MATHEMATICAL PROFICIENCIES DISPLAYED BY GRADE 12 LEARNERS IN THEIR RESPONSE TO FINAL EXAMINATION QUESTIONS IN THE CONTEXT OF SOUTH AFRICAN SCHOOLS IN THE GAUTENG PROVINCE

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Abstract: This paper is based on a dissertation study submitted to the Faculty of Education at the University of Johannesburg in fulfillment of the Master's degree in Mathematics Education. The dissertation focused on an analysis of grade 12 examination scripts on sequences and series, differential calculus, analytical geometry and trigonometry in the Gauteng Province in South Africa. The review of the question papers revealed that questions on sequences and series and differential calculus does not demand higher order reasoning. The analysis revealed three categories of learners' mathematical proficiencies, not proficient, yet to be proficient and proficient.

Keywords: mathematical proficiency, conceptual understanding, strategic competence, adaptive reasoning.

IN TRODUCTION

The Department of Education (1996, 2002 and 2007) are policy statements that recently addressed educational reform in South Africa but despite these changes the grade 12 final Mathematics examination results in South Africa remain low. Leung (2005) highlighted the2003 TIMMS studies that focuses on mathematics achievement in developing and developed countries, showed that South Africa was the lowest in mathematics achievement among the countries that participated in the study in 2003.

Taylor and Vinjevold (1999) find some challenges in Mathematics education, such as, attitudes of teachers towards learner-centeredness, achievement of outcomes, teachers' low level of conceptual knowledge, domination of teacher-centeredness as opposed to the structure of curriculum, lack of learning structures in classrooms that promote the development of higher order skill and improper use of everyday examples by teachers to relate to their teaching. The DoE (1995, 1996 and 2007) addresses concerns of learner-centeredness, achievement of outcomes, resources allocation and holistic development of learners but these issues still remain a challenge in various classrooms.

The nature of mathematical proficiencies

Allen, Donaldson, Edenfield, Fox, Gleason, Gold, Kelly, Kilpatrick, Wilson and Wilson (2009) conceptualizes the framework for mathematical proficiencies into three categories, mathematical proficiency with content, mathematical proficiency in learning and teaching and proficiency in mathematical activity. This study used mathematical proficiency with content that was discussed by Kilpatrick, Swafford and Findell (2001)and National Research Council (2001 and 2002) and the first four mathematical proficiencies were used; procedural fluency,

conceptual understanding, strategic competence and adaptive reasoning, these are intertwined, interdependent, and interconnected.

Danley (2002) and Schoenfeld (2007) argue that procedural fluency and conceptual understanding form mathematical knowledge, strategic competence and adaptive reasoning are mathematical skills.

Statement of the problem

Stein *etal*. (1996) focuses on well structured mathematical tasks, key issues raised are; task features are that mathematical tasks must promote students thinking and reasoning, provision of students' sense making, existence of multiple solution strategy and existence of multiple representations. Furthermore mathematical tasks must be characterized by the following cognitive demands; memorization and use of algorithms with or without concept understanding, complex thinking and reasoning, tasks must demand explanation, justification and conjecturing. Some conceptual issues arise in the quest to identify mathematical proficiencies that learners display in their response to examination questions. Schoenfeld (2007) explains that for learners to be proficient in procedural knowledge and conceptual knowledge the following theoretical constructs must be addressed: are they using multiple solution strategy in their response to examination questions? Do students produce mathematical explanations and justifications? Do students engage in high level thinking and reasoning?

The purpose of the study was to find mathematical proficiencies that are intended for mathematics learning, achieved by learners during Mathematics learning and detected as learners finish high school when they are examined. The study undertook the following objectives to achieve the aim; A review of the 2007 Grade 12 Senior Certificate Mathematics final examination question papers to identify mathematical proficiencies that the examination questions demand, analyze the2007 grade12 Senior Certificate Mathematics final examinations learners' scripts to identify mathematical proficiencies that learners display in their response to examination questions.

The study responded to the following research question to achieve its aims and objectives: What mathematical proficiencies do grade 12 learners display in their response to examination questions on sequences and series, differential calculus, analytical geometry and trigonometry? At the beginning of the study I assumed that these are the questions that address the whole spectrum of proficiencies and would provide me with the necessary data.

