THE PRIMACY OF TEACHING PROCEDURES IN SCHOOL MATHEMATICS

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Currently, it is unpopular to propose that procedural fluency should be accorded high priority in the teaching of school mathematics. In this paper a case is made for teaching for procedural fluency. It is argued that procedural knowledge is the major component of the legitimate school mathematics knowledge as constituted by high-stakes school mathematics examinations such as the National Senior Certificate Mathematics examination. Furthermore it is argued that the teaching of procedures is not only about the step-by-step actions to accomplish whatever results are to be accomplished. Rather, examination of mathematics teachers' activity shows that there is evidence of seeds for exploitation to enhance the mathematicalness of learners by focusing on the teaching of procedures.

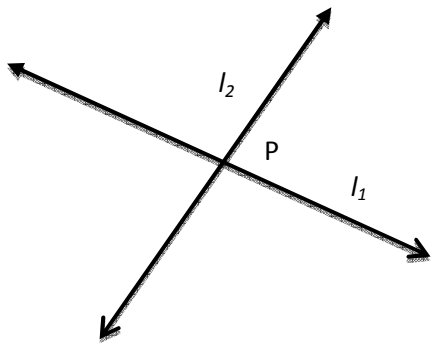
INTRODUCTION

"Mathematics as an educational task", the title of Fredunethal's seminal work, conjures up many images and begs many questions. For example, what is at play? An education in Mathematics? The role of Mathematics in school education? Both? It is without doubt that Freudenthal through the Realistic Mathematics Education (RME) programme had a major influence on both the direction of the development of school mathematics curricula and research in Mathematics Education.

RME a comprehensive mathematics research and development programme, with its core "reality is the source and domain of applications of Mathematics." It spread from Netherlands to various parts of the world. In South Africa there was the Realistic Mathematics Education in South Africa (REMESA) project (Julie, et al, 1999), the USA had the Mathematics in Context (MiC) project (Romberg and Shafer, 2002) and in Indonesia there is the still operative Pendidikan Matematika Realistik Indonesia (PMRI) project (Sembring, Hadi & Dolk, 2008).

In a sense RME was initially a dissident development against the prevailing direction of school mathematics developments in particularly the then Western countries. These nations were, to put it mildly, shocked when the Soviet Union launched Sputnik in 1957 and they sought renewal and modernisation of their school mathematics enterprise. A seminar was held at Royaumont in 1959 (Organisation for European Economic Cooperation (OEEC), 1961) to discuss the direction of this renewal and modernisation. Vigorous debate and interchange characterised this seminar with Freudenthal and Dieudonné apparently at each other's throats. Dieudonné was advocating a school mathematics curriculum founded on the

formal structures of Mathematics. His quest is characterised by the famous statement “Euclid must go!” Freudenthal was agitating for a different starting point: school mathematics embedded in the experiential world of the child. During the interchange it is reported that Freudenthal shouted at Dieudonné by stating “Don’t shout at me, for I can shout louder than you—and in more languages.” (Howson, 2004). However, the outcome of the seminar was that school mathematics curricula should be firmly based in the formal structures of Mathematics. This culminated in the “New Maths” school curricula characterised by set theory as the underpinning for school mathematics. Emblematic of the content of school mathematics textbooks during the era was as illustrated in Figure 1 below.



The intersection of the lines l_1 and l_2 is the set of points common to all points common to both lines. Since P is the only point in the intersection, we write $l_1 \cap l_2 = \{P\}$.

Figure 1: Excerpt from a "New Maths" textbook (School Mathematics Study Group, 1968, p 11)

South Africa also adopted the “New Maths” programme and an exemplar of the textbook entries of the day was as presented in Figure 2.

Example.—Draw the diagram of the relation *is double* on the set {2, 3, 4, 6, 8, 9} (Figure 6).

This example suggests that one or more elements may not appear in the relation. In the above case, the element 9 is not used.

