Title: Centre of mass of polygons and other shapes

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Abstract:

For an object of uniform composition (mass, density, etc.) the centroid of a body is also its centre of mass. Practical demonstrations are used to determine the centre of mass of the various pieces of lamina in the shape of polygons. We then use the Cartesian plane (with the aid of the Geometers Sketch-pad) to locate and calculate the coordinates of the centroids. Further, the centre of mass of other shapes is investigated and various techniques (e.g. using lines of symmetry) are illustrated.

Introduction:

I grew up in the days when "mathematics" meant "school mathematics" divorced from the real world. My high school algebra course consisted of pure symbolic manipulation, for example, and my geometry course focused on producing proofs. When instruction in maths focuses almost exclusively on mastery of facts and procedures, students are not likely to develop higher-order skills necessary for using mathematics (Schoenfield 1985).

According to Stewart (2009), the art of teaching is directing our students in the art of guided discovery. To this end the emphasis is on conceptual understanding rather than rote learning and the "Rule of three" should be implemented: topics are presented numerically, algebraically and geometrically. Also, verbal and descriptive processes should be encouraged.

Further, the reform movement in mathematics education requires the teacher to provide tasks that are stimulating, relevant and foster growth in knowledge. This can be achieved by helping students connect mathematical ideas and concepts to real life situations (National Council of Teachers of Mathematics, 1991).

Terminology: centroid, centre of mass, symmetry and balance.

a) Refer to fig 1.

   Consider a triangular piece of lamina and we wish to determine the centroid (the centre of mass. We can demonstrate it practically by suspending the triangle in a vertical plane and using a plumb line. After obtaining the centroid, we show how the triangle is balanced horizontally. Now we can investigate the intersection of the medians, the centriod etc.

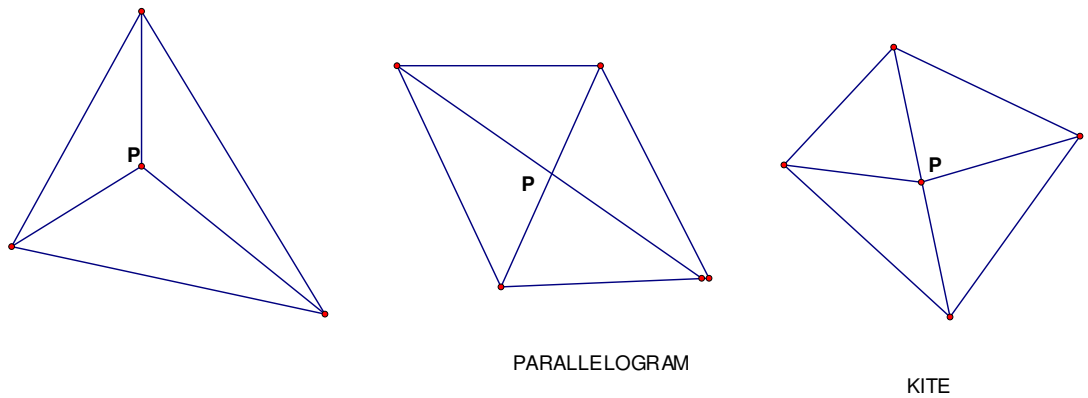

Fig 1

Refer to Fig 1: Let us consider P to be an interior point so that each polygon is divided equally in areas. So, P is also the centre of gravity. However, this is not true in the case of the KITE! Students should be given this task to investigate. We assume they have access to geometrical software like *Geometer's Sketchpad.* Such an investigation may proceed along the following lines.

