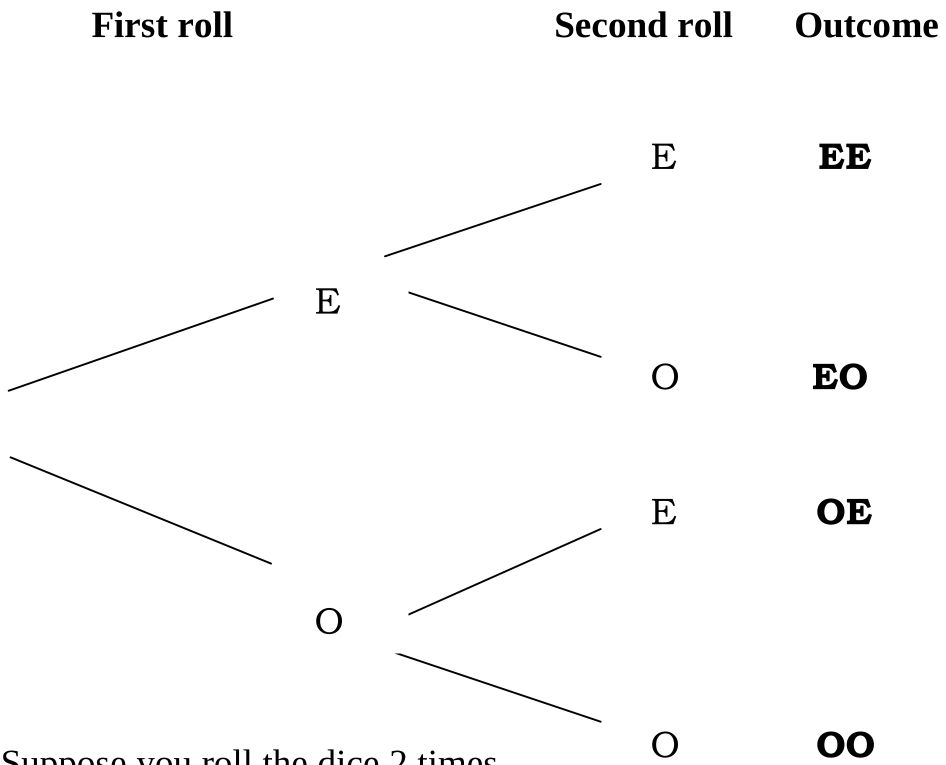
FINDING PROBABILITIES USING A TREE DIAGRAM

A tree diagram can help you to help you list all the possibilities of an event.

Example: 1

Suppose you are interested in whether you get an even number (E) or an odd number (O) when you roll a dice.

If you roll the dice a second time the tree diagram grows to this:



1. Suppose you roll the dice 2 times.

   What is the probability of getting even and odd numbers?

   Solution: *P (odd & even)* $\frac{n(O\,\&\,E)}{n(A)} = \frac{2}{4} = \frac{1}{2}$

2. Suppose you roll the dice 3 times.

   What is the probability of getting 1 even and 2 odd numbers?

   Solution: This means that *n (A) = 8 and n (1 even & 2 odd) = 3*

   P (1 even & 2 odds) = $\frac{3}{8}$

MUTUALLY EXCLUSIVE EVENTS

Mutually exclusive events are events that cannot happen at the same time.

The **addition law** for probabilities only works for mutually exclusive events. The law states:

> If A, B and C …. are **mutually exclusive** events then:
>
> The probability of one event **OR** another happening is found by **adding** the probabilities of the individual events.
>
> *P (A* **or** *B* **or** C…) = *P (A)* + *P (B)* + *P (C)* +….

Example: 2

Joseph has 12 coins in his pocket. If he takes a coin at random from his pocket, what is the probability that it is (5c =2, 10c=1, 20c=4, 50c=3, R2=2)

Either 5 cents or 20 cents coin?

Solution: $P\ (5\ cent) = \frac{2}{12} = \frac{1}{6}$ $\qquad P\ (20\ cents) = \frac{4}{12} = \frac{1}{3}$

$\therefore P\ (5\ cent\ \textbf{or}\ 20\ cents) = P\ (5\ cent) + P\ (20\ cents) = \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$

A 5 cent or 10 cents or 20 cents or 50 cents or R2?

Solution: $P\ (5\ cent) = \frac{2}{12} = \frac{1}{6}$ $\quad P\ (10\ cents) = \frac{1}{12}$ $\quad P\ (20\ cents) = \frac{4}{12} = \frac{1}{3}$

$P\ (50\ cents) = \frac{3}{12} = \frac{1}{4}$ $\quad P\ (R2) = \frac{2}{12} = \frac{1}{6}$

$\therefore\ P\ (5\ cent\ \textbf{or}\ 10\ cents\ \textbf{or}\ 20\ cents\ \textbf{or}\ 50\ cents\ \textbf{or}\ R2)$

$= \frac{2}{12} + \frac{1}{12} + \frac{4}{12} + \frac{3}{12} + \frac{2}{12} = 1$

NB: The sum of all probabilities of mutually exclusive events is **1**. With two mutually exclusive events it is **certain** that one will happen and the other will not.

Another way of thinking of this is using complementary events where

P (A) + P (not A) = 1.

INDEPENDENT EVENTS

Two events are said to be ***independent*** when the result of one event has no effect on the result of the other. Suppose you toss a coin and roll a dice. The result of tossing the coin (heads or tails) has NO affect on the outcome of rolling the dice. These events are ***independent***.

Example: 3

Supposed you roll a dice and toss a coin.

The outcomes from these two combined events can be shown in a two-way table:

Toss a coin	**H**	(H; 1)	(H; 2)	(H; 3)	(H; 4)	(H; 5)	(H; 6)
	T	(T; 1)	(T; 2)	(T; 3)	(T; 4)	(T; 5)	(T; 6)
		1	**2**	**3**	**4**	**5**	**6**
		Roll a dice					

What is the probability that you will find a head and 3 dice if you toss a coin and roll a dice?

Solution: Looking at the probabilities of the separate events you have:

$P\ (H) = \frac{1}{2}$ and $P\ (3) = \frac{1}{6}$

If you multiply these probabilities you get: $P\ (H) \times P\ (3) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

This is called the ***multiplication law for probabilities***: The multiplication law for probabilities only works for independent events. The law states:

> If A, B, C ….are **independent events**
>
> The probability of an event **followed** by another event is found by **multiplying** together the probabilities of the individual events.
> $P\ (A \text{ and } B \text{ and } C\ldots) = P\ (A) \times P\ (B) \times P\ (C) \times\ldots$

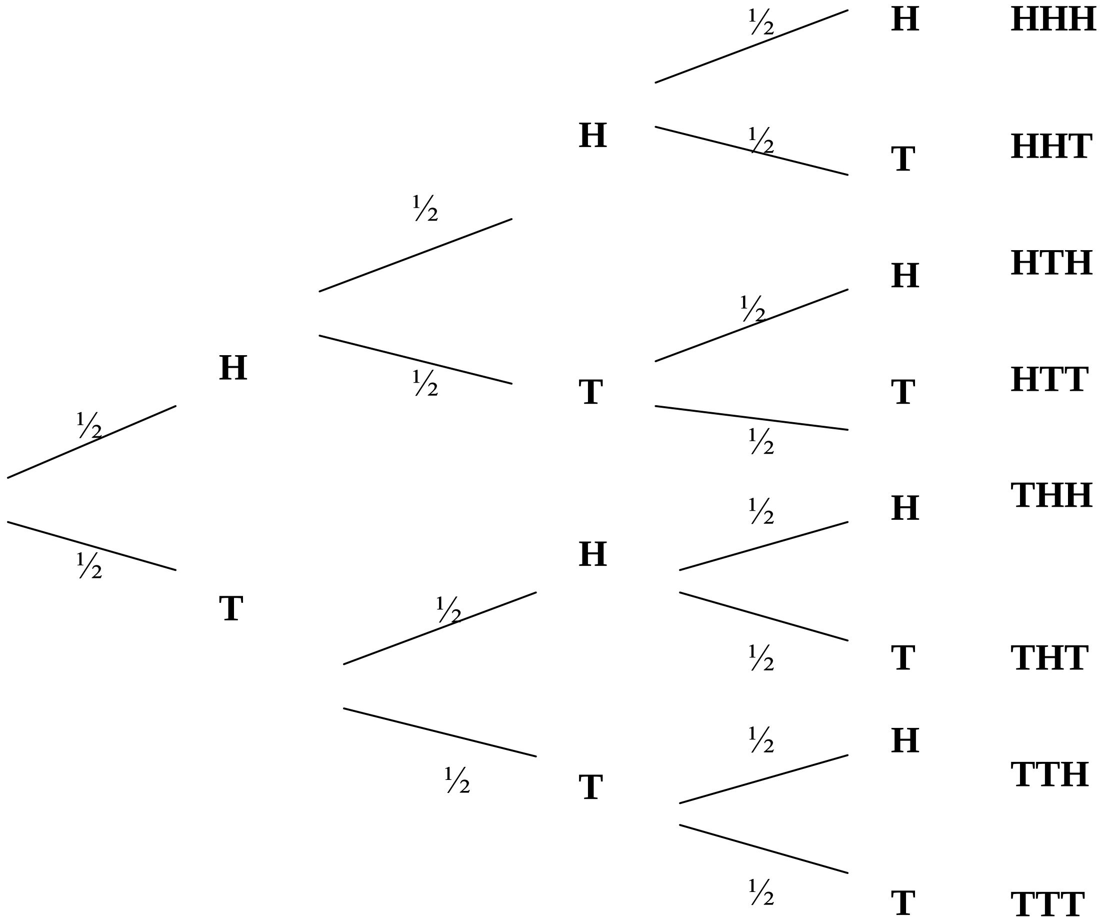
TREE DIAGRAMS AND INDEPENDENT EVENTS

The multiplication law can be used when you draw tree diagrams of independent events.

Example: 4

Suppose you toss a coin 3 times. You know $P\ (H) = \frac{1}{2}$ and $P\ (T) = \frac{1}{2}$

You can draw a tree diagram showing these probabilities along the branches like this:



Suppose you wanted to work out the probability of getting a tail, followed by two heads – i.e. *P (THH)*. These are independent events so you are finding *P (T and H and H)*

$$P\ (THH) = P\ (T\ \textbf{and}\ H\ \textbf{and}\ H) \quad = P\ (T) \times P\ (H) \times P\ (H)$$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

Suppose you want to find the probability of tossing 2 heads and 1 tail (i.e. not tails first, followed by 2 heads, but any combination of 2 heads and 1 tail); you would use the **law of addition.**

$$P\ (2\ heads\ and\ 1\ tail) = P\ (HHT\ \textbf{or}\ HTH\ \textbf{or}\ THH)$$

$$= P\ (HHT) + P\ (HTH) + P\ (THH)$$

$$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$$

A tree diagram allows you to use both the **Addition law** of probabilities (for mutually exclusive events) and the **Multiplication law** (for independent events)

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FROM FACTORS TO FACTORIZATION

Maggie Naidoo
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INTRODUCTION:

I have chosen to share my ideas on how I teach factorization with the senior phase teachers as I teach in the FET phase and find that we do not have enough time to drill concepts like factorization. Since this topic is found in the senior phase, I though that if pupils are exposed earlier to the methodology, then teachers in FET phase can concentrate on multiplying, dividing, adding and subtracting algebraic fractions without having to spend time teaching and re-teaching factorization.

CONTENT:

GRADE 7: 7.1.3 factors including prime factors of 3-digit whole numbers

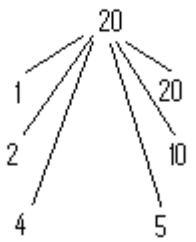
GRADE 8: 8.1.2 multiples and factors

GRADE 9: 9.2.9 uses factorization to simplify algebraic expressions

In grade 7 and 8 pupils are asked to list the factors of numbers.

Eg 1. Find the factors of 20

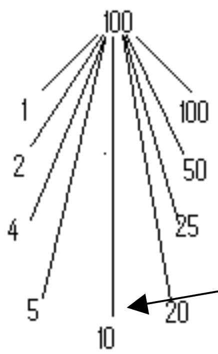
Try to get pupils to list the factors as pairs ie. product of two numbers



Therefore the factors of 20 are:

$F_{20} = \{1;2;4;5;10;20\}$

Eg 2. Find the factors of 100



Therefore the factors of 100 are:

$F_{100}=\{1;2;4;5;10;20;25;50;100\}$

Note: A square no will always have one factor that repeats itself , write it down once.

Practice examples

Find the factors of : 1. 25

2. 72

3. 12

Finding prime factors: Revise prime numbers.

Use prime numbers to find prime factors

Eg1. Find the prime factors of 20

2	20
2	10
5	5
	1

Therefore the prime factors of 20 are:
$PF_{20}=\{2^2x5\}$

Eg1. Find the prime factors of 100

2	100
2	50
5	25
5	5
	1

Therefore the prime factors of 100 are:
$PF_{100}=\{2^2x5^2\}$

Now find the prime factors of practice exercise one.

FINDING LCM AND HCF:

To find LCM and HCF we use the method of writing to ones

Eg1. Find the LCM and HCF of 20 and 100

2*	**20**	**100**
2*	10	50
5*	5	25
5	1	5
	1	1

LCM= 2X2X5X5=100
HCF= 2X2X5=20

Eg2. Find the LCM and HCF of 8,12 and 50

2*	**8**	**12**	**50**
2	4	6	25
2	2	3	25
3	1	3	25
5	1	1	25
5	1	1	5
	1	1	1

LCM= 2x2x2x3x5x5=600
HCF= 2

Practice examples: Find the LCM and HCF of:

1. 10, 12, 18
2. 9, 12, 15
3. 6, 8, 12
4. 15, 24, 36

TRINOMIALS:

- Always look for a common factor
- Create common brackets if there is four terms
- If it is a binomial-does it have minus, are the numbers square numbers.(Difference of two squares)
- Trinomial

Remove a common factor:

Eg 1. $a^2 + ab = a a + a b = a(a + b)$

Eg2. $3ac - 6c^2 = 3 a c - 2 \times 3 c c = 3c(a-2c)$

Eg3. $2a+3ab-4b^2-6b^3 = a(2+3b) - 2b^2(2+3b)$
$= (2+3b)(a-2b^2)$

Difference of two squares:

Eg4. $4x^2-9y^2=(2x-3y)(2x+3y)$

Trinomial:

Eg5. $a^2 - 5a + 6$
$=(a-2)(a-3)$

find the factors of 6 in pairs

	Add		Add
1x6	7	-1x-6	-7
2x3	5	-2x-3	-5

Eg6. $x^2+5x-45$
$=(x-5)(x+9)$

Find the factors of -45 in pairs

	Add		Add
1x-45	-44	-1x45	44
3x-15	-12	-3x15	12
5x-9	-5	-5x9	5

Eg7. $2x^2-9x+10$
$=(2x-5)(x-2)$

Find the factors of 20 (2x10)

Add		Add	
1x20	21	-1x-20	-21
2x10	12	-2x-10	-12
4x 5	9	$\frac{-4}{2}$ x -5	- 9

Since we multiplied by two we need to divide by two to find factors.

Practice exercise: Find the factors of :

1. $mx-2x^2$
2. a^2b^2+abc
3. am-ab + 2m-2b

4. $5(a+b) - fa - fb$
5. x^2-y^2
6. $49m^2-81n^2$
7. a^2-36
8. $x^2+7x+12$
9. $x^2-8x+12$
10. $a^2+2x-15$
11. $2m^2-14m-36$
12. $2x^2+3x+1$
13. $5m^2-16mn+3n^2$
14. $15-a-6a^2$
15. $6x^2-17x+12$

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Platonic Solids

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INTRODUCTION:

Children seldom explore the geometrical space of their environment. Their thinking is often limited to two dimensions, because of this the introduction of depth often presents a barrier when challenged to think outside of their comfort zone. Linear to 3D though is not always easy for children. It is for this reason that charts, ideas, thoughts and analysis of the 3 dimensional world should always be displayed in every mathematics classroom. In order to expand children's knowledge and to motivate them to love the world of geometry, I have adopted a practical approach to introducing this topic in the grade 8 classroom.

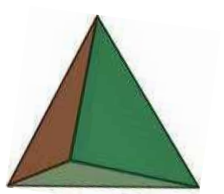
CONTENT:

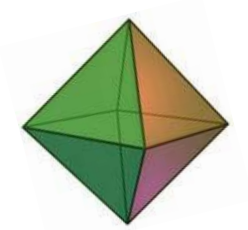
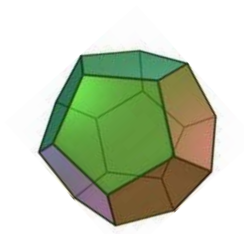
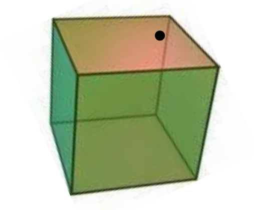
1. If children can make something then they will take ownership of it and like it, so my approach is to build the models so they are able to identify what makes up three dimensional shapes and discover things about the shape. The shapes we are going to examine all have straight edges (not like a crumpled ball of paper). That is we are working with ***Platonic soilds or regular polyhedrons*****.**

- ***Polyhedron (Poly =many; hedro = sides)***
- ***Platonic soilds (discovered by Plato - Greek Mathematician)***

Plato found that there were only five regular polyhedrons and he assigned some important words from nature to them. Let us see what he meant:

- *tetrahedron icosahedron octahedron cube dodecahedron*



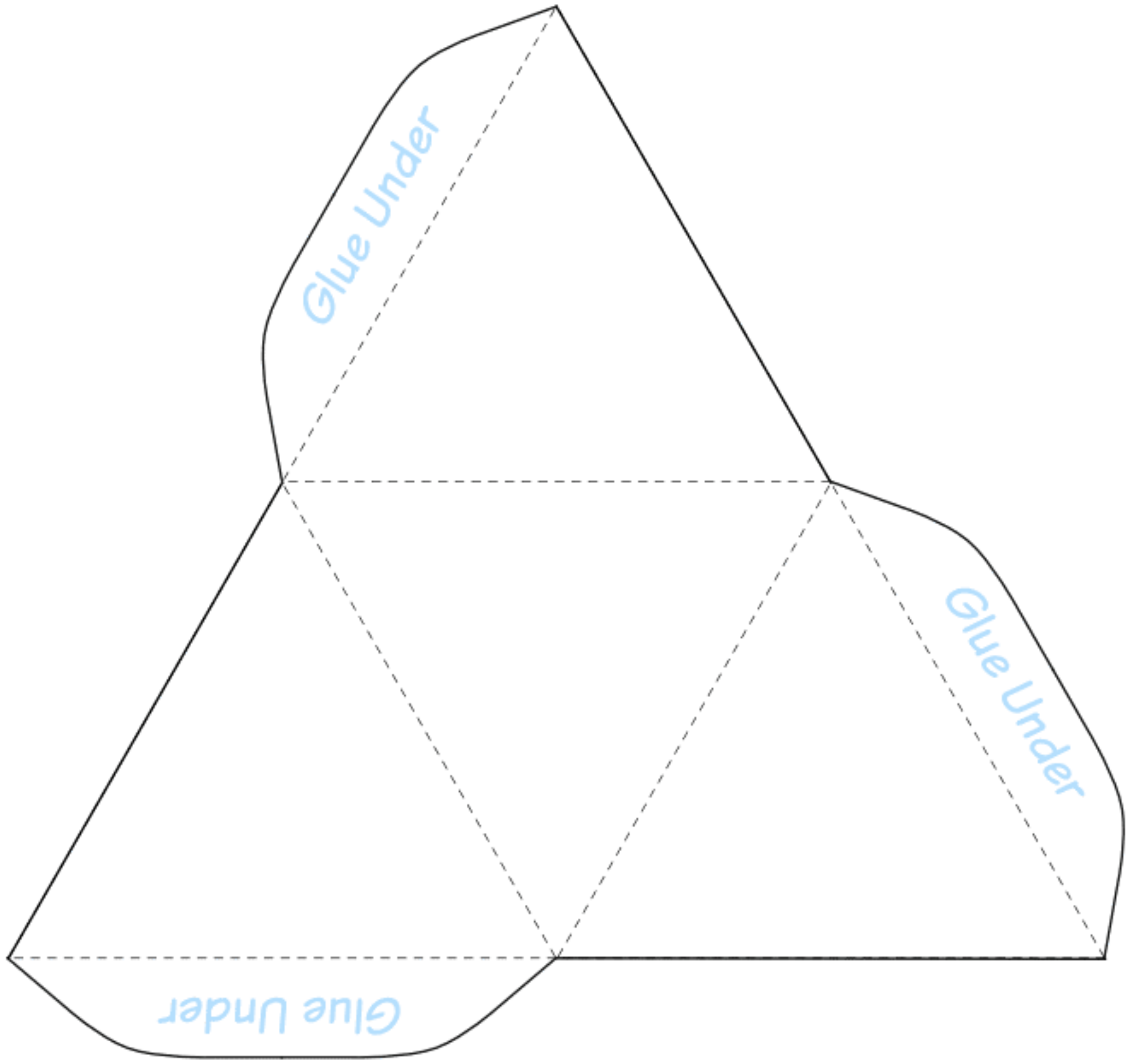




FIRE WATER AIR EARTH COSMOS

Notice that he named the first 3 shapes in terms of the most essential elements for the survival of mankind. The other two represent our planet and the universe/cosmos.

Templates will be used to build the five shapes. Pupils are then asked to find the number of faces, number of edges and number of vertices.

TETRAHEDRON



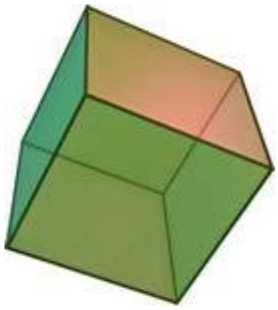
PLATONIC SOLIDS - WHY FIVE?

In a nutshell, it is impossible to have more than 5,
because any other possibility would violate simple rules about
the number of edges, corners and faces you can have together.

EULER'S FORMULA

Do you know about "Euler's Formula"? It says that for any convex polyhedron (which includes the platonic solids) that the Number of Faces plus the Number of Vertices (corner points) minus the Number of Edges always equals 2

It is written: F + V - E = 2



Try it on the cube:

A cube has 6 Faces, 8 Vertices, and 12 Edges,

so:

6 + 8 - 12 = 2

FACES MEET

Next, think about a typical platonic solid. What kind of faces does it have, and how many meet at a corner (vertex)?

The faces can be triangles (3 sides), squares (4 sides), etc.

 Let us call this "s", the number of sides each face has.

Also, at each corner, how many faces meet? For a cube 3 faces meet at each corner. For an octahedron 4 faces meet at each corner.

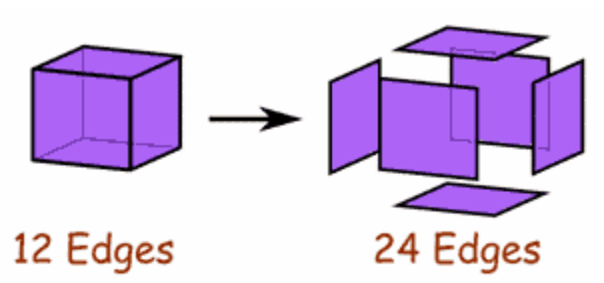
 Let us call this "m" (how many faces meet at a corner).

(Those two are actually enough to show what type of solid it is)

EXPLODING SOLIDS!

Now, imagine we pulled a solid apart, cutting each face free.

You would have all these little flat shapes. And there would be twice as many edges (because you cut along the edge).



Example: the cut-up-cube would now be six little squares.

Each square would have 4 edges, for 24 edges in total (versus 12 edges when joined up to make a cube).

So, how many edges? Twice as many as the original number of edges "E", or simply 2E

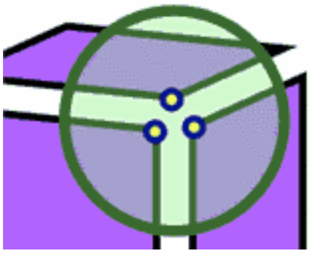
But this will also be the same as counting all the edges of the little shapes. There will be s *(number of sides per face)* times F (number of faces).

- This can be written as sF = 2E

Likewise, when we cut it up, what *was* one corner will now be *several corners*.

In the case of a cube there will be three times as many corners.

But for every corner, there is an edge! So it will also equal 2E.



- This can be written as mV = 2E

BRING EQUATIONS TOGETHER

That is all the equations we need, let us use them together:

$$sF = 2E, \text{ hence } F = 2E/s$$
$$mV = 2E, \text{ hence } V = 2E/m$$

Now let us put those into "F+V-E=2":

$$F + V - E = 2$$
$$2E/s + 2E/m - E = 2$$

Next, some rearranging ... divide the lot by "2E":

$$1/s + 1/m - 1/2 = 1/E$$

Now, "E", the number of edges, cannot be less than zero, so "1/E" cannot be less than 0:

$$1/s + 1/m - 1/2 > 0$$

Or, more simply:

$$1/s + 1/m > 1/2$$

So, all that remains is to try different values of:

- "s" (number of sides each face has, cannot be less than 3), and
- "m" (number of faces that meet at a corner, cannot be less than 3),

and we are done!

THE POSSIBILITIES!

The possible answers are:

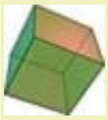
s	m	1/s+1/m	> 0.5 ?
3	3	0.666...	✓
3	4	0.583...	✓
4	3	0.583...	✓
4	4	0.5	✗
5	3	0.533...	✓
3	5	0.533...	✓
5	4	0.45	✗
4	5	0.45	✗
5	5	0.4	✗
etc...	...	...	✗

Result: There are only 5 that work! All the rest are just not possible in the real world.

> Example: s=5, m=5
>
> "1/s + 1/m - 1/2 = 1/E" becomes "1/5 + 1/5 - 1/2 = 1/E" or "-0.1 = 1/E",
>
> which makes E (number of edges) = -10, And you can't have a negative number of edges!

REAL?

And the last step is to see if those solids are real:

s	m	what it means	solid	
3	3	triangles meeting 3-at-a-corner	tetrahedron	
3	4	triangles meeting 4-at-a-corner	octahedron	
4	3	squares meeting 3-at-a-corner	cube	
5	3	pentagons meeting 3-at-a-corner	dodecahedron	
3	5	triangles meeting 5-at-a-corner	icosahedron	

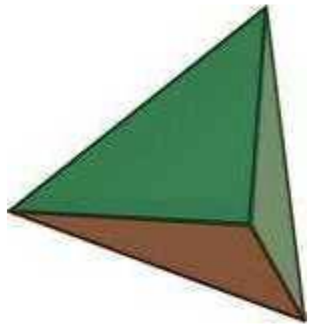
So, only 5, and they all exist.

Tetrahedron

Tetrahedron Facts	
Notice these interesting things:	
	It has 4 Faces
	Each face has 3 edges, and is actually an Equilateral Triangle
	It has 6 Edges
	It has 4 Vertices (corner points)
	and at each vertex 3 edges meet
And for reference:	
	Surface Area = $\sqrt{3} \times (\text{Edge Length})^2$
	Volume = $(\sqrt{2})/12 \times (\text{Edge Length})^3$

The tetrahedron also has a beautiful and *unique* property ... all the four vertices are the same distance from each other!

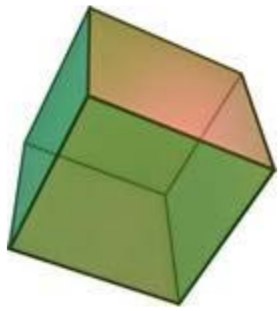
And it is the only Platonic Solid with no parallel faces



Cube

Cube (Hexahedron) Facts	
Notice these interesting things:	
	It has 6 Faces
	Each face has 4 edges, and is actually a square
	It has 12 Edges
	It has 8 Vertices (corner points)
	and at each vertex 3 edges meet
And for reference:	
	Surface Area = $6 \times (\text{Edge Length})^2$
	Volume = $(\text{Edge Length})^3$

A cube is called a hexahedron because it is a polyhedron that has 6 (*hexa-* means 6) faces.



Cubes make nice 6-sided dice, because they are regular in shape, and each face is the same size.

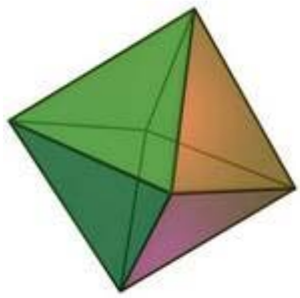
In fact, you can make fair dice out of all of the Platonic Solids.

Octahedron

Octahedron Facts	
Notice these interesting things:	
	It has 8 Faces
	Each face has 3 edges, and is actually an Equilateral Triangle
	It has 12 Edges
	It has 6 Vertices (corner points)
	and at each vertex 4 edges meet
And for reference:	
	Surface Area = $2 \times \sqrt{3} \times (\text{Edge Length})^2$
	Volume = $(\sqrt{2})/3 \times (\text{Edge Length})^3$

It is called an *octahedron* because it is a polyhedron that has 8 (*octa*-) faces, (like an *octopus* has 8 tentacles)

If you have more than one octahedron they are called octahedra

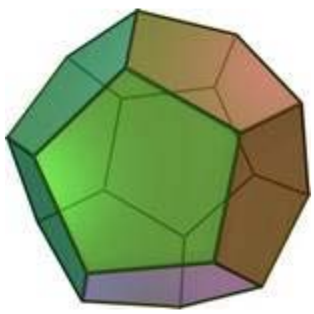


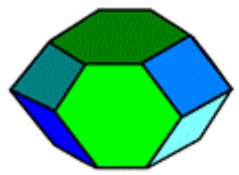
Dodecahedron

Dodecahedron Facts	
Notice these interesting things:	
	It has 12 Faces
	Each face has 5 edges, and is actually a pentagon
	It has 30 Edges
	It has 20 Vertices (corner points)
	and at each vertex 3 edges meet
And for reference:	
	Surface Area = $3\times\sqrt{(25+10\times\sqrt{5})} \times (\text{Edge Length})^2$
	Volume = $(15+7\times\sqrt{5})/4 \times (\text{Edge Length})^3$

It is called a *dodecahedron* because it is a polyhedron that has 12 faces (from Greek *dodeca*- meaning 12).

If you have more than one dodecahedron they are called dodecahedra





When we say "dodecahedron" we often mean "regular dodecahedron" (in other words all faces are the same size and shape), but it doesn't *have* to be - this is also a dodecahedron, even though all faces are not the same.

12-Sided Dice? Yes! A dodecahedron which has 12 equal faces has an equal chance of landing on any face.

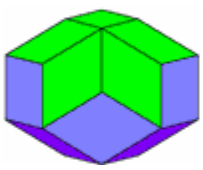
In fact, you can make [fair dice]() out of all of the Platonic Solids.

Icosahedron

Icosahedron Facts	
Notice these interesting things:	
	It has 20 Faces
	Each face has 3 edges, and is actually an Equilateral Triangle
	It has 30 Edges
	It has 12 Vertices (corner points)
	and at each vertex 5 edges meet
And for reference:	
	Surface Area = $5 \times \sqrt{3} \times (\text{Edge Length})^2$
	Volume = $5 \times (3+\sqrt{5})/12 \times (\text{Edge Length})^3$

It is called an *icosahedron* because it is a polyhedron that has 20 faces (from Greek *icosa-* meaning 20)

If you have more than one icosahedron they are called icosahedra



When we say "icosahedron" we often mean "regular icosahedron" (in other words all faces are the same size and shape), but it doesn't *have* to be - this is also an icosahedron, even though all faces are not the same.

20-Sided Dice? Yes! An icosahedron that has 20 equal faces has an equal chance of landing on any face.

In fact, you can make [fair dice]() out of all of the Platonic Solids.

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HOW I TEACH TRANSLATIONS USING AUTOGRAPH

Nathi Ndlovu

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INTRODUCTION

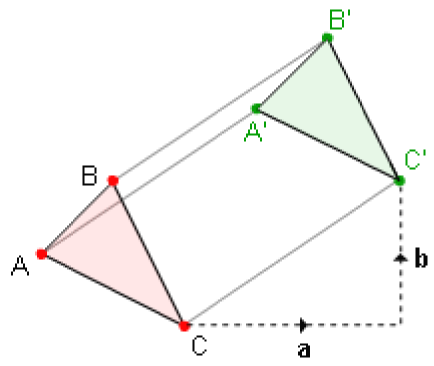
In the community where I'm serving, the kids are all dying to go to university. Because mathematics is a bursary subject, they know doing well will help them improve their chances. There is so much they can do with a Maths degree. Mathematics is known to learners as a scary thing. I use a laptop loaded with mathematics teaching programmes. It is this machine that I use in my Maths class to teach topics like, amongst others, Transformation Geometry. This enables my learners to understand the topic quickly.

Presentation

I shall show how Autograph does the following:

Translation

All points of an object on a plane move the same distance and in the same direction under translation.

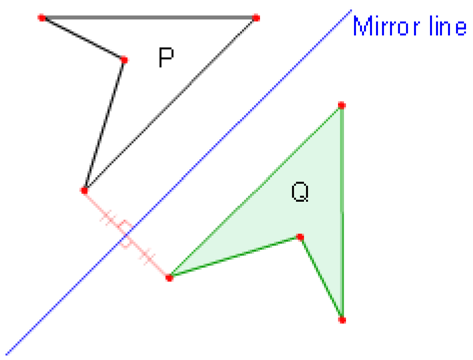


The image of a point P (x ; y) after translation a units horizontally and b units vertically is P′ (x + a ; y + b).

Translation preserves **orientation**, **shape** and **size**

Reflection

Under reflection the object and its image are symmetrical about the mirror line (line of reflection).

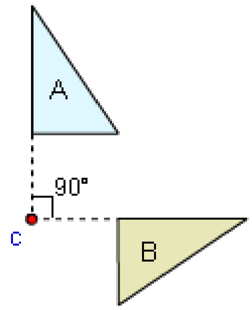


Corresponding points on the image and object are at the same perpendicular distance from the mirror line.

The image of the point P(x ; y) after reflection about
- the x−axis is P′(x ; −y)
- the y−axis is P′ (−x ; y)
- the line y = x is P′ (y ; x)
- the line y = −x is P′ (−y ; −x).

Reflection preserves **shape** and **size** but doesn't preserve orientation.
Points on the mirror line do not move.

Rotation



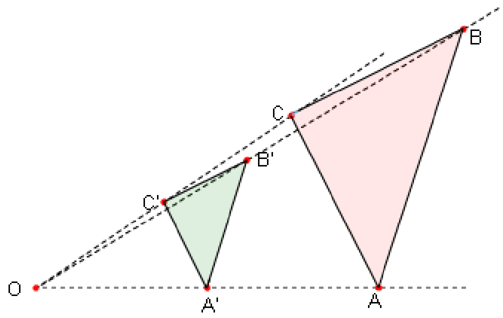
Under rotation, every point on the object is rotated through the same angle (clockwise / anti clockwise) about the **centre of rotation.**

The image of a point P(x ; y) after rotation about the origin through

90° is P′ (−y ; x) anti-clockwise direction:
180° is P′ (−x ; −y)
−90° is P′ (y ; −x) clockwise direction:
−180° is P′ (−x ; −y)

Rotation preserves **shape** and **size** but not orientation.
The center of rotation is the only invariant point.

Enlargement



An enlargement is defined by its center and **scale factor**.
Scale factor refers to the ratio of the corresponding sides.

The enlargement by a scale factor of **k**, through the origin, takes every point (a ; b) to the point (**k**a ; **k**b).

For **k > 1** the image gets enlarged
For **−1 < k < 1** the image gets diminished
For **k < 0** the image lies on the other side of the centre of enlargement

Under enlargement with scale factor **k,** area of the image = k^2 × area of the object.

Conclusion

Integrating technology into the classroom has converted mathematics from a scary thing to an exciting subject to my learners. Having the laptop and data projector supplied by the Vula Mathematics Project at Hilton College in my maths classroom has changed my approach to teaching the subject.

HOW I TEACH DATA HANDLING

PRESENTER: Mrs Nkanyane M.G

INSTITUTION: Mananga Primary school Capricorn (Limpopo Province)

Duration: 60 MINUTES

Target Group: Grade 2 and 3 Educators

No of Participants: 30

Introduction:

- Data is very important in our daily lives so it is therefore important for every one to pay attention despite the age. Sometimes, we foundation phase educators in particular do not go much deeper when dealing with such topics forgetting that this will lay a firm foundation for the future statisticians.

Description of Content:

Learning Outcome 5: Data Handling

The learner will be able to collect, summarise, display and critically analyse data in order to draw conclusion and make predictions and to interpret and determine chance variation.

AS – Sorts physical objects according to one attribute chosen by the teacher
AS –Draws pictures and constructs pictographs that have a 1-1 correspondence between own data and representations.
AS –describes own or a peer's collection of objects, explains how it was sorted and answers questions about it.

MOTIVATION FOR RUNNING THE WORKSHOP

There is a clear indication that we live 'statistics' Data is collected almost daily, Statistics is now taken as a subject on its own and therefore our learners must be sharpened from an early age.

I have realised that many educators encounter problems in newly introduced concepts just like when Breakthrough or OBE were introduced, it was difficult to kick start. I am quite sure that they will benefit a lot from this Data handling presentation although it is not a newly introduced concept. They will be able to move on with ease in this Learning Outcome as it will be thoroughly explained with all of us being actively involved.

PRESENTATION.

Activity 1

In a class of 30 learners they were divided into two groups of 15 learners. The 1st group was asked to bring a flower each and the 2nd group was asked to bring a leaf each. Now the group have to sort their collections.
At first they didn't understand how to sort until I give them a direction ;

- The flowers to be sorted according to colour.
- The leaves to be sorted according to shape.

The learners are asked to report their findings back to the whole class.
<u>Group 1.</u>
Sorting outcomes
There were five different colours flowers;

- Yellow Flowers = 4
- Red flowers = 2
- Pink flowers = 3
- White = 1

<u>Group 2</u>
Sorting outcomes
There were two different shapes;

- Round leaves = 3
- Elongated = 7

A learner from each group to explain their findings.

Activity 2

Looking at the learners' birthday chart, they are asked to display such data graphically.

January ▪ Kgetha ▪ Mina ▪ Naledi ▪ Onnicca	February ▪ Olga	March ▪ Dineo ▪ Eunice ▪ Karabo ▪ Lebo ▪ Lorraine	April ▪ Lesedi ▪ Letago ▪ Tumi
May ▪ Vicky ▪ Geneva	June ▪ Nakedi ▪ Moshe ▪ Senyane ▪ Vincent	July ▪ Zinzi	August ▪ Kopedi ▪ Kgotso

September	October	November	December
▪ Junior ▪ Nyiko ▪ Mehloti ▪ Melida ▪ Nakedi	▪ Ovinia	▪ Peter	▪ Phillemon ▪ Reuben

Graphical representation of data

9												
8												
7												
6												
5			☺						☺			
4	☺		☺			☺			☺			
3	☺		☺	☺		☺			☺			
2	☺		☺	☺	☺	☺		☺	☺	☺		☺
1	☺	☺	☺	☺	☺	☺	☺	☺	☺	☺	☺	☺
	Jan	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**

Looking at the graph, answer the following questions;

1. Which months have the most birthdays?
2. Which months have the least birthdays?
3. It was said earlier that the class has 30 learners, but 31 birthdays have been represented on the graph. What might be the reason?

NB. The last part of learning outcome is not yet attended to; which says

...critically analyse data in order to draw conclusions and to make predictions and to interpret and determine chance variation (probability/randomness)

<u>The purpose of the critical analysis of data</u>

- To draw **conclusions**
- To make **predictions**
- To determine **chance variation**
- To interpret **chance variation**

Activity 3

<u>Critical analysis of data in order to make predictions and to interpret and determine chance variation.</u>

N.B Chance **– randomness, prediction & probability.**

- Playing a game of chance- A raffle in order to let the learners understand **randomness.**

  - Tossing of a coin to let them understand **prediction.**
  - Giving statements that will be commendable by possible,
  - impossible, certain and uncertain for **Probability**.

.

Maybe = not sure, being uncertain, it may happen.

<u>*Certain*</u> = it will definitely happen.

<u>*Possible*</u> = it can happen.

<u>*Impossible*</u> = it will never happen.

EVENTS	**Maybe**	**Certain**	**Possible**	**Impossible**
➢ All the learners can share the same birth day				
➢ Learners and an educator can share a birth day				
➢ People like dogs more than cats				
➢ All the learner will pass at the end of the year				

➢ All the educators will go shopping tomorrow				
➢ Some cattle eat meat				
➢ It will rain milk one day				

Conclusion

Activities 1 & 2 are common ones but activity 3 appears to be uncommon though not new, we have to dwell much on this part. It is a matter of language and motivation, remember to make it fun- be in form of a coin or die throwing or a raffle. This will be a laying of a firm foundation for statistics in a way.

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Jenkins .T et al *Classroom Mathematics* Grade 2

Dada .F et al *New Day-by-Day* Numeracy Grade 2

PATTERNS IN TRANSFORMATIONS: HOW I TEACH THEM

Nonhlanhla Nxele

Nottingham Road Combined School and the Vula Laptop Project

INTRODUCTION

My laptop is part of my life – like my other arm. It can't teach my Maths class but it helps me to teach my Maths class. I should have a notice on my door that says

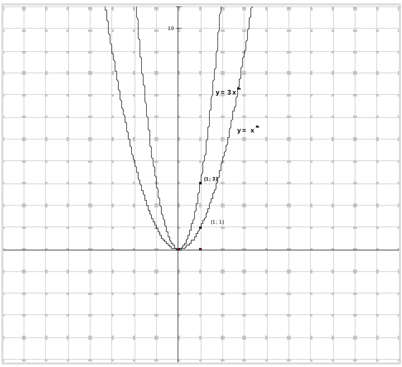
I ♥ MY LAPTOP

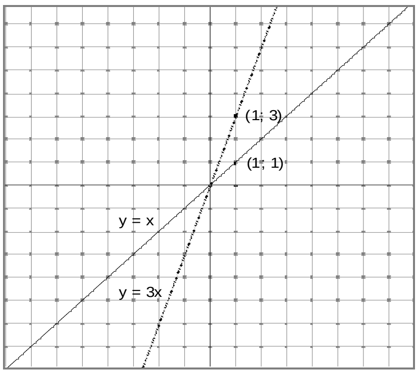
Some people think that computers are good in teaching Maths. They are wrong. They are good at helping good teachers to teach good Maths. More especially if they have got good Maths software. In my classroom we use Excel, Geometers Sketchpad and Autograph.

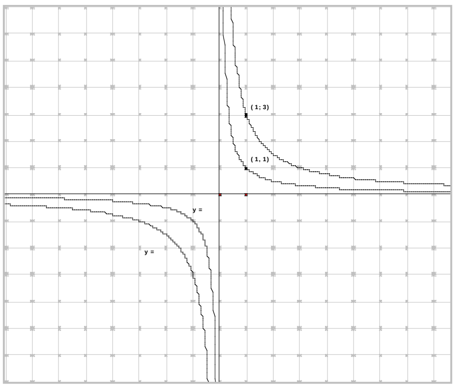
In my presentation I show how I use Maths software to transform graphs. I use Geometers Sketchpad for stretching and shifting (a, p and q) and Autograph for combinations e,g. $y = a(x+p)^2 + q$ and $(x-a)^2 + (y-b)^2 = r^2$.

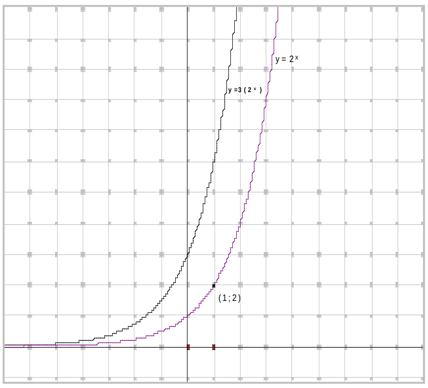
I always start with a seed graph which is the dotted line.

Stretching – the effect of *a*



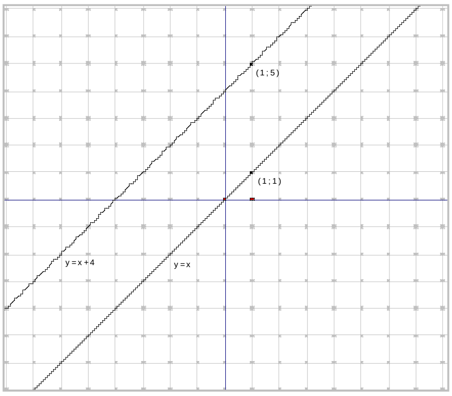


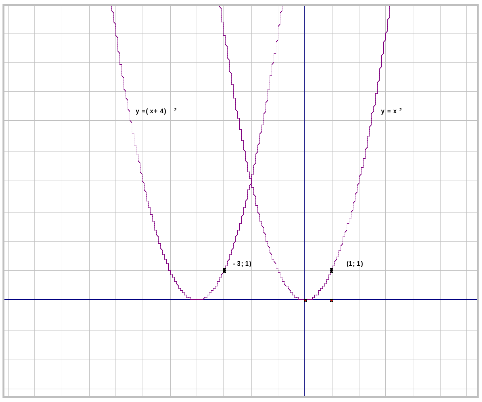


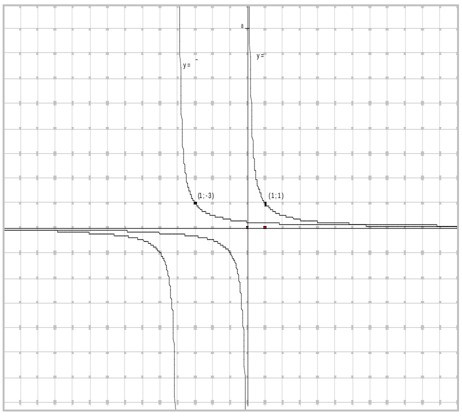


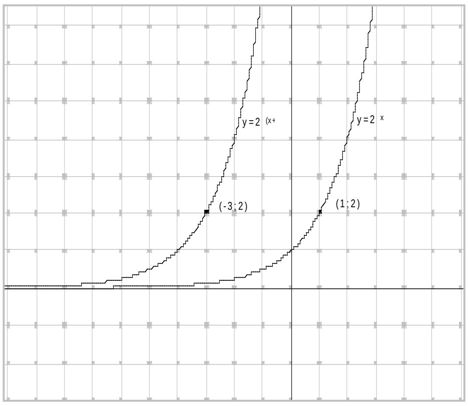
a has the **same** stretching effect on all the graphs

Horizontal shift – the effect of *p*









p makes all the graphs move the **same** way by the **same** amount - horizontally

q makes all the graphs move the **same** way by the **same** amount - vertically

Conclusion

Because transformations using Maths software is quick and efficient, my learners grasp the general concepts first. They are then confident enough, and curious and excited enough, to start drawing their own graphs. Next they draw the graphs on flip charts using different colours, This is followed by an oral presentation. After some (class) consolidation they paste their flip charts on the wall.

Acknowledgement

Mrs S. Southwood , Vula Mathematics Project – Hilton College

THE INTRODUCTION OF MATHEMATICAL LITERACY IN SOUTH AFRICAN SCHOOLS: A FOUNDATION TO SOCIO-ECONOMIC DEVELOPMENT

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ABSTRACT

South Africa currently faces serious high incidents of crime which many see as direct consequence of poor quality education provided for majority of the country's citizens before the democratic dispensation of 1994. Education and for that matter schools were organized and funded on racial lines during the era of apartheid. The best curriculum, schools, equipment and quality teachers went to the white only schools, while the black child attended the most improvished school. While white schools studied mathematics and science in addition to other subjects, black schools generally did not offer those subjects in their curriculum. This discriminatory and undemocratic organization of education turned out two categories of graduates: those who were all round and equipped with basic skills to work or study further for better careers and those who could merely read and write with very little opportunity for further studies or any employment. The democratic government which came to power in 1994 realized the harm apartheid has done to the socio-economic development of the country and started to transform education to make it accessible to all the country's citizens. In today's world mathematics cuts across the daily lives of people and all development endeavours. To this end Mathematical Literacy and allied subjects such as Accounting, Physical Science and Technology, which are seen as the foundation for socio-economic development of both individuals and communities, were introduced in all schools recently. This paper explores the value of Mathematical Literacy in the school curriculum as the foundation stone for modern careers, job opportunities and over all social and economic advancement of South Africa.