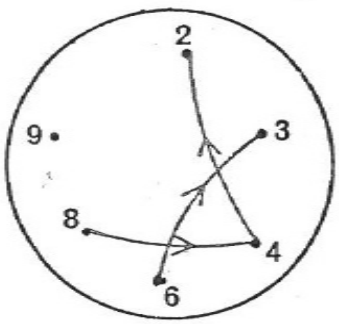


FIG. 6

Figure 2: Excerpt of "New Math" South African textbook example (Malan and Smith, 1967, p 16)

Freudenthal, however, continued with his project and this sense he was swimming against the stream of formalistic structural mathematics as the starting point for school mathematics.

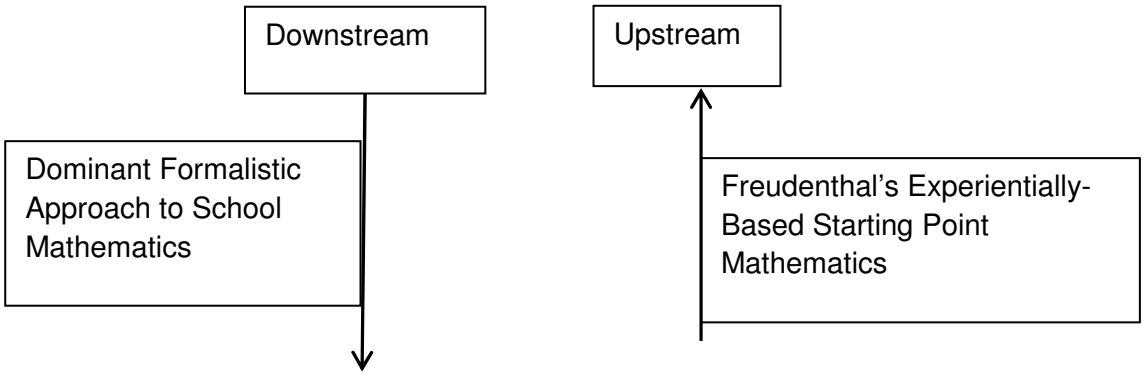


Figure 3: "New Maths" and Freudenthal's 'oppositionary' approach

Freudenthal's curriculum project for school mathematics was not the only one. There were others such as Papert's Logo-programming with its defining "Teaching children to be mathematicians vs teaching them mathematics" (Papert, 1972) and Davis's Madison project (Davis, 1984) with its "assimilation paradigm". Variants of these school curriculum mathematics orientations eventually replaced the "New Maths" and today are the dominant paradigmatic orientations for school mathematics curricula more or less across the world. These orientations favour the development of the processes of doing mathematics with the formal aspects of mathematics coming into play only later. This is diagrammatically depicted in Figure 4.

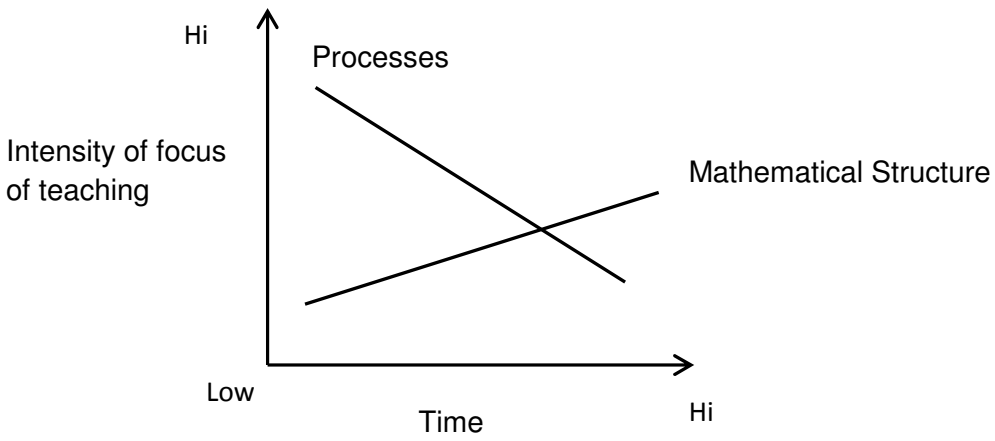


Figure 4: Current dominant focus of school mathematics approaches

The issue pursued in this paper is "Is it time, in South Africa with her highly unsatisfactory performance in school mathematics, to swim against this stream and accord procedural fluency a higher priority?" This question is considered by firstly discussing what legitimate school mathematics knowledge is followed by an exposition of the content of this knowledge. Thirdly a model for the focus for teaching for the proposed legitimate school mathematical knowledge is presented. The penultimate section provides examples on how teachers are near to realizing this model in their activity of teaching procedures and it is concluded that

this might lead to ways of mathematics classroom pedagogies to arrest the unsatisfactory performance in high-stakes school mathematics examinations.

