<u>Title:</u> Bringing context to mathematical problems: should we or shouldn't we?

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Abstract

This paper addresses issues related to presenting secondary school mathematical problems within a real-life context. Notwithstanding certain concerns from research, it is argued that teachers have many opportunities to incorporate every-day situations into their teaching, either by imposing those situations onto the problems expected in the curriculum, or by finding real-life problems that fit the mathematics. Some guidelines are proposed for making decisions on which contexts to use, and some examples with discussions of their appropriateness in the classroom are discussed.

Introduction

Traditionally, learning mathematics took the form of mastering the manipulations and procedures in isolation of any context. In recent times, many believe that, "… everyday topics are seen in a new light when considered in mathematical terms" (Land, 1975, p. ii). Many teachers believe, as Land does, that students of mathematics come to appreciate mathematics more when it is presented in the form of every-day problems.

The question that we need to ask ourselves is, "Do we describe the everyday situation through the mathematics, or do we fit the mathematics to the everyday situation?" I suggest that we should be doing both! And, whichever way you do this, you bring the contextual knowledge into the fray.

Comment from the literature

"Contextual knowledge is related to everyday-life problems in the real world and enters the school situation through the presentation of the problem in a context with its own story. It does not appear when problems are presented using only mathematical symbols with numbers, operators, and variables" (Sáenz, 2008, p. 125).

What Sáenz is suggesting here is that, devoid of a situation, a problem appears in pure mathematical form, containing for the students unfamiliar symbols in which you apply certain rules and procedures to "get the answer". By incorporating a context we possibly give meaning to these otherwise 'strange' symbols.

However, there are those who caution us when using context in mathematical problems. Galbraith (2005) warns teachers that placing problems in context is not necessarily akin to mathematical modelling. He categorises certain types of word problems that teachers use in their classrooms. *Inter alia*, these include "unreal word problems", in which the real solution to the problem is much simpler than what the mathematics of the problem requires, and "silly word problems" in which the answer is unrealistic. In both of these instances the

mathematics loses its credibility for the students because it gives rise to situations with which the students cannot identify.

Cooper and Dunne (2000) found in their research that the context of a mathematical problem can impede the student, either because the context is too different from what that student has experienced (thus making it foreign to the student), or because the context brings extraneous information into the problem that distracts the student from what is relevant to the problem solution.

I believe that Galbraith (2005) is correct when he warns against conflating context and mathematical modelling in problems presented at secondary school level. 'Context' does not mean 'modelling', as seems to be suggested by our current Curriculum and Assessment Policy Statement in South Africa (for example, see DoE, p. 35 and p. 48). However, I do believe that as teachers, we can bring a better appreciation of mathematics into our classrooms by including context. The trick is to understand what the context does for the problem, and to be sensitive to the affects that the context brings to bear on the student in terms of making the problem more or less accessible.

I now turn to an examination of mathematical problems that have a context that I have used in my teaching. As teachers, making decisions about what will enhance learning in our classrooms, I believe it will be useful to interrogate these problems with the following points in mind:

- Does the context make the problem more interesting?
- Is the context real enough so as not to make the mathematics seem absurd? (Galbraith, 2005)
- Does the context mislead the students?
- Does the context bring an appreciation for mathematics in every-day life?
- Does the context make the mathematics 'messy'? (i.e. unwieldy decimals, unmanageable numbers, etc.)

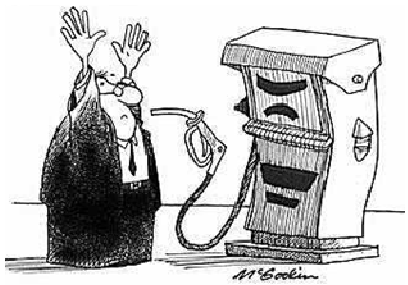
Contextual problems for the classroom

1. **Opportunities with number**

Placing a problem within a context may seem to some to be contrived, and in certain cases this is so. However, it can give the problem a little more dimension than if it were presented purely mathematically. Take for example this problem:

If 10,2 is increased to 10,48 , what percentage increase has taken place?

