An exploration of Mathematical Literacy teachers' perceptions of, and performance in Mathematical Literacy tasks based on algebra

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Abstract

This paper reports on an exploration of teachers' perceptions with regard to their performances as they engaged with contextualised tasks of Mathematical Literacy (ML), and to examine their performance in ML tasks based on algebra. The participants of the study were a class of 24 students, 17 of whom who completed the ACE (ML) programme at the University of KwaZulu-Natal (UKZN) for the 2007 to 2008 academic years. They formed part of a group of 691 practising mathematics teachers from throughout KZN. We used four sources of data. Firstly, data was generated from teachers' reflection about certain tasks, the solution of which required the use of algebra. A second data collection instrument was an open-form questionnaire and the third instrument was two unstructured interviews with two teachers. The final instrument was the analysis of the teachers' examination scripts. The theoretical framework for the study was derived from the PISA cycle of mathematisation which specifies 5 aspects of mathematisation. The findings reveal that although the teachers conveyed varying understandings of the ML curriculum, they believed that knowledge of basic algebra was necessary and adequate for them to deal with ML problems. Furthermore the teachers believed mathematical teaching experience contributes to improved problem solving in ML. It was found that for this sample of teachers, the mathematics specialist teachers performed better than their non-mathematics specialist counterparts in ML problems which required algebraic skill.

Introduction

With the advent of the school subject Mathematical Literacy in South Africa in 2006, there have been numerous training initiatives for practising teachers who want to teach Mathematical Literacy. There are many challenges experienced by teacher educators who design professional development programmes for practicing teachers. Designing suitable and relevant programmes that could help teachers effectively facilitate the learning of Mathematical Literacy poses even greater challenges. There is fuzziness about what constitutes Mathematical Literacy pedagogical knowledge. How much of mathematics does a Mathematical Literacy teacher need to teach Mathematical Literacy knowledge for teaching? Can any teacher be retrained to teach the subject? In this paper we briefly look at some issues related to knowledge for teaching Mathematical Literacy, specifically dealing with the use of algebra in solving ML tasks. The purpose of the study reported in this paper, was to explore teachers' perceptions about, and performance in Mathematical Literacy tasks based on algebraic concepts.

Literature Review

One of the interventions decided by the Department of Education was to introduce the subject Mathematical Literacy (ML) as a fundamental subject in the Further Education and Training (FET) band in order to help develop numeracy skills among South African citizens. ML seeks to produce learners who are participating citizens, contributing workers and self managing people (DoE, 2008). Its purpose is not for learners to do more mathematics, but more application and to use mathematics to make sense of the world.

One challenge experienced throughout the world is the shortage of qualified mathematics teachers. This challenge was noted by the RAND as a critical impediment towards the initiative of development (RAND 2001, p. xiv). In the USA system of education there were imbalances in terms of the qualifications of teachers. These inequalities in teachers' qualifications meant inequalities in their knowledge base. It became a priority to address the issue of teachers' knowledge base, because if in the process some teachers have a restricted view of proficiency and are not themselves proficient in mathematics, their efforts to improve their students mathematical knowledge may be limited (RAND 2001, p. 9). This is similar to the situation in South Africa as Clark and Linder (2006, p. 7) contend: "South Africa is a country with a system of educational provision that spans a wide range of contexts, staffed by teachers whose levels of pre-service training differ markedly, and whose experiences (and expectations) of teaching and learning differ in turn by extremes".

In a study conducted at Rhodes University by Brown and Schafer (2006, p. 50), teachers were trained based on mathematical modelling and it was observed that "teachers with weaker mathematical skills took considerable longer to master the contexts and skills developed in these activities. This suggests that a teacher's level of mathematical skill is an important determinant of success". They further went on to admit that, "due to the pressure of time on the programme, it was not possible to include consolidation work that was appropriate for the less skilled teachers, for all the contexts considered" (Brown et al., 2006, p.51). Despite these studies which suggest that ML teachers need deep mathematics understanding, the DoE has in many cases encouraged teachers with just a Grade 12 mathematics background (in some cases even without this) to enrol for inservice programmes to retrain as Mathematical Literacy teachers. This study will show for a small sample of teachers, the teachers with mathematics teacher training experience performed much better than those teachers who were retrained from other specialisations.