Key words: curriculum, school, mathematics, mathematical literacy, foundation, socio-economic, development.

HOW I TEACH TALLY CHARTS OR BAR GRAPHS TO SENIOR PHASE LEARNERS

PRESENTER: Ms LANGA PHUTI

INSTITUTION: BOKWIDI PRIMARY SCHOOL

LIMPOPO PROVINCE: (WATERBERG DISTRICT)

MATHEMATICS EDUCATOR

Target Group: Grade 4 Educators

INTRODUCTION

Educators are encouraged to be proactive in developing learner's content knowledge despite circumstances that they are faced with. Learner's acquisition of content knowledge shouldn't be measured by the availability of resources in school or schools situated in advantaged areas. The study was conducted with the aim of encouraging educators who are in disadvantaged areas with fewer resources to use what is available to the best interest of the learners. Despite all challenges that educators especially in rural areas are facing, learner's knowledge can still be developed.

DESCRIPTION OF CONTENT

Learning Outcome 5: Data Handling

The learner will be able to collect, summarise, display and critically analyse data in order to draw conclusions and make predictions, and to interpret and determine chance variation.

Assessment standard: **(4.5.1)**

We know this when the learner:

- Collects data (alone and/or as a member of a group or team) in the classroom and school environment to answer questions posed by the teacher and the class.
- Organises and records data using tallies and tables.
- Draws a variety of graphs to display and interpret data (ungrouped) including:
  - Pictographs with a one-to-one correspondence between data and representation (e.g. one picture = one shoe);
  - Bar graphs.

TEACHING TECHNIQUES

Co-operative learning

- Pupils were arranged into group size of 4.
- Group members were engaged in problem solving and discussion.
- Face – to – face interactions enable group members to collaborate effectively.
- Individual accountability is also encouraged.

SKILLS, KNOWLEDGE AND VALUES (SKV)

Skills:
- Representation and interpretation;
- Estimation and calculation;
- Reasoning and communication;
- Investigation and analysing.

Knowledge
- Data handling : pictograph and Bar graph

Values

The focus in teaching and learning of data handling in the Intermediate Phase is on gaining the skills to gather and summarise data so that they can be interpreted and predictions made from them.

Presentation

The whole lesson presentation was video recorded with the aim of giving participants the exact picture of what exactly transpired in the classroom. Proper procedures were followed before the whole process takes place. Firstly the principal was consulted and permission was granted. Parents of those learners taking part (Grade 4) were also consulted to grand their children the permission to participate freely in this lesson presentation.

ACTIVITY

Make a survey of shoe sizes worn by Grade 4 learners in a class. Shoe sizes were written in the table shown below:

S$_{G11}$	**S**$_{G12}$	B3	G3	B1	G1
G1	G1	G4	3	B3	G1

G2	B2	$\mathbf{S}_{G12}$	B1	G2	B2
B1	G3	G1	B4	B1	G2
B4	B1	B2	G1	$\mathbf{S}_{B12}$	G1

Note:

B → Represent the Boys

G → Represent the Girls

S → Represent the Small

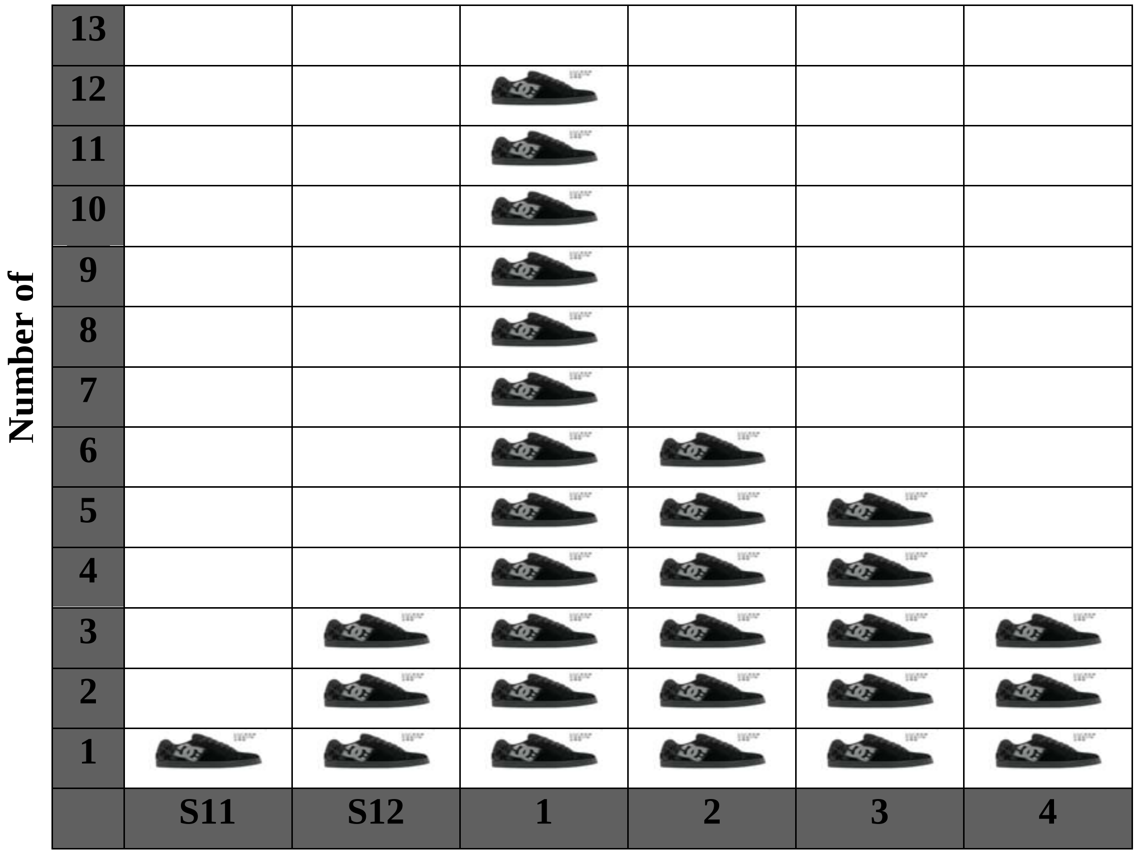
Data analysis

Data can be organised and analysed by means of a table of tallies:

SHOE SIZE	**Tallies**	**Frequency**
Small 11	/	1
Small 12	///	3
1	~~////~~ ~~////~~ //	12
2	~~////~~ /	6
3	~~////~~	5
4	///	3
TOTAL		**30**

A big number chard was placed on the floor were by each learners place his or her shoe next to the corresponding shoe size.

Example of a **NUMBER CHARD** used to gather data:



Shoe sizes

Question from the graph:

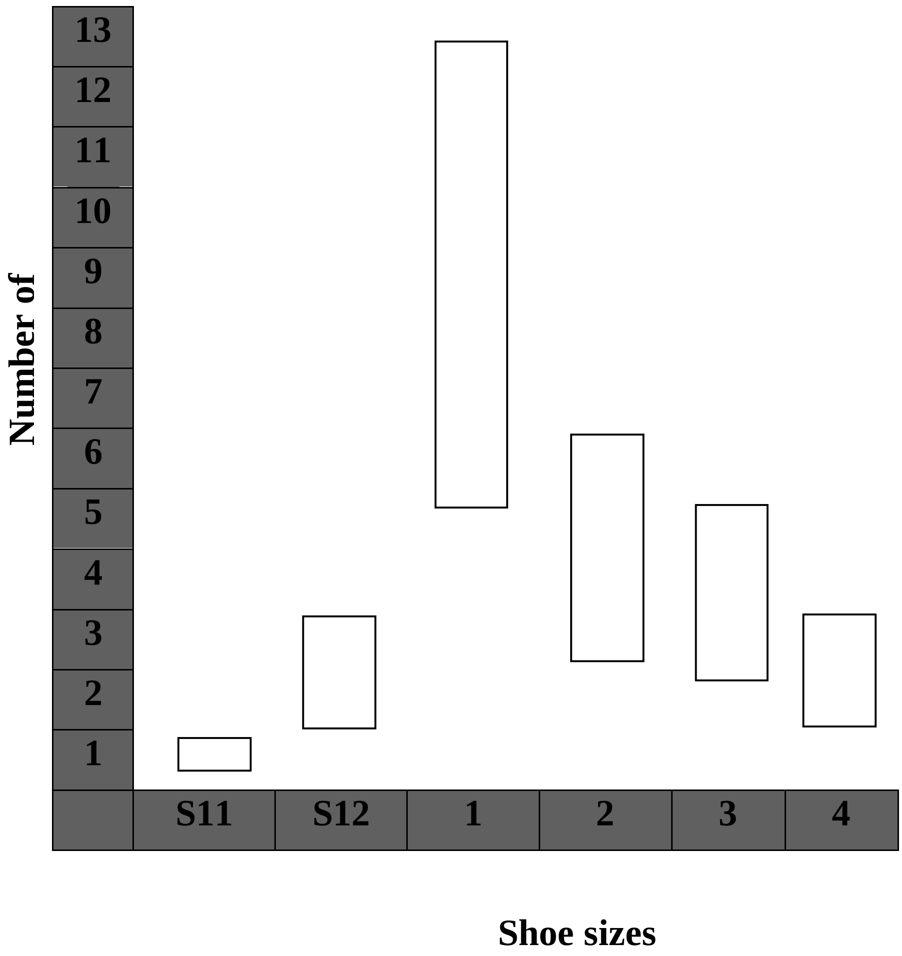
1. Which shoe size is worn by most learners in this class?
2. How many learners wear the smallest shoe size?
3. Which is the smaller size in this class?
4. Which is the bigger size in this class?
5. How many learners wear the bigger shoe size?
6. What conclusion with regard to this class shoe size can you deduce?

BAR GRAPH

Another way of representing this data is by drawing a bar graph. A bar graph has special kinds of diagrams called bars used to represent data. Instate of each unit being represented as an individual shape, the whole group can be shown in the form of a bar. Bar graph can be shown in two ways:

- Vertical bar graph: bars are drawn vertically.

- Horizontal bar graph: bars are drawn horizontally.



Question from the graph:

1. What does the bars represents?
2. Which shoe size is worn by most learners in this class?
3. How many learners wear the smallest shoe size?
4. Which is the smaller size in this class?
5. Which is the bigger size in this class?
6. How many learners wear the bigger shoe size?

CONCLUSION

Participants should remember at all times when trying to contextualised content knowledge learners world of living or the surrounding environment should be considered very important. Educators should avoid using context that is not familiar to learners.

REFERENCES

1. Department of Education (2002) Revised National Curriculum Statement Grades R – 9 (Schools) Mathematics.

FROM KITE TO LOGS: HOW I TEACH INVERSES

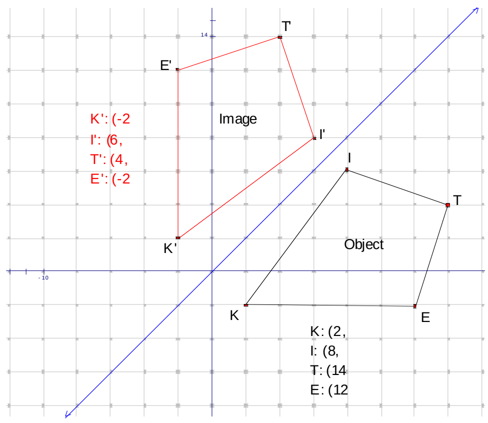
NOSISA SOSIBO

DUMABEZWE HIGH SCHOOL,
and
VULA LAPTOP PROJECT, HILTON COLLEGE

INTRODUCTION

Like most of the Mathematics in our new curriculum, transformations follow a routine and predictable pattern. Inverses are no exception: ‘you just swap x and y.’ Instead of being threatened by inverses, my learners enjoy them - even log graphs! It is important for learners to be able to picture the y↔x reflection process on the Cartesian plane. Using a laptop, data projector, Geometers Sketchpad and a pre-prepared lesson, we transform the points on a kite, straight line, parabola, hyperbola, and an exponential function. The shapes and graphs are generated by joining points.

Start with a quadrilateral



The lesson proceeds, step by step, something like this:

Educator: What shape is this?

Learners: A kite?

Educator: Why

General discussion follows

Educator: What are the coordinates of its vertices?

Various learner-type guesses follow

Educator: Where should I draw the line y = x?

More discussion

Educator: What do you remember about reflections?

Educator: What happens if we reflect the kite across y = x?

More guessing

Educator: Let’s let Sketchpad do it . . . were you correct?

Educator: What are the new coordinates of the image kite?

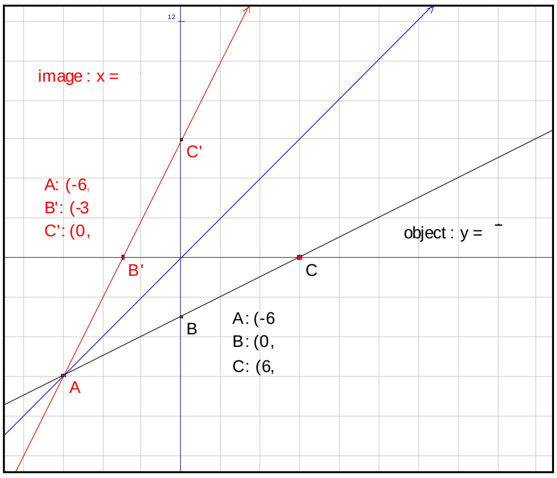
Etc etc etc

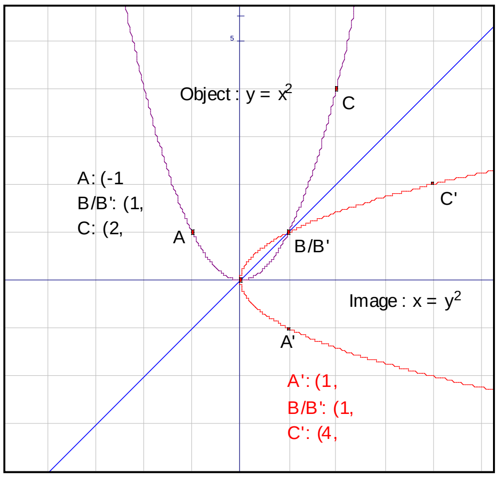
Educator: How have the coordinates changed?

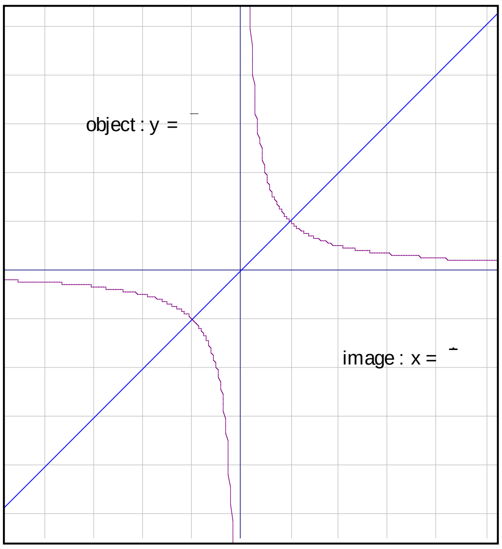
More discussion . . . development of (y; x) ↔ (x; y) concept

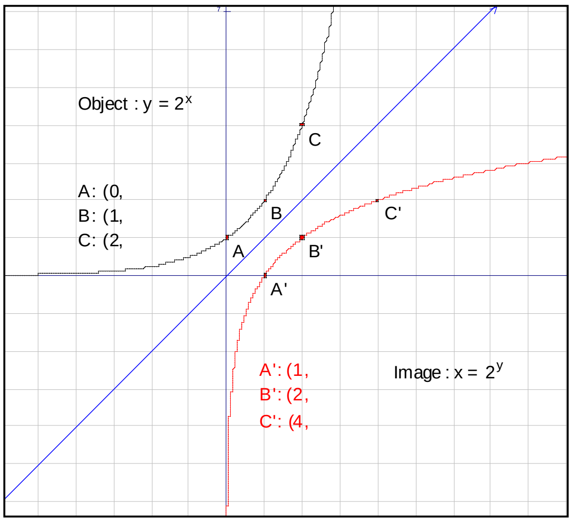
When we have established the principle of this type of reflection, we go on to the different functions and show the sameness of the process every time.

With this technology, we are able to do this in Grade 10 and the x = y reflection becomes just one of the transformations. When we repeat the exercise in Grade 12, we concentrate on the function equation in its y = . . . form and introduce the necessity for the word 'log'.









CONCLUSION

The pre- prepared lesson may be used by anyone. However in my classroom the demonstration is constantly interrupted by questions and lively discussion. Learners have to think, anticipate and commit themselves to an answer before the technology gives them the correct position of the image. This approach makes the development of the log function a natural and logical process.

NOTES

1. My thanks to Sue Southwood who helped me prepare this document
2. The GSP lesson will be available on www.hiltoncollege.com/Vula

QUADRATICS CAN BE FUN!

A WORKSHOP ON QUADRATIC EQUATIONS

Ian L Atteridge, Dainfern College

Solving a quadratic equation is the corner stone of solving polynomial equations. There are a multitude of available techniques to solve and manipulate an equation of degree two. Some of the techniques will work for higher powers and are particularly useful. Being able place quadratics in context is essential for understanding.

Introduction

The aims of the workshop are:

- to give the educator teaching resources and ideas which can be used
- to give the educator two quadratic based tasks so as to teach for understanding
- to introduce educators to a few games and alternate tasks which instill confidence and enthusiasm in learners

Content

The workshop will take the form of an introduction and reasoning behind the alternate activities followed by hands on playing of a game and a catapult activity. There will be time for discussion during the workshop. Possibilities for extension activities will be suggested and discussed.

Conclusion

The workshop will appeal to grade 10 and 11 educators who wish to move beyond the textbook and develop critical thinking, problem-solving, collaborative, and communication skills in their learners. These activities may also allow learners to apply their knowledge in new situations.

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IMPLEMENTING MATHEMATICAL INVESTIGATIONS IN THE FOUNDATION PHASE

SHANBA GOVENDER

AUSTERVILLE PRIMARY SCHOOL – DURBAN -KZN

TARGET AUDIENCE: GRADES 1 - 3

DURATION: 1 HOUR

MAXIMUM NUMBER OF PARTICIPANTS: 50

1. Motivation for Workshop
   Mathematical Investigations enhances children's learning.
   This sessions aims at providing guidance to the teacher on including investigations as a tool in teaching and learning.

2. Description of Content of Workshop

   Investigations present curiosity provoking situations, problems, and questions that are intriguing and captivate children's interest and attention.
   At the outset, students are unable to solve the problem because they are complex.
   There is more than one way to approach or solve problem.
   Investigations are often designed to be tackled by learners working in pairs or teams over a period of time.
   Investigations provide children with opportunities to engage in the authentic practices of mathematicians as they discover, invent and use mathematics to understand the world

   ACTIVITY :
   Investigation 1 *: <u>How many buttons in the bottle?</u>*
   - Investigate the numerical contents of the bottle filled with buttons .
   - Predict the number of buttons.
   - Explore the distribution of colours.
     - representing the number of each colour button on a table / graph, answering questions, and comparing their results.

Investigation 2 : ***Discovering patterns by placing discs on the counting chart.***

FINDINGS OF INVESTIGATIONS :

2 X

The 2 x table makes a straight line pattern

in every second column.

Vertical Bar Pattern

Multiplication by 2 does not end with 1. 3, 5, 7 or

9 in the units colum

3 X

Slanting pattern.

Every alternate number is an even no. (3, 6, **9,** 12, **15,** 18 **)**

Slanting pattern also has increase and decrease in tens and ones column.

4 X

Bar pattern.

Multiplication by 4 does not end with 1. 3, 5, 7 or 9

in the units column - ends with 0, 2, 4, 6, 8.

(4, 8, 12, 16, 20**, 2**4**, 2**8**, 3**2 **)**

5 X

Vertical bar

Ends in 0 or 5.

CONCLUSION : **Success Criteria.**

On completion of the workshop delegates would have gained an understanding of the importance of including investigations and the use of concrete apparatus in the mathematics lessons to make teaching and learning effective.

LEARNING MATHEMATICS THROUGH FUN AND GAMES

SHANBA GOVENDER

AUSTERVILLE PRIMARY SCHOOL – DURBAN -KZN

TARGET AUDIENCE: GRADES 1 - 3

DURATION: 1 HOUR

MAXIMUM NUMBER OF PARTICIPANTS: 50

3. Motivation for Workshop
   Children enjoy playing games. Experience tells us that games can be very productive learning activities.

4. Description of Content of Workshop

Mathematical games are 'activities' which:

- involve a challenge, usually against one or more opponents
- governed by a set of rules and have a clear underlying structure
- normally have a distinct finishing point;
- have specific mathematical cognitive objectives.

Benefits of Using Games

The advantages of using games in a mathematical teaching and learning :

- Meaningful situations - for the application of mathematical skills are created by games
- Motivation - children freely choose to participate and enjoy playing
- Positive attitude - Games provide opportunities for building self-concept and developing positive attitudes towards mathematics, through reducing the fear of failure and error;
- Increased learning - in comparison to more formal activities, greater learning can occur through games due to the increased interaction between children, opportunities to test intuitive ideas and problem solving strategies
- Different levels - Games can allow children to operate at different levels of thinking and to learn from each other. In a group of children playing a game, one child might be encountering a concept for the first time, another may be

developing his/her understanding of the concept, a third consolidating previously learned concepts

- Assessment - children's thinking often becomes apparent through the actions and decisions they make during a game, so the teacher has the opportunity to carry out diagnosis and assessment of learning in a non-threatening situation
- Home and school - Games provide 'hands-on' interactive tasks for both school and home
- Independence - Children can work independently of the teacher. The rules of the game and the children's motivation usually keep them on task.

Few language barriers - an additional benefit becomes evident when children from

non-english-speaking backgrounds are involved. The basic structures of some games are common to many cultures, and the procedures of simple games can be quickly learned through observation. Children who are reluctant to participate in other mathematical activities because of language barriers will often join in a game, and so gain access to the mathematical learning as well as engage in structured social interaction.

Hints for Successful Classroom Games

- Make sure the game matches the mathematical objective
- Use games for specific purposes, not just time-fillers
- Keep the number of players from two to four, so that turns come around quickly
- The game should have enough of an element of chance so that it allows weaker students to feel that they a chance of winning
- Keep the game completion time short
- Use five or six 'basic' game structures so the children become familiar with the rules - vary the mathematics rather than the rules
- Send an established game home with a child for homework
- Invite children to create their own board games or variations of known games.

ACTIVITY : Participants engage in playing a series of mathematical games.

CONCLUSION: **Success Criteria.**

On completion of the workshop delegates would have gained an understanding of the importance of including investigations and the use of concrete apparatus in the mathematics lessons to make teaching and learning effective.

PROBLEM SOLVING IN THE FOUNDATION PHASE

SHANBA GOVENDER

AUSTERVILLE PRIMARY SCHOOL – DURBAN -KZN

TARGET AUDIENCE: GRADES 1 - 3

DURATION: 1 HOUR

MAXIMUM NUMBER OF PARTICIPANTS: 50

5. Motivation for Workshop

   Problem Solving is one of the critical outcomes of the NCS.

   As per the Government Gazette 30880 – 14 March 2008 - time is allocated for Problem Solving and Investigations in the Foundation Phase.

   Grades one and two 15 minutes daily and 20 minutes daily for grade three.

However many of us educators shy away from this strategy that empowers a child to become a critical thinker. This apprehension that we sometimes feel needs to be eliminated. This can only be done if we ourselves explore and understand the process of finding solutions.

6. Description of Content of Workshop

   Problem Solving - Envisages learners who are able to :

- Identify and solve problems and make critical decisions using critical and creative thinking.
- Work effectively with others As members of a team, group, organisation and community.
- Organise and manage themselves and their activities responsibly and effectively.
- Collect, analyse, organise and critically evaluate information.
- Communicate effectively using visual, symbolic and / or language skills
- Use science and technology effectively and critically, showing responsibility towards the environment and the health of others.
- Demonstrate an understanding of the world as a set of related systems by recognising that problem-solving contexts do not exist in isolation.

Types of Problems

- ❖ Routine Problems
- ❖ Non – Routine Problems

Routine Problems are

- ❖ Basic Word Problems

  a) I have 14 toy cars. Mom gave me 3 more toy cars.
  How many toy cars do I have altogether ?

  b) There are 44 children in our class.
  21 children go out to play soccer.
  How many are left in the class ?

Non – Routine Problems

- ❖ Involves greater critical thinking and analysing.
- ❖ May have more than one basic operation to solve the problem.

a) I am a two digit number.
The sum of my digits is 9.
My tens digit is 5 more than my
units digit

Useful Strategies for Problem Solving

- Look for key words. (What is given and required)
- Look for patterns.
- Make a drawing.
- Construct a table.
- Trial and Improvement.
- Guessing and Checking.
- Work Backwards.
- Make the problem simpler.
- Visualise the situation.

- Change the strategy.

ACTIVITY : Solve problems that are suitable for the foundation phase
Learner using various strategies. (Handout included).

Acknowledgements / Reference

- 1. Mr V Naidoo - Problem Solving
- 2. TIPS - Mr Tony Reddy
- 3. TIPS - Mr H S Govender
- 4. NCS Document - Mathematics - DOE.

TOPIC: POLYGONS

PRESENTER: MRS.SHOBANA GOVENDER

SENIOR PRIMARY HEAD OF DEPARTMENT: CROSSMEAD PRIMARY

TARGET AUDIENCE: GRADES 4-7

DURATION: 1 HOUR

MAXIMUM NUMBER OF PARTICIPANTS: 50

1. Motivation for Workshop

Learning Outcome 3 involves the teaching of Space and Shape(Geometry)where the learner will be able to describe and represent characteristics and relationships between two-dimensional shapes and three-dimensional objects in a variety of orientations and positions. I find that many grade 4-7 mathematics educators are not familiar with this learning outcome as they are not maths.specialists.

Learning Outcome focus

The study of space and shape improves understanding and appreciation of the pattern, precision, achievement and beauty in natural and cultural forms. It focuses on the properties, relationships, orientations, positions and transformations of two-dimensional shapes and three-dimensional objects. The study of space and shape enables the learner to:

- develop the ability to visualise, interpret, calculate relevant values, reason and justify; and
- interpret, understand, classify, appreciate and describe the world through two-dimensional shapes and three-dimensional objects, their location, movement and relationships.

The learner should gain these skills from experiences with concrete objects, through drawing and construction, and in the abstract justification of spatial relationships. It is important that the study of two-dimensional shapes and three-dimensional objects be contextualised to include the study of natural and cultural forms and artefacts.

Contexts should be selected in which the learner can study space and shape in a way that can be used to build awareness of other Learning Areas, as well as human rights, social, economic, cultural, political and environmental issues. For example, the learner should be able to:

- use national flags to demonstrate transformations and symmetry in designs;
- investigate and recognise the geometrical properties and patterns existing in traditional and modern architecture;

- use maps in Geography as specific forms of grids; and
- investigate geometric patterns in art.

Intermediate Phase focus

The learner's experience of space and shape in this phase moves from recognition and simple description to classification and more detailed description of features and properties of two-dimensional shapes and three-dimensional objects.

Learners should be given opportunities to:

- draw two-dimensional shapes and make models of three-dimensional objects; and
- describe location, transformations and symmetry.

2. Description of Content of Workshop: Polygons

GEOMETRY

Geometry is all about shapes and their properties. The two most common subjects are:

	Two dimensional shapes[2D] is about flat shapes like circles and triangles ... shapes that can be drawn on a piece of paper because they only have length and breadth.
	Three dimensional objects[3D] include cubes and pyramids because they have length, breadth and height.

TOPIC: PROBABILITY

PRESENTER: MRS.SHOBANA GOVENDER

SENIOR PRIMARY HEAD OF DEPARTMENT: CROSSMEAD PRIMARY

TARGET AUDIENCE: GRADES 4-7

DURATION: 1 HOUR

MAXIMUM NUMBER OF PARTICIPANTS: 50

1. Motivation for Workshop

In learning outcome 5 which deals with data handling the learner will be able to interpret and determine chance variation.

Many educators shy away from teaching the aspect of probability because they are not familiar with the following:

- **Certainty, uncertainty and impossibility**
- **Random experiments**
- **Frequency and relative frequency**

Through the study of chance, the learner will also develop skills and techniques for making informed choices, and coping with randomness and uncertainty.

The study of chance (probability) develops awareness that:

- **different situations have different probabilities of occurring; and**
- **for many situations there are a finite number of different possible outcomes.**

In this phase, the learner is not expected to calculate the probability of events occurring.

This aspect can be taught effectively through games. The activities will enable the educators to implement a learner-centred classroom environment where learners work in pairs or groups. Learners show little fear ad discover for themselves whist enjoying the excitement of playing a game.

2. Description of Content of Workshop: Probablity

1. WHAT IS PROBABLITY?

Probability is the likelihood or chance that a given event will occur. Probability is usually expressed as a ratio of the number of likely outcomes compared with the total number of outcomes possible.

2. CERTAINTY; UNCERTAINTY AND IMPOSSIBILITY

- **Ceratinty:** It will definitely happen or take place.
- **Impossible:** It will never happen or take place.
- **Uncertainty**: If we are unsure if something will happen or take place.

Examples:

1. **It is raining outside. I go outside but do not have an umbrella. Will I get wet?**
   Answer: CERTAIN
2. **I roll a die. I get the number 7.**
   Answer: IMPOSSIBLE – The die has only the numbers 1-6.
3. **I place 4 red balls and 8 green balls in a container. I will draw out a red ball in my 1st pick.**
   Answer: UNCERTAIN – I may get a red or a green ball.

Try this out:

Place discs into cups as follows:

Cup 1: 5 Red

Cup 2: 5 Blue

Cup 3: 1 Red ; 1 Blue

Draw 1 disc from each cup and note if a red disc is drawn in each case.

It is certain that a red disc is drawn from cup 1.

It is impossible to draw a red disc from cup 2.

It is uncertain whether a red disc is drawn from cup 3.

Various games can be played in the classroom using the following:

- **Dice**
- **Playing cards**
- **Discs of different colours**
- **Coins**

Ordering Uncertainty [grade 5 upwards]

Levels of uncertainty can be garded. We cannot predict what will happen. We can only say that some outcomes are more likely than others.

Example: Place discs as follows:

Cup 1 : 3 blue discs

Cup 2: 3 red discs

Cup 3: 2 red and 1 blue

Cup 4: 1 red and 2 blue

Draw one disc from each cup and note the colour.

CUP	1	2	3	4
COLOUR				

Repeat this task several times and note the outcomes.

CUP	1	2	3	4
COLOUR				
CUP	1	2	3	4
COLOUR				

CUP	1	2	3	4

COLOUR				

CUP	1	2	3	4
COLOUR				

CUP	1	2	3	4
COLOUR				

CUP	1	2	3	4
COLOUR				

<u>You will realize by now that the level of uncertainty actually varies.**</u>**

3. <u>RANDOM EXPERIMENT</u>

Let's look more closely at situations where there is uncertainty, that is, where there are various outcomes fro a given game or situation under discussion.

Random experiment refers to a process which could result in various possible outcomes, that a certain procedure is going to lead without us knowing what this result is going to be until we actually do the procedure.

<u>EXAMPLES</u>:

1. **A die is rolled. What will the outcome be?**
2. **A cup has 2 red, 2 blue and 2 green smarties in it. I randomly choose 1 smartie?**

   What colour is it?
3. **I choose a red hearts picture card from a full pack of playing cards. What can did**

   I choose?
4. **I toss a coin. What do I get?**

5. **Bafana Bafana playing in the second round of the FIFA world Cup.**

In each case we do not know or we are uncertain what the outcome will be until we have actually performed the experiment. These are examples of a random experiment.

Every random experiment has a set of possible outcomes[known as the sample space].

{.} is used to denote the set of all possible outcomes.

<u>ANSWERS: The set of all possible outcomes of a random experiment as shown in the above examples:</u>

1. **{1;2;3;4;5;6}**
2. **{red; blue; green}**
3. **{jack; queen; king; ace}**
4. **{head; tail}**
5. **{Yes; No}**

<u>Further examples</u>:

1. Select 6 children from your class – 3 boys and 3 girls – Kiara; Chanel and Krivasha are the girls and

Adhir; Akshay and Cliffy are the boys. Cover another learners's eyes[Penny] with a cloth so that

she cannot see. Make the 6 learners stand in a circle. Penny must now choose 1 learner.

1.1 What gender was chosen? {male; female}

1.2 Which learner was chosen? { Kiara; Chanel; Krivasha; Adhir; Akshay; Cliffy}

1.3 What kind of hair di the learner have? {long hair; short hair; medium length hair}

2. <u>Let learners roll die in pairs</u>.

2.1 List the possible outcomes which would result in an odd number being rolled. {1; 3; 5}

2.2 List the outcomes which results in all the even numbers being rolled.{2;4;6}

2.3 List the outcomes which would result in a number less than 5 coming up when the die is

rolled. {1;2;3;4}

2. Use <u>a pack of playing cards. Get a learner to choose</u>:

2.1 a black card

{spade ace; spade 2; spade 3; spade 4; spade 5; spade 6; spade 7 ; spade 8; spade 9;

spade 10; spade jack; spade queen ; spade king; club ace; club 2; club 3; club 4; club 5;

club 6; club 7; club 8; club 9; club 10 ; club jack; club queen; club king}

2.2 a king card

{spade; club; heart; diamond}

4. <u>FREQUENCY AND RELATIVE FREQUENCY</u>

Here, we are going to compare the likelihood of various outcomes of a random experiment which are initially predicted and then checked by examining the frequencies of the particular outcomes.

Every random experiment has various outcomes. If the experiment is repeated a number of times, then the frequency of any particular outcome is defined to be the number of times that particular outcome occurred.

<u>Look at this example</u>:

Roll a die. List the possible outcomes. {1; 2;3;4;5;6}

Now roll the die 15 times: The following sequence was rolled:

1 6 3 5 2 4 3 6 2 3 1 4 6 6 1

The 1 has a frequency of 3

The 2 has a frequency of 2

The 3 has a frequency of 3

The 4 has a frequency of 2

The 5 has a frequency of 1

The 6 has a frequency of 4

The frequency of each outcome varies as the experiment is repeated. Try the same experiment again to check.

Some outcomes are more likely than others. The more likely outcomes lead to hgher frequencies when the experiment is repeated.

<u>EXAMPLE</u>:The educator places 5 black marbles and 2 red marbles in a cup. She asks the learners the following questions:

1. Suppose I draw a marble from the cup, what colour will I get?

<u>ANSWER: We are uncertain: BLACK or RED</u>

2. If I draw a marble from the cup, note the colour and replace it and draw again until I have drawn

25 times. What colour marble do you predict will be drawn more often[have a higher frequency]?

Why?

<u>ANSWER: The black marble should be drawn more often</u>.

Now perform the actual experiment.

<u>Draw a table like this and complete after each draw</u>.

DRAW	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
COLOUR																									

Now complete a tally table.

It is obvious from the tally table that the black marble has a higher frequency as should have been predicted.

COLOUR	TALLY	FREQUENCY
BLACK		
RED		
TOTAL		25

The more chance of something happening, the higher the frequency, that is, if we repaeat the experiment than we can expect the outcome of the experiment which is 'more likely' or has more 'chance' to occur more often.

The frequency of a particular outcome which is merely 'how often' this outcome occurs.

The relative frequency = $\dfrac{\text{frequency}}{\text{number of repeats of the experiment}}$

<u>EXAMPLE</u>:

The educator rolls a die 10 times to calculate the raltive frequency of each possible outcome.

Suppose these are the outcomes: <u>1 2 3 5 6 4 3 2 2 6</u>

Frequency of 2 is 3 **Relative frequency of 2 is $\frac{3}{10}$.**

Frequency of 6 is 2 **Relative frequency of 6 is $\frac{2}{10}$.**

Frequency of an even number is 6 **Relative frequency of an even number is $\frac{6}{10}$.**

1. Begin the lesson by asking students to define probability (the likelihood or chance that a given event will occur). Probability is usually expressed as a ratio of the number of likely outcomes compared with the total number of outcomes possible. Ask students if they can give an example of probability.

2. To help students understand probability, work on the following problem as a class: Imagine that you have boarded an airplane. The rows are numbered from 1 to 30, and there are six seats per row, three on each side of the isle. Seats in each row are labeled A through F. Using that information, work together as a class to solve the problems listed below.

   a. How many seats are in the airplane?*180 seats*
   b. What are your chances of sitting in row number 7?*6/180, or 1/30*
   c. What are your chances of sitting in a window seat?*There are two window seats per aisle, for a total of 60 window seats. Your chances of seating at a window would be 60/180, or 1/3.*
   d. What are your chances of sitting in an "A" seat?*There are 30 A seats, so your chances are 30/180, or 1/6.*
   e. What are your chances of sitting in an even-numbered row?*Of the 30 rows, 15 are even-numbered, so your chances are 15/30, or 1/2.*

3. To figure out each problem, students must set up a ratio between the total number of outcomes—in these problems either the total number of seats or rows—and the specific question asked. Tell students that they will write their answer as a fraction, decimal, and percentage. Example: The chance of sitting in seat 7A is 1/180, .00555, or .555 percent. The ratio presented as a percentage helps make it clear if the probability of an event is great or small.

4. Distribute the Classroom Activity Sheet and tell students that they are going to work on several probability problems in class, expressing the answer as a fraction, decimal, and

percentage. Students may work individually or with partners. The problems and the answers are listed below:

a. Your sock drawer is a mess. Twelve black socks and six red socks are mixed together. What are the chances that, without looking, you pick out a red sock? What are the chances of picking a black sock?*The total is 18 socks, and one-third of them are red (6/18, or 1/3, or .333, or 33.3 percent). The probability of picking a red sock is 1/3, or 33.3 percent. Because two-thirds of the socks are black (12/18, or 2/3, or—rounding up—66.7 percent), the probability of picking a black one is higher—2/3, or 66.7 percent, compared with 1/3, or 33.3 percent.*
b. You are rolling a regular die. What is the probability of rolling a 3?*Of the total of six outcomes, 3 is one outcome. The probability is the ratio 1/6, .1666, or 16.66 percent.*
c. If you are rolling a regular die, what is the probability of rolling an even number?*Of the six possible outcomes, half, or three outcomes, could be an even number. The probability is 3/6, 1/2, .5, or 50 percent.*
d. You are randomly choosing a card from a deck of 52 cards. What is the probability that the card you pick will be a king?*Of the 52 possible outcomes, four outcomes are kings. The probability is 4/52, 1/13, .076, or 7.6 percent.*
e. You are visiting a kennel that has three German shepherds, four Labrador retrievers, two Chihuahuas, three poodles, and five West Highland terriers. When you arrive, the dogs are taking a walk. What is the probability of seeing a German shepherd first?*Out of a total of 17 dogs, 3 are German shepherds. The probability of seeing a German shepherd is 3/17, .176, or 17.6 percent.*
f. Two out of three students in Mr. Allen's class prefer buying lunch to bringing it. Twenty students prefer buying lunch. How many students are in Mr. Allen's class?*Students can set up the following problem: 20/30, or 2/3, of the total number of students (X) buy lunch (20). To express that mathematically, 2/3 (X) = 20. Solve for X, which equals 30, so there are 30 students in Mr. Allen's class.*

MEASURE OF DISPERSION AROUND THE MEAN

Presenter: Tlou Robert Mabotja
Institution: Waterberg District (Limpopo Province)
Curriculum Advisor "Mathematics"
Robetlou@yahoo.com

Target Audience: FET Band
Duration: 1 hour (30 participants)

***NB**: Participants must have a **scientific calculator** for this workshop. In these notes a **CASIO fx-82ES plus** is used.*

DESCRIPTION OF WORKSHOP

This is a one hour workshop discussion: the aim of this workshop is to share with participants ideas on measure of dispersion through revision of basic calculations of the mean, variance and standard deviation. Participants are requested to bring along scientific calculators i.e. **CASIO *fx-82ES* plus.** Calculator usage will in this case play a vital role in the workshop, as we aim to bring teachers up to speed with the latest technological instruments in teaching mathematics. The aim is not to replace mathematics, but rather to supplement it. Participants are also briefed about their expectation after the workshop and that the workshop is designed in such a way that is hands on "that everyone is given an opportunity to participate in working out solutions from the worksheets".

MOTIVATION FOR RUNNING THE WORKSHOP

This workshop is designed to demonstrate how measures of dispersion around the mean can be taught using various techniques, i.e. mathematical and scientific calculator to Grade 11 and 12 learners. It will also include activities from Learning Outcome 4 that are exam related in order to show participants sections that are examinable as outlined in the core assessment standard at the end of the year. It also intends to demonstrate a simple strategy of meeting the assessment standards and how to provide an opportunity to start thinking about the role played by **CASIO *fx-82ES* plus** to find the standard deviation. Those strategies will encourage educators to design similar activities on their own. Finally this workshop was designed to demonstrate how much fun, finding standard deviation can be to both educators and learners.

LEARNING OUTCOME AND ASSESSMENT STANDARD COVERED

Learning Outcome 4: Data Handling and Probability

Assessment Standard 11.4.1 (a)

Calculate and represent measures of central tendency and dispersion in univariate data by calculating the variance and standard deviation of sets of data manually (for small sets of data) and using available technology (for larger sets of data) and representing results graphically using histograms and frequency polygons

Measures of central tendency (or averages) are very important as they can give a picture of the group that they represent. However, taken by themselves, they give a very limited view of the whole picture. As well as the average, you need to know how the rest of the data are grouped around the average – whether it is closely grouped or scattered more widely. You need to consider a **MEASURE OF THE SPREAD** or **DISPERSION** of data items around the middle values.

You can find a measure of **Dispersion around a mean** or **a median.** In this workshop we consider dispersion around a mean.

1) VARIANCE

The **mean** is the balance point of a distribution of data. There are various ways you can measure the spread of a distribution around its mean. A measure that gives an idea of the spread of a data set is the **deviation from the mean** – i.e. how far away from the mean each data item is:

- The differences from the mean are written $(x - \bar{x})$, where x is an element of the set of data and $\bar{x}$ is the mean of the set of data.
- The **variance** $= \dfrac{\sum (x - \bar{x})^2}{n}$ is the average of the squares of the deviations of each data item from the mean

Note:

- This measure of spread takes into account all data items.
- It is a measure of the **variability** of the data items.
- If the value of the variance is large, then the data items are widely spread. If the value of the variance is small the data items are closely clustered around the mean.

WORKSHEET 1

Suppose two Grade 11 learners, Robert and Tlou, each have written three tests.

- Robert's marks (rounded off to the nearest mark) are: 26, 28 and 30
- Tlou's marks (rounded off to the nearest mark) are: 21, 20 and 73.

1) Calculate:
   a) The mean marks of Robert
   b) The mean marks of Tlou

2) Work out the deviations from the mean of each data item by completing the table:

Robert		
Marks x	Deviation $(x-\bar{x})$	Deviation^2 $(x-\bar{x})^2$
26		
28		
30		
Total = $\Sigma(x-\bar{x})^2$		

Tlou		
Marks x	Deviation $(x-\bar{x})$	Deviation^2 $(x-\bar{x})^2$
21		
20		
73		
Total = $\Sigma(x-\bar{x})^2$		

3) Find the mean (average) of squared deviations.

2) STANDARD DEVIATION

The most common measure of dispersion is the **standard deviation**. It is simply the square root of the variance and is usually represented by the letter σ (lower case "sigma")

Please note:

In Grade 12 you will learn about sampling (examined in Paper 3) where you can estimate the standard deviation of a large population by calculating the standard deviation of a random sample of that population using the formula:

$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$. This formula gives an estimated standard deviation for the entire population.

To find the **standard deviation** in Paper 2 the formula $s = \sqrt{\text{variance}} = \sqrt{\frac{\sum(x-\bar{x})^2}{n}}$ will be used because you are dealing with entire populations rather rather than random samples of populations.

Note:

- The standard deviation has the same units as the data items and as the mean.
- A small standard deviation tells you that the data items are closely clustered around the mean, while a large standard deviation tells you that the items are more spread out.
- The standard deviation is the most commonly used measure of dispersion.
- Although it looks as though the standard deviation is complicated to calculate it really takes little time and is very easy if you use a scientific calculator (**CASIO *fx-82ES* plus**).
- The standard deviation can be used to compare different sets of data.

(3) USING THE CALCULATOR TO FIND THE STANDARD DEVIATION

It is easy to use a calculator to find the standard deviation. Remember to first get into STAT mode on the calculator.

To work out the standard deviation on the calculator all you need to do is enter the data and then enter:

[SHIFT] **[1]** **[4: VAR]** **[3: x$^{\sigma}$ n]** **[=]**

Look again at Robert and Tlou's marks

Robert's marks are: 26, 28 and 30

Tlou's marks are: 21; 20 and 73

To work out the Standard Deviation for Tlou's marks, press the following keys

[MODE] [2: STAT] [1: 1–VAR] 21[=] 20 [=] 73 [=]

[SHIFT] [1] [4: VAR] [3: $x\sigma n$] [=]

- You should find that the standard deviation of Robert's marks = 1, 63
- Similarly you should find the standard deviation of Tlou's marks = 24,75
- What do these values tell you about the spread of the data?

(4) USING THE STANDARD DEVIATION TO REACH CONCLUSIONS:

Provided that the sample size is reasonably large and the data is not too skewed (that is, it does not have some very large or very small values), it is possible to make the following approximate statements:

- About **66%** of the individual observations will lie within **one standard deviation** of the mean.
- For most data sets, about **95%** of the individual observations will lie within **two standard deviations** of the mean.
- **Almost all** of the data will lie within **three standard deviations** of the mean.

WORKSHEET 2
The office manager of a small office wants to get an idea of the number of phone calls made by the people working in the office during a typical day in one week in June. The number of calls on each day of the (5-day) week is recorded. They are as follows: Monday **– 15**; Tuesday **– 23**; Wednesday **– 19**; Thursday **– 31**; Friday **– 22** 1. Determine: a) The mean number of phone calls per day b) The standard deviation (correct to 1 decimal place).

2. On what percentage of the days is the number of calls within one Standard Deviation of the mean?

One Standard Deviation from the mean is:

$\bar{x} \pm \sigma =$ ………………………………

So the interval is (…… − ……; …… + ……) = ……………………

The phone calls on ………………………………………… fall within the interval.

$\frac{\quad}{\quad} \times$ 100% = ……………………

So the number of phone calls on ………. of the days lies within one Standard Deviation of the mean.

(5) WORKING WITH UNGROUPED AND GROUPED DATA IN A FREQUENCY TABLE

<u>Grouped Data:</u>

- Multiply the data item or score (x) by the frequency (f) and record this in an extra column in the frequency table (fx).
- Calculate the arithmetic mean of fx i.e. calculate ($\bar{x}$) = $\dfrac{\text{Total of scores}}{\text{Total frequency}}$

$$= \frac{\sum fx}{n}$$

NB. Where f is the frequency, x is the value of the data item, $\bar{x}$ is the mean and n is the total number of values

WORKSHEET 3

Midyear report of Lethabo Mabotja, a Grade 11 learner reflect the following performance results:

Sepedi – **45**, English – **30**, Life Orientation – **51**, Mathematics – **45**, Accounting – **51**, Economics – **23** and Business Studies – **43.**

1) Organize this data in a frequency table.

