Title: Calcudoku – the new addictive number puzzle

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Calcudoku is a wonderfully addictive number puzzle which can be solved using a combination of logical reasoning and simple arithmetic. The puzzle has similarities with Sudoku, but in addition to logic requires the solver to explore aspects of simple addition, subtraction, multiplication and division. Exploring Calcudoku puzzles in the classroom setting has the potential to open up interesting discussions around important mathematical issues such as logical argument, proof and justification, as well as provide opportunities to practice simple mental arithmetic in a fun and constructive context. This workshop is intended as an introduction to Calcudoku. The rules of the puzzle will be explained, some basic strategies to solving the puzzle will be explored, and the pedagogical potential of the puzzle will be discussed.

Target audience: GET and FET teachers.

Motivation for the workshop

As mathematics teachers and educators, we should constantly be on the lookout for fun, constructive and engaging activities that have pedagogical value. Calcudoku puzzles, with their emphasis on logical reasoning and simple mental arithmetic, represent just such an activity.

WORKSHOP CONTENT

Introduction

Originating from Japan, Calcudoku is one of many names used for this particular puzzle. In addition to proprietary trademark names such as KenKen and KenDoku, this puzzle is variously known as Newdoku, Rekendoku, MathDoku, Calkuro, Kashikoku-Naru, Minuplu and Calcudoku. Of these, Calcudoku is perhaps the most well known.

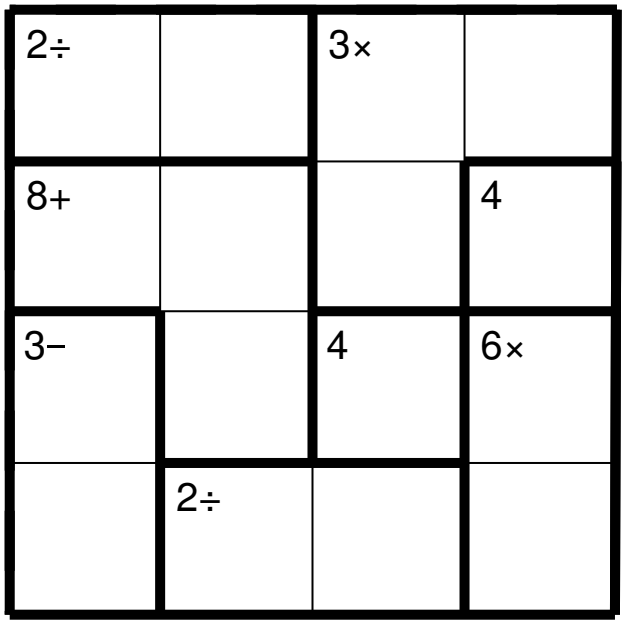
Calcudoku has similarities with Sudoku, but in addition to logic requires the solver to explore aspects of simple addition, subtraction, multiplication and division. A Calcudoku puzzle is a square $n \times n$ grid of cells subdivided into a number of cages. Similar to Sudoku, each column and each row of the grid should contain each of the digits from 1 to n exactly once, in any order. So, in a 4×4 grid each row and each column should contain each of the four digits 1, 2, 3, and 4 exactly once.

The subdivision of the grid into cages, some of which may be irregularly shaped, marks the difference between Calcudoku and Sudoku. Each of these cages contains a *target number* as well as an *operation*. The operation can be either $+$, $-$, $\times$ or $\div$. If the cage contains the

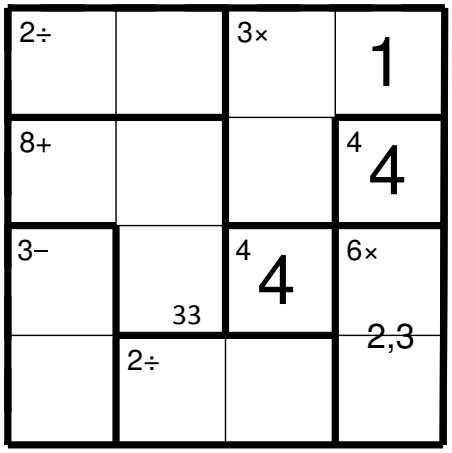
operation addition $(+)$, then all the numbers in the cage need to *add up* to the target number. Similarly, if the operation is multiplication $(\times)$, then the *product* of all the numbers in the cage must give the target number. The operations of subtraction and division only occur in cages containing two cells. If the operation is subtraction $(-)$, then the *difference* between the two numbers in the cage (in any order) must give the target number. If the operation is division $(\div)$, then the *quotient* of the two numbers in the cage (in any order) must give the target number. Cages that comprise only a single cell contain only a target number since in such cases the target number itself uniquely specifies what number should be placed in the cell.

