AN ANALYSIS OF GRADE 12 MATHEMATICS LEARNERS' UNDERSTANDING OF OGIVES.

Makhalanyane Phillip Moeti

Vuyanimawethu Secondary School, Khuma, Matlosana Municipality, North West Province

AMESA North West

Abstract

The study investigates Grade 12 Mathematics learners' understanding when they answer questions based on cumulative frequency curve (ogive) and tables. Learners have shown weakness therefore it is lost opportunity to have learned ogives in Grade 11. The statistical reasoning, mathematical understanding and graph comprehension are concepts that frame this study. The learners' responses on written task are studies as case study applying qualitative approach. The findings of the study are: (a) learners have inadequate ability to read data from ogive, (b) learners lack graph interpretation skills but (c) they have sufficient graph construction. The study recommends that learners should be given statistical tasks that develop their interpretation and analysis of data embedded in statistical graphs and *statistical graphs should elicit both instrumental understanding and relational understanding from learners*

Introduction

The research project is about Grade 12 Mathematics learners' performance on answering question related to cumulative frequency curve, known as ogive and cumulative frequency tables. Grade 12 Mathematics learners have shown a weak performance on reading and plotting of cumulative frequency tables and curve in their classworks, homeworks and tests where this topic is assessed. The learners' performance may affect their final examination if action is not taken to analyse their comprehension of ogive. Their poor performance is also a lost opportunity for learning ogives and related topics from Grade 11. The study attempts to investigate how and why these learners fail to comprehend and correctly read and plot problems related to ogives.

As secondary mathematics teacher, I observed that learners have challenges in plotting and reading of statistical graphs (pie chart, box plot, histogram, ogive; etc.). The challenges depend on the type of the graph. For example, learners find it easy to plot graphs with single axis than those that have two axes (Friel & Bright, 1995, 1996; Wu, 2004). An ogive is a

statistical graph that has two axes. Another observation is on the type of data collected. Learners find it easy to plot graphs that need discrete values (box plot, pie charts, and bar graphs) than those that need cumulative frequency values (Wu, 2004). For example, a histogram and an ogive are not easy to ploy when compared to bar graph and pie chart (Wu, 2004). An ogive uses cumulative frequency data. The Grade 12 learners are chosen because they have done ogives in Grade 11 (see DoE, 2003, 2007) and exhibit a lack of comprehension of ogives. Moreover, ogives are examinable in previous Grade 12 final examination papers. If, as mathematics teacher, can better understand learners ways of working with ogives, I can adapt my teaching to help them.

Background to the study

Data Handling is new to the South African school context and was introduced in Grade 10 – 12 (FET band) in Curriculum 2005 and National Curriculum Statement (NCS) (DoE, 2003). No adequate study of statistical education existed in the past in order to capacitate both teachers and learners (Steen, 2001). Data Handling intends to develop learners' statistical capacity and can motivate learners to choose statistics for career purpose. These assertions are significant to call for the research that will contribute to statistical education that develops statistical capacity and utilitarian value to both learners and teachers (Steen, 2001; Wu, 2004).

The purpose and research questions

The purpose of this study is to investigate the understanding of Grade 12 Mathematics learners when they read and plot data using ogive. The general question is: how and why these learners fail to comprehend and correctly read and plot problems related to ogives. The following sub-questions underpin the research problem and are investigated:

- What understanding and reasoning do Grade 12 mathematics learners have when they read and extract information from cumulative frequency tables and ogive?
- What understanding and reasoning do Grade 12 mathematics learners have when they plot an ogive and draw a cumulative frequency table?
- What reasoning do learners have when they interpret the ogives?

The study attempts to analyze and describe graph comprehensions, mathematical understanding and statistical reasoning learners apply when they engage with ogives. The study does not intend to investigate teaching strategies/approach a teacher may employ to advance learners' understanding of ogives.

Theoretical background

Theoretical background reviews three concepts, namely, statistical reasoning taxonomy, mathematical understanding and statistical graph comprehension. The study will not review literature on how teachers can develop learners' statistical reasoning, mathematical understanding and graph comprehension.

Curriculum demand in terms of statistical reasoning, mathematical understanding and graph comprehension: Curriculum demand on Data Handling envisage a learner who is "able to collect, organize, analyze and interpret data to establish a statistical probability models to solve related problems." (DoE, 2003, p. 38). Among its Critical Outcomes, the NCS (2003) stipulate that the learner envisage should be able to "solve and make decisions using critical and creative thinking" and "communicate their ideas by using visual, symbolic and language skills in various models." (DoE, 2003, p. 2). For learners to be able to analyze, interpret, be critical and creative thinkers, and communicate effectively; they should have a deep understanding and higher reasoning capacity on Statistics. Graph comprehension (discussed below) enables learners to organize, analyse and communicate their ideas using graphical representations where informed decisions can be made (Friel & Bright, 1995; Friel et al., 2001)

Statistical reasoning: Understanding and reasoning are concepts that are central to teaching and learning discourse. Statistical Reasoning, Statistical Thinking and Statistical Literacy are current concepts on Statistics education and serve as critical outcomes for teaching and learning statistics at school level (Garfield, 1998, 2002; DelMas et al., 2002; Tempelaar et al., 2006; Lee & Lee, 2010). The three concepts are interrelated, overlap and arrange in hierarchical order from Statistical Literacy, Statistical Thinking and Statistical Reasoning (Garfield, 2002; DelMas et al., 2002). DelMas et al. (2002) asserted that each of the three statistical concepts is an outcome on itself and can be assessed separately

Garfield (2002) define statistical reasoning as learners' ability to manipulate statistical content and the skill learners demonstrate in using statistical concepts in a specific problem solving. The learners have Statistical Reasoning competence when they are able to analyze, interpret, make inference and judgments about statistical information that is embedded in graphical representations (histogram, an ogive, pie chart, etc), summaries of data (correlation, population, sampling, probability) and statistical measure (spread and central tendency) (Garfield, 1998, 2002; Tempelaar et al., 2006; Lee & Lee, 2010). Central to the definition of Statistical Reasoning is that learners should be able to communicate statistical concepts and content within a specific context using various statistical modes (computations,

representation, summaries, etc) (Garfield, 2002). In this study, learners possess statistical reasoning competency when they are able to interpret and make inferences from computation of data and the reading of data from ogive.

The competence of why and how statistical investigations are conducted is referred as Statistical Thinking (Tempelaar et al., 2006; Lee & Lee, 2010). Statistical Thinking is realized when learners understand which method is suitable to collect data and why a particular representation (graphs, table) is used to display a collected information or data. Lee & Lee (2010) further assert that Statistical Thinking includes the recognition and understanding of entire investigation process of collecting, organizing, analyzing and interpreting data. An ogive is a model of data collection and representation and it can be used to analyse and interpret organized data. The transition from cumulative frequency table to an ogive, and vice versa; are aspects of statistical thinking. Learners should recognize items that a particular for the transmitting data from one model to another.

Statistical Literacy embodies understanding and application of basic statistical language and tools (Garfield, 1998, 2002; delMas et al., 2002; Tempelaar et al., 2006; Lee & Lee, 2010). Learners are perceived to have Statistical Thinking competence when they know how to calculate statistical measures (spread and central tendency, sampling and population, probability, etc.), know the meanings of definitions of statistical terms and can differentiate them (especially mean and median, sampling and population), know how to analyze and interpret data from graphs and answers; and be able to use statistical symbols and formulae. In this study, the task given to learners does not assess Statistical Thinking but statistical language and tools will assist the learners to answer questions that demand statistical reasoning. The appropriate use of a tool depends on the learners' knowledge of the meaning of the terms (language). For example, learners can be able to calculate quartiles if they know what are quartiles.

A task should have a multiple representation of date, interrelationship between or among concepts and different statistical ideas that develop statistical reasoning (Garfield and Ben-Zvi, in press). The task given to Grade 12 learners for the purpose of this study have both multiple representation of data (cumulative frequency table and ogive) and interrelationship among ogive, five number summary and percentage calculations; and comparisons of two ogives where inference is sought (see Appendix A). The statistical task, therefore, entails aspects of Statistical Literacy, to less extend Statistical Thinking and mainly Statistical Reasoning.

Mathematical understanding: Skemp (1976) identified two types of mathematical understanding as instrumental and relational understandings. Learners have instrumental understanding if the task elicits the application of rules, technique and procedures through the memorization of cues, symbols and formula (Skemp, 1976). Relational understanding is when learners have conceptual understanding (Abdul-Rahman, 2005), that is, learners know how to apply rules, why they apply particular rules in different contexts (Skemp, 1976) and they are able to derive meaning from their answers. An instrumental mathematics does not have context except skill of computation, and relational mathematics have real or constructed contexts that which learners are required to make sense of relationships, descriptions, judgments and inferences embedded in the context.

Taxonomy to assess statistical reasoning: The Statistical Reasoning Assessment (SRA) is taxonomy for statistical reasoning published with different focus areas on statistical concepts by Garfield (1998, 2002) and DelMas et al. (1999). Garfield‘s SRA assesses learners’ correct and incorrect reasoning for statistical concepts (Tempelaar et al., 2006). The types of statistical reasoning entailed in the model are idiosyncratic, verbal, transitional, procedural and integrated process reasonings (see Garfield, 2002, p. 8). They are listed in ascending levels from 1 to 5 respectively; and no indication that they are hierarchical and/or interrelated (cf. Table 2.). Levels 1, 2, 3 & 4 have connotations with instrumental understanding.The implication may be that learners know the meaning of statistical words (frequency, cumulative frequency), formulae and application (quartiles) of concepts, calculating (procedural reasoning), and can move data from cumulative frequency tables to ogives and vice versa (transitional reasoning) but they cannot understand their meanings and application within an ogive. Learners also can know computational rules, formulae and procedural competence but they cannot integrate and interpret the results (DelMas et al., 1999). Levels 3, 4 & 5 are consistent with rational understanding because the learner understands both the procedural rules, definitions of concepts and formulae; and how to explain the results and make judgments based on data on the ogive.

