EXPLORING LEARNER UNDERSTADING OF MATHEMATICS USING CONCEPT MAPS

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Understanding in mathematics learning is a complex and challenging process for many learners in rural schools. It is so often defined as links and bonds, yet teachers do not teach bonds but are responsible for ensuring that understanding occurs. The study reports on the utility of concept maps in exploring learners' understanding of the "links and bonds" of mathematics. A multiple case study method involving Grade 10 learners in three high schools was used and it included empirical investigation in which concept mapping and reflective interviews were used as instruments for data collection. Through the use of concept maps, lack of vocabulary for expressing mathematics concepts was found to be one of the causes for poor understanding. Exposure of mathematics teachers in the use of concept maps as an assessment and teaching instrument may contribute towards better understanding in mathematics learning.

INTRODUCTION

Learning in mathematics and understanding are two processes that are directly proportional (Rubel & Chu, 2012). Learning may further be viewed as a function of understanding. Concept map is a teaching tool that may play a pivotal role in promoting understanding and establishing links and bonds that are critical in order for meaningful understanding to occur in a classroom situation. Understanding has different meanings to people depending on the context in which it is used (Yen, Lee & Chen, 2012). Garegae (2001) defines understanding as an agreement or a contract, an act of sympathy, comprehension, or a mental state. Star (2007) describes understanding as involving the following words: know, explain, procedure, rote, problem solving, network, bonds, links, web and schema. Simple and common as it may look, understanding in mathematics learning is seemingly very problematic and in some instances difficult to achieve. Concept mapping is seemingly a non- occurrence since it is not prescribed for use in any textbooks which is where mathematics authority seems to lie. Conventional assessment methods are less able to portray the lapses in understanding and that paralyses the teachers' effort in helping learners to develop adequate understanding.

The metaphorical description of understanding as a link in many studies is not helpful enough because teachers do not teach connections. The work of Skemp (1979) represents a landmark in the identification and distinction of relational and instrumental understanding. The Curriculum and Assessment Policy Statement (CAPS) FET Phase (Department of Education, 2011) documents that the study of mathematics contributes to personal development through deeper understanding and successful application of its knowledge and skills. The CAPS through its assessment standards presents understanding as an output of mathematics to the world and not as an input in the making of mathematics. Manipulative phrases, like 'are we together' are sometimes used to check if learners are attentive. Unfortunately, the yes response is often mistaken for understanding. Traditional teachers believe that mathematical reasoning and explanations are matters of intellectual courtesy

which comes only after mastering procedures. Hence, the problem the study particularly focuses on is the efficacy of concept mapping in identifying the lapses in learners' mathematical understanding.

Problem Statement

Learners have a great difficulty in constructing adequate understanding in mathematics learning. Reports on learning performance which is a possible indicator of how well understanding has been constructed suggest the high prevalence of inadequate understanding among mathematics learners. For instance, in 2003, the Third International Mathematics and Science Studies (TIMSS) tested grade 8 learners' mathematical proficiency and South Africa came last of the 50 participating countries. Maree, Aldous, Hatting, Swanepool and Van den Linde *(*2006:3) note that the failure rate in mathematics in South African schools remains unacceptably high. The poor performance in mathematics as indicated by TIMSS and Maree *et al.,* is an indicator that understanding as the goal of mathematics teaching and learning has never been adequately realized. The causes of this problem need to be amicably resolved lest it continue to cast gloom over the future aspirations of many learners and subsequently the economy of the country.

LITERATURE REVIEW

The focus is on describing what understanding is, and identifying the role that concept mapping may play in the portrayal of understanding. Studies suggest that concept mapping has the potential of making concept relations more transparent (Ross & Wilson, 2012). Accordingly, there seems to be confluence of the fact that both mathematical understanding and concept mapping are centered on interconnections, networks and interrelation between concepts (Wu, et. al., 2012).

WHAT IS UNDERSTANDING?

Van de Walle (2009) views understanding as something that exists along a continuum. According to the author, instrumental understanding has fewer and weak connections and gradually changes to relational understanding which has connections with more knowledge. Studies describe instrumental understanding as an exercise of memory, knowing the rule and being able to use it, and as a product of near sighted approach to learning (Tseng, et al. 2012). Devlin (2006) and Manon (2005) believe that instrumental understanding is a necessary condition for the development of relational understanding despite it being badly described. Relational understanding which is occasioned through a search for coherence in ideas is characterized by active enquiry and investigation. This study views understanding as a mental activity change that occurs through a search for coherence in mathematical ideas and can be modeled graphically by using concept maps.