Because the context and content are intertwined in ML, this may present certain challenges. Steen (1999, p. 12) points out that "the test of quantitative literacy, as of verbal literacy, is whether a person naturally uses appropriate skills in many contexts. Teachers know all too well the common phenomenon of compartmentalization, in which skills or ideas learned in one class are totally forgotten when they arise in a different context. Students need to learn numeracy in multiple contexts". According to Steen "all teachers need to help students think of mathematics not just as tasks on school worksheets but as something that arises naturally in many contexts". He also points that "teachers need to broaden their goals to encompass more than just the narrow arithmetic-algebra track that has dominated USA mathematics programs. In particular, they need to vigorously develop several parallel and highly interconnected strands of quantitative thinking".

In a study conducted by Graven and Venkat (2008, p. 7) in Gauteng, three teachers from one school were given a similar ML task. It was however found that all teachers interpreted

the task very differently, which is an indication that there exists a tension in relation to the understanding of the relationship between the content and the context. These different understandings may pose a significant barrier towards the implementation of the subject ML, which is "not less than or even part of mathematics, but something more than mathematics" being "the ability to situate, interpret, critique, use and perhaps even create mathematics context" (Bowie & Frith, 2006, p. 30).

The OECD/PISA (2003, p. 30) provides three components that describe the extent to which the problem-solver in Mathematical Literacy can handle mathematics in a well-founded manner when confronted with real-world problems.

- The first component is the *situations or contexts* in which the problems are located.
- The second component is the *mathematical content* that has to be used to solve the problems, organised by certain overarching *ideas*.
- The third component is the *competencies* that have to be activated in order to connect the real world, in which the problems are generated, with mathematics, and thus to solve the problems.

Any discussion of mathematical skills and knowledge necessary for success in ML tasks is not complete without a discussion of the role played by algebra in the solution of such tasks. In order to attain mathematical proficiency, the problem solver has to be fluent in various algebraic procedures because algebra is the most commonly used tool employed to solve mathematical problems. Algebra is therefore considered in this study for two particular reasons. Firstly, algebra is regarded as "foundational in all areas of mathematics because it provides the tools (language and structure) for analyzing and representing quantitative relationships, for modeling situations, for solving problems, and for stating and proving generalizations" (RAND 2001, p. xx). Secondly, problems in Mathematical Literacy are largely immersed in **context.** In the previous mathematics curriculum, problems of this nature were simple referred to as 'word problems'. Efficient use of algebra in this research is determined by the level of success in solving problems which require algebraic skill. Engaging with word problems demands from the problem-solver a great deal of interpreting within and across different representations. Interpreting in mathematics involves writing, in most cases converting the English written statement into a mathematical statement. According to Ntenza (2004) citing Vygotsky, "writing requires particular demands from the writer who has to engage in the deliberate structuring of the web of meaning". For Vygotsky "writing seems to increase the cognitive actions executed by students, and the inner speech is critical in the writing process" (Ntenza 2004, p. 14).

We end this section with a brief discussion of the role played by practice in attaining mathematical proficiency**.** According to Alibali, Rittle-Johnson and Siegler (2001, p. 346) "procedural knowledge is the ability to execute action sequences to solve problems and conceptual knowledge is an implicit or explicit understanding of the principles that govern a domain and of the interrelations between units of knowledge in a domain". In order to engage successfully in ML problems that necessitate the attainment of these strands of mathematical proficiency (procedural fluency and, conceptual understanding) people need opportunities for practice. Van de Walle (2007, p. 69) looks at "practice" as referring to different problem-based tasks or experiences, spread over numerous class periods, each addressing the same basic ideas. He is of the idea that if learners are provided with ample and varied opportunities to reflect on or create new ideas through problem-based tasks, there is an increased opportunity to develop conceptual ideas and more elaborate and useful connections. The work by Chase and Mayfield (2002, p. 105) where they "compared

three different methods of teaching five basic algebra rules involving some kind of practice, they found that cumulative practice of component skills is an effective method of training problem solving". Again, the study conducted by Lovett and Rosenberg-Lee (2006, p. 4) did reveal that pure procedural practice does lead to improvements in conceptual knowledge.

