ANALYZING THE RELATIONSHIP BETWEEN CONCEPTUAL AND PROCEDURAL KNOWLEDGE IN A DEPARTMENTALLY PROVIDED NUMERACY WORK BOOK.

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1. Introduction

Mathematical knowledge, in its fullest sense includes significant, fundamental relationships between conceptual and procedural knowledge (Hiebert & Lefevre, 1986, p. 6). However for many years conceptual understanding and procedural fluency have been competing for attention in school mathematics. Scholars in the field of mathematics education and those in psychology have been concerned about the relationship between these two strands. This has led to a lively debate on how these two concepts of mathematical proficiency are developed, how are they related to each other, which one enhances the development of the other, and also if either strand promotes higher cognitive demands than the other. However, so far, this debate shows no sign of finding a common ground for all parties or researchers involved (Schneider & Stern, 2011).

A review of the literature reveals a debate where the two strands are presented as competing with each other, and they are usually discussed as if they either exist as dichotomies or co-exist on a continuum (some arguing that one comes first before the other, while other scholars view them as co-existing but at par.) This debate has also led to the questioning of which of the two strands is more important and whether developing skills with symbols leads to the development of conceptual understanding or whether the presence of mathematics concepts enhance symbolic representation and skills practice (Sowder, 1998, as cited in Star, 2005).

Hiebert and Lefevre (1986) suggest that the relationship between conceptual and procedural knowledge is the key to further advances in our understanding of how children and adults do mathematics and how they think about what they do. With this debate in mind, this paper aims to show how both procedural and conceptual understanding are equaly important for learners to solve any mathematical problem.

To show how the two forms of knowledge are equally important, I have analysed two tasks from a South African Grade 3 departmental numeracy workbook. I present the results of the

two activities and show how the two tasks promote conceptual understanding and procedural fluency and how this might impact on teaching and learning. To do this, I have used a modified rubric.

2. Analytical Framework

The theoretical framework, used for this paper is based on a number of authors' views on the relationship between conceptual and procedural knowledge. Hibert and Levreve (1986) suggest that the relationship between conceptual and procedural knowledge is the key to further understanding of how conceptual and procedural knowledgen and of how children and adults do mathematics and how they think about what they do. Additionaly Schneider and Stern (2011) have also argued that the understanding to this relationship between the two knowledge forms, can facilitate, our general understanding of the human mind and at the same time it can also allow us to come up with the learning environment where knowledge is to be conveyed.

However, different scholars make different predictions as to the interrelations between conceptual and procedural knowledge. There are those scholars who view conceptual knowledge as a *source* of childrens' procedural knowledge while others assume the opposite to be true or posit bi-directional causal relations.

Rittle-Johnson and Alibali (1999), proposed the view that points to a bidirectional causal relationship between conceptual and procedural knowledge. This means that an increase in one kind of knowledge will, then, prompt an increase in the other one as well. This implies that an increase in conceptual knowledge will prompt an increase in the procedural. Rittle-Johnson and Alibali (1999) also argue that conceptual and procedural understanding lie on the same continuum and they cannot be seperated but the two ends of the continuum represents two different types of knowledge. The dominant assumption is that one end of the continuum features procedural fluency while the other end conceptual understanding.

Rittle-Johnson and Alibali (1999) have defined conceptual knowledge as explicit or implicit understanding of the principles that govern a domain and of the interrelations between pieces of knowledge in a domain and procedural knowledge as action sequences for solving problems. According to Rittle-Johson and Alibali (1999) the two forms of knowledge do not develop independently, they view conceptual knowledge as forgrounding the procedural knowledge. For example, they argued that conceptual understanding influences learners procedures. This is also supported but some theories of acquisition which argue that the generation of procedural knowledge is based on the conceptual understanding (Rittle-Johnson & Alibali ,1999). It is argued that in fact children use their conceptual understanding

to constrain procedural discovery and adapt their existing procedures to novel tasks (Gelman & Galistel, 1978 as cited in Rittle-Johnson & Alibali, 1999).

In addition we have two views as to a starting point in describing the relationship: the view of concept-first theory and the procedure first theory. (Rittle- Johnson, Siegler, & Alibali, 2001). According to concepts-first, children will initially acquire conceptual knowledge, for example by listening to verbal explanations, and will then, by practice, derive procedural knowledge from it. In contrast according to the procedures-first theory, it posits that children initially acquire procedural knowledge in a specific domain, for example by trial-and error learning, and then gradually abstract conceptual knowledge from it by reflection.