Marks x	Frequency f
45	
30	
51	
23	
43	
Total:	

2)

a) Get into the STAT mode for univariate data
b) Set up a frequency table by pressing the following keys

[SHIFT] [SETUP] Scroll down using ▼ arrow

[3:STAT] [1:ON]

c) Enter data into the first column
d) Enter data in the second column by using the arrows to get to the first cell in the Frequency column
e) Press **[AC]**

3) Calculate, correct to the nearest whole number:

a) The mean

b) The Standard Deviation

c) What percentage of the marks lies within one Standard Deviation of the mean?

Grouped Data:

- First find the midpoint (x) of each group or class of intervals
- Multiply each midpoint (x) by the frequency of that group (f) and record this in an extra column in the frequency table (fx).
- Calculate the arithmetic mean of fx i.e. calculate ($\bar{x}$) = $\frac{\text{Total of scores}}{\text{Total frequency}}$

$$= \frac{\sum fx}{n}$$

NB. Where f is the frequency of the interval, x is the midpoint of the interval, $\bar{x}$ is the mean and n is the total number of values

WORKSHEET 4

The following table (grouped frequency distribution) shows the mark obtained by 220 learners in a Mathematics examination

Percentage	**1 –**	**11**	**21 –**	**31 –**	**41 –**	**51 –**	**61 –**	**71 –**	**81 –**	**91 –**

1) Complete the following table

	x		fx		
1 – 10	*5*	*2*	$5 \times 2 = 10$	2445.30	4890.61
11 – 20	*15*	*6*	$15 \times 6 = 90$	1556.30	9337.82
21 – 30					
31 – 40					
41 – 50					
51 – 60					
61 – 70					
71 – 80					
81 – 90					
91 – 100					
Total					

2) Calculate the approximate mean marks correct to the nearest whole number
3) Find an approximate value of the Standard Deviation correct to 1 decimal place.

Using the CASIO *fx-82ES* plus

Get into Stats mode	[MODE][2:STAT][1:1-VAR]
Set up a frequency table	[SHIFT][SETUP] Scroll down using the ▼ arrow [3:STAT][1:ON]
Enter midpoints in the first column	5[=] 15[=] 25[=] 35[=] 45[=] 55[=] 65[=] 75[=] 85[=] 95[=]
Enter data in the second column	Use the arrows to get to the first cell in the frequency column: 2[=] 6[=] 11[=] 22[=] 39[=] 59[=] 45[=] 20[=] 11[=] 5[=]

Find the mean	[SHIFT] 1 [4:VAR] [2: $\bar{x}$] [=] *The answer is……………….*
Find the standard deviation	[SHIFT] 1 [4:VAR] [3: $x\sigma n$] [=] *The answer is …………………..*

CONCLUSION

The presenter conduct this workshop to support Grade 11 and 12 educators, subject advisors and other stakeholders with respect to the use of various techniques to calculate the measure of dispersion especially the standard deviation. It also assists participants in acquiring an understanding of accurate use of a scientific calculator (**CASIO *fx-82ES* plus**) to find standard deviation. This is not aimed at changing the way of teaching mathematics but it supplement the manual way of teaching. Technology at present era plays an important part in our life, therefore it important to introduce the technology in our teaching as recommended by the National Curriculum statement in order to give guide to the implementation of this new curriculum.

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VEDIC SQUARES

VISHNU NAIDOO

PRINCIPAL: BUFFELSDALE SECONDARY

TARGET AUDIENCE: GRADES 4-7

DURATION: 1 HOUR

MAXIMUM NUMBER OF PARTICIPANTS: 50

7. Motivation for Workshop

   This workshop is the departure from the stereotyping in Mathematics. The Mathematics shows an important link between numbers and shapes. The NCS makes mention about integration among learning outcomes. Three learning outcomes are linked. Learning Outcome One(Numbers); Two(Patterns) and Four(Shape) are linked using Vedic Squares.

   Educators struggled with this aspect of Mathematics.

8. Description of Content of Workshop

   Vedic Mathematics is Mathematics that has its origins in India.

   Vedic Mathematics comes from the Vedic tradition of India. The Vedas are the most ancient record of human experience and knowledge, passed down orally for generations and written down about 5,000 years ago. Medicine, architecture, astronomy and many other branches of knowledge, including mathematics, are dealt with in the texts. Perhaps it is not surprising that the country credited with introducing our current number system and the invention of perhaps the most important mathematical symbol, 0, may have more to offer in the field of mathematics.

   The remarkable system of Vedic Mathematics was rediscovered from ancient Sanskrit texts early last century. The system is based on 16 sutras or formulae, such as: "by one more than the one before" and "all from nine and the last from 10". These describe natural processes in the mind and ways of solving a whole range of mathematical problems. For example, if we wished to subtract 564 from 1,000 we simply apply the sutra "all from nine and the last from 10". Each figure in 564 is subtracted from nine and the last figure is subtracted from 10, yielding 436.

   Vedic Mathematics could be described as very ‘profound mathematics’.

There are two types of vedic squares. One type is used in architecture and the other type is purely mathematical.

Our focus will be on the mathematical one.

8.1 Vedic Squares

Let us consider a 4 x 4 vedic square.

We will consider the product of single digit numbers.

The 4 x 4 vedic square

X	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	3
4	4	8	3	7

In the above multiplication the answer must be a single digit. If the answer is not a single digit then the sum of the digits must be found.

Examples of this: 4 x 3 =12

12 = 1+ 2 = 3

Join all the digits which are same.

Which is the most frequent digit?

Three (3) is the most frequent.

If we join the 3's what shape will emerge? A quadrilateral.

If we join the 4's what shape will emerge? A triangle.

Some interesting questions:-

Can we get a 0? Why?

What is the highest digit?

What is the lowest digit?

We will construct the following vedic squares:-

5 x 5 ; 6 x 6 ; 7 x 7; 8 x 8; 9 x 9.

Another interesting feature is that of a sequence which will be used to generate 'vedic worms'.

9. Activities for Workshop

9.1 Complete the following vedic squares

9.1.1 5 x 5 vedic square

X	1	2	3	4	5
1	1	2	3	4	
2	2	4	6	8	
3	3	6	9	3	
4	4	8	3	7	
5					

9.1.2 6 x 6 vedic square

X	1	2	3	4	5	6
1	1	2	3	4		
2	2	4	6	8		
3	3	6	9	3		
4						

	4	8	3	7		
5						
6						

9.1.3 7 x 7 vedic square

x	1	2	3	4	5	6	7
1							
2							
3							
4							
5							
6							
7							

9.1.4 8 x 8 vedic square

X	1	2	3	4	5	6	7	8
1								
2								
3								
4								

5								
6								
7								
8								

9.1.5 9 x 9 vedic square

x	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

9.1.6 Join the digits which appear 5 or more times.

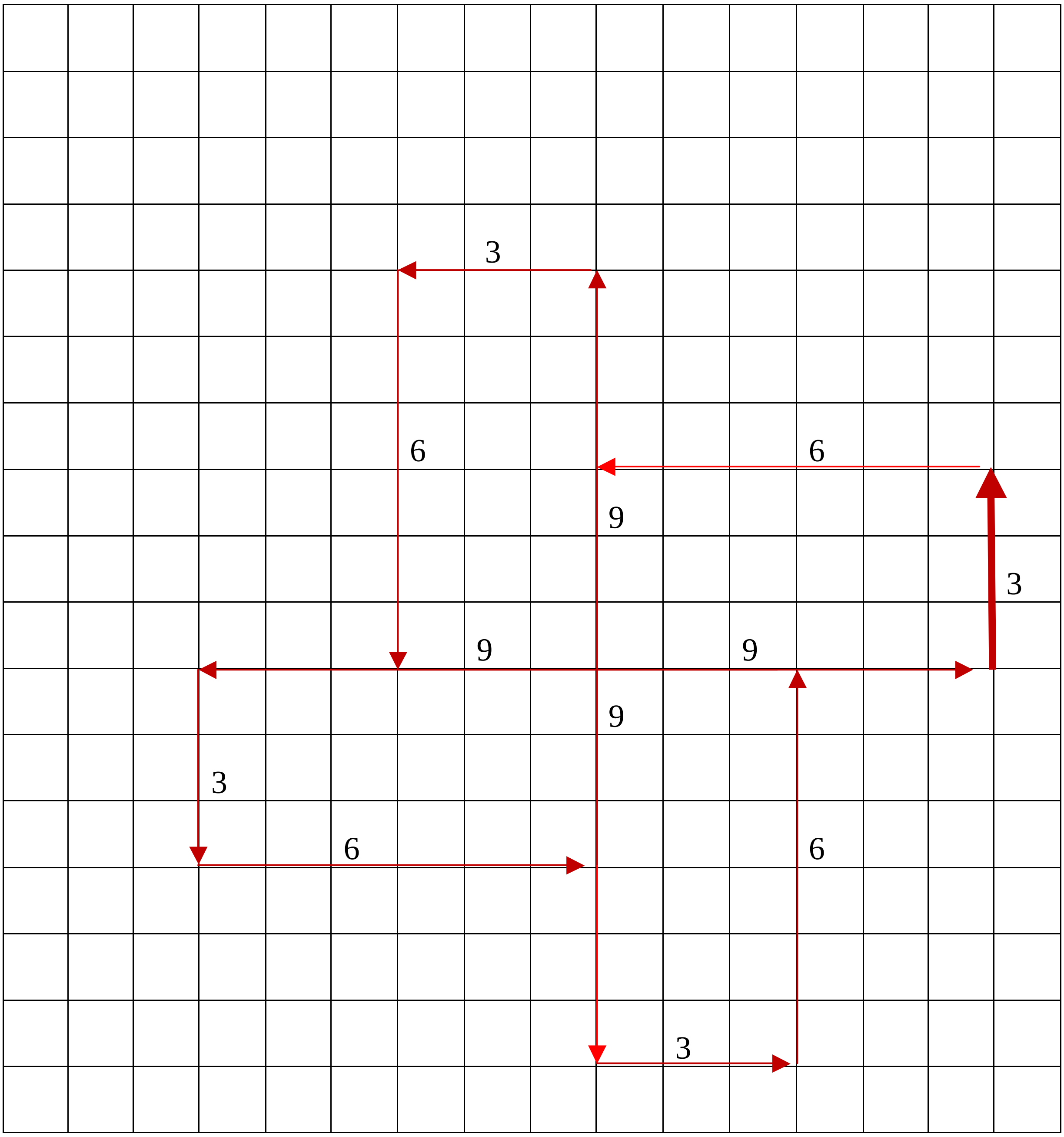
9.1.7 Join each vertex to every other vertex.

10. Consider the sequence from the 9 x 9 square.

4.1 3; 6; 9; 3; 6; 9 ; 3; 6; 9

Grid paper will be use from an arbitrary point.

Use two colours for alternating lines.



4.2 Four(4) other sequences will be done

CURRICULUM CHALLENGES IN MATHEMATICS

VISHNU NAIDOO

PRINCIPAL: BUFFELSDALE SECONDARY

TARGET AUDIENCE: FET PHASE

DURATION: 1 HOUR

MAXIMUM NUMBER OF PARTICIPANTS: 50

1. Motivation for Workshop

   The Mathematics Curriculum is written as Learning Outcomes and Assessment Standards. The Assessment Standards specify the minimum. Educators are allowed to 'extend' their learners. Within the curriculum itself some aspects of Mathematics is left to interpretation. The issue of minimum has implications for the examination paper. The process cannot go beyond the assessment standard example – the curriculum mentions find the derivative of $\frac{1}{x}$. This is not the same as $\frac{a}{x}$ where $a \in Z$; and a$\neq$ 0.

2. Description of content of Workshop

   The various topics in NCS FET Mathematics will be considered for the purposes of the workshop.

   2.1 Calculus

   Rules for differentiation does not preclude the teaching and assessment of certain problems.

   Find $\frac{dy}{dx}$ if t = $\frac{3}{y+1}$ and x = $\frac{1+t}{t}$

   We need to write y in terms of x.

   2.2 Mathematics of Finance

   There are many aspects which are challenging. If these problems are ignored then it is obvious that the curriculum is being compromised. Consider this problem for Grade 10.

   2.2.1 ABC investments wishes to invest a certain amount of money at a bank earning interest at 10 % p.a compounded annually. After how many years will this investment double?

   This problem translates to

   $A = P(1+i)^n$

   Let x be the investment and therefore the accumulated amount is 2x.

   $2x = x(1+10\%)^n$

   $2 = (1,1)^n$

   This is problematic for the grade 10 learner who has not done logarithms. Here is the challenge to the educator.

   Our LHS = 2. If we let our RHS = f(x) we can use the calculator and set up a table.

Our equation will now become

$f(x) = (1,1)^x$

Using the FX 82 ES PLUS we can proceed.

The values that we are looking for are integral. If we obtain fractional answers then how do we proceed remembering the compounding is annually.

2.2.2 Retirement and future value of annuities.

Suppose a lady receives a gratuity of R2 million at retirement. She invests R1 million in a fund for 10 years which earns interest at a rate of 9,6% p.a compounded monthly. How much will she paid monthly for the 10 years without leaving a balance?

Is this problem within the scope of the Grade 12 curriculum. The answer is yes since present and future values of annuities are done.

2.3 Trigonometry

In trigonometry the 3 reciprocal ratios are not part of the curriculum. The implications are very serious. These ratios are going to come through the ‘back door’. Instead of saying cosec θ we are going to say $\frac{1}{sin\theta}$. All previous problems could be changed and then learners are forced to use first principles each time.

2.3.1 Compound angle theorem

The theorem which is the starting point is now given as the definition. Why should it be cos(A-B)?

Let us look at an alternative:

Prove that cos(A+B) = cosAcosB –sinAsinB

2.3.2 A lovely ‘back door’ problem

Without using a calculator find the value of:-

$$\frac{\frac{cos65^0}{sin65^0}+tan200^0}{1+\frac{cos70^0}{cos70^0}tan155^0}$$

3. Activities for Workshop

The following problems will be done at the workshop.

3.1 Find $\frac{dy}{dx}$ if t = $\frac{3}{y+1}$ and x = $\frac{1+t}{t}$

3.2 ABC investments wishes to invest a certain amount of money at a bank earning interest at 10 % p.a compounded annually. After how many years will this investment double?

3.3 Retirement and future value of annuities.

Suppose a lady receives a gratuity of R2 million at retirement.

She invests R1 million in a fund for 10 years which earns interest at a rate of 9,6% p.a compounded monthly. How much will she paid monthly for the 10 years without leaving a balance?

3.4 Prove that cos(A+B) = cosAcosB –sinAsinB

3.5 Without using a calculator evaluate

$$\frac{\frac{cos65^0}{sin65^0}+tan200^0}{1+\frac{cos70^0}{cos70^0}tan155^0}$$

It is obvious that before the finalization of the curriculum these challenges must be addressed with all stakeholders. There are many other such challenges.

REAL LIFE MATHEMATICS

VISHNU NAIDOO

PRINCIPAL: BUFFELSDALE SECONDARY

TARGET AUDIENCE: FET MATHEMATICAL LITERACY

DURATION: 1 HOUR

MAXIMUM NUMBER OF PARTICIPANTS: 50

11. Motivation for Workshop

    Real life problem can pose a challenge to learners and educators. The calculations can become tedious and problematic. Not all real life problems lends itself to standard type solutions. The only time one realizes the nature of Mathematics involved in a problem is when one encounters such a problem in real life. The kind of mathematical modeling involved will be dependent on the problem.

12. Description of Content of Workshop

    Consider the following problems:

    12.1 English cucumbers are grown scientifically and under controlled conditions.

    A school has 5 tunnels. Each tunnel can accommodate 480 plants. Seedlings , which are planted in plastic bags, take 4 weeks before cucumbers are produced. Cucumbers grow for 10 to 12 weeks. Each plant produces 12 and 15 cucumbers during the period given. After the expiry of 12 weeks the tunnel is revamped for another two weeks befor it is ready for the new crop.

    To unpack this problem is obviously very challenging. One of the mantras we chant in problem solving is to simplify the problem or to look at a simpler problem.

    This problem in particular may not have a perfect solution.

12.2 Metro Water uses the following table to determine the account of a consumer:-

9,6 kl	-	R0,00
17,6 kl	-	R8,47
5,3 kl	-	R11,29
13.0 kl+	-	R17,41

Fixed charge if quantity exceeds 9,6 kl – R81,24

Items exclude vat.

12.3 A hall measures 60 metres by 24 metres. You have to cover the roof with flat chromodek sheets measuring 2700mm by 760 mm. The cost is R35 m^2. Each sheet weigh 5kg. As a result of overlap of sheets you calculate 5% of the area will be lost.

12.4 Mrs Dube constructs cylindrical water tanks. Each tank has a height of 2m and a radius of 0,5m.

12.5 A publishing company printed a commemorative brochure. The fixed costs for printing and packaging amounted to R16000. The cost of printing brochures as per the following quantities :

R50 per brochure for $n < 5000$

R35 per brochure for $n \geq 5000$

13. Activities for the Workshop

The problems listed in 2 will be done.

13.1 English cucumbers are grown scientifically and under controlled conditions.

A school has 5 tunnels. Each tunnel can accommodate 480 plants. Seedlings , which are planted in plastic bags, take 4 weeks before cucumbers are produced. Cucumbers grow for 12 to 15 weeks. Each plant produces 12 and 15 cucumbers during the period given. After the expiry of 12 weeks the tunnel is revamped for another two weeks befor it is ready for the new crop.

To unpack this problem is obviously very challenging. One of the mantras we chant in problem solving is to simplify the problem or to look at a simpler problem.

This problem in particular may not have a perfect solution.

13.1.1 Suppose we have two tunnels and we wish to have a continuous supply of cucumbers then give an illustration or graphical representation of production for 1 year.

What is the maximum production?

What is the minimum production?

13.1.2 How will the situation change if we use
- 13.1.2.1 3 tunnels?
- 13.1.2.2 4 tunnels?
- 13.1.2.3 5 tunnels?

13.1.3 Suppose the average cost of producing one cucumber is R1,50 and the selling price to the 'middle man' is R3 then determine the
- 13.1.3.1 the total cost for one year.
- 13.1.3.2 the total revenue for the year.
- 13.1.3.3 the total profit for 1 year.

3.1.1 Discussion:

Let seedling phase be: S

Growing phase be : G

Preparation phase be: P

Weeks	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Tunnel 1	S	S	S	S	G	G	G	G	G	G	G	G	G	G	G	G	P	P
Tunnel 2	S	S	S	S	G	G	G	G	G	G	P	P	S	S	S	S	G	G
Tunnel 3							S	S	S	S	G	G	G	G	G	G	G	G
Tunnel 4																		
Tunnel 5																		

Weeks	18	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Tunnel 1	S	S	S	S	G	G	G	G	G	G	G	G	G	G	G	G	P	P
Tunnel 2	G	G	G	G	G	G	G	G	G	G	P	P	S	S	S	S	G	G
Tunnel 3	G	G	G	G	P	P	S	S	S	S	G	G	G	G	G	G	G	G
Tunnel 4																		
Tunnel 5																		
Weeks	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52		
Tunnel 1	S	S	S	S	G	G	G	G	G	G	G	G	G	G	G	G		
Tunnel 2	G	G	G	G	G	G	G	G	G	G	P	P	S	S	S	S		
Tunnel 3	G	G	G	G	P	P	S	S	S	S	G	G	G	G	G	G		
Tunnel 4																		
Tunnel 5																		

3.1.1 Observations

Two Tunnels: Total number of weeks of production = 52 + 18 = 70 Lowest production =70 x 480 = 33600 Maximum production = 70x $15^2/12^2$ x 480 = 52 500
Three Tunnels (Same approach) Total number of weeks of production =

13.2 Metro Water uses the following table to determine the account of a consumer:-

9,6 kl	-	R0,00
17,6 kl	-	R8,47
5,3 kl	-	R11,29
13.0 kl+	-	R17,41

Fixed charge if quantity exceeds 9,6 kl – R81,24

Items exclude vat.

13.2.1 Les's bill is R584,08. What quantity of water did Les use.

13.2.2 Suppose Les use the following quantities then determine the amount to be paid:-

13.2.2.1 35 kl

13.2.2.2 75 kl

13.2.3 Assuming the threshold levels increases by 10% then how would this have affected 2.2.2

13.3 A hall measures 60 metres by 24 metres. You have to cover the roof with flat chromodek sheets measuring 2700mm by 760 mm. The cost is R35 m^2. Each sheet weigh 5kg. As a result of overlap of sheets you calculate 5% of the area will be lost.

13.3.1 Calculate how many sheets you need to order.

13.3.2 What is the total cost of the sheets?

13.3.3 What is the cost of delivery if transport cost is R5 per kg ?

13.4 Mrs Dube constructs cylindrical water tanks. Each tank has a height of 2m and a radius of 0,5m.

13.4.1 Calculate the volume of each tank.

13.4.2 How many litres of water does each tank hold ?

13.4.3 How many tanks does she need to supply a village with 20 000 l per day.

13.4.4 Calculate the surface area of each tank.

13.5 A publishing company printed a commemorative brochure. The fixed costs for printing and packaging amounted to R16000. The cost of printing brochures as per the following quantities :-

R50 per brochure for $n < 5000$

R35 per brochure for $n \geq 5000$

3.5.1 Write down 2 defining equations to capture the above information.

3.5.2 The amount realized from advertisements amounted to R20 000. If the amount received from donations amounted to R40 per book then determine firstly the defining equation for this situation.

3.5.3 Determine the break - even point

3.5.4 If the non-governmental organization responsible for all the logistics and if they wish to realize an amount of at least R40 000 then determine the quantity of brochures that will realise such a target.

3.5.5 If the potential number of recipients of this brochure is 100 000 then determine the optimum net revenue.

It is clear from the first problem that strategy is quite different for each problem.

We will try to attempt as many problems as we can.

NUMBER PATTERNS IN THE FET(NCS) PHASE BY EXPLORING THE SUM OF 3 CONSECUTIVE NUMBERS AND FINDING THE LAST DIGIT ON A LARGE EXPONENTIAL NUMBER WITH BASE 3. BY B NCUBE

Learning Area	:	Mathematics Number Patterns
Topic	:	The Number 3
Time	:	1 Hour
Date	:	28 March to 1 April 2010
School	:	Ilanga Secondary School
Target Audience	:	FET
Congress	:	National Level
Maximum No. of Participants	:	30 Participants

Motivation for the Workshop

This presentation will make the participants explore other ways of teaching number patterns. It will also show other ways of solving problems and this would in turn help the learners to be able to solve some problems like in the Mathematics Olympiad in a nutshell it would make the participants to think outside the box.

Description of the content of the workshop

The presenter would start by relating how important the number 3 is. The presentation would show the significance of the number 3 both in the past and in our present day life.

The next thing would be to explore what the sum of the 3 consecutive numbers is. If the answer is found then check whether it works for all numbers alternatively the use of general terms would be done and would give the answer faster.

The participants would be asked to work in groups of 3 – 5 to find the sum of any 6 consecutive numbers and check whether it is also a multiple of 3.

Lastly the presenter would ask the group what the last digit of 3^{2010} would be and then demonstrate how it can be found. Having found the answer the group would be asked the question, “Does this work for 3 only or its multiples? Then there would be challenged to try for other numbers, make a conjecture and prove:

Time allocation would be as follows:

Introduction	5 minutes
First Presentation	10 minutes
Group Activity	10 minutes
Second Part of Presentation	10 minutes
Group Activity	10 minutes
Conclusion	5 minutes

<u>ABSTRACT</u>

The workshop is meant for the FET phase. It would be a presentation on number patterns which would be interaction between the presenter and the participants as there would be activities for the participants to do while the presenter checks their progress.

THE NUMBER THREE

I was born on the 3rd Is it of any significance this number 3?

The Greeks used the number 3 a lot. There were the three Fates, three Graces, three Gordons and the three Furies. Even Apollo's Pythia sat on a three legged chair (tripod) and Cerberus was a three headed dog. The trimester, the Holy Trinity and some others and you have a world-wide phenomenon.

Geometrical figure – two straight lines cannot possibly enclose any space, or form a plane figure; neither can two plane surfaces form a solid. Three lines are necessary to form a plane figure, and three dimensions of length, breadth, and height are necessary to form a solid. Three stands for that which is solid, real, substantial, complete and entire.

All things that are complete and stamped with this number 3.

God's attributes are three: Omniscience, Omnipresence and Omnipotence. There are three great divisions completing time i.e. Past, Present and Future. Three degrees of comparisons complete our knowledge of qualities. It was on the third day that Jesus rose again from the dead. Earth is the third planet from the sun. There are three primary colors.

What can you say about the sum of 3 consecutive numbers?

1 + 2 + 3 = 6

2 + 3 + 4 = 9

20 + 21 + 23 = 63

$50 + 51 + 52 = 153$

$x + (x + 1\ ) + (x + 20\ \ =\ \ 3x + 3$

$= \ 3(x + 1)$

The answer is a multiples of 3 and always 3 times the middle number. The sum of any 3 consecutive numbers is a multiple of 3.

Q The sum of any 6 consecutive numbers is a multiple of 3. Prove?

What is the one's digit or the last digit of 3^{2010}?

$3^3 = 237$ the one's or last digit is 7

$3^0 = 1$	$3^8 = 6561$
$3^1 = 3$	$3^9 = 19683$
$3^2 = 9$	$3^{10} = 59049$
$3^3 = 27$	$3^{11} = 177147$
$3^4 = 81$	$3^{12} = 531441$
$3^5 = 243$	$3^{13} = 1594323$
$3^6 = 729$ `	$3^{14} = 4782969$
$3^7 = 2187$	$3^{15} = 14348907$

Last digit	exponents							
1	0	4	8	12	16	20	24	4n
3	1	5	9	13	17	21	25	4n + 1
9	2	6	10	14	18	22	26	4n + 3
7	3	7	11	15	19	23	27	4n + 3

Tn = 4n + 2

2010 = 4n + 2

2010 – 2 = 4n

208 = 4n

$$\frac{2008}{4} = \frac{4n}{4}$$

n = 502

3^{2010} last digit is 9

Does this work for 3 only or its multiples? Try for other numbers, make a conjuncture; and prove.

LINEAR PROGRAMMING

Presenter: Mr L.L. Ramashala
Institution: Ramollwane Secondary School (Bakenberg Cluster)
Head of Department: Mathematics

Duration: 1 hour

Target Group: Grade 11 and 12 Educators

No of Participants: 30

INTRODUCTION

Linear programming has been and is still a major problem to both educators and learners in the FET band. This has been a problem ever since even in the old curriculum because learners were not performing well in linear programming. Even now in the National Curriculum Statement (NCS) linear programming is still a problem that requires immediate interventions before it becomes late. Therefore these kinds of workshop with try to provide simpler solution those problems that occurred for decades.

DESCRIPTION OF CONTENT:

Learning Outcome 2: Functions and Algebra

The learner is able to investigate, analyze, describe and represent a wide range of functions and solve related problems.

11.2.8	12.2.8
a) Solve linear programming problems by optimizing a function in two variables, subject to one or more linear constraints, by numerical search along the boundary of the feasible region. b) Solve a system of linear equations to find the co-ordinates of the vertices of	Solve linear programming problems by optimizing a function in two variables, subject to one or more linear constraints, by establishing optima by means of a search line and further comparing the gradients of the objective function and linear constraint boundary lines.

the feasible region.	

MOTIVATION FOR RUNNING THE WORKSHOP

To ensure that participants learn how to:

- Draw graphs defined by linear inequalities.
- Find the simultaneous graphical solutions of a system of linear inequalities.
- Translate conditions involving physical quantities into system of first – degree relationship.
- Model constrains for real life problems that need to be optimized.
- Solve linear programming problems.

Linear programming is a simultaneous graphical solution of a system of linear inequalities. It is used to solve optimized problems. Linear inequalities are called constrains. If the constrains are less or equal (≤) shading is below the linear graph and if constrains are greater or equal (≥) shading is above the linear graph. The shaded region which represents the simultaneous graphical solution is called the feasible region.

REAL LIFE SITUATION

We will consider how real life situation may be modeled as a set of limited quantities, i.e. a set of inequalities. A region on a Cartesian plane can then represent the situation and facts about the real life quantities can be deduced.

<u>(ACTIVITY) – Game of dice</u>

Consider the following game involving the throwing of two die, one red and the other green. The rules of the game are as follows:

- The number on each die must be 3 or more
- The sum of the two numbers must be 9 or more
- The winner should get the highest score

Method 1

The results of each throw of the dice are recorded in a table. If the throw meets the rules, it is accepted. The point (*x; y*) represents the number on the red die (*x*) and the number on the green die (*y*).

(*x; y*)	**Red (*x*)**	**Green (*y*)**	**(*x*+ *y*)**	**Meets the conditions**
(*1; 1*)	1	1	2	No
(*1; 2*)	1	2	3	No
(*1; 3*)	1	3	4	No
(*1; 4*)	1	4	5	No
(*1; 5*)	1	5	6	No
(*1; 6*)	1	6	7	No
(*2; 1*)	2	1	3	No
(*2; 2*)	2	2	4	No
(*2; 3*)	2	3	5	No
(*2; 4*)	2	4	6	No
(*2; 5*)	2	5	7	No
(*2; 6*)	2	6	8	No
(*3; 1*)	3	1	4	No
(*3; 2*)	3	2	5	No
(*3; 3*)	3	3	6	**Yes**
(*3; 4*)	3	4	7	**Yes**
(*3; 5*)	3	5	8	**Yes**
(*3; 6*)	3	6	9	**Yes**
(*4; 1*)	4	1	5	No
(*4; 2*)	4	2	6	No
(*4; 3*)	4	3	7	**Yes**
(*4; 4*)	4	4	8	**Yes**
(*4; 5*)	4	5	9	**Yes**
(*4; 6*)	4	6	10	No

(*5; 1*)	5	1	6	No
(*5; 2*)	5	2	7	No
(*5; 3*)	5	3	8	**Yes**
(*5; 4*)	5	4	9	**Yes**
(*5; 5*)	5	5	10	No
(*5; 6*)	5	6	11	No
(*6; 1*)	6	1	7	No
(*6; 2*)	6	2	8	No
(*6; 3*)	6	3	9	**Yes**
(*6; 4*)	6	4	10	No
(*6; 5*)	6	5	11	No
(*6; 6*)	6	6	12	No

The winner is the person who gets the highest score (S) based on the rules:

$S = 2x + y$ (the sum of double red number and the green number)

We will draw a table with the accepted results as well as the scores for each throw based on the scoring rule. $S = 2x + y$ (called the objective function)

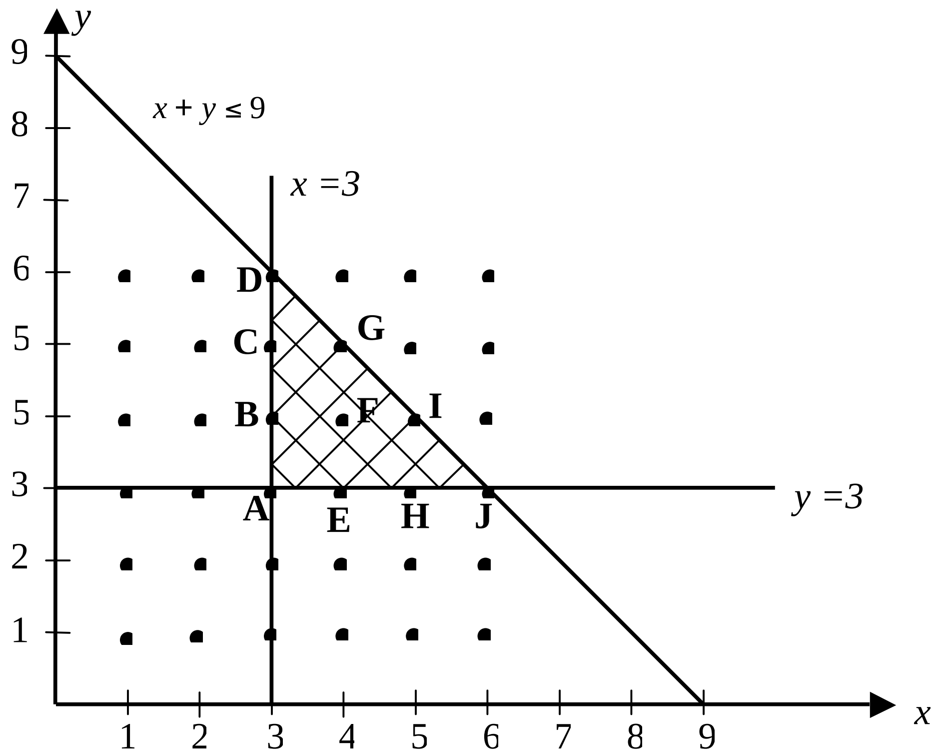
(x; y)	**Red (x)**	**Green (y)**	**S = 2x+ y**
A (*3; 3*)	3	3	**2(*3*) + *3* = *9***
B (*3; 4*)	3	4	**2(*3*) + *4* = *10***
C (*3; 5*)	3	5	**2(*3*) + *5* = *11***
D (*3; 6*)	3	6	**2(*3*) + *6* = *12***
E (*4; 3*)	4	3	**2(*4*) + *3* = *11***
F (*4; 4*)	4	4	**2(*4*) + *4* = *12***
G (*4; 5*)	4	5	**2(*4*) + *5* = *13***
H (*5; 3*)	5	3	**2(*5*) + *3* = *13***
I (*5; 4*)	5	4	**2(*5*) + *4* = *14***
J (*6; 3*)	6	3	**2(*6*) + *3* = *15***

Clearly the throw (6; 3) yields the highest score in this situation.

Method 2

We can represent the results of this game graphically. The rules of the game can be represented as linear inequalities:

- The number on each die must be 3 or more $x \geq 3$ $y \geq 3$
- The sum of the two numbers must be 9 or less $x + y \leq 9$



To find the maximum value:

$S = 2x + y$ at point J (6; 3)

$= 2(3) + 3$

$= 15$

Representing inequalities graphically.

In order to solve linear programming examples, you should be able to graph regions specified by inequalities in two forms: $y > mx + c$ or $ax + by + c > 0$. The inequality can be < or >; or ≤ or ≥.

Procedure:

a) Draw the boundary line $y = mx + c$ or $ax + by + c = 0$ first.
   - To include points on the line (inequality is ≤ or ≥) draw a solid line.
   - To exclude points on the line (inequality is < or >) draw the line as dashed.

b) Decide which side of the line to shade in. Test a point not on the line, by substituting its x - and y - coordinates into the inequality. If this satisfies the inequality (makes it true), shade the side that contains the point. But if it makes the inequality false, shade the other side.

ACTIVITY 1

Sketch the feasible region bounded by the following constrains in each case, hence determine the maximum and the minimum vales of the given expression P in each case:

$y \le 0$

$3y + 2x \ge 24$

$x - y \le 0$

$4 \le x$

$P = 3x + y$ (is called the **Objective function**)

Translating the words into mathematics.

Linear programming problems often involve lengthy statements in words — you have to extract the ***constraints*** and the ***objective function*** (expression to optimize) from the question, and then ***optimize the function***.

Procedure

c) Convert any constraints to mathematical inequalities.
d) Draw the feasible region.
e) Evaluate the objective function at each vertex.

Here are some useful hints:

- Read the question through carefully, and identify the two variables first.
- Label the variables with appropriate letters, but then later convert them to *x* and *y*. This makes the graph-drawing easier.
- If you convert the variables in alphabetical order, it is also more straight-forward to check your work.
- Then identify the objective function (expression to be optimized) — look for wording such as :

  At least (≥); At most (≤); Not more than (<); Not less than (>); a minimum (≥) and a maximum (≤).

- Remember that some constraints are implied (***implicit constraints)***. Many quantities cannot be negative, and so often two constraints will be $x \geq 0$ and $y \geq 0$.

Optimizing the function

The ***objective function*** is the function which is used to find a maximum (or minimum) value. The aim is to optimize the objective function $ax + by$ subject to several constraints. In Grade 11 the learners do this by doing a "numerical search along the boundary of the feasible region" – the Vertex Method. In Grade 12 the learners are asked to do this "by means of a search line and further comparing the gradients of the objective function and linear constraint boundary lines".

Procedure to follow when using a search line:

a) Draw the region formed by the constraints (the ***feasible region***), the intersection of several inequalities.
b) Use the ***objective function*** to identify which vertex (or boundary line segment) gives the optimal value of S, where $S = ax + by$.

   Step 1: Make y the subject of the formula in the objective.

   Step 2: Write down the gradient of the objective function.

   Step 3: To draw one of these parallel lines, use the following methods:

   - Let the numerator represent the y-axis and the denominator represent the *x*-axis.
   - Put dots on the axis and then draw a dotted line on the graph. This represents what we call a "Search line".

   Step 4:

- Move the line parallel with itself until it touches the feasible region for the first time. The point represents the ***minimum value***.
- The moving line continues to move across the feasible region and the last vertex touches will represent a ***maximum value***.

ACTIVITY 2

The owner of a clothing factory plans to manufacture shorts and trousers. A maximum of 60 pairs of shorts and 40 pairs of trousers can be manufactured daily, while in total not more than 70 pieces of clothing can be manufactured daily. It takes 5 hours to manufacture a pair of shorts and 10 hours for a pair of trousers, while there are at most 500 working hours available per 5 – day week. The profit is R15 on a pair of shorts and R25 on a pair of trousers. Maximize the profit using linear programming.

CONSTRAINS:

CONCLUSION

Thinking and hoping this intervention will impart change to educator's method of teaching and also to the reflection of outcomes at the end of the year. Let the concepts not be a disaster but something that can be approached with easy and confidence.

REFERENCES

1) Department of Education (2003) *National Curriculum Statement* **Grade (10 - 12) (Mathematics)**
2) Phillip, M.D, Basson, J & Botha, C. *Mathematics Textbook and Worksheet* (2007) Grade 11 FET
3) Phillip, M.D, Basson, J & Botha, C. *Mathematics Textbook and Worksheet* (2007) Grade 12 FET

OER workshop – AMESA 2010

1. Motivation for running the workshop

- Title of the workshop :

Open Educational Resources for mathematics teachers – teaching maths in diverse classrooms in South African schools.

- Name of presenter(s): Ingrid Sapire
- Institution where you are employed: University of the Witwatersrand
- Target audience: All phases

- Duration: 1 hour
- Maximum no. of participants: 30

- Motivation for the workshop: This workshop will introduce to teachers a set of materials which include content on how to teach maths in South African schools. The materials were developed for use in ACE courses by academics from 9 different tertiary institutions in South Africa, have been piloted and used in several institutions and are freely available to everyone on the OER Africa website. A teacher who was asked to critically read and comment on the materials said that "all teachers in South Africa should have the opportunity to read these materials". This workshop will take the materials to a few more teachers out there.

- Description of content of workshop: Teachers will hear about the materials but they will also do a practical activity based on content from Unit 6 of the materials, which is entitled "Teaching all children mathematics".

Times:

15 min – intro to the materials and brief background to their development

15 min – navigating the website (using a CD on which content is saved so that using it simulates navigating the site on the web) – introduce the idea of multiple entry points to an activity

30 min – adapting an activity to make it more accessible to all learners in a diverse classroom.

- The activities and worksheets to be used in the workshop (maximum 8 pages)

The presenter will need a data projector to demonstrate the OER Africa page and the OERs. If sufficient participants bring laptops, or if the workshop can be arranged in a computer lab, all groups can use computers to navigate the site. Preferably the workshop should be in a computer lab. Then the abstract in the programme wont need the note about laptops.

(see appendix A – outline of activity and separate file for four open source activities selected for adaptation from the MALATI materials. Web reference: http://academic.sun.ac.za/mathed/Malati/Products.htm)

1. An abstract describing the level, nature and content of the workshop (200 words)

In this workshop participants will be introduced to a set of materials which include content on how to teach maths in South African schools. The materials were developed for use in ACE courses by academics from 9 different tertiary institutions in South Africa, have been piloted and used in several institutions and are freely available to everyone on the OER Africa website. The materials were produced with intermediate and senior phase teachers in mind, but have been used by teachers from foundation phase to FET level. There will be a practical activity based on content from Unit 6 of the materials, which is entitled "Teaching all children mathematics". This activity will give participants insight into developing tasks which have multiple entry points and are accessible to learners who are on different levels in a diverse class. A copy of the CD with the full set of materials (and other supporting information) will be available to participants in the workshop. (A laptop at the session would be useful but not essential for participation.)

Appendix A: Worksheet for workshop (see Unit 6, SAIDE ACEMaths OERs)

Activity	Adapting number activities for diverse learners 1. What are the problems presented by the activity which call for adaptation of the activity? 2. Give details of how you would adapt the activity for a diverse learner group to address the problems you mention above. 3. Does your adaptation address all of the problems you identified? 4. Have you noticed any further problems which learners may encounter when attempting the activity after discussing your adaptation with other groups?
	Workshop: 1) Choose and adapt an activity 2) Present adaptation briefly to the group 3) Note comments raised in discussion – this is an on-going activity!

DRILL WORK ACTIVITIES REGARDING MENTAL MATHS AS INDICATED IN THE FOUNDATIONS FOR LEARNING CAMPAIGN

Presenter: Mokgoko Sebela (Curriculum Adviser)

Institution: Capricorn District Office (LIMPOPO)

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mokgoks@iafrica.com

015-2237405 / 0828820214

- Target audience: Foundation Phase and Intermediate phase teachers.
- Duration: 1-hour.
- Maximum number of participants: 35 – 45.
- Motivation for the workshop: Educators in the Primary Schools are struggling to teach their learners mental mathematics for the 10 minutes as stated in the FFLC (Foundations for Learning Campaign) policy. This workshop will enlighten the educators on some of the activities that could be useful in a mathematics classroom as mental mathematics activities.
- Description of the workshop:

1. Drill work activities for the Grade 1's and 2's:

- Number bonds – this will be the 1st activity presented
- Number bond of 5; e.g. 3 + ? = 5 done in a playful way.
- Playfully with Number bond of the following:
- 9; 10; 19; 17; 20.

2. Mental mathematics on simple counting, addition or subtraction:

• *I have 2 dots.* • *Who has 3 dots?*

- Materials: these are a deck of cards, and on a card for each statement there is a description at the end and a learner must respond.

- Activity: Shuffle the cards and dish them out to the entire class. Have one child begin by reading his/her question. The person who answers is the one whose “I HAVE” statement is the correct response. In this way, each child must do each mental calculation as the questions loops around the class. When the loop is complete, collect the cards, reshuffle and play repeatedly.

  - ACTIVITIES WILL BE DONE AS IN THE FOLLOWING:

- WHO HAS.............................??

- Statements: 1. I have 2 dots. Who has 3 dots?

2 I have 3 dots. Who has 6 dots?
3. I have 6 dots. Who has 1 dot?
4. I have 1 dot. Who has 8 dots?
5. I have 8 dots. Who has 5 dots?
6. I have 5 dots. Who has 7 dots?
7. I have 7 dots. Who has 10 dots?
8. I have 10 dots. Who has 2 dots/
9. I have 2 dots.

- Variations: You could use different colours or dots and therefore word the statements including colour and number. i.e. I have 2 red dots.

  - Statements:

1. I have a red circle. Who has a blue square?
2. I have a blue square. Who has a yellow circle?
3. I have a yellow circle. Who has a green square?
4. I have a green square. Who has an orange circle?
5. I have an orange circle. Who has a white square?
6. I have a white square. Who has a purple circle?
7. I have a purple circle. Who has a black square?
8. I have a black square. Who has a red circle?

9. I have a red circle.

-
- Variations: Addition of a triangle and repeat colours with different shapes: i.e. red circle, blued circle, green triangle, etc.
    - ---
-
- Number recognition.   Who Has........?
-
- Statements:

1. I have the numeral 6.	Who has the numeral 3?
2. I have the numeral 3.	Who has the numeral 9?
3. I have the numeral 9.	Who has the numeral 0?
4. I have the numeral 0.	Who has the numeral 4?
5. I have the numeral 4.	Who has the numeral 5?
6. I have the numeral 5.	Who has the numeral 1?
7. I have the numeral 1.	Who has the numeral 11?
8. I have the numeral 11.	Who has the numeral 12?
9. I have the numeral 12.	

- This exercise can be extended through 20 as the children learn the numerals. It can also be extended with seals for various holidays.
- i.e. I have 6 hearts (Valentine's day).
- Or i have 6 bunnies (Easter).
- ....
-
- WHO HAS ....???
- (Addition – Subtraction – Multiplication   Drill)
-
- Statements:

1. I have3.	Who has 2 more?
2. I have 5.	Who has 1 less?
3. I have 4.	Who has this times 2?
4. I have 8.	Who has 6 less?
5. I have 2.	Who has this times 5?
6. I have 10.	Who has 2 more?
7. I have 12.	Who has 3 less?
8. I have 9.	Who has this times 1 less 3?

9. I have 6. Who has this times 3?
10. I have 18. Who has 5 less?
11. I have 13. Who has 2 less?
12. I have 11. Who has 4 less?
13. I have 7. Who has this times 2?
14. I have 14. Who has 1 more?
15. I have 15. Who has 15 less?
16. I have 0. Who has 3 more?
17. I have 3.

WHO HAS ....???

(Fraction Drill)

Statements:	1. I have 10.	Who has 2 of this?
	2. I have 5.	Who has 5 of this?
	3. I have 1.	Who has 4 times as many?
	4. I have 4.	Who has 2 of this?
	5. I have 2.	Who has this times 3?
	6. I have 6.	Who has 2 of this?
	7. I have 3.	Who has this times 4?
	8. I have 12.	Who has of this?
	9. I have 8.	Who has this times 2, less 2?
	10. I have 14.	Who has 2 of this?
	11. I have 7.	Who has this times 2 and 4 more?
	12. I have 18.	Who has 2 of this?
	13. I have 9.	Who has this and as many more?
	14. I have 15.	Who has of this?
	15. I have 10.	

WHO HAS ....???????

(Decimals, - Basic Operations)

Statements:

1. I have 3,4. Who has 1 less than this?
2. I have 2,4. Who has half this much/
3. I have 1,2. Who has 0,5 less than this?
4. I have 0,7. Who has this times 3?
5. I have 2,1. Who has this divided by 7?
6. I have 0,3. Who has this plus 3,7?
7. I have 4. Who has 2,2 less than this?
8. I have 1,8. Who has double this much?
9. I have 3,6. Who has 1,6 less than this?
10. I have 2. Who has this and 1,8 more?
11. I have 3,8. Who has 2,5 less than this?
12. I have 1,3. Who has 1,5 more than this?
13. I have 2,8. Who has 2,4 less than this?
14. I have 0,4. Who has this divided by 2?
15. I have 0,2. Who has this times 3?
16. I have 0,6. Who has this divided by 6?
17. I have 0,1. Who has 1,4 more than this?
18. I have 1,5. Who has 12,7 more than this?
19. I have 14,2. Who has 10,1 less than this?
20. I have 4,1.

- FOR THE SENIOR PHASE (GRADE 8)

- WHO HAS ……………….??????

- (Integers, Basic operations)

- Statements:

1. I have -2. Who has this plus 2?
2. I have 0. Who has this minus 5?
3. I have -5. Who has this minus 3?
4. I have -8. Who has this times 2?
5. I have -16. Who has this divided by -4?
6. I have +4. Who has this times 3?
7. I have +12. Who has this minus 13?
8. I have -1. Who has this plus 11?
9. I have +10. Who has this divided by 2?
10. I have +5. Who has this plus -8?
11. I have -3. Who has this times -3?
12. I have +9. Who has this minus -5?
13. I have +14. Who has this divided by -2?
14. I have -7. Who has this plus 8?

15.	I have +1.	Who has this times 6?
16.	I have +6.	Who has this minus 12?
17.	I have -6.	Who has this plus 8?
18.	I have +2.	Who has this times -9?
19.	I have -18.	Who has this plus 7?
20.	I have -11.	Who has this minus -19?
21.	I have +8.	Who has this plus 16?
22.	I have +24.	Who has this minus 17?
23.	I have 7.	Who has this minus 4?
24.	I have +3.	Who has this plus -7.
25.	I have minus -4.	