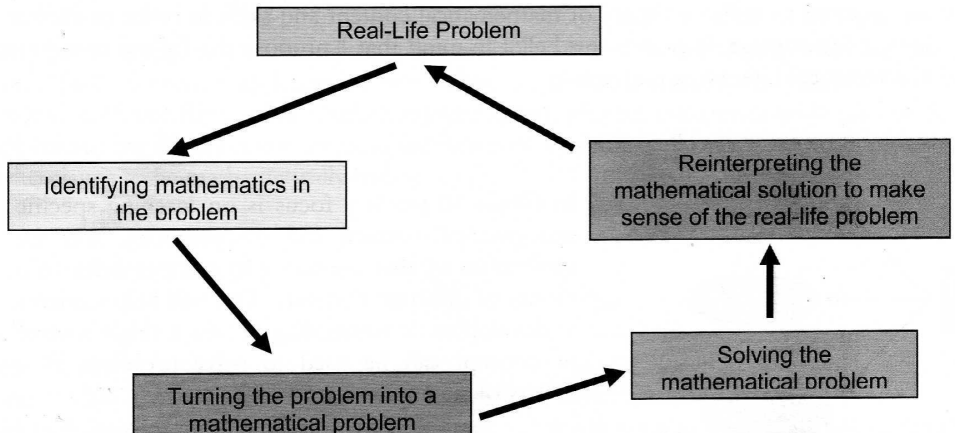
Theoretical Framework

Solving problems in ML requires a great deal of movement within various representations (verbal, algebraic, symbolic, and graphical). According to OECD/PISA (2003, p. 26) context-based problems can be solved by following the general strategy used by mathematicians, which the PISA mathematics framework - refers to as **mathematizing.** Mathematising can be characterized as having five aspects:

- Starting with a problem situated in reality.
- Organizing it according to mathematical concepts and identifying the relevant mathematics.
- Gradually trimming away the reality through processes such as making assumptions about what are the important features of the problem, generalizing and formalizing, which promote the mathematical features of the situation and transform the real-world problem into a mathematical problem that faithfully represents the situation.
- Solving the mathematical problem.
- Making sense of the mathematical solution in terms of the real situation, including identifying the limitations of the solution.

The following figure shows the cycle of mathematisation.

Figure 1 : Schematic Diagram showing the process Cycle of Mathematisation



(Adapted from: OECD. 2003. p.38)

The table below shows how certain aspects of mathematisation are linked to specific demands of the context or the mathematics activity required to engage in that aspect of mathematisation.

Table 1: Aspects of mathematisation as a process

Aspect of mathematisation	Description of the mathematisation process	Mathematical or contextual demands of the particular aspects
1	Problem situated in a real life context	Knowing definitions, rules or semiotic representation used in context
2	Organizing it according to mathematical concepts and identifying the relevant mathematics.	Knowledge of relevant mathematical concepts, and conditions under which a concept can be applied
3	Gradually 'trimming away' reality through processes such as making assumptions about what are the important features of the problem, generalizing and formalizing, which promote the mathematical features of the situation and transforming the real-world problem into a mathematical problem that faithfully represents the situation.	Making assumptions to render problem solvable. Identifying relevant process/formula or rule that is suitable. Deciding whether the problem requires a direct, inverse or multi-step approach. Formulating (if necessary) the required model or equations.
4	Solving the mathematical problem.	Carrying out the computation, correct substitution, simplification, algebraic manipulation, obtaining a solution (procedural fluency), conversion of units.
5	Making sense of the mathematical solution in terms of the real situation, including identifying the limitations of the solution.	Interpreting answer within the context, to see if the solution makes sense in the context. Engaging in contextual reasoning.