Theoretical framework and literature review

The study used constructivism as a lens to view learners' acquisition of mathematical proficiencies. Bruner (1966) highlights that constructivism is when learners actively construct their own understanding and knowledge of the world, through experiencing things and reflecting on those experiences, reconcile new experiences with old ones with the help of a knowledgeable instructor. Vygotsky (1978) further discusses that constructivism is when learners' socially construct knowledge linguistically through interacting with signs and symbols under the guidance of a knowledgeable peer or adult. Ausubel (1985) explains that constructivism is an active social construction of meaning through hypothesis generation, testing and generalizations. Lastly, Watts and Bentley (1991) defines constructivism as the theory that brings about the knowledge of reality through perception, language, memory and

the ability to go beyond given information. Constructivism is conceptualized by the discussion of the Zone of Proximal Development as follows*;*

'This difference between twelve and eight, or between nine and eight, is what we call the zone of proximal development. It is the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under the guidance or in collaboration with a more capable peer.' Vygotsky (1978: 86).

Davydov (1995) explains that actual development level is the level of the development of mental functions that has been established as a result of finished developmental cycles which enables learners to solve problems independently, and potential development is the ability to solve problems with the help of the teacher or more knowledgeable peer. In the context of this study, the ZPD shows the mathematical proficiencies that learners display in their response to examination questions, which is the actual development because final examinations are written by learners without the help of a peer or educator.

The study was on mathematical proficiencies that learners display in their response to examination questions. Kilpatrick *etal.* (2001) explains that mathematical proficiency is knowledge, expertise, competence and the facility of mathematics. Danley (2002) argues that mathematical proficiency is the ability to engage in Mathematics with understanding, computing, applying, reasoning and engaging. N.R.C (2002) shows that mathematical proficiency is based on a conception of what it means to be competent in Mathematics. Kilpatrick etal. (2001) and the N.R.C (2001) displays how mathematical proficiencies are intertwined, interconnected and interdependent.

The study used four mathematical proficiencies, conceptual understanding, procedural fluency, strategic competence and adaptive reasoning. Allen *etal*. (2009) specifies that conceptual understanding and procedural fluency form mathematical knowledge, strategic competence and adaptive reasoning are mathematical skills). (Kilpatrick *etal*. (2001) and Zakaria and Zaini (2009) explain that conceptual understanding is the ability to comprehend mathematical concepts, operations and relations. Star (2001) and Danley (2002) argue further that it is the *knowing why* of Mathematics knowledge which involves the ability to explain answers and identify mathematical relationship between procedures. Kilpatrick *etal*. (2001) defines procedural fluency as the learners' skills of carrying out mathematical procedures in a manner that is accurate, flexible and appropriate. Star (2001) explains that procedural fluency is '*knowing how'* of mathematical knowledge which involves learners ability to quickly recall and accurately execute procedures.

NRC (2001and 2002) argues that strategic competence is the learners' ability to formulate, represent and solve mathematical problems. Allen *etal.* (2009) elaborates that learners must experience the use of multiple strategies of solving a mathematical problem taking into cognizance that some strategies are specific to solving a particular problem. Star (2001) illuminates that learners must understand the context of the problem, the key elements of the problem and when representing the problem must discard irrelevant features. Kilpatrick *etal*. (2001) explains that adaptive reasoning is the learners' ability for logical thought, reflection, explanation and justification. It involves the learners' capacity to think with logic about the relationship between mathematical concepts and relations.

(Huitt (2009); Forehand (2005) and Schoenfeld (2007) shows that examination questions have action verbs from the Bloom's taxonomy such as define, illustrate, solve, compare, develop and justify, they determine the mathematical proficiencies that learners must display in their response to the examination questions. In this study mathematical proficiencies that learners display in their response to examination questions are identified and classified with the help of the Bloom's taxonomy.

RESEARCH METHODS

The study was purely qualitative; the data is from learners' examination scripts already collected by the University of Johannesburg project called Script analysis. Volkwein (2001) argues that qualitative research involves the process of generating theory by the researcher through the interpretation of evidence. Bogdan and Biklen (1998) explains that qualitative research puts emphasis on viewing variables in the natural setting in which they are found. Creswell (2005) and Cohen and Manion (1994) shows that detailed data is gathered through open ended questions that provide direct quotations and analysis of data is characterized by inductive reasoning by the researcher. A thick description of learners' mathematical proficiency was provided by the researcher; hence the study is purely qualitative.

