Teachers can and do: Examining specialist teacher knowledge in the teaching of secondary school algebra

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Abstract

This paper is intended to open up discussion surrounding how teachers employ pedagogical content knowledge to pre-empt and remediate common errors and misconceptions that emerge when students learn secondary school algebra. As a theoretical basis the paper uses Shulman's notion of pedagogical content knowledge (as well as the theories of other authors) to understand the specialist knowledge that teachers draw on. This specialist knowledge is examined using actual student errors and misconceptions that occur commonly in our secondary algebra classrooms, and this paper looks at how teachers make pedagogical decisions drawing from their specialised knowledge in order to redress or pre-empt these errors and misconceptions. The intention of the paper is to show that dedicated teachers are not only people who "can", but are also people who "do", thus dispelling the inherent myths contained in the famous quotation by Shaw.

Introduction

"He who can, does. He who cannot, teaches." (George Bernard Shaw, *circa* 1915.)

Notwithstanding the era in which this statement was made it is often quoted today, much to the chagrin of modern-day teachers. The statement implies two things: firstly, teachers do not fall into the category of society "who can" (dichotomous as this may seem), and secondly, teaching is not something that one "does". To dispel the inherent myth in the latter, ask anyone who has actively engaged a group of students in the learning of something and I am certain that the responses will be unanimous in one respect: teachers certainly "do". To dispel the former myth, *viz*. that teachers come from those in society who "cannot", I will examine how teachers employ specialised knowledge to teach effectively. I will use certain examples of student mathematical reasoning to scrutinise how teachers draw on their knowledge-base to make decisions that will enhance or remediate student conceptions, or possible hinder learning. In doing so I hope to demonstrate that teachers "can" and "do" in very specialised ways.

Before turning to examples from the classroom, I will look at how research will inform our examination of teacher knowledge for the purposes of this paper.

Theoretical perspectives

Historically, teacher knowledge has been gauged by the individual's tertiary achievement in mainly academic courses (Carpenter *et. al.,* 1988). This mind-set is still quite widespread in

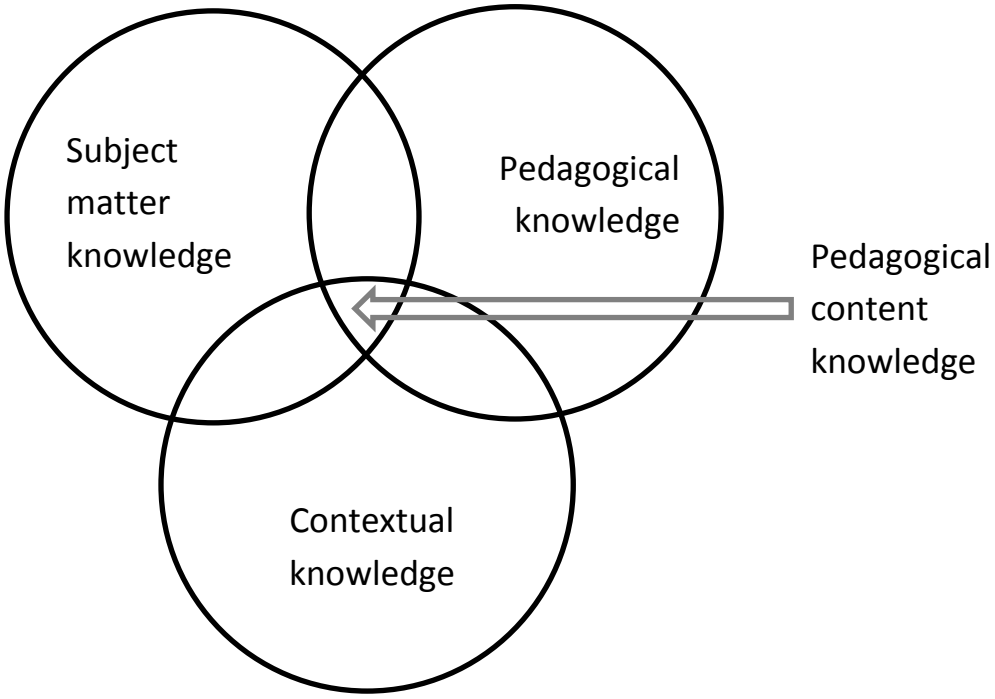
our profession today even though educationalists generally accept that academic performance is not the only pre-requisite in developing competent teachers.