Statistical Reasoning Assessment	**Examples based on ogive**
Level 1: Idiosyncratic reasoning. The student knows some statistical words and symbols, uses them without fully understanding them, often incorrectly, and may scramble them with unrelated information.	A learner knows the terms frequency and cumulative frequency but uses them incorrectly, that is, for a question that requires a reading of cumulative frequency a learner gives a frequency as answer.
Level 2: Verbal reasoning. The student has verbal understanding of some concepts, but cannot apply this to actual behaviour.	A learner can explain the skewness of data distribution base on the mean and median but do not understand why.
Level 3: Transitional reasoning. The student	A learner can construct an ogive from

is able to correctly identify one or two dimensions of statistical process without fully integrating these dimensions.	cumulative frequency table, and vice versa.
Level 4: Procedural reasoning. The student is able to correctly identify the dimensions of a statistical concept or process but does not fully integrate them or understand the process.	Learner knows how to find five number summary but cannot identify and locate them in the ogive.
Level 5: Integrated process reasoning. The student has a complete understanding of a statistical process, coordinates the rules and behavior. The student can explain the process in his or her own words with confidence	Learner has complete understanding of calculating statistical measures by using relevant formulae and procedures. The learner can explain, justify, interpret and infer the results so as to communicate them using the ogive.

Table 1: Model for statistical reasoning. (Adopted from Garfield, 2002, p. 8).

Graph comprehension: The ability of a learner to read, plot and interpret statistical graph is referred as graph comprehension (Curcio, 1987, Fiel et al., 1995, 2001). Graph comprehension entails graph reading, graph construction and graph interpretation (Friel et al., 2001; Wu, 2004). Numerous researchers used Curcio's taxonomy for assessing leaners' graph reading when answering statistical graph (see Curcio, 1987, p. 384; Friel & Bright, 1995, p. 3; Friel et al., 2001, p. 130). Curcio's taxonomy for graph reading lists reading the data, reading between data and reading beyond data. Reading the data is when information asked is extracted directly from the graph and/or table; and it is explicit and obvious from the graph, that is, learners need not to calculate nor analyse in order to answer.

Reading between data is when relationships are made between data in the table and/or graph and mathematical operations (addition, subtraction, etc.). Learners are required to make simple calculations (it may be percentages) and answers are read from the table or graph. Friel and Bright (1995) explained that reading between the graph include questions where learners are required to make use of mathematical inequalities (e.g., greater than, greatest, less that, etc.). In cumulative frequency tables or an ogive, learners may be required to read number of participants that got 30% and less.

Reading beyond data includes interpolation, extrapolation and predicting the effects of data based on the graph (Friel & Bright, 1995; Friel et al., 2001). Learners are expected to give answers to questions that require prior knowledge that is related to the graph (Friel & Bright, 1995). Learners are required to manipulate and analyse data in order to suggest the trend of data, that is, to give data meaning. In an ogive, when learners are required to determine top 25% of participants in the test who are going to be given calculators; learners are expected

to calculate the cut-off mark (when the total of the test is not 100) and state the number of participants who will get calculators (cf. Appendix A).

Apart from graph reading, graph comprehension includes graph construction and graph interpretation (Friel et al., 2001, Wu, 2004)). Wu (2004) study included graph evaluation as the fourth skill that can assess learner's graph comprehension skills when answering statistical graphs. The descriptors for graph construction are shape, labels, scales and specifiers (Friel et al., 2001; Wu, 2004). In the cumulative frequency table, class interval denotes the scale. In the ogive, a scale refers to values assigned to vertical and horizontal qualities. A vertical scale represents cumulative frequency values and horizontal scale represent upper limit of class intervals of the quality/context understudy in the task. The shape of an ogive exhibits an increasing curve that increases at different rates at different points because of cumulative nature of frequencies. Graph evaluation is determined by whether the graph plotted and/or table are correct or not (Wu, 2004). If the characteristics (labels, scales, specifiers, shape) of an ogive are correct in a plot then an ogive is evaluated as correct.

Wu (2004) describe that graph interpretation deals with inference made from the data shown in the graph based on given condition. Graph interpretation may assess learners' comparisons of two tests by using common statistical measures and learners are expected to interpret the results based on the ogive.

Reading data directly from the graph and table is the easiest skill of graph reading (Curcio, 1985; Friel & Bright, 1995, 1996; Friel et al., 2001; Wu, 2004). Reading between data and reading beyond data are a challenge for the learners because learners are required to make integrations of computations, comparisons, extending the data and analyse data from the graph (Friel & Bright, 1995, 1996; Friel et al., 2001).