The study has two components, the review of the 2007 grade 12 higher grade Mathematics final examination questions and the analysis of the grade 12 final examination scripts from the Gauteng Department of Education. Questions were selected from the 2007 grade 12 higher grade Senior Certificate Examination and they are: Paper 1; Question 5, 6, and 7; Paper 2; Question 1, 3, 4, 5, and 6. There were 90 randomly selected scripts from batches of 1000 scripts that were used in a larger project called script analysis in the Faculty of Education at the University of Johannesburg and marks ranged as shown below; group A (GA) from 0-50, group B (GB) from 51-90 and group C (GC) from 90-200 marks.

Data Management and analysis

The review of the 2007 Higher Grade Mathematics final examination is summarized in table 1 below:

TOPICS	Procedural Fluency	Conceptual Understanding	Strategic competence	Adaptive reasoning	TOTAL
Sequences and Series	7	2	7	0	16
Differential calculus	13	6	13	0	32
Analytical Geometry	6	5	6	1	18
Trigonometry	18	13	18	4	53

Table 1: A summary on the provision of mathematical proficiencies in the 2007 higher grade mathematics final examination questions.

The review showed that all (100%) of the examination questions on all the topics demand procedural fluency and strategic competence. Two out of seven examination questions (28.6%) on sequences and series demand conceptual understanding. Zero out of seven examination questions (0%) on sequences and series demand adaptive reasoning. Six out of thirteen examination questions, (46.2%) on differential calculus demand conceptual understanding and no questions (0%) examination questions on differential calculus demand adaptive reasoning. The review also showed that five out of six (83.3%) of the examination questions on analytical geometry demand conceptual understanding. One out of six (16.7%) examination questions on analytical geometry demand adaptive reasoning. Lastly the review showed that thirteen out of eighteen (72.2%) examination questions on trigonometry demand adaptive reasoning. Four out of eighteen (22.2%) examination questions on trigonometry demand adaptive reasoning.

The mathematical proficiencies that learners display in the 2007 higher grade Mathematics final examination scripts were distinguished by codes that have been developed using theoretical constructs on the mathematical proficiencies. The codes are shown in table 2 below;

Mathematical Proficiencies	**Performance By learner**	**Codes**
Conceptual Understanding	No response. It is either learners only wrote the question number or left a blank space.	CU0
	Learner displays no knowledge of connecting procedures & rules.	CU1
	Learner connects Mathematics procedures, concepts and algorithms with no justification.	CU2
	Learner connects Mathematics procedures, concepts and algorithms with minimal skill of justification.	CU3
	Learner connects & reconnects mathematical procedures, concepts and algorithms with sound mathematical justifications and explanation.	CU4
Procedural Fluency	No response. It is either the learner only wrote the question number or left a blank space.	PF0
	Learner displays no knowledge of the procedure.	PF1
	Learner shows insight of a procedure with major mathematical errors.	PF2
	Learner shows acceptable knowledge of procedure with minor errors.	PF3

	Learner carries out mathematical procedures accurately, appropriately and flexible.	PF4
Strategic competence	No response to the question. It is either learners only wrote the question number or left a blank space.	SC0
	No knowledge of formulating, representing and solving mathematical problems.	SC1
	Formulate, represent and solve mathematical problems with some computational errors (no creativity).	SC2
	Formulate, represent and solve mathematical problems showing reasons of 'how' and minimal reasons 'why' of mathematical knowledge (minimum creativity).	SC3
	Formulate, represent and solve mathematical problems showing both the 'how' and 'why' of mathematical knowledge (creative response).	SC4
Adaptive Reasoning	No response to the question. It is either learners only wrote the question number or left a blank space.	AR0
	Uses methods to display logical thought, reflection and justification	AR1
	Uses empirical methods to display logical thought, reflection and justification.	AR2
	Uses abstractions to display logical thought, reflection limited to mathematical reflection and explanation.	AR3
	Uses abstractions to display logical thought with mathematical reflection, explanation and justification.	AR4

Table 2: codes for mathematical proficiencies.

