Exploring moderating effects of reflection and mathematics confidence on the performance of a mathematics task

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Abstract

This paper focuses on reflection as a metacognitive skill and on mathematics confidence. The context of the inquiry is mathematics problem solving as an educational task. The purpose of this paper is to present the results and findings of a qualitative inquiry into Grade 8 and 9 learners' reflection and mathematics confidence as experienced during problem solving in Mathematics. The interpretivist approach that was followed employed a convenient sampling of four participants who were interviewed and video recorded while solving a mathematics problem. Verbatim transcriptions were analysed by means of a priori coding that identified the role of reflection and mathematics confidence. The researchers' analysis indicates that participants implemented metacognitive strategies through reflection on the task and the personal self. Through such reflection, participants displayed evidence of the dimensionality metacognition and mathematics confidence.

Introduction

This is an account of a descriptive study on the role that mathematics confidence and reflection play during mathematics problem solving. The qualitative study involves interviews held with four participants in senior phase mathematics and considers their reflective skills and the mathematics confidence they displayed.

Concern is often expressed about learners' mathematics achievement and the small number of learners that enrol for Mathematics in the Further Education and Training (FET) phase (Maree, Olivier & Swanepoel, 2004:54; DoE, 2010:13). According to Maree et al. (2004:85), South African learners perform inadequately due to a number of traditional approaches towards Mathematics teaching and learning. Maree (1997b:95) also classifies cognitive factors, external factors, internal and intra-psychological factors, as well as the facilitation of subject content as problems that affect study orientation.

One psychological factor in the Study Orientation in Mathematics (SOM) questionnaire developed by Maree, Prinsloo and Claassen (1997b) measured the level of mathematics confidence of grade 7 to 12 learners in a South African context. Sherman and Wither (2003:138) documented a case where a psychological factor, mathematics confidence, caused the impairment of mathematics achievement.

The phenomenon that learners reflect on their own achievement with fear, low self-motivation, distraction and some mental disorganisation (regarded as aspects of their higher-

order reasoning and reflective questioning) undermines confidence to engage with the mathematics task.

Purpose of and motivation for the research

The aim of this study is to explore the reflective strategies learners in the senior phase implement and to foster the mathematics confidence that these strategies entail. Research investigating the relationship between mathematics confidence and mathematics problem solving concludes that low confidence results in poorer performance (Maree; Prinsloo & Claassen, 1997b; Sherman & Wither, 2003:138; Ashcraft & Kirk, 2001). There is a paucity of literature on confidence in mathematics, the awareness of the metacognitive strategy, and reflection on mathematics (Jain & Dawson, 2009). This study attempts to overcome this scarcity in literature by focusing on individual differences in mathematics confidence and reflective thinking as a facet of metacognition. (Lesh & Zawojewski, 2007:764). It seems as though mathematics concept development and the development of problem-solving abilities such as knowledge and regulation should be part of mathematics education, and that metacognitive and affective factors should be taken into account.

Description of significant terms

A mathematics problem, defined as a mathematical-based task, indicates realistic contexts in which the learner creates a model for solving the problem in various circumstances (Chalmers, 2009:3). Making decisions within these contexts is only one of the elementary concepts of human behaviour.

Though the nature and development of mathematics problems are widely researched (Lesh & Zawojewski, 2007:768), especially with the focus on how learners see and approach mathematics and mathematical problems, Polya-style (heuristic) problems involve strategies such as picture drawing, working backwards, looking for a similar problem or identifying necessary information (English, Lesh & Fennewald, 2008:1; Lesh & Zawojewski, 2007:768). Confirming the use of these strategies, Zimmerman (1999:8-10) describes dimensions for academic self-regulation by involving conceptual-based questioning using a technique called prompting (asking questions on questions). Examples of these prompts are questions starting with why, how, what, when and where, in order to provide scaffolding for information processing and decision making. These prompts are also known as metacognitive strategies aimed at the regulation of knowledge and skills as a form of reflective thinking in the problem-solving process.

Individuals who experience panic, helplessness and mental paralysis when they are required to solve Mathematics problems, have low mathematics confidence. Reducing anxiety builds self-confidence and sets the basis for a favourable attitude towards doing mathematics. Strawderman (2010:7) explains that individuals have certain avoidance behaviour strategies that cause them to replace their engagement with mathematics with avoiding mathematics in everyday life.