The difficulty in graph construction is due to the type of the data embedded in the graph and/or table. Friel et al. (2001) and Wu (2004) found that it is easier for learners to plot graphs that require discrete values than frequency values. In this study, an assumption is made that an ogive and its table are special cases of frequency values and are therefore difficult to construct. The difficulty in frequency value graphs is as the result of effects of scale and specifiers of the graph (Wu, 2004). It is also found that single axis graphs are easier to plot than two axes graph (Curcio, 1985; Friel & Bright, 1995, 1996; Friel et al., 2001; Wu, 2004). An ogive is a two axes graph therefore it may be difficult for learners to plot. The graph interpretation is the most difficult skill of graph comprehension (Friel & Bright, 1995; Friel et al., 2001; Wu, 2004). It is difficult because learners are required to explain,

justify or compare data from graphs and also infer their results base on the data in context (Wu, 2004).

Methodology, design and data collection

The research applies qualitative approach to analysis Grade 12 Mathematics learners when they read and plot the Cumulative frequency tables and curves (ogive). The qualitative approach may reveal multiple perspectives held by learners (Leedy & Ormrod, 2005) on their written work. Secondly, the study attempts to describe and interpret (Leedy & Ormrod, 2005; Opie, 2010) learners' understanding and reasoning when they deal with ogives. The Grade 12 learners are a case study because the study wants to have in-depth understanding (Leedy & Ormrod, 2005; Opie, 2010) of learners' understanding and reasoning on ogives. Case study is undertaken on "real situation, with real people in the environment often familiar to the researcher" and the "interactions of events, human relations and other factors are studied in the unique location" (Opie, 2010, p. 74). The researcher is familiar with the Grade 12 learners as their teacher and learners' response are analysed and interpreted from their written work. The case study will categorise learners' responses on the statistical task of cumulative frequency tables and ogive, interpret each response in terms of performance and identify patterns on Grade 12 Mathematics learners' understanding and statistical reasoning of ogive and cumulative frequency tables (Leedy & Ormrod, 2005).

This researcher has chosen his Grade 12 learners of the school that he is teaching as his case study because learners are easily accessible to him to collect data. The findings of this study cannot be generalized to other Grade 12 learners of other school but the focus of this research is to investigate salient factors that stymie Grade 12 learners' comprehension of ogives.

Participants: There are 31 Grade 12 Mathematics learners who participated out of 76 because of parents' consents and learners' voluntary participation in this study. The learners are both males (n=13) and females (n=18) between the ages of 16- 19 years. They study either Natural Science (n=23) or Commerce Streams (n=8). Learners passing percentage of Grade 11 Mathematics ranges from 19 % to 67%. The Grade 12 Mathematics learners are purposefully selected because they will avail the appropriate information needed for this research project.

Collection of data: Learners are given a statistical task to write. It will take sixty (60) minutes to allow learners an ample time to complete the task. A marking guide (Appendix B) is used to assess learners' response. The researcher marked learners' answer sheets and

his colleague moderated the whole batch to maintain consistence and credibility of data collection. The statistical task is more structured so as to collect data of all learners' responses in the same way and at the same time (Leedy & Ormond, 2005). These responses are analysed to evaluate learners' understanding of cumulative frequency. The task has question 1 and 2 (cf. Appendix A).

The task cognitive demands are classified using the National Curriculum Statement Cognitive levels (DoE, 2007). The cognitive descriptors are awarded weighted percentages and the Mathematical task is deemed to be standardized if it has approximately the same percentages listed per descriptors/levels. Table 2 depicts the classification of cognitive demand of statistical task cognitive demands. The classification was done by the research and his colleague independently and there was no discrepancy.

Procedure of collecting data: The ogive is examined in Grade 12 as Grade 11 previous/acquired knowledge. The statistical task will be given to learners after school hours so as not to disturbed teaching and learning contact time. The learners responses will be marked the same as per marking guide.

Cognitive descriptors	**Question number**	**Marks allocated**	**Percentage**
Knowledge (25%)	1.1; 1.2; 1.3; 1.7; 2.1;	6	23.1
Routine Procedures (30%)	1.4; 2.3;1.5; 2.4; 2.6; 2.7	8	30.8
Complex procedures (30%)	1.6;1.8; 2.2; 2.5;	9	34.6
Solving Problems (15%)	2.8	3	11.5

Table 2: Task cognitive demand.

Results

The summary of results of learners' responses is shown on Table 3. Learners got an average percentage of 37.7% on graph reading (A). Questions 2.4 and 2.6 demanded routine procedure and Question 1.2 and 2.1 required recall of knowledge. Learners' performance in reading between data (A2) is moderate at an average score of 55.5%. They got 100% in question 1.1 because its cognitive demand is a recall of knowledge, have

transitional reasoning and learners uses instrumental understanding. No achievement is recorded in question 2.7 because it demands a routine procedure and procedural reasoning and learners should have relational understanding. Reading between data and reading beyond data are a challenge for learners and concurs with findings of Friel and Bright (1995, 1996), Friel at al. (2001).