LEGITIMATE SCHOOL MATHEMATICS KNOWLEDGE

The choices of the content to be included in school mathematics and what to make its focus for teaching are not always clear. Mathematical content other than the current selected ones with different foci for teaching could as well have been chosen. School mathematics is a particular kind of mathematics. (Julie, 2002). It is a hybrid of pure mathematics, mathematical modelling, the application of mathematics and probability and statistics, the traditional disciplines in higher education institutions. A further characteristic of school mathematics is that it is wholly situated on the established knowledge side of a knowledge continuum as illustrated in the Table 1. The traditional disciplines in higher education institutions are more towards the frontier side in as far as its research and research training concern.

Established	Frontier
Stable (current state of intransitivity). Vetted by the knowledge custodial community. State of intransitivity has a long existence. Generally publicly visible and recognisable Is teachable having gone through didactical transposition processes	Transitive (Unstable and fragile). Prone to refutation and falsification. Visibility and recognisability restricted to that sector of the custodial knowledge community who concerns themselves with the particular area of knowledge. Is not teachable but only co-explorable by the immediate community.
School Knowledge	

Table 1: A Knowledge Continuum

School mathematics is in essence the mathematics that is legitimized by tests and examinations. This notion of valid disciplinary knowledge in general is convincingly argued by Kvale (1993, pp. 219-220) who states that "…the examination itself contributes to establish the valid knowledge of a discipline…The examination committee defines what is valid and legitimate knowledge in a field." The contention in this paper is thus that legitimate school mathematics knowledge is the knowledge that is tested in high-stakes examinations such as the National Senior Certificate (NSC) Mathematics examination.

This notion of legitimate knowledge as the knowledge constituted by high-stakes examinations for obtaining a certificate of worth is common in many other practices. The

knowledge necessary to obtain a learner's driver's license is a less esoteric common example. Of all the knowledge around about driving a motor vehicle, what counts and focused on to obtain this initial certificate of worth for driving a motor vehicle legally, is the knowledge that will be tested as encapsulated in the K53 manual. A less esoteric example is the knowledge required to be admitted to various professions such as a certified financial risk analyst or a chartered accountant.

The above constitution of legitimate school mathematics has long been realized by schools serving learners from high socio-economic status (HSES) environments. These schools are characterized by high achievement and an aspect of their practice to attain this high achievement is portrayed as follows for the Australian situation.

Private establishments…exploit the socially discriminating potential of school subjects to the full. [They do this] by interpreting and formalizing its cognitive demands of [a curriculum] into a programme of group work in which concepts and operations are mastered pragmatically, that is, as a set of practices that represent ends in themselves, the "business" of the classroom and the exam.

… These classes can be worked to a high standard of competitive performance, following the bureaucratic procedures of syllabus-stripping (reduction to examinable topics), chalk and talk, worked examples, worksheets, past papers and exam rehearsals. To this participation in national and university mathematics contests add luster and encouragement, as does the annual release of exam results and the media parade of top students. (Teese, 2000, pp 186 - 188).

This pragmatic approach to reform in curricula is also evident in South Africa. With the introduction of Outcomes-Based Education, schools serving learners from HSES environments re-interpreted of the curriculum to not lose the focus on the high-stakes examinations. Harley and Wedekind (2004, p 202) captures this re-interpretation as follows:

But there is another imperative, no less powerful than in all other secondary schools: the Senior Certificate 'exit' qualification known as 'matric'. Accordingly, the OBE programme has a time allocation of only of only three periods per week, and the responsibility rests with the school librarian. For over 90 percent of the available time, learners are engaged in traditional subject lessons.