Shulman (1986) recognised that teachers exhibit specialist knowledge and he termed this *pedagogical content knowledge* (p 26). This knowledge, he believed, was in addition to other recognised teacher knowledge, such as subject matter knowledge and curriculum knowledge. Shulman developed a much more comprehensive categorisation of different aspects of teacher knowledge, but for the purposes of this paper I keep the description very simple.

Gess-Newsome (1999, p. 12) proposed a useful model for coming to terms with the different knowledge categories used by teachers (see figure 1 below).

In the Venn diagram the set on the top left depicts *subject matter knowledge*. This pertains to what teachers know about their subject from an academic perspective. In the set at top

Figure 1. Categories of knowledge depicted in a Venn diagram (Gess-Newsome, 1999, p. 12)



right, *pedagogical knowledge* is shown. This is what teachers know about how to teach their subject area. The intersection of *subject matter knowledge* and *pedagogical knowledge* indicates to us that these are not definitive aspects of teacher knowledge. These knowledge domains are not bounded, but rather infuse and are sometimes called upon simultaneously by teachers as they go about the task of teaching.

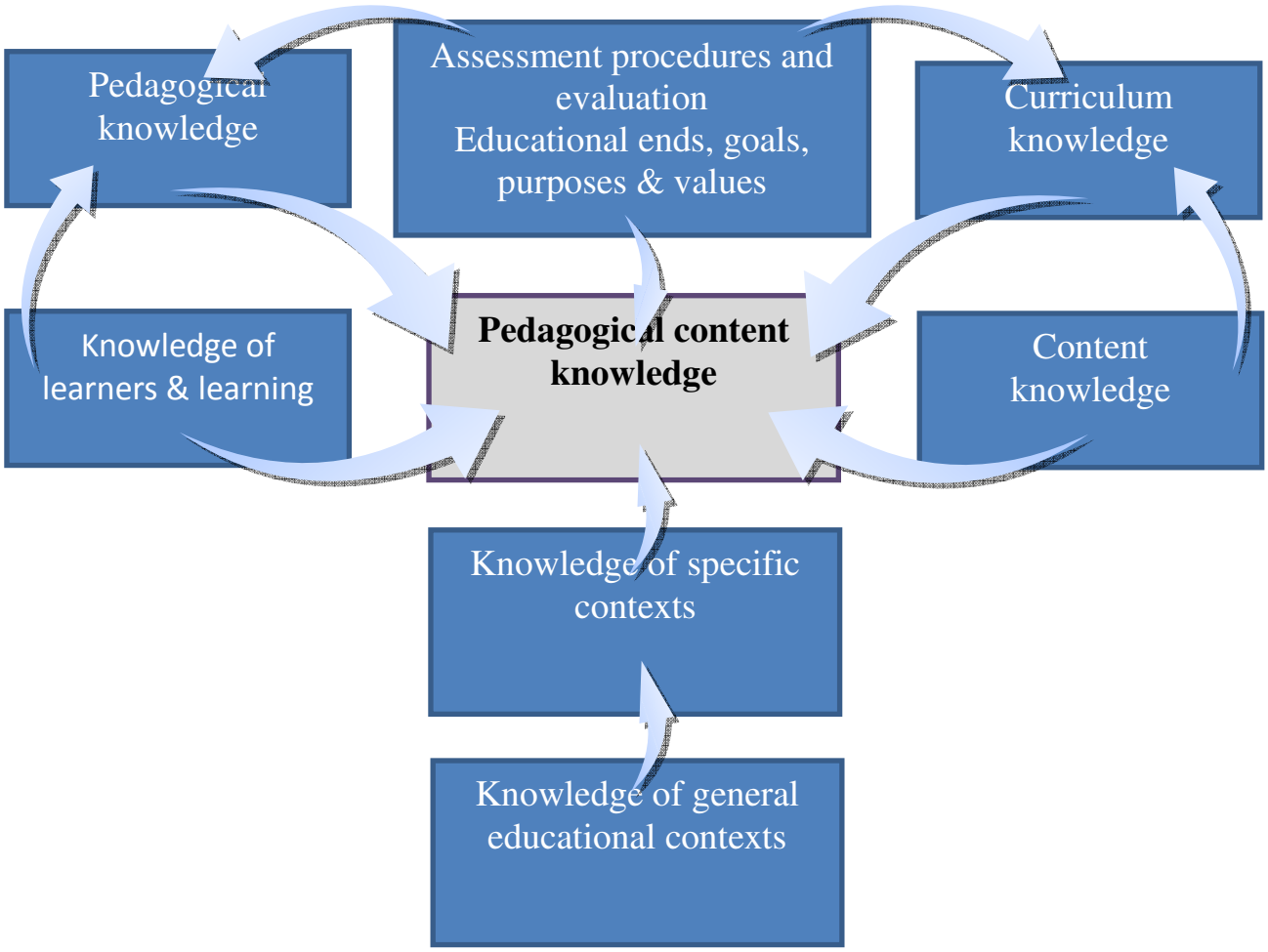
The set at the bottom shows *contextual knowledge*. This refers to, *inter alia*, what teachers know about the circumstances surrounding their students, the school in which they teach and the broader educational community to which they belong. Again, there is an intersection to show that these categories of knowledge are not mutually exclusive.