Kilpatrick *etal*. (2001) showed that mathematical proficiencies are interwoven, interconnected and interdependent. A Summary of mathematical proficiencies displayed by learners in their response to chosen examination questions is shown below. The mathematical proficiencies were shown in form of strands. A question by question analysis of mathematical proficiencies displayed by learners in their response to the chosen examination questions is shown below:

EXAMINATION QUESTIONS	CATEGORIES OF MATHEMATICAL PROFICIENCIES					
	NOT PROFICIENT		YET TO BE PROFICIENT		PROFICIENT	
	PR0CEDURAL	CONCEPTUAL	PROCEDURAL	CONCEPTUAL	PROCEDURAL	CONCEPTUAL

SEQUENCES AND SERIES	PF0+SC0 PF1+SC1	PF0+SC0+CU0 PF1+SC1+CU1	PF3+SC2 PF2+SC1 PF2+SC2	PF3+SC2+CU2 PF2+SC1+CU1 PF2+SC2+CU1	PF4+SC4	PF4+SC4+CU4
TOTAL	143	69	33	11	47	10
DIFFERENTIAL CALCULUS	PF0+SC0 PF1+SC1	PF0+SC0+CU0 PF1+SC1+CU1	PF3+SC2 PF2+SC1 PF2+SC2	PF3+SC2+CU2 PF2+SC1+CU1 PF2+SC2+CU1	PF4+SC4	PF4+SC4+CU4
TOTAL	195	126	31	52	44	92
ANALYTICAL GEOMETRY	PF0+SC0 PF1+SC1	PF0+SC0+CU0 PF0+SC0+CU0+AR0 PF1+SC1+CU1 PF1+SC1+CU1+AR1	PF3+SC2 PF2+SC1 PF2+SC2	PF3+SC2+CU2 PF2+SC1+CU1 PF2+SC2+CU1	PF4+SC4	PF4+SC4+CU4 PF4+SC4+CU4+AR4
TOTAL	2	61	0	53	43	41
TRIGONOMETRY	PF0+SC0 PF1+SC1	PF0+SC0+CU0 PF0+SC0+CU0+AR0 PF1+SC1+CU1 PF1+SC1+CU1+AR0	PF3+SC2 PF2+SC1 PF2+SC2	PF3+SC2+CU2 PF2+SC1+CU1 PF2+SC2+CU1 PF2+SC2+CU1 +AR1	PF4+SC4	PF4+SC4+CU4 PF4+SC4+CU4+AR2
TOTAL	44	286	38	155	33	144

Table3: Strands of mathematical proficiencies from the analysis of learners' scripts.