- (*From Mathematics in Michigan, Volume XIX, No. 5, June 1980.*)
- *Presented in South Africa by Gloria Sanok from USA*.

CUMULATIVE FREQUENCY TABLE AND DRAWING AN OGIVE OF GROUPED AND UNGROUPED DATA

PRESENTER: Mr M.D. Tlhako and Ms M.P. Gwangwa

INSTITUTION: Dennis Matlhaba Secondary School

Limpopo Province: Mathematics Educators

Duration: 1 hour

Target Group: Grade 11 and 12 Educators

No of Participants: 30

INTRODUCTION

When one first look at the data set, all he or she sees is a jumble of information. One needs to sort, record and represent data in a way that makes more sense. Some data are easy to sort into a list but others need to be grouped if are of high quantity. Other data can be sorted into tables; those tables can be used to count of the number of times a particular piece of data occurs.

DESCRIPTION OF CONTENT

Learning Outcome 4: Data Handling and Probability

Assessment Standards:

10.4.1 (a) Collect, organise and interpret univariate numerical data in order to determine measures of dispersion, including quartiles, percentiles and the interquartile range

11.4.1 (a) Calculate and represent measures of central tendency and dispersion in univariate numerical data by:

- Box and whisker diagrams
- Ogives

MOTIVATION FOR RUNNING THIS WORKSHOP

This form of intervention was developed with the aim of assisting participants with various techniques on how grouped and ungrouped data can be analyzed and represented. Various activities are included which allows participants to be actively involved during the presentation by taking part in finding solutions. On the other hand this paper will serve as a tool when developing daily planning and preparations.

FREQUENCY TABLES

In a **frequency table** you keep count of the number of times a data item occurs by keeping a tally. The number of times occurs is called the **frequency** of that item.

In a frequency table you can also find a "running total" of frequencies. This is called the **cumulative frequency**. It is useful to know the running total of the frequencies as this tells you the total number of data items at different stages in the data set.

CUMULATIVE FREQUENCY TABLE WITH UNGROUPED DATA

Suppose you kept a record of the **mathematics marks obtained by learners in a test in Grade 11.** The test is out of a 10 marks and you need to draw a frequency table with marks show the below:

5	**3**	**6**	**8**	**7**	**4**	**6**	**8**	**2**	**6**	**4**	**7**	**3**
7	**9**	**8**	**5**	**9**	**8**	**7**	**5**	**8**	**9**	**6**	**6**	**5**
5	**8**	**7**	**1**	**7**	**2**	**6**	**8**	**7**	**8**	**4**	**7**	**4**
6	**7**	**10**	**7**	**9**	**6**	**8**	**7**	**6**	**9**	**8**	**6**	**2**
4	**3**	**6**	**5**	**7**	**4**	**7**	**5**	**7**	**2**	**5**	**7**	**4**

Tally of the marks are shown in the frequency table below:

Marks	Tally	Frequency	Cumulative frequency
0	-	0	0
1	/	1	1
2	////	4	5
3	///	3	8
4	~~////~~ //	7	15
5	~~////~~ ///	8	23
6	~~////~~ ~~////~~ /	11	34
7	~~////~~ ~~////~~ ~~////~~	15	49
8	~~////~~ ~~////~~	10	59
9	~~////~~	5	64
10	/	1	65

Note:

- The final total in the cumulative frequency column is the same as the total number of learners. This helps you check that your working is correct.
- The **frequency** tells you how many learners **scored a certain mark**
  The **cumulative frequency** tells you how many learners **scored that mark or less.**

CUMULATIVE FREQUENCY TABLE WITH GROUPED DATA

(a) **Grouped Discrete Data:** is data that has a certain exact value (often whole numbers) and is often collected by counting. E.g. A set of data collected about shoe sizes would be discrete.
Discrete data can be shown by points on a number line.

Note:

- The **frequency** tells you how many learners scored **between certain marks**
- The **cumulative frequency** tells you how many learners scored the **upper limit or less than the upper limit** of the group.
- **Points** in this case is the combination of **upper boundary and Cumulative frequency**

Worksheet 1

The following are the percentages obtained by all Grade 11 learners at Polokwane Secondary School for their end of term Mathematics Examination.

55	61	32	43	65	78	82	51	46	37
52	41	80	57	62	43	58	54	45	52
63	65	44	47	54	41	31	28	84	53
46	57	66	63	68	54	49	57	58	54
16	31	85	66	41	72	66	37	65	63
76	27	35	58	67	46	48	59	68	79

Cumulative frequency table:

Marks %	**Tallies**	**Frequency**	**Cumulative frequency**	**Points**
0 – 10	-	0	0	(10 ; 0)
11– 20	/	1	(0 + 1) = 1	(20 ; 1)
21– 30	//	2	(1 + 2) = 3	(30 ; 3)
31– 40				
41– 50				
51– 60				
61– 70				
71– 80				
81– 90				
91– 100				
Total				

(b) **Grouped Continuous Data:** can be any value within a certain range (not exact values) and is found by measuring. For example, a set of data relating to the masses of all under 14 boys would be continuous data.

Continuous data can be shown by a continuous number line.

Note:

- The **frequency** for each group tells you how many under14 boys are **between the limits of a particular Mass group in kilograms**
- The **cumulative frequency** for each group tells you how many under14 boys **are less than or equal to the upper boundary of the group.**
- **Points** in this case is the combination of **Upper boundary** and **Cumulative frequency**

<u>Worksheet 2</u>

The following are the Masses of all the under 14 boys playing rugby at Polokwane Secondary School (to the nearest *kg*)

27	35	42	35	36	51	43	30	51	37
36	48	41	38	40	46	37	46	42	38
42	42	46	41	26	42	34	38	39	38
31	35	37	43	33	39	33	54	40	36
36	31	29	37	37	45	46	44	31	46
36	40								

a) Determine suitable class interval (use 8 intervals)
b) Draw up a frequency table.

Solution:

Masses In *kg*	**Tallies**	**Frequency**	**Cumulative frequency**	**Points**
$20 < x \leq 25$	-	0	0	(25 ; 0)
$25 < x \leq 30$	//	2	2	(30 ; 2)
Total				

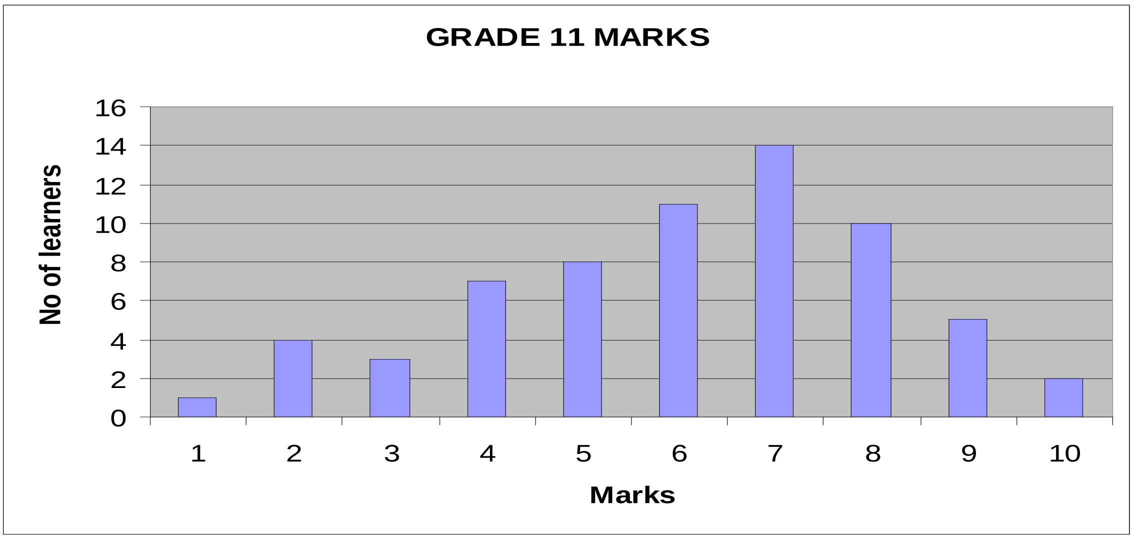
REPRESENTING DATA FROM A FREQUENCY TABLE

This frequency table shows Grade 11 mathematics marks

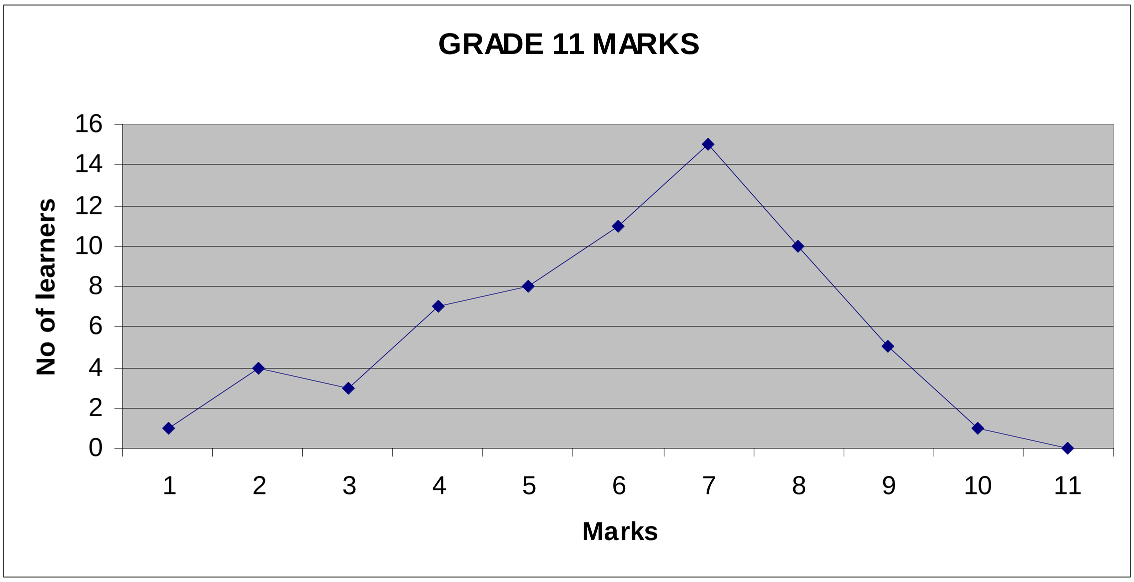
Marks	**Frequency**

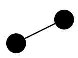
0	0
1	1
2	4
3	3
4	7
5	8
6	11
7	15
8	10
9	5
10	1

This data can be represented by drawing a **HISTOGRAM**.



This same information can be represented in a **frequency polygon** by joining the midpoints of the bars of the histogram and then erasing the bars. It must start and end on the horizontal axis. This represents no problem on the left, but on the right side the graph had to be continued down to the midpoint of the imaginary next column.





Notice: The graph begins and ends where the frequency is zero

A **frequency polygon** is a line graph of the data. It is a useful way of representing the data as it gives another way of reading frequency. Frequency polygons are often used to compare different frequency distributions. You may have seen line graphs in the newspaper that are actually frequency polygons.

CUMULATIVE FREQUENCY CURVE OR OGIVE

Another way of representing the data from the frequency table is to draw a **cumulative frequency curve**. When you draw the graph of a cumulative frequency distribution you obtain a curve that has a characteristic shape. This curve is called a **cumulative frequency curve** or an **Ogive**.

DRAWING AN OGIVE OF UNGROUPED DATA.

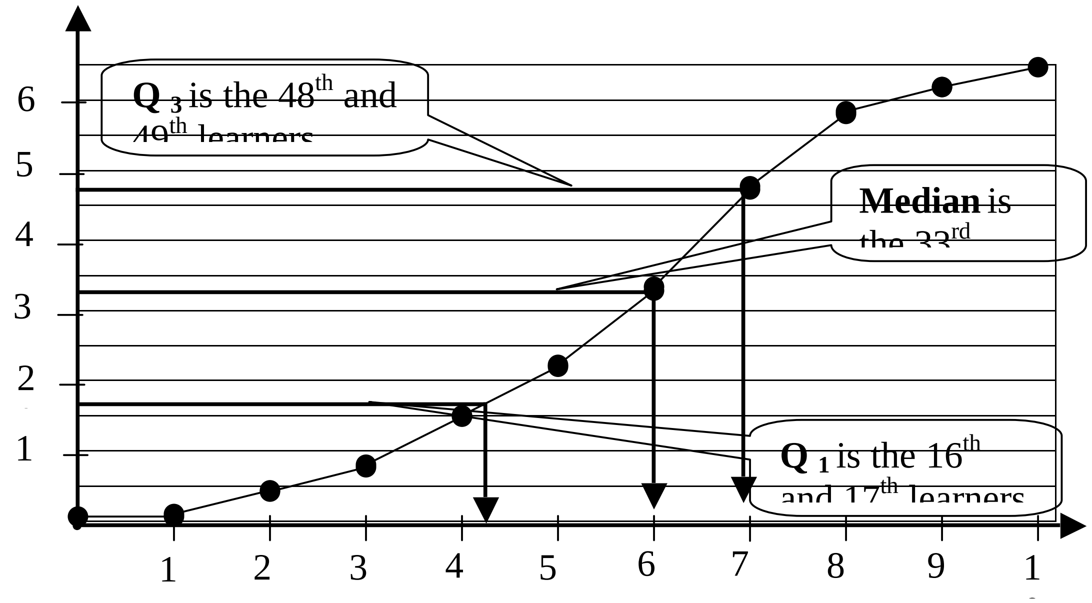
Marks	**Frequency**	**Cumulative frequency**	**Points**
0	0	0	(0 ; 0)
1	1	1	(1 ; 1)
2	4	5	(2 ; 5)
3	3	8	(3 ; 8)
4	7	15	(4 ; 15)
5	8	23	(5 ; 23)
6	11	34	(6 ; 34)
7	15	49	(7 ; 49)
8	10	59	(8 ; 59)
9	5	64	(9 ; 64)
10	1	65	(10 ; 65)
Total	**65**		

The 16th and 17th learners are in here $Q_1 = 5$

The 33rd learner is in here $M = 6$

The 48th and 49th learners are in here $Q_3 = 7$

Finding the median and quartiles from Cumulative frequency table



Note:

- If you join the points with a smooth line you get a cumulative frequency curve.
- The graph starts on the horizontal axis – i.e. where the cumulative frequency is zero. i.e start the curve at **(0 ; 0)**
- The shape of the curve is a leaning **S shape**. If you get any other shape you have made a mistake.

DRAWING AN OGIVE OF GROUPED CONTINUOUS DATA

For a cumulative frequency curve of continuous data,

- **The first element** of the ordered pair is **the upper limit of the interval.**
- The **second element** is the value of the **cumulative frequency.**

Worksheet 3

The following are the percentages obtained by all Grade 11 learners at Polokwane Secondary School for their end of term Mathematics Examination.

Marks %	**Tallies**	**Frequency**	**Cumulative frequency**	**Points**
0 – 10	-	0	0	(10 ; 0)
11– 20	/	1	(0 + 1) = 1	(20 ; 1)
21– 30	//	2	(1 + 2) = 3	(30 ; 3)
31– 40				
41– 50				
51– 60				
61– 70				
71– 80				
81– 90				
91– 100				
Total				

a) Use data above to draw an Ogive
b) Use the curve to determine the median, lower and upper quartiles.

Note:

- The graph starts at the point where the cumulative frequency is zero (at the lower end of the first group. So you plot the point **(10 ; 0)**).
- When you join the points on the graph you are assuming that the items are evenly spread throughout the groups. Data items are not necessarily evenly spread on a graph so any reading from a frequency curve is an **estimate**, not an exact reading.
- Always draw lines on the graph to show where you took the readings

REFERENCES

1. Department of Education (2003) *National Curriculum Statement* **Grade (10 - 12) (Mathematics)**

2. Scheiber, J. and Dickson, M. (2007). Data Handling, RADMASTE Centre, University of the Witwatersrand.

So arm span length can help me predict my height?

<u>Phase : GET LO 5</u>

Using Census@School questionnaires in collecting personal Data in class , displaying the data and interpreting it to INSPIRE TEACHERS TO PERSONALIZE DATA COLLECTION IN ORDER TO ENHANCE LEARNER INVOLVEMENT AND INTEREST IN STATISTICS IN THE MATHEMATICS CURRICULUM

<u>Presenter</u>: L.T.J.Van Rensburg

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I am the Free State maths4stats provincial coordinator. I taught mathematics to GET and FET learners for more than 14 years in the rural areas where resources are scarce and learners are poor. When Data is treated effectively in the class room it can make a huge contribution to better Mathematics Results.

This workshop will help teachers to gather data without cost or effort since it can be done by using Census@School questionnaire.

1- <u>Hour Workshop</u>
2- <u>GET Phase</u>
3- <u>Only 35 teachers can be accommodated.</u>

<u>Outcomes: </u>Selects, justifies and uses appropriate methods for collecting and representing data.

The exercise aims to inspire teachers to use the Census@School Questionnaire used in the Census@ School Survey of Statistics South Africa in the beginning of 2008 with 1500 schools that were sampled in all 9 provinces.

This questionnaire gathers data in three categories:

Personal Data

Household Data

School Data

Data about School **E**very participant will be issued with a Statistics South Africa C@S questionnaire as well as a frequency table and graph paper.

Statistics South Africa
Preferred supplier of quality statistics

Census@school

CensusAtSchool Form 2009 **Grades 3 - 7**

EMIS ☐☐☐☐☐☐☐☐☐ Class ☐☐ Learner number ☐☐☐☐

Learner name

SECTION A: ABOUT YOU

1. Are you a

1 ☐ Boy? 2 ☐ Girl?

2. What is your date of birth?

☐☐☐☐☐☐☐☐
D D M M Y Y Y Y

3. What grade are you in at school?

Grade ☐☐
e.g. Grade 10

4. Where were you born?

1	Western Cape	6	North West
2	Eastern Cape	7	Gauteng
3	Northern Cape	8	Mpumalanga
4	Free State	9	Limpopo
5	KwaZulu-Natal	10	Outside South Africa

5. Are you right-handed, left-handed or ambidextrous? (An ambidextrous person is able to use the right and left hands equally well.) **Mark the appropriate box with 'X'.**

1 ☐ Right-handed
2 ☐ Left-handed
3 ☐ Ambidextrous

Questions 6, 7 & 8 will be filled in by educator

6. How tall are you without your shoes on? **Answer to the nearest cm**

☐☐☐ centimetres

7. What is the length of your right foot, without a shoe? **Answer to the nearest cm**

☐☐☐ centimetres

8. What is your arm span? (Open arms wide, measure the distance across your back from the tip of your right hand middle finger to the tip of your left hand middle finger). **Answer to the nearest cm**

☐☐☐ centimetres

9. What is the colour of your eyes?

1	Brown	2	Green
3	Blue	4	Other

10. In which languages can you hold an everyday conversation?

1	Afrikaans	7	Sesotho
2	English	8	Sepedi
3	IsiNdebele	9	Setswana
4	Siswati	10	Tshivenda
5	IsiXhosa	11	Xitsonga
6	IsiZulu	12	Other

11. What is your favourite sport that you have played this year? Use the sport coding list to code your answer, or put 0 if you do not have a favourite sport.

Sport code ☐☐

12. What sport would you like to participate in? Use the sport coding list to code your answer or put 00 if there is no sport you would like to participate in.

Sport code ☐☐

13. How important are the following issues to you? **Mark the appropriate box with 'X'.**

	Not	Ave	Most	Don't Know
Reducing pollution				
Recycling our rubbish				
Conserving water				
Conserving energy				
Owning a computer				
Access to the internet				

Teachers will be grouped into 7 groups of 5 each.

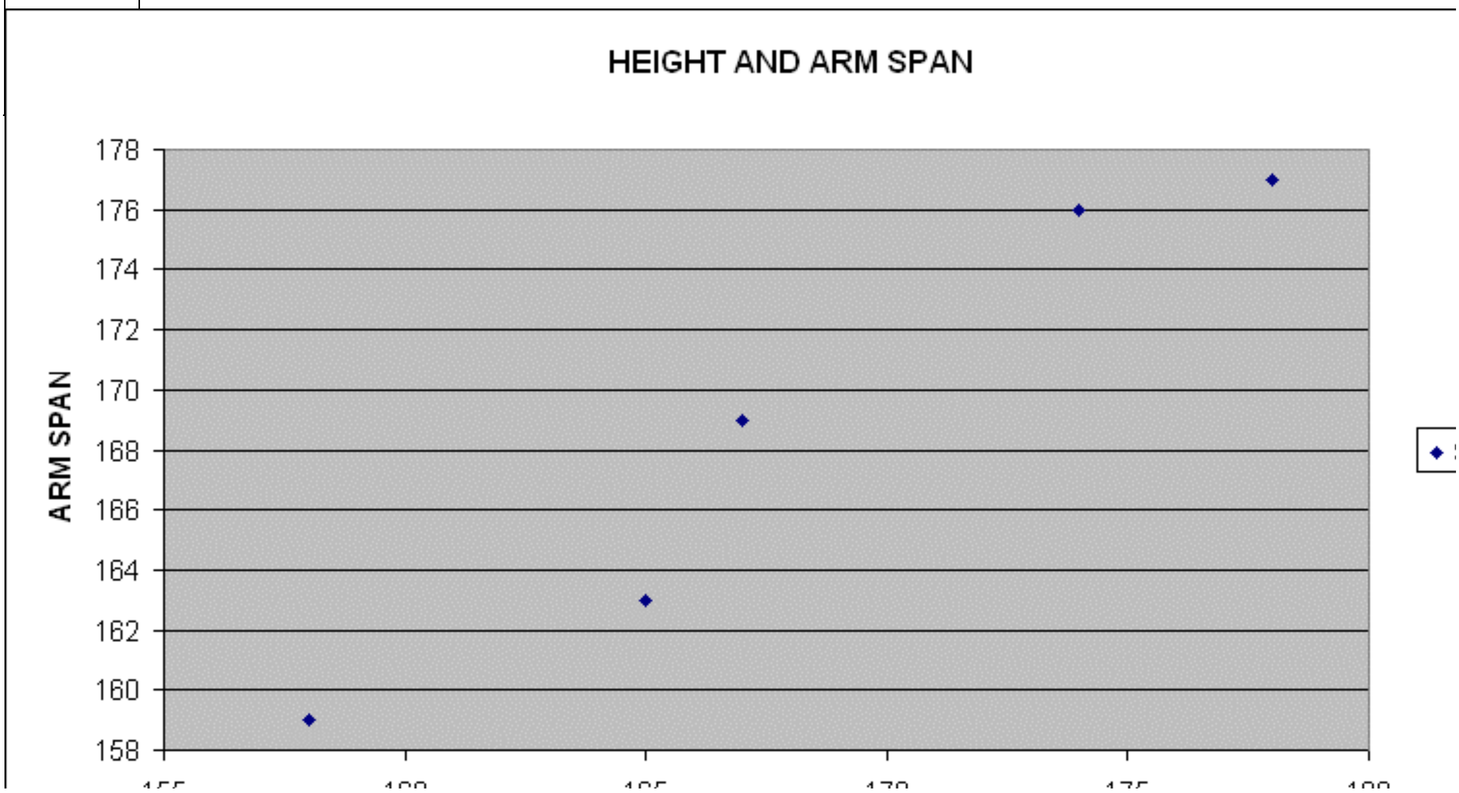
Only 2 questions of Section A will be completed namely 6 and 8. The teachers will be given measuring instruments as used in C@S project by Statistics South Africa to measure their arm span as specified on questionnaire and also to measure their height.

These two variables will be plotted on a printed table provided and data will then be ordered by using a tally table and a stem and leaf plot just to illustrate these tools also

The 5 groups will consolidate the data in a table so that it can be plotted on a scatter plot on the graph paper provided.

x height					
y arm span					

X HEIGHT	Y ARM SPAN
158	159
167	169
178	177
165	163
174	176



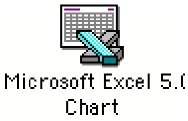
Deduction:

There is a strong positive correlation between a person's arm span and height.

DEDUCTION;

There is a strong positive correlation between a person's height and length of arm span.

This workshop will also make teachers aware of career opportunities in STATISTICS SOUTH AFRICA.



AN

INTRODUCTION TO

MATHEMATICS IN

EXCEL

Dennis Almeida, Senior Lecturer, Canterbury Christ Church University, UK

CONTENTS

1. WHAT IS EXCEL?

EXCEL is a computer spreadsheet. A spreadsheet provides an area divided into rectangular

cells where data (numerical or otherwise) can be entered, analysed and graphically represented at

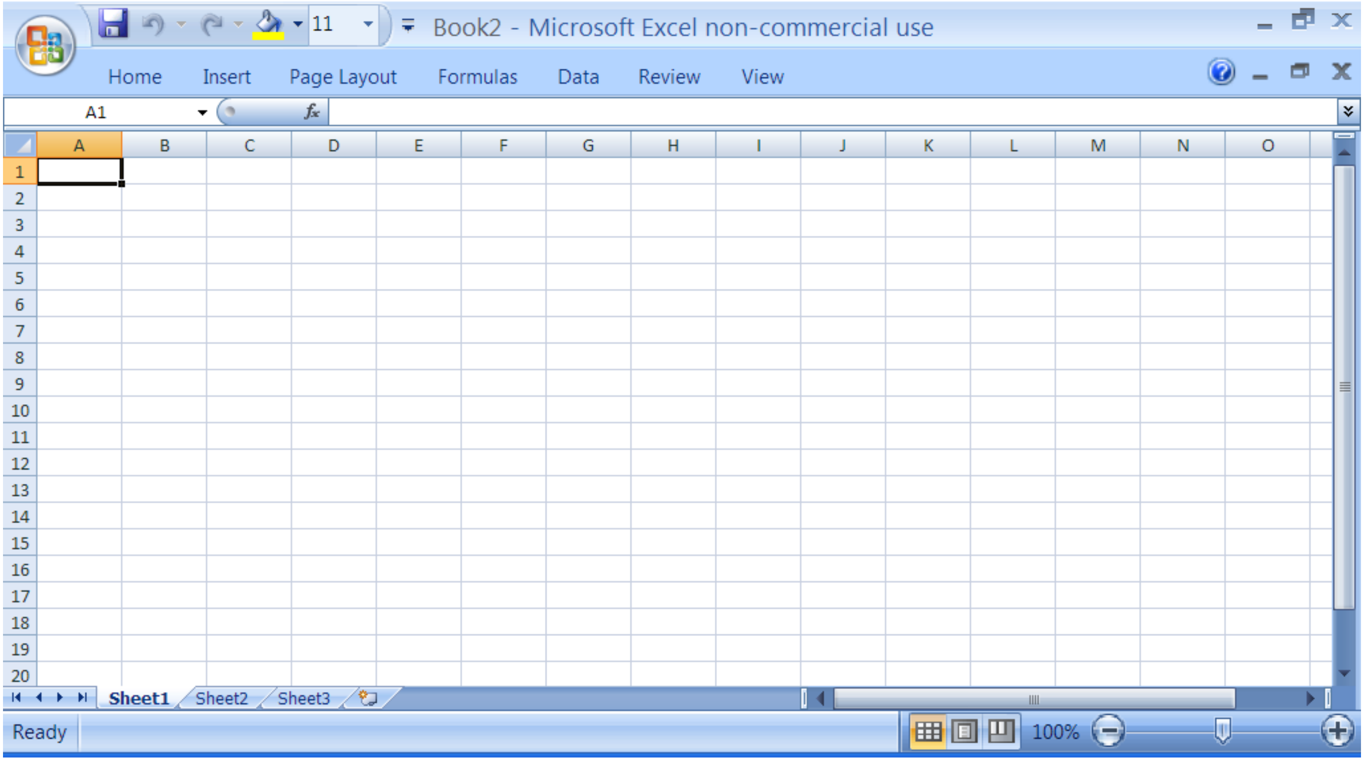
a rapid rate.

Each cell has a label determined by the *row* and *column* that it occupies- thus a cell will have a label like A1, B3, C4, etc. (see the EXCEL screen below).

In addition to storing data in cells, it is possible to use *formulae* or *functions* in each cell. These formulae can be used to transform or analyse data in other cells. For example, numerical data in cells can be squared by using the squaring formula ‘^2’ or you could test whether this numerical data was less than 10 by using an ‘if ....then....’ formula : some of these formulae will be introduced to you during this course.

Although EXCEL is essentially a business-oriented software package it has tremendous applications in statistics, probability, numerical analysis, and other branches of mathematics. In this guide some relevant numerical techniques and applications of EXCEL to mathematics will be introduced.

2. THE EXCEL SCREEN



GETTING STARTED

In what follows the symbol → will be used to signify 'the next step is'.

A. Accessing EXCEL:

Using the mouse place the cursor (the cross) on the EXCEL icon. → Double click on the EXCEL icon: the EXCEL screen will shortly appear - the image on the screen is called a *worksheet*.

B. Entering data in an EXCEL worksheet: Let's see how to enter the data 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 in cells A1 to A10. When you place the cursor (the cross) in a cell it becomes an *active cell*. An active cell is *highlighted* - the border of the cell is bolder than normal.

Using the mouse place the cursor in the cell A1 - this makes cell A1 active.

→Type 1 into the cell A1 and press the return key (↲): the data 2 is now entered in cell A1

→ type 2 into the cell A2 and press ↲

→ type 3 into the cell A3 and press ↲

.

.

→ type 10 into the cell A10 and press ↲

C. Making a correction in a worksheet: Just highlight the cell where you have inadvertently entered the wrong data by using the arrow keys ← ↑ → ↓ or the mouse. Then enter the correct data.

Suppose cell A10 required the number 20 instead of 10. The procedure for correction is:

Highlight/place the cursor in cell A10

→ enter the number 20

D. Making multiple corrections: Suppose the entries in cells A1 to A10 in part B were to be the prime numbers 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 and not the first 10 natural numbers. In this case you can effect the correction by *highlighting the array of cells* A1 to A10 and then pressing the delete key. The procedure is given below:

Highlight the cell A1

→ whilst the shift ⇑ key is pressed down, press the ↓ key until the cells A2 to A10 is coloured black (this has highlighted cells A1 to A10)

→ press the delete key

→ enter the data 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 in cells A1 to A10 as in part B.

N.B. An alternative method of highlighting an array of cells is given in the summary at the end of this section.

E. Making other cells in the worksheet active/highlighted: You can use the arrow keys or the mouse to make active/highlight other cells anywhere in the worksheet.

Other ways of doing this are listed below but it is best to keep things simple in the beginning!

i) use the mouse to click on the *scroll bars* on the left side of the worksheet.

ii) use the *Page Up/Page Down* keys to move the active/highlighted cells up or down the worksheet.

iii) simultaneously press the *Ctrl* and *Home* (respectively *End*) keys to access the top (respectively the end) row of the worksheet.

F. Saving your worksheet (on prime numbers, for example) on a floppy disc:

Click on **File**
→ click on **Save as**
→ click on the arrow below **Drives** → click on drive **a**
→ give your worksheet a name (not more than 7 characters) in the space below **File name**
→ now press the enter key or click on OK: your sheet is saved on the floppy disc under the name you've chosen.

G. Closing your worksheet:

Click on **File**
→ click on **Close**
→ click on OK or press the enter key.

H. Opening a new worksheet:

Click on **File**
→ click on **New**
→ click on OK or press the enter key.

G. Retrieving or opening a saved worksheet (say the prime number worksheet):

Click on **File**
→ click on **Open**
→ Select the appropriate drive - drive **a**
→ Click on the appropriate file name
→ click on **OK:** the prime number worksheet will now appear on screen.

<u>Well done! you have now successfully engaged with EXCEL.</u>

SUMMARY OF PROCEDURES

Opening a new worksheet : Use the mouse to point the cursor at **File** and click the mouse button (click on **File**). Next move the cursor to **New** and click the mouse button.

Saving a worksheet on the hard disc: Click on **File**. Then click on **Save**. Give your worksheet a name in the space below **File name.** Then click on **OK.**

Saving a worksheet on an external memory device: Click on **File**, click on **Save as**, click on the required location**.** Now give your worksheet a name in the space below **File name.** Then click on **save.**

Opening a saved worksheet : Click on **File** and click on **Open**. Select the appropriate location. Click on the appropriate file name**.**

Entering data: You can type in data into the *active cell* - this is the cell whose sides are bolder than that of other cells. You can make any cell active by clicking on it using the mouse or using the arrow keys. Type the entry into the active cell and press the return key (↵).

***Accessing cells*:** Use the mouse or the arrow keys to access cells in the other parts of the worksheet.

Other ways of doing this are: clicking on the bold arrows in the *scroll bars* enables you to access other parts of the sheet; using the *Page Up/Down* keys on the keyboard to move up and down the worksheet; Simultaneously using the *Ctrl* and *Home* (respectively *End*) keys to access the top row (respectively, the end row) of the worksheet.

Highlighting an array of cells : Highlighting an array of cells will be necessary in order to delete (or copy them, to implement a formula or function, etc). To do this click on the *top left most cell* in the array and, whilst holding *Ctrl* key down press the ↓ key until all desired cells are highlighted. Alternatively hold the mouse button down, drag the cursor down to the bottom right most cell of the array and then release the button. This requires some getting used to but practice makes perfect.

Deleting an array of cells : Highlight the array of cells to be deleted. Then press the *delete* key on the keyboard (and click on *OK* at the prompt if required).

4. GENERATING SEQUENCES

• Before commencing your exploration of mathematics in EXCEL note that the following key

strokes in EXCEL correspond to these common mathematical symbols:

Symbol in EXCEL	+	-	*	/	**SQRT**	***x^y***
Mathematical symbol	+	-	×	÷	√	x^y

• Each iteration procedure is followed by exercises (whose answers are given at the end of the

following page). The symbol ↲ denotes the enter key.

• Subsections or exercises marked with an * may be delayed until a later occasion.

4.1 Entering a sequence of numbers. If a sequence of numbers is such that the each term of the sequence is related to the previous term then it is possible to enter the entire sequence in EXCEL by using just three main steps:

entering a *formula* → copying → pasting.

A *formula* is an operation on numbers or letters. The *formula* you will initially use is = A1+1, this will be used to generate the natural numbers 1, 2, 3, 4, ...,20.

Here each term of the sequence (apart from the first) is one more than its predecessor: i.e. the

$(n + 1)^{st}$ term = (n^{th} term + 1). This explains why you have to use the formula =A1+1. The following procedure will give the sequence 1, 2, 3, 4, ...…..,20.

In cell A1 enter the number 1

→ in cell A2 type = A1+1 (the **formula** = A1+1 will appear in the formula bar)
→ press ↲ (the number 2 will appear in A2)
→ with cell A2 *active* click on **edit** and then on **copy** OR press **Ctrl** and **C** (cell A2 will *flash*)
→ highlight cells A2 to A20 as shown in section 3 (cells A3 to A20 will be shaded but A2 will not)
→ click on **edit** then on **paste** OR press simultaneously **Ctrl** and **V**

(3 to20 will appear in cells A3 to A20).

Note (i) that you can copy and paste also by dragging the lower right indented corner of the cell in which you have inserted a formula and (ii) that a formula in a cell can be corrected by making that cell active, clicking on the *formula bar*, and then making the correction in the *formula bar*.

The formula that you use depends on the sequence being considered. For example, = A1+2 will give the arithmetic sequence 1, 3, 5, 7, ...; = 2*A1 will give the sequence 1, 2, 4, 8, ...

Delete the entries in the A cells prior to attempting each of the following exercises.

Ex 4.1.1 Generate the following sequences (each with 25 terms) using an appropriate formula

(i) 1, 3, 5, 7, ...,49

(ii) 0, 1, 2, 3, ...,24 (start with 0).

(iii) 0, 2, 4, 6, ...,48.

(iv) 1, 3, 7, 15, 31,......,33554431 [double and add 1]

(v) 0, 2, 6, 14,........,33554430 [similar to (iv): double and add 2]

4.2 Entering a linked sequence of numbers. The sequence 1, 4, 9, 16, 25, 36,......., 400 may be linked to the sequence 1, 2, 3, 4, 5, 6, ..., 20: each term of the first sequence is the *square of* the corresponding term of the second. In this sub-section you will learn how to generate such *linked* sequences of numbers.

Begin with the sequence 1, 4, 9, 16, 25, 36,......., 400 stated above:

In cells A1 to A20 enter the consecutive numbers 1 to 20 (as in 4.1) → in cell B1 enter the formula = A1^2 and press ↲ (the number 1 will appear) → with cell B1 *active* click on **edit** and then on **copy** (cell B1 will *flash*) → highlight cells B1 to B20 (cells B2 to B20 will be shaded but B1 will not) → click on **edit** then on **paste** (4, 9, 16, 25, 36,......., 400 will appear in cells B2 to B20).

The formula you use will obviously be different for other *linked* sequences. For example,

= A1^3 will give the sequence 1, 8, 27, 64,...……

= A1/2 + 1 will give the sequence 1.5, 2, 2.5, 3,……...

Ex 4.2.1 By using an appropriate formula, generate the following sequences which are all linked to the sequence 1, 2, 3, 4, 5, 6, ...,25. You need not generate the linked sequences in the B cells; they may be generated in the C cells or D cells etc. Delete the entries in the B cells if you want to generate them there.

(i) 2, 8, 18, 32, ..., 1250 (2×squares)

(ii) 1, 8, 27, 64, ..., 15625 (cubes)

(iii) 1.5, 2, 2.5, 3,............., 13.5 [(natural numbers÷2)+1]

(iv) 1, 3, 6, 10,................., 325 [triangle numbers: $n(n+1)/2$]

Ex 4.2.2 The following two sequences are linked to a sequence of integers which needs to be generated in the A cells prior to generating the two linked sequences.

(i) -125, -64, -27, -8, -1, 0, 1, 8, 27, 64, 125

(ii) 25, 16, 9, 4, 1, 0, 1, 4, 9, 16, 25.

***4.3 Direct generation of linked sequences**. It is sometimes possible (and convenient) to generate a *linked* sequence directly in just a single column (rather than in two columns). Some thought will be needed before this can be achieved. For example, each term apart from the first in the sequence 1, 4, 9, 16, 25, 36,......., 400 is connected to it predecessor by the relation *((square root the previous number) + 1)*^{2} . Thus this sequence can be generated in the A cells as follows:

Enter 1 in A1
→ in A2 enter the formula = (SQRT(A1) + 1)^2 (the number 4 will appear)
→ Copy and paste the formula in cells A3 to A20 (the numbers 9, 16, 25, ...,400 will appear).

Ex 4.3.1 For the 4 linked sequences in 4.2.1 give the formula that will directly generate the sequence utilising a single column.

Answers 4.1.1 (i) A1+2, (ii) A1+1 (A1=0), (iii) A1+2 (iv) 2*A1+1 (v) 2*A1+2

5. ANALYSING AND HANDLING NUMERICAL DATA

A mathematician faced with a set of numbers will naturally attempt to make some sense of it by analysing it some way; for example, by summing the data, by ordering the data, or by finding the mean and standard deviation of the data. In this section you will be introduced to EXCEL techniques for analysing data quickly and effortlessly.

5.1 Summing a sequence of numbers. Summing a large set of numbers is tedious and fraught with error (if you've tried checking the total in your long supermarket till receipt you'll know the extent of this problem!). Providing you enter the data accurately EXCEL can be great assistance in summing large sets of numerical data. The following procedure will give you the sum of the sequence 1, 2, 3, 4, ...50.

Enter the numbers 1, 2, 3, 4, ...,50 in cells A1 to A50 as in section 4.1
→ in cell B1 (or any other cell) type the formula = SUM(A1:A50)
→ press ↲ : the sum 1275 will appear

Note: Although it is more convenient to use columns you can sum <u>across rows</u> as well. The procedure below shows how to sum the numbers 1, 2, 3, ...,25 in the first row; the first part of which describes the way to generate the numbers 1, 2, 3, ..., 25 in the row A1 to Y1.

Enter the number 1 in A1
→ In B1 enter the formula = A1 +1
→ copy and paste the formula in cells C1 to Y1
→ in cell Z1 (or any cell different from A1 to Y1) type the formula = SUM(A1:Y1)
→ press ↲ : the sum 325 will appear

Ex 5.1.1 Sum the 5 sequences in Ex 4.1.1.

5.2 Cumulative sums of a sequence. Often a mathematician will want to know how the sum of a large set of numbers gradually accumulates to its total: for example, s/he will want to know the cumulative sums 1, (1+2), (1+2+3),........(1+2+3+....+50) of the sequence 1, 2, 3, 4, ...50. You can see that in each of these 50 cumulative sums the initial term 1 is **fixed** while the last term in the sums **varies.** Now, in an EXCEL formula prefixing the initial cell label with $ signs **fixes** that cell, leaving the terminating cell label as normal allows it to **vary**. Work through the procedure below to make sense of this:

Enter the numbers 1, 2, 3, 4, ...50 in cells A1 to A50 (as in sec 4.1)

→ in cell B1 type in the formula = SUM($A$1:A1)

→ copy and paste this formula in the cells B1 to B50

→ press ↵ (the cumulative sums/triangle numbers 1, 3, 6, 10,.........,1275)

Observe how the formulae in B2 to B50 change from =SUM($A$1:A2) to =SUM($A$1:A50).

Note that you will find later on that just one $ sign is necessary in the formula above – for the moment using two $ signs will cause no problems.

Answers 4.2.1 (i) 2*(A1^2) (ii) A1^3 (iii) A1/2 + 1 (iv) A1*(A1+1)/2

4.2.2 Sequence -5, -4,......,4, 5 in A1 to A11 and = A1^3 in (i) = A1^2 in (ii)

4.3.1 (i) =2*((SQRT(A1/2)+1)^2) (ii) =(A1^(1/3)+1)^3 (iii) =A1+0.5

(iv) =(1+SQRT(1+8*A1))* (1+SQRT(3+8*A1))/8

Ex 5.2.1 By first generating the appropriate (linked) sequence of 25 numbers determine the cumulative sums of the following sequences

(i) 1, 4, 9, 16, ...,625

(ii) 1, 3, 5, 7, ..., 49. Do you notice anything about the cumulative sums?

(iii) 1, 9, 25, 49,..................., 2401 (odd squares)

(iv) 1, 8, 27, 64,........,15625 (cubes). Do you notice anything about the cumulative sums?

*Ex. 5.2.2 (i) Enter the numbers 1, 2, 3, ..., 100 in the A cells. Enter the reciprocals of these 100 numbers in the B cells (the reciprocal of a number *n* is the number 1/*n*). In the C cells determine the 100 cumulative sums

1, (1+1/2), (1+1/2+1/3), (1+1/2+1/3+1/4), ..., (1+1/2+1/3 +..............+1/100).

(ii) Extend the range in the A cells from 100 to 150. Correspondingly extend the ranges in the B and C cells. Make a note of the value of the 150th cumulative sum (1+1/2+1/3 +..............+1/150).

(iii) Extend the range in the A cells from 100 to 200. Correspondingly extend the ranges in the B and C cells. Make a note of the value of the 200th cumulative sum (1+1/2+1/3 +..............+1/200).

(iv) Let S_n denote the cumulative sum (1+1/2+1/3 +..............+1/*n*).

By continuing the extensions in (ii) and (iii) or otherwise try to answer the following question:

As *n* gets larger and larger does S_n reach a ‘limiting’ number or does it increase inexorably?

5.3 Making tables. On occasion a mathematician may have to make tables which refer to a set of initial values; for example, when compiling a table of squares, cubes, fourth powers, and fifth powers of the first 10 natural numbers. After entering the numbers 1, 2, ...,10 in cells A2 to A11 and the numbers 2, 3, 4, 5 in cells B1 to E1, a procedure to make this table might make use of these formulae:

= $A2^B$1 copied and pasted in cells B2 to B11 (gives the squares),

= $A2^C$1 copied and pasted in cells C2 to C11 (gives the cubes),

= $A2^D$1 copied and pasted in cells D2 to D11 (gives the fourth powers),

= $A2^E$1 copied and pasted in cells E2 to E11 (gives the fifth powers).

Notice that the A label in the first part (A2) of each formula is **fixed.** Also the cell number 1 in the second part (B1, C1, D1, E1) is also **fixed.** This is why these labels are prefixed with the $ sign. Further, because the second letter (the B) in =$A2^B$1 varies, we need just one formula

= $A2^B$1. Work through the procedure below to make sense of this:

Enter the numbers 1, 2, 3, ..., 10 in cells A2 to A11

→ Enter the numbers 2, 3, 4, and 5 in cells B1 to E1

→ Enter the formula = $A2^B$1 in cell B2 (the number 1 will appear)

→ copy and paste this formula in the rectangular array

B2	**E2**
B11	**E11**

(highlight this array before pasting): the required table will now appear.

Answers 5.1.1 (i) 625 (ii) 300 (iii) 600 (iv) 67108837 (v) 67108812

Ex 5.3.1 Make a table which shows the 100 different sums of pairs of the first 10 natural numbers. Enter the numbers 1, 2, 3, ..., 10 in cells A2 to A11 **and** in the cells B1 to K1. Now use a suitable formula in cell B2. Copy and paste this formula in the rectangular array of the table.

Ex 5.3.2 Make a table which shows the 100 different sums of pairs of **squares** of the first 10 natural numbers. Enter the numbers 1, 2, 3, ..., 10 in cells A2 to A11 **and** in the cells B1 to K1. Now use a suitable formula in cell B2 to copy and paste in the rectangular array of the table.

Ex 5.3.3 Make tables of your own choice.

5.4 Bounds of data. To numerically order or to find the maximum or the minimum number in a set D of numbers, say D = {1, 7, 6, 3, 2, 5, 7}, the following two procedures may be used:

Enter the data D in cells A1 to A7
→ Highlight cells A1 to A7 → click on DATA
→ click on SORT → click on *ascending* or *descending* order
→ click on OK (the list of numbers 1, 2, 3, 5, 6, 7, 7 or 7, 7, 6, 5, 3, 2, 1 will appear)

or

Enter the data D in cells A1 to A7
→ In B1 type the formula = MAX(A1:A7) → press ↲ (7 will appear)
→ In B2 type the formula = MIN(A1:A7) → press ↲ (1 will appear)

Practice these procedures on data of your own.

5.5 Random numbers. The following procedure generates 50 random numbers between 0 and 1

In cell A1 type the formula = RAND( )
→ copy and paste the formula in cells A1 to A50 (50 random numbers between 0 and 1 appear)

Using the *integer part* function INT generates random numbers between 1 and 6 as follows:

In cell A1 type the formula = INT(1 + 6*RAND( ))   → copy and paste the formula in cells A1 to A50 (50 random numbers between 1 and 6 appear)

Note: These random numbers are 'sensitive' in that any key stroke will given an entirely new set of 50 numbers.

Ex 5.5.1 What formula will generate random numbers between 1 and *n*, where *n* is any positive number?

Answers 5.2.1 (i) 1, 5, 14, 30,.....,5525 (ii) 1, 4, 9, 16,......, 625: sq numbers

(iii) 1, 10, 35,......, 20825 (iv) 1, 9, 36, 100, ..., 105625: sq. triangle numbers

5.2.2 Cumulative sums are 1, 1.5, 1.833.. , 2.0833... , ..., 5.18737752. The 150th cumulative sum is 5.59118059, the 200th cumulative sum is 5.87803095. The cumulative sums eventually reach 'infinity'. For information, the 227th sum exceeds 6, the 616th sum exceeds 8, the 1674th sum exceeds 9, the 4550th sum exceeds 10, etc.....

5.3.1 = $A1 + B$1 **5.3.2** = $A1^2 + B$1^2

6. CHARTING OR GRAPHING DATA

6.1 Data can represented pictorially. As an example, let's see how we can represent the following unemployment figures for 8 successive months, January to August:

3000000, 3200000, 3300000, 3500000, 3400000, 3250000, 3100000, 3000000

The procedure below should be followed carefully:

Enter the numbers 3000000, 3200000, ..., 3100000, 3000000 in the cells A1 to A8

→ Highlight A1 to A8

→ Click on the chart icon (a bar chart in the top right)

→ Place the cursor where you want the chart. Next drag the cursor down and to the right with the

mouse to obtain a rectangle in which you want the chart placed

→ Click on NEXT

→ Click on LINE

→ Click on NEXT

→ Click on option 2

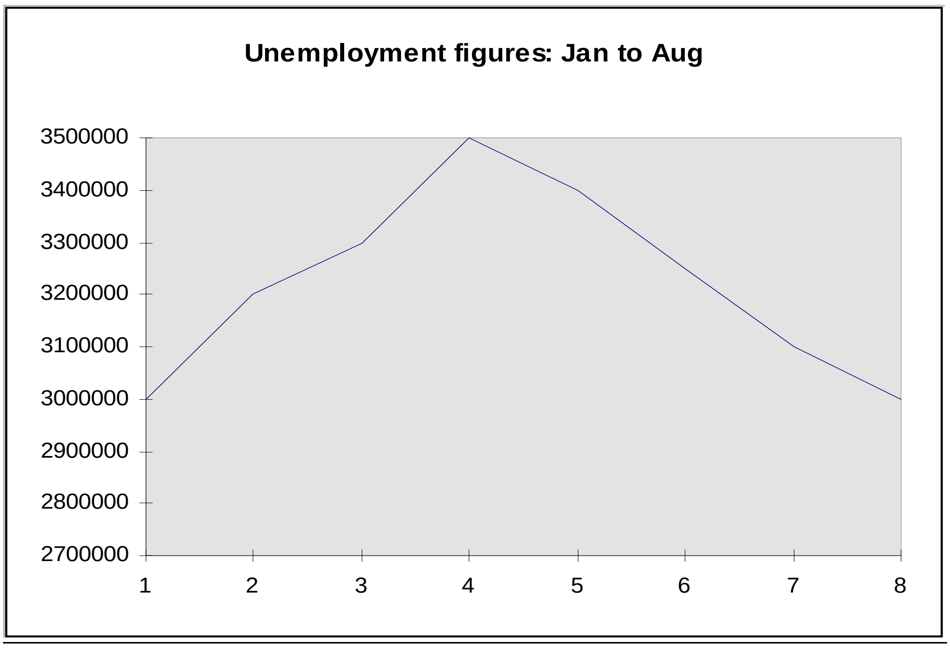
→ Click on NEXT

→ Click on NEXT

→ Click on the 'No' option to 'Add a legend?'

→ Type 'Unemployment figures: Jan to Aug' in CHART TITLE box

→ Click on FINISH (the line graph like the one below will appear)



Note: You can make the graph large or smaller by clicking **on** the perimeter of the rectangle containing the graph and then dragging the bold points with the mouse.

Notice that, in the graph just drawn, the horizontal axis is labelled by the numbers 1 to 8. Further the vertical axis has no description. To label the horizontal axis with the months January to August and to label the vertical axis with the descriptor: ‘Numbers unemployed’, follow this procedure:

Enter the months Jan, Feb, Mar, April, May, June, July, Aug in cells A1 to A8

→ Enter the numbers 3000000, 3200000, ..., 3100000, 3000000 in the cells B1 to B8

→ Highlight the rectangle of cells with vertices A1,B1, A8 and B8.

→ Click on the chart icon (a bar chart in the top right)

→ Place the cursor where you want the chart and click with the mouse to obtain a rectangle in

which you want the chart placed

→ Click on NEXT → Click on LINE → Click on NEXT

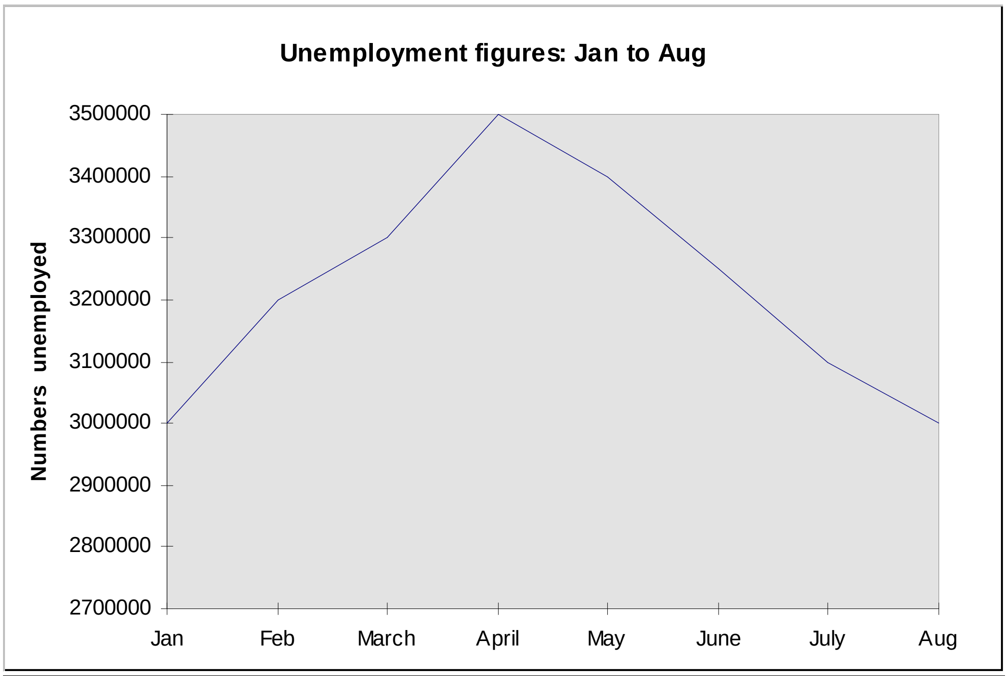
→ Click on option 2 → Click on NEXT → Click on NEXT

→ Click on the ‘No’ option to ‘Add a legend?’

→ Type ‘Unemployment figures: Jan to Aug’ in CHART TITLE box

→ Type ‘ Numbers unemployed’ in the Y AXIS box

→ click on FINISH (the line graph like the one below will appear)

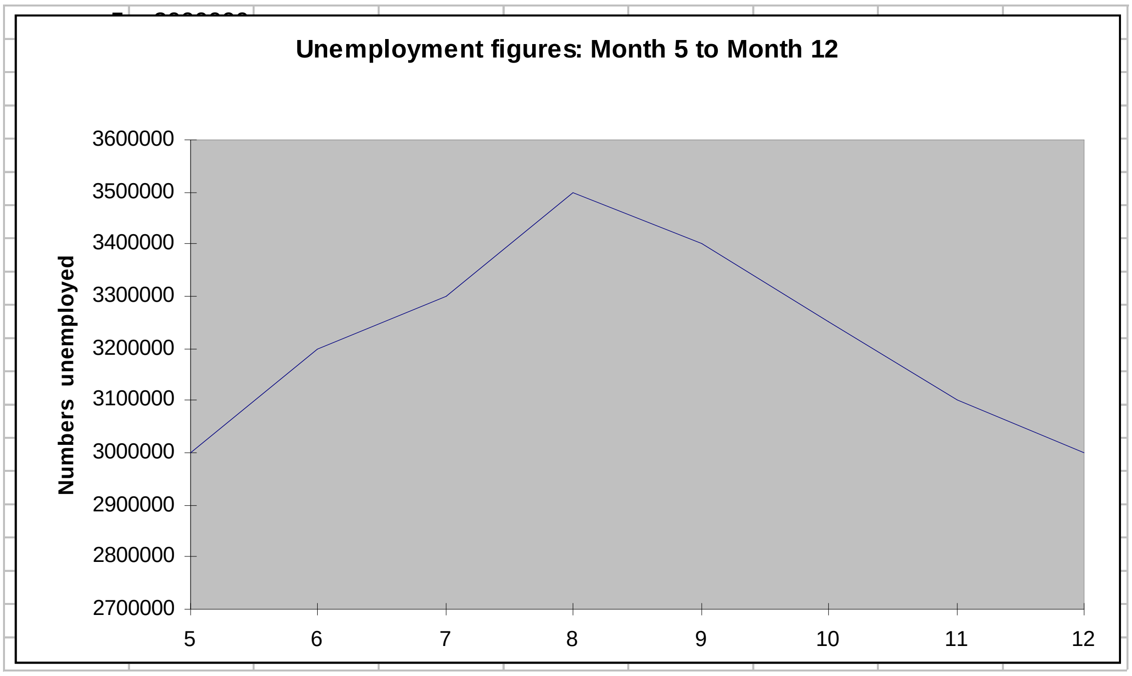


Ex 6.1.1 Practice the procedure on graphs of your choice.

Note: You can insert a chart into a Microsoft Word document by clicking on INSERT then on OBJECT then on CREATE FROM FILE. Next select the EXCEL file which contains the required chart. Finally click on OK.

6.2 Inserting x-axis values in general. When the x-axis values are numerical then the procedure above will not work. To illustrate the procedure when both x and y-axis values are numerical consider the case when the unemployment figures relate to the last 8 months of the year labelled 5, 6, 7, 8, 9, 10, 11, and 12. The following procedure will enable you to label the x-axis with the numbers 5, 6, 7, 8, 9, 10, 11, 12:

Enter the months 5, 6, 7, 8, 9, 10, 11, 12. in cells A1 to A8
→ Enter the numbers 3000000, 3200000, ..., 3100000, 3000000 in the cells B1 to B8
→ Highlight B1 to B8
→ Click on the chart icon (a bar chart in the top right)
→ Place the cursor where you want the chart and click with the mouse to obtain a rectangle in which you want the chart placed
→ Click on NEXT → Click on LINE → Click on NEXT
→ Click on option 2 → Click on NEXT → Click on NEXT
→ Click on the ‘No’ option to ‘Add a legend?’
→ Type ‘Unemployment figures: Month 5 to Month 12’ in CHART TITLE box
→ Type ‘ Numbers unemployed’ in the Y AXIS box
→ click on FINISH (the line graph like the one below will appear)
→ Click the mouse with the cursor **inside** the rectangle containing the line graph (perimeter is hatched)
→ Click the mouse with the cursor **on** the line graph itself
→ Click on X VALUES → Click in X VALUES box
→ Double click on cell A1 and drag the cursor down to cell A8
→ Click OK (you should now have a graph like the one below)


Unemployment figures: Month 5 to Month 12
Numbers unemployed
3600000
3500000
3400000
3300000
3200000
3100000
3000000
2900000
2800000
2700000
5
6
7
8
9
10
11
12

7. ANALYSING MEASURES OF LOCATION AND SPREAD

Now that you have some familiarity with EXCEL, column and row headings should be used to signify the meaning of calculations you will perform.

7.1. Mean. The mean is a measure of location. Means of numerical data can be calculated rapidly and easily in EXCEL. The mean of the data **D** = {1, 3, 6, 10, 15} is computed in EXCEL as follows:

In cell A1 enter the heading MEAN

→ In cell A2 enter the heading DATA

→ In cell B2 enter the heading **D**.

→ In cells B3 to B7 enter the numbers 1, 3, 6, 10, 15.

→ In cell B1 determine the average of data **D** by typing the formula = AVERAGE(B3:B7)

(You should get the following output)

MEAN	7			
DATA	**D**			
	1			
	3			
	6			
	10			
	15			

7.2 Properties of the mean. How does translation of the data affect the mean? To explore this is EXCEL you could adopt this procedure. Here the following convention is used:

(**D** + 1) = {2, 4, 7, 11, 16}: this is data **D** with 1 added to each number.

(**D** + 2) = {3, 5, 8, 12, 17}: this is data **D** with 2 added to each number. And so on.

In C2, D2, E2, F2, G2 enter the headings (**D**+1), (**D**+2), (**D**+3), (**D**+4), (**D**+5) respectively.

→ In C3, D3, E3, F3, G3 enter the formulae = B3+1, = B3+2, = B3+3, = B3+4, =B3+5 resp.

→ Simultaneously highlight cells C3 to G3

→ Copy and paste cells C3 to G3 into the rectangular array

C3.........G3

C7..........G7

→ Calculate the means of (**D**+1), (**D**+2), (**D**+3), (**D**+4), (**D**+5) by and copying and pasting the

formula in B1 into cells C1 to G1. (You should get the following output).

MEAN	7	8	9	10	11	12
DATA	**D**	**(D+1)**	**(D+2)**	**(D+3)**	**(D+4)**	**(D+5)**
	1	2	3	4	5	6
	3	4	5	6	7	8
	6	7	8	9	10	11
	10	11	12	13	14	15
	15	16	17	18	19	20

These calculations show that the mean of (**D** + k) = (mean of **D**) + k

Ex 7.2.1 (i) By altering the data **D**, the universality of rule [mean of (**D** + *k*) = (mean of **D**) + *k*] can be easily be justified. Just alter the entries in cells B3 to B7 to see this.

(ii) The above can made even more easy by entering random numbers in cells B3 to B7 by entering the formula = INT(1+20*RAND()) in cell B3 and then copying and pasting into cells B4 to B7. Then successive pressing of the F9 key will yield successive values of the data **D**.

Ex 7.2.2 *k***D** is the data in **D** multiplied by *k*. Verify the rule [mean of (*k***D**) = *k*(mean of **D**)] by adapting the procedure above. Change the headings in C2 to G2 to 2**D**, 3**D**,......,6**D** respectively. Change the formulae in C3, ..., G3 to = B3*2, ..., =B3*6 respectively.

*7.3 Visual illustration of properties of the mean** To literally <u>see</u> the rule [mean of (**D** + *k*) = (mean of **D**) + *k*] you could follow this procedure.

Label cells A1 and A2 with the headings k and DATA respectively.

→ Label cells B2, C2, D2 with the headings **D,** (**D** + k), and MEAN respectively.

→ Initially enter 0 in cell B1, enter any data in cells B3 to B7.

→ Make entries in cells C3 to C7 by using the formula = (B3 + $B$1) and copying and pasting.

→ In cells D3 to D7 copy and paste the formula = AVERAGE($B$3:$B$7)

→ Enter 30 in cell D10 (this is needed to preserve scale)

→ Highlight the rectangular array of cells from C3 to D10

→ Obtain a graph (located, say, in columns E to I) of entries in the C and D columns

→ Click on COMBINATION

→ click on NEXT

→ click on OPTION 1

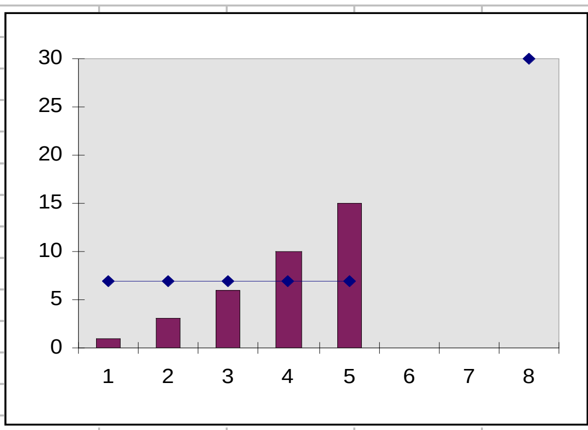
→ click on NEXT

→ click on NEXT

→ click on FINISH (you should get something like the worksheet below)

→ Alter the value in B1 from 0 to 10 in succession and you will see the translational effect on the mean.

k	0		
DATA	D	D+k	Mean
	1	1	7
	3	3	7
	6	6	7
	10	10	7
	15	15	7
			30



7.4 Standard Deviation. The standard deviation is a measure of spread. It is the square root of the average of the squares of the deviations from the mean (the mean is subtracted from each item in the data – each of these numbers is squared - finally the square root of the mean of these squares is computed). Standard deviations of numerical data can be calculated rapidly and easily in EXCEL by using the formula = STDEVP(cell label : cell label).

The procedure in **7.1** is slightly changed to find the standard deviation of the data

D ={1, 3, 6, 10, 15}.

In cell A1 enter the heading STD.DEV

→ In cell A2 enter the heading DATA

→ In cell B2 enter the heading **D**.

→ In cells B3 to B7 enter the numbers 1, 3, 6, 10, 15.

→ In cell B1 determine the average of data **D** by typing the formula = STDEVP(B3:B7)

(You should get the following output)

STD. DEV	5.01996			
DATA	**D**			
	1			
	3			
	6			
	10			
	15			

Ex 7.4.1 Adapt the procedure in **7.2** to verify the rule [Std.dev of (**D** + *k*) = Std.dev of **D**] for different sets of data **D**.

Ex 7.4.2 Adapt the procedure in Ex **7.2.2** to verify the rule [Std.dev of (*k***D**) = *k*(Std.dev of **D)**] for different sets of data **D**. For the data **D** = {1, 3, 6, 10, 15} and *k* = 2, 3, 4, 5, and 6 you should get:

STD.DEV	5.01996	10.03992	15.05988	20.07984	25.0998	30.11976
DATA	**D**	**2D**	**3D**	**4D**	**5D**	**6D**
	1	2	3	4	5	6
	3	6	9	12	15	18
	6	12	18	24	30	36
	10	20	30	40	50	60
	15	30	45	60	75	90

8. SIMULATING SIMPLE GAMES

The mathematical rationale for simulating games in EXCEL is twofold:

i) a game is played according to a set of logical rules.

ii) it allows for a comparison between empirical and theoretical probability.

The simulations will require the use of logic statements such as:

=IF(A2>3, 1, 0): this returns 1 if cell A2 contains a number greater than 3 and 0 otherwise.

=IF(A2=2, “WIN”, “LOSE”): this returns WIN if cell A2 =2 and LOSE otherwise.

=IF(AND(A2=3,B2=4),1, “”): this returns 1 if A2=3 and B2=4 and a blank cell otherwise.

=IF(OR(A2=3,B2=4), “WIN”, “LOSE”): this returns WIN if A2=3 or B2=4 and LOSE otherwise.

etc.

8.1 Simulating die games. The following procedure simulates and analyses series of 100 games involving throwing a die: you win if the die number is less than 4.

Label cells A1, B1, and C1 by DIE THROW, OUTCOME and TOTAL WINS

→ Generate 100 die throws in cells A2 to A101 by using the formula = INT(1 + 6*RAND())

→ In cell B2 type the formula = IF(A2<4, 1, 0)

→ copy and paste the last formula in cells B2 to B101

→ In C2 type = SUM(B2:B101) to obtain the number of games you have won

→ Press F9 (all 100 numbers change to generate another set of 100 die throws at one key stroke!)

Using words instead of numbers to identify wins and losses requires the use of the COUNTIF formula. To make sense of this formula do this simulation

Label cells A1, B1, and C1 by DIE THROW, OUTCOME and TOTAL WINS

→ Generate 100 die throws in cells A2 to A101 by using the formula = INT(1 + 6*RAND())

→ In cell B2 type the formula = IF(A2<4, “WIN”, “LOSE”)

→ copy and paste the last formula in cells B2 to B101

→ In C2 type = COUNTIF(B2:B101, “WIN”) to obtain the number of games you have won.

Note: If you win with a 2 or 3 with a die throw then the single formula

= IF(AND(A1>1, A1<4), 1, 0) will suffice.

8.1.1 Repeat the above example for games where you win if the die throw is

(i) less than 3 (ii) equal to 1 (iii) bigger than 4 (iv) equal to 6

8.1.2 In each of the 4 cases in 8.1.1 what is the mean number of wins in a sequence of 100 games?

(Optional) Is each mean close to the theoretical probability of winning in each game?

8.1.3 A coin is thrown and you win if it lands heads. Denote a head by 1 and a tail by 2. Simulate 100 coin throws and determine the number of wins. Is the mean number of wins in agreement with your intuition about coin throws?

8.2 Simulating games with 'complex' criteria for winning. To simulate die games where you win if the die throw is 2 or 5 requires the use of an IF OR logic statement. The following procedure shows how to simulate a series of 100 such games

Label cells A1, B1, and C1 by DIE THROW, OUTCOME and TOTAL WINS
→ Generate 100 die throws in cells A2 to A101 by using the formula = INT(1 + 6*RAND())
→ In cells B2 to B101 copy and paste the formula = IF(OR(A1= 2, A1=5),1, 0)
→ In cell C2 the formula = SUM(B2:B101) will indicate the number of games you've won

8.2.1 In each of the 2 cases below simulate 100 games and determine the mean number of wins. Use may be required of a double OR statement as shown in part (i).

(i) You win if the die throw is 1, 2 or 4 (use = IF(OR(OR(A2=1,A2=2),A2=4),1,0).

(ii) You win if the die throw is 2, 4 or 6.

8.3 Simulating games with multiple events. A simple example of such a game is when you throw a coin and a die - you win if the coin lands heads (heads = 1, tails = 2) and the die throw is less than 4. Simulating such a game may involve using an IF AND formula:

Label A1, B1, C1, and D1 with COIN, DIE, WIN?, TOTAL WINS
→ Generate 100 coin throws in cells A2 to A101 by using the formula = INT(1 + 2*RAND())
→ Generate 100 die throws in cells B2 to B101 by using the formula = INT(1 + 6*RAND())
→ In cells C2 to C101 copy and paste the formula = IF(AND(A1<2, B1<4), 1, 0)
→ In D2 use = SUM(C2:C101) to determine the total number of wins

Ex 8.3.1 In each of the 2 cases below simulate 100 games and determine the mean number of wins.

(i) You win if the coin lands heads and the die throw is greater than 2

(ii) You win if the coin lands heads and the die throw is 1

Ex 8.3.2 A coin and a die are thrown. You win if the coin lands heads and the die throw is 1 or 2.

[*]Ex 8.3.3 The popular playground hand game ‘Scissors, Stone, Paper’ is played by two players as follows: At a given signal each player makes a hand signal to signify either a pair of Scissors, a Stone or a sheet of Paper. Your Scissors beats your opponents Paper, Your Stone beats your opponents Scissors, Your Paper beats your opponents Stone; reverse situations are wins for your opponent; identical hand signals are draws.

[*]Ex 8.3.4 Five Finger Morra is played by two players as follows: At a given signal each player shows between 1 to 5 fingers of the right hand and the **sum** of the fingers shown is recorded. If the sum is divisible by 3 then the game is a draw, if there is a remainder 1 when the sum is divided by 3 then you win, otherwise your opponent wins. [Hint: consider using the formula =mod(C2,3): this returns the remainder upon division by 3].

Ans 8.1.1 (i) =IF(A1<3, 1, 0) (ii) =IF(A1=1, 1, 0) (iii) =IF(A1> 4, 1, 0) (iv) =IF(A1=6, 1, 0)

8.1.3 =INT(1+2*RAND()) simulates coin throws; =IF(A1<2, 1,0) is one way to identify wins

9. AUTOMATING CALCULATIONS.

In this section fundamental methods for automating calculations are described. The methods essentially consist of

> i) Giving a set of columns (or rows) of a worksheet a set of headings which describe either the data that is to be entered in the columns (data cells) or the way in which the data is handled (handling cells).
>
> ii) Formatting the handling cells by using a logic statement of the kind
>
> =IF(data cell="", "", formula)
>
> If a data cell is blank (i.e. the entry is "") then the corresponding handling cell is also blank (i.e. the entry is also ""). If a data cell has an entry then it is handled in the corresponding handling cell by the 'formula' you have specified.

Work through the following two subsections to make sense of this.

9.1 Running total. Here a worksheet is formatted so that you can keep track of your monthly entertainment budget of £300. After formatting your expenditure is to be entered daily - the total remaining is automatically calculated.

Make the heading 'Aug budget (£)' in cell A1
→ Enter the amount 300 in cell B1
→ Make following headings: 'Date' in A3, 'Spending' in B3, 'Money left' in C2
→ Enter the formula = B1 in cell C3
→ Enter the formula = IF(B4="", "",1) in cell A4
→ Enter the formula = IF(B5="", "",A4+1) in cell A5. Copy + paste formula to cells A6 to A33
→ Enter the formula = IF(B4="", "",C3-B4) in cell C4. Copy + paste formula to cells C5 to C33

Now when you enter your expenses day by day in the data cells B4 to B33, the total remaining automatically appears in cells C4 to C33. Enter some figures successively in cells B4 to B33 to see how this works.

Ex 9.1.1 Add an additional handling column D to the above worksheet which automatically calculates the running average expenditure; i.e. after day 2 it shows the average for the first 2 days, after day 3 it shows the average for the first 3 days, etc.

Ex 9.1.2. Now add an additional handling column E which automatically returns 'CUT DOWN' if your running average is greater than 10 and 'OK' otherwise. [You may need to use a 'nested' IF statement - IF(D4="", "", IF(...., "....", "....."))]

Answers 8.2.1 (ii) = IF(OR(OR(A2=2,A2=4),A2=6),1,0) OR =IF(MOD(A2,2)=0, 1,0).

8.3.1 i) =IF(AND(A2=1,B2>2),1,0) (ii) =IF(AND(A2=1,B2=1),1,0).

8.3.2 =IF(AND(A2=1,B2<3),1,0).

8.3.3 Possible simulation: Let 1 = Scissors, 2 = Stone, 3 = Paper. You win with the following combinations (1, 3), (2, 1), and (3, 2): here your outcome is shown first. The difference between the 'coordinates' is -2, 1, and 1 respectively. Other differences are 0, -1 and 2. So you win with the difference 1 or -2. Use the statement = IF(OR(C2=1, C2 =-2),1,0)

8.3.4 Use the statement = IF(MOD((A2+B2),3)=1,1,0)

9.2 Mean and Standard Deviation of pooled data. The second example automatically calculates the mean and standard deviation of data that has been pooled into class intervals (e.g. 3 people whose heights are in the range e [1.5, 1.6), 5 people with heights in the range [1.6, 1.7), etc).

The procedure below applies to data that has been pooled in up to 25 intervals (it can easily be extended). The interval size for the procedure below is 10 and the smallest interval has left hand value 5; however both of these can be altered at any time. Note that column widths may

need to be enlarged to accommodate certain headings (or use a smaller font size).

Enter the title: 'Mean and Standard Deviation of pooled data' in cell A1

→ Make the following headings: 'MEAN' in H1, 'Total: f' in D2, 'Total: f*x' in F2, 'Total: f*x^2' in G2, 'Int. length' in A3, 'STD.DEV' in H3, 'Interval' in B4, 'frequency' in D4, 'Int. mid-pt' in E4, '(freq)(mid-pt)' in F4 [column width may need to be enlarged or use a smaller font size], '(freq)(mid-pt^2)' in G4, 'a' in B5, 'b' in C5, 'f' in D5, 'x' in E5, 'f*x' in F5, 'f*x^2' in G5

→ In B3 enter 10 (this interval length can be changed at any time)

→ In B6 enter 5 (this minimum length can be changed at any time)

→ In H2 enter the formula =IF(D3=0, "", F3/D3) (this will calculate the mean)

→ In D3 enter the formula =SUM(D6:D30) (this will compute the total frequency)

→ In F3 enter the formula =SUM(F6:F30) (this will compute the total of freq*x)

→ In G3 enter the formula =SUM(G6:G30) (this will compute the total of f*x^2)

→ In H4 enter the formula =IF(D3=0, "", G3/D3-H2^2) (this will calculate the std.dev)

→ In B7 enter the formula =IF(D6="", "", C6). Copy+paste onto cells B8 to B30

→ In C6 enter the formula =IF(D6="", "", B6+$B$3). Copy+paste onto cells C7 to C30

→ In E6 enter the formula =IF(D6="", "", (B6+C6)/2). Copy+paste onto cells E7 to E30

→ In F6 enter the formula =IF(D6="", "", D6*E6). Copy+paste onto cells F7 to F30

→ In G6 enter the formula =If(D6="", "",D6*E6^2). Copy+paste onto cells G7 to G30

All you have to do now is to enter the frequencies in the cells D6 to D30. You don’t have to use all 25 formatted cells - just use the ones you require. As you can see the computations are completely automatic. Observe that the interval lengths can be specified in cell B3 and the smallest interval left end point can be specified in cell B5.

Ex 9.2.1 Add the heading ‘MODE?’ in cell A5. Create handling cells A6 to A30 which return ‘>‘ when the corresponding interval is the modal class (the class with the greatest frequency) and a blank cell otherwise.

Ex 9.2.2 Add the heading ‘MEDIAN?’ in cell H5. Create handling cells **H7** to H30 which return ‘<‘ when the corresponding interval is the median class (the class in which the median lies) and a blank cell otherwise. [Hint: Use an IF AND statement which incorporates the cumulative total SUM(D$6:D6) which is compared by < and > signs with, $D$3/2, half the total cumulative frequency]

* Ex 9.2.2 Amend the automatic calculation so that it can deal with non-butting intervals such as (5, 9), (10, 14), (15, 19) , etc.

Answers 9.1.1 =IF(B4=“”, “”, AVERAGE($B$4:B4))

9.1.2 =IF(D4=“”, “”, IF(D4>10, “CUT DOWN”, “OK”))

*10 GRAPHING FUNCTIONS.

10.1. Graphing single functions Any function may be graphed or charted. The following procedure for graphing a line chart of $y = x^2$ may be adapted for other cases.

In Cells A1 to A21 enter the numbers -10 to 10
→ In Cells B1 to B21 enter the values $(-10)^2$, $(-9)^2$, ... 9^2, 10^2 by using the formula = A1ˆ2
→ Highlight Cells B1 to B21
→ follow the procedure of **6.1**

Note If you want the chart/graph to appear on a new sheet (i.e. a sheet different to the one containing your data) then follow the slightly modified procedure below:

In Cells A1 to A21 enter the numbers -10 to 10
→ In Cells B1 to B21 enter the values $(-10)^2$, $(-9)^2$, ... 9^2, 10^2 by using the formula = A1ˆ2
→ Highlight Cells B1 to B21
→ Click on INSERT → Click on CHART → Click on AS NEW SHEET
→ follow the procedure of **6.1**

Ex 10.1.1 Construct the line graphs of the following functions for x = -10, -9, ...9, 10.

(i) $y = x^3$ (ii) $y = 2x^2 + 1$ (**be sure to use** = 2*(A1^2) + 1)

Ex 10.1.2. Construct the graph of $y = 500e^{-x}$ for x = 0, 0.05, 0.10, 0.15, ..., 1.95, 2.00 : this will occupy cells A1 to A41. Use = A1 + 0.05 in the A cells, use = 500*EXP(-A1) in the B cells.

10.2 Multiple graphing. The following procedure shows how to obtain the line graphs of more than one function in the same chart. Below the line graphs of $y = x^2$, $y = x^2/2$, $y = 2x^2$ will be constructed in the same chart.

Enter the values 0 to 4 in increments of 0.1 in A1 to A41

→ In cells B1 to B41 enter values x^2 : use = A1^2

→ In cells C1 to C41 enter values $x^2/2$: use = (A1^2)/2

→ In cells D1 to D41 enter values $2x^2$: use = 2*(A1^2)

→ Highlight the rectangle of cells B1 C1 D1

B41 C41 D41

→ follow the procedure of **6.1**

Ex 10.2.1 Obtain the line graphs of the following quartet of functions in the same chart.

$$y = 10e^{-x} = 10\exp(-x)\ ;\ y = 10\exp(-x^2);\quad y = x^2 + 1\ ;\quad y = x^3 - 1$$

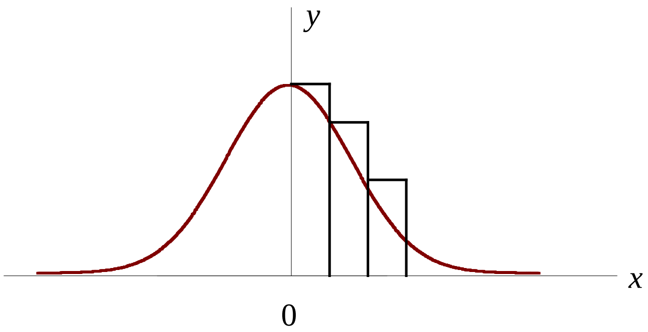
Take values $x = 0$ to $x = 2$ in increments of 0.05.

Answers 9.2.1 Possible formula =IF(D6=MAX(D$6:D$30), “>“, “”)

9.2.2 Possible formula =IF((AND(SUM(D$6:D6)<D$3/2, SUM(D$6:D7)>D$3/2), “<“, “”) in cell **H7**. Copy and paste in cells H8 to H30

*11. APPROXIMATE AREAS UNDER GRAPHS

The area under the graph is approximated by rectangles of equal width but whose length is given by appropriate *y* coordinates as shown below:



The following procedure to find the area *A* under the curve $y = x^2$ between $x = -3$ and $x = 3$ may be adapted to other cases such as the one above where $y = \exp(-x^2)$.

Enter the numbers between -3 to 3 in increments of 0.05 in cells A1 to A121: Use = A1 + 0.05 → Enter the corresponding values of x^2 in cells B1 to B121: Use = A1^2 → In cells C1 to C121 enter the areas of the rectangles that 'make up' the area *A*: Use = B1* 0.05 → In cell D1 obtain the area *A* by using = Sum (C1: C120) (the approx value 18.0025 will appear)

Ex 11.1.1 How does the approximate value 18.0025 compare with value for *A* using **integration**?

Ex 11.1.2 Using values of *x* from 0 to 2 in steps of 0.01 (this will occupy cells A1 to A201)

determine an approximation for the area under $y = x^3$ between $x = 0$ and $x = 2$.

Ex 11.1.3 Using values of *x* from 0 to 1 in steps of 0.01 determine an approximation for the area under the curve $y = \exp(-x^2)$ between $x = 0$ and $x = 1$.

Answers 11.1.1 Actual value is $A = \int_{-3}^{3} x^2 dx$ = $[x^3/3]^{x=3}$ - $[x^3/3]^{x=-3}$ = 18

11.1.2 Approx area = 3.9601 (actual = 4)

11.1.3 Approx area = 0.749979 (actual = ??)

ADDITIONAL ACTIVITIES

1. Exploring the sum of odd and even numbers. Here a worksheet is formatted with headings and formulae so that a pupil can explore the additive relationships between odd and even numbers.

In A1 Enter the title 'SUM OF ODD AND EVEN NUMBERS' (or something similar)

→ Highlight cells A3 to B3 and change the width's of columns A to C to size 12

(format→column→width→change size)

→ Enter these headings: In each of A3 and B3 – 'Enter number', in C3 – 'Sum'

→ In C4 enter the formula =IF(B4= "", "",A4+B4)

→ In A5 enter =IF(A4= "", "", IF(MOD(A4,2)=0, "even", "odd")); copy+paste to C5

→ Highlight the rectangle of cells with vertices A4, C4, A5, C5

→ Copy+paste this rectangle of cells up to the row A25, B25, C25, say

When a pupil enters numbers cells A4 and B4 one of the following relations: Odd+Odd=Even, Even+Even=Even, Odd+Even=Odd, Even+Odd=Odd: will be verified. Numbers are entered in the even numbers cells A2n, B2n, C2n.

<u>2. Exploring the product of odd and even numbers.</u> This is just a slight variation of activity 2: SUM becomes PRODUCT and the formula in C4 changes to =IF(B4= "", "",A4*B4). The relations that are verified are: Odd×Odd=Odd, Even×Even=Even, Odd×Even=Even, Even×Odd=Even

<u>3. One multiplication table: 2×10 to 999×10</u>. Here a worksheet is formatted so that all the multiplication tables from 2×10 to 999×10 are given in ONE table. All the pupil has to do is to specify in a given cell the multiplication table s/he requires to see.

Change the widths of columns A to E to size 4

→ In A1 enter the heading ‘Multiplication table for the number’

→ In G1 enter the number 7 and align to the left (highlight the cell G1 and press the click on left button). The combined A1+G1 should read ‘Multiplication table for the number 7’

→ In B3 enter ‘x’ and align centrally. Copy+paste to cell B12.

→ In D3 enter **exactly** as seen ‘= and align the = sign centrally. Copy+paste to cell D12.

→ In A3 enter =$G$1. Copy+paste to A12.

→ In C3 enter 1 and align left; in C4 enter =C3+1 and align left; copy+paste to C12

→ In E4 enter =A3*C3. Align left. Copy+paste to E12. The 7×7 table appears.

To obtain other multiplication tables the entry in G1. The tables can be extended easily to give 2×*n* to 999×*n*.

<u>4. Addition quiz.</u> This EXCEL quiz is designed to test a pupils knowledge of his/her addition facts. Encouragement is given if s/he gets the wrong answer.

Highlight row 3 up E3 and change the cell alignment to ‘justify’

(format→cells→alignment→justify→OK)

→ In A1 enter the heading ‘DO YOU KNOW YOUR BIG SUMS?’

→ In each of cells A3 and C3 enter ‘Number bigger than 20’

→ In B3 enter **exactly** as seen ‘+. Align centrally. Copy+paste to B25, say.

→ In D3 enter **exactly** as seen ‘=. Align centrally. Copy+paste to D25, say.

→ In E3 enter ‘Your answer’

→ In F4 enter the formula below

=IF(E4="","",IF(E4=A4+C4, " correct",IF(E4>A4+C4," too big - try again ","too small-try again"))) copy and paste to cell F25

A pupil now enters numbers in columns A and C and gives his/her answers in the E column. If it is right then EXCEL responds “correct” otherwise “too big - try again”, or “too small - try again”.

5. Multiplication quiz. Essentially the same as activity 4 but with multiplication. Here the following changes have to be made: + becomes x in the B cells; + becomes * in the formula in cell F4 (remember to copy and paste to F25). Title also changes and there no need for a restriction on size of numbers, so headings in A3 and C3 change.

6. Factors of a number quiz. Here a pupil is tested about his/her knowledge of factors of a number less than or equal to 60.

Change the widths of columns B to AE to size 2.5

→ Justify the alignment in cells A3 and A4 (see activity 4)

→ In A1 enter the heading ‘FACTORS OF THE NUMBER’

→ In G1 enter 24 (combined heading is ‘FACTORS OF THE NUMBER 24’

→ In A3 enter the heading ‘Natural numbers’

→ In A4 enter the heading ‘Is it a factor? Enter y if it is.’

→ Enter the numbers 1 to 30 in the row of cells B3 to AE3

→ In B5 enter =IF(B4="","",IF(AND(B4="y",INT($G$1/B3)=$G$1/B3),"ok","no!"))

copy and paste to cells AE5

EXCEL responds in the way specified in the formulae in the 5th row: “ok” if the number selected is a factor and “no!” if it isn’t. Of course, the number whose factors are to be identified can be changed – just alter cell G1 to show a positive number less than or equal to 60.

7. Automatic pooling of data from a statistics project.

As a pupil enters data into this type formatted worksheet the data is pooled into prescribed classes. Here the data is a positive integer less than or equal to 40 and is pooled into the closed intervals:

1-10, 11-20, 21-30, 31-40. The frequency and cumulative frequency is automatically computed.

Change the widths of columns A to G to size 10

→ Justify the alignment in cells B3 to G3

→ In A1 enter the heading ‘Data pooled into intervals 1-10, 11-20, 21-30, 31-40

→ Enter these headings: In A3 – ‘Data’, in B3 – ‘Interval number’, in D3 – ‘Interval’,

in F3 – ‘Frequency’, in G3 – ‘cumulative frequency’

→ Enter 1,11, 21, 31 in the column of cells D4 to D7

→ Enter 10, 20, 30, 40 in the column of cells E4 to E7

→ In B4 enter =IF(A4="","", IF(INT(A4/10)=A4/10, INT(A4/10), INT(A4/10)+1))

copy+paste to cell B50, say.

→ In F4 enter =IF(A$4="","",COUNTIF(B$4:B$50,E4/10)); copy and paste to cell F7

→ In G4 enter =IF(A$4="","",SUM(F$4:F4)). Copy+paste to cell G7.

As the data is entered into the column headed ‘Data’, it is automatically pooled and the frequency distribution computed. Subsequently, as shown in the EXCEL guide, the mean and standard deviation can be calculated and the cumulative frequency graph be constructed.

Optional Exercise A. How could you amend this calculation to deal with non-integer data. Try exploring the function wizard (Insert→function). A rather laborious formula to use in B4 is

=IF(MOD(ROUND(A4,0),10)>0, INT(ROUND(A4,0)/10)+1,INT(ROUND(A4,0)/10))

See whether you can discover a simpler one.

Optional Exercises.i) How could you amend this calculation to deal with other types of intervals?

ii) Design a worksheet which automatically gives quartiles from pooled data.

FURTHER SPREADSHEET ACTIVITIES ON MOTIVATING GENERALIZATION

1) Odd and square numbers: what is the connection?

Type ‘Odd Numbers’ in cell A1 (alter the width of column A1 if necessary to make text fit.

→ Enter the number 1 in cell A2

→ Enter the formula = A2+2 in cell A3. Copy and paste this formula in cells A4 to A26. You'll get the odd numbers from 1 to 49

→ Type 'Successive sums of odd numbers' in cell B1 (alter the width of column A1 if necessary to make text fit.

→ In cell B2 type the formula =sum($A2:A2). Copy and paste this formula in cells B3 to B26.

→ What can you conjecture about the successive sum of odd numbers starting from 1?

→ How can you explain this?

2) When the product of two numbers always equal it sum? And why?

Type 'A' in cell A1

→ Type 'A/(A-1)' in cell B1

→ Type 'A + A/(A-1)' in cell C1

→ Type 'A x A/(A-1)' in cell D1

→ In cell A2 enter the formula = int(100*rand()) + 1. This will generate a non-zero random natural number between 1 and 101.

→ In cell B2 type the formula =A2/(A2-1).

→ In cell C2 type the formula =A2 + B2.

→ In cell D2 type the formula =A2 * B2.

→ What do you notice about the sum (A2+B2) and the product (A2 *B2)?

→ Press F9 to change the value of A. Is your conjecture confirmed? Press F9 repeatedly to be sure

→ Think of a reason why this conjecture should be true.

3) Moving graphs 1.

Enter the number -10 in cell A3

→ Enter the formula = A3+0.5 in cell A4. Copy and paste this formula in cells A5 to A43. You'll get the numbers from -10 to 10 in steps of 0.5

→ In cell B2 type y = x^2

→ In cell B3 type the formula =A3^2. Copy and paste this formula in cells B4 to B43. You'll get the square of the numbers from -10 to 10 in steps of 0.5

→ In cell C1 type the number 2

→ In cell C2 type y = (x+C1)^2

→ In cell C3 type the formula =(A3+$C$1)^2. Copy and paste this formula in cells C4 to C43. You'll get the (numbers from -10 to 10 in steps of 0.5 **plus** C1) squared.

→ Highlight the cell array B3 to B43; C3 to C43 and draw the graphs that relate to the data in these 2 columns. You will get the graph of y = x^2 and y = (x+C1)^2. The latter graph is series 2.

→ What do you notice about the relative locations of the 2 graphs?

→ Change the value in cell C1 to -2. What do you notice?

→ Experiment with other values in C1 – make these multiples of 2 in the first instance.

→ What can you conjecture about the relative location of the graphs of y = x^2 and y = (x+C1)^2?

4) Moving graphs 2.

Enter the number -10 in cell A3

→ Enter the formula = A3+0.5 in cell A4. Copy and paste this formula in cells A5 to A43. You'll get the numbers from -10 to 10 in steps of 0.5

→ In cell B2 type y = x^2

→ In cell B3 type the formula =A3^2. Copy and paste this formula in cells B4 to B43. You'll get the square of the numbers from -10 to 10 in steps of 0.5

→ In cell C1 type the number 20

→ In cell C2 type y = x^2+C1

→ In cell C3 type the formula =A3^2+$C$1. Copy and paste this formula in cells C4 to C43. You'll get the square of the numbers from -10 to 10 in steps of 0.5 **plus** C1

→ Highlight the cell array B3 to B43; C3 to C43 and draw the graphs that relate to the data in these 2 columns. You will get the graph of y = x^2 and y = x^2+C1. The latter graph is series 2.

→ What do you notice about the relative locations of the 2 graphs?

→ Change the value in cell C1 to -20. What do you notice?

→ Experiment with other values in C1 – make these multiples of 10 in the first instance.

→ What can you conjecture about the relative location of the graphs of y = x^2 and y = x^2+C1?

SHARED BIRTHDAYS INVESTIGATION.

The number 24 is insignificant when compared to 365. However when you consider a certain kind of event then it is not so. The event is recording the birthdays of 24 pupils in a class.

We will simulate the birthdays of the 24 pupils using the random number function Excel. To make the simulation simple, the 365 days of the year (leap years banned!) are labelled 1 to 365: so 1 is January 1, 32 is Feb 1, etc.

As birthdays are likely to occur in any day of the year, the birthday of a pupil in a class is a random number between 1 and 365. A way of producing this number is to use the function **=INT(1 + 365*RAND()).**

The steps required to simulate the birthday problem in Excel are given in the boxes below.

<u>Simulating the birthdays of one class of size 24</u> The birthdays a class of 24 pupils can now be generated by *copying* and *pasting* the function **=INT(1+365*RAND())** in a row of 24 cells. For example, this can be done in the array of cells from A2 to X2 by entering the function **=INT(1+365*RAND())** in cell A2; a way of *copying* and *pasting* here involves *pointing the mouse at cell A2* next *simultaneously pressing the keys Ctrl and C* then *highlighting the cells A2 to X2 by dragging the mouse across these cells* and finally *simultaneously pressing Ctrl and V*.

STEP 2

Simulating the birthdays of 100 classes of size 24 Next the birthdays of 100 classes of 24 pupils can be generated by *copying and pasting* the *row* A2 to X2 onto the 100 rows which end with the row A101 to X101: *highlighting the cells A2 to X2* next *simultaneously press Ctrl and C* then *highlighting the cells A2 to X101* finally *simultaneously pressing Ctrl and V*.

STEP 3

Identifying if a class has shared birthdays The problem of identifying shared birthdays in such a vast amount of numerical data was inventively solved by the students. Their method, which is the epitome of simplicity, uses the fact that if there are shared birthdays in a class then the set of 24 numbers (birthdays) has a mode (a non-unique number which occurs with the greatest frequency). The function used is **=IF(MODE(A2:X2)>0,1,0)** which gives a value 1 when the class has at least one shared birthday and #N/A otherwise. This is *entered in cell Y2* and *copied and pasted in cells Y3 to Y101* (the method is exactly the same as described previously).

STEP 4

Determining the relative frequency of shared birthdays An estimate of the probability of shared birthdays in a class of 24 may be given by the relative frequency of this event from the simulation above. The most efficient method that the students used for doing this was by employing the function **=COUNTIF(Y2:Y101,1)/100**. This first counts the number of 1's in cells Y2 to Y101 and then divides this number by 101 thus giving the relative frequency of shared birthdays in a class of 24. The function may be *entered in cell Z1.*

DO NOT LOOK AT SUBSEQUENT PAGES UNTIL YOU HAVE FINISHED STEP 4!

A sample simulation is shown below:

FIGURE 1

= INT(1+365*RAND())   =IF(MODE(A2:X2)>0,1,0)

=COUNTIF(Y2:Y101,1)/100

BIRTHDAYS OF 100 CLASSES OF 24 STUDENTS																									0.46
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**Test**	**REL.**
59	279	276	111	249	98	33	6	61	329	341	110	187	164	4	297	4	215	194	40	305	203	61	15	1	**FREQ.**
219	219	109	110	115	240	92	297	177	150	95	256	257	336	256	82	248	296	250	296	70	242	241	163	1	
292	46	328	283	197	83	245	49	141	85	236	210	350	295	54	243	221	190	140	172	335	276	292	255	1	
103	153	285	42	139	75	57	93	48	224	337	25	76	121	352	9	301	258	184	20	235	320	257	320	1	
101	26	293	186	27	308	33	250	114	155	86	361	87	235	342	246	318	69	97	264	191	172	220	80	#N/A	
91	13	314	96	202	42	299	38	43	282	63	361	343	48	346	111	277	355	266	45	215	358	346	197	1	
69	337	237	178	34	67	304	36	348	69	79	360	263	159	27	135	194	110	322	114	49	171	190	289	1	
196	135	136	39	42	199	363	321	349	150	90	36	158	46	25	101	264	162	115	329	173	159	26	189	#N/A	
195	97	260	194	162	122	75	349	338	255	279	308	31	293	135	172	114	180	121	109	16	59	211	217	#N/A	
42	202	70	92	269	218	361	331	343	13	235	173	72	118	53	324	174	176	175	35	119	247	36	327	#N/A	
234	77	157	15	27	270	312	58	338	189	276	28	176	219	162	34	279	336	317	120	306	184	280	257	#N/A	

The surprise that greets the relative frequency of shared birthdays in a class of 24

(0.46 in the simulation shown above) is genuine and brings great joy to the mathematics teacher: it is seldom that the general body of students feel *equally* enthusiastic for a piece of mathematics as the teacher!

Students' enthusiasm increases when they are able to verify the consistency of the relative frequency by easily implemented repeat simulations:

<u>Repeated simulations of the birthdays of 100 classes of 24</u> Because the normal Excel spreadsheet is not protected - random numbers change with every additional input - all it takes to conjure up another set of birthdays of 100 classes of 24 is to *press key F9* (any key will suffice but F9 is the recommended one).

Each simulation will give a relative frequency close to 0.5. For 23 or less pupils in a class the relative frequency will be, on average, less than 0.5; it is only when the class size is 24 or more that the relative frequency will, on average, exceed 0.5. This is the reason for choosing a class size of 24 for the exploration.

The mathematics of this investigation and further spreadsheet explorations.

Having consistent verification that the relative frequency of shared birthdays in a class of 24 is about 0.5 gives students a motivation to understand 'why' this probability is so high.

The mathematics that followed the spreadsheet explorations then seems reasonable. A summary of the steps to computing the theoretical probability for this event are as follows:

i) Probability of at least 2 people's birthdays matching = 1 - Probability that all birthdays different

ii) Probability that all birthdays different = $\dfrac{\text{Number of ways that birthdays can all be different}}{\text{Total no. of possibilities for 24 birthdays}}$

no. of choices for 1st person no. of choices for 2nd person ...... no. of choices for 24th person

$$= \frac{365\times364\times363\times\ldots\ldots\ldots\ldots\times342}{365^{24}} = 0.462$$

Total no. of possibilities for 24 birthdays

iii) Hence the probability of at least 2 people's birthdays matching = 1 - 0.462 = 0.538

Further spreadsheet exploration of shared birthdays The initial spreadsheet exploration involved only in determining whether there were shared birthdays not in how many pupils had shared birthdays or in how many *different* birthdays were involved. A secondary exploration can be done with the aim of completing the following table:

No. of pupils sharing birthdays	2	3	4	4	5	5	6

No. of birthdays involved	1	1	1	2	1	2	2
Relative frequency							

The following scheme shows how this exploration may partly be done. It is by no means the only method nor the most efficient.

=FREQUENCY($A2:$X2,LARGE($A2:$X2,A$1)): this ranks the birthdays in according to numerical size; matches are identified by joint rankings

=IF(SUM($Z2:$AW2)=SUM($A$1:$X$1)+A$1,1,0)

: returns 1 if exactly two people share 1 birthday

=IF(AND(SUM($Z2:$AW2)=SUM($A$1:$X$1)+3, COUNTIF($Z2:$AW2,MODE($Z2:$AW2))=3),1,0)

: this returns 1 if exactly 3 people share 1 birthday

SECONDARY INVESTIGATION OF MATCHING BIRTHDAYS																								R. Freq.	0.38	0.01	0.14	0	0	0	0
																								Sharing							
																								N=	**2**	**3**	**4**	**4**	**5**	**5**	**6**
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	n=	**1**	**1**	**2**	**1**	**1**	**2**	**2**
24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	b'days	0	0	0	0	0	0	0
24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1		0	0	0	0	0	0	0
24	23	22	21	20	19	19	17	16	15	14	13	12	12	10	9	8	7	6	5	4	3	2	1		0	0	1	0	0	0	0
24	23	22	21	20	19	18	17	17	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1		1	0	0	0	0	0	0
24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1		0	0	0	0	0	0	0
24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1		0	0	0	0	0	0	0
24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1		0	0	0	0	0	0	0
24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1		0	0	0	0	0	0	0
24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1		0	0	0	0	0	0	0
24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	5	3	2	1		1	0	0	0	0	0	0
24	23	23	21	20	19	19	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1		0	0	1	0	0	0	0

<u>Figure 2</u>

The simulation above shows typical relative frequencies:

No. of pupils	2	3	4	4	5	5	6

sharing birthdays							
No. of birthdays involved	1	1	1	2	1	2	2
Relative frequency	0.35	0.02	0	0.08	0	0	0

These relative frequencies are interesting - they show that it is more likely that 4 people will share 1 or 2 birthdays than 3 people share one birthday. The relative frequencies obtained by the spreadsheet simulations can be found to be roughly in agreement with theoretical probabilities as shown in the following mathematical analysis:

no. of ways 2 people can share a birthday

no. of ways 22 people can have different birthdays

$$P(\text{2 people share 1 birthday}) = \frac{(365\times{}^{24}C_2)\times(364\times363\times\ldots\ldots\times343)}{365^{24}} = 0.3725$$

$$P(\text{3 people share 1 birthday}) = \frac{(365\times{}^{24}C_3)\times(364\times363\times\ldots\ldots\times344)}{365^{24}} = 0.008$$

$$P(\text{4 people share 1 birthday}) = \frac{(365\times{}^{24}C_4)\times(364\times363\times\ldots\ldots\times345)}{365^{24}} = 0.0001$$

$$P(\text{4 people share 2 birthdays}) = \frac{(365\times{}^{24}C_2)\times(364\times{}^{22}C_2)\times363\times362\times\ldots\times344)}{2\times365^{24}} = 0.1254$$

Title: 3-D constructions: Getting the view right 2010

Name: **Stanley A. Adendorff**

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Description of content of 2 hour workshop:

This investigative activity is an integrated exercise. Integration by nature occurs within mathematics, and in this case it addresses a variety of learning outcomes, such as:

- Number: The use of number to find the amount of blocks used in a construction; the number of levels related to the height of the construction.
- Functions: Patterns are developed in terms of the number of terms and blocks used to construct each term in a particular pattern (identifying number patterns, such as multiples of 3 and multiples of 2); tables (completing number tables); equations (once a formula has been established to change the subject of the formula) and graphs (using graphical representations depicting linear relationships).
- Space and Shape: Handling / rotating and investigating the properties of a variety of constructions; drawing perspective views; match perspective drawings; etc

Simultaneously, it also integrates other Learning Areas such as:

- Technology (constructing spatial figures/structures).

What will be done in the workshop?

The content relates to learning outcome 3 (Space and Shape) in the RNCS assessment standards that run from grade R to grade 8, including:

- *Building 3-D objects using concrete materials (e.g. building blocks);*
- *Observing and building 3-D objects …;*
- *Recognizing 3-D objects from different positions;*
- *Recognizing and describing objects from different positions;*
- *Describing changes in the view of an object held in different positions;*
- *Drawing and interpreting sketches of 3-D objects from different positions (perspectives).*

What will the participants be doing in the workshop?

Participants will

- do a short baseline activity
- peer / self assessment of the baseline activity
- read and engage with the modular material;
- get involved in small group discussions;
- use wooden blocks to construct 3-D figures;
- draw different perspectives or viewpoints, such as front, back, left, right or aerial views and ground plan;
- engage with analytical activities to determine the number of blocks used, number of levels, etc for particular constructions;

How will the time slot be broken up?

- Introduction (reference to educational theory): *10 minutes*
- Baseline activity: What does the audience know about the topic? *20 minutes*
- Discussion of the relevance of workshop content and activities with respect to the NCS: From grade R to G *10 minutes*
- Read and engage with the modular material; making 3-D constructions, examine and analyse constructions on paper, draw different perspectives or viewpoints, such as front, back, left, right or aerial views and ground plan; engage in small group discussions *60 minutes*
- Related discussions *20 minutes*

Motivation for running workshop?

Why is this workshop important?

Many learners reach matric with obvious problems visualizing, interpreting and representing views(points) / perspectives of 3-dimensional figures. Often this can be ascribed to under-developed visual concepts and skills. The concept *visual skill* relates to the learner's ability to accurately interpret diagrams and sketches of simple 3-dimensional objects or constructions, such as recognizing objects or figures that have been turned around, inverted or viewed from a different direction, also imagining or visualizing what something looks like after some transformation has been performed on it.

Thus, intimately linked to learners' ability to visualize, is their ability to draw or construct accurately what they think they see or observe.

Dealing with Space and Shape appears to be almost always only a pen / pencil and paper exercise. The main purpose of this workshop is twofold:

- to emphasize a practical approach to teaching of mathematical content dealing with 3-dimensionality
- to introduce teachers to appropriate activities to enhance the learning of this particular content.

How will it help participants?

Participant teachers would be exposed to practical teaching methodologies based on constructivist and realistic mathematics approaches: working with 3-dimensional wooden construction blocks and contexts through four levels of mathematization, namely from the situational to the formal (Drijvers, 2003:54; Gravemeijer, 1994:102). Throughout the workshop teachers will thus be exposed to mathematics material that facilitates self-exploration and self-activity, thus occupying pupils constructively - in a way that enhances learning and understanding. This material is by nature quite flexible and allows mathematics teachers to arrange chunks of content in such a way so as to compile tutorials, investigations, projects, formative as well as summative tests, etc. At the end of the workshop teachers would have developed particular skills and knowledge to help teach this content effectively.

References

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Van der Walle, J.A. & Lovin, L.H. 2006. *Teaching student-centred mathematics: grades 5 – 8.* Pearson: Boston

Some Strategies to Encourage Critical Thinking in our Everyday Teaching

Marc Ancillotti: AMESA Conference 2010

Teaching Critical Thinking / Problem Solving

1) Create the framework – why changes to the new curriculum – see quotes – international trends
2) Why critical thinking?
3) New curriculum – what is different? Is there a difference?
4) So how do we teach critical thinking if it is so important?
5) What makes us different to any other school? What is the opportunity that presents itself to us as a school?
6) Choosing the correct questions is critical.
7) Getting kids to be comfortable being uncomfortable. Creating the correct intellectual (risk-taking) environment.
8) Teaching kids how to scaffold, and then getting them to apply their own scaffolding – what is scaffolding?
9) Understanding that modeling and problem solving is contextual – but we can certainly apply the skills learnt in the classroom.
10) Getting the kids to make connections beyond the text book.

Some important points to consider:

Choose questions carefully – open ended questions where process is as (more) important than the answer. Try to avoid having a 'right' answer – which often reinforces that there is a wrong answer.
Do this in context

Remember that critical thinking cannot happen unless facts have already been taught/are known.

Allow for investigation in order to develop and test hypothesis – directed and open investigation nurture thought processes and develop confidence.

WHAT MAKES A GOOD TEACHER? (From grade 11 pupils)

1. They genuinely care – they have invested in their learners.
2. They challenge us. They make us think.
3. They instill confidence – we don't feel stupid when we ask questions.
4. They love their subject.

SKILLS RELATED TO CRITICAL THINKING

Across subject areas and levels, educational research has identified several discrete skills related to an overall ability for critical thinking. These are:

- Finding analogies and other kinds of relationships between pieces of information
- Determining the relevance and validity of information that could be used for structuring and solving problems
- Finding and evaluating solutions or alternative ways of treating problems

There are several generally recognized "hallmarks" of teaching for critical thinking:

- Promoting interaction among students as they learn - Learning in a group setting often helps each member achieve more.
- **<u>Asking open-ended questions</u>** that do not assume the "one right answer" - Critical thinking is often exemplified best when the problems are inherently ill-defined and do not have a "right" answer. Open-ended questions also encourage students to think and respond creatively, without fear of giving the "wrong" answer.
- **<u>Allowing sufficient time for students to reflect</u>** on the questions asked or problems posed - Critical thinking seldom involves snap judgments; therefore, posing questions and allowing adequate time before soliciting responses helps students understand that they are expected to deliberate and to ponder, and that the immediate response is not always the best response.
- Teaching for transfer - The skills for critical thinking should "travel well." They generally will do so only if teachers **<u>provide opportunities for students to see how a newly acquired skill can apply to other situations</u>** and to the student's own experience.

Problem-solving is an excellent group activity, particularly if two or more groups work on the same task independently and then come together to compare strategies. In this way, each student has the benefit of exposure to several ways of solving the problem.

ENHANCING THE ENVIRONMENT (after Keefe & Walberg, 1992)
Critical thinking in the classroom is facilitated by a physical and intellectual environment that encourages a spirit of discovery. An effective teacher does not always have a passive, receptive class. Remember that we need to create an environment where risk taking can take place. We want our kids to feel comfortable being uncomfortable.

All educators are interested in teaching critical thinking to their students.
"We should be teaching students how to think. Instead, we are teaching them what to think." Clement and Lochhead, 1980, *Cognitive Process Instruction.*

“The first goal of education, "what to think," is so traditionally obvious that instructors and students may focus all their energies and efforts on the task of transmitting and acquiring basic knowledge. Indeed, many students find that this goal alone is so overwhelming that they have time for little else. On the other hand, the second goal of education, "how to think" or critical thinking, is often so subtle that instructors fail to recognize it and students fail to realize its absence.”

Over the last decades there has been a well-known decline in the math and science ability of students in our country compared to other countries. SIMSS & TIMMS show that we rank among the lowest in the world in math and science achievement.

Children are not born with the power to think critically, nor do they develop this ability naturally beyond survival-level thinking. Critical thinking is a learned ability that must be taught. Most individuals never learn it. Critical thinking cannot be taught reliably to students by peers or by most parents. Trained and knowledgeable instructors are necessary to impart the proper information and skills.

The easiest, least time-consuming, and the least expensive method to teach critical thinking is to simply modify one's teaching and testing methods slightly to enhance critical thinking among one's students. **Stay with your subject matter, but present this in such a way that students will be encouraged to think critically about it.** This is accomplished during class by questioning the students in ways that require that they not only understand the material, but can analyze it and apply it to new situations.

Homework: Both traditional reading homework and special written problem sets or questions can be used to enhance critical thinking. Homework presents many opportunities to encourage critical thinking.

Quantitative Exercises Mathematical exercises and quantitative word problems teach problem solving skills that can be used in everyday life. This obviously enhances critical thinking.

Term Papers: The best way to teach critical thinking is to require that students write. Writing forces students to organize their thoughts, contemplate their topic, evaluate their data in a logical fashion, and present their conclusions in a persuasive manner. **Good writing is the epitome of good critical thinking.** Getting students to write more is the best, and perhaps the easiest, way to enhance critical thinking. Writing forces students to organize their thoughts and think critically about the material.

Critical thinking is an active process, while, for most students, listening to lectures is a passive activity. The intellectual skills of critical thinking - analysis, synthesis, reflection, etc. must be learned by actually performing them. Classroom instruction, homework, term papers, and exams, therefore, should emphasize active intellectual participation by the student.

Enhancement of critical thinking can be accomplished during class by periodically stopping and asking students searching and thoughtful questions about the material you have just presented, and then wait an appropriate time for them to respond. Learn students' names as quickly as possible and ask the questions of specific students that you call upon by name. If an individual cannot answer a question, help them by simplifying the question and leading them through the thought process: ask what data are needed to answer the question, suggest how the data can be used to answer the question, and then have the student use this data in an appropriate way to come up with an answer.

It is probably wise to begin asking the factual type of question so that students will realize that they have to pay attention. However, the goal of critical thinking requires that you eventually ask questions that require students to think through a cause and effect or premise and conclusion type of argument.

Rather than condition students to value only what the instructor says, get them to think deeply about the topic and value what they think and feel. Teach so that students think their ideas matter. Ask them to make connections and recognize patterns. They will experience a responsibility for their own education and think about what they learn and read. Students will be involved with their own learning, will feel deeply about it, and learn to value and trust their own thoughts and ideas.

In class, encourage questions from students. Always respond positively to questions; never brush them off or belittle the questioner. Instead, praise the questioner (for example, say "Good question!" or "I bet a lot of you want to know that"). Questions from students mean they are thinking critically about what you are saying; encourage that thinking!

During class, bring in historical and philosophical information about math and science that enables students to understand that all scientific and mathematical knowledge was gained by someone practicing critical thinking in the past, sometimes by acts of great courage or tedious painstaking work in the face of seemingly insurmountable difficulties.

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WHAT'S MY RULE?

Michele Demetriou micheled@jamesralph.com **CASIO**

The aim of the workshop is to share ideas, methods and tips on teaching Number Patterns up to Grade 12. The workshop includes: discovering patterns (the Beauty of Mathematics!), using calculators to define and explore patterns, and investigating explicit and recursive functions. The purpose is to inspire and empower teachers to be proficient in conveying their knowledge passionately. Calculator usage will play a role in the workshop, as we aim to bring teachers up to speed with the latest technological aids in teaching mathematics. The aim being not to replace mathematics, but rather to supplement it: it's conventional mathematics, new methods.

A WHAT COMES NEXT?

1. 0; -1; 2; -3; 4…

2. 10; 10; 12; 16; 22…

3. 400; 361; 324; 289; 256; 225…

4. $3; 1; \frac{1}{3}; \frac{1}{9}; \frac{1}{27}\ldots$

5. $1; \frac{1}{2}; \frac{1}{3}; \frac{1}{4}; \frac{1}{5}\ldots$

6. $0; \frac{1}{2}; \frac{2}{3}; \frac{3}{4}; \frac{4}{5}; \frac{5}{6}\ldots$

7. 1; 1; 2; 3; 5; 8…

HOW CAN WE USE OUR CALCULATORS?

Calculator Keys: MODE 2: STAT

KEY	MENU	MODEL EQUATIONS	STAT CALCULATION TYPES
1	1-VAR		Single Variable/ data handling
2	A + Bx	A + Bx	Linear Regression
3	_ + Cx^2	A + Bx + Cx^2	Quadratic Regression
4	ln x	A + Blnx	Logarithmic Regression
5	e^x	Ae^{Bx}	e Exponential Regression
6	A.B^x	AB^x	AB Exponential Regression
7	A.x^B	Ax^B	Power Regression
8	1/x	A + $\frac{B}{x}$	Inverse Regression

Choose your pattern's function

8. 0; 1; 2; 3; 4…

a) Determine the general term.

TECHNOLOGY TIP!

1. **Determine, by first and second differences, what type of sequence we are working with.**
2. **Use the calculator table above, to pick that particular type of function.**

AC

SHIFT

SELECT	OPTIONS	EXPLANATION

5 Reg	**1 A**	Regression **co-efficient of A**
	2 B	Regression **co-efficient of B**
	3 r	Correlation co-efficient r
	4 x (x hat)	Estimated **value of x (number of term)**
	5 □ (y hat)	Estimated **value of y (value of term)**

9. 10; 10; 12; 16; 22…

a) Write down the n^{th} term.

b) What is the next term?

c) What is the value of term 14?

d) Which term has a value of 9712?

10. 400; 361; 324; 289; 256; 225…

a) Define T_n

b) Find the 7^{th} term.

c) Find the 11^{th} term.

d) Which term has a value of 1?

e) What then happens at term 21?

11. $3; 1; \frac{1}{3}; \frac{1}{9}; \frac{1}{27} \ldots$

a) Determine the general term.

b) Find the next five values in the sequence.

TECHNOLOGY TIP!

Try using MODE 3: Table, to determine all the values simultaneously!

c) Which term has a value of $\frac{1}{177147}$?

12. $1; \frac{1}{2}; \frac{1}{3}; \frac{1}{4}; \frac{1}{5} \ldots$

a) Define the n^{th} term.

13. $0; \frac{1}{2}; \frac{2}{3}; \frac{3}{4}; \frac{4}{5}; \frac{5}{6} \ldots$

a) Determine T_n

14. $1; 1; 2; 3; 5; 8\ldots$

Fibonacci Sequence!

B EXPLICIT & RECURSIVE FORMULAE

MATHS MATTERS!

When working with an iterative/ inductive/ recursive formula, one must know the previous term(s) in order to evaluate the present term.

3; 6; 9; 12… is a first order recursive formula (we only need the previous term to calculate the present term).

So $T_{n+1} = T_n + 3$ where $n \geq 1$ and $T_1 = 3$

Fibonacci Sequence is a second order recursive formula (we need to know both the previous terms to calculate the present term)

Find the explicit and recursive formulae of the following:

1. 2; 4; 6; 8; 10…

Explicit Formula: Recursive Formula:

HELPFUL HINT!

ARTHIMETIC SEQUENCES CAN BE WRITTEN AS RECURSIVE FROMULA BY:

$T_{n+1} = T_n + d$ (where d is the common difference, and is actually derived from $T_{n+1} - T_n = d$)

RECURSIVE FORMULA HAVE 3 REQUIREMENTS:

1. T_{n+1} =

TECHNOLOGY TIP!

Recursive formulae are easily checked on the calculator (Mode 1: Comp)

Eg. $T_{n+1} = T_n + 2$ $T_1 = 2$ $n \geq 1$

1. Save $T_1 = 2$ into the calculators ANS memory by pressing 2

2. 2; 4; 8; 16…

Explicit formula: Recursive formula:

HELPFUL HINT!

GEOMETRIC SEQUENCES CAN BE WRITTEN AS RECURSIVE FROMULA BY:

$T_{n+1} = rT_n$ (where r is the common ratio, and is actually derived from $\frac{T_{n+1}}{T_n} = r$)

3. 1; 4; 9; 16…

Explicit formula: Recursive formula:

HELPFUL HINT!

QUADRATIC SEQUENCES CAN BE WRITTEN AS RECURSIVE FROMULA BY:

$T_{n+1} = T_n$ + (general term of the linear

RTP that the following is also a recursive formula for the sequence 1; 4; 9; 16…

$T_{n+1} = (\sqrt{T_n} + 1)^2$ $T_1 = 1$ $n \geq 1$

4. 1; -2; 3; -4; 5…

Explicit formula: Recursive formula:

BACK TO FIBONNACI…

The explicit formula for Fibonacci Sequence is

$$T_n = \frac{\phi^n - \varphi^n}{\sqrt{5}} \qquad \text{where} \qquad \phi = \frac{1}{2}\left(1+\sqrt{5}\right)$$

$$\varphi = \frac{1}{2}\left(1-\sqrt{5}\right)$$

Let’s check on our calculators:

Mode 3: Table

$$\text{F(x)} = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^x - \left(\frac{1-\sqrt{5}}{2}\right)^x}{\sqrt{5}}$$

Start? 1

End? 30

Steps? 1

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MOVES 4 MATHS – MATHS 4 MINORS

Michele Demetriou micheled@jamesralph.com **CASIO**

The aim of the workshop is to show teachers how to introduce calculator skills appropriately at primary school level. The intention is not so that the learners become dependent on the calculator for valuable "mental maths", but rather that they become acquainted with the technology before the big leap to high school. The purpose is to inspire and empower teachers to themselves be proficient in calculator usage, but also to lead teachers to be creative in facilitating the learners to make their own discoveries in mathematics, by using the calculator. The aim being not to replace mathematics, but rather to supplement it: it's conventional mathematics, new methods. Our aim is to empower teachers, through knowledge in mathematics and skills in technology.

ACTIVITY 1: SKIP COUNTING

- Counting in threes

3 + = = = = =

Ask learners to predict next number in the sequence, before checking on the calculator

Once sequence is developed, have class read it out loud to help in the sequence's memorization.

- Programme your calculator to count in 10's from 6732 to 7312.

- What's my rule? Allow learners to work in pairs. One player programmes the calculator and passes the calculator to the other player. The second player must determine the first player's rule by pressing = on the calculator until the pattern becomes obvious.

ACTIVITY 2: THE FACTOR GAME

- 2 players, one calculator

Player one enters a whole number, player two predicts a factor, then tests the factor on the calculator. Correct factors get one point, incorrect factors lose one point.

Player	Prediction	Enter	Screen	Comment
1		328	328	Find a factor
2	2	÷ 2	164	1 point
1	2	÷ 2	82	1 point
2	4	÷ 4	20,5	Oops. You try.
1	2	82 ÷ 2	41	2 points

- When is the game over?

- Variation: find two factors, and multiply to check on the calculator.
  E.g. 24 =

  1 x 24, 2 x 12, 3 x 8, etc.

  Points: 2 for 1 pair, 4 for pairs, 8 for 3 pairs

ACTIVITY 3: SPACE INVADERS

- Enter any number into the calculator. The digits in the number are space invaders and must be shot into a zero.

E.g.

Player	Press	Screen	Comment
1	7453	7453	Shoot the 4
2	- 400	7053	Shoot the 7
1	-7 000	53	

The player who misses, loses.

- Now try to shoot the digits into zero, by adding.

E.g.

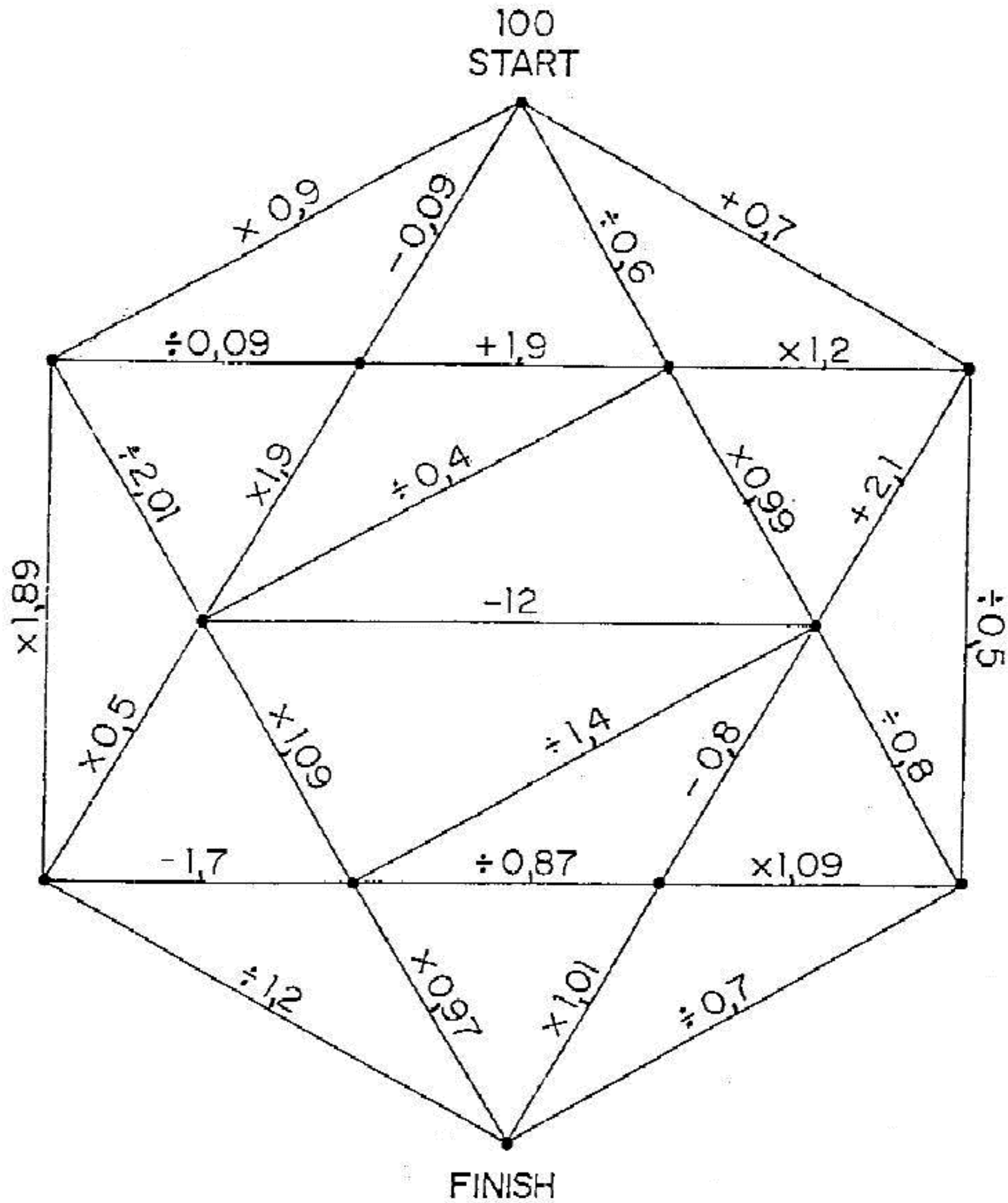
Player	Press	Screen	Comment
1	7453	7453	Shoot the 4
2	+ 600	8053	Shoot the 5
1	+ 50	8103	Now there's a 1?

- Now try using decimals

E.g. 67,143

ACTIVITY 4: MAZE

- Follow the maze starting at 100. Try to get the highest total by the time you reach "finish". You may not use the same route twice.

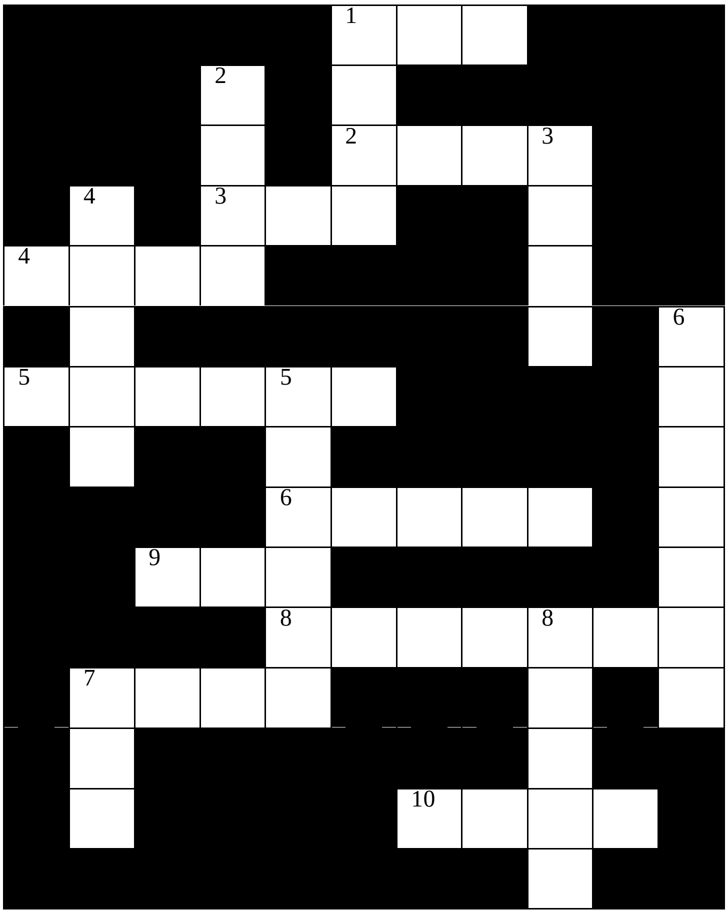


ACTIVITY 5: CROSS NUMBERS

Solve the calculations, then turn your calculator up side down and fill the word you see on your screen in on the crossword.

Across:

1. $6^2 + 700$
2. 3 x 1879
3. 257 + 257 + 257
4. 2 x 53 x 73
5. $5^3 \times 3691$
6. $2^3 \times 7 \times 631$
7. $\sqrt{9272025}$
8. 23 x (8038 + 156381)
9. 3 x 13 x 17
10. 4700 – (5 x 17)



Down:

1. $\frac{23148}{3}$
2. $\sqrt{50481025}$
3. $3^2 \times 5 + 7290$
4. $\frac{3867}{5000}$
5. (2 + 2) x 2 x 179 x 263
6. 21515232 ÷ 4
7. (30 – 7) x (37 – 2)
8. 6 x 6053

ACTIVITY 6: MAGIC 4

- Write down a number
- Add one less than your number
- Add nine
- Divide this sum by 2
- Subtract your original number

The answer is always 4!

There are more of these, look out for them!

ACTIVITY 7: WHAT'S MY RULE?

Explore patterns:

- 1 + 2 + 1 = 4
  1 + 2 + 3 + 2 + 1 = 9
  1 + 2 + 3 + 4 + 3+ 2 + 1 = 16
  1 + 2 + 3 + 4 + 5 + 4 + 3+ 2 + 1 = 25

- (9 x 9) + 7 = 88
  (98 x 9) + 6 = 888
  (987 x 9) + 5 = 8 888
  (9 876 x 9) + 4 = 88 888…
  (9 876 543 x 9) + 1 = 88 888 888

- 33 x 37 = 1 221
  333 x 37 = 12 321
  3 333 x 37= 123 321
  33 333 x 37= 1 233 321…
  333 333 x 37 = 12 333 321

- (1 x 8) + 1 = 9
  (12 x 8) + 2 = 98
  (123 x 8) + 3 = 987
  (1 234 x 8) + 4 = 9 876
  (12 345 x 8) + 5 = 98 765…
  (123 456 78 x 8) + 8 = 98 765 432

- 11 x 11 = 121
  111 x 111 = 12 321
  1 111 x 1 111 = 1 234 321
  11 111 x 11 111 = 123 454 321…

- Multiply a two digit number by 11:

11 x 35 = 385

11 x 27 = 297

11 x 24 = 264

Does the pattern hold for all two digit numbers? When is the pattern broken?

ACTIVITY 8: BROKEN KEYS

- Suppose one of the keys on the calculator key pad is broken and nothing happens when you press it.

E.g. If the 5 key is broken, we can calculate questions such as

115 + 47 = 116 – 1 + 47 or 112 + 3 +47 = 162

Try the following, without the 4 key:

1. 145 + 126 =
2. 14 x 3 =
3. 162 – 124 =
4. 167 + 94 =
5. 44 x 104 =

Now try without using the 2 or 7 keys:

6. 107 + 121 =
7. 978 – 234 =
8. 78 x 2 =
9. 96 -27 =
10. 27 x 3 =

- A variation of the game: pretend that the ÷ key is broken. Learners would have to make good use of their estimation skills.

E.g. 16 x ___ = 208

Try 16 x 12 = 192 (too small)

Or 16 x 15 = 240 (too big)

Try 16 x 13 = 208 - Yes! We were looking for 13!

11. 8 x ___ = 112
12. 72 x ___ = 360
13. ___ x 11 = 715
14. ___ x 14 = 2590
15. ___ x 82 = 1886

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ALGEBRA TILES: RESOURCES FOR TEACHER DEVELOPMENT

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At the previous AMESA conference, Miranda (2009) delivered a presentation on a research study which investigated how teachers may develop ways of helping learners realize the need to make adequate meanings of algebra. One of the highlights of the presentation was the use of manipulative resources, algebra tiles, in explaining the several algorithms used in the manipulation of polynomials. The audience showed a particular interest in the algebra tiles and suggested that at the next AMESA conference, a workshop on algebra tiles be given to teachers, subject advisors, educators and researchers who may be interested in school algebra. This workshop attends to the need, to use manipulative resources as a way to help learners become attentive observers and make adequate meanings of mathematics (Miranda, 2007; Namukasa & Kaahwa, 2007). Algebra tiles are rectangular geometric shapes which can be made from paper or hard plastic materials. They are useful in the teaching and learning of algebra when used to model integers and variables. In particular, due to limited time, this workshop only explores tiles for 1, x, and x^2. Workshop participants are given kits of algebra tiles and guided to use them in representing, expanding, factorizing algebraic expression and completing a square.

WORKSHOP SYNOPSIS

Participants in this workshop will be led through a series of algebra-based activities involving the use of algebra tiles. We will mainly explore how these tiles can be used to represent, expand, and factorize linear and quadratic expressions as well as to complete a square. The purpose of this workshop is to share with mathematics teachers, and other interested parties, another way of helping learners develop an understanding of the meanings behind some of the algorithms we use in the teaching of algebra, for example, FOIL, and the steps involved in completing the square.

Time Frame for the Workshop

This workshop will be about two hours long. Questions and points of discussion are welcome during and after the workshop. The last 10 minutes will be reserved for reflection on classroom implications of the workshop content and other possibilities. Number of participants and Structure: Because of limited resources, participants will be requested to work in groups—three to five people per group.

Skill Level Requirement of Participants:

This workshop would be useful to (junior and senior) secondary mathematics in-service and pre-service teachers, mathematics advisors, mathematics educators, and other researchers interested in school algebra.

Tasks to be covered:

Representing, expanding, factorizing algebraic expressions and completing the square using algebra tiles.

INTRODUCTION: WHAT ARE ALGEBRA TILES?

Algebra tiles are rectangular manipulatives used in the teaching and learning of algebra to model variables and integers. Other names used for these manipulatives are such as algetiles, polynomial tiles, math tiles or virtual tiles. They provide a useful way of introducing polynomials in any grade level of the school mathematics curriculum. These tiles can be made out of soft, poster, cardboard paper or hard plastic material of different colors. The number of sets in each kit depends on what variables are represented in the kits. There is no fixed way of naming the tiles but still there is need for the class to adopt names by which they can refer each piece to when they are talking about them and using them in algebra. For example, the different pieces can be used to model: $1, x, y, z, x^2, y^2, z^2, xy, xz, yz$, however, this workshop will concentrate on one variable.

The first time I introduced algebra tiles to the Namibian classroom was in 2007 when I was carrying out a research study for my Doctoral research. The study (Miranda, 2009) involved a group of Namibian mathematics teachers in in-service learning activities exploring different ways of helping learners develop a deeper understanding of algebra. Based on the evaluation of the project, there is still need for more school based studies in which teachers are engaged in inquiring into their own learning and their learners' learning of mathematics. This workshop is an extension of what has inspired in the mentioned teacher development project and the role concrete materials like algebra tiles can play.

Why Use Algebra Tiles?

There seems to be reluctance in African classrooms to use manipulatives for multiple representations in the teaching and learning of mathematics (Namukasa & Kaahwa, 2007). Yet, it is widely accepted that using multiple representations allows learners to understand mathematical concepts from different perspectives (see e.g., Duval, 2006; NCTM, 2000; Arcavi, 1999; and Duval, 1999). Many African mathematics curricula do not explicitly point to the need to include manipulatives or other forms of representation in the instruction of mathematics. Concrete materials such as algebra tiles will help learners see what is not readily accessible by vision alone and make possible more meaningful operations on abstract symbols

Learners experience difficulties when they add, subtract, multiply or divide polynomials. Algebra tiles help them visualize algebraic concepts such as expressions, equations, factorizing, expanding, and completing the square. Algebra tiles can also help learners understand the concepts and actions of 'collecting like terms' that many learners struggle with when solving equations.

Distributive Property

Several authors have described algebra as generalized arithmetic (see, e.g. Kieran, 1992; Mason, 1996; Stacey & MacGregor, 2000). Therefore it might be useful to introduce algebra to the learners by linking it to their understanding of concepts they have previously learned in arithmetic. One thing that is common to algebra and the manipulation of polynomials is the distributive property. Let us for example imagine multiplying 5 by 16. One of the algorithms that readily come to the learner's mind is to think of this multiplication as $5 \times (10 + 6)$. This can also be represented by the use of base ten blocks as follows in **Figure 1**.

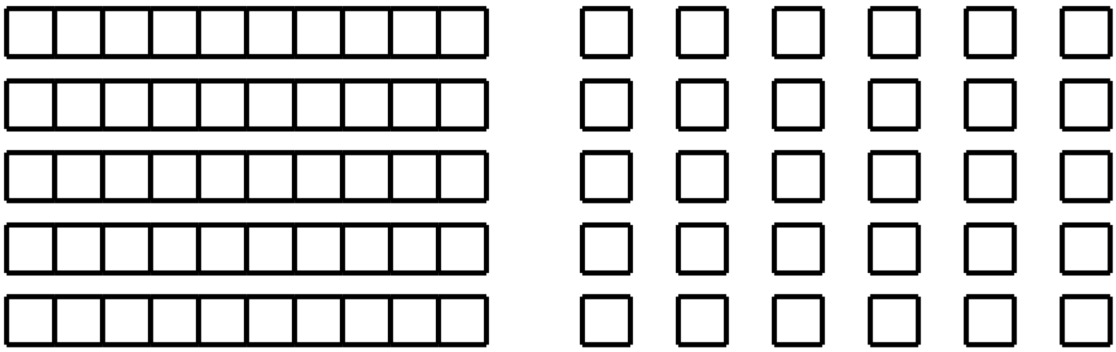

Figure 1: Representing the multiplication of 5 by 16

As can be seen from the representation, we have modeled five groups of ten and five groups of six. This gives us (5 x 10) + (5 x 6) = 50 + 30 = 80. We can also try to represent this as an area of a rectangle measuring 5 by 16 in **Figure 2**. In this diagram, there are five rows of ten (long rectangles) and five rows of six (small squares).

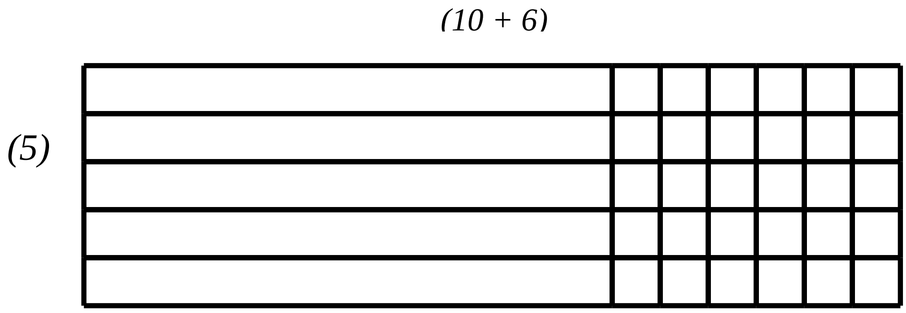

Figure 2: Representing the Multiplication of 5 by 36 as an Area

We can now move on to generalize and use this to represent $5(x+6)$. In this case we are taking the dimensions of the long rectangle to be 1 by x units as can be seen in **Figure 3**. The area of each small square is *1* by *1*.

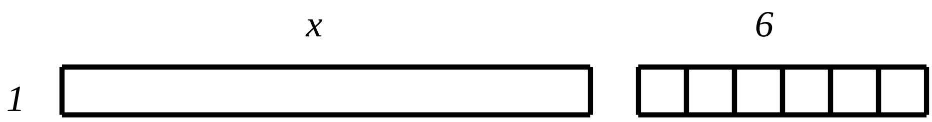

Figure 3: Using the Long Rectangle to Represent the Area of x

This method can be extended and applied to the multiplication of an algebraic term and a binomial, e.g. $x(x+2)$, see **Figure 4** below. That is, we have a rectangle whose length is x and width $x+2$. As can be seen from the diagram, the area of the big square is x^2, the area of each long rectangle is x therefore the area of the whole rectangle is x^2+2x.

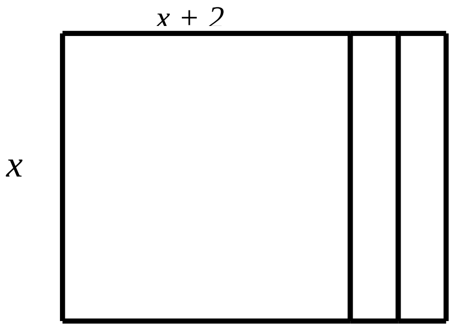

Figure 4: Representing the Area x(x+2)

We can even apply this method to multiplication operations involving negative entities, e.g., 2x (x-3), to represent the area of a rectangle of dimensions $2x$ and $x-3$. As can be seen in **Figure 5,** the area of each big square is x^2, and the area of each long rectangle is $-x$. Hence the total area of the whole rectangle is $2x^2-6x$. The regions with negative values are indicated with broken lines and those with positive values are indicated with solid lines.

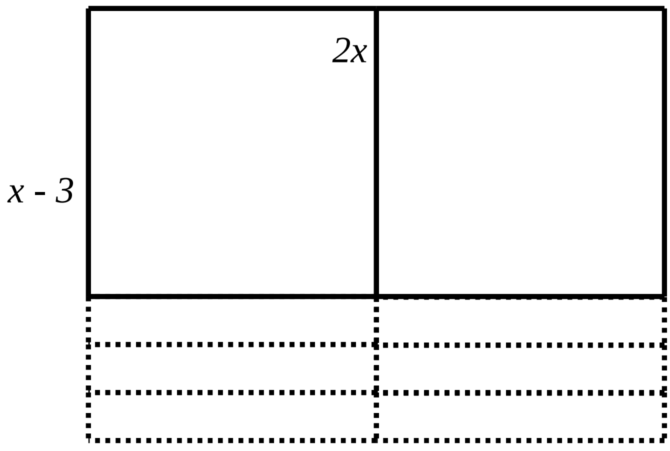

Figure 5: Representing the Area 2x(x-3)

Try a few more examples below:

1. $5\ (x - 4)$
2. $2x(3x + 6)$
3. $5x\ (2x - 3)$
4. $3x\ (2x + 5)$

EXPANDING BINOMIALS

When the learners are comfortable with using the tiles and drawing their actions on the tiles, one can then move on to apply the methods above to the multiplication of binomials. Take for example the multiplication of two binomials $(x+1)(x+2)$. To expand this will be the same as determining the area of a rectangle measuring $(x+1)$ by $(x+2)$, see **Figure 6**. By counting the total pieces making up the whole rectangle, one can see that we have one big square (x^2), three long rectangles ($3x$) and two small squares (2), Hence the total area of the rectangle is x^2+3x+2.

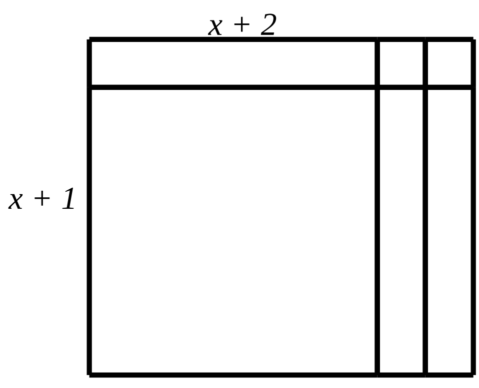

Figure 6: Expanding (x+1) (x+2)

More examples to try:

1. *(x + 2) (x + 4)*
2. *(x + 3) (x - 4)*
3. *(2x + 4) (x + 3)*
4. *(3x - 5)(x - 2)*

FACTORIZING

Just as we have used the tiles to determine the total area of a rectangle, we can also use them to reverse this, in determining the dimensions of a given rectangle with a known total area. But let's first look at this example, **Figures 7** and **8.** If we have a rectangle made up of a total of 30 square units, we can re-arrange it in many different ways; two of them are shown in **Figure 7** as *10 x 3* and in **Figure 8** as (*6x5*).

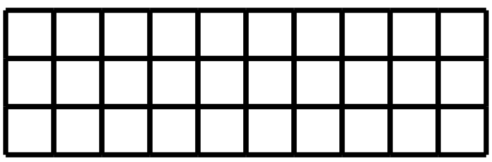

Figure 8: Representing 30 as 10 x 3

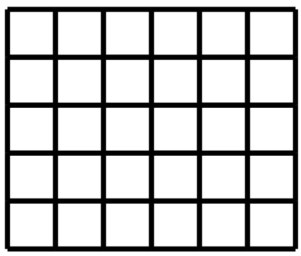

Figure 7: Representing 30 as (6 x 5)

We can now apply this to determining the dimensions of rectangles involving variables. Let is consider a rectangle whose area is $2x + 4$. if we arrange all the pieces together and form a rectangle, its dimensions are going to be ($x + 2$) by (2). Hence $2x + 4 = 2(x + 2)$.

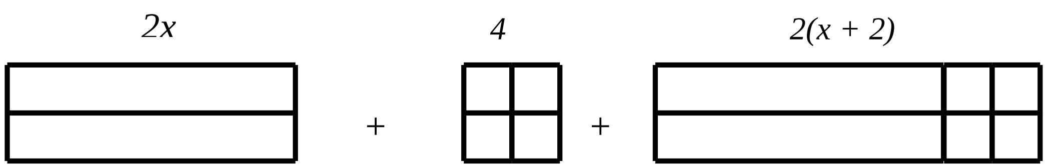

Figure 9: Representing $2x + 4$

Let us now look at an example involving negative values, for example, $(5x - 10)$. Representing this (see **Figure 10**), we get a rectangle which is 5 units long and ($x - 2$) units wide. Learners must always be reminded to pay attention to the signs of each term so that they obtain appropriate results. In this case, each long rectangle measures ($+x$) by ($+1$) and the small squares measure (-1) by ($+1$).

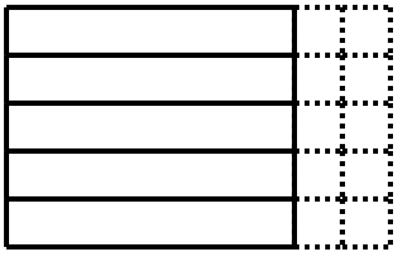

Figure 10: Representing $(5x - 10)$

At this stage learners can now move onto factorizing quadratic expressions, for example $x^2 + 7x + 10$. Again, we can start by working out how many pieces we need for each value and then try to regroup them into a rectangle whose dimensions we need to determine. This is represented in **Figure 11**. One can always check to see if expanding gives back the original expression by counting the total pieces making up the rectangle.

Let us now look at the case where the middle term is negative, for example $x^2 – 5x + 6$. These are represented and re-arranged in **Figure 12** below, giving us a rectangle measuring $(x - 3)$ by $(x - 2)$. Hence the factors of $x^2 – 5x + 6$ are $(x - 3)$ and $(x - 2)$. It must be noticed here that even we have arranged the long rectangles of $(-5x)$ along the length of the bigger rectangle, this does not affect the dimension of the length of the resulting rectangle.

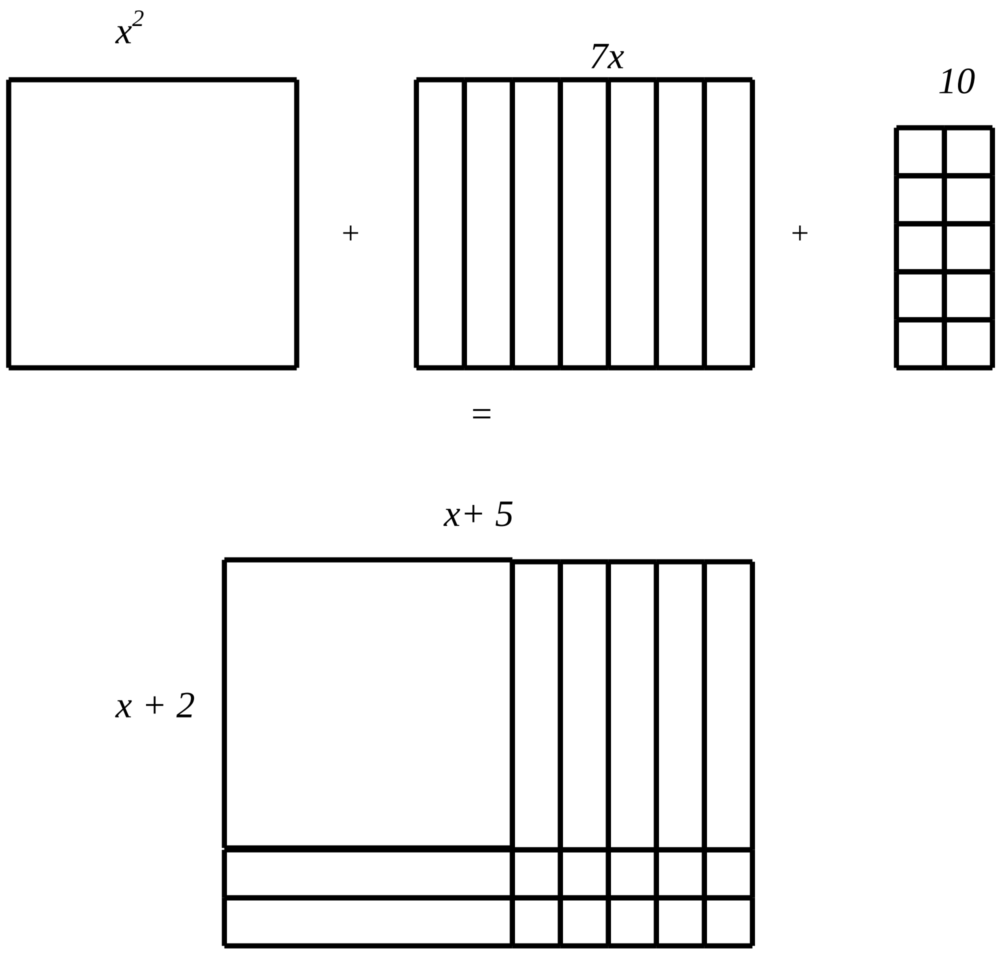

Figure 11: Representing Factoring of $x^2 + 7x + 10$

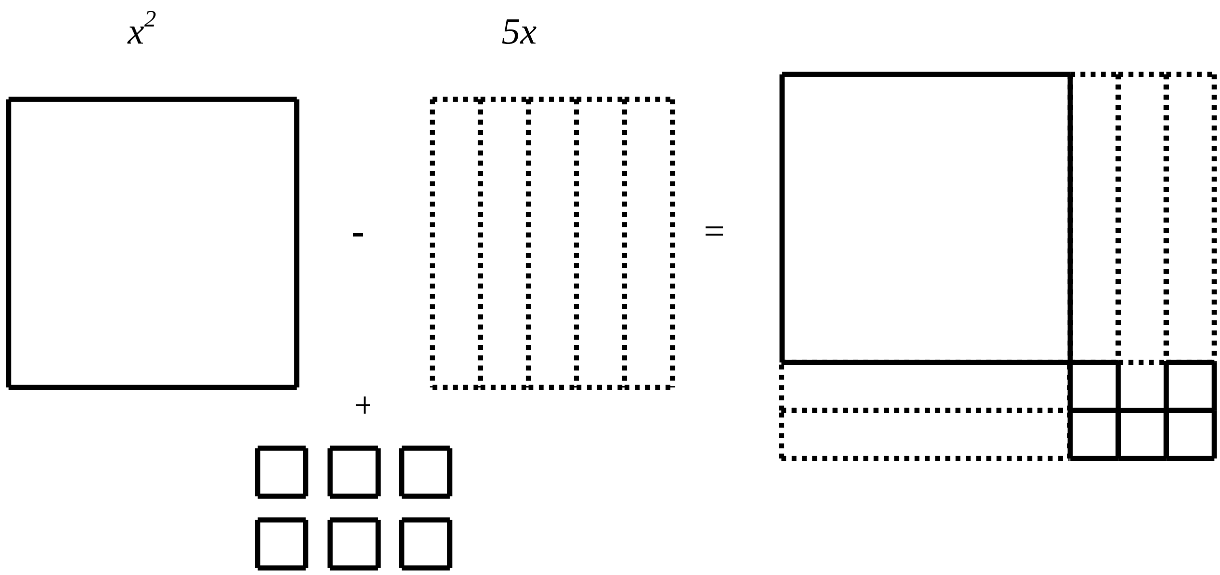

Figure 12: Factoring $x^2 – 5x + 6$

Factorize the following examples:

1. $3x + 12$
2. $x^2 – 5x$
3. $2x^2 + 10x$
4. $3x^2 – 9x$

COMPLETING THE SQUARE

One of the things that the use of algebra tiles enable in a mathematics classroom is the explanation of the algorithms that we use when manipulating algebraic expressions and solving equations. Many learners fail to understand the logic behind things such as FOIL for expanding, and the steps involved in factorizing. Let's look at a simple example ($x^2 + 5x$) in which a learner may be asked to factorize by completing the square. With the use of algebra tiles, the notion of completing a square becomes very clear here because we are trying to use the available pieces to form a rectangle which is a perfect square. As can be seen in **Figure 13**, the piece for x^2 is already a square and now we must arrange the long rectangles ($5x$) making sure that both sides (length and width) get an equal share. Since dividing $5x$ *into* two equal parts gives us $2x$ *and* $\frac{1}{2}x$, that is how much each side must get.

The next step is to determine how much area we still need to fill in order to complete our rectangle into a perfect square. These extra pieces are indicated with dashed lines. As can be seen in **Figure 14,** if we work out the total area of the extra pieces we added to complete our square we get $6\frac{1}{4}$. The convention that is commonly used in Namibian mathematics classroom when completing the square is to halve the coefficient of x and square the result. If we apply this algorithm to this case we get $\left(\frac{5}{2}\right)^2 = \frac{25}{4} = 6\frac{1}{4}$.

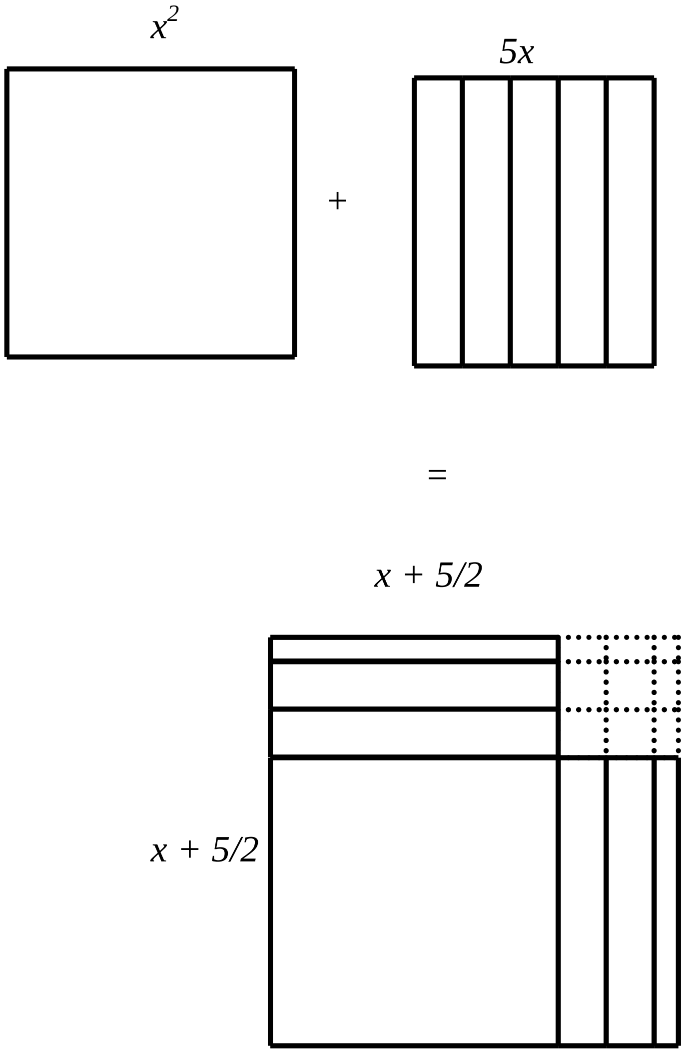

Figure 13: Completing the Square to Factor $x^2 + 5x$

Factorize the following by completing the square:

1. $x^2 - 8x$

2. $x^2 + 3x + 1$

3. $x^2 + 10x$

4. $x^2 + 6x$

FURTHER EXPLORATIONS WITH THE TILES

Algebraic tiles can also be used to factor quadratic trinomials and to solve both linear and quadratic equations. Try solving the following equations and see how the tiles work for you. By this time you should be able to manipulate tiles by drawing and not only using the concrete tiles.

1. $2x - 3 = x + 2$
2. $x + 3 = 11 - 3x$
3. $4(x - 2) = 2(x + 10)$
4. $x^2 + 5x + 6 = 0$

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GENERATE GRAPHS AND GENERALISE THE EFFECTS OF A, P AND Q USING EXCEL 2007

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Capricorn District
motubatse@absamail.co.za

INTRODUCTION

Excel is almost available in most schools as it come packaged with MS Office. This is an excellent and immensely powerful tool when it comes to the onscreen generation of graphs. For the purpose of this paper some of the graphs listed in NCS will be generated and the effects of a, p and q changes be investigated

Step 1: Investigate $y=x^2$ (The basic parabola or base model). They suggest that if you understand this parabola then you are well on your way to understand all parabolas.

Step 2: Investigate $y=ax^2$ by means of slider control. Here we ask the question . What happens if you multiply x^2 by a constant?".

Step 3: Investigate $y=a(x+p)^2$ by means of slider control
The key question
"What happens if you add a constant p
\
Step 4: Investigate $y=a(x+p)^2+q$ by means of slider control
The key question
"What happens if you add a constant q

Whatever tool is generally available (MS Office, Open Office, etc) it is possible for the busy teacher to create accurate and useful diagrams, and beautiful mathematical expressions - (using Equation Editor, Excel or equivalent).

Delegates will be shown how to use and access developer tools in Excel 2007 and draw graphs and how to approach LO 2 across the FET Band

10.2.2, 11.2.2 and 12.2.2 Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures about the effect of the parameters *k*, *p*, *a* and *q* for functions including: $y=a(x+p)^2+q$
Among other things they will be shown the process by showing them the following

- Accessing developer tools in Excel 2007
- Changing values in a cell using the scroll bar[form control]
- Drawing Scatter Chart/X_Y Chart
- Protecting sheet from accidental editing
- Interchanging Domain and Range
- Using the names manager
- Changing values of a, p, q in a scatter graph to see their effect.
-

Reference :

http://academic.sun.ac.za/mathed/

NCS Mathematics Subject Statement

Radmaste material[Mastec : Jackie Scheiber]

www.**marilynvossavant**.com/articles/**gameshow**.html

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http://www.mjyoung.net/misc/doors.htm

http://en.wikipedia.org/wiki/Monty_Hall_problem

PROBABILITY[CONDITIONAL] A GAME OF CHANCE DEMONSTRATED BY THE MONTY HALL PARADOX

Diale Motubatse

Bahananwa Capricorn

[motubatse@absamail.co.za]

The Monty Hall problem is a probability puzzle based on the American television game show Let's Make a Deal. The name comes from the show's host, Monty Hall. The problem is also called the Monty Hall paradox, as the result appears absurd but is demonstrably true. Monty Hall is a game in which apparent common sense is misleading.

WHAT IS PROBABILITY? A CHALLENGE TO MAXIMIZE WINNINGS?

..

- **Which of the following is an outcome?**
  - Rolling a pair of dice, Landing on red., Choosing 2 marbles from a jar.
- **Which of the following is an Experiment ?**
  - Tossing a coin. Rolling a single 6-sided die. Choosing a marble from a jar.

The Problem

Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a car, behind the others, goats. The car and the goats were placed randomly behind the doors before the show. You are allowed to open one door, and will win whatever is behind the door you open. You pick a door, say number 1, and the host, who knows what's behind the doors, opens another door, say number 3, which has a goat. He says to you, "Do you want to pick door number 2?" Is it to your advantage to switch your choice of doors?

The rules of the game show are as follows:

- After you have chosen a door, the door remains closed for the time being.
- The game show host, Monty Hall, who knows what is behind the doors, now has to open one of the two remaining doors,
- And the door he opens must have a goat behind it.
- If both remaining doors have goats behind them, he chooses one randomly.

- After Monty Hall opens a door with a goat, he will ask you to decide whether you want to stay with your first choice or to switch to the last remaining door.

Activity

- The participants will be divided into groups and different activities will be done based on the above grounding scenario “The Monty Hall Problem”. The results of the activities will be added to get an overall conclusion to show whether it is advisable to switch or to stay. One member of the group will act as the Show Host “Monty” and one as the “Contestant” and the others as “Audience” who will be recording the results.

Probability Trial 1

Three inverted cups/glasses/containers 1, 2, and 3, one of which hides a valuable object are randomly mixed while the Contestant looks away. A Contestant chooses one of the three cups/glasses/containers at random (Move One). At this point, the probability of success, i.e., choosing the valuable is calculated and recorded which is …………………….

The Show Host who knows where the valuable is, must eliminate one of the empty, unchosen cups, leaving only two cups on the table (Move Two).

The Contestant does not switch cups through the process and the experiment is repeated 50 times and results recorded on the given worksheet. (Move Three), The experiment is repeated and the Contestant always switches cups and results recorded on the given worksheet and done 50 times. (Move Four).

Ratio is now calculated : Stay : Switch =……... : ……….

Probability Trial 2

Three labeled boxes 1, 2, and 3 are supplied. While the contestant looks away, the host randomly hides a coin/valuable in the box and next, the contestant randomly points to a box which contains a coin. At this point, the probability of success, i.e., choosing the coin is calculated and recorded which is …………………….

Then the host purposely opens up a losing box from the two unchosen. Lastly, the contestant "stays" and opens up his original box to see if it contains the coin. The process is repeated 50 times. Play again this time the contestant always " switching" fifty times and keep track of how often the contestant wins

Ratio is now calculated : Stay : Switch =……... : ……….

Probability Trial 3

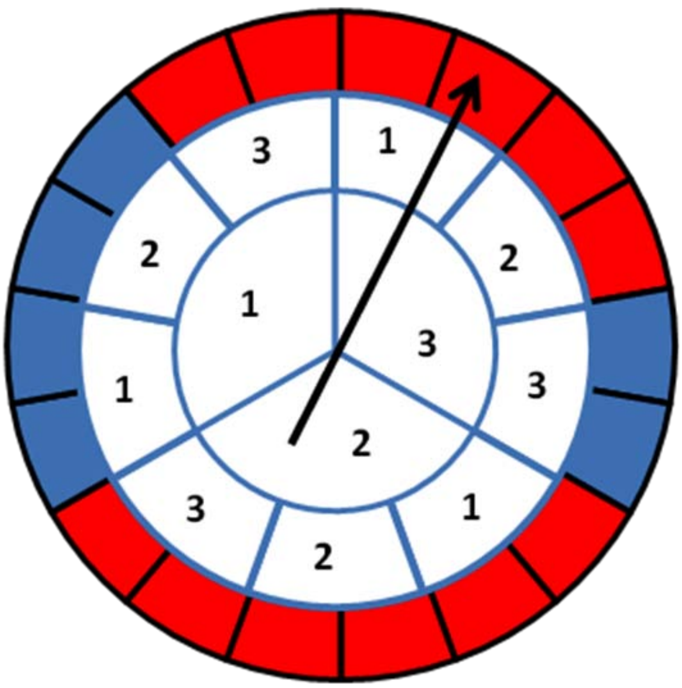
Using a pack of cards, two royalties and one ace are taken out, mix up the cards randomly then lay them out face down in a row while the contestant is looking away and the host should know the ace. Allow the contestant an ace. A contestant chooses one of the cards randomly (Move One). At this point, the probability of success, i.e., choosing an ace is calculated and recorded as……………………

The Show Host who knows where the Ace is, must eliminate one of the Royalties, unchosen Royalty, leaving only two cards on the table (Move Two).

The contestant <u>does not </u>switch cards through the process and the experiment is repeated 50 times and results recorded on the given worksheet. (Move Three), The experiment is repeated but this time the contestant <u>always</u> switches cards and results recorded on the given worksheet and repeated fifty times. (Move Four).

Ratio is now calculated : Stay : Switch =……... : ……….

To analyze this problem we represent this scenario as a random variable on a roulette wheel. The roulette wheel below simulates the Let's Make a Deal game. The inner wheel represents the number of the door that the car is behind, the middle wheel represents the door that is selected by the contestant, and the outer wheel represents the door Monty Hall can show. Spinning this roulette wheel once is equivalent to playing the game once. The outer wheel also tells you what your strategy should be to win. The red means that in order to win the contestant needs to switch doors, and the blue means that the contestant should not switch.

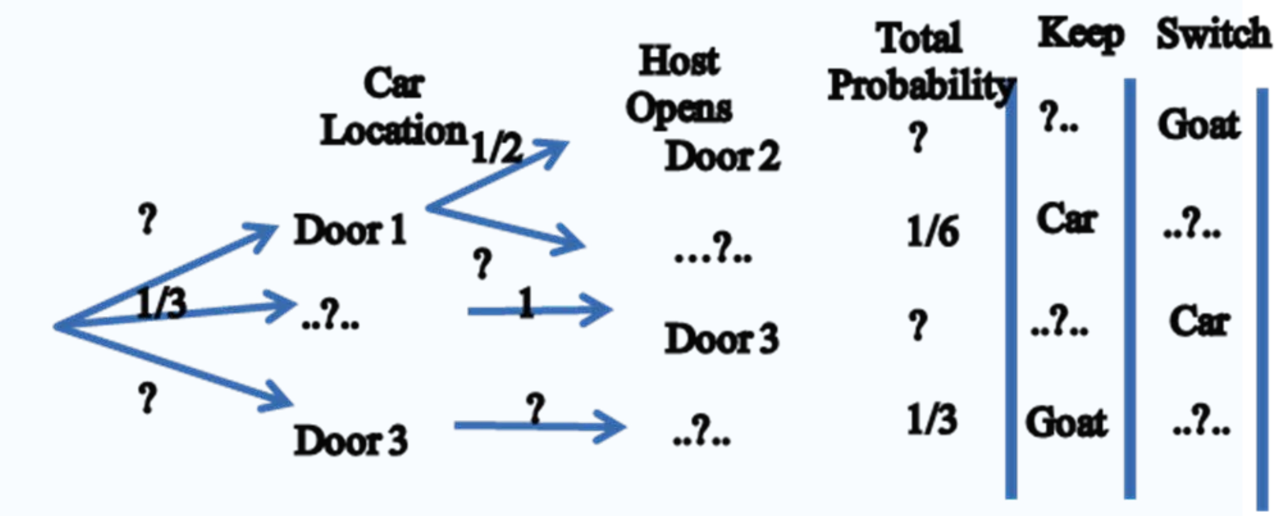


1. **Complete the Wheel[1, 2 OR 3 on the coloured sections of the wheel]**
    - Compare the red sections and the blue sections in the outer wheel
    - What does this say to you?

2. **Complete The Table**

Keep Choice[Door]	W(in) 1	1	1	2	2	2	3	3	3
Switch Prize[Location]	L(ose) 1	2	3	1	2	3	1	2	3

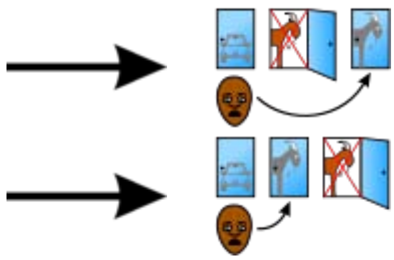
3. Complete the Tree Diagram



4. Pictures[Complete]

1.

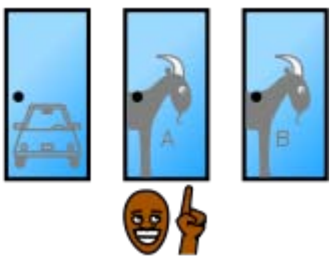
Host reveals either goat



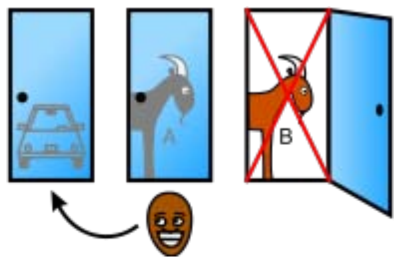
Player picks a car
(probability =……………….)

Switching Loses or Wins ?
…………………………..

2.



Host must reveal Goat B



Player picks a Goat A
(probability =……………….)

Switching Loses or Wins ?
…………………………..

3.

Player picks a Goat B
(probability =………………..)

Switching Loses or Wins ?
…………………………..

Is it always to your advantage to switch your choice of doors? Explain

Reference :

http://www.codingthewheel.com/archives/21-and-the-monty-hall-paradox

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*math.ucr.edu/~jdp/**Monty_Hall**/**Monty_Hall**.html*

Title: **MODELLING WITH SKETCHPAD**

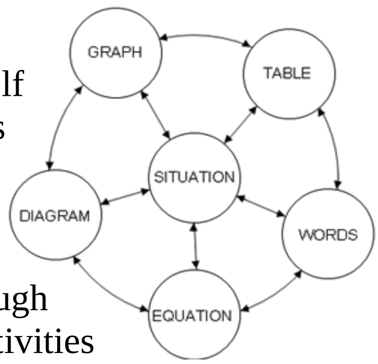
Presenter: Sue Southwood

Affiliation: VULA Mathematics Project, Hilton College

Target audience: Educators in the Senior and FET Phases who subscribe to the philosophy “Students learn when they are actively engaged.” and are interested in different ways of exposing their learners to the important concept of modelling. Familiarity with Sketchpad would be an advantage, but is not necessary.

Audience size: Thirty

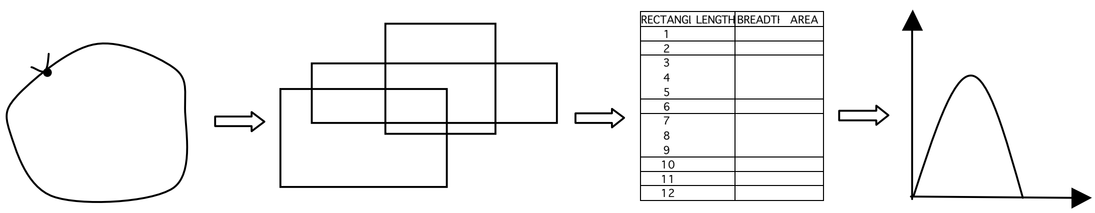
Time: Two hours



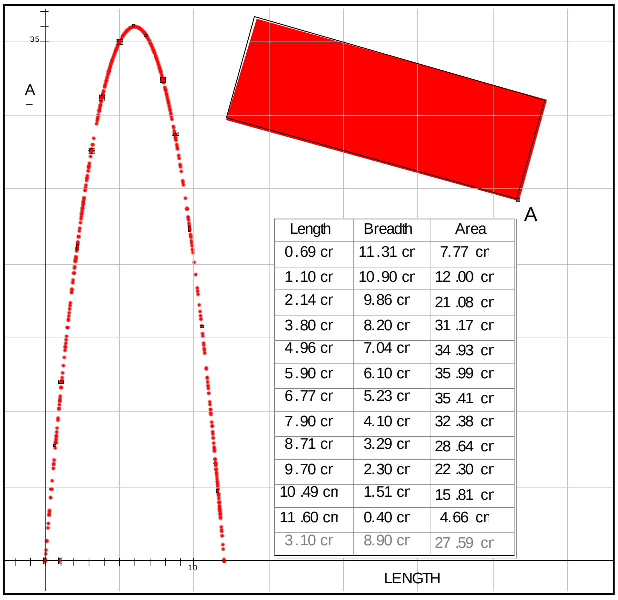
Mathematical modelling is a theme that winds itself through the curriculum statements. The three activities allow the participants to progress through the various stages of a model: from practical, hands–on exercises using string and measurement or the construction of a geometric object using Geometers Sketchpad, through calculations and tables, to graphs. The first two activities investigate the relationship between the dimensions of a rectangle and its area. The third activity investigates the relationships between the number of sides of a polygon and its area.

Part 1: 30 minutes

Rectangles from string: Using string and rulers and measurement, participants make a series of rectangles with the same perimeter, measure their dimensions, calculate their areas and enter the results in a table. They draw a graph from the entries.



Part 2: **40 minutes**

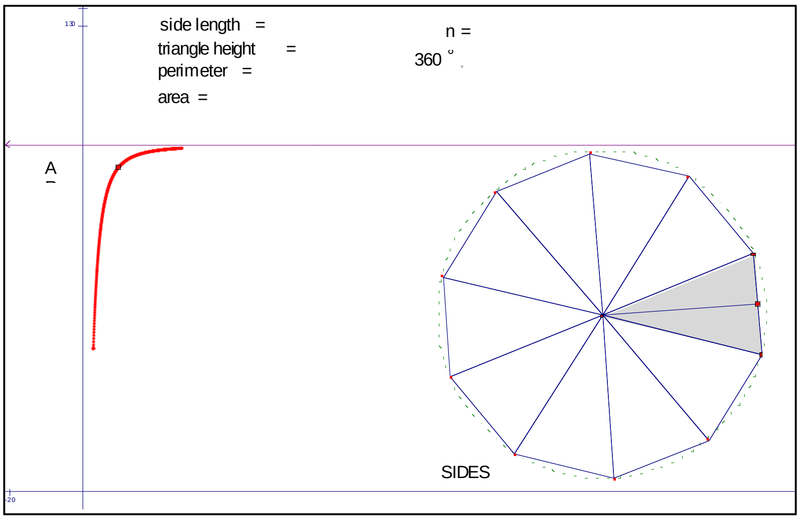


Length	Breadth	Area
0.69 cr	11.31 cr	7.77 cr
1.10 cr	10.90 cr	12.00 cr
2.14 cr	9.86 cr	21.08 cr
3.80 cr	8.20 cr	31.17 cr
4.96 cr	7.04 cr	34.93 cr
5.90 cr	6.10 cr	35.99 cr
6.77 cr	5.23 cr	35.41 cr
7.90 cr	4.10 cr	32.38 cr
8.71 cr	3.29 cr	28.64 cr
9.70 cr	2.30 cr	22.30 cr
10.49 cn	1.51 cr	15.81 cr
11.60 cn	0.40 cr	4.66 cr
3.10 cr	8.90 cr	27.59 cr

Rectangles with Sketchpad:
Participants repeat the previous activity using Geometers Sketchpad. They construct a dynamic rectangle of constant perimeter, measure its dimensions and calculate its area, create a table and enter the results in to the table. They then draw a graph from the table: area against length. Sketchpad emphasizes the connections between the different phases of the model.

Part 3 **40 minutes**

Polygons with Sketchpad:



Participants construct a regular polygon that is controlled by a parameter (*n*) which is used to change its number of sides. Using the dimensions of one of its component isosceles triangles, they calculate the area of each polygon. They then draw a graph from the results: number of sides against area. The exercise also illustrates the fact that the area of an *n*-gon tends towards the area of its circumscribed circle as *n* tends towards infinity.

Reference:

Tamblin, Michael A. **Multiple Representations Using Technology**, Mathematics Teacher September 2009, Volume 103, Issue 2, Page 146

WORKSHOP: USING GEOGEBRA (OPEN SOURCE SOFTWARE) IN THE MATHEMATICS FET CURRICULUM

Gerrit Stols

University of Pretoria

Target audience: FET mathematics teachers

Duration: 2-hour workshop

Maximum no. of participants: Depends of the number of computers in the IT lab

Motivation for the workshop: GeoGebra a free software that the teachers and learners can use to enhance understanding of graphs and transformation of graphs.

Description of content of workshop: Teachers will work through a series of activities that will help them to understand transformations of graphs while simultaneously learn them how to use GeoGebra.

ABSTRACT

Functions and transformations are topics in the mathematics curriculum which could be use as unifying themes in school mathematics. The FET (grade 10–12) curriculum also emphasize the integration and an understanding of transformations and functions. According to the National Curriculum Statement the learners have to generate as many graphs as necessary, supported by available technology, to make and test conjectures about the effect of the parameters for different function. The aim of this workshop is to use GeoGebra to do exactly this. The DoE also encourage the use of open source software in the schools. That make the use of GeoGebra even more relevant.

THE ACTIVITIES AND WORKSHEETS TO BE USED IN THE WORKSHOP

Drawing graphs

You can create and modify algebraic coordinates and equations by using the Input Bar at the bottom of the GeoGebra window.

Activity 1: Construct the following graphs

a) $3x + 2y = 6$

b) $y = 3x^2 - 4x - 6$

c) $x^2 + 3x - 2y^2 - 3y = 25$

d) $y = \frac{3}{x-2} - 3$

e) $y = 2.3^{x+2} - 1$

If you need help here are some examples:

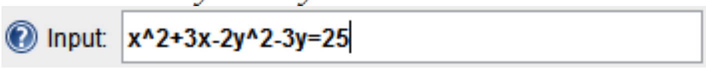
① Click on the Input Bar on the bottom of the GeoGebra window.

② Use the keyboard and the dropdown menus to type the equation:

$3x + 2y = 6$

$y = 3x^2 - 4x - 6$

$x^2 + 3x - 2y^2 - 3y = 25$



$y = \frac{3}{x-2} - 3$

$y = 2.3^{x+2} - 1$

③ Press the enter key on the keyboard after typing each equation.

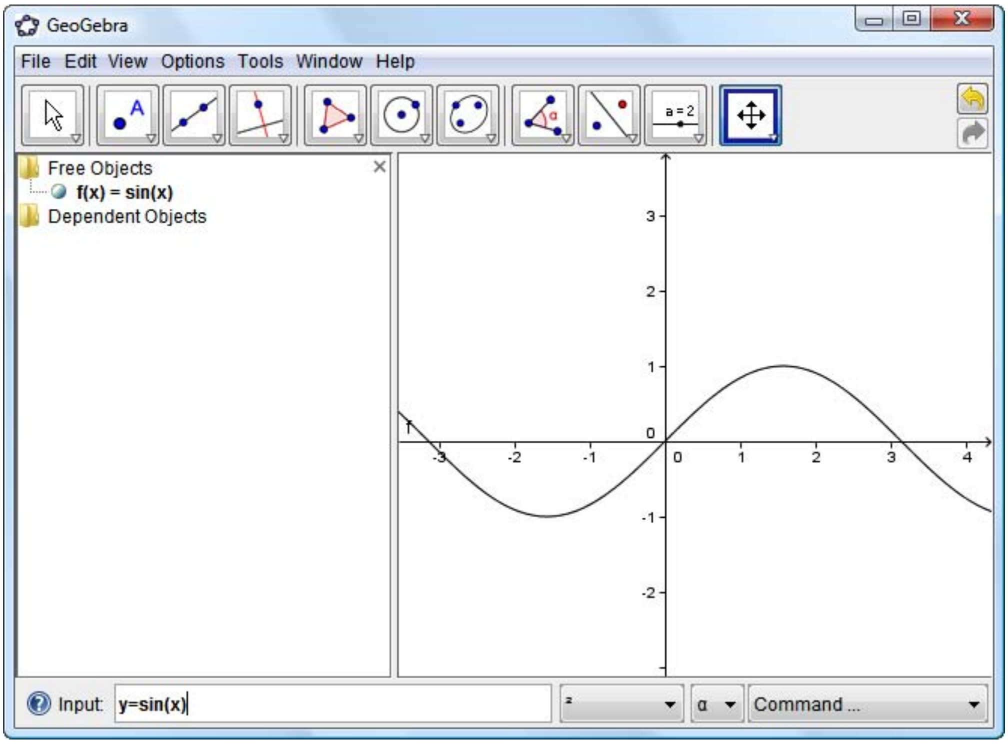
You can create and modify trigonometric equations by using the Input Bar at the bottom of the GeoGebra window. You can use radian measure or degrees. The default mode is radian measure.

Construction of a trigonometric graphs (in radian measure)

① Click on the Input Bar on the bottom of the GeoGebra window.

② Use the keyboard and the dropdown menus (next to the Input Bar) to type the equation: $y = \sin x$

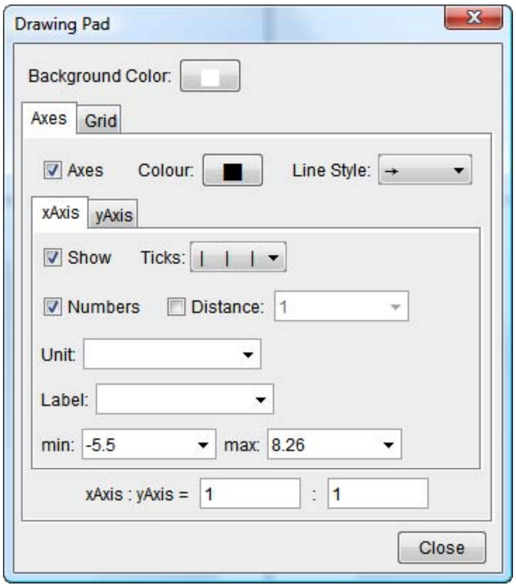
③ Press the enter key on the keyboard



Activity 2: Construct: $y = \sin x$ using degrees

① Move the cursor to the *x*-axis. Press the right button on the mouse (right click).

② The following screen will appear:



From the dropdown list select degrees:

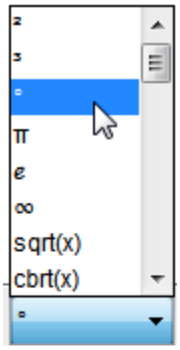
Adjust the minimum and maximum *x*-values:

Change the distance between the *x*-values:

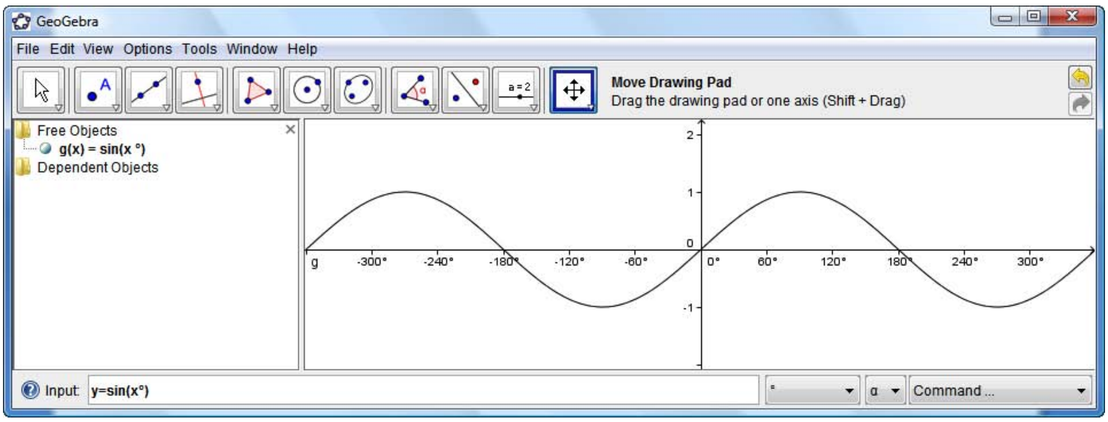
③ Close the window and click on the Input Bar on the bottom of the GeoGebra window.

④ Use the keyboard and the dropdown menus (next to the Input Bar) to type the equation:

Use the dropdown list for the degree sign:



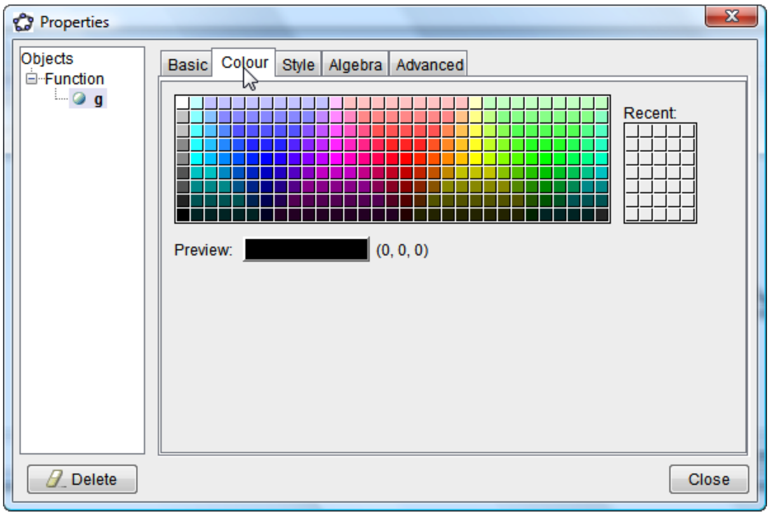
⑤ Press the enter key on the keyboard.



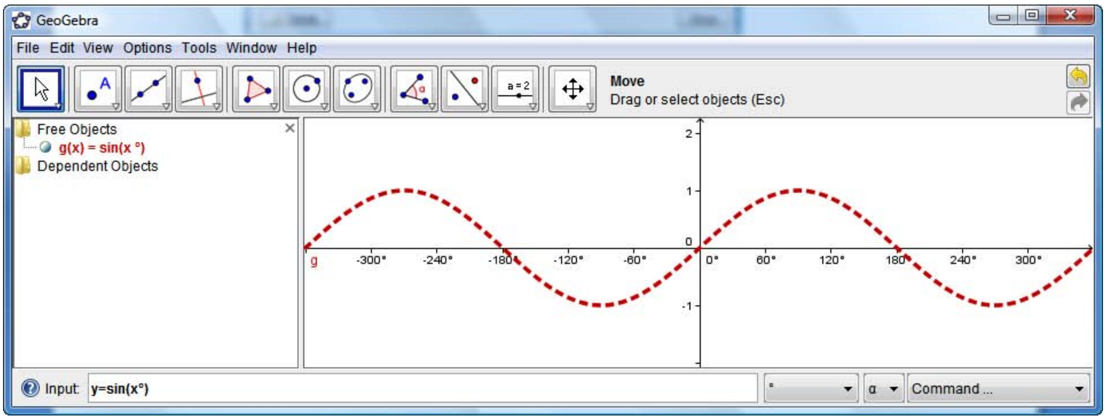
⑥ If you want to you can change the appearance of the graph:

Right click on the graph and select properties.

Click the Colour tab and select any colour.



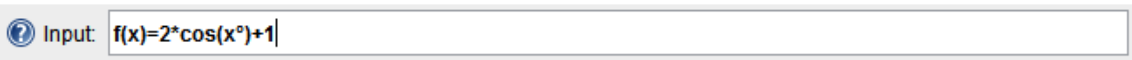
Click the style tab and select the line thickness and style.

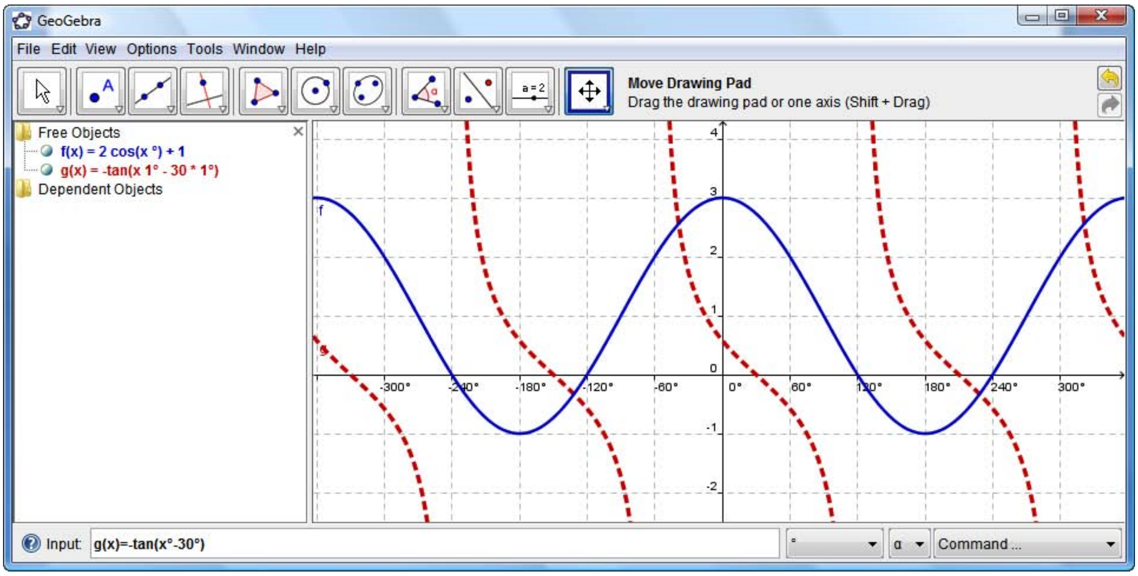


Activity 3: Construct the following graphs using degrees

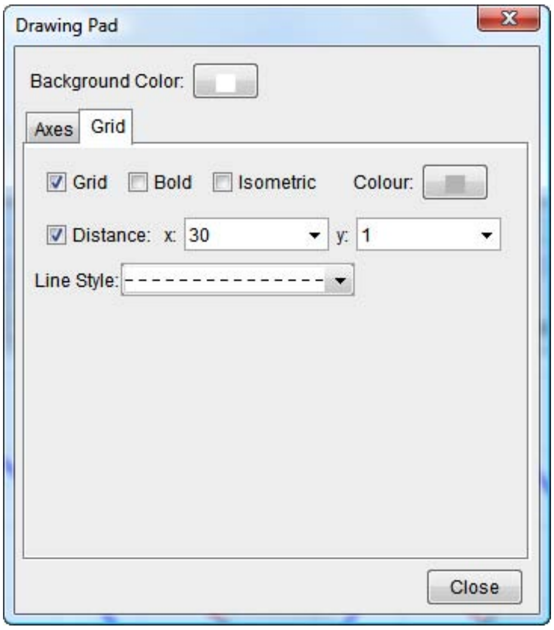
$f(x) = 2\cos x + 1$ and $g(x) = -\tan(x - 30°)$

Follow steps 1 to 4 in the previous section, but type:





To add a grid as you noticed in the background of the previous sketch right click the *x*-axis and make the following selections:

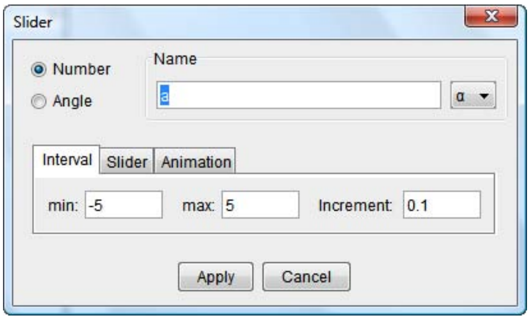


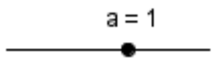
USE SLIDERS TO TRANSFORM GRAPHS

You can create and use sliders to change the coefficients of the equations of graphs.

① Select the Slider tool from the Construction Tools:

② Click where you want to locate the slider. The following window will appear:





③ Click the Apply button and a slider will appear.

④ Go to the Construction Tools and select the Arrow

Use the arrow to drag the point a on the slider. You will notice the value of point a on the slider will change.

⑤ Repeat steps 1 to 4 to create more sliders but rename them k, p and q.

Activity 4: Create sliders in GeoGebra and use it to illustrate the effect of a, k, p and q on the graphs

a) $y = a(x+p)^2 + q$

b) $y = a.2^{x+p} + q$

c) $y = \frac{a}{x+p} + q$

d) $y = a.\sin k(x+p) + q$

e) $y = a.\cos k(x+p) + q$

f) $y = a.\tan k(x+p) + q$

Describe in your own words the effect of a, k, p and q on these graphs

WORKSHOP: THE TEACHING AND LEARNING OF TRIGONOMETRY: STARTING FROM A REAL LIFE SITUATION EVOLVING INTO INDUCTIVE AND DEDUCTIVE REASONING

Sandra Van Niekerk

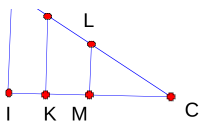
Uplands Outreach

svanniekerk@uplandscollege.org

In this workshop you will experience how to develop concepts of trigonometry using a problem-solving approach.

MOTIVATION FOR RUNNING WORKSHOP

I used to introduce concepts of trigonometry by the following example.



LC	MC
1.47 cm	1.24 cm

LM	MC	LM
0.531	0.848	0.626

JK	KC
1.19 cm	1.90 cm

JK	KC	JK
0.531	0.848	0.626

HI	IC
1.53 cm	2.45 cm

HI	IC	HI
0.531	0.848	0.626

Where angle C = 32^0

Learners had to:

1. First identify all the different right angled triangles.
2. Investigate the tables with a lot of guidance
3. Developed terminology for the sides of a triangle as they are situated with respect to the angle C
4. They then had to relate the conventional sides names to their descriptions
5. etc.

Needless to say if the lesson was not properly structured kids found it confusing. I off course new exactly what I want them to learn and eventually gave the game away and introduced the ratio names with clear description. They did the following activities.

Activity 1.

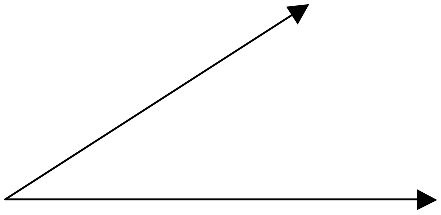
DESCRIPTION OF CONTENT OF WORKSHOP

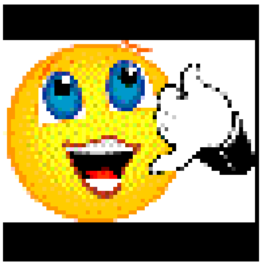
- Investigating the “chance” of tossing a coin 8 times with a head as an outcome
- Getting familiar with the excel function “RANDBETWEEN” Generate data, converting to text, counting the successful events, creating charts
- Investigating the outcomes on a dice.

Activity One

Acute Angle:

"A pretty Angel" Why?





Please write down what you know about angles.

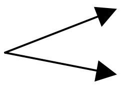
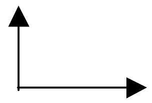
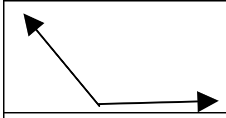
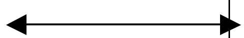
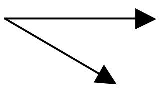
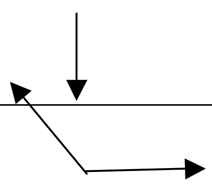
Cut out the arrows on the table

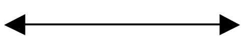
Join them with the pins as demonstrated.

Complete the following table

Mathematical name	Make a sketch(es) of the angles that have	Describe the angle in terms of

	these properties	degrees
Acute angle		
Right angle		
Angle of reflection		
Reflection angle		
revolution		

Drawing	**Mathematical Name**	**Write properties in words**	**Properties described mathematically**
	**Acute Angle**		**$0^0 <$ angle $< 180^0$**
	**Right Angle**		**Angle $= 90^0$**
	**Obtuse angle**		**$90^0 <$ angle $< 180^0$**
	**Straight angle**		**Angle $= 180^0$**
	**Angle of reflection**		**$180^0 <$ angle $< 360^0$**
	**Revolution**		**Angle $= 360^0$**

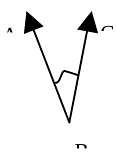
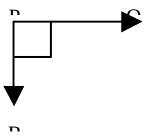


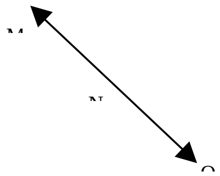
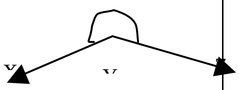
What about angles less than 0^0 ☺		**Clockwise rotations give negative angles**
What about angles bigger than 360^0 ☺		**Anti clockwise rotations give positive angles**

Write down the attributes of an angle

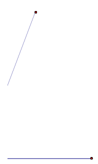
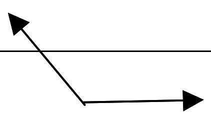
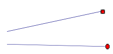
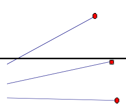
What do we use to measure the size an angle?

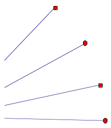
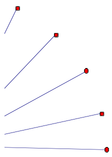
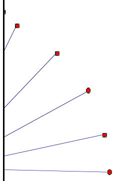
Use the protractor to measure the following angles in the first column of the following table and complete the table:

Angle	**Name of angle in symbols**	**Mathematical Name**	**Size of Angles**
			
			

An investigative Activity

Stage	Sketch	No of angles	The first difference	The second difference
1				
2 				
3				

4				
5				
6				
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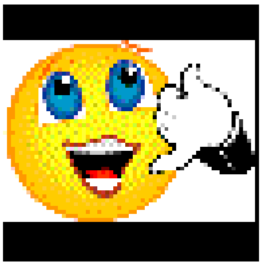
AMESA 2010

Workshop



Today we are going to solve a problem.

You are the chairperson of the body corporate for your building, *The Heights*. The committee has decided to buy a new ladder which can reach the top of the building. The building is 30 m high. So you have the money to buy the ladder, but your problem is to know *how long should the ladder be? How do you decide?*

Problem solving Polya's 4 steps

- Understand the problem
- Devising/ making a plan
- Carrying out the plan
- Looking Back

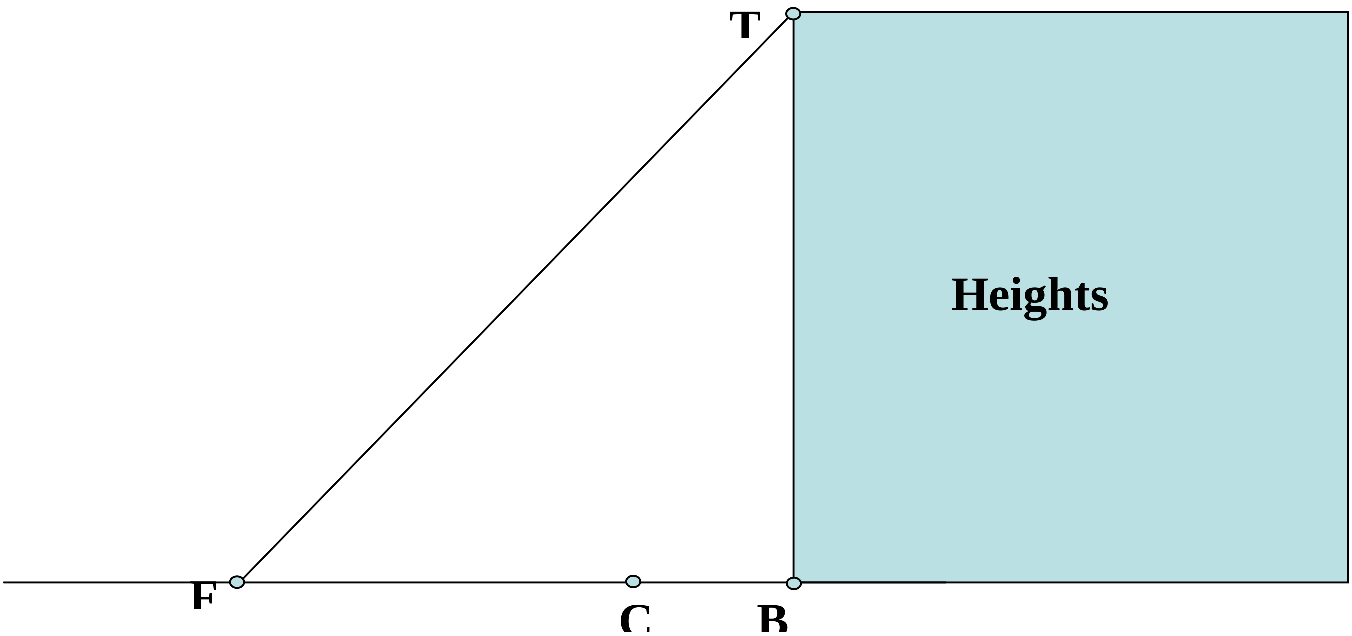
1. Understand the problem

Analysing the problem.

…. what information do we need, what information do we have, what assumptions can we make? Can we recognise mathematical content that can help us solve the problem

Let's represent the situation in a sketch: we imagine we have a ladder TF that reaches the top T of the

30 m high building TB. We also know ∠TBF = 90° (it is not the leaning tower of Pisa!).



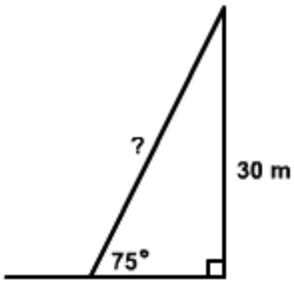
We now realise that the problem of how long the ladder must be to reach T, depends on the problem of what is the best position of F on the ground. Here real "physics", the context of the problem must be considered: If F is too close to B, the ladder may fall backwards, and if F is too far from B, F may slide along the ground and the ladder will slide along the wall and fall. So what is the best position for F? Surely, you want F as close as possible to B, to give the shortest ladder (why do you want the shortest possible ladder?). But how close is safe?

An assistant at the hardware shop advises you that angle TFB should not be bigger than 75° to prevent the ladder from falling backwards. So, of course you are going to choose the ladder that can reach the top at an angle of 75° (i.e. the shortest possible ladder)

☺ ☺☺

We now have all the information. You can now re-formulate your practical problem as a pure, abstract mathematical problem:

In a triangle with angles of 90° and 75° and a side of 30 m, how can you calculate the length of the hypotenuse?



2. Making a plan.

What knowledge do you bring to the situation?

Show how to find the length of the ladder.

I wonder… Can It be calculated? A well known Mathematicain Descartes “said if it can be constructed and measured it can be calculated”

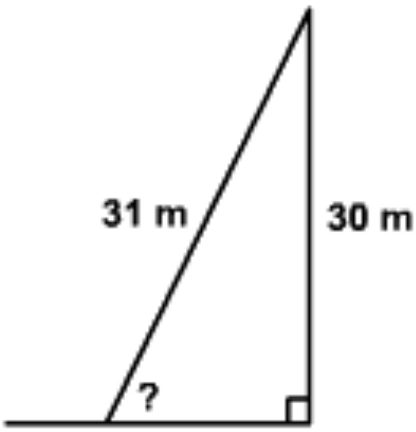
New strategy, new problem

You also learn at the hardware shop that they can make ladders in units of 1 m, i.e. you can order a ladder of 30 m, or 31 m, or 32 m. You know the 30 m ladder is too short. But will the 31 m be long enough, or should you buy the 32 m ladder? There is quite a price difference. How to decide?

This is a new mathematical problem type, which is the *inverse* of the original:

If you took the 31 m ladder, you could work backwards and check if the angle with the ground is less than 75°.

But how to calculate it? Again, Pythagoras will not help us!
Can you make a plan?



Discussion
We will find that we actually do have previous knowledge that can solve the problem: The essential structure of the situation is that *for a specific angle, different lengths of ladders are parallel to each other, and this means that all the different right-angled triangles are **similar**, so the ratio of corresponding sides are equal.*

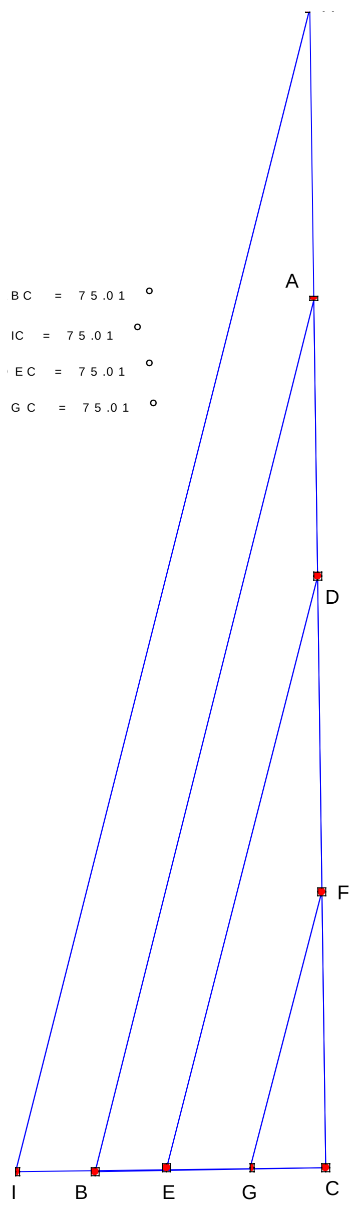
Do you agree with the following?

For a specific angle, the *ratio* of corresponding sides of similar triangles are equal.

This means that the ratio of these sides is independent of the size of the triangles.

The key to your ladder problem is that each angle has its own specific constant ratio that is different from other angles.

The fact that every angle has its own ratio value means that these ratios are *functions of the angle*.



Let's investigate:

1. How many triangles do you see here?

2. What do you know about all these triangles?

3. Complete the following table?

In triangle	Measure the length of the Side opposite	Measure the length of the Hypotenuse	Ratio
HIC	Angle I = (75^0) CH =17	IH =17.5	$\frac{CH}{IH} = \frac{17}{17.5} =$ 0.96

In triangle	Measure the length of the Side opposite	Measure the length of the Hypotenuse	Ratio
HIC	Angle H = 25^0 CI =	IH =	$\frac{CI}{HI} =$
ABC	Angle A = 25^0 BC =	AB =	$\frac{BC}{AB} =$

DEC	Angle D = 25^0 EC =	DE =	$\frac{EC}{DE} =$
FGC	Angle G = 25^0 FC =	FG =	$\frac{FC}{FG} =$
Complete the table as indicated.			
In triangle	Measure the length of the Adjacent side to	Measure the length of the Hypotenuse	Ratio
HIC	Angle I = (75^0) IC =	IH =	$\frac{IC}{IH} =$
ABC	Angle B = (75^0) BC =	AB =	$\frac{BC}{AB} =$
DEC	Angle E = (75^0) EC =	DE =	$\frac{EC}{DE} =$
FGC	Angle G= (75^0)		$\frac{GC}{FG} =$

	GC =	FG =	

What do you notice?

In triangle	Measure the length of the Side opposite	Measure the length of side adjacent to	Ratio
HIC	Angle I = (75^0) CH =	Angle I IC =	$\frac{CH}{IC}$ =
ABC	Angle B = (75^0) AC =	Angle B BC =	$\frac{AC}{BC}$ =
DEC	Angle E= (75^0) DC =	Angle E EC =	$\frac{DC}{EC}$ =
FGC	Angle G= (75^0)	Angle G	

	EC =	GC =	$\frac{EC}{GC} =$

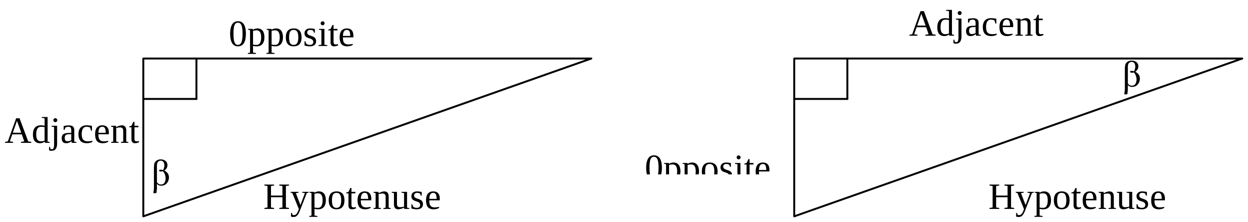
What do you notice?

Naming and extending

In order to facilitate communication about these ideas, we need to introduce a common vocabulary, It is prudent to begin with referring to the sides of the right triangle as *opposite*, *adjacent* and *hypotenuse*. This means that for our work on the ladder problem we can say that the ratio $\frac{opposite}{hypotenuse}$ for a specific angle, e.g. for 30° or 75° in a right triangle is constant for all sizes of the triangle, and these values are different for 30° and 75°.

Now we can turn our attention also to the combination of other ratios in the triangle, and investigate them in a similar way. There are 6 different ratios, because there are 6 different combinations of two sides of a triangle:

$\frac{\text{opposite}}{\text{hypotenuse}}$ and $\frac{\text{hypotenuse}}{\text{opposite}}$	sin of an angle in a right angled triangle
$\frac{\text{adjacent}}{\text{hypotenuse}}$ and $\frac{\text{hypotenuse}}{\text{adjacent}}$	
$\frac{\text{opposite}}{\text{adjacent}}$ and $\frac{\text{adjacent}}{\text{opposite}}$	cos of an angle in a right angled triangle tan of an angle in a right angled triangle



Now let us investigate

Use your calculator to complete the following table>

X	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
sin(X)										
cos(X)										
tan(X)										

What do you notice?

Problem solved

It is time to use our newly constructed knowledge to solve our original ladder problem!

By now we know that for the angle of 75°, the ratio $\frac{opposite}{hypotunuse}$ has a specific value, no matter how long the ladder –

We can obtain the specific value $\frac{opposite}{hypotunuse}(75) = 0{,}966$ from a calculator or through construction and measurement. Then:

$$\frac{\text{opposite}}{\text{hypotenuse}}(75°) = \frac{SB}{SF}(75°) = \frac{TB}{TF}(75°) = 0{,}966$$

$$\Rightarrow TF(75°) = \frac{TB}{0{,}966} = \frac{30}{0{,}966} = 31{,}06 \text{ m}$$

This means you have to buy the 32 m ladder.

Question: If you decide not to use the ladder at an angle bigger than 65°, how long a ladder should you buy?

The inverse problem

We also wanted to check if you buy a ladder of 31 m, what angle it would make with the ground. This is the inverse problem of the previous. In this case we can calculate the value of sin(β):

$$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{30}{31} = 0{,}96774$$

On a calculator, we use the inverse trigonometric function named arcsine. Usually there's a button on the calculator labeled "inv" or "arc" or "$\sin^{-1}$" that you press. In our case, if you enter $\sin^{-1}(0{,}96774)$, the calculator gives us 75,41°, which shows that this ladder was too short.

DEVELOPING ALGEBRAIC REASONING IN THE PRIMARY SCHOOL

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Target audience: Grades 3 to 8 teachers

Duration: 2 hours

Max. number of participants: 50

Description of workshop content:

Participants will
- discuss the question: "what is algebra?" (15 min)
- discuss the question: "what is algebraic reasoning?" (5 min)
- analyse the NCS (GET) to determine what the curriculum prescribes regarding algebra and algebraic reasoning (15 min)
- receive a short presentation on the above activities (20 min)
- do activities that can develop primary school children's (and participants') algebraic reasoning (45 min)
- design their own activities that can develop primary school children's algebraic reasoning (15 min)
- closure (5 min)

Motivation for running the workshop:

For many years now, mathematics educators have been concerned about the quality of learners' knowledge and understanding of elementary algebra. In the 1980's and 1990's, numerous research projects had been undertaken internationally, and a wealth of knowledge regarding learners' misconceptions in algebra had been accumulated. Based on these insights, researchers have made a number of recommendations to improve the practice of elementary algebra teaching. These not only included a change in the approach to teaching elementary algebra in the early high school years, but also essential groundwork to be done in the primary school years. Curricula implemented in a number of countries since 2000, notably Australia, New Zealand, the USA and South Africa, reflect this need for the development of algebraic reasoning in the lower grades, before learners formally encounter algebra in the high school.

This workshop hopes to assist participants to understand what algebraic reasoning is, and how it can be developed in the primary school, by analyzing the NCS (GET) for Mathematics, exposing them to typical activities, and to engage them in designing their own activities.

WORKSHEET 1

1. In groups of about 4, discuss the question: "What is algebra?"

Note: Formulate a final answer for the group, and be prepared to be called upon to share your answer with all the participants.

Total time: 15 min

2. In groups of about 4, discuss the question: "What is algebraic reasoning)?"

Note: Formulate a final answer for the group, and be prepared to be called upon to share your answer with all the participants.

Total time: 10 min

3. Analyse the NCS (GET) to determine what it states regarding:

(Total time: 15 min)

3.1 What is algebra?

3.2 What should be done in the primary school to develop learners' algebraic reasoning (focus on LO1 and LO2)

4. Short presentation on algebra, algebraic reasoning, and the “Big Ideas of Algebra” (10 min)

WORKSHEET 2

Activities to develop learners' algebraic reasoning

Work in groups

Number patterns:

1. If you start with 3 and count in fours, you get the following sequence:

   3; 7; 11; 15; 19; …

1.1 Write down the next three numbers.

1.2 Find the 20^{th} and 50^{th} numbers in the sequence.

1.3 Will 487 be a number in the sequence? If yes, what is its position in the sequence?

1.4 What "Big Idea(s)" of algebra is/are emphasized by 1.1 to 1.3?

2.1 Is it true that the sum of two even numbers is always an even number? Investigate.

2.2 Is it true that the product of an even number and an odd number is always an odd number? Investigate.

2.3 Is it true that the sum of any number of consecutive odd numbers is always a square number? Investigate.

2.4 Is it true that the sum of two consecutive numbers (e.g. 1 and 2) is a prime number?

2.5 What "Big Idea(s)" of algebra is/are emphasized by 2.1 to 2.4?

3. Study the following pattern:

Row 1		1		
Row 2	2	3	4	

Row 3		5	6	7	8	9		
Row 4	10	11	12	13	14	15	16	

3.1 How many numbers are in Row 50?

3.2 What is the first number in Row 21?

3.3 What “Big Idea(s)” of algebra is/are emphasized by 3.1 and 3.2?

WORKSHEET 3

Activities to develop learners’ algebraic reasoning

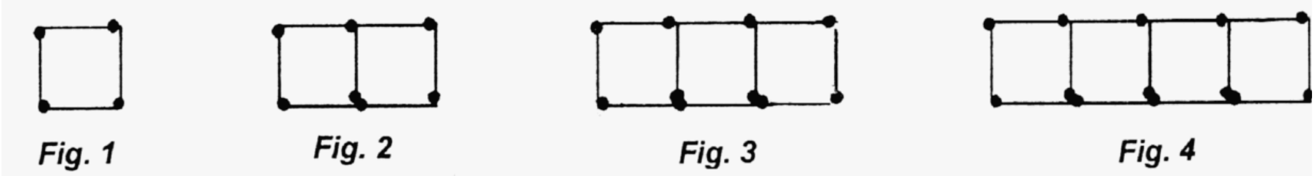
Work in groups

Geometric patterns:

Each of the following shapes is made from matches. In each case (1 to 4)

- draw the next two pictures
- describe in words how many matches are required to move from one picture to the next picture
- complete the table that follows after the shapes

1.



2.

3.

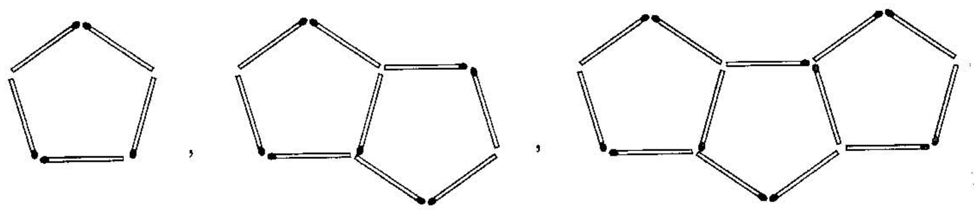


Fig. 1 *Fig. 2* *Fig. 3*

4.

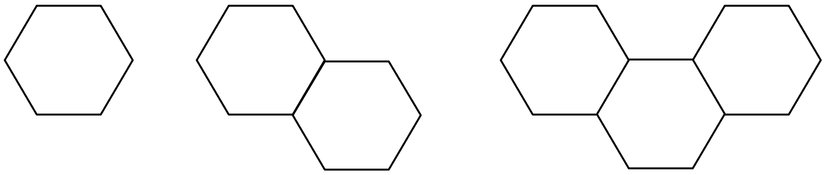


Fig. 1 *Fig. 2* *Fig. 3*

	Figure number	**1**	**2**	**3**	**4**	**8**	**100**	**rule**
1	Number of matches							
2	Number of matches							
3	Number of matches							
4	Number of matches							

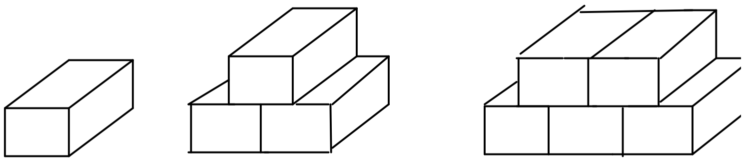
What "Big Idea(s)" of algebra is/are emphasized by these activities?

WORKSHEET 4

Activities to develop learners' algebraic reasoning

Work in groups

1. Brick wall:



Instructions for your learners:

Study the "growing" brick wall. 1 2 3

1.1 Describe in words how you continue the brick wall.

1.2 How many bricks will be necessary for

(a) Brick wall 4?
(b) Brick wall 5?
(c) Brick wall 10?
(d) Brick wall 50?

1.3 Which brick wall can be built with (a) 55 (b) 101 (c) 200 bricks?

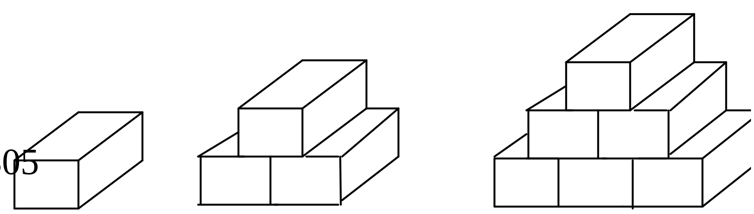
Questions for workshop participants:

1.4 In what grade(s) could this activity be given?

1.5 Can this activity be used to develop learners' algebraic thinking? If yes, how?

1.6 Which of the "Big Ideas of Algebra" are addressed here?

2. Another brick wall:



Repeat questions 1.1 to 1.6 for this brick wall.

1 2 3

WORKSHEET 5

Activities to develop learners' algebraic reasoning

Work in groups

Activity 1:

Often the opportunity to develop learners' algebraic thinking arises spontaneously in class.

A Gr 2 learner makes the following comment:

"Miss, I have discovered that 3 + 5 = 5 + 3".

1.1 How will you respond to this learner?

1.2 Is there an opportunity here to develop learners' algebraic thinking? If yes, how?

1.3 Which of the "Big Ideas of Algebra" are addressed here?

Activity 2:

Grade 3 or 4 learners are given the following problem: "Farmer Jackson plants young orange trees. He plants 18 rows, each containing 32 trees. How many orange trees did he plant?"

You observe that several learners answer the question by "breaking down" one or both of the numbers, e.g.

Learner 1: 32 = 30 + 2. 18 x 30 = 540 and 18 x 2 = 36.

Therefore 18 x 32 = 540 + 36 = 576

Learner 2: 18 = 20 – 2. 32 x 20 = 640 and 32 x 2 = 64.

Therefore 32 x 18 = 640 – 64 = 576

Learner 3: 32 = 30 + 2 and 18 = 20 – 2. 30 x 20 = 600, 30 x 2 = 60, 2 x 20 = 40 and 2 x 2 = 4. Therefore 32 x 18 = 600 – 60 + 40 – 4 = 576

2.1 Are the methods of all three learners acceptable?

2.2 Is there an opportunity here to develop learners' algebraic thinking? If yes, how?

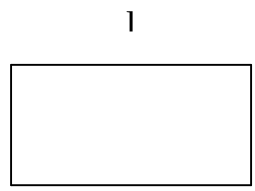
2.3 Which of the "Big Ideas of Algebra" are addressed here?

WORKSHEET 6

Activities to develop learners' algebraic reasoning

Work in groups

1. Perimeter (distance around the perimeter of a shape):



We as teachers know that the perimeter of any rectangle is given by the formula P = 2 x (length + breadth), or then 2(l + b).

1.1 Where does this formula come from?

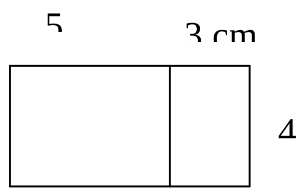
1.2 How should we teach this formula?

1.3 Can this formula be used to develop learners' algebraic thinking? If yes, how?

1.4 Which of the "Big Ideas of Algebra" can be addressed in the process?

2. Area of a rectangle:

2.1 Do you think learners can calculate the area of the rectangle shown alongside in different ways?



2.2 If yes in 2.1, show the different methods.

2.3 Is there an opportunity here to develop learners' algebraic thinking? Discuss.

2.4 Which of the "Big Ideas of Algebra" (if any) can be addressed in the process?

WORKSHEET 7

Activities to develop learners' algebraic reasoning

Work in groups

Seeing the generality through the specifics, using technology:

1.1 What would you prefer when you buy an item at a 20%-discount sale:

That the 20% discount is subtracted first from the price, and then 14% VAT is added,

Or

That the 14% VAT is first added to the price, and then the 20% discount is subtracted?

Investigate by studying specific cases by hand, or preferably generated by an Excel spreadsheet.

Your conclusion:

1.2 Will your conclusion in 1.1 only apply for 20% discount and 14% VAT (or for any %)

1.3 Is there an opportunity here to develop learners' algebraic thinking? Discuss.

1.4 Which of the "Big Ideas of Algebra" (if any) can be addressed in the process?

NOTES

1. What is algebra?

- Algebra is the language most often used for investigating, communicating and problem solving in Mathematics. Algebra can be seen as generalised arithmetic, and can be extended to the study of functions and other relationships between variables. A central part of this outcome is for the learner to achieve efficient manipulative skills in the use of algebra. (NCS (GET)).
- Algebra enables us to express and manipulate generalized numerical statements.

2. What is algebraic reasoning?

Looking for, and identifying regularities/patterns/relationships, generalizing them, formulating conjectures using a suitable symbolic system and investigating/ justifying/refuting them.

Algebraic reasoning can be developed and supported through the use of "algebrafied" tasks (in arithmetic and geometry/measurement) that help children look for regularities/patterns/ relationships.

3. What are some of the "Big Ideas" of algebra that need to be developed as part of algebraic thinking?

1. GENERALISING

Every mathematical technique or procedure, such as addition, extending a pattern, solving a linear equation, etc. is a "general method" for resolving a class of similar problems. Thus, every mathematics lesson affords opportunities for learners to generalize for themselves.

It is important that learners articulate what they do/did during the generalising tasks in order to become aware of the generality of their actions. Teachers can use "standard" arithmetic and geometric tasks to develop and support learners' awareness of generalization. In these tasks learners do not only focus on finding the (correct) answer to the problem, but become aware that the same procedure is carried out each time a similar problem is solved, e.g.

"Standard" task	Generalising the "standard" task
What is the next number in the row of numbers: 1; 3; 5; 7; ...?	What is the next three numbers in the row of numbers: 1; 3; 5; 7; ...? And the 50^{th} number? And the 100^{th} number?

Investigate whether the following is true: $3 + 4 + 5 = 3 \times 4$	Investigate whether the following are true: $6 + 7 + 8 = 3 \times 7$ $11 + 12 + 13 = 3 \times 12$ Can you make more such statements? Can you make similar statements with five numbers, e.g. $1 + 2 + 3 + 4 + 5 = 5 \times 3$? Will it work for 7 numbers? 4 numbers?
Find the sum of 24 and 42.	Find the sum of 16 and 61, 35 and 53, 27 and 72. What do you see in each case?

2. PATTERNS and RELATIONSHIPS

While developing learners' notion of relationships, the notions of variables/variability and generality are simultaneously developed.

"Standard" task	Generalising the "standard" task
What is the next number in the row of numbers: 1; 3; 5; 7; ...?	What is the next three numbers in the row of numbers: 1; 3; 5; 7; ...? And the 50^{th} number? And the 100^{th} number?
Blocks are stacked to form a wall, as shown below. How many block are needed to build a wall 5 levels high?	Blocks are stacked to form a wall, as shown. How many block are needed to build a wall 5 levels high? And 10 levels high? And 50 levels high?
Addition and multiplication tables to find individual answers	Addition and multiplication tables to find relationships between rows and columns.
Calenders are used to find dates	Calenders are used to find relationships between dates.

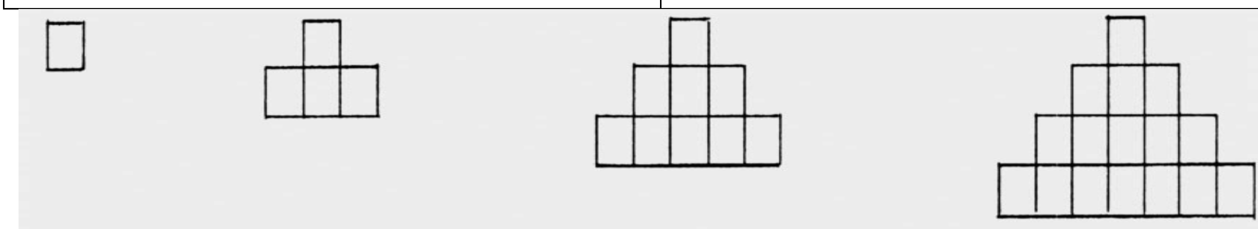


Fig. 1 *Fig. 2* *Fig. 3* *Fig. 4*

3. STRUCTURE and EQUIVALENCE

While developing learners' notion of structure in arithmetic, the notions of equivalence and generality (and often relationships) are simultaneously developed.

Here learners must become explicitly aware of the properties of operation, in particular the distributive, commutative and associative properties. This can be achieved by allowing learners to become aware of what they often intuitively do when doing arithmetic calculations, such as breaking down (decomposing) numbers and regrouping them. For example, in a relatively simple calculation such as 18 + 27, learners would often proceed as follows: 18 + 27 = 10 + 8 + 20 + 7 = 10 + 20 + 8 + 7 = 30 + 8 + 2 + 5 = 30 + 10 + 5 = 45. Implicit in this are both the commutative and associative properties. Learners must articulate what they are doing.

Concurrent to developing an increasing awareness of the properties of operation, the notion of equivalence is developed, i.e. that different (numeric) expressions are equal in value, for example:

3 + 5 = 5 + 3 (commutative property)

3 x (5 x 7) = (3 x 5) x 7 (associative property)

6 x (20 + 7) = 6 x 20 + 6 x 7 (distributive property)

4. LANGUAGE

The NCS only requires learners for the first time in grade 8 to "describe, explain and justify observed relationships or rules in own words or *in algebra*". Therefore, the symbolic language of algebra need not be introduced in the lower grades; rather, learners should express their generalizations in words. However, where learners spontaneously start replacing words or traditional symbols such as □ with algebraic symbols such as *x* or *a*, they should be allowed to do so. They should however be required to clearly articulate why they do so, and what the meaning of the *x* or *a* is, and the teacher should satisfy herself that the learner understands this "new" symbol as a placeholder for any suitable number.

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A PRACTICAL APPROACH TO TEACHING BAR GRAPHS

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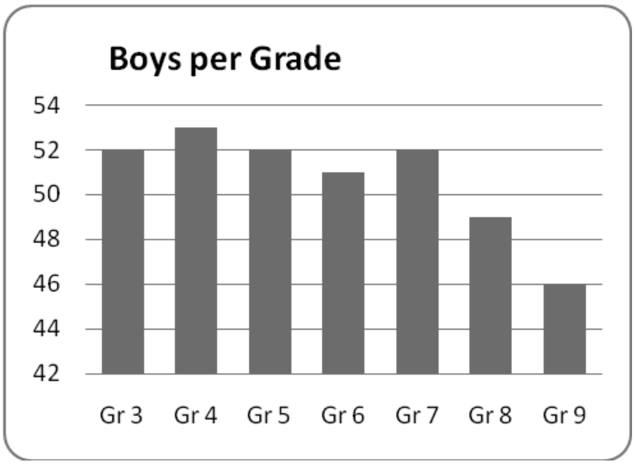
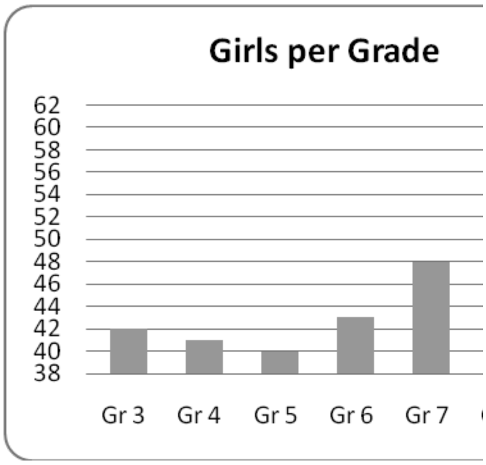
SENIOR PHASE

- **INTRODUCTION**
- This presentation is based on the Grade 7 – 9 Assessment Standards : Draw a variety of graphs by hand/technology to display and interpret data (grouped and ungrouped) including bar graphs and double bar graphs
- Interpreting statistical graphs is a very important skill in statistics as it gives a quick glance at the main features of the data set. Graphs can also be used to compare different data sets.
- **PRIOR KNOWLEDGE**
- Before doing these activities, learners must be able to complete a tally / frequency table
- **ACTIVITY 1:**
- The following set of data is given regarding the number of learners in a grade from one of the schools as collected during the Census@school survey (2009) in the Eastern Cape:

	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 9
Boys	52	53	52	51	52	49	46
Girls	42	41	40	43	48	58	60

- Use the above information and draw a bar graph for each gender (one for the boys and one for the girls) on separate bars



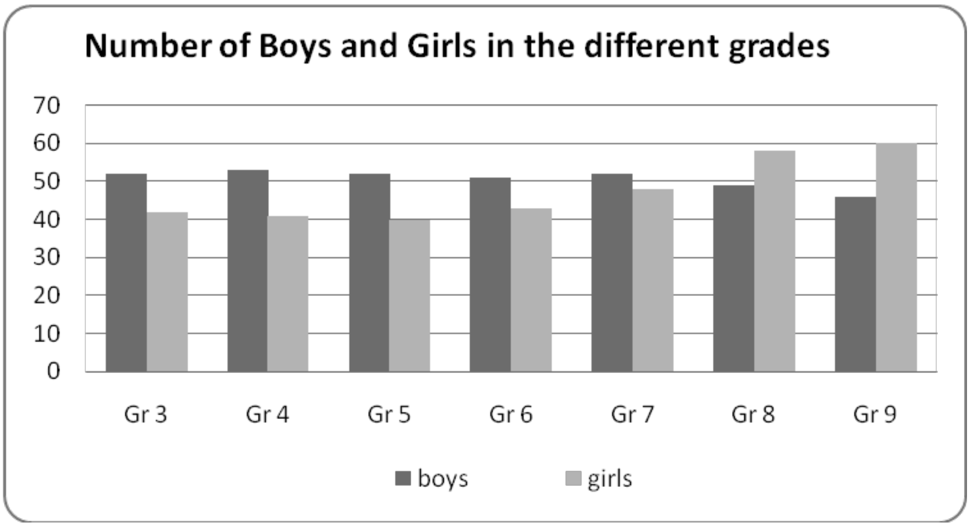


Questions that one could ask:

- 1. In which grades are there more boys than girls?
- 2. Why do you think there are less boys in grade 8 and 9 than girls?

- **ACTIVITY 2**

- Use the given information to create a compound bar graph.
- A compound bar graph can be used to compare different sets of data
- If you look at separate bar graphs it is not easy to compare them. The bar graphs can be combined into one diagram. The bars in each group are put next to each other. This makes it much easier to compare the data sets.
- Method:
- Draw the graph of the boys leaving a gap to place the relevant girls "bars" next to the boys. One can either draw the "bars" of the girls next to the boys or cut the girls "bars" out from Activity 1 and place them next to the boys

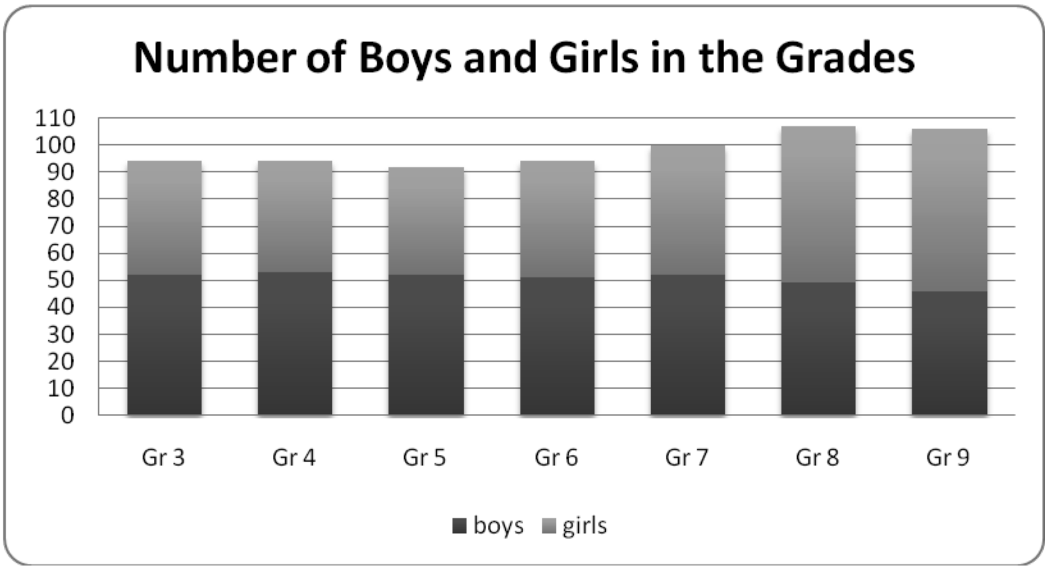


- Again questions can be set on the above data, e.g.

- 1. How many more boys than girls are there in Grade 3?
- 2. What is the difference between the number of boys and girls in Grade 9?
- 3. In general, would you say that there are more girls than boys in grade 8 and 9?

ACTIVITY 3

- Use the given information of Activity 1 to form a sectional bar graph
- We use sectional bar graphs when we have two, or more, different sets of information on the same topic. They are particular useful when we are also interested in the total of the two or more bars.
-



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- Again questions can be set on the above data, e.g.
-
- 1. In which grade were the most number of learners?
- 2. In which grade were the least number of learners?
- 3. What trends can you read from this graph?

CONCLUSION:

It is important to make sure that learners understand the bar graph before moving onto the compound and sectional bar graphs. Learners also need to be made aware of how to interpret bar graphs

- **REFERENCES:**

- Jackie Scheiber, 2009, Data Handling in the GET Band, RADMASTE Centre, University of Witwatersrand.
- Classroom Mathematics (Laridon et. Al)

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TEACHING OF THE FUNDAMENTAL COUNTING PRINCIPLE

YVONNE VAN DER WALT

Eastern Cape Maths4stats Provincial Coordinator

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FET PHASE MATHEMATICS GRADE 12

INTRODUCTION:

- There are different techniques for counting the number of ways an event can occur. The Counting Principle is one of the ways to find the number of ways an event can occur
-
- **DEFINTION:**
- The Fundamental Counting Principle is if one event occur in "M" ways and a second event can occur in "N" ways, the number of ways the two events can occur in sequence is (M x N). This rule can be extended for any events occurring in sequence
-
- **USING THE FUNDAMENTAL COUNTING PRINCIPLE**
-
- **ACTIVITY 1:**
- a) You decided to purchase a new car. How many different ways can you select one manufacturer, one car size and one color if you have the following choices ?
-
- Manufacturer: GM, Ford (F), Kia (K)
- Car size: Small (S), Medium (M)
- Color: White (W), Green (G), Blue (B),
-
- Solution: There are 3 choices of manufacturers, 2 car sizes and 3 colors.
- Applying the Fundamental Counting Principle we then have:
-
- 3 x 2 x 3 ways = 18 ways
-

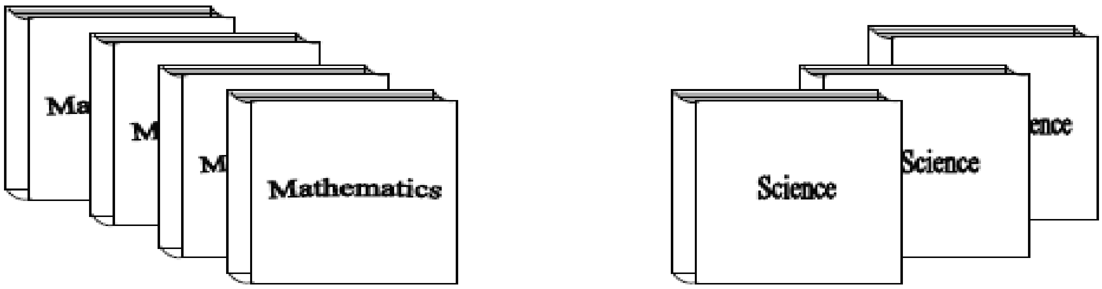
- One could use a tree diagram to see the number of choices. This is not always suitable to the different scenarios as it could become very complicated when drawing the ‘branches’ of the tree diagram
-
- b) Your choices now include Toyota, a large car, Silver of Gold in color. Find the number of ways the event can occur now.
-
- Solution:
- Number of manufactures: 4 choices
- Number of car sizes: 3 choices
- Number of colors: 5 choices
-
- Therefore the number of ways: 4 x 3 x 5 ways = 60 ways
-
- **ACTIVITY 2:**
- The access code for a house’s security system consists of four digits. Each digit can be 0 through to 9. How many access codes are possible if
-
- a) each digit can be repeated?
- b) each digit can only be used once and not repeated?
-
- Solution for a)
- Determine how many tasks are involved in compiling the codes
-
- Task 1: First number there are 10 choices (number 0, 1, 2, 3, 4, 5, 6, 7, 8, 9) = 10
- Task 2: Second number there are 10 choices (number 0,1, 2, 3, 4, 5, 6, 7, 8, 9) = 10
- Task 3: Third number there are 10 choices (number 0, 1, 2, 3, 4, 5, 6, 7, 8, 9) = 10
- Task 4: Fourth number there are 10 choices (number 0, 1, 2, 3, 4, 5, 6, 7, 8, 9) =10
-
- Using the Fundamental Counting Principle;

- Number of ways = 10 x 10 x 10 x 10 ways = 1 000 ways
-
- Solution b)
- There are still 4 tasks involved but with a difference
-
- Task 1: First number there are 10 choices (number 0,1, 2, 3, 4, 5, 6, 7, 8, 9) = 10
- Task 2: Second number there are only 9 choices now as the number that was chosen for task 1 cannot be used again in task 2, e.g. say that number 6 was used in for the first number then the number that are left are (0, 1, 2, 3, 4, 5, 7, 8, 90 = 9
- Task 3: Third number there are now only 8 numbers left as tho number have been used, one for task 1 and one for task two, e.g. say that number 4 was used now, then the numbers that are left are (0, 1, 2, 3, 5, 7, 8, 9) = 8
- Task 4: Fourth number has only 7 numbers left, e.g. three numbers form task 1, 2 and 3 and chosen number e.g 7 to be used. The numbers that are left then are
- (0, 1, 2, 3, 5, 8, 9) = 7
-
- Using the Fundamental Counting Principle: 10 x 9 x 8 x 7 = 5040 ways
- **ACTIVITY 3:**
- In how many different ways can we choose 3 cards from 13 cards in one suit (for example spades) without replacement
-
- Task 1 : 13 choices
- Task 2: 12 choices
- Task 3: 11 choices
-
- Number of ways : 13 x 12 x 11 = 1 716 different ways without replacement
-
- ACTIVITY 4:
- In how many ways can the number plates of Eastern Cape be arranged?
-

•	• LETTERS (26 – 6)	• NUMBERS (0 -

•	• 20	• 20	• 20	• 10	• 10	•
• TASK	• 1	• 1	• 1	• 2	• 2	

- Task 1: number of letters: 20 (A, E, I, O, U and Q are not used)
- = 20 x 20 x 20 = 8 000 letters.
- Remember there are 3 letters used for number plates.
- Task 2: number of number: 10 = 10 x 10 x 10 = 1 000 numbers.
- Remember there are 3 numbers used for number plates
- Task 3: the abbreviation EC = 1
- Number of ways = 20 x 20 x 20 x 10 x 10 x 10 x 1 = 8 000 000 car registrations
- **ACTIVITY 4**: (Exemplar 2008 Mathematics paper 3)
- Four different mathematics books and three different science books were left on the table. You need to place these books on a shelf



- 4.1 If you decide to place any book in any position, in how many different ways can you arrange the books on the shelf?
- Solution: 7 x 6 x 5 x 4 x 3 x 2 x 1 = 7! = 5040 different ways
- 4.2 If two particular books must be placed next to each other, in how many different ways can you arrange the books on the shelf?

-
- Solution:
- Two books can be arranges in 2 x 1 = 2 different ways
- We have 6 objects left and not 5. The two books is counted as one object plus the remaining 5 books
- Number of ways : 6 x 5 x 4 x 3 x 2 x 1 = 6! = 720 ways. Therefore the total arrangement of these books can take place in 2 x 720 = 1440 different ways
-
- 4.3 If all the mathematics books must be placed next to each other and all the science books must be placed next to each other, in how many different ways can you arrange the books on the shelf?
-
- Solution:
- Mathematics books: 4 x 3 x 2 x 1 = 4 ! = 24 different ways
- Science books: 3 x 2 x 1 = 3! = 6 different ways
- Combining the Mathematics and Science books : these can be arranged in 2 different ways.
- Therefore the total arrangement of these books can take place 24 x 6 2 = 288 different ways
-
- **CONCLUSION**: When working through this section of work one needs to read each question carefully and analyse what information do I have and what is required.
-
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