For Gess-Newsome (1999), the area of overlap of these three knowledge categories is where we will find the specialised teacher knowledge referred to as *pedagogical content knowledge*. Thus, it is not simply knowledge of the subject, and it is not limited to how one teaches the subject. This specialist teacher knowledge comes out of a combination of

knowing the subject matter, knowing how it should be taught, as well as knowing the circumstances in which this teaching will happen.

Further to this, Shulman (1986) sees pedagogical content knowledge as the specialised knowledge that teachers use to transform the content into a 'teachable' form, so that it becomes more accessible to the students. However, from the model shown below in figure 2, taken from Morine-Dershimer and Kent (1999, p. 23), we see a picture of the complexity of this specialised knowledge. Briefly, this model shows us how the top-down influence of educational issues is thought to feed into teacher knowledge: on the one hand, in terms of pedagogy (how to teach, and this is influenced also by who the students are); and on the other hand in terms of curriculum (what is taught, which is influenced by the subject content). The bottom-up influences also feed into teacher knowledge, and these relate to the context (i.e. where that teaching takes place).

Figure 2. A model showing the complex make-up of pedagogical content knowledge by Morine-Dershimer and Kent (1999, p. 23)



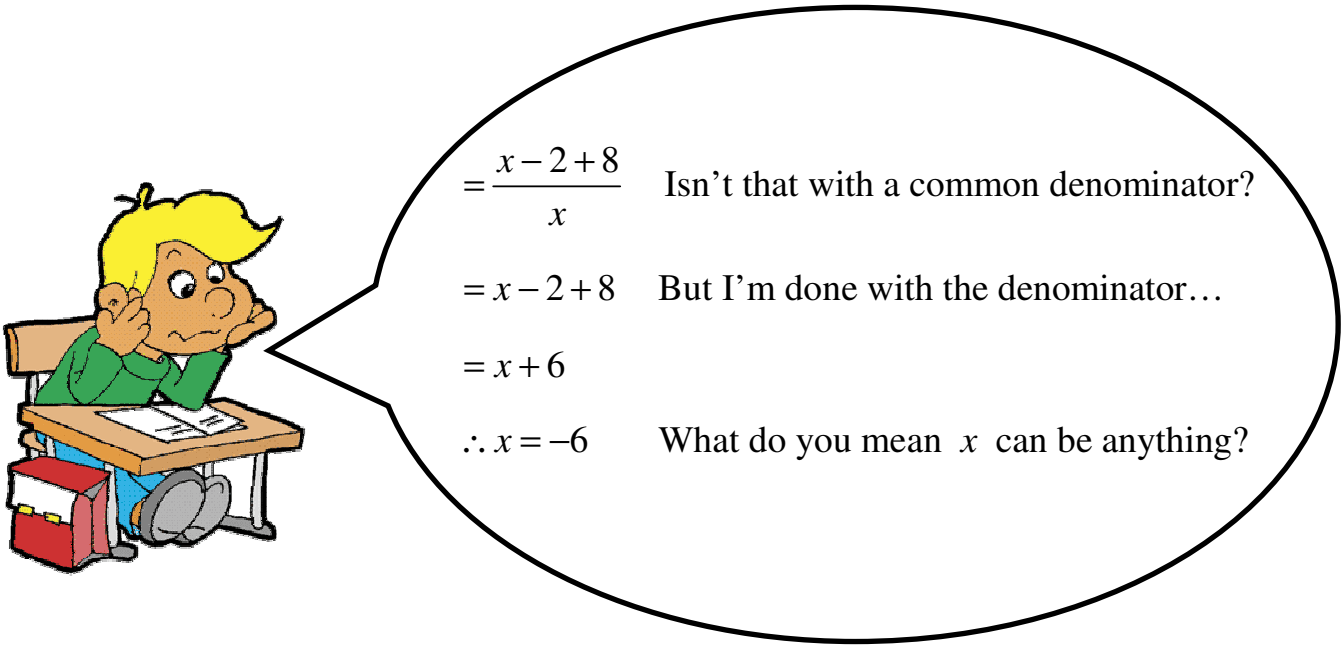
Loosely translated, pedagogical content knowledge is thus the knowledge that a teacher gleans from pedagogy, curriculum and context to make subject matter presentable and more accessible to students. As Gess-Newsome (1999) puts it, pedagogical content knowledge is "…the integration and transformation of other forms of knowledge" (p. 4). Using our understanding of how the assimilation of different knowledge categories manifest as a teacher's pedagogical content knowledge (as per Morine-Dershimer and Kent's model in figure 2) I now turn to a discussion of the errors and misconceptions that teachers must deal with in the classroom.

A look at what happens in the classroom

All this theory sounds great. But what are the implications for teachers? What do secondary teachers of mathematics face in the classroom when it comes to their knowledge?

In terms of mathematical understanding, students of secondary mathematics sometimes exhibit the strangest logic. Take this hypothetical case to place the theory discussed above in context and to begin our discussion:

The student is the steps used in answering the question: Simplify: $x-2+\frac{8}{x}$



How does the teacher respond to this reasoning in order to correct the errors? More importantly, though: how does the teacher instil the correct mathematical thinking and procedures?