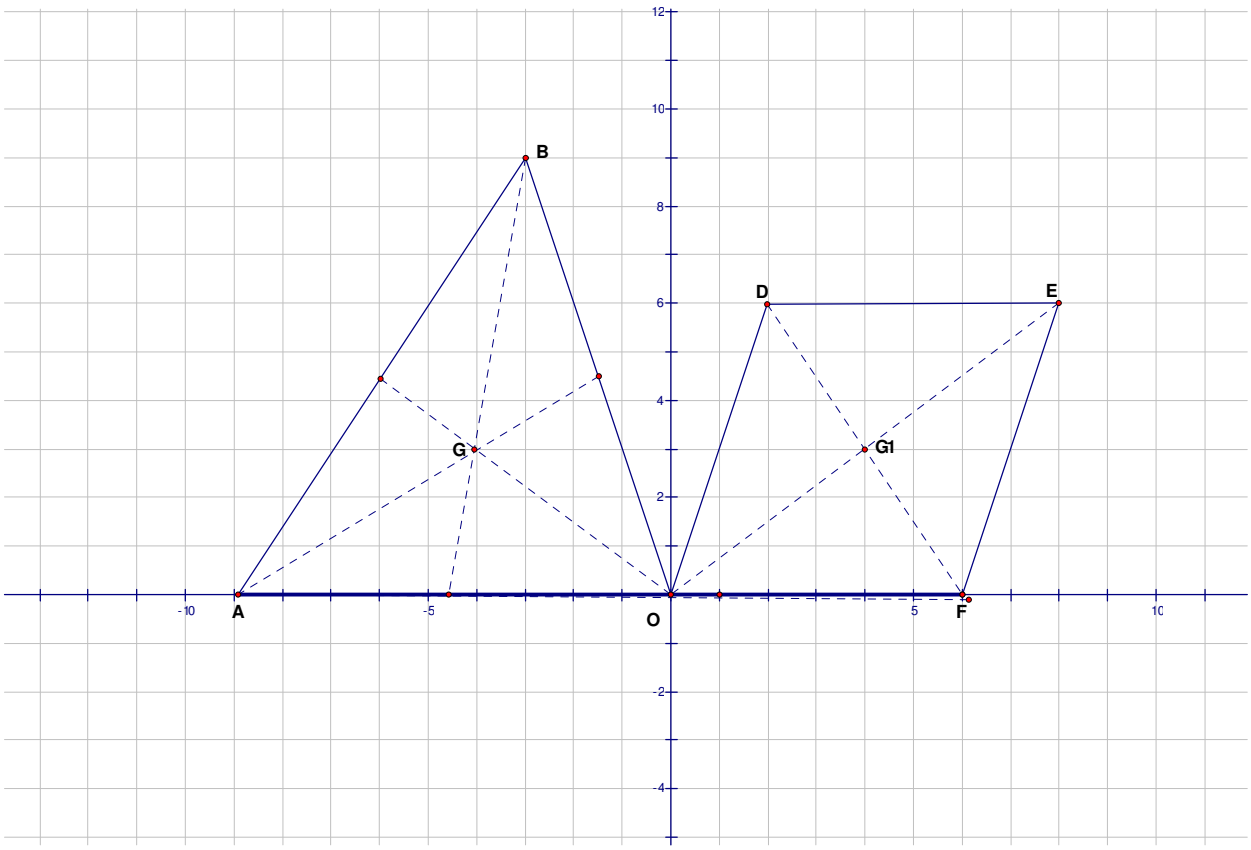

Fig 2

With the aid of the sketch pad and using the cartesian plane two examples are illustrated (does not constitute proof). Visualisation should enhance the students' conceptual understanding.

In Δ AOB, the medians are drawn and point G is the centroid. G is the point (-4,3),

A(-9,0), B (-3,9) and O (0,0). The centroid can be obtained by calculating the average of the

x and y coordinates of the vertices of the Δ ABC.

Similarly, G1 is the centroid of parm DEFO. The centroid is the point (4,3) and the vertices of the parallelogram are (0,0), (6,0), (8,6) and (2,6). The centroid, is also the average of the coordinates of the vertices of the parm.

However, this is not the case for the Trapezium and the Kite (in figure 2).