Clearly, the focus is on getting the learners to succeed in high-stakes examinations. This situation generally holds in high-performing schools serving learners from HSES environments as is evident from these schools finishing the syllabus during the second quarter of the grade 12-year and focusing attention on preparing students to succeed in the NSC examination by, in a structured way, working through past and imagined papers drawn up by teachers. Of course, it has to be borne in mind that there are academic, cultural and economic capital in HSES environments which account for learners to acquiring the necessary wherewithal to participate in the in the "national and university mathematics contests" mentioned Teese. The so-called progressive ways of doing mathematics is so near to what happens in their extra-school environments that the school environment need not to be organized to spend much time on these issues. Thus in designing teaching for success in schools in LSES environments the practice of examination-focused organization should not be looked upon as retrogressive since success in high-stakes examinations, as Muller (2000: 62) argues, provide learners from LSES backgrounds with a necessary "article of universally recognized cultural capital such as a school diploma [which] confers symbolic power on the holder."

By claiming that the legitimate knowledge is the knowledge that is prioritized and constituted by the examination, does not imply that the examination questions are the knowledge. It is the knowledge inherent in such questions that are at stake.

THE CONTENT IN HIGH-STAKES SCHOOL MATHEMATICS EXAMINATIONS

Consideration of the two lists, with some exemplar examination questions, in Table 2 reveals that different “objects” of mathematical knowledge are privileged in the two situations. With privileging viewed as question types having more than 50% of the questions of the kind given in the lists, in the two examinations, then different kinds of knowledge are accorded high priority in the two situations. Consequently, the intensity of the focus of teaching will differ for the two situations.

List A	List B
Take any three-digit number whose digits increase as you read from left to right. Make another number by reversing the order of the digits, and subtract the smaller from the larger. Reverse the order of the digits of the difference, and add it to the difference. What is the answer and why?	Find the difference between 732 and the number formed by reversing the digits.
Take any fraction. Make a new fraction with the new numerator the difference between the numerator and denominator and the new denominator the sum of numerator and the denominator. Do the same to your new fraction. What do you notice?	Simplify $\frac{54}{104}$
Is "OTTO" always divisible by 11?	Show that 5665 is divisible by 11.
Comment on the method below for the factorisation of quadratic expressions. $2x^2 - 7x + 3$ $x^2 – 7x + 6$…Divide 2 by 2, multiply 3 by 2 $x^2 – x – 6x + 6$… Split middle term so that there are common factors for first and last term $x(x – 1) – 6(x – 1)$…common factor $(x – 1)(x – 6)$…common factor $(x-\frac{1}{2})(x-\frac{6}{2})$ …Divide both 1 and 2 by 2 $(x-\frac{1}{2})(x-3)$ …Simplify $(2x – 1)(x – 3)$… Multiply the first factor by 2. (The second is multiplied by 1 since the denominator of 3 is 1)	Factorise $2x^2 - 7x + 3$

Table 2: Examination Question Types

The Curriculum and Assessment Policy Statements (CAPS) document and current high-stakes examinations such as the NSC examination put much emphasis on questions of the type B. It is stated that

Knowledge	Routine Procedures	Complex Procedures	Problem Solving
20%	35%	30%	15%

Table 3: Percentage Distribution of Knowledge Problem Types for Mathematics Examination (DBE, 2011)

This shows that the legitimate knowledge is of a procedural nature with the accompanying concepts embedded in these procedures. Emphasising procedural adeptness, although sometimes suspiciously viewed, is well-established in East-Asian countries. In these environments "Teaching and learning stress both fundamental techniques (via repeated practice) and fundamental knowledge…embedded in a cultural belief that skilfulness (ability to do certain things well) can bring about cleverness and creativity." (Leung, 2010, p 366).

TOWARDS A MODEL FOR TEACHING PROCEDURAL KNOWLEDGE

Given the proposition that the legitimate knowledge that ought to be taught in schools is those privileged by the high-stakes examinations and that, for the South African situation, this school mathematical knowledge is procedural in nature, it is proposed that teaching should emphasise mastery of mathematical procedures and the concepts involved in these procedures as given in Figure 5 below.

