TOWARDS A MODEL FOR INTENTIONAL TEACHING FOR IMPROVING ACHIEVEMENT IN HIGH-STAKES MATHEMATICS EXAMINATIONS

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'Teaching and learning' has become the catchphrase to capture the crucial aspect of teaching of teachers' classroom work. This developed in a direction of reducing teaching to a subsidiary position. However, there were, and still are, voices against this untenable stance. As part of the quest to re-insert discussions and deliberations about teaching into the cauldron of proposals linked to the enhancement of teaching, I present a model for teaching based on intentionality, assessment for learning and insights gained from working with teachers to develop their teaching. The evolution of the model is described and the extent to which teachers' current teaching attunes with the model is presented.

INTRODUCTION

A cursory perusal of descriptions, considerations and prescriptions for improving the state of school mathematics renders that much attention is accorded to learning. This is understandable given advances in research on learning, how people come to know, the processes of how mathematical knowledge is constructed by experts and the socio-economic conditions conducive for effective learning. To a certain extent deliberations on teaching have been backgrounded with the most popular notion being advanced that teaching should be some kind of facilitation. Davis (1997) draws attention to this invisible-making of teachers [and by implication teaching] in his analysis of a textbook series for school mathematics. He found that

> From this point [early in the textbook] on the teacher is banished from the text and the teaching of mathematics is left to the student [and that] the text does not merely signify that all students are competent autodidacts but, crucially, selects them along gender lines from the pool of readers. (Davis 1997: 6, 14).

Ten years later Morrow (2007: 1) captures the invisibility of teaching as

> … in our policies and plans we think very little about teaching … we think that it is better to talk about 'facilitation' or 'instruction' … perhaps we think that teaching is no longer needed because of 'learner-centred education' … Perhaps this silence [about teaching] is due to the fact that in South Africa we no longer have any teachers, but, instead, now have 'classroom educators'.

These two instances point in the direction that teaching should be more foregrounded in deliberations about the classroom work of teachers. Recently some attention has been paid to teaching as is evident by the titles of books such as *Retrieving teaching: Critical issues in curriculum, pedagogy and learning* (Shalem & Pendlebury 2010)

Julie, C. (2013). Towards a model for intentional teaching for improving achievement in high-stakes mathematics examinations. In Z. Davis & S. Jaffer (Eds.), *Proceedings of the 19th Annual Congress of the Association for Mathematics Education of South Africa*, Vol. 1. (pp. 86 – 96). Cape Town: AMESA.

which is a festschrift in honour of Morrow and the authors engage with the ideas on teaching, amongst others, proffered by him.

In this paper I propose a model for the teaching of school mathematics. The evolution of the proposed model and the mechanisms that triggered it are discussed. It is concluded that the implementation of such explicitly-articulated models might be helpful for engaging teachers to develop their teaching.

A NOTE ON METHODS AND THEORY

The format of this article is more in the form of a story in the sense of a story documenting "a given milieu in an attempt to communicate the general spirit of things. [A]nd is the first cut at understanding enough to see if a case study is worth doing." (Denny 1978: 2). In trying to craft some sort of a story, Freudenthal's (1991: 161) dictum of reporting our research "…so candidly that it justifies itself, and that [the] experience can be transmitted to others to become like their own experience." is adhered to. The story being told is about the realisation of a product called a model of teaching for school mathematics. The context of its emergence and development is a continuous professional development project, the Local Evidence-Driven Improvement of Mathematics Teaching And Learning Initiative (LEDIMTALI). The core quest of this project is the development of teaching mathematics.

The data drawn on are the observation notes of teaching, recorded interactions in workshops and teacher institutes and other primary documents such as newsprint sheets and print-recorded work generated during project activities. The data were collected during the 2012 academic year. In line with the story flavour, data are primarily used to support the flow of the story and not in the strict sense of some research approaches in an analysed form.