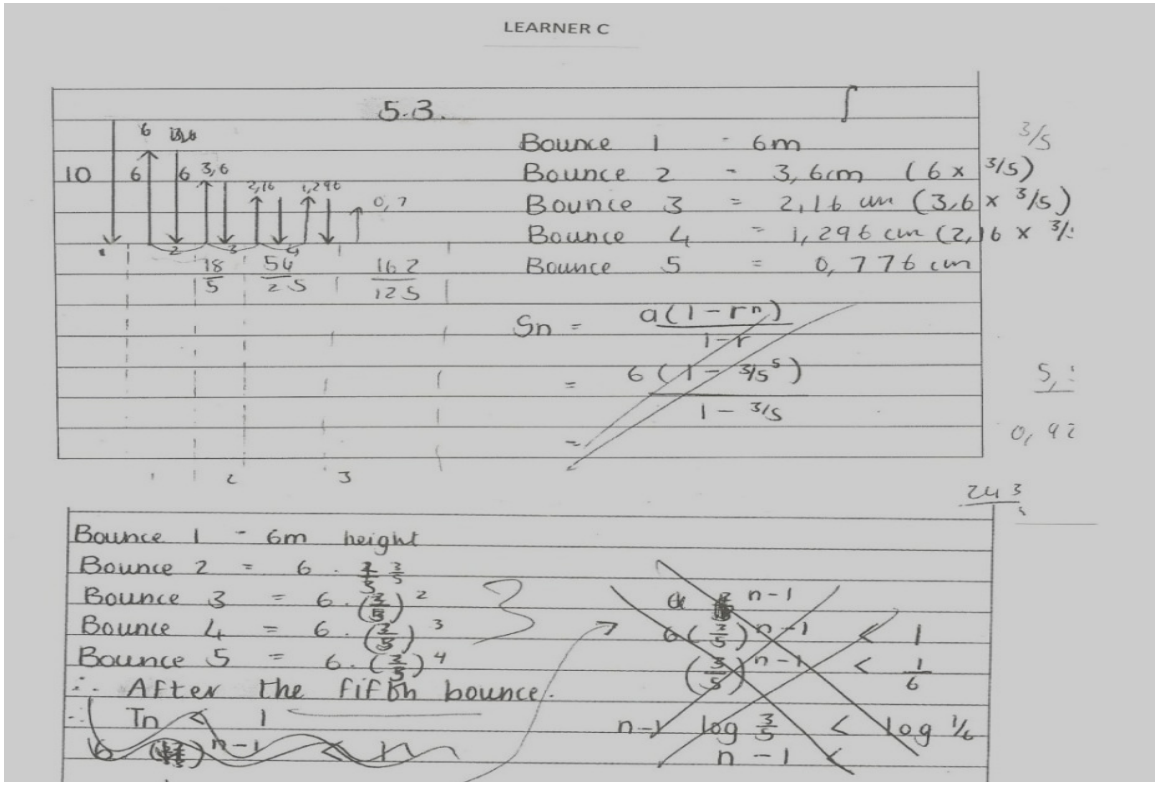

Figure 1: Learners' responses for question 5.3 on sequences and series.

The vignette above shows one example of learner's work on an examination question on sequences and series that was classified as PF3+SC2+CU1. This is one of the 11 responses of learners that were classified as yet to be proficient on examination questions on sequences and series that demand conceptual knowledge. Using the memorandum this learners' response has PF3 which shows that the learner has an acceptable knowledge of the procedure but could not use the formula, SC2 shows that the learner substituted with correct values the simplification did not lead to the required answer. Lastly to reach the required answer the learner could have comprehended with the procedures of logarithm thus the response is classified as CU1 to show that the learner has no knowledge of connecting procedures.

The vignette below shows two examples of learner's responses on examination questions on differential calculus that demand conceptual knowledge. Learner A's response is one of the 126 classified as not proficient and was one of those coded as PF1+SC1+CU1.These were learners who have no knowledge of the procedure of finding the derivative, simplify wrong and could not comprehend the derivative rule with simplification of quadratic equations to reach the required solution in accordance with the memorandum. Learner B's response was one of the 92 classified as proficient, one of those coded as PF4+SC4+CU4. These were learners who carried out the procedure of finding the derivative accurately, appropriately and flexible. These learners firstly simplify quadratic equation then took the derivative which is evidence of comprehending procedures.