The role of understanding in mathematics learning

It is a known fact that understanding supports improved performance for all learners at all levels. Manon (2005) argues that understanding breeds confidence and engagement whilst the lack thereof results in disillusionment and disengagement. According to the author, understanding supports improved performance for students at all levels. Van de Walle (2001) notes that understanding is intrinsically rewarding and self-generating; it enhances memory, improves problem solving abilities and attitudes; and facilitate the learning of new concepts and procedures. Understanding should be an ever present goal of mathematics

learning since its absence could lead to disillusionment and disengagement with disastrous consequences for mathematics.

Teaching for understanding

According to Davis and Simmt (2003), mathematics teachers are expected to occasion the emergence of a complex collective whose interactions and products are mathematical. Van de Walle (2001), agrees that instruction should be structured in a way that includes students actively constructing their own knowledge with understanding whilst the relationship among concepts, skills and problem solving is emphasized. Selinger (1994) suggests that tests can be set in such a way that instrumental understanding cannot be useful for questions and relational understanding is required for all questions. Sponsel (2003) observes that providing students with the formula $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ does not instantly create understanding. The literature suggests that procedures learned without understanding tend to be error prone, are easily forgotten and do not transfer easily to novel problem types.

Concept maps

Huerta, Galan and Granell. (2009) define a concept map as a graphical representation consisting of nodes (concepts) and labeled lines.

According to them (Huerta *et al.,* 2009), lines denote a relation between a pair of concepts. Brinkmann (2009) describes concept maps as powerful tools for identifying students' knowledge structures, especially misconceptions and other conceptions. According to the author, concept maps facilitate meaningful learning, they aid organizing and understanding of new subject matter. Barolos (2009) and Kersting *et al.* (2012) have elaborated on concept maps as a tool for promoting understanding. The above excerpt is an example of a concept map constructed by one of the subjects.

Poor understanding is largely attributed to memorization which according to skemp (1979), arouses no interest, gives learners no pleasure, work is boring and difficult, and learners do it because they have to. Cohen (1993) agrees that rote learning is a bankrupt or near sighted approach to learning. Mwakapenda (2004) acknowledged that lack of expertise in expressing mathematics and inadequate understanding in mathematical language and mathematics itself are likely to constrain student's ability to describe the links between concepts. Manon (2005) notes that language, material and symbols are mathematical tools which play an important role in learning mathematics with understanding.

RESEARCH DESIGN AND METHODOLOGY

The study was guided by the following research questions: (1) How do individual learners construct their understanding of mathematics concepts? and (2) What are the causes of poor understanding in mathematics learning? A multiple case study method was selected to determine how the subjects in each case have constructed their understanding of the given sets of concepts and to check if causes of poor understanding were similar (Creswell, 2007).

PARTICIPANTS IN THE STUDY

A group of Grade 10 Mathematics learners (n=12) were selected from three schools in Taledi central cluster (in Dr Ruth Segomotsi Mompati District) through purposive sampling methods. The three schools are classified as cases in this study. Each school had one mathematics teacher for the grade 10 learners. The educator concerned was requested to participate in the process of identifying participants (Snowball sampling [Creswell, 2007]).

PROCEDURE

Qualitative research instruments were used in the process of exploring learners' understanding of mathematics learning. Concept maps, interviews and observations were used for collecting qualitative data from the participants.

Concept maps

Each learner was given two sets of concepts from algebra and geometry respectively. Each set had 15 content concepts which are critical in achieving Learning Outcome 1 and 3 respectively. Learners were asked to independently design two distinctive concept maps using the given sets of concepts. Learners were encouraged to draw the maximum possible links between the concepts within each setting. Sample concept map was to demonstrate to learners what a concept map was. The following concept sets were given to learners.

Set one (LO1 concepts): number systems, fraction, rational numbers, real numbers, integer/s, ratio, decimals, percentages, natural numbers, whole numbers, proportions, exponent, Inverse, denominator and identity.