One of the key constructs in this paper is around a model for teaching and not a method of teaching. The word 'model' carries its own baggage. Wartofsky (1979) views models as tools and clarifies them as

> The cognitive artifacts we create [as] representations to ourselves of what we do, what we want, and of what we hope for. The model is not, therefore, simply a reflection or a copy of some state of affairs, but beyond this, a putative mode of action, representation of prospective practice, or of acquired mode of action. (op.cit.: xv).

I use model in this sense with a deep realisation that tools continually evolve, are used differently in different circumstances of need (an axe is sometimes used as a hammer) and users do adaptations to tools to suit the pursuits they are engaged in.

One of the vexing issues in doing and reporting research currently is that the theoretical framework which underpins the work must be stated upfront. An extensive discussion regarding this issue is not delved into. A meaning accorded to this prescription is that the constructs generated by some guruistic figure, such as for example, Vygotsky, must be referred to and employed to render support that the work qualifies as theoretically sound. There is nothing wrong with this and is needed at times. There, however, are instances when a *bricoleur* (Cobb 2007) stance is taken with regard to the underlying theoretical constructs of import for research. This stance is, in my contention, appropriate to relay the evolutionary construction of models which by its very nature draws on different theoretical orientations and is adopted in this paper.

WHY FOCUS ON A MODEL FOR TEACHING?

The relegation of teaching to a subordinate position in boundary objects such as policies, curricula and textbooks in the school mathematics teaching enterprise has been alluded to above. Whereas before teachers had available specific descriptions of models of teaching this has virtually disappeared from the radar. Models that teachers had access to and knew about were, for example, cognitively-guided instruction, the problem-centred approach to teaching, diagnostic, discovery and expository teaching. A variety of reasons can be found for the virtual disappearance of the explicit awareness-making of models of teaching. Two of these come to mind. The first is the erroneous view that fundamental pedagogics was an apartheid socio-technology to devise and justify a racially-based educational dispensation. Models of teaching enacted through didactics and subject didactics as sub-disciplines of education were viewed as belonging to this discourse. Secondly, the notion of constructivist teaching was offered and taken as the mantra for teaching. Within this notion learning was foregrounded. This led to, as Davis (1997) convincingly demonstrates, learners in essence being put in the position of autodidacts. As an aside, it should be noted that this was targeted primarily at learners from poor and marginalised backgrounds and teachers from or teaching in schools in schools in similar socio-cultural and socio-economic milieus. Although the policy interpretation of diverting teaching to learners was for all schools, Harley & Wedekind (2004: 202) found that in schools inhabited by learners from middle and upper socio-economic class backgrounds "… the OBE programme has a time allocation of only three periods per week, and the responsibility rests with the school librarian. For over 90 percent of the available time, learners are engaged in traditional subject lessons." One may daresay that this engagement was primarily through traditional teacher-guided instruction.

Morrow (2007: 3) defines teaching "as an activity guided by the intention to promote learning" which he later changed to "the organising of systematic learning" due to "the weight carried by the word 'intention'". In his exemplification of the implementation of this notion of teaching, he takes readers into a course he offered at honours level to a large class of post-graduate students in Education. What comes through in his exemplified description of "the organising of systematic learning" and discussions I had with many of the students who enrolled for the course are at least two things. First, the course had as a principle that "the conceptual and practical organisation of the course should be strong, unambiguous and explicit…" (Morrow 2007: 22). Secondly, the during the contact component of the course a dialogical form of teaching was adopted. The discussions were on material that was pre-read by the course participants and these were supplemented by tutorials. The point I am demonstrating is that whether we want to admit it or not some model of teaching always underlies teaching and the content should be systematically organised to provide students epistemological access to knowledge. In what follows I offer a model for the systematic organisation of school mathematics as a particular genre of mathematics (Julie 2002). Before moving to the model *per se*, the issues that provided inspiration for it is described.

INSPIRATIONS

Normally things that are constructed do not unexpectedly fall from the air. They are triggered by real and imaginary encounters one has in particular contexts. The contextual triggering of ideas is not a strange phenomenon in Mathematics. