<u>Title:</u> Proofs in Geometry and Trigonometry: bringing back the old and looking at the relevance in today's classroom

Name of presenter: Douglas C Schuster

Institution: Military Academy (Department Faculty of Military Science, Stellenbosch University), Saldanha.

Target Audience: Workshop is aimed at FET phase

Motivation for the workshop:

The reintroduction of Euclidean Geometry as compulsory content in the new FET curriculum (CAPS) and its added specifications that the proofs of theorems (as well as the proofs of the Trigonometric Rules) are examinable have addressed some of the concerns (regarding mathematics specifically) that have been raised by, amongst others, tertiary institutions citing the under-preparedness of students arriving at these institutions and in the workplace. The formal teaching for deductive logic (mostly done through Euclidean Geometry at FET level) has unfortunately by and large fallen by the wayside during the last few years with Euclidean Geometry having been relegated to the 3rd paper and the confines of lesser importance, less teaching time, after-hours teaching time and 'self-study' for those few students and teachers who have persisted with its 'optional content'. The consequences are manifesting themselves clearly:

...*a large percentage of the students who passed Grade 12 will struggle to pass in the first years of university, not only in fields that required mathematics and science* [where it is especially problematic]. *University lecturers will tell you that in their experience, students over the years have gotten weaker even as the matriculation results on the outside get stronger. ... We worry, as university leaders, about the large numbers who drop out or repeat courses and years, simply because we made the mistake of believing that a pass in Grade 12 means a student is qualified to study at university.* Prof. Jonathan Jansen, Vice-Chancellor and Rector: University of the Free State – 9 January 2012.

This brings us to the question of what qualifies a student to study mathematics at university:

Pure mathematics is the class of all propositions of the form 'p implies q' where p and q are propositions ... Bertrand Russell

I believe that it is safe to say that the ability to *think mathematically* is a fundamental prerequisite to the [further] study of mathematics. If it is not fostered at school level it becomes an almost unbridgeable handicap:

[Mathematical] *Logic is not formally included as content in the school mathematics curriculum. But it should be, and teachers should make sure that their learners understand the basic logical principles. Erroneous logical generalisations are not only apparent in proofs ..., but it affects children's everyday mathematical activity.* (Olivier, n.d., p.68.)

I am told that teachers are often uncertain and even scared of teaching Euclidean Geometry and advanced Trigonometry and that they believe that it is too difficult for mainstream classrooms. With this workshop I hope to contribute in some small way to the larger efforts

of providing teachers (who feel that they can benefit from it) with skills, tips, ideas and encouragement in the teaching of the above mentioned contents and principles.

Description of the content of workshop:

With this workshop I intend to present (for consideration) elements of my personal classroom practice in teaching for deductive reasoning:

- The tools of the trade (theorems, rules and identities): "*If I can I prove the theorems by making sense out of them (vs. trying to learn them by rote), then I understand them and I can apply them.*"
- The value of analysis: Constructing valid arguments/proofs as well as being able to criticise such arguments are fundamental to doing mathematics. However students find the construction of (deductive) proofs very difficult. Therefore it is necessary to look at the logic underlying valid proofs and the process of constructing them (within contexts).
  A synthesis shows that every step is true, but does not explain why this step was taken [and not another]. *A synthetic proof convinces the reader that the fact to be demonstrated is true, but it does not reveal to him the real plan of the demonstration, does not tell him why this sequence of arguments was selected. Proofs are not discovered by synthetic methods, and if forgotten, synthetic demonstrations are most difficult to reconstruct. But synthetic proofs are usually short and elegant, and are in place when no pedagogical conditions need to be considered.*
  An analysis, on the other hand, is lengthy and not elegant, but is the only method that accounts fully for each step in the demonstration. It is the only method by which students can hope to discover proof, or to re-discover them after forgotten. Analysis is the method of discovery, synthesis (proof) the method of concise and elegant presentation. Schultze (1912, p. 36)
- The value of **noticing**: how the use of colour has improved my teaching practice in this regard.

I will present problems that I have given students in the past (in conjunction with some solutions they have provided to these problems) for critical consideration and discussion:

- Misconceptions surrounding the tools (such as 'identities').
- Misconceptions surrounding the (deductive) logic.

I will briefly make a case for the value of teaching for these principles:

- Transferability of the underlying reasoning skills to other [non-mathematical] disciplines
- Tertiary study
- Career choices

I will present problems requiring both analysis and synthesis designed to challenge *educators* mathematically.

Programme of activities:

Introduction – 5 min

1. Activity 1: Proving Trigonometric Identities

    - Working on the activity – 10 min
    - Presenting students' solutions – 10 min
    - Group discussion – 10 min

2. Activity 2: Deriving the Area Rule (*method of discovery*)

    - Working on the activity in small groups – 20 min
    - Group discussion – 5 min

3. Activity 3: Analysis and the value of *noticing*

    - Presenting problem as example – 20 min
    - Discussion – 5 min

4. Activity 4: Working on problem(s)

    - Working on the activity in small groups – 30 min
    - Group discussion – 5 min

Conclusion and 'Homework' – 5 min

References

Olivier, A.I. (nd). *Aspects of mathematical knowledge for teaching. Analysing mathematical thinking: A problem-centred approach*. [Unpublished course notes] Available from http://www.sun.ac.za/mathed/174/InduksieDeduksie.pdf

Scultze, A. (1912). *The teaching of mathematics in secondary schools*. New York: The Macmillan Company.