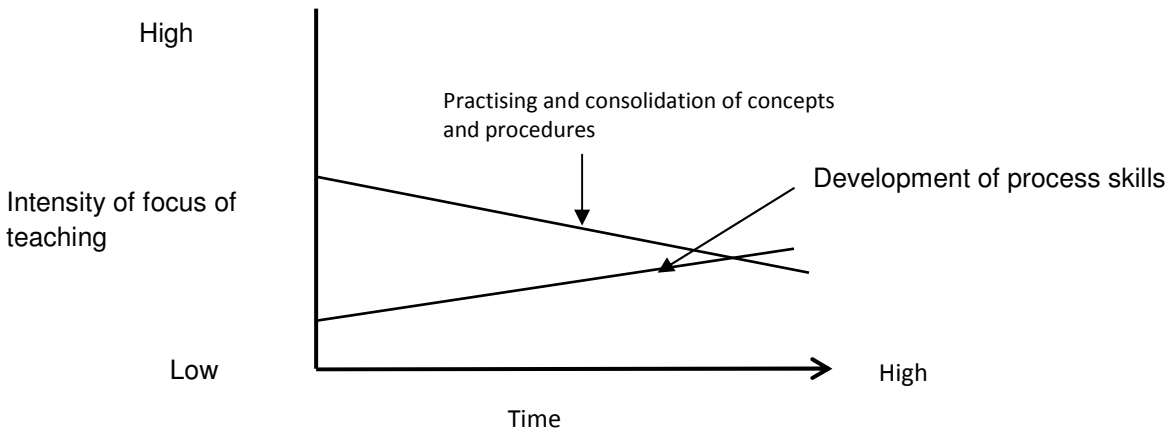


Figure 5: Focus of teaching on procedures

In this configuration importance is accorded to procedures and by implication calculation. This is not such a bad thing. It can be argued that one of, if not the primary, the purposes of Mathematics is the quest for efficient and effective methods of calculation. For example, the pursuit for solving the Riemann hypothesis is to find ways of calculation. Sabbagh (2002, 222) alludes to this by asserting "cryptographers have to devise even better methods of encryption, and these will rely upon the

discovery of more and more sophisticated mathematical methods, some of them derived from a proof of the Riemann hypothesis."

A narrow view of intentional teaching focusing on "Practising and consolidation of concepts and procedures" is not advocated. Rather, where appropriate, process skills should be developed during this kind of teaching. In this regard, the following scheme for organizing the knowledge objects of school mathematics is proposed.

Conventions, representations and notations

The way things are written, the symbols that are used. Examples: "+" is used to indicate "addition"; "√" is used as the "square root" sign.

Concepts and definitions

A concept is a mathematical "thing" with the property that there are certain elements which cannot be changed so that it remains the "thing" that it is. Examples: Solution; equation; midpoint; area and real number.

Procedures and techniques

Procedures are step-by-step actions to obtain a desired result. Example: Factorisation of quadratic trinomials; constructing the altitudes of a triangle and simplifying algebraic fractions.

Relationships between concepts

Links between different concepts and combinations, mostly operationally (addition, subtraction, multiplication, division, etc.) linked, of concepts. Theorems are the most commonly known relationships.

Figure 6: Organisational scheme for school mathematics

Proposing a primary focus on mastery of procedures should not be viewed as the exclusion of other processes. As stated by Mason and Davis (1991, 78) "To master a topic…it is necessary to have a sense of what the techniques does and the kind of questions it answers [and] be aware of various ways of thinking about constituent ideas…"A narrow view of solely focussing on drill and practice of the step-by-step actions is thus not advanced. It is a myth that when there is talks about mastery of procedures then other aspects of doing Mathematics cannot be dealt with. Watson and Mason (1998) provide a scheme for incorporating aspects of doing Mathematics when dealing with mastery of procedures. In line with the objects of school mathematics given above, I have adapted it for a teaching development project as given in the table 3. The left-hand side is a list of the "various ways of thinking" or processes, aspects of being mathematical or mathematicalness.