Set two (LO 3 concepts): angle, line, acute, obtuse, right angle, area, prism, volume, equilateral, triangle, polygon, isosceles, degree/s, sides and length

Interviews

The following open-ended questions were asked and follow-up probes were made to get greater detail of information of how knowledge was organized:

a) How did you start drawing the map?

b) Why did you leave out some of the concepts?

c) Is there any concept that you know a lot about, and why? And

d) Is there any concept that you think you don't know much about? [And why …?]

Observation

The observations were recorded as field notes and analyzed for content. The researcher employed a simple observation method in the current study. The researcher employed active listening during interviews. The use of mathematical language was observed in the process.

RESULTS

The results of the study are categorized as responses to the research questions which were formulated to guide the study.

How do individual learners construct their understanding?

A learner understands a thing well if he or she can do the following: identify its types, uses and components; and can provide relevant examples. Literature does not view the types, uses, and components as defining the concept understanding but broadly embraces it as knowledge.

The understanding constructed by learners was a continuum consisting of the following stages: Non-understanding (confusion and incorrect use of language), incoherence and illogical clustering, coherent and logical abstractions (Chunking). The initial stage of continuum is termed non-understanding because agents of understanding are mixed up. Non-understanding is not the total absence of understanding but represents a situation where things that are not related were linked directly. For example, indicating that fractions were components of a set of natural numbers. During interviews one of the learners remarked that whole numbers included integers.

Learner understanding occasioned was twofold: it was largely instrumental than relational. Non-understanding and incoherence illogical cluster are recursively organized to form instrumental understanding. Coherent and logical abstraction emerges from incoherence and illogical cluster through re-organization of components which is effected through further learning. The organization of learner understanding is such that it flows from a more inclusive concept to the least inclusive concept which suggests that the least inclusive concept could be a component of a more inclusive one. Learners were mostly starting the concept maps with the concepts which were seen to be big [i.e. the more inclusive concept].

What are the causes of poor understanding in mathematics learning?

Learners lack the language skills for expressing the mathematical relation. For example, concept map above has the following proposition: “Area is used to measure capacity and angle to calculate”. It also includes inappropriate positioning of concepts at the end of a connector. The propositions are not appropriately verbalized. For example, number system is a type of rational number. This suggests that the learner concerned is confused. Confusion is a major cause of poor understanding

DISCUSSION AND CONCLUSION

The following may be viewed as the key aspects that may need to be taken into consideration:

How do individual learners construct their understanding of mathematics concepts?

The content embedded in any mathematical theme should include types, uses and components, and examples to initiate instrumental understanding. Pedagogical practice should be context based, organized logically and coherently to include the integratedness of concepts to ensure that relational understanding is occasioned. For example,the teacher may comprehensively describe a fraction as follows: A fraction has three types, namely: Proper, improper and decimals; it can be expressed as a ratio; are used to create proportions; can be written as powers.

What are the causes of poor understanding in mathematics learning?

If assessment had included concept mapping, educators would have discovered the deficiencies in the learners’ inability to comprehend mathematical concepts. The omissions and concepts that are improperly connected reveals deficiencies in learner’s ability to comprehend such concepts. Subsequent teaching could have been used to redress the lapses identified. For this reason, concept mapping is recommended for use in classroom practices both as a tool of assessment and pedagogy. All mathematics teachers need to be

trained on concept mapping for them to use it effectively as a tool to promote understanding and also for assessment.

The omissions in concept mapping were due to inadequate exposure to mathematical concepts embedded in a specified learning outcome. All participants did not use the concept identity in their respective concept maps. During the interview some indicated that their teacher has never used it in class. Given the integrated nature of learning and that comprehensively learning was done in isolated bits or pieces, it is worth recommending that teachers should always have the integratedness of concepts whenever they are teaching.

Deliberate effort to reach out to all teachers to increase their use of mathematical language in mathematics teaching is recommended. Mathematics teachers should be encouraged to prescribe language based activities for learners. Poorly developed language skill aggravates the misconceptions of the questions asked on externalized question papers and these result in underperformance. The effort to improve learners' language competence could aid the understanding of mathematical concepts, improve mathematics learning performance and put an end to the hue and cry that follow the release of the end of year results.