To determine the level of cognitive demand at which assessment tasks are posed, it is useful to use a hierarchy or taxonomy. The questions were categorised according to the taxonomy levels of the SAML school subject. It is clear that the aspects of mathematisation are different from the skills and demands listed in the SAML taxonomy (DoE, 2008).. The DoE

designed and introduced an assessment taxonomy that could be used to classify the cognitive demand of tasks (DoE, 2008).The MLSAG taxonomy is described below:

- Level One: Knowing
- Level Two: Applying routine procedures in familiar contexts
- Level Three: Applying multi-step procedures in a variety of contexts
- Level Four: Reasoning and reflecting

Methodology

This qualitative study utilised an interpretive approach. Within the interpretative paradigm, we opted to utilize a case study approach. The case in this study is the group of 17 teachers who enrolled at UKZN for the ACEML course. The research questions underpinning the study are: What are the teachers' perceptions of ML tasks based on algebra? What is the nature of the performance of the teachers in ML tasks based on algebra? The research instruments that were used were the teachers' written responses to examinations, the teachers written reflections, a semi-structured questionnaire and interviews with two teachers. The two interview respondents are referred to as Teacher 1 and Teacher 2

The ACE programme under discussion in this paper was made up eight modules consisting of six ML-specific modules and two generic modules. Four of the six modules are devoted to the development of content knowledge skills in ML while two modules are devoted to the development of pedagogic content knowledge skills in ML. The two generic modules focus on general professional development and the roles of the educator (teacher).The four content modules formed part of this study.

For the purpose of this research, a participant was considered as a "mathematics specialist" (MS) if s/he studied mathematics up to tertiary level, while a participant was considered as "non-mathematics teacher" (NMS) if s/he studied mathematics only up to Grade 12 level. Furthermore mathematics specialists are represented by A, B, D, L, C, O, H & F; while the non-mathematics teachers are represented by G, I, N, K, Q, J, P, M & E..

Results and Discussion

Teachers' perceptions of the role of algebra in teaching and learning ML

The first theme that emerged from an analysis of the reflections, questionnaire and interviews was the importance of fundamental knowledge of algebra. The analysis of the teachers' reflections revealed that they felt basic mathematical knowledge, (which was mainly algebra in this case), was necessary when dealing with this task of ML. Teacher **E** said "*but because of the basic knowledge of pure mathematics I have learnt before, I tried to answer some questions. I then strictly say this mathematical literacy needs the understanding of pure mathematics".* This notion was also supported by teacher **D** as she explains "*The task was challenging….but through prior knowledge from [mathematics] learning area I was able to attempt".* This was also felt by J, as he said "…*if we did mathematics it will be easier to understand maths literacy*". Teacher A is quoted as saying "*Background (mathematical one) is the key element in making learners to pass this course. So lack of this background is not good*". Again, B supports the idea as she points out that, "*Some of the students do not have adequate pre-knowledge background. Sometimes some contexts require the mathematical skills of which some of us don't have because we didn't major with it".*

The need to draw on basic algebraic knowledge was also seen by both interview respondents as a gateway towards better problem solving in ML, was the first theme that emerged from the analysis of the interviews. Teacher 1 acknowledged the role of prior knowledge when asked about role of the grade 12 algebraic knowledge on the teaching and learning ML. She had this to say: "… *like the prior knowledge that you have got will assist you like in teaching of Mathematical Literacy especially when you are dealing with…if you do not know anything about algebra so it becomes difficult for you…therefore its adequate [meaning necessary in this case]"*. This feeling was also confirmed by Teacher 2 when she said, "*That [the module] was difficult for person with only grade 12 mathematics because there with the experience that you had, you would easily identify the formula to use..."*.Teacher 1 also viewed the algebra knowledge needed for mathematics problem solving as being similar to that required for solving mathematical literacy tasks: "*Ya I view the Mathematical class the same way…the difference is in the structuring of questions…like in Mathematical Literacy class the questions are context-based rather than in Maths…so but the algebra is the same".* Teacher 2 however felt that the algebra required for ML was simpler than that required for Mathematics as she added, *"No it is not similar, in Mathematical Literacy class they are using simple and simple algebra but in Maths class they are dealing with more complex situations and most of them you cannot relate those problems in a real life situation…".*