In both perspectives above, conceptual knowledge is taken to be knowledge of the core concepts and principles and their interrelations in a certain domain. Thus assumed to be stored in some form of relational representation, like schemas, semantic networks or hierarchies (Byrnes & Wasik, 1991 as cited in Schneider, Rittle-Johnson; Star, 2011). Because of its abstract nature and the fact that it can be consciously accessed, it can be largely verbalized and flexibly transformed through processes of inference and reflection. It is, therefore, not bound up with specific problems but can in principle be generalized for a variety of problem types in a domain (Baroody, 2003 as cited in Schneider et al, 2011). Whereas on the other side, procedural knowledge, is seen as the knowledge of operators and the conditions under which these can be used to reach certain goals (Byrnes & Wasik, 1991 as cited in Schneider et al, 2011). Further, it allows people to solve problems quickly and efficiently because it is to some degree automated which can be accomplished through practice and allows for a quick activation and execution of procedural knowledge, since its application, as compared to the application of conceptual knowledge, involves minimal conscious attention and few cognitive resources. As a consequence, it is coupled to specific problem types (Baroody, 2003 as in Schneider et al, 2010).

However, Star (2005) and others view the relationship between conceptual and procedural knowdge as co-existant and as both being equally important to each other for the devalopment of mathematical knowledge. Thus Star (2005), has proposed the re-conceptualization of conceptual understanding and procedural fluency. Star and others have opposed the ideas of classifying conceptual understanding and procedural knowledge based on whether they contain rich connectedness or not. That idea, he argues, is making things difficult to classify the knowledge which belongs to the deep procedural aspects. In fact both conceptual and procedural understanding can have either a superficial or deep quality understanding (Star, 2005). There are different ways that one can know the key conceptual

ideas and procedures of mathematics and to Star (2005) skilful execution of mathematics could mean two different things. On one hand application of skills involves use of procedures automatically or by rote. Whereas on the other hand being skilled means being able to select appropriate procedures for particular problems, modifiying when needed or apply those procedures in a different context.

Star further emphasized two important ideas when one is to define what procedural knowledge or skills are: on the one hand, skilful execution involves being able to use procedures rapidly, efficiently, with minimal error, and with minimal conscious attention; in other words, to execute a procedure automatically or by rote (Anderson & Lebiere, 1998 as cited in Star, 2002). Whereas on the other hand, being skilled refers to one's ability to select appropriate procedures for particular problems, modify procedures when conditions warrant, and explain or justify one's steps to others; that is, to execute a procedure thoughtfully or deliberately (Star, 2005)

Moreover, Star (2005) has suggested that procedural knowledge can be executed in ways that involve deep understanding (e.g indicative procedures) other than rote (memorization) and this is the same with conceptual understanding. In fact Star and Seifert (2002) have argued that it is possible for someone to execute qualitative mathematical concepts through memorised first before they are understood. According to Greeno (1978) those scholars who argued that procedural understanding is all about rote learning might neglect the other ways in which a procedure can be executed other than by rote, some of which could be characterized as "intelligent" or even as indicative of "procedural understanding" as seen by Star (2005). This shows how most of those studies seems to considered procedural outcomes other than rote knowledge and turn a blind eye on its development (Star, 2005). In fact in executing a procedural skill requires that one have knowledge of its component steps and the order in which these steps should be applied. But not all performances of a skill are the same. One could say that the whole process of conceptual acquisition and mastering of procedures in mathematics, does not take place in isolation but it take place as a connection. This is supported by Baroody et al. (2007, p.123) who argue that "depth of understanding entails both the degree to which procedural and conceptual knowledge are interconnected and the extent to which that knowledge is otherwise complete, well structured, abstract and accurate".

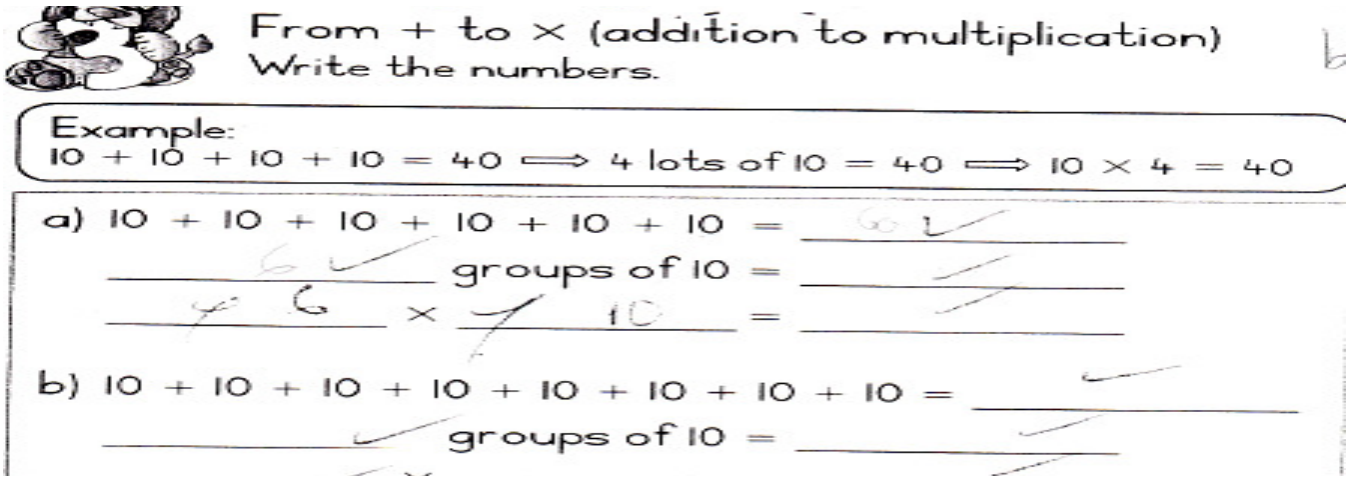
Hence conceptual undersrtanding and procedural fluncy co-exist. This implies that deep understanding entails both the degree to which procedural and conceptual knowledge are interconnected and the extent to which that knowledge is complete, well structured, abstract