Mathematicalness activity	Example
Exemplifying Specialising	Write an explanation on how to solve for x if $-1 \leq 2 - 3x \leq 8$. (i) Write a test question on the simplification of algebraic fractions.
Completing Deleting Correcting	To find the solution of $4\sin 2\theta - 1 = 3$ for $\theta \epsilon [0^o; 90^o]$, add 1 to each side to get $4\sin 2\theta = 4$; complete this. Delete the unnecessary steps To sketch the graph of $y = 2x^2 + 6x - 3$ (ii) Factorise to find the x-intercepts (iii) Find the y-intercept (iv) Find the co-ordinates of the turning point (v) Calculate a table of values (vi) Plot and connect the points
Comparing Sorting Organising	What is the same and what is different about solving simultaneous linear equations in two unknowns using substitution or using graphs? Put in order some questions to help you choose which method of factorisation to use.
Changing Varying Reversing Altering	Factorise $ak - (k + a) + a^2$ in two different ways. Find out if you can calculate the distance between two points by only using the coordinates of one of the points.
Generalising Conjecturing	Of what is $\frac{6^{2x}.11^{2x}}{22^{2x}.3^{2x}} = \frac{(6.11)^{2x}}{(22.3)^{2x}} = 1$ an example? When can the signs of terms be changed during solving linear inequalities?
Explaining Justifying Verifying Convincing	How can we sure that that $2 \leq x < 5$ if $5 \leq x + 3 < 8$? Is it always true that to draw a graph $y = 2\cos\theta$ it is only necessary to set up a table of values for $\theta = 0^0$, 30^0 and 45°, plot and connect the points?

Table 3: Mathematicalness, procedures and techniques

A similar list can be constructed for the other objects mentioned in the organisational scheme of mathematics taught in schools in Figure 6 above. Thus the proposal for teaching focused on mastery of procedures as the bulk of the legitimate knowledge for school

mathematics is not a position that excludes the development of the mathematicalness of learners. Rather, the contention is that such mathematicalness can be fostered through focusing on the mastery of procedures and its accompanying concepts.

POTENTIAL BENEFITS OF FOCUSSED TEACHING OF PROCEDURES

One of the dilemmas in South Africa is that too few learners pass with acceptable levels of achievement in the NSC examination for Mathematics. Although there is no magic formula to turn the situation around, it is generally accepted that when foci are clear and an entire system knows what the goals are then the chances are that there will be collective and concerted effort to achieve such goals. The contention is that a firm pursuit for mastering the legitimate knowledge as articulated above has the potential for more learners achieving a higher level of quality of success than is currently the case.

There is widespread agreement that learners who do school mathematics are not equally interested in it. They have diverse motives or rationales for wanting or not to do school mathematics. In a recent conversation with Mathematics teacher, he related about a girl in grade 12 performed above average in Mathematics in grade 11 changing to Mathematical Literacy in her current grade. The reason she offered for her switching was that she could obtain a higher level of pass in Mathematical Literacy which will enhance her chances for access into further studies for a low mathematically-based career. Her motive for opting out of Mathematics is different from that of a learner whose sights are set on a highly mathematically-based career. Drawing on the work of Mellin-Olsen (1987) and its further development by Goodchild (2001), a hierarchical (in terms learners reasons for doing Mathematics) taxonomy of rationales is:

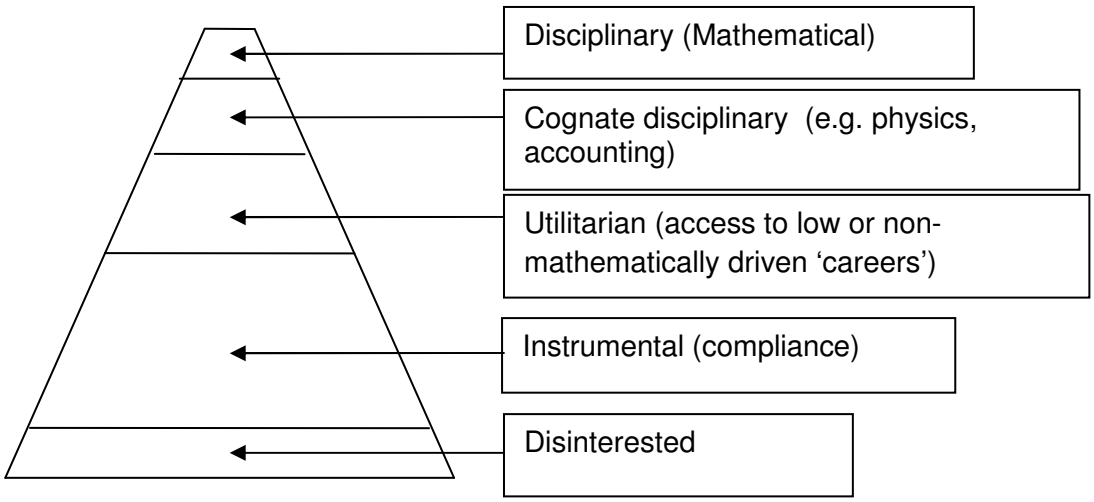


Figure 7: Learners' motives for doing school mathematics

The teaching model's initial focus on consolidation of flexible understanding of concepts and procedural aspects will address the needs of learners holding utilitarian and instrumental

rationales, who are majority of learners in non-specialised schools, and contribute to enhancing their achievement. It is also possible that learners with these two rationales might change their rationales to the upper two ones. This results from the common notion of "success breeds success". Accompanying this common sense notion is a current strong view that the relationship achievement → self-esteem is stronger than the one self-esteem → achievement. This counters the notion that the development of the self-esteem of learners from low socio-economic status (LSES) environments must first be developed before they can achieve at much higher levels. Catering for the needs of learners holding the two rationales does not necessarily jeopardise chances of learners in the two upper bands. Learners in these bands are normally self-directed and goal-focused as can be gleaned from the success stories of in the media of learners from LSES environments who perform above normal expectation. In fact, the chances are high that such learners will achieve even higher given the development of learners' mathematicalness through a strong focus on the procedural aspects and their accompanying concepts. Given these considerations the possibility is high that an increase in the number of learners offering Mathematics as a NSC examination might result and so counter the unhealthy practice of shepherding learners away from Mathematics to boost the overall pass rates of schools.

A last potential benefit is that the focus on a particularly highly examination-prioritized aspect of mathematics more definitive guidance is provided for teaching in the sense of intentional teaching as teachers doing their business with "specific outcomes or goals in mind for children's development and learning." (Epstein, 2007, p. 1). Intentionality—doing things on purpose—having been identified as the singular attribute of outstanding teachers. (Slavin, 2000, p. 7).

WHAT HAPPENS IN CLASSROOMS?

This question is engaged with drawing on experiences of a project, the Local Evidence-Driven Improvement of Teaching And Learning Initiative (LEDIMTALI), which is based on the considerations mentioned above. Examples of how teachers participating in this project deal with teaching procedures are used to illustrate that there are traces of shifting the teaching of procedures towards one of incorporating the development of learners' mathematicalness.

It is widely recognized that teachers accord a lot of time to the teaching of procedures.The normal style of teaching procedures when they are introduced is a demonstrative exposition of the steps of the procedures with its unfolding written on the chalkboard as the demonstration proceeds. This is interspersed with questions directed at learners which are either answered by the teacher him/herself or learners. Upon completion of the demonstration of a few key examples by the teacher, the learners are presented with similar or near-similar problems to work through. During this phase the teacher monitors learners' progress and understanding and provides assistance to learners who are struggling or request further assistance. At the end of the period the learners are given similar problems for homework. At the start of the session following the introduction, the given homework is checked and selected learners are requested to work through the solutions of the homework problems on the chalkboard. During this process, the teacher checks the work of the learners at the chalkboard whilst also checking whether learners did complete the assigned homework. The responses provided are the used by the teacher to draw learners' attention

to aspects of their ways of working. Some these that were observed during visiting more than 30 classrooms during the first and second quarters of this year are:

Drawing attention to the 'action words' in mathematical problem statements: It is a fairly normal occurrence that when learners are engaged with particular procedures they attend only to the relevant steps required to obtain a particular outcome—the final answer. They imitate these steps as explained by the teacher or in the textbook and do not attend to whatever 'action words' such as, for example, 'find', 'simplify', 'solve', 'test'. This non-attendance might lead to learners experience problems later when the immediate cue of 'we are busy with it' is not available and they have to respond to the 'action words' in the formulation for executing the procedure. During classroom visitations, it was observed that there are slight indications of teachers drawing learners' attention to such 'action words'. For example, a teacher was dealing with the factorisation of trinomials in a grade 10 class. After her exposition of the procedure, the demonstration by a learner of a problem given as practice to the class, the teacher brought to the learners' attention that the instruction given for these kinds of problems is "Factorise fully" or "Find the factors". In another instance, dealing with exponents the teacher stressed that answers must be in their simplest form and the instruction to these problems will be "simplify". A learner who did not "simplify fully" was reminded firmly that the answer must be in simplest form. It is contended that drawing learners' attention to explicit 'action words' contributes towards a greater awareness of the language machinery inherent in working with procedures. For example when dealing with a question such as "Evaluate $\frac{x^3+1}{x^2-x+1}$ if x = 7,85 without using a calculator. Show all your work" [Thanks to Mr Raymond, chief curriculum advisor of the Western Cape Education Department for the problem], learners will probably be able to discern what procedural steps must be followed to arrive at the solution as per the prescriptions in the problem statement.

Reference to common errors/misconceptions: During the feedback on homework the attention of learners is also drawn to common errors/misconceptions they commit related to topic. It is surmised that these are errors/misconceptions which teachers have identified with learners' work during their experience. In a lesson dealing with exponents, the simplification of $\left[\frac{2x^{-1}}{(2x)^{-1}}\right]^{-3}$ was dealt with. The teacher drew the learners' attention to "-3 has to do with the whole bracket". The "with the whole bracket" was illustrated with

$\left(\frac{1}{2}\right)^{-3} = \frac{(1)^{-3}}{2^{-3}}$ where it was stressed that the exponent, -3, has to do with the "top" and "bottom" of the bracket. $\left(\frac{1}{2}\right)^{-3} = \left(\frac{2}{1}\right)^{3}$ was presented and learners were reminded that the negative index indicates the "reciprocal". They were then asked if the answer is "6". The teacher was thus explicitly and, with the example at hand, drawing learners' attention to an oft-repeated misconception.

Working from the general structural representational format of a problem type: Solution of exponential equations was the topic being dealt with and checking of homework related to $ab^{cx}+d=0$ and variants were completed. The teacher announced that lesson of the day will be on equations of the type $ax^{\frac{m}{n}}-b=0$, which was written on the chalkboard. He explained the features of this general representation and stressed that $ax^{\frac{m}{n}}$ means $a.x^{\frac{m}{n}}$, highlighting the multiplication (.). After the explanation of the general structural representational format of the problem type an exemplar problem $8x^{3/4}-27=0$ was demonstratively solved with constant reference to how this specialized form was linked to the general structural representational format.

The three examples above are representative of ways teachers deal with procedures. Although the primary focus is on the demonstrative exposition of the steps pertinent for the procedures, there are, as alluded to above, traces of reference to elements of mathematical ways of working with procedures given in table 3 above. I content that it is these traces that can be fruitfully exploited to bring them to a level of explicitness. For example, with respect to the common errors and misconceptions alluded to above, Mason (2000, p 10) suggests, amongst others, that these are converted to exercises in which learners are asked to "explain what is wrong about" the particular way of working.

CONCLUSION

There has been a long-standing debate about the prioritization of procedures as the objects of teaching in school mathematics. This debate is characterized by a view that the teaching of procedural fluency will degenerate into mindless drill-and-practice and hence mitigate against the development of the mathematicalness of learners. However, there have also been voices calling for a sense of balance. (Brownell, 1987 (first printed in 1956); Sfard, 1991). My contention is that the balance must be tilted in favour of procedural fluency with the enthusiasm for a focus on the process aspects of school mathematics replaced by a form of pragmatism. This pragmatism is not about moving forward to the past in the sense of 'back to the basics'. It is, as argued above, about legitimate school mathematics. It is about what can reasonably work in schools in its current context of massive socio-political challenges that the majority of teachers face. It is also about affording more learners the opportunity to perform at higher levels of achievement in high-stakes mathematics examinations than is currently the case—a dire need in South Africa.

ACKNOWLEDGEMENTS

This work is supported by the National Research Foundation under Grant number 77941. Any opinions, findings and conclusions or recommendations expressed in this material are those of the author and do not necessarily reflect the views the National Research Foundation.

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