THE NOTION OF 'TOPIC' IN SCHOOL MATHEMATICS AND THE PROBLEM OF ISOTOPIC COHERENCE

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I draw on Eco's (1984) notion of topic and isotopy to develop methodological resources for describing mathematics content that emerges in the mathematical work of teachers and students. Using two Grade 9 lessons on the teaching of exponents, I show how use of the notions of topic *and* isotopy *enables a description of the content that emerges in pedagogic situations. In particular, the paper establishes a proposition, mooted in the work of Davis (2011a), that mathematics topics do not remain stable across pedagogic situations.*

INTRODUCTION

The general problematic within which this paper is located is the constitution of mathematics in the pedagogic situations of schooling, specifically what is constituted as mathematics and how in school mathematics lessons. Drawing on Eco's (1984) use of the notion of topic in semiotics, this paper develops the notion of topic as a methodological resource for describing the constitution of mathematics in pedagogic situations of schooling. I interrogate the assumption that mathematics topics remain stable across pedagogic contexts because what is realised as content under a specific topic name does not necessarily reflect the content one expects to be associated with the topic name. The idea that topic is not stable across contexts has been used in other work (see Davis, 2011a; Chitsike, 2011), but a more careful elaboration is now required to establish it as a proposition.

WHAT IS A TOPIC?

The notion of *topic* is conceptualised in semiotics as referring to the subject of a text in relation to its levels of coherence. The notion of *isotopy* first introduced by Greimas (1970) and later adapted by Eco (1984), is associated with the level of coherence of a text. In reference to literary texts, Eco describes topic as

an abductive schema that helps the reader to decide which semantic properties have to be actualised whereas isotopies are the actual textual verification of that tentative hypothesis (Eco, 1979 as cited in Eco, 1984: 189).

Eco regards topic as a pragmatic resource deployed by the reader to focus the interpretation of the meaning of a text and isotopy as "a level of possible semantic actualisation of the text" (Eco, 1984: 189). The glossing of the notion of isotopy as 'repetition' in a text, due to Greimas, was found to be problematic by Eco (1984), prompting him to redefine the idea.

Indeed, isotopy almost always refers to constancy in going in a direction that a text exhibits when submitted to rules of interpretative coherence […] What should be clear in any case is that the identification of the topic is cooperative (pragmatic) movement guiding the reader to individuate the isotopies as a semantic feature of a text. (Eco, 1984: 201)

Topic imposes a rule of interpretative coherence and isotopy refers to the semantic result of that coherent interpretation (Eco, 1984: 193). Further, Eco recognises the "the actualized isotopy as the 'objective' content of the expression" (Eco, 1984: 193).

Eco's (1984) notions of topic and isotopy are semiotic resources for describing and analysing literary texts. This paper focuses on mathematics texts produced by teachers and students in the pedagogic contexts of schooling. The texts referred to by Eco include conversations, comic books, films, poems and so forth, that may be used as pedagogic resources but clearly differ from pedagogic texts. A pedagogic text, defined as "an utterance within a context of a pedagogic relationship" (Dowling, 1998: 112), includes sequences of verbal, written or gestural significations such as teacher and student speech, worked examples or notes written on a chalkboard or a textbook. Pedagogy is to be understood as necessarily evaluative. Pedagogic evaluation distinguishes legitimate from non-legitimate knowledge statements for students and reveals criteria for the recognition and realisation of mathematical objects and procedures in pedagogic contexts. Such evaluations are, of course, not identical across pedagogic contexts, and so the criteria that are posited could, in principle, be rather different across contexts. The paper attempts to develop Eco's notions of topic and isotopy as resources for describing the constitution of mathematics exhibited in the pedagogic texts produced by teachers and students in the context of schooling.

According to Eco (1984), the reader posits the topic of a literary text in generating a theory about its semantic content and isotopies verify or disrupt the reader's construction of the topic. In the latter case, the topic is altered to conform to the isotopies. In the reading of literary texts the reader usually has a store of cultural information that can be selected from to actualise a topic. With pedagogic texts, however, the student is routinely confronted with content that is unfamiliar, and has to rely on the evaluative criteria operative in the pedagogic situation to constitute the topic. It is also the case that the student cannot but draw on their store of mathematics in their attempts to constitute the topic. Isotopy, which, in the case of mathematics, the student reads off from the operational activity deployed in the pedagogic situation, confirms (or disrupts) the student's construction of the content of the topic.

The pedagogic organisation of a content area and an elaboration of the content reveals the topic. An examination of the elaboration of the content in the pedagogic context reveals the isotopy constituted by the teacher's operational activity and the isotopy confirms (or disrupts) the content of the topic for the student. The operational activity in a pedagogic situation is described by considering the operations that emerge in that situation along with the collections of objects over which the operations are performed. In addition, the criteria regulating the selection and sequencing of the operations are to be described (Davis, 2011b).