Firstly, the teacher needs to know how to do the problem at hand (the content knowledge). This entails such knowledge as knowing what we mean when we talk about “simplifying”; accomplishing this simplified form by finding a lowest common denominator through the creation of equivalent fractions, so that we can combine like terms (fractions) into a single fraction.

Secondly, (from a curriculum perspective) the teacher needs to consider the knowledge that students at this particular level are expected to have mastered prior to this point, and to what extent they can cope with the content required by this problem. That is, does the prior curriculum content make provision for working with equivalent fractions and developing understanding of why we need a common denominator to add or subtract fractions in a numerical context. What other mathematical procedures and concepts may have interfered with the student’s ability to correctly handle this problem? (Ball & Bass, 2000). For example, has this student confused procedures for solving equations with procedures for simplifying a fractional expression? In addition to this, the teacher needs to know how the students will need to use this particular mathematical procedure in future years.

Thirdly, the teacher needs to know how this individual student coped with the fractions that he encountered in previous experiences. Another consideration is that the teacher must establish the extent to which this student is reliant on technology, such as modern scientific calculators, for example, which may obscure the mechanical procedure of operating with fractions that algebra teachers often assume have been mastered in prior years. These all form part of the milieu that the teacher must construct to understand the context in which the learning is taking place.

This commentary gives some idea of the spectrum of knowledge that a teacher draws on to be in a position to address the errors exhibited by this student. Before decisions can be made about how to best rectify the errors this knowledge needs to be assimilated and a plan of action best suited to this student must be formulated. And, this often has to be undertaken spontaneously in a busy classroom where there are many other demands!

I now turn to an analysis of some common errors and misconceptions that have occurred in my classroom (as I am sure that these have occurred in many algebra classrooms). In doing so I will reflect on how teachers possibly draw on content knowledge, knowledge of curriculum and their understanding of the students and the general context they find themselves in, to formulate a specialised knowledge that makes the content presentable and accessible.