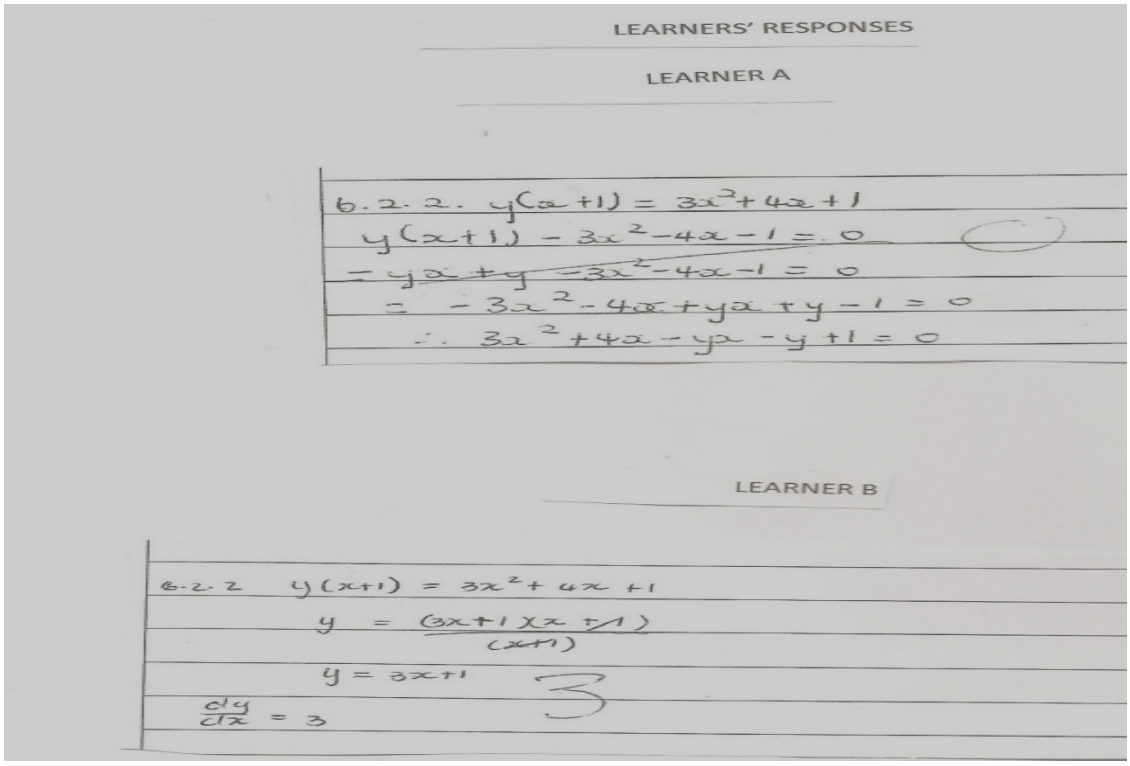

Figure 2: Learners' responses for question 6.2.2 on differential calculus.

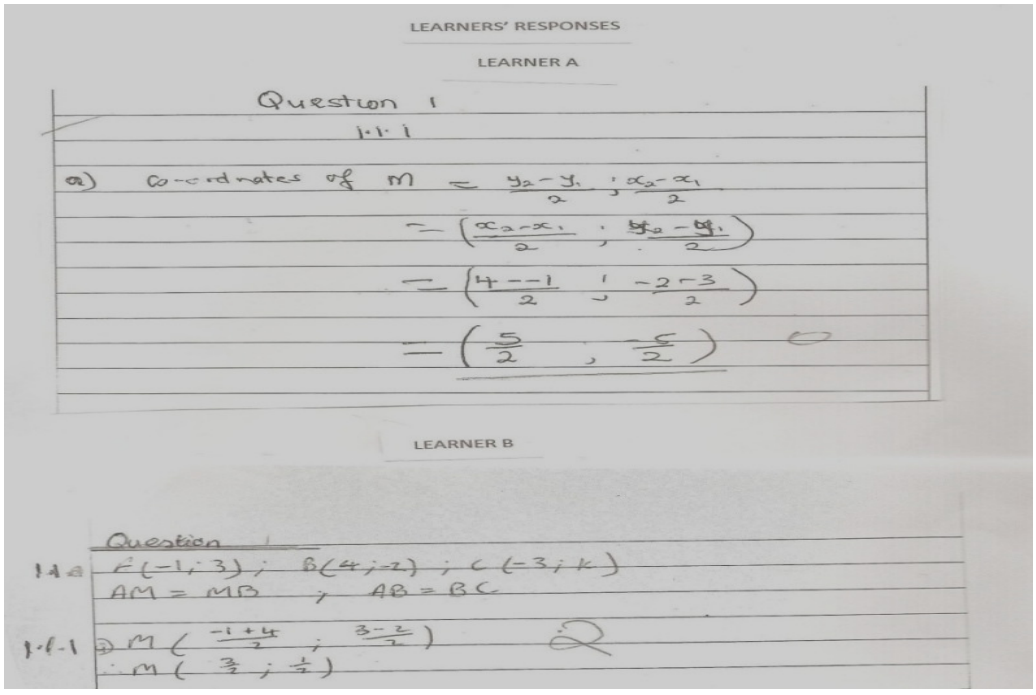

Figure 3: Learners' responses for question 1.1.1 on analytical geometry.

Another vignette above shows two learners' responses for examination questions on analytical geometry that demand procedural knowledge. Learner A's response was one of the 2 classified as not proficient and one of those coded as PF1+SC1. These were learners

with no knowledge of the procedure on finding the co-ordinates of the mid-point and simplify with wrong values. Learner B was one of the 43 classified as proficient, one of those coded PF4+SC4. These were learners who carried out the procedure of finding the mid-point accurately, appropriately and flexible. These learners chose the correct formula, substitute correct values and simplify correctly to reach the required solution as in the memorandum.