DIVISION OVER EXPONENTIAL EXPRESSIONS

In a Grade 9 lesson the teacher announced that the lesson topic was division and proceeded to use an example, $7^3 \div 7^2$, to illustrate procedures for solving problems involving division of exponential expressions. The worked example $7^3 \div 7^2$ served as an exemplar of the class of problems that deal with the simplification of statements of the form $\frac{a^m}{a^n}$, where $m, n \in \mathbb{N}$.

The teacher presented a method for dividing exponential expressions, which she referred to as the “long method” (see Figure 1). The teacher’s procedure for the “long method” is shown in an extract from the lesson transcript:

Teacher:	So here, how many sevens do you have?
Learners:	Two.
Teacher:	No.
Learners:	Three.
Teacher:	It’s seven, seven, seven. Nhe? {Right?}
Learners:	Yes.
Teacher:	Then you divide by how many sevens?
Learners:	Two.
Teacher:	So it is seven, seven. The long method is a primary. It’s a primary method, not a high school method. But here you are not .. the same. Then what you do, you cancel seven and seven nhe? {right?}
Learners:	Yes.
Teacher:	Then seven no {and} seven. Then lo ushiyekileyo uzakuba yintoni? {Then what happens to the seven that is left?}
Learners:	[Indistinct]
Teacher:	Yi-answer. {It is an answer.} The one that is not being cancelled is your?
Learners:	[Chorus] Answer.

The teacher’s first question in the extract is ambiguous. The learners appear to interpret the question as referring to the number of sevens in the expression $7^3 \div 7^2$ but the teacher is referring to the number of sevens in the transformation of 7^3 to 7 multiplied by itself three times. The teacher does not refer to exponentiation as repeated multiplication, but that appears to be what is intended. Instead, the exponent signifies the number of 7s that have to be repeated. In the transformation from 7^3 to 7.7.7 the dots between the sevens represent multiplication for the teacher but the operation multiplication is absent from the teacher’s speech and as such remains implicit at the level of the spoken word. Similarly, the use of cancelling implicitly refers to division. But, again, the operation division is absent from the teacher’s speech when performing the cancellation operation.

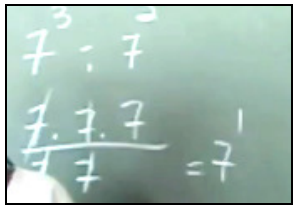

Figure 1. A teacher’s “long method” for dividing exponential expressions

Let us now look more closely at the operational activity of the teacher. The procedure involves a series of transformations from the initial expression $7^3 \div 7^2$ to the terminal expression, 7. Each transformation is legitimate irrespective of the operations employed as long as the transformation produces an expression equal in value to the initial expression.

The expression 7^3 in the teacher's procedure signifies a collection of objects with 3 indexing the cardinality of the set and 7 indicating the type of objects in the collection. So 7^3 refers to a set *A* with cardinality 3. There are an infinite number of sets for any given *n* ($n \in \mathbb{N}$), thus forming an equivalence class associated with that natural number. However, in this case we know that the set is to populated by 7s. So the particular set referred to with the cardinality of 3 is the set *A* = {7;7;7}. Similarly, the expression 7^2 represents a set with cardinality 2, made up of 7s; that is, we have a set, *B*, where *B* = {7;7}. The listing of 7.7.7 and 7.7 seems to suggest that the teacher's procedure renders numbers as objects to be counted.

From the expression $\frac{7.7.7}{7.7}$, the teacher generates the expression 7 by cancelling 7s from the numerator and denominator, $\frac{\cancel{7}.\cancel{7}.7}{\cancel{7}.\cancel{7}}$, leaving a single 7 remaining in the numerator of the expression. We might think of the process of eliminating 7s by cancelling as generating the set *C*, consisting of the object 7; that is, *C* = {7}. *C* is then represented by the teacher as 7^1.

The effect of the procedure is to transform the problem of division of exponential expressions to a problem about counting. The teacher's operational activity, therefore, constitutes counting as the isotopy. The operations, such as "how many sevens", serve as the isotopic markers for students. Consequently, counting becomes the content associated with the topic, division of exponential expressions. Counting serves as the primary regulative resource underpinning the teacher's procedure for solving problems involving division of exponential expressions. The regulation of the students seems to be motivated by the teacher's image of the competence of her students: "The long method is a primary. It's a primary method, not a high school method. But here you are not .. the same." Secondly, the teacher's procedure seems to be strongly regulated by the iconic features of mathematical expressions, what Davis refers to as "the-usually distorting-over-determining effect that arises when mathematical expressions are treated as a source of imagistic data for the purposes of mathematical processing" Davis (2011b: 313).