In graph construction (Category B), average percentage is 79.85. A substantial achievement of 61.3% is recorded for specifiers (B4). Specifiers of cumulative frequency table recorded 58.1% while for an ogive is 64.5%. Learners were unable to write class intervals correctly therefore lack of relational understanding. A 58.1% compared to 64.5% reflect that learners are more competent in plotting ogives from cumulative frequency tables than drawing a cumulative frequency table from an ogive. Learners' transitional reasoning is bias in favour of plotting an ogive. Most of textbooks activities (exercises) require learners to plot graphs from tables and rarely the way around.

Code	Correct response	Wrong response	No attempt	Statistical Reasoning Assessment	Total	Correct Average %
A1: Reading data	54	64	6		124	43.6
1.2	27	4	0	None	31	87.1
2.1	20	11	0	None	31	64.5
2.4	6	22	3	Procedural	31	19.4
2.6	1	27	3	Procedural	31	3.2
A2: reading between data	117	66	3		186	55.5
1.1	62	0	0	Transitional	62	100
1.3	23	8	0	Procedural	31	74.2
1.4	20	11	0	Procedural	31	64.5
1.5	12	18	1	Procedural	31	38.7
2.7	0	29	2	Procedural	31	0
A3: reading beyond data	36	136	15		186	14.5
1.6	5	26	0	Procedural	31	16.1
1.7	4	24	3	Procedural	31	12.9
2.3	27	57	9	Procedural	93	29
2.5	0	29	3		31	0
Overall average percentage of graph reading is 37.7%						
B1: shape (1.8)	27	6	0	Transitional	31	87.1
B2: labels	54	8	0	Transitional	62	87.1
1.8	26	5	0		31	83.9
2.2	28	3	0		31	90.3

B3:scales	52	7	3	Transitional	62	83.9
1.8	27	3	1		31	87.1
2.2	25	4	2		31	80.7
B4:specifiers	38	24	0	Transitional	62	61.3
1.8	20	11	0		31	64.5
2.2	18	13	0		31	58.1
Overall average percentage of graph construction is 79.85%						
C1: graph evaluation	42	20	0		62	67.7
1.8	22	9	0		31	70.9
2.2	20	11	0		31	64.5
Overall average percentage of graph evaluation is 67.7%						
D1: graph interpretation (2.8)	5	82	6	Integrated process	93	5.4
Overall average percentage of graph interpretation is 5.4%						

Table 3: Summary of results of learners' responses.

Graph evaluation (Category C) has an overall average percentage score of 67.7%. The correctness of an ogive scored 70.9% and cumulative frequency table is 64.5%. These scores are consistent with learners' scores on specifiers (B4) because the correctness of the graph depends on how well the graph was constructed. In graph interpretation (Category D), learners got an average score of 5.4%.

Category D requires learners to make an inference by comparing data from two Questions, i.e. 1 & 2 (cf. Appendices A and B, Questions 1.6, 2.5 and 2.8). No learner got full three (3) marks. The cognitive demand of Question 2.8 is solving problems, which has relational understanding. Question 2.8 therefore, required learners to apply their integrated process reasoning by comparing data or results from Question 1 and 2. Learners have challenge on relational understanding and integrated process reasoning on the task that required problem solving.

Errors. The study observed many errors as classified in Table 4.2 Table shows five error categories and their percentage of learners who committed each error category at least once.

Error category	Error average percentage
E1: comprehension errors	62.1
E2: Calculation errors	43
E3: Plotting an ogive	20.2
E4: Drawing table errors	23.7
E5: Interpretation errors	94.6

Table 4: Category of learners' errors

Category E1 is about learners' comprehension skills. E1 are those shown by learners who do not understand what a particular question asked for at least once. Comprehension errors are mostly committed in reading beyond data and reading between data with 18 learners who do not attempt answering questions based on them. Reasons attributed to comprehension error is that questions, although fair (in terms of phrasing/language) for Grade 12 learners, are not familiar as textbooks activities. Category E2 is about learners performing a mathematical manipulation and errors are committed at least once. Calculation errors are mostly observed in reading between data (53.2%) and reading beyond data (81.1) because the two themes require calculations (procedural reasoning). No learner got Question 2.7 correct and no pattern is observed. Learners gave different answers and this may be attributed to comprehension skill. Other learners gave various answers with and without calculations. The learners' calculation errors may be attributed to both misunderstanding of the questions and computational errors. There are no error patterns for Questions 1.7 and 2.3. The learners' errors may be attributed to misunderstanding of the questions more so Question 2.3 which has 9 non-attempts. Question 2.3 is fair because learners had calculated median, lower and upper quartiles in Grade 11.

