TEACHERS' UNDERSTANDING OF A KEY ALGEBRA CONCEPT

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Abstract

This paper looks at how working in a professional learning community can result in a shift in teachers' understandings of adding variables and the concept of a variable. As four teachers in a professional learning community in the Data Informed Practice Improvement Project (DIPIP) were designing an algebra lesson, their own understanding of a key Algebra concept developed further. The teachers' development took some time and required the support and the safety made possible in the professional learning community.

Introduction

Research has shown that working with variables can be challenging for teachers and learners. Sfard and Linchevski (1994) looked at the history and teaching of algebra and suggested that for a lot of learners, algebra is nothing more than a string of symbols to which well defined manipulations are applied. Many learners do not make meaning in algebra classes.

Professional learning communities as a model for teacher learning are being increasingly developed and researched internationally, both in mathematics education and more generally (Brodie, 2011). Successful professional learning communities create environments where teachers can explore their strengths and weaknesses with colleagues, develop collaborative solutions to problems of practice and implement new ideas collectively for the benefit of learners.

In this paper we discuss how teachers in one professional learning community learned to teach algebra more meaningfully.

DIPIP: Professional and Networked Learning Communities

The Data Informed Practice Improvement Project (DIPIP) is a professional development project where teachers form professional learning communities. In 2011 DIPIP worked in seven schools in Gauteng, involving twenty-two mathematics teachers, who formed four professional learning communities[14]. During school terms a facilitator would meet with the teachers of each professional learning community on a weekly basis at the school. The facilitators worked closely with the teachers and guided the process strongly. The idea is that after the first year, the facilitators will gradually withdraw guidance and the professional learning communities will continue to work together in their schools.

Katz et al (2010) argued that having a practice that is the focus of the professional community supports successful teacher learning in a professional learning community. In DIPIP this focus is working with learners' errors. We acknowledge that errors make sense to learners in terms of their current knowledge and try to support teachers to understand why the errors make sense to learners.

As part of the DIPIP activities, network meetings were organised where two or more communities came together for a combined meeting. In 2011 the first network meeting provided opportunities for each professional learning community to account for the reasoning behind learners' errors in a common test that their learners wrote and the teachers analysed. Each group gave a ten-minute

[14] One community involved three schools.

presentation and about thirty minutes were used for discussions where members of other groups as well as the project team members could comment, question, challenge and give feedback. The facilitator of the network learning community meeting worked towards hearing the teachers' views, putting them into conversations with each other, and challenged the teachers to think more deeply about their practice as well as the reasoning behind learner errors (Brodie & Shalem, 2011).

Analysing learners' errors from tests

In 2011 DIPIP used a standardised, international test[15] as one focus of the professional learning communities' meetings. One class of learners per participating teacher at each school in the project wrote the paper. After the test was written, the professional learning community recorded all the errors made by learners. Teachers found it interesting that common errors occurred throughout all grades. Each professional learning community identified five common errors to work with. These errors were analysed by describing the content addressed in the specific test item, analysing the correct response and analysing the error(s) and the reasoning behind them.

The test addressed questions in algebra. One question asked learners to "Write more simply, where possible", a number of expressions such as $2a + 5a$, $2a + 5b$, $2a + 5b + a$. The focus of this paper is how the teachers in one professional learning community came to understand their learners' errors in simplifying the expression $2a + 5b$.

Four of the classes in the school (two Grade 10 and two Grade 11 classes) wrote the test. The distribution of answers was as follows:

Observed answer	**% of learners**
$2a + 5b = 2a + 5b$ or leaving answer open (correct answer)	10%
$2a + 5b = 7ab$	70%
Other incorrect answers	20%

The teachers of this professional learning community were concerned about the error $2a + 5b = 7ab$ and the number of learners making it.

Herscovics and Linchevski (1994) looked at the gap between arithmetic and algebra in a large-scale assessment carried out in Great Britain, involving three thousand secondary school learners. They stated "when asked to add 4 onto $3n$, 36% had a correct answer $3n + 4$, but 31% answered $7n$ while 16% gave 7 as the answer" (Herscovics & Linchevski, 1994, p. 62). They argue that getting the incorrect response of $7n$ might be as a result of what Sfard and Linchevski (1994) call the process-product dilemma. The dilemma lies in the fact that where we work with an expression $7n$, the $7n$ indicates both the process of multiplying 7 and n and the answer when the 7 and n are multiplied. It could also be difficult for learners to accept the lack of closure in an answer of $3n + 4$. This difficulty stems from the arithmetic requirement that elements connected by operations be replaced by a single one (e.g. 3 + 2 = 5) (Herscovics & Linchevski, 1994).

[15] This test was initially developed in the United Kingdom as part of the research programme "Concepts in Secondary Mathematics and Science" (CSMS) (Kûchemann, 1981). The test was extended and used by the "Increasing Competence and Confidence in Algebra and Multiplicative Structures" (ICCAMS), project based at Kings College London. We are using a slightly modified version of the ICCAMS test.