REFERENCES

Baralos, G. 2009. Concept mapping as evaluation tool in mathematics learning. Available: http://www.google.co.za/search?hl. Date of access:05Oct.2010

Brinkmann, A. 2009. *Knowledge maps- Tools for building structures in Mathematics.* Available: http://www.google.co.za/search?hl. Date of access: 15Oct.2011

Cresswel, J.W. (2007). *Qualitative inquiry and research design: Choosing among five approaches.* Lincoln:University of Nebraska

Cohen, A. 1993. A new educational Paradigm. *Phi Delta Kappan;* 791-795, June. Department of Education. (2011). *Curriculum and Assessment Policy Statement (CAPS) GET Mathematics.* Pretoria: Department of Education

Devlin, K. 2006. How do learn mathematics? *Devlin' s angle.* MAA on line. . Available: http://www.google.co.za/search?hl. Date of access: 03 March 2011

Davis, B. & Simmt, E. 2003. Understanding Learning Systems: Mathematics Education and Complexity Science. *Journal for Research in Mathematics Education,* 34(2):137-167.

Garegae, K.G. 2001, A Quest for Understanding Understanding in Mathematics Learning: Examining Theories of Learning. The teachers' beliefs about mathematics, its teaching and learning. Available: http://www.google.co.za/search?hl. Date of access: 15 Oct. 2011.

Huerta, M.P., Galan, E., Granell, R. 2009. Concept maps in Mathematics Education: A possible framework for student assessment. Available: http://www.google.co.za/search?hl. Date of access: 15 Oct. 2011.

Kersting, N.B., Givvin, K.B., Thompson, B.J., Santagata, R. & Stigler, J.W. (2012). Measuring usable knowledge: Teachers' analyses of mathematics classroom videos predict teaching quality and student learning. *American Educational Research Journal*, 49 (2)

Manon, J. (2005). Making sense: Teaching and learning mathematics with understanding. http://www.google.co.za/search?hl Date of access: 10 March 2011

Maree, M., Aldous, C., Hatting, A., Swanepool, A. & Van Den Linde. 2006. Predictors of learner performance in Mathematics and Science according to a large scale study in Mpumalanga. *South African Journal of Education,* 26(2): 229-252.

Mwakapenda, W. 2004. Understanding student understanding in mathematics. *Pythagoras,* 60: 28-35.

Noel, M-P. & Rouselle, L. (2012). How understanding mathematical learning disability may guide mathematics teaching. *BJEP Monograph Series 11,* No. 8 1 (1), 63-82

Rubel, L.H. & Chu, H. (2012). Reinscribing urban: teaching high school mathematics in low, income urban communities of color. *Journal of mathematics teacher education.* 15 (1), 39-52

Selinger, M. (1994). *Teaching mathematics.* New York: Routledge

Skemp, R.R. 1979. *The Psychology of Learning Mathematics*. New York: Penguin.

Sponsel, B. 2003. Midterm- Mathematical understanding: Maths791. 1-10

Star, J. 2007. What is mathematical understanding? How can we measure it? Theoretical and imperial reflections. (A seminar presentation- Department of Elementary Education Seminar, Middle East technical University, Ankara, Turkey, 27 August 2007) on line: http://gseacademic. edu /~ starjo. Date of access: 29 March 2010

Tall, D. 1978. The dynamics of understanding mathematics. *Mathematics teaching.* 81:50-52.

Tseng, K-H, Chang, C-C, Lou, S-J., Tan, Y., & Chiu, C-J. (2012). How concept mapping perception navigates student knowledge transfer, performance. *Educational Technology and Society,* 15 (1), 102-115

Van de Walle, A.J. 2001. *Elementary and middle school mathematics: Teaching developmentally.* 4th ed. New York: Longman.

Van de Walle, A.J. 2009. Main ideas: (Discussion paper. p. 1-3) http://wps.ablongman.com/ab_vandewalle_maths_6/54/13859/3547980.cw/content/ind...(Date of access: 05 Jan 2011)

Wu, P-H., Hwang, C-J., Milrad, M., Ke, H-R. & Huang, Y-M. (2012). An innovative concept map approach for improving students' learning performance with an instant feedback mechanism. *British Journal of Educational Technology,* 43 (2), 217-232

Yen, J., Lee, C. & Chen, I. (2012). The effecs of image-based concept mapping on the learning outcomes and cognitive processes of mobile learners. *British Journal of Educational Technology,* 43 (2), 307-320