Category E3 (plotting an ogive) refers to learners' errors on shape, scale and specifiers exhibited on ogives. Learners' errors are mostly on specifiers (35.5%). Most E3 depicts an ogive that is not grounded, specifiers (coordinates) and scales are not correct. The reason for error is that both specifier and scales are determined by cumulative frequency which Wu (2001) find that cumulative frequency poses a challenge for learners. The errors on specifiers and scale may be attributed to calculations. Category E4 (drawing table errors) are on labels, scales and specifiers. About 9 learners did not write label: cumulative frequency. Another factor that affected errors on scale and labeling is that learners wrote the frequency and/ cumulative frequency values without writing class intervals. Category E5 (Interpretation errors) deals with errors observed in comparing two Questions (Q1 & Q2). The error average

is 94.6% and the highest on errors committed by learners. No learner got full three marks. Learners did not used answers from Questions 1.6 and 2.5 of which the comparison should have been inferred. About 83% of learners wrote yes or no. Others have written only the choice without substantiating. The question was fair because it required learners to use context and calculations. The reasons for learners' performance in these questions may be comprehension skills, calculation and interpretation skills.

Reflecting on research sub-questions

- What understanding and reasoning Grade 12 mathematics learners have when they read and extract information from cumulative frequency table and ogive?

Learners had difficulty with reading beyond data (A3) where learners scored 14.5%. Question on A3 require learners to compute data and compare or analyse the results base on the ogive and/or table (Cursio, 1987; Friel & Bright, 1995, 1996; Friel et al., 2001). Errors committed by learners suggest that they lack both comprehension and computation skills. The skills required learners to apply procedural reasoning and have relational understanding. In questions 1.6 and 2.5 (16.1% & 0%, respectively), learners were unable to apply procedural reasoning where they could have procedurally explain which procedure/process to apply to answer the questions. Again learners know the meaning and calculation of median, lower quartile and upper quartile from Grade 11 because these measures appear in five number summary of box and whisker plot. Category A3 required learners' relational understanding. Learners did not display relational understanding because they know how to calculate but cannot link their answers to either the ogive or cumulative frequency table nor do they understand the meanings of their answers.

Learners got an average of 37.1% in reading the data (A1). Category A1 (reading data) entails the extraction of data from the printed page to answer a question that is on the graph (Friel &Bright, 1995, 1996; Friel et al., 2001). It is expected that A1 should not be difficult for learners because reading the answer from the graph and table is a skill learners have from other graphs (e.g. algebra). Learners' errors on A1 reveal lack of comprehension skills. Learners' poor performance is reflected in their lack of procedural reasoning where instrumental understanding has to be applied. Learners also have a prior knowledge of reading data from other statistical graphs (e.g. pie charts, bar graph, histogram, box plots, etc.) encountered in lower grades (Curcio, 1987). Friel et al. (2001) alluded on sequencing of graphs develops graph comprehension. Therefore, learners' instrumental understanding is not yet developed when they read data from an ogive and tables. It hampers the development of relational understanding because it depends on instrumental for it to be

effective (Skemp, 1976). Learners seem to have a good grounding in transitional reasoning because they completed the cumulative frequency table with 100%. They still lack procedural reasoning when the read data from an ogives. These may be affected by cumulative, scales and specifiers (Friel & Bright, 1995; Kelly et al., 1997; Friel etal., 2001; Wu, 2004).

- What understanding and reasoning do the Grade 12 Mathematics learners have when they plot an ogive and draw a cumulative frequency table?

The sub-question is assessed by employing graph construction (B) and graph evaluation (C). Learners exhibit an understanding of plotting an ogive and table as evident in an overall score of 79.85% and 67.7% for Categories B and C respectively. Learners who were able to draw an shape of an ogive, labeled horizontal and vertical axes and jotted points accordingly (specifiers); were most certainly to get graph evaluation correct. The plotting of an ogive and drawing of table errors (20.2% and 23.7% respectively) suggest that learners have few mistakes or misunderstanding of plotting and drawing of cumulative frequency. Learners depicted both instrumental and relational understanding on graph construction. Instrumental understanding is manifested when they labeled an ogive and cumulative frequency table correctly; and they were able to draw shape of an ogive. Their relational understanding is realized in both scales and specifiers. Errors committed were less, that is, 18.9% and 11.9% respectively. Learners know that any graph (including algebraic functions) has axes, they must be labeled, specifiers be identified and the particular shape of the graph be maintained. This suggests that learners effectively applied transitional reasoning because they used these graph components to represent data in ogive and be able to move data from tables to graph and vice versa.

- What reasoning learners have when they interpret data from an ogive?