The last vignette below shows three learners' responses for questions on trigonometry that demand conceptual knowledge. Learner A's response was one of those 286 classified as not proficient and one of the coded PF1+SC1+CU1. These were learners with no knowledge of the procedure of finding the angle β, substitute with wrong values and the simplification did not give the answer shown in the memorandum. Learner B's response was one of those 155 classified as yet to be proficient, one of the coded as PF2+SC2+CU1. This learner had some insight with the procedure of finding the value of angle β showed some major errors like dividing by root three but still keep it on one side of the equation. This shows failure to comprehend surds and trigonometric ratios. Lastly, Learner C's response was one of those 144 classified as proficient and one of those coded PF4+SC4+CU4. These learners chose the correct procedure, substituted with the correct surds, simplified correct to reach the required solution as shown in the memorandum.

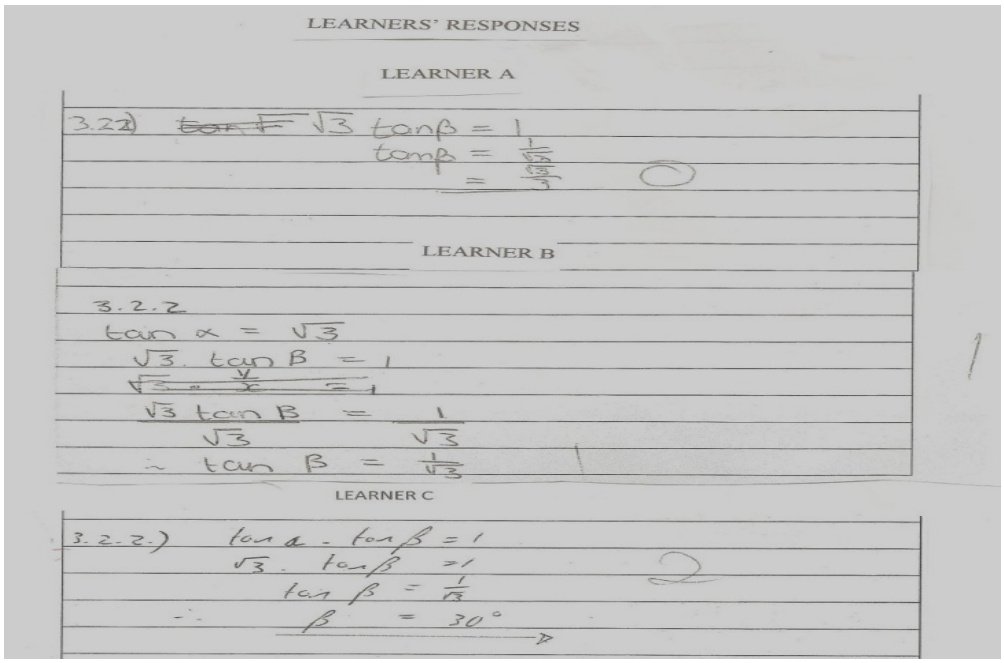

Figure 4: Learners' responses for question 3.2.2 on trigonometry.

DISCUSSION

There are three categories of mathematical proficiencies that are displayed by learners in their response to examination questions on all four topics analyzed for questions that demand either procedural or conceptual knowledge. For learners who were not proficient, the strands showed learners who left blank spaces and those that used wrong mathematical procedures resulting in their responses to be mathematically wrong. For the category yet to be proficient, there were a variety of indicators that showed this category; some learners only wrote the correct formula, some wrote the correct formula, substituted correct and

simplified wrong, some chose correct formula, substituted correct, simplified correct and wrote wrong conclusion, some chose correct formula, failed to comprehend procedures, some comprehended correct and failed to justified their answers. For learners who were proficient, these learners used relevant procedures, comprehended procedures, substituted correct, simplification is correct and reached the correct answer or conclusion.