Reflection, as defined by Glahn, Specht and Koper (2009:95), is an active reasoning process that confirms experiences in problem solving and relates to social interaction. Reflection can be seen as a transformational process based on previous experiences, affecting our objective way of thinking (Garcia, Sanchez & Escudero, 2009:1). According to Van der Walt (2008:79), all metacognitive knowledge is actively and collectively unified by reflection.

Concise literature review

English (2007:123-125) proposes powerful ideas for developing mathematics towards the 21st century. Some of these ideas include a social constructivist view of problem solving, planning, monitoring and communication; effective and creative reasoning skills; analysing and transforming complex data sets; applying and understanding school mathematics; and explaining, manipulating and forecasting complex systems through critical thinking and decision making.

In a technology-based information age, computation, conceptualisation and communication are basic challenges South Africans have to face (Maree, Prinsloo & Claassen, 1997; Lesh & Zawojewski, 2007). These challenges are constantly changing and have led to different ways of mathematical thinking and approaches to problem solving (English, Lesh & Fennewald, 2008:1). Developing problem-solving abilities for academic success should stretch even beyond school level. According to Kleitman and Stankov (2003:2), managing uncertainty in one's understanding is essential in mathematical problem solving and research into mathematical problem solving is becoming increasingly more complex than before (Lester & Kehle, 2003:510). Therefore, problem solving continues to gain consideration in the policy documents of numerous organisations, both nationally (DoE, 2010:3) and internationally (TIMSS, 2003; Moloi & Strauss, 2005).

Anxiety, an aspect of neuroticism, is often linked to personality traits such as conscientiousness and agreeableness (Morony, 2010:2). This negative emotion manifests in faulty beliefs that cause anxious thoughts and feelings (Thijsse, 2002:17). Some examples of anxiety are general anxiety, test or evaluation anxiety, problem-solving anxiety and mathematics anxiety. The latter is a widespread phenomenon and relates to moderating

effects in metacognition (Legg & Locker, 2009:30). In the presence of a supporting teacher, *in a transmission-type* classroom, mathematics anxiety reduces. In fact, Thijsse (2002:25) states: *Math anxiety*[8] *often begins with the teacher.*

Mathematics is probably the only subject causing this adverse reaction, as it is a clear and concentrated example of intellectual learning. Distinguishing between three domains of mathematics confidence, namely social, psychological and intellectual, Strawderman (2010:2) provides a range to compare Sheffield and Hunt's (2006:2) *math* anxious and non-*math* anxious learners.

It is essential that learners take control of their mathematics performance. This will encourage them to take control of other aspects of their lives as well. According to Ashcraft (2002:2) the experience of mathematics anxiety is related to gender, teaching approaches, ethnic background, age, attitude and previous experiences in the subject. It appears we all have some degree of apprehension towards mathematics. Stuart (2000:331) declares: *Anxiety is nothing else but a lack of confidence.*

Strawderman (2010:1) states that mathematics confidence is multidimensional. Both Newstead (1999:4) and McLeod (1993) argue that the root of mathematics confidence lies in the nature of classroom experiences as documented by Newstead (1999:7), McLeod (1993), Tobias (1978) and Stodolsky (1985). More recently, Legg and Locker (2009:42) conclude that metacognition has a moderating association with mathematics anxiety, and in turn, with mathematics confidence. Ashcraft and Kirk (2001) associate mathematics confidence with achievement in mathematics.

Lesh and Zawojewksi (2007:778) agree with Livingston (1997:2) who believes that mathematics concepts (cognitive) and higher order thinking (metacognitive) should be studied correspondingly and interactively. Identifying individual trends, behaviour patterns and feelings, could relate to success in mathematics problem solving (Lesh & Zawojewksi, 2007:778; Lesh & locker, 2007:28).

While some research claims that *seeing* and *doing* mathematics is useful in the interpretation of and decision making in problem-solving processes (Lesh & Zawojewski, 2007), a more affective approach would involve *feelings* or the feelings about mathematics (Sheffield & Hunt, 2006), in other words, affective factors. High levels of negative metacognitive beliefs

[8] Math anxiety – a colloquial, general term in literature – refers to mathematics anxiety. For the purpose of this study, which takes place in a South African context, math anxiety is rather understood as insufficient mathematics confidence.

are associated with various affective factors or a lack of confidence (Reeder; Rexhepi-Johansson & Wykes, 2009:50).