The results suggest that learners have failed to compare and interpret data displayed in Question 1 and Question 2. Learners failed to use answer of Questions 1.6 and 2.5 to infer which test between mathematics and programming was easier to get calculators. Learners were unable to comprehend what the question required because most of them arbitrarily chose one test without explaining their choice. The error analysis suggests that the difficulty is on comprehension, computational and interpretation skill. Learners had missed an opportunity to use a graph/ogive to analyze data and to communicate the information embedded on it (Friel et al., 2001). The purpose of statistical graphs and tables is to reduce a large data, organize it and to make interpretation and inference so that a meaningful decision can be made (Curcio, 1987; Garfield, 1998, 2002). The learner performance (5.4%)

therefore suggest that learners do not possess an integrated process reasoning because they cannot explain, justify and make an informed decision based on the manipulation of statistical measures to make effective communication. Learners lack relational understanding because unable to calculate, and if they do; they cannot integrate their results within the context. They cannot infer on their calculations to say what information their calculation suggests.

Findings

The graph interpretation tasks were challenging for the learners' because the tasks required learners to apply statistical reasoning based on data that is embedded in the graph(s). Secondly, as Wu (2004) suggests, graph interpretation is also inhibited by learners' language difficulties in expressing themselves. The comprehension errors (62.1%) are language related because learners have to read a question with comprehension.

Graph components (shape, labels, scales and specifiers) serves as prior knowledge to any statistical graph. Learners have an adequate transitional reasoning of moving data from table to an ogive and vice versa.

The ogive uses cumulative frequency data and Wu (2004) found that frequency data was difficult to read than discrete values from graphs. Cumulative frequency makes reading between data and reading beyond data difficult for learners.

Conclusion and implication for teaching practice

The study concludes that: (a). learners have difficulty in reading between data and reading beyond data which required procedural reasoning and relational understanding; (b). learners have adequate skills on graph construction and sufficient transitional reasoning; and (c). lastly, learners have a serious difficulty of graph interpretation and integrated process reasoning.

The findings of the study have implication to teaching of statistical graphs. I recommend that teacher should assist learners in graph reading especially reading between data and reading beyond data. Teachers may either or both discuss how data can be read from statistical graphs or/and emphasise graph reading aspects during learners' responses on classworks and/or homeworks. The study further recommend that learners should be given tasks on statistical graphs that develop their analysis and interpretation of data embedded in the graphs; and statistical graphs should elicit both instrumental understanding and relational understanding from learners. The graph comprehension, statistical reasoning and

mathematical understanding are skills and pedagogical concepts that are also manifested in other function/graphs (algebra, trigonometry). Secondly, these skills and concepts have propensity to develop an envisage learner stipulated in NCS (2003). Among its Critical Outcomes, the NCS stipulates that the learner envisage should be able to "solve and make decisions using critical and creative thinking" and "communicate their ideas by using visual, symbolic and language skills in various models." (DoE, 2003, p. 2).

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APPENDIX A

Question 1

The following frequency table shows the performance number of learners in Grade 12 Mathematics Test.

Marks	Frequency	Cumulative frequency
$0 < x \leq 10$	5	
$10 < x \leq 20$	8	
$20 < x \leq 30$	17	
$30 < x \leq 40$	15	
$40 < x \leq 50$	3	
$50 < x \leq 60$	2	

1.1 Copy and complete the table.

1.2 How many learners wrote the test?

1.3 How many learners got 20 and less?

1.4 How many learners got more than 40?

1.5 How many learners got between 10 and 40?

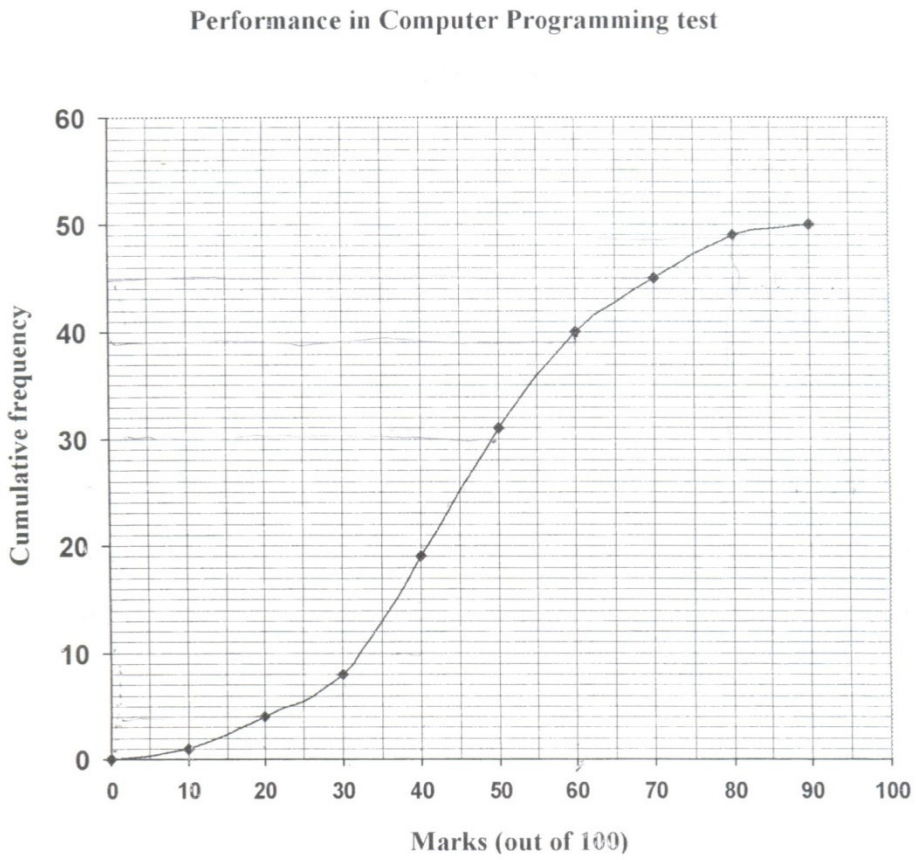
1.6 Only the top 25% of the learners are going to be given calculators. Determine the cut-off mark to determine the top 25%.