Consider now the problem amended as follows:



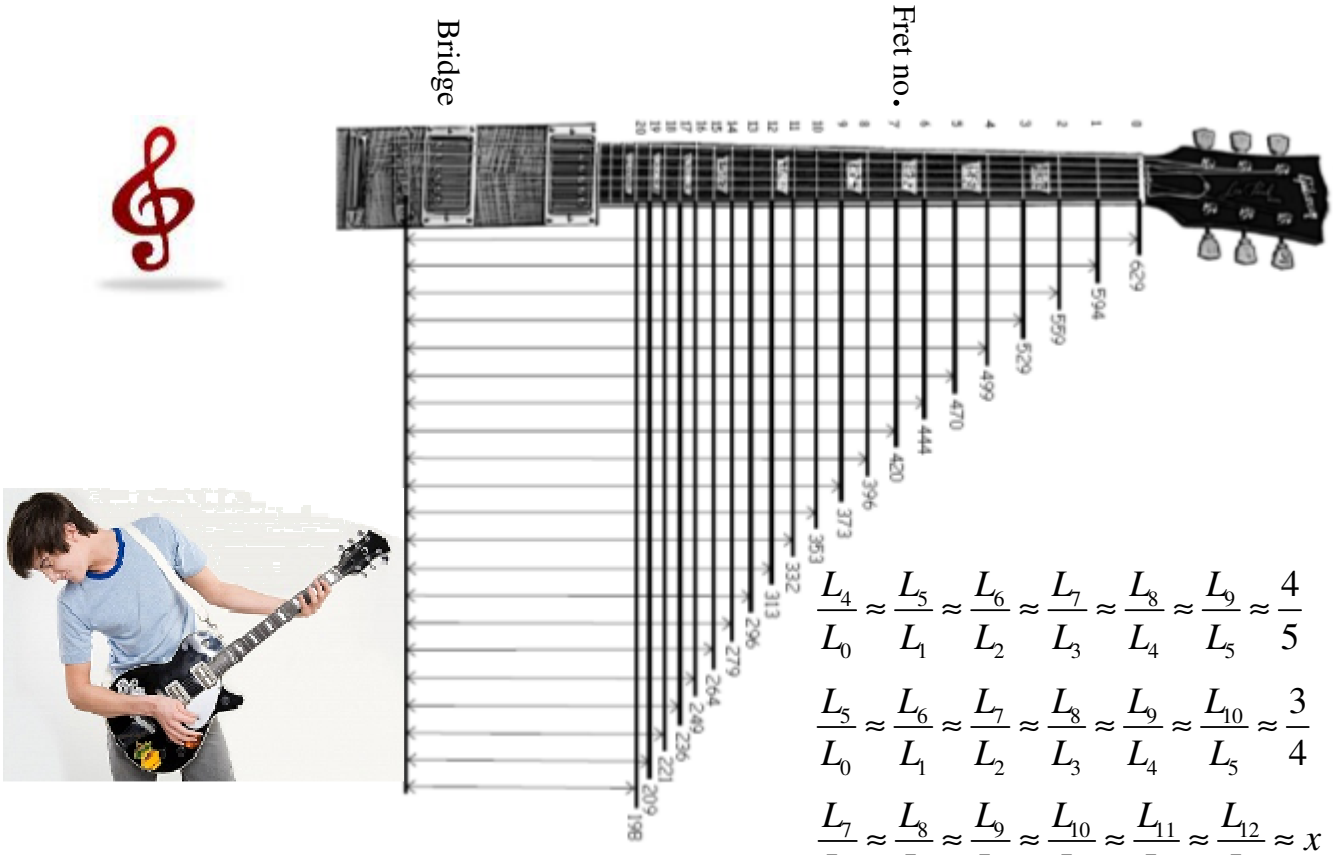
The price of petrol was R10,20 and has now been increased to R10,48. What percentage increase will motorists now pay for petrol?

All that has happened here is that a purely numerical question has been set within a context. Students are now given the opprtunity to be aware that mathematics *can* serve a function in everyday life. With the introduction of a context, one can also include a picture that further enhances the quality of the question, and students can think about the mathematics and the context more deeply, as, for example here, the man in the street is being held to ransome by increases in petrol price. Thus, the student is not only involved in a mathematical proceedure, but is also able to relate it more generally to a situation in society.

There are numerous opportunities to this, especially when dealing with number. Below are two pictures that set the context very nicely. On the left we have a photo enlargement of a housedust mite, showing the dimensions of the actual photo. Using ratios, and the enlargement factor students can do calculations to find the actual size of the mite. On the right, we see a picture of the Pneumonia bacteria. The size is given as $0{,}5\,\mu m$, where μm is the unit for micrometres, and $1\,\mu m = 1m \times 10^{-4}$, thus giving the teacher opportunity to work with scientific notation within a real life context.