Metacognitive knowledge utilises planning in a goal-driven approach. Before, during and after the cognitive task, metacognition acts upon cognition and regulates cognition's product: performance (Jacobs, 2010:1). Literatures regarding metacognition confirm difficulty defining this higher (meta-) approach to cognition and cognitive tasks. With over thirty decades of investigating and exploring this realm of the mind, some researchers still quote Flavell (1979) by stating metacognition is simply *thinking about thinking* (Flavell, 1979; Legg & Locker, 2009:471; Lai, 2011:33).

Reeder, Rexhepi-Johansson and Wykes (2009:49) confirm the two dimensions of metacognition as knowledge of cognition and the regulation of cognition. Knowledge about one's own cognition includes knowledge regarding the functionality of the mind and insight. There are three sub-dimensions of this knowledge, namely knowledge of the person, task and strategies. The process of regulation refers to the control of cognition or the thinking process (Legg & Locker, 2009:473). It merges the processes of monitoring, planning and evaluation. Metacognition, as a skill, seems to rely on the reflective skills of the individual, thus completing the three stages: before, during and after the task reflection. Therefore, in this study, reflection is seen as parallel with metacognition. Reflection is in a sense the fuel that kindles metacognition.

After the three stages, any individual (as a reflective practitioner) experiences moments where synchronous reflection facilitates the so-called *aha* moment, which is a moment of acquired insight (Schon, 2010:3). This acquirement is a result of metacognitive reflection.

Reflection as a high-level cognitive thinking process can help with the solving of mathematics problems and understanding mathematics (Kaune, 2006:351). Kaune (2006:350) quotes Dubinsky (1991) when stating …*that we somehow move into another dimension when we reflect on what we have done.*

The following conceptual framework, derived from the theoretical framework, portrays the literature and the concepts for the current research design.

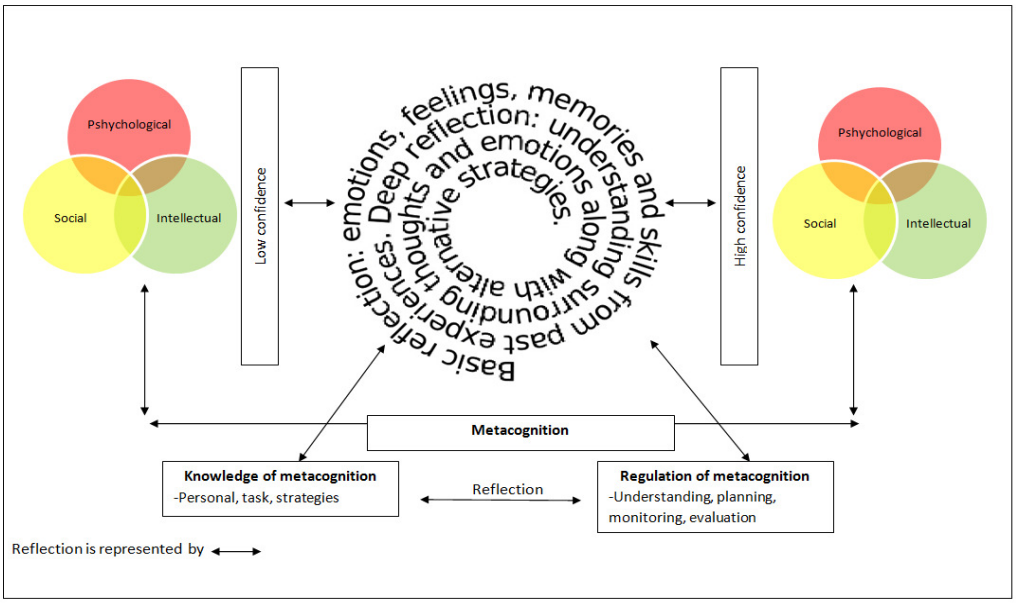

Figure 1: Concept map of mathematics confidence and metacognitive reflection

Source: Adapted from Strawderman (2010), Gibbs (1998) and John (2000)

The social domain involves external factors outside the individual's control such as family, friends and teachers (Strawderman, 2010:2). The psychological field linked with this domain is the behaviour of individuals, and ranges between involving oneself and avoidance of situations like mathematics activities or classes.