and accurate. Hence deep understanding of both conceptual and procedural knowledge should be our major aim and priority for all mathematics educators. (Star, 2005) added that flexible problem solving requires that students integrate their conceptual knowledge of principles in the domain with their procedural knowledge of specific actions for solving problems. This integrated knowledge allows students to identify prior knowledge that is relevant including potentially relevant procedures and to adapt existing procedures to solve novel problems. Thus, integrated knowledge is not inert rather; it can be used to flexibly solve both routine and novel problems accurately and efficiently.

3. Data presentation and analysis

Mathematics tasks can be examined from different perspectives, that includes the number and kinds of representation evoked, the variety of ways in which they can be solved, and their requirements for communication (Stein et al, 2009). The tasks were randomly selected from the Grade 3 workbook which was the subject of this analysis. Figure 1 showns a sample of tasks.

Figure 1: Sample of learner tasks taken from Grade 3 workbook[13] (Task A and Task B)

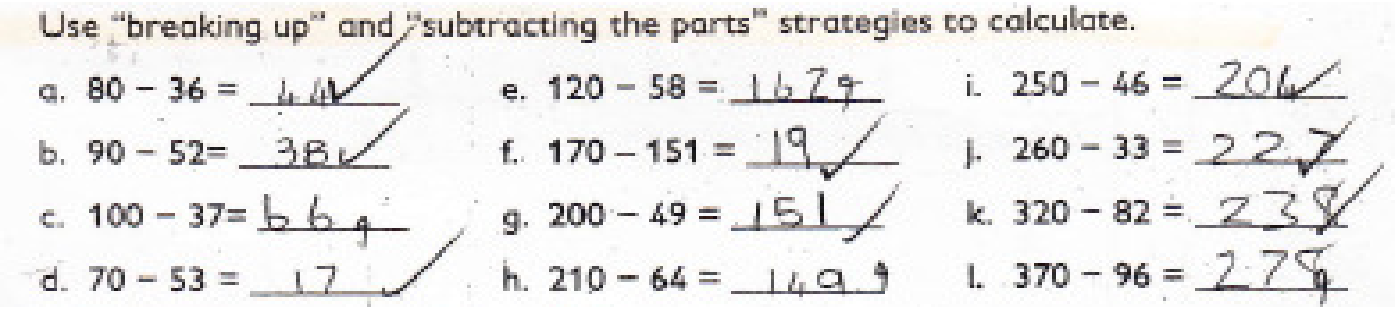

From + to × (addition to multiplication)
Write the numbers.

Example:
10 + 10 + 10 + 10 = 40 ⟹ 4 lots of 10 = 40 ⟹ 10 × 4 = 40

a) 10 + 10 + 10 + 10 + 10 + 10 = _____
_____ groups of 10 = _____
_____ × _____ = _____

b) 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 = _____
_____ groups of 10 = _____

[13] Department of Basic Education (2011) Numeracy workbook Grade 3, DBE, Pretoria, South Africa



Use "breaking up" and "subtracting the parts" strategies to calculate.

a. 80 − 36 = 44	e. 120 − 58 = 1629	i. 250 − 46 = 206
b. 90 − 52= 38	f. 170 − 151 = 19	j. 260 − 33 = 227
c. 100 − 37= 669	g. 200 − 49 = 151	k. 320 − 82 = 238
d. 70 − 53 = 17	h. 210 − 64 = 149	l. 370 − 96 = 278

In this paper I present my analysis of two tasks to establish the relationship between how conceptual understanding and procedural fluency are promoted and whether they are seen (in the design of the workbook task) as separate or presented as if they co-exist. To do that I looked at what type of understanding the learner might need to have to be able to solve the problem and what a learner needs to know in terms of conceptual understanding and procedural fluency.