From the insert below, we see an opportunity to show how mathematics and music are

connected through ratios. L_0 through L_{20} are the string lengths for each fret number.



$$\frac{L_4}{L_0} \approx \frac{L_5}{L_1} \approx \frac{L_6}{L_2} \approx \frac{L_7}{L_3} \approx \frac{L_8}{L_4} \approx \frac{L_9}{L_5} \approx \frac{4}{5}$$

$$\frac{L_5}{L_0} \approx \frac{L_6}{L_1} \approx \frac{L_7}{L_2} \approx \frac{L_8}{L_3} \approx \frac{L_9}{L_4} \approx \frac{L_{10}}{L_5} \approx \frac{3}{4}$$

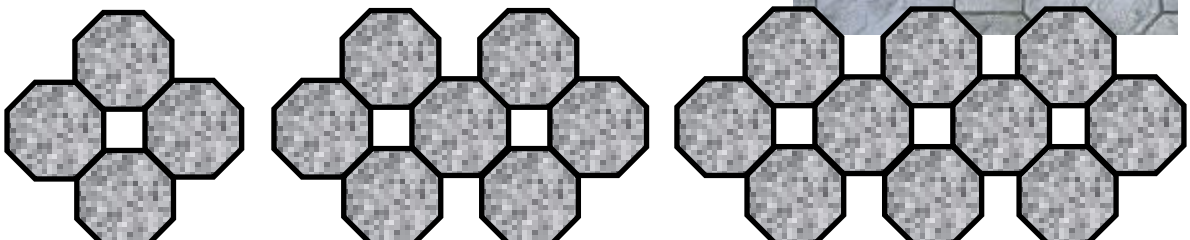
$$\frac{L_7}{L_0} \approx \frac{L_8}{L_1} \approx \frac{L_9}{L_2} \approx \frac{L_{10}}{L_3} \approx \frac{L_{11}}{L_4} \approx \frac{L_{12}}{L_5} \approx x$$

The world is full of problems involving number and we as teachers can pick and choose. Not all of the problems given here are suitable in all classrooms. The teacher needs to gauge the students and decide what contexts will be appropriate based upon whether or not the learning will be hindered or enhanced by the context. For example, in the guitar problem above, the mathematics is more complex than simple ratios because the relationships between the ratios of the lengths are only *approximately* equal. If students are going to get tripped up with approximating the ratios with 0,8 and 0,75 to deduce the answer for x, then perhaps this context is not suitable for those students. In that case, there are many other opportunities out there that will be more suitable.

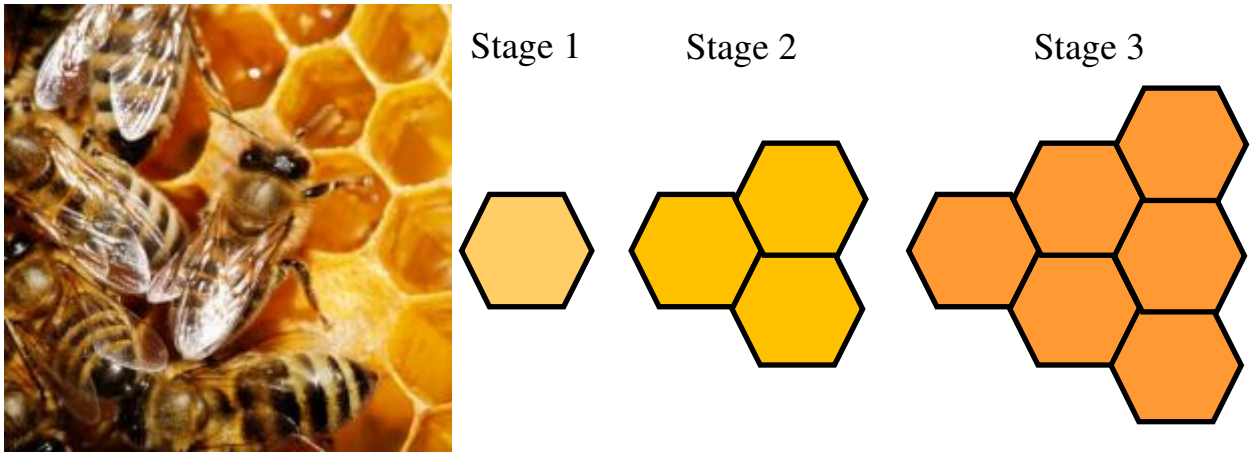
2. The world is full of patterns

These days, teachers are fortunate because nearly all have a phone with a reasonable quality camera. Look out for interesting patterns in the world and take a pic. Then bring these into the classroom and use them to teach your students about first and second order difference patterns, as well as geometric sequences.