Identifying a conceptual problem

One of the facilitator's roles is to bring research-based knowledge into the communities' discussions so that they do not become solipsistic (Jackson & Temberley, 2008). Two research papers on algebra were discussed at the professional learning community meeting. The first paper by Booth (1999) reports on possible causes for children's difficulties in learning algebra and highlights how a teacher can assist learners to interpret symbols. In algebra the + sign can be an operation, as in $2a + 5a$, but it can also be in the result of an operation, like in $2a + 5b$. Usiskin (1999) suggest that the different conceptions of algebra are related to the different uses of variables. He is of the opinion that there is a lot of confusion amongst learners about the different uses of the variable. These papers helped the teachers to see that their learners' errors were not only local – other learners in other places also make them - and thus the teachers were encouraged to investigate the learners' thinking behind the error and to think of ways of addressing them.

At the first network meeting in 2011, the professional learning communities in one district met. Each professional learning community prepared a discussion on the five common errors identified in their learners' tests. Each community shared their discussion on the item with the other communities. All of the communities identified the error $2a + 5b = 7ab$ and there was extensive discussion on this.

At one point a teacher made a comment about the learners being unsure about the collecting of like terms and suggested using the metaphor of having two apples and five bananas - which stay as two apples and five bananas when you put them together. The project leader made a comment that the metaphor is actually not a good one to use. The learners can, and often do, say that two apples and five bananas are seven fruit. More important is the fact that *a* does not represent an apple, it represents a number. The project leader then asked the network meeting if $2a$ can be added to $5b$. Initially some teachers said that it could not be, but after some wait-time on the part of the project leader, other teachers responded by saying that it can be added and that the answer is $2a + 5b$.

Development of Teachers' Understanding

During the first professional learning community meeting after the network meeting the community discussed the network meeting, including the metaphor of apples and bananas. Towards the end of the discussion, one of the teachers, Elelwani, remarked that while she accepts the view that the metaphor is mathematically incorrect, her experience was that it works when it is used. She further indicated that it was with the same metaphor that she was taught when she was at school. Initially the three other teachers agreed with her, but then one teacher, Jabu, challenged Elelwani. He asked her why, in terms of the metaphor, we cannot add apples and bananas, but we can multiply them, since $2a + 5b$ cannot be simplified further but $2a \times 5b = 10ab$. His initial challenge was around the use of the metaphor when multiplying $2a$ and $5b$. Immediately after that he followed up his own comment and said "No man, the *a* is not for apples – it is for a number and it cannot be the number of apples if there are 2 apples". There was not a long discussion at that time and it did not seem that Elelwani was convinced.

At the next meeting the activity was to design a lesson drawing on the issues that had come up in the error analysis and the network meeting. All four teachers would teach this lesson. When the facilitator asked the community what the critical issue in the lessons should be, Elelwani said that it should be for learners to understand that a variable represents a number and not an object. This

comment contradicted her comment during the previous meeting when she defended using two apples and five bananas as a metaphor for $2a + 5b$.

The above discussion suggests that Elelwani felt safe enough to have an opinion about the metaphor of apples and bananas – even though it meant that she disagreed with the project leader and the facilitator. There is evidence of teachers using a metaphor of apples and bananas to help learners understand the adding of unlike terms (Chalouh & Herscovics, 1999; Philipp, 1999). Elelwani defended the metaphor saying that she knows why it cannot be used, but still thinks that it is a good metaphor. It was only when she was challenged in the professional learning community and the challenge indicated that the *a* cannot be for apples that Elelwani and the other teachers in the professional learning community really agreed that it should not be used and understood why. The reflection on the network meeting in the professional learning community meeting caused a critical reflection on the metaphor and helped to develop consensus that it should not be used. This consensus generated a new understanding, that the *a* and *b* represent numbers. The consensus also brought a commitment to teach with this new understanding and the teachers found a new way to help learners understand the meaning of a variable.

Conclusion

Cook and Brown (1999) distinguish between knowledge and knowing. Knowledge is something that can be possessed, while knowing implies using knowledge in action. Kazemi and Hubbard (2008) use the distinction between knowledge and knowing when looking at teachers' learning in practice. Teachers possess knowledge, but just having the knowledge is not enough to be good at what they do (Kazemi & Hubbard, 2008). Knowing here refers to knowledge becoming part of the practice of the teacher. Elelwani had the knowledge about the metaphor not being a good way of teaching. Through the challenge and discussion at the network and professional learning community meetings it became part of her practice and the practice of the other teachers. There was a movement from knowledge to knowing.

This paper argued that working in a professional learning community with a strong focus on learners' errors assisted in bringing about a deeper understanding of a variable representing a number in the four members of one professional learning community. The key features of the professional learning community were that teachers were safe to express their opinions, reflected on challenges from their colleagues and were given time to build their understandings for themselves in relation to what they had learned.

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