Worked example

Consider the following 4×4 Calcudoku puzzle:



We can immediately put 4 into both of the single-cell cages since they both have a target number of 4. The 6× cage must contain the numbers 2 and 3 (although we don't as yet know in which order they go) since this is the only pair of numbers from 1 to 4 whose product is 6. This means that the only number unaccounted for in the 4th column is 1, so this must go in the remaining cell at the top of the 4th column. This gets us to the following point:



Focusing on the 3× cage, we can reason the cage must contain two 1s and a 3. Since the two 1s can't be in the same row or column, the 3 must be placed at the top of the 3rd column with the 1 immediately below it. The bottom cell in the 3rd column must thus be a 2. This gets us to:

2÷		3× 3	1
8+		1	4 4
3−		4 4	6× ~~2,3~~
	2÷	2	

Since we now have a 2 in the bottom row, this means that the 6× cage (which has to contain a 2 and a 3) cannot have the 2 in the bottom row. The 3 must thus be in the bottom row with the 2 directly above it.

The three digits in the 8+ cage could be 4, 3, 1 or 4, 2, 2 or 3, 3, 2. The first option (4, 3, 1) is not possible since the 2nd row already contains both a 1 and a 4 so it is impossible to fit the numbers 4, 3, 1 into the cage without repeating a number in the 2nd row. Alternatively one could argue that since the 2nd and 3rd rows already contain a 4, and since the 8+ cage is restricted to the 2nd and 3rd rows, it is not possible to put any combination of three numbers into the cage if one of those numbers is a 4. This reasoning also rules out the second option of 4, 2, 2. This means that the 8+ cage must contain the numbers 3, 3, 2, and since the two 3s can't be in the same row or the same column, there is only one possible arrangement that will work:

2÷		3× 3	1
8+ 3	2	1	4 4
3−	3	4 4	6× 2
	2÷	2	3

The 3rd row only has one number missing, namely 1. The top cell of the $3-$ cage must thus be a 1 which means the other number in the cage must be a 4 in order for the difference of the pair to be 3. The bottom cell of the 2nd column must thus be 1 (the only number missing from the bottom row), and the remaining cells are then easily filled in:

2÷ 2	4	3× 3	1
8+ 3	2	1	4 4
3− 1	3	4 4	6× 2
4	2÷ 1	2	3

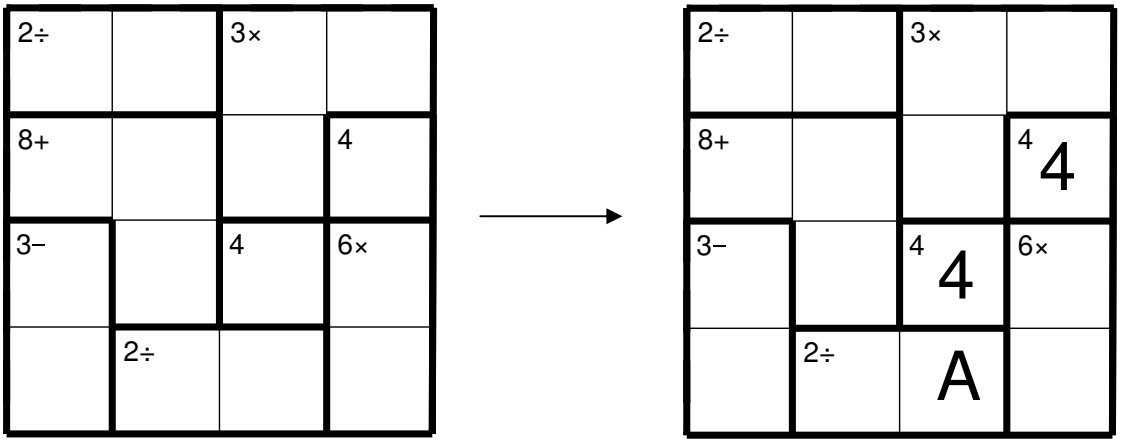
There are of course a variety of solution paths to any given puzzle. The solution path described above is not necessarily the quickest or most economical/efficient, but it highlights the kind of reasoning process required in solving the puzzle. Now that you've had an introduction to solving Calcudoku puzzles, try solving the following puzzle on your own:

3−	7+	2	6×
		2−	
1−	1		3−
	2÷		

Further strategies

Other than general strategies of logical reasoning, there are a number of specific strategies that are often useful to consider. If we are solving a 4×4 Calcudoku puzzle (for example) then we know that the sum of the four cells in any row or column must add up to $1+2+3+4=10$. Similarly, the product of the four cells in any row or column must equal $1\times2\times3\times4=24$. This strategy can be applied to multiple rows or columns as well, and this is where this particular strategy can often be rather useful.

If we return to our original 4×4 Calcudoku puzzle (shown below), then we can immediately fill in the two single-cell cages that both have a target number of 4. Let's now focus on the bottom cell of the 3rd column (labelled A in the diagram below).



Since we are working with a 4×4 grid, the product of all the cells in a given row or column must be $1 \times 2 \times 3 \times 4 = 24$. Using this strategy we can reason that all the cells in the 3rd and 4th columns must have a product of $24 \times 24 = 576$. Although we don't know the actual value of some of the cells, we do know the products of various cages. We thus know that $3 \times 4 \times 4 \times 6 \times A = 576$. Solving for A, which can be accomplished relatively quickly with a bit of prime factorization, yields $A = 2$ (which is what we obtained previously using a completely different approach).

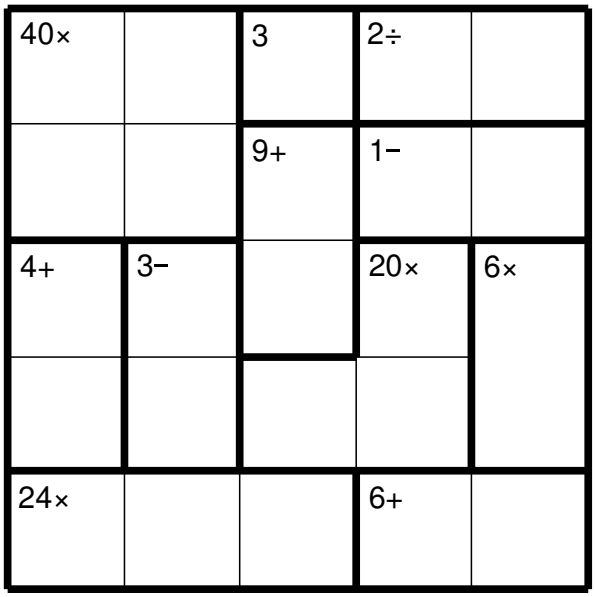
For larger grids it is also sometimes helpful to consider parity arguments. Since we know that $even + even = even$, $odd + even = odd$, $odd + odd = even$, $even \times even = even$, $odd \times even = even$ and $odd \times odd = odd$, one can sometimes use this to limit the possible entries for certain cages.

Pedagogical value

Exploring Calcudoku puzzles in the classroom setting has the potential to open up interesting discussions around important mathematical issues such as logical argument, proof and justification, as well as provide opportunities to practice simple mental arithmetic in a fun and constructive context.

A final example

Below is an example of a 5×5 Calcudoku puzzle. See how you manage solving it on your own.



Now that you're hooked, you'll be looking for more puzzles to enjoy! There are a number of Internet sites that offer daily updated free Calcudoku (KenKen) puzzles of varying degrees of difficulty. Two good sites are listed below:

http://www.kenken.com/

http://www.nytimes.com/ref/crosswords/kenken.html

Summary of time allocation

Introduction to Calcudoku	10 minutes
Worked example No. 1	10 minutes
Individual exercise No. 1	15 minutes
Discussion of strategies	10 minutes
Individual exercise No. 2	10 minutes
Concluding comments	5 minutes

REFERENCES

Davis, T. (2010). *Kenken for teachers.* Retrieved March 6, 2012, from http://www.geometer.org/mathcircles/kenken.pdf