Using pedagogical content knowledge to make a difference

In analysing the examples presented here I will look at content and curriculum knowledge that could be drawn upon to make decisions about how best to manage the misconceptions and errors. For the purposes of this paper, I have omitted details of the scenario, such as the learner’s grade, level of competence, and other *contextual knowledge*, that is integral to the Gess-Newsome model (1999). This has been done to facilitate more expedient

discussion of the specialist teacher knowledge required in each example (focussing on curriculum and content knowledge), and in no ways intends to minimalise the important role that context knowledge plays in pedagogic decisions that teachers make. Suffice it to say that teachers have a far more intimate understanding of their students' contexts than can ever effectively be shared in a forum such as this.

Scenario 1

A student presents the following: $3x = 2$

$$\therefore x = \frac{2}{-3}$$

The secondary curriculum makes provision for the solution of this type of linear equation from as early as grade 8, and students are expected to master these skills quite quickly. It seems that this student has solved by dividing by 3 on both sides (reversing the operation of '3 times *x*' that appears on the left in the first step). However, the sign has been changed to negative. It is possible that this student is using a rule-bound approach: *viz*. 'Take the term across the equals and change the sign.' To pre-empt this type of formulation which rests upon a misconception the teacher needs to be conscious that a literal interpretation of a rule could lead to this error. With this in mind, the teacher might teach another rule: 'What you do to the left, you must also do to the right to keep the equation true or balanced.' In other words, we do not 'take the term across the equals and change sign', but rather 'do the same to both sides of the equation'. If the rule 'doing the same to both sides' is used, the teacher can explain to the student in the example above that on the left the term has been divided by 3, and on the right it has been divided by -3.

There is also possibly another misconception that this student exhibits: he has taken the 'term' across the equals and changed the sign. But the –3 is not a term in this equation, and hence cannot be dealt with by this 'rule', *viz*. 'take the term across the equals and change the sign.' The question is, does the teacher address the misunderstanding of what a term is now (in this problem situation), or is it best left for another occasion? Addressing it here may cloud the immediate problem (i.e. doing the same to both sides when solving an equation), but we all know that it is important for this student to be able to distinguish between a term and the components of a term. This is what I call a pedagogical decision that the teacher must make. The teacher draws on pedagogical content knowledge in order to make this kind of decision.

The teacher, as part of curriculum knowledge, needs to know how the student will be using these 'rules' for manipulating equations. Later in the secondary school system, when asked to 'solve for *x*', one may encounter a student doing the following:

$$x+3\sqrt{x+2}=2$$
$$\Rightarrow\ 3\sqrt{x+2}=2-x$$
$$\Rightarrow\ \ \sqrt{x+2}=-1-x\ldots$$

It seems that this student in the third step has 'taken the 3 across the equals and changed sign', exhibiting a misunderstanding of the rule as mentioned above. However, here it becomes important to impress upon the student that the '3' is a component of a term, and not a term on its own, and that what has happened in the third step is 'division by 3' on the left, with 'subtraction of 3' on the right. This begs the question: would talking the issue of the 3 not being a term in itself in the linear equation earlier in the student's schooling have made any difference to how that student approached the problem involving the root above? Or, would confronting the issue of the 3 not being a term have clouded the issue of doing the same to both sides of the equation to maintain the equality?

Scenario 2

The problem is to simplify the expression $\dfrac{x^2-2x-3}{x-3}$.

A student gives the following working: $\dfrac{x^2-2x-3}{x-3}=x-2x=-x$.

The algebra in this working makes no sense, so the first thing the teacher needs to ascertain is what the student did. The student cancelled as follows:

$$\frac{x^{\cancel{2}}-2x-\cancel{3}}{\cancel{x}-\cancel{3}}=x-2x$$

Now the teacher needs to know where this misconception may have developed. It seems likely that in prior years the student has been taught something like this:

$$\frac{x^2}{x}=x$$

This understanding is most likely being transferred directly to the problem above. It is also possible that the use of the term 'cancel' has caused the student to apply the concept of 'cancelling' as in the problem above. In terms of the curriculum, the student has probably been taught equivalent fractions in earlier years, in which numerically we divide both top and

bottom of the fraction by the same number and end up with an equivalent fraction. However, when taught algebra this is often done using the terminology 'cancel', and the student may not make a connection between what happened with number and what is expected now in the algebra.