The isotopy generated by the teacher's procedure produces a context for counting and portrays content of the topic, exponential division, as counting. The content associated with the topic, division of exponential expressions, in this pedagogic context differs from what is usually associated with the topic.

EXPRESSIONS WITH NEGATIVE EXPONENTS

In another grade 9 lesson, a teacher introduced the topic negative exponents using the example, 2^{-1}. Up to that point the teacher had discussed exponential expressions of the form a^m, where $m \in \mathbb{N}$. However, that the indices were natural numbers remained implicit to the criteria used by the teacher. In the extract below the teacher explained that expressions with negative exponents were not in an appropriate form and should be converted to expressions with positive exponents.

Teacher: <u>Sijonge kengoku xa sine-negative</u> exponent. {Let's look at when we have a negative exponent.} Most of the time we do not like to give our answers with negative exponents. <u>Nhe</u>? {Right?}

After a discussion with students, some of whom stated that the answer should be written as $\frac{1}{2}$, the teacher insisted on an explanation for the answer.

Teacher: We change that power and write it as a fraction, but i-fraction ke ayizusuka kude, asizuthatha {the fraction wont be taken afar and we wont take it} from other numbers from outside, amanani siwasebenzisa {we use the numbers} from here. Siyavana? {Do we understand each other?}

The teacher's motivation for the procedure described below appears to be regulated by the need to convert the exponential expression with a negative exponent to one with a positive exponent. Secondly, it also seems that fraction, as the form of the answer, is another central regulative resource. The first stage of the procedure involved converting 2^{-1} to 1×2^{-1} (see Figure 2).

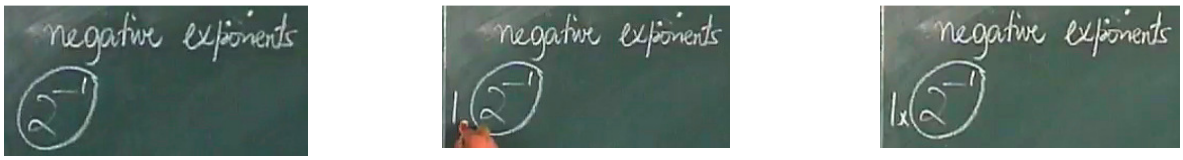

Figure 2. A teacher demonstrates the transformation from 2^{-1} to .

Teacher: [Draws a circle around 2^{-1}] Do we have any number that is written in front of this power [pointing at the circled 2^{-1}]?

Learners: No.

Teacher: And if we don't have a number we take that the number that is there is equal to?

Learners: [Chorus] One.

Teacher: One. We take for granted. In fact it is a rule that the number that is always here [pointing at the space preceding 2^{-1}] is equal to one.

The motivation for converting 2^{-1} to 1×2^{-1} soon becomes clear as the teacher sets up a spatial frame for the solution (See Figure 3). The transformation to the expression 1×2^{-1} essentially serves to generate parts of an expression to populate the spatial frame.

Teacher: So we are going to change this power to a fraction.

Learners: Yes.

Teacher: [Inaudible] A fraction has got a [Draws the division line for a fraction] ….numerator [Pointing to the space for numerator] and a denominator [Pointing to a space for the denominator]

Learners: [Chorusing with teacher] Numerator and denominator.

Teacher: But how are we going to write that fraction. [Erases fraction division line]

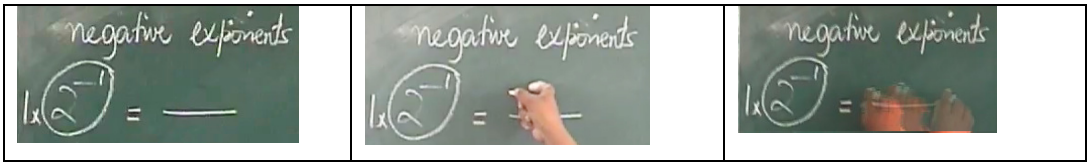

Figure 3. Teacher sets up the spatial frame for the solution.

The teacher agreed with the students that the answer was $\frac{1}{2^1}$ and proceeded to explain how different parts of the expression 1×2^{-1}were to be spatially distributed in the fraction spatial frame (see Figure 4).

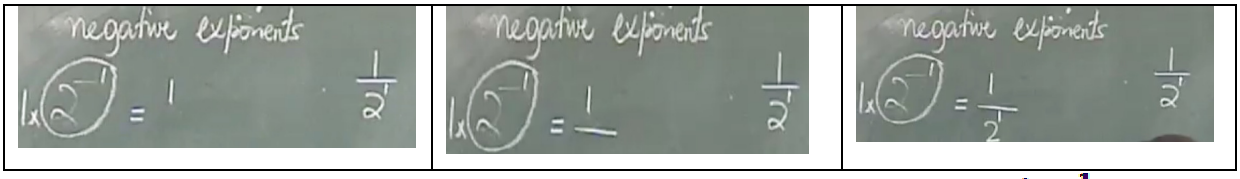

Figure 4. A teacher demonstrates the transformation from 1×2^{-1} to $\frac{1}{2^1}$.