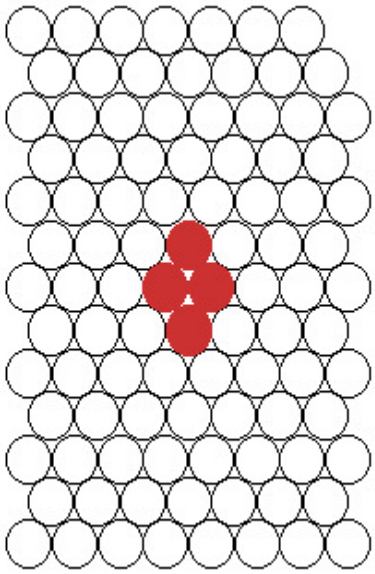
Take a look at the image of paving shown. The teacher can construct a simple pattern using the number of octogons, and / or squares required, something like what is posed below.



A similar problem, but one that leads to a simple second order difference, comes from a beehive, and (depending on how you build the hive) can generate the triangular numbers (as shown below).

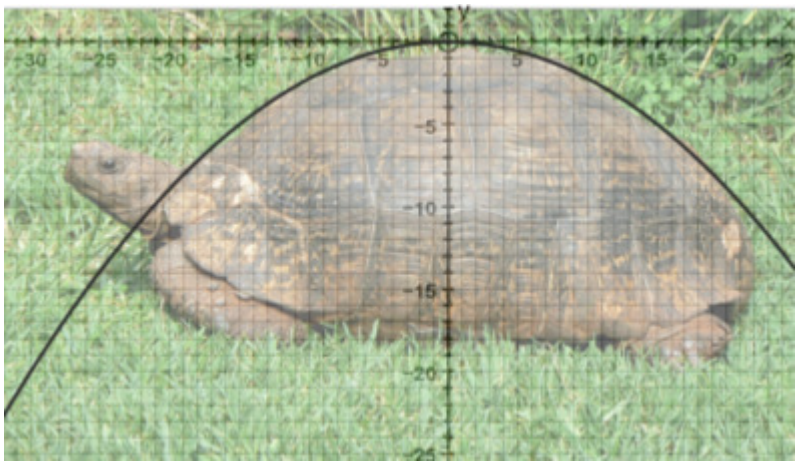


The trick to making these contexts applicable to our classrooms is to identify one or more patterns that are contained in the picture (examples of which are shown in the preceding two contexts given in the first three stages). This might become a little tricky if you use a context like the beadwork, shown below. However, situations like this sometimes give wonderful opportunities for allowing students to explore the patterns, such as number of red (grey in this picture) beads and the number of black beads.



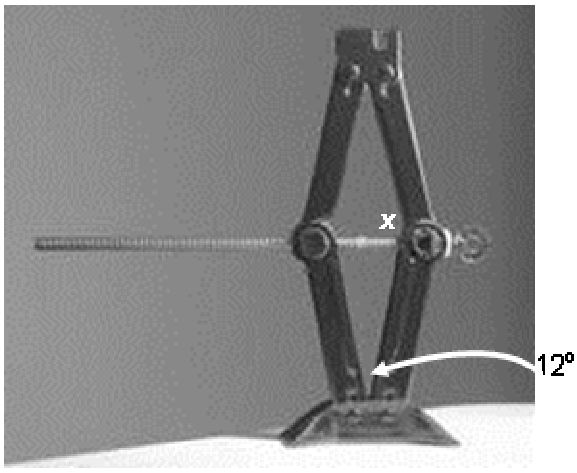
3. **Bring the mathematics to life using shape**

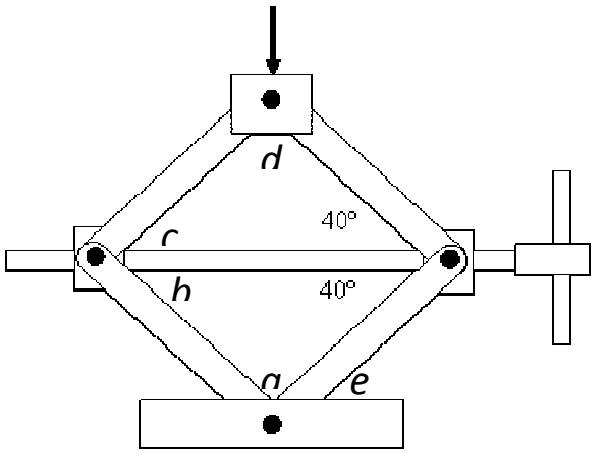
Take a look at the picture on the right. Is this a coincidence? Is there a