In addition to this, why did this student not do the following?

$$\frac{x^{\cancel{2}}-2x-\cancel{3}}{\cancel{x}-\cancel{3}}=\frac{x-2x-1}{1-1}$$

If given a numerical fraction it is likely that the student will divide top and bottom correctly, and when dividing a quantity by itself, arrive at the answer of 1. However, in the algebraic context the student does not seem to be able to make these connections. Does the teacher address this problem here? This is another pedagogical decision that needs to be made based on what the teacher thinks is the best means of helping this student to understand the algebraic procedures.

Pre-empting the errors and misconceptions

Ideally, teachers would like to teach in such a way that students do not formulate misconceptions, and that they do not end up making errors. If the teacher is to have a chance of doing this then a very special knowledge is necessary to anticipate certain student behaviour when dealing with the mathematics. This could result in carefully choosing the way in which the content is presented, possibly excluding certain types of examples initially if they might cloud issues for the students.

To demonstrate, I look at common errors that students make when working with powers.

Scenario 3

When confronted with $2\,.\,3^x$ students sometimes misguidedly give 6^x. This error could be as a result of the student not fully understanding that operating with powers (*viz.* numbers represented as repeated multiplication) is different to operating with ordinary numbers. The astute teacher might realise that this could be a problem and choose to avoid examples such as $2^x\,.\,3^x$. The problem that this teacher is anticipating here is that this example correctly results in 6^x, and is therefore likely to reinforce the misconception contained in $2\,.\,3^x = 6^x$. Drawing on pedagogical content knowledge, the teacher might choose to leave the teaching of this concept until students are more adept at the manipulation of powers and ordinary numbers, thus introducing the following at a later date:

$$2^x\,.\,3^x = (2\,.\,3)^x = 6^x$$

Similarly, introducing a problem such as $\frac{32^x}{4^x}$ may influence students into incorrectly tackling a problem like $\frac{32^{x-2}}{4^x}$. The students are quite likely to produce the following:

$\frac{32^{x-2}}{4^x} = 8^{x-2-x}$, working on the basis that $\frac{32}{4} = 8$ and $\frac{a^{x-2}}{a^x} = a^{x-2-x}$. The teacher must understand that *both* assumptions that the student is making when arriving at this answer are the result of prior learning, firstly in the context of division of number, and then in the concept of division of powers. Drawing on this knowledge of the student, together with the content knowledge (about number and powers) the teacher is able to predict certain behaviours, and thus avoid particular examples (at least initially) in order to try to help students to more easily assimilate the different concepts.

Concluding remarks

Teachers are faced with varied and often unpredictable responses from students that may contain errors and demonstrate misconceptions. Also, the teacher often needs to respond to these 'in the moment', and they draw on a specialist knowledge base in order to do this.

We can say that certain pedagogical decisions are made by teachers in order to circumvent the onset of any misconceptions or to correct errors and misconceptions that happen subsequent to prior learning. Either way, these decisions are not made in isolation, but are the result of an accumulated specialist knowledge that the teacher draws upon, based on what the teacher knows of the students, what the teachers knows about the prior learning and where the student is headed mathematically, and what the teacher knows about the subject of mathematics. These decisions result in the content being presented in a particular way so as to make the mathematics more accessible to the students.

This specialist knowledge has enabled teachers to make important pedagogical decisions, and this is what has been termed pedagogical content knowledge (Shulman, 1986).

From the foregoing discussion, it seems reasonable to conclude that competent teachers know their subject matter, are familiar with the curriculum and have a good understanding of the context in which their students find themselves. In their day-to-day activities, teachers make informed decisions regarding how to make their subject matter presentable and accessible based upon their pedagogical content knowledge. Surely, this places those teachers firmly into a category of society of people who *can*, and *do*? I therefore propose an amended quote, applicable to the many dedicated and able teachers out there:

"Those who can, do. Those who can with insight and caring, teach."

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