Despite arguing that procedural fluency and conceptual understanding are inseparable, for analytic purposes I will engage with them separately. I used a checklist to identify criteria relevant to each knowldge (namely conceptual and procedural understanding) to assess learners' mathematical tasks for grade 3. The criteria were adapted from those developed by Suh (2007). I then assessed the learners' mathematics activities and used the checklist as an instrument to tease out the differences between conceptual and procedural understanding.

3.1 Assessment Rubric

The assessment rubrick was developed to guide the analysis of two tasks in a grade 3 learners' numeracy workbook. The rubric used focuses mostly on how the tasks promote and develop conceptual and procedural understanding by looking at its demands (e.g what a learner needs to know in terms of conceptual and procedural understanding). For example, a problem might be presented as 4 + 3 = 8. To analyse this task I look at the types of procedures i.e counting as well as putting things together that would be required for a learner to solve this problem. And I also look at the concept such as number, addition (putting together) as well as equal sign (equivalent). For example in this case a learner needs both conceptual and procedural understanding to get the answer.

Although it was my aim initially to also analyse this task based on the cognitive demands they promote, it was not possible. This is because to judge a mathematical task based on the deepness or the level of cognitive demands it poses, one needs to observe learners under taking the process and to get a learner to explain how she/he gets the answer.

I used the rubric by completing it in the following way: a tick or cross was placed in the appropriate box. A judgment was made based on the following: if the majority of the ticks were placed in the conceptual understanding boxes then the activity is considered to be a concepts based activity. If the majority of the ticks were placed in the procedural fluency boxes then the activity was considered to be a mainly skills oriented and a procedures activity that required the use and application of concepts covered.

3.2 Data presentation and analysis

My findings showed that both tasks promote the development of both conceptual and procedural understanding .

3.2.1 Addition and multiplication:

The fist task focuses on addition and multiplication. In this task, a learner is expected to solve different problems involving addition and multiplication e.g 10 +10 +10+ 10 = 40 ↔ 4 lots of 10 = 40 ↔ 10 x 4 = 40. In this task there are different mathematical concepts ranging from a concept of number, addition (putting together), multiplication (as repeated addition), grouping, place value, equal sign (equivalent). But at the same time there are different procedures that are needed by a learner to solve this problem such as counting in groups, forward and in multiples. While on the other hand this task introduced learners to multiplication as repeated addition of equivalent sets using number 10, the task required learners to find the answer of 10 +10 +10+ 10 = 40 ↔ 4 lots of 10 = 40 ↔ 10 x 4 = 40. In that way the task promotes the connection between the two operations. It is also testing the multiplication skill by drawing on learners' pre knowledge of addition. For example, if a learner was asked to complete the number statement using addition and multiplication. From this activity, we can see that there is a presence of both conceptual and procedural knowledge that is being promoted or demonstrated by the learner but one cannot tell which knowledge comes first before the other one.

Looking at the learner's answers, this activity promotes mathematical understanding/knowledge in terms of the above mentioned concepts. In this case a learner needs to draw on those concepts such as mathematical operation to be able to find the answer. One can see that this learner was able to provide the answer by using a procedure of her/his choice or the prescribed one (e.g horizontal addition). Not only that but for this learner to be able to find the right answer she/he has demonstrated procedures that are accurate and appropriate otherwise he/she could not get the answer. Based on this learner's work, the learner has demonstrated fluency with the addition combinations up to 10 + 10.

To promote conceptual and procedural fluency, the task introduced concepts (e.g., meanings of multiplication) as addition. It is possible for a learner to get the answer of any addition problem without understanding but for multiplication it might not be easy. Hence to explore the concept of decomposing numbers using groups of 10 made it easier for the learner. But a learner required both procedural and conceptual understand this includes the meanings of multiplication equally to be able to find the answer, which might be the problem if one's knowledge is lacking.

3.2.2 Breaking up and subtraction

The ability to visualize what is happening in a subtraction situation is an important foundation for understanding the operation and how it works, and for making good decisions about strategies for computation (Stein et al, 2009). Within this task there are also different concepts and procedures involved for one to deal with the sums or problems. In this task a learner is introduced to the understanding of different types of subtraction problems and to compute using any strategy of their choice for subtracting whole numbers. Although the task seems to be predominantly influenced learner's procedural fluency in subtracting accurately and efficiently there are also concepts involved and without an understanding of those concepts it might also not be easy for the learner to find the answer. The mathematical concepts involved include: whole number operations, addition and subtraction, and many more.

4. Conclusion

In conclusion the paper shows from the task analysis of the sample workbook that the two forms of knowledge, conceptual knowledge and procedural fluency, cannot exist in isolation and both are equally needed by a learner in learning of mathematics. Thus in both tasks I have analysed, the way that a learner achieves correct answers requires both the application of his/her procedural and conceptual understanding.

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