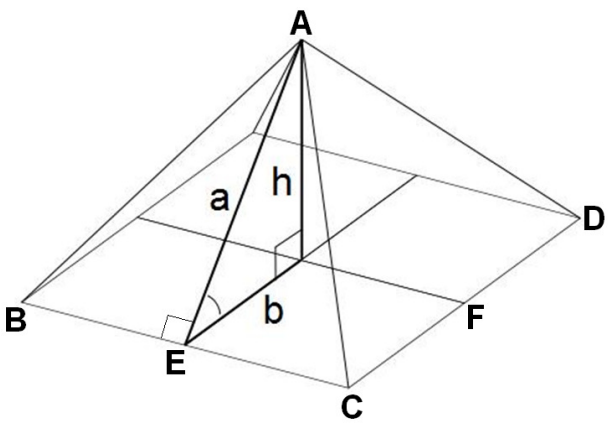
reason why the mid-sectional view of the tortoise shell is “very nearly” parabolic? Similarly, is there a reason why a cyclist’s helmet is “very nearly” parabolic in the same way? These connections in the real world are there waiting for us to draw upon.

By finding things in our world that we can put into a mathematical form, we allow the real world to enter our classroom. Even if these problems seemed somewhat contrived (often because we wish to keep them at an elementary level, and we wish to keep the mathematics “neat”). But, they still serve the purpose of allowing students the opportunity to see mathematics in the world. Take the geometry problem posed for grade 8 students below. It is a very elementary mathematical task, but it brings home the notion of angles and parallel lines inherent in an ordinary car jack.





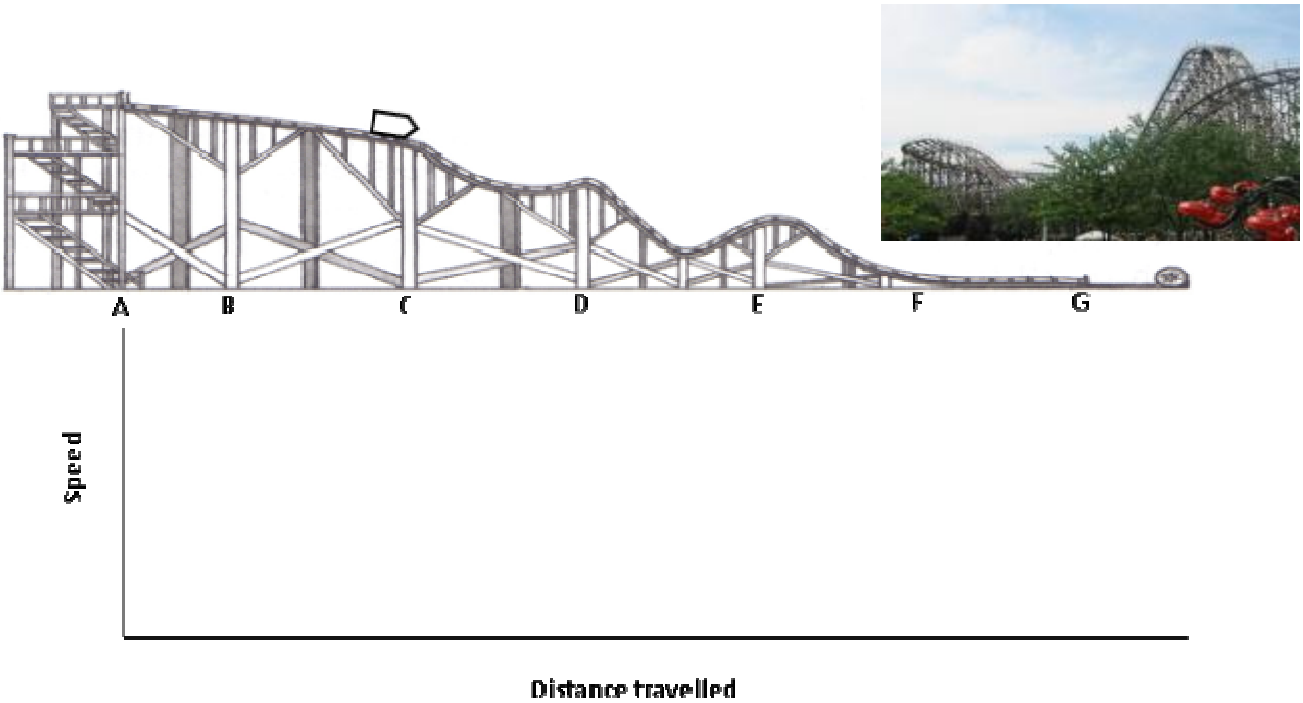
The problems can become more intricate, as with the example of the Louvre Glass Pyramid shown below. The actual measurements can be found on the Internet: perpendicular height from the apex is 21,64m and the square base has sides of 35,42m. That’s about all you need to know to work out a whole lot about this architectural feature.