The intellectual domain entails cognitive influences. This includes skills and knowledge of the problem-solving procedures and strategies. Personal performance is measured in this domain and is associated with the field of personal achievement and the perception thereof (Strawderman, 2010:3). The psychological domain extends further because of affective factors. Emotional history, familiar experiences and stimulus reactions are associated with the individual's feelings of confidence, anxiety or discomfort, and pleasurable experiences.

Research question(s)

The primary question this study sought to answer is: what role does reflection and mathematics confidence play during mathematics problem solving in the senior phase?

The study consequently asks the following secondary research questions:

(i) Which reflection strategies or skills do learners in senior phase mathematics implement, if any?

(i) What does the mathematics confidence experienced by learners entail?

Research design

An exploratory qualitative research design was implemented. The researcher employed an interpretivist paradigm, as it seems fit to the description of the typology by Cecez-Kecmanovic (2011:5) of research paradigms in social sciences. Aiming to understand the social reality[9] as a phenomenon and to interpret the reasons for participants' actions and thoughts, the researcher did not comment on participants' attempts during the interview. The interpretation and explanations of the qualitative data were of a social constructivist nature. Approaches ranged between the solving of a mathematics problem, video recordings, field observations, and individual interviews.

Sample

Four participants were invited to take part in this qualitative study. The participants were learners in grade 8 and 9 who took Mathematics as part of their school subjects. Purposive convenient sampling (Maree et al., 2010:79) of the participants resulted in two learners with high achievement in Mathematics and two learners with average achievement. The specific participants were invited with the purpose of constituting a desired group with regard to mathematical achievement, gender and race.

The researcher, in partnership with the participants (Maree et al., 2010:41), explored the phenomena considering the variables, mathematics confidence and reflection during problem-solving activities, as internal processes.

Methodology

Data was collected in two stages consisting of individual interview sessions. In the first session (stage 1), participants solved a Mathematics problem based on the area of circles. The participants' attempt to solve the problem was video recorded. Once the problem was solved, the video recording was played back and occasionally paused, rewound and stopped to ask the participant semi-structured, open-ended questions about what he or she was doing or thinking at a particular moment. The participants answered questions about what they had done in each step, what they had been thinking and why they had done what they had done. The aim of this interview session was to answer the secondary research question, namely to explore which reflection strategies or skills learners in senior phase Mathematics were implementing.

[9] The social reality in the present study is learners' confidence in doing mathematics and their reflection on mathematical processes.

In the second interview session (stage 2), participants answered semi-structured questions regarding their experiences with Mathematics, currently and in the past. This was done in order to explore the mathematics confidence that learners develop based on their experience(s) with Mathematics. Three types of qualitative questions were asked: main questions, probes or exploring questions, and follow-up questions (Bormotova, 2010:69). The qualitative open-ended questions asked during the second session were adapted from questions by Simmer (2011:44-45). Some examples of these questions include the following:

(a) *Did the teachers you had for Mathematics have any effect on your fondness or dislike of Mathematics? How so? (Please give as much detail as possible.)*

(b) *In your opinion, what makes Mathematics difficult or easy to understand?*

(c) *What do you think being confident means?*

The use of semi-structured interviews had the following specific purposes: to identify the important variables; to formulate breakthrough questions about the variables, and to generate a hypothesis for further investigations (Welman, Kruger & Mitchel, 2009:197). Findings are represented in the two stages discussed below.

Findings

Stage 1: Interviews on the reflective skills that learners implemented during problem solving

Each participant read through the Mathematics word problem more than once and then drew three circles relating to the information provided. All four participants read, reread and double-checked their work as they wrote the diameters in/or next to each of their respective circles. They used lines and half lines inside the circles to illustrate diameters or radii. However, when they started with the actual calculations, each participant had his or her own peculiar difficulties from then onwards. For this reason, each participant's progress throughout the problem-solving process is discussed separately[10].