The second theme emerged when teachers in the interviews revealed that mathematics **teaching experience** played a positive role in problem solving in ML. Firstly, it appeared that their additional training experience helps them to solve problems more easily. Teacher1 was quoted as saying*: "… then a person with experience of solving problems finds himself in an advantageous position and is also able to teach better than the one who has not got a chance to teach and has been exposed to problem solving"*. Teacher 2 shares a similar sentiment as she pointed out, "*I think you cannot separate the algebraic experience with Mathematical Literacy because…the one who has been teaching algebra or with the experience in teaching will….with confidence handle the problem".* Responses in the questionnaire by teacher J (who did not get a DP) indicates that he saw his lack of mathematics teaching experience and knowledge as a hindrance to his success. In confirming this claim, participant I said "…*some of us are not teaching mathematics where they are and they find themselves getting delayed in solving problems*".

One of the reasons attributed by the teachers to the positive influence of mathematics teaching experience was the fact that mathematics teachers have been teaching mathematics and thus been exposed to mathematical thinking more recently than NMS group. The long period between when the NMS group had last studied mathematics added to their challenges, because they had not worked or thought about basic mathematics procedures for a long time. This is confirmed by Teacher 1 said, "*I can say the one who is currently teaching is fresher than the one who last did it at grade12 in terms of practicality…so it becomes easier for the person who is teaching to solve problems…you start by struggling but you eventually get there"*. On a similar note Teacher J (who did not get a DP) said : "*We need to know all those things so it is difficult for a person who did maths in ten years back to do this".*

The third theme related to the use of practice in acquiring skills. One teacher emphasized that practice and familiarity helped them improve their problem solving skills in ML. Teacher 1 said, "*No, like when I did it for the first time I thought it was very difficult but now when we were doing remedial work in class and then my peers had showed me how to do it, it was*

not that difficult…but the first time I looked at it I thought no it was going to be difficult and I couldn't do it".

Teachers' performance in ML tasks requiring algebraic skills

It was found that the mathematics teachers performed better than the non-mathematics specialists in the four content modules. The results of the first two modules are presented below in Table 2, with the mathematics specialist (MS) shaded and the non-mathematics specialists (NMS) unshaded.

Student's Name	Numbers & Operations	Data Handling
G	61	69
I	0 (No DP)	61
A	62	68
N	60	65
K	0 (No DP)	57
B	72	75
D	75	65
Q	63	73
L	72	62
J	0 (No DP)	54
C	38 (Fail)	58
O	61	65
P	34(Fail)	51
M	35(Fail)	55
H	43(Fail)	62
F	60	65
E	50	52

Table 2: Performance of the MS and the NMS groups in two modules

An examination of Table 2 reveals the overall better performance in the modules by the MS group except for the two teachers C and H who performed poorly from the MS group and teacher Q from the NMS group who performed well. Teacher Q is an accounting teacher who has never taught mathematics, while C has a primary teachers' diploma and H has a secondary teachers diploma. An initial analysis as it appears in Appendix B reveals that out of 17 participants, three did not get a Duly Performed (DP) certificate, and these were all from the NMS group of teachers. Furthermore, 4 participants (2 mathematics specialists; and 2 non-mathematics teachers) failed, which makes a total of 5 non-mathematics teachers who did not succeed in the module Numbers and Operations.

Thereafter the in-service teachers' achievements in the examination of the four content modules were analyzed to try to identify specific areas of differences in problems requiring algebraic skills according to the four levels of the Mathematical Literacy taxonomy (DoE, 2008). The modules were Numbers and Operations in context; Data Handling; Functional Relationships; and Shape, Space and Measurement. These scores provided in Table 3 do not represent the teachers' results in the whole papers, since we only targeted the algebraic-rich questions within the four modules, respectively, although it is true that they also performed better overall in the whole modules as well. One of the striking findings is the difference in performance between mathematics specialists and non-mathematics teachers in the various algebraic-rich problems.