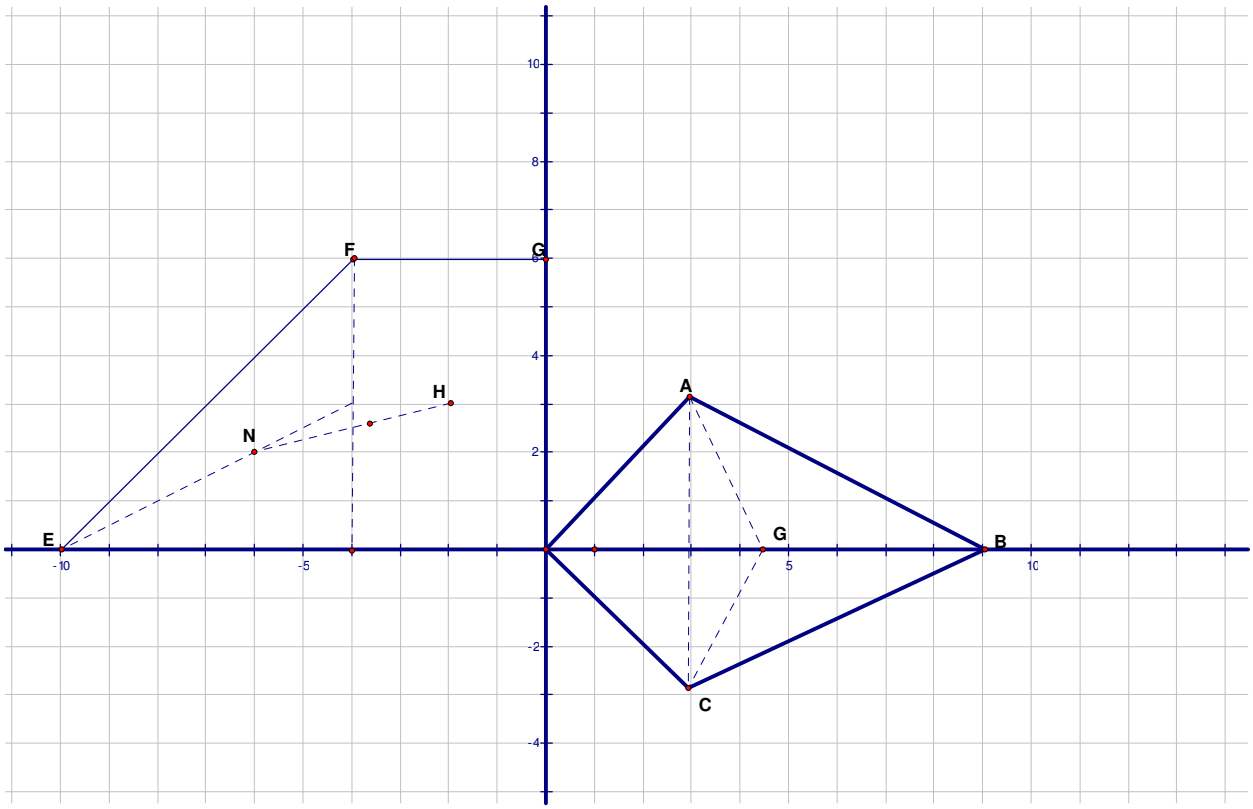

Fig 3

Kite: The line of symmetry is OB and the centroid should lie on OB.

The coordinates of the vertices are (0,0), (3,3), (9,0) and (3,-3). The average of the coordinates is the point (3.75, 0), which is on the line of symmetry. We also note that the mid-point of OB is (4.5,0).

If we partition the Kite into two Δ AOC and Δ ACB whose centroids are (2,0) and (5,0) and their areas are 18 and 90 (square units) respectively.

Let $G_x = \frac{ax_1 + bx_2}{a+b} = \frac{2\times 9 + 5\times 18}{9+18}$ = 4. Similarly $Gy = 0$. Therefore the centroid of the kite is (4,0).

The centroid of the trapezium is for the students to do as an exercise.

Conclusion:

Hence, mathematics tasks in which students engage; projects, problems, constructions, application exercises and so on and the resources must provide students with the opportunities for learning. These tasks must be stimulating and should be a means to an end. Students should be able to develop skills, concepts, apply and make connections to the real world. I hope that this integrated approach would be refreshing and stimulating for both learners and teachers. I would appreciate some feed- back from delegates at the conference.

Reference:

National Council of Teachers of Mathematics. (1991). Professional standards for teaching mathematics. Reston, VA: National Council of Teachers of Mathematics.

Schoenfeld, A, H. (1985). *Mathematical Problem Solving.* New York: Academic Press, Inc.

Stewart, J.(2009).*Calculus.* Belmont: Brooks/Cole.

Appendix:

Worksheet: Determine the centre of mass of each of the figures below.