Learner A took 5 minutes to solve the given word problem. This learner monitored her work continuously and evaluated her answer using three different, but incorrect, strategies. The first difficulty this learner had was to remember the formula for the area of a circle. *I couldn't remember the formula, and so I just said the area of, and then I drew the circle and then I said length times breadth. I didn't think it was that.*

[10] Due to the summative nature of the paper, the analysis of findings will be represented and discussed by focusing only on one phase or facet of each participant's problem-solving attempt.

Having had trouble remembering the formula, the learner frowned and read the question again, this time in smaller parts, as if looking for something. Eventually she tried a different approach: *So I tried half of the diameter and then I put a question mark because I didn't know what it was. And then, I drew another circle and I thought area is, like shaded area is the whole circle and then I remembered what that formula was.*

She coloured in a fourth circle, not one of the original three, and remembered the formula, incorrectly: *I figured out that half of the diameter is not the formula and I think, I remembered what the formula was.* Feelings of confusion were aroused as she read the question repeatedly: *I was confused about what the formula was.*

She regulated her understanding and planning by writing and underlining the word *area* just before she remembered

I remembered the formula for the area. I couldn't remember what was the difference between circumference and area, and then I remembered that area has a squared at the end. I'm looking at the information that I wrote here (pointing with her finger*). I remembered that the shaded area is the area. And I just drew a picture for myself and that's when I remembered what exactly the formula was.*

Learner B solved the word problem in 6 minutes. Checking her work and the information given in the problem, she constantly managed to use self-talk to guide herself through the process. She realised, after monitoring her every step, that she was unsure about what to calculate, what formula to use and whether what she was doing was right.

What is the area of circle c? I wanted to write my formula so I went back to check what are we doing. Area? Perimeter or volume? So then I paused to double check; okay what is the area? So I specifically went to look for, if its area. I remember it from when we were doing circles. First I was looking at the pi and I was thinking is this right? … I first thought it was right so I went down to equals. And then I carried on. I think while I was reading the question I think I was…, wait – what was I thinking? I think I was thinking if the pi is right. If I put the r or if I had to do it a different way. Then I thought it was right so I carried on.

The learner realised that she had made a mistake, and after *checking* and correcting it, she paused:

Like every time you do it, a step, you go back to the beginning, so I think I recognized it after I changed the formula. (Looking down at her work and nodding her head*). I was thinking wait is the r supposed to come before or am I not supposed to put the multiple multiplication sign*

or am I supposed to put it so I wrote and then I thought wait – Is this right? I did long multiplication; I thought it would be much easier.

Learner C solved the word problem in the least amount of time, only four minutes. It appears that she too monitored her work, but not as much as learner A and B.

I'm calculating two comma five times two comma five. Now, I'm working out what is three comma one four times two comma two five. I was checking if it was a six, I was making sure what the rest of the number was, so I didn't write it wrong. I'm looking at this, (pointing towards her solution's answer*), because I wasn't sure if I should write the square or not, sir. Yes sir* (smiling shyly). At the end, the learner did not evaluate her answer: *I went back to the question a lot, like I didn't know I do that. I did not look at my answer again.*

Learner D took the longest time of all – altogether 8 minutes – to solve the problem. She identified a lack of strategy: *I have a problem with making sure my adding and subtracting is right, I was going over my remainder.* This learner had knowledge of other strategies that might help, including monitoring and evaluation. However, when reflecting on previous experiences, she realised that she didn't understand: *I knew the answer was wrong. I wasn't sure if the answer was right, so I was checking my remainder again to make sure so I got 77 and then I was checking basically the remainders, just to make sure it was right. Well I knew that was wrong once I remembered 8, but I just wanted to see if I did that, would it be right? Then I have to go back to see if I did add or subtract properly, the first time I got 3 and I don't understand why.*

From this brief overview of four participants' problem-solving attempts, it is evident that each utilised metacognitive strategies. The strategies included learners' reflecting on their own knowledge of skills, their person and tasks, as well as reflection on regulation of planning, understanding, monitoring and evaluation.

Discussion of Stage 1

Learners drew conclusions from the information in the problem, made generalisations and solved, at least partially, the non-routine problem by applying their knowledge of geometry in complex procedures. However, it appears that the four participants attained these benchmarks by reflecting on their available knowledge and skills in an attempt to complete the task.