Teacher:	The numerator will be the number of this power. How many powers are here?
Students:	One.
Teacher:	One. So this one [points at 1 in 1 x 2^{-1}] will be? The numerator andithi? {is it not?}[Writes 1 as numerator of fraction.]
Students:	Yes.
Teacher:	And then we are going to change this multiplication [points at x in 1 x 2^{-1}] to a division. [Draws fraction line.]
Students:	Division.
Teacher:	And then what ever comes after that will also change. Then we will take all this [points at 2^{-1} in 1 x 2^{-1}] to the what?
Students:	Denominator.
Teacher:	Denominator and change this negative [points at - in 1 x 2^{-1}] to
Students:	Positive.

In general terms, the teacher's procedure for substituting an expression with a negative exponent a^{-n} (where $n \in \mathbb{N}$) by an expression with a positive exponent ($\frac{1}{a^n}$) can be written as follows: (1) Write the expression a^{-n} as 1 x a^{-n}. (2) Convert the expression to fraction form, $\frac{p}{q}$, by doing the following: a) change multiplication to division; (b) move the "1" in the expression 1 x a^{-n} to the numerator space of the fraction; (c) change the sign of the exponent from a^{-n} to a^n as a^{-n} moves to the denominator of the fraction; and (d) write the solution as $\frac{1}{a^n}$.

As discussed above the isotopy constructed from the teacher's operational activity is strongly regulated by operations implicated in the distribution of parts of the mathematical expression to a spatial frame. Johnson & Davis (2010: 135) refer to such a spatial frame as a *character distribution matrix* that they define as "a resource for the regulation of the mathematical activity demanding the use of very particular spatial distributions of symbols in the organisation and presentation of transformations from one mathematical expression to another". Consequently, the spatial distribution of parts of the expression suggests an

existential shift from numbers to symbols or characters since numbers cannot move in the way suggested by the teacher.

In the mathematics encyclopaedia, a^{-n} (where $n \in \mathbb{N}$) is defined as $\frac{1}{a^n}$. The teacher substitutes the definition of a^{-n} with a procedure for converting expressions with negative exponents to expressions with positive exponents. Consequently, she realises the definition of a^{-n} inductively. The substitution of definitions with operations is not surprising in the GET curriculum context where definitions are not explicitly taught. The teacher's operational activity constitutes symbol generation and symbol distribution as the isotopy, with the sign of the exponent in the exponential expression and the location of the expression either in the numerator or the denominator position, as the isotopic markers for students. Consequently, how to 'change' a negative exponent to a positive exponent through symbol distribution becomes the content associated with the topic, expressions with negative exponents.

On a home-schooling website, *Home School Math Net*, the topic of exponential expressions with negative exponents is introduced through patterns: $2^3 = 8$; $2^2 = 4$; $2^1 = 2$; $2^0 = 1$; $2^{-1} = \frac{1}{2} = \frac{1}{2^1}$; $2^{-2} = \frac{1}{4} = \frac{1}{2^2}$. In this case, the content of the topic, expressions with negative exponents, are likewise realised inductively and replaces the formal definition of a^{-n} (where $n \in \mathbb{N}$). Here the isotopy relies on successive divisions by two. The isotopy constituted by the website pedagogic text differs from the isotopy constituted by the teacher discussed above. The content associated with the topic, exponential expressions with negative exponents, differs across the two pedagogic situations and differ from what we expect to associate with topic.

CONCLUDING REMARKS

The two examples discussed above illustrate that for a given topic the associated content that emerges in the pedagogic situation may differ substantially from each other and from the content usually associated with the topic. Consequently, content cannot be directly inferred from the topic name. Secondly, content associated with a topic name is not stable across different pedagogic situations. Since pedagogy is necessarily evaluative, the realisation of the isotopies and consequently the content associated with a topic is dependent on the evaluative criteria that emerge in pedagogic situations. Variations in isotopies and therefore topic content is possible because operations are functions and function rules can vary as long as they produce the same output for a given input (Davis, 2011a). Thus, content of a topic can only be established by examining the isotopies that emerge from a description of the operational activity present in the pedagogic situation.

Typically, the two examples discussed above would be referred to as procedural as opposed conceptual knowledge. The topic-isotopy as a methodological resource provides an alternate to the procedural-conceptual distinction to describe mathematics constituted in pedagogic contexts.

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