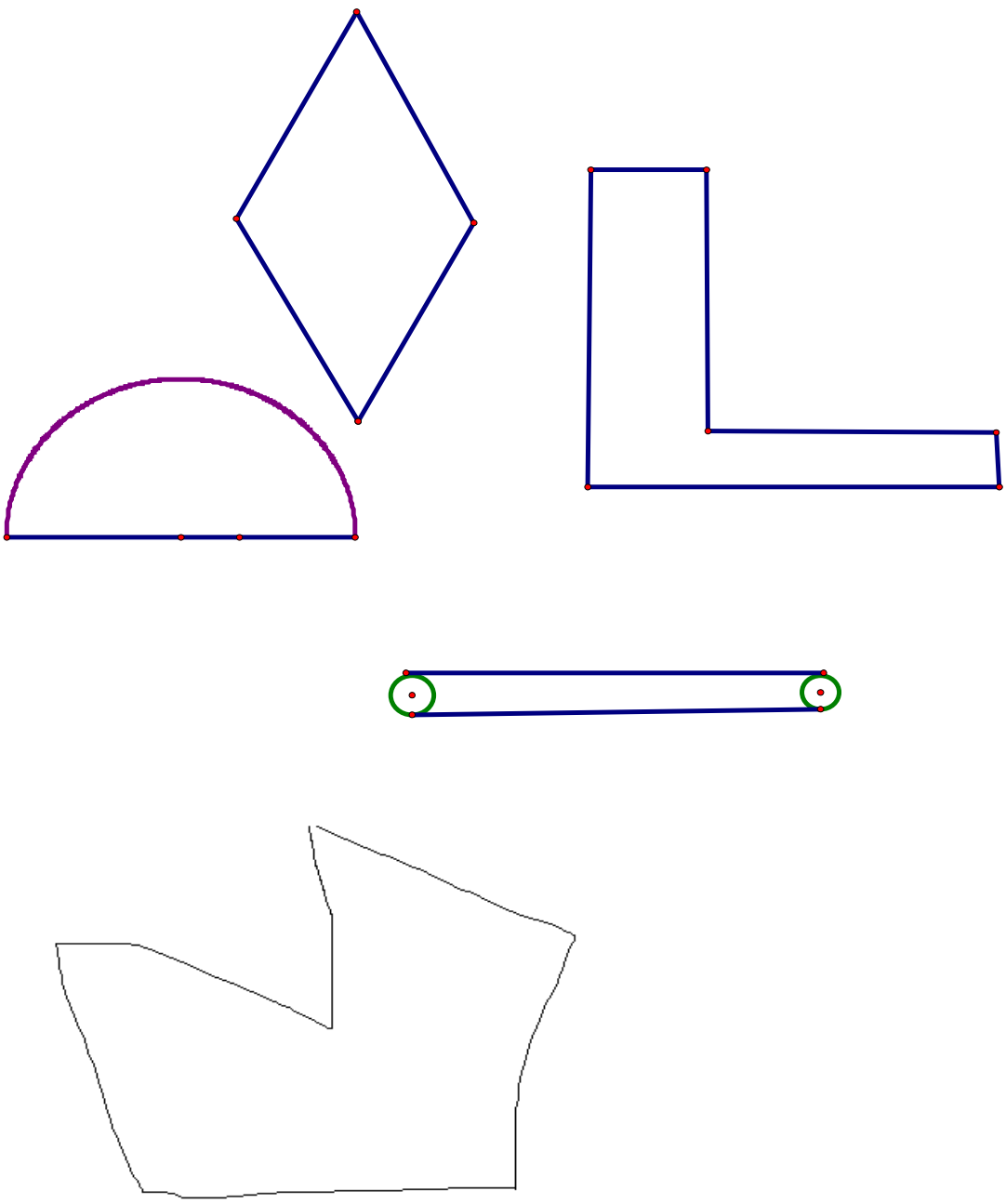
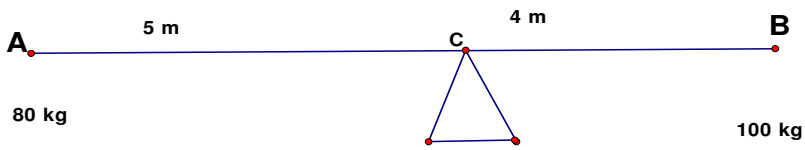
Fig 4.