For sequences and series, there were learners who are *not proficient, yet to be proficient* and *proficient.* There were 63.13% of learners' for examination questions that demand procedural knowledge were not proficient. There were 76.67% of learners' for examination questions that demand conceptual knowledge are not proficient. Again the strands revealed that these learners left blank spaces and others used wrong mathematical procedures resulting in their responses to be wrong. There were 14.78% learners who were yet to be proficient on questions that demand procedural knowledge and 12.22% learners for examination questions that demand conceptual knowledge were yet to be proficient. The study showed 21.08% learners for examination questions that demand procedural knowledge were proficient and, which 11.11% learners for examination questions that demand conceptual knowledge are proficient.

In the questions on differential calculus, three categories are shown, *not proficient, yet to proficient and proficient.* The study showed that 72.22% of learners for examination questions that demand procedural knowledge are not proficient and 46.67% of learners for questions that demand conceptual knowledge are also not proficient. There were 11.48% learners for questions that demand procedural knowledge are yet to be proficient and 19.26% of learners for questions that demand conceptual knowledge were yet to be proficient. Lastly, there were 16.3% learners for questions that demand procedural knowledge were proficient and 34.07% learners for questions that demand conceptual knowledge (see figure 2 for these proficiencies).

For the questions on analytical geometry, there were 4.44% learners for questions that demand procedural were not proficient and 39.35% learners for questions that demand conceptual knowledge are also not proficient. No learners were yet to be proficient on questions that demand procedural knowledge and 34.19.learners for questions that demand conceptual knowledge were yet to be proficient. Lastly, 95.56% learners for examination questions that demand procedural knowledge were proficient and 26.45% learners for examination questions that demand conceptual knowledge are proficient. (See figure 3 for these proficiencies).

Lastly, for trigonometry there are 57.04% learners for questions that demand procedural knowledge, were not proficient and 48.89% learners for questions that demand conceptual knowledge were also not proficient. There are 26.76% learners for questions that demand procedural knowledge were yet to be proficient and 26.3% learners for questions that demand conceptual knowledge were also yet to be proficient. There was also 23.34% learners for questions that demand procedural knowledge were proficient and 24.62% learners for questions that demand conceptual knowledge was proficient (see figure 4 for these proficiencies).

CONCLUSION

The study has revealed that a majority of learners leave high school not proficient in sequences and series for examination questions that demand both procedural and conceptual knowledge. This was contrary to a study by Zakaria and Zaini (2009) whose findings was that a majority of learners were not proficient in conceptual knowledge and proficient in procedural knowledge. The study has also revealed that there were learners who leave high school yet to be proficient in examination questions in sequences and series. A low number of learners leave high school being proficient in examination questions on sequences and series that demand procedural knowledge or conceptual knowledge. This implied that a majority of learners leaving high school will struggle in tertiary courses that demand the knowledge of sequences and series.

The study has also revealed that a majority of learners leave high school not proficient in examination questions on differential calculus that demand procedural knowledge and this is contrary to studies on procedural knowledge such as Star (2001) that shows that learners cope with procedural knowledge. Almost half learners leave high school not proficient on examination questions on differential calculus. There were a considerable number of learners who leave high school yet to be proficient in differential calculus. These learners have performed errors, misconceptions and problems on notation as shown in the study by Luneta and Makonye (2010). There was a low number of learners leaving high school proficient in examination questions on differential calculus that demand procedural knowledge and a slightly higher number of learners leaving high school proficient on differential calculus that demand conceptual knowledge. This was contrary to studies on conceptual and procedural knowledge such as that by Simpson and Zakaria (2004) and Schneider and Stern(2010).

The study revealed that a very low number of learners leave high school not proficient in examination questions on analytical geometry that demand procedural knowledge and a higher number of learners who left high school not proficient in examination questions that demand conceptual knowledge as shown in a study by Zakaria and Zaini (2009). There were also learners who left high school yet to be proficient in examination questions on analytical geometry that demand both procedural and conceptual knowledge. A majority of learners left high school proficient in examination questions on analytical geometry that demand procedural knowledge and a low number of learners left high school proficient in examination questions that demand conceptual knowledge.

This research study has revealed that on average half the learners that left high school were not proficient in examination questions on trigonometry that demand both conceptual and procedural knowledge. There was also a considerable number of learners who left high school that are yet to be proficient in examination questions on trigonometry that demand both procedural and conceptual knowledge. There was a low number of learners that left high school proficient in examination questions on trigonometry that demand both procedural and conceptual knowledge.

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