Then why not extend the problem to the Great Pyramid of Khufu at Giza, and calculate the volume of the bricks that have eroded over time. Well, maybe? But the opportunities are out there!

4. **Where there's motion, there's mathematics**

The relationships between distance and speed, and speed with respect to time are wonderful opportunities to get students to engage with functions. Take your students through a rollercoaster ride by getting them to make mathematical connections with elements of motion, as in the example below.



Then, we can always use stories from the media. Gebrselassie's consecutive marathon world records at the Berlin Marathon gave us wonderful opportunities to bring human

Rested Gebrselassie Tops His World Record

By JERÉ LONGMAN

Published: *The New York Times*, September 28, 2008

Haile Gebrselassie of Ethiopia again showed why many consider him the greatest distance runner ever, breaking his own world record in the marathon in Berlin on Sunday and becoming the first person to run the 26.2-mile race under 2 hours 4 minutes.

endeavour into the mathematics classroom (see insert below).

5. Trends in the world

Statistics are the order of the day, and why not utilise these to enhance our students' mathematical understanding. Just keep an eye and ear open for modern trends, such as the following two examples that were reported recently. The first context looks at the trend of breeding pairs of African penguins over the years, and gives students to opportunity to apply knowledge of mathematical trends in an actual situation reported by SANCCOB.

African penguin with chick on Malgas Island

Index year	Estimated no. of breeding pairs
0	100
1	70
2	80
3	75
4	75
5	45
6	65
7	65
8	50
9	55
10	60
11	50

An estimated 2500 breeding pairs were recorded in 1956…

Adapted from figures given by SANCCOB, 2011

Below, we see an article reported by Venter on the earth's population that is not only topical, but also gives wonderful opportunity to discuss trends. Use the mathematics to justify or negate Venter's observations using functions, and allowing students to choose appropriate trends.

Our ecosystems, our future…

On Monday 31st October [2011] we crossed the 7 billion human mark. It took us around 200 000 years to reach the 1 billion mark in 1805. 123 years later we reached the 2 billion mark (1927), 33 years later the 3 billion mark (1960), 14 years later the 4 billion mark (1974), 13 years later the 5 billion mark (1987), 12 years later the 6 billion year mark (1999) and another 12 years to reach the 7 billion mark. The good news is that the 7 billion mark is probably the turning point, as the rate at which we're growing is slowing and our population is projected to peak at around 10.2 billion in 100 years' time.
Andrew Venter (CEO Wildlands Conservation Trust), November 2011

Conclusion

The teaching of mathematics has moved towards presenting more of the problems within a real life context. However, research has shown that there could be inherent problems associated with the way in which we contextualise the mathematics in terms of how students perceive the problem (i.e. how 'real' is the context), as well as in terms of factors in the context that may mislead students (Cooper and Dunne, 2000). I believe that there are many opportunities for teachers to use contextual problems that will enhance the students' mathematical experience, whether the situation is put into an appropriate mathematical form, or whether the mathematics is 'fitted' to a context.

List of references

Cooper, B and Dunne, M. (2000). Assessing children's mathematical knowledge: social class, sex and problem-solving. Buckingham: Open University Press.

DoE. (2011). Curriculum and assessment policy statement. Department of Basic Education. Pretoria: Government Press.

Galbraith, P. (2005) *Applications and mathematical modelling: Meeting the challenge*. Paper presented at a Marang PhD Weekend, University of the Witwatersrand, Johannesburg.

Land, F. (1975). *The language of mathematics*. Norwich: John Murray Publishers.

Sáenz, C. (2008). The role of contextual, conceptual and procedural knowledge in activating mathematical competencies. *Educational Studies in Mathematics*, 71 (2), 123 – 143.