1.7 Approximately, how many learners will get free calculators?

1.8 Draw a cumulative frequency curve. (ogive).

Question 2

The ogive (cumulative frequency graph) shows the performance of students who took a test in basic programming skills. The test had a maximum of 100 marks.



2.2. Construct a frequency table for the information given in the ogive.

2.3. Determine the median, lower quartile and upper quartile.

2.4. If the passing percentage is 40%, how many learners who “failed “the test?

2.5. Only the top 25% of the learners are going to be given calculators. Determine the cut-off mark to determine the top 25%. How many learners who got top 25%?

2.6. What percentage range is received by 30 to 40 learners?

2.7. How many learners who got between 30 to 70%?

APPENDIX B.

Marking guideline.

Question 1.

<table>
<tr><td>An item</td><td>Marks allocation</td></tr>
<tr><td>1.1

<table><tr><td>Marks out of 55</td><td>Frequency</td><td>Cumulative frequency</td></tr><tr><td>$0 < x \leq 10$</td><td>5</td><td>5</td></tr><tr><td>$10 < x \leq 20$</td><td>8</td><td>13</td></tr><tr><td>$20 < x \leq 30$</td><td>17</td><td>30</td></tr><tr><td>$30 < x \leq 40$</td><td>15</td><td>45</td></tr><tr><td>$40 < x \leq 50$</td><td>3</td><td>48</td></tr><tr><td>$50 < x \leq 60$</td><td>2</td><td>50</td></tr></table></td><td>1: cumulative frequency (correct/wrong)

1: correct</td></tr>
<tr><td>1.2. 50 learners wrote the test</td><td>1</td></tr>
<tr><td>1.3. 30 learners</td><td>1</td></tr>
<tr><td>1.4. 5 learners</td><td>1</td></tr>
<tr><td>1.5. 40 learners</td><td>1</td></tr>
<tr><td>1.6. 40 marks</td><td>1</td></tr>
<tr><td>1.7. 5 learners</td><td>1</td></tr>
<tr><td></td><td></td></tr>
</table>

Question 2.

<table>
<tr><th>An item</th><th>Marks allocation</th></tr>
<tr><td>2.1. 50 learners</td><td>1</td></tr>
<tr><td>2.2. Performance in computer Programming test
<table>
<tr><th>Marks out of 55</th><th>Frequency</th><th>Cumulative frequency</th></tr>
<tr><td>$0 < x \leq 10$</td><td>1</td><td>1</td></tr>
<tr><td>$10 < x \leq 20$</td><td>3</td><td>4</td></tr>
<tr><td>$20 < x \leq 30$</td><td>4</td><td>8</td></tr>
<tr><td>$30 < x \leq 40$</td><td>11</td><td>19</td></tr>
<tr><td>$40 < x \leq 50$</td><td>12</td><td>31</td></tr>
<tr><td>$50 < x \leq 60$</td><td>9</td><td>40</td></tr>
<tr><td>$60 < x \leq 70$</td><td>5</td><td>45</td></tr>
<tr><td>$70 < x \leq 80$</td><td>4</td><td>49</td></tr>
<tr><td>$80 < x \leq 90$</td><td>1</td><td>50</td></tr>
<tr><td>$90 < x \leq 100$</td><td>0</td><td></td></tr>
</table></td><td>1: label

1: scale (class intervals)

1: specifiers (cumulative frequency values corresponding to upper limits of class interval)</td></tr>
<tr><td>2.3. Median= 31
Lower quartile= 6
Upper quartile= 46</td><td>1
1
1</td></tr>
<tr><td>2.4. 19 learners</td><td>1</td></tr>
</table>

2.5. 4 learners	1
2.6. 22%	1
2.7. 37 learners	1
2.8. Top 25% cut off % for Mathematics test is 66% with 5 learners and for programming test is 75% with 4 learners. It was easier to get a calculator in Mathematics test because of 66% of cut off %	1: 66% for mathematics test 1: 75% for programming test 1: for saying mathematics test