We notice that for the four different content modules, and at various levels of taxonomy, the mathematics specialists performed better than the non-mathematics teachers for questions requiring algebraic skills. This is shown in Table 3 below.

Numbers and Operations in context							
Level 1		Level 2		Level 3		Level 4	
Math	Non Math	Math	Non Math	Math	Non Math	Math	Non Math
56	48	78	61	49	40	56	41
Difference = 8		Difference = 17		Difference = 9		Difference = 15	

Functional Relationships							
Level 1		Level 2		Level 3		Level 4	
Math	Non Math	Math	Non Math	Math	Non Math	Math	Non Math
66	38	65	45	70	41	52	39
Difference = 28		Difference = 20		Difference = 29		Difference = 13	

Data Handling					
Level 1		Level 2		Level 3	
Math	Non Math	Math	Non Math	Math	Non Math
87	81	85	71	40	26
Difference = 6		Difference = 14		Difference = 14	

Shape, Space and Measurement			
Level 2		Level 3	
Math	Non Math	Math	Non Math

65	43	50	26
Difference = 22		Difference = 24	

Table 3: Performance difference between Mathematics and non-mathematics specialists studying Mathematical Literacy in various levels of taxonomy.

Further analysis shows that the NMS group experienced particular problems with aspect 3 of mathematising which requires, identifying the necessary mathematics, and setting up equations if necessary. Problems were experienced with respect to modeling, recognizing the differences between inverse and direct problems and carrying out multi-step solutions. Although the MS group experienced problems in these sections too, it was to a smaller extent. Furthermore these questions involve the following concepts:

- Percentage calculation: problems involving the calculation of transfer duty.
- Substitution: in using the compound increase/decrease formula.
- Ratio formulation: mixing quantities using the knowledge of fractions.
- Conversion: from degrees celcius to farhenheit.

Many of these concepts have been part of the GET and FET mathematics curriculum prior to the introduction of ML which the MS teachers would have been teaching. We argue that the reasons for this may be attributed to three reasons. Firstly, certain questions were related to the mathematics content which the MS teachers would have been teaching in their classrooms. A second reason is that some of these questions require the third aspect of mathematisation which involves setting up a model and solving it. This is a higher level of algebraic thinking as compared substituting into known formula (questions at Levels 1 and 2 of the taxonomy). A third reason was put forward by a teacher in the NMS group who pointed out that the long period between when they had last studied mathematics added to their challenges, because they had not worked or thought about basic mathematics procedures for a long time. Thus the non-mathematics group experienced a higher level of difficulty than their counterparts, particularly in those which required much mathematisation.A fourth reason for this is that the weighting of the paper was biased towards level 3 and level 4 questions,. The weighting for the module examination in term s of the SAML taxonomy was that the weighting of the percentage for Levels 1, 2, 3 and 4 were 8, 23, 46 and 23 % respectively. This would have disadvantaged the non-mathematics teachers who coped better with the lower level procedural type of questions which did not require much mathematisation.

Concluding Remarks

The findings reveal that although the teachers conveyed varying understandings of the ML curriculum, they believed that knowledge of basic algebra was necessary and adequate for them to deal with ML problems. Furthermore the teachers believed mathematical teaching experience contributes to improved problem solving in ML and that practice and familiarity helped teachers improve their problem solving skills in ML. Within the group, it was found that Mathematics specialist teachers performed better than the non-Mathematics teachers.

Some reasons for this were identified as Mathematics specialists having more exposure to particular concepts as a result of their mathematics teaching experience. Also, non-mathematics teachers have had a gap period (since their school mathematics experiences) during which time they have not been doing any mathematics, while the mathematics specialists have had ongoing mathematical teaching experience and were consequently learning more. Another reason could be that the paper was more heavily loaded towards the higher level of questions.

We hope that the findings of the paper has contributed to an understanding of the ways in which algebraic skills and knowledge is used in solving ML tasks. Furthermore we hope that it has added to knowledge about the knowledge required to teach the subject ML. By also identifying areas of difficulty experienced by ML teachers, the study also points to these areas as possible areas of difficulty for ML learners.

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