The participants in the study read the problem more than once, indicating their regulation of understanding. This also shows that they reflected on their understanding and planning of the task. Drawings of pictures served as proof of their planning and reflection on their

understanding, knowledge and memories of strategies that they had learnt in the past. When faced with uncertainty and low mathematics confidence in the intellectual domain, they all monitored more frequently and employed other metacognitive behaviours. These behaviours include regulation by reflecting on other strategies and using memories of skills previously acquired. The learners realised what strategies or knowledge they did not have and tried to find other ways, other formulas, pictures, rewriting and underlining words to broaden their understanding. Each learner experienced moments of low and high confidence when reflecting on his or her work. Alternating between low and high mathematics confidence may depend on the level (basic or deep) of reflection (i.e. whether the participant has the necessary knowledge or strategies to reflect on or not). It also depends whether the learner understood or merely memorised the steps. One learner in particular mentioned feeling confused after reflecting on her knowledge of the formula. Nevertheless, she did not give up (high mathematics confidence) and later remembered the formula.

Constant monitoring by reflecting on their work was observed for all four participants. They monitored their performance after every step by looking back at the question, pictures, pointing with the finger or the pen towards something they had already written down. They also rechecked their answers on the calculator. They made sure that the values they substituted into their formula were correct.

Balancing the complexity of the problem is important. The difficulty level should be not too low or too high, as this might de-motivate the learner and achievement will suffer (Schunk, Pintrich & Meece, 2008:256). In this study, it seems as though learners did reflect. However, it is not sure what they reflected on. When asked what they were thinking (probing a reflective thought), participants probably answered what they remembered (actual reflection on task, person or strategy), what they thought they remembered (reflecting on newly acquired knowledge of the task), or what they ought to have remembered. They probably even reflected on their reflection. This requires the ability to move from basic reflection on emotions, feelings, memories and skills to a deeper reflection where they understand the connection between their emotions, alternative strategies and surrounding thoughts. It appears from the study that the four participants actually monitored their progress and alternated between states of confidence. Reflecting by monitoring seems to have caused uncertainty in those participants who lacked a sufficient amount of strategies or knowledge. They ended up reflecting only on what they knew they could do. Some strategies that might have worked in the past were either forgotten or not understood in the first place.

Stage 2: Interviews focusing on what the mathematics confidence of learners entails

Participants were asked questions regarding their experiences with Mathematics so as to determine what their mathematics confidence entailed. The three domains of Strawderman (2010) supported, as part of the conceptual framework, the identifying of aspects of learners' mathematics confidence. Their responses were categorised in two extremities, distinguishing between low and high mathematics confidence. Each end indicated confidence in the social, psychological and intellectual domain. The subcomponents for the social domain were identified as family matters, friends, teacher factors, and societal influences. Feelings, level of confidence, comfortableness and likes or dislikes were part of the psychological domain, whereas skills, strategies, success and failures regarding achievement were recognised as components of the intellectual domain.

Teachers, understanding and feelings appeared to play a crucial and integrative[11] role in all three domains. Learner C[12] mentioned that *the teacher just made the maths fun. The class was small, we were all close together. She would play around with us. Sometimes she gave us work to do; she would play music, nice music. We get a project to do, a fun project, a group project, creative and stuff.*

It is clear that this learner enjoyed the subject – however, she commented negatively on another teacher: *We got this other teacher. He was a male. I think it was the teacher that made me dislike maths. You don't like the teacher, so you don't concentrate.*

This transcription clearly shows how the three domains interactively affected each other and to what extent (low or high) mathematics confidence was experienced.

Figure 2 demonstrates this connection. It contains an explorative summary of one participant's feelings and views the learner's low and high mathematics confidence traits.

[11] Although other aspects of the domains were also explored, only three examples are given in this paper.

[12] Account is given of one participant only.

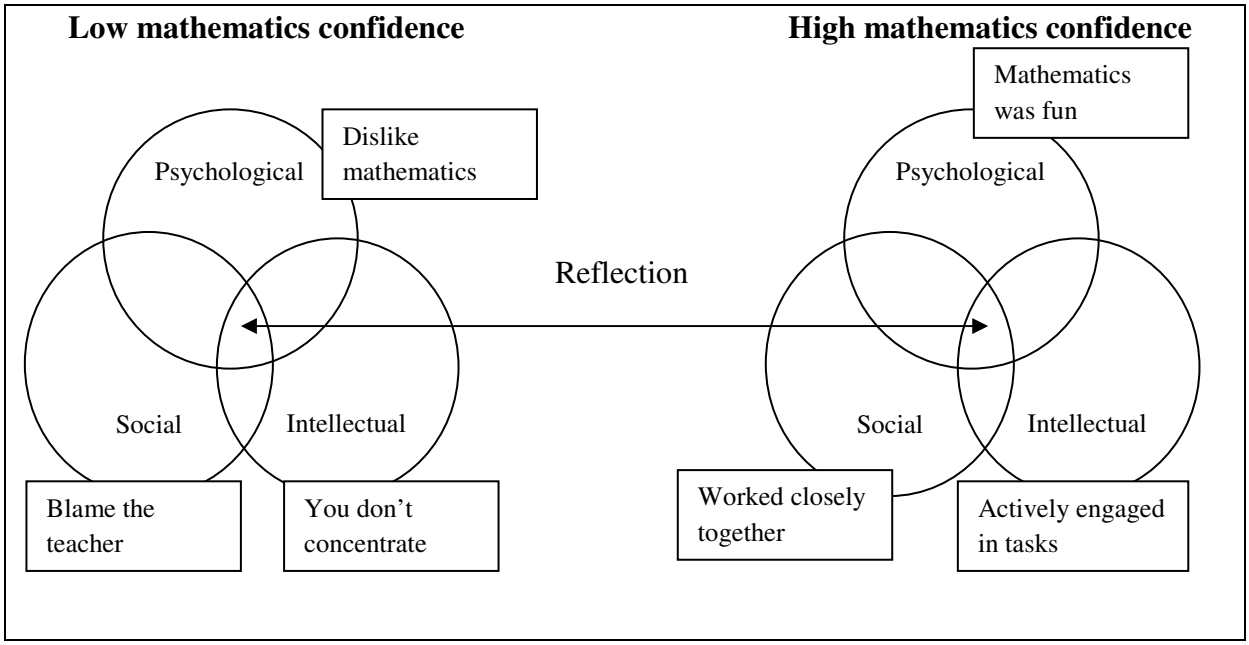

Figure 2: Two extremes of mathematics confidence

Discussion of Stage 2

The barrier between low and high mathematics confidence is clear. Disliking mathematics and blaming the teacher are warning signs that understanding (intellectually) is lacking (Refer to Stage 1 - findings for learner C). During problem solving, the learner reflected on knowledge and strategies previously learnt and eventually on the memories of prior experiences. Avoidance symptomatically describes why the learner did not evaluate her own answer in Stage 1. The literature confirms this as a trademark of low mathematics confidence.

On the other hand, while reflecting on experiences with Mathematics that were fun and enjoyable, the learner engaged only in the tasks she liked. Ultimately, she only acquired some skills and knowledge, and therefore solved the problem despite experiences in low confidence. This is confirmed in Stage 1 when learner C said: *I wasn't sure. It was like I couldn't remember*.

Concluding remarks and recommendations

Some of the limitations of this research included time constraints and the small number of participants. Findings are limited to the specific described sample and therefore great care was taken not generalise outside the sample. Results may well vary depending on paradigmatic perspectives.

Evidence obtained from the interviews showed that participants reflected on their knowledge, regulation and confidence with regard to Mathematics. When a learner displayed low mathematics confidence, he or she initially reflected on experiences and skills that relate to the context of the problem, and in due course reflected on strategies utilised to solve the problem. Their confidence in the correctness of the solution was linked to their initial evaluation and monitoring of knowledge and skills. When reflecting on strategies and monitoring performance, participants showed low or high confidence in Mathematics depending on whether they had the necessary knowledge and strategies available. When reflecting on strategies or knowledge that was sufficient and helpful, participants who monitored their work appeared to be confident. The opposite was also true.

More research, especially in South African contexts, is necessary to supplement the limited literature on metacognitive and affective roles during mathematics problem solving. Teacher training regarding the importance and role of metacognition and affect is essential, as teachers are the sole role models for metacognitive reflection. The findings in this study confirm that there should be an acute awareness of the role of reflection and mathematics confidence in the mathematics classroom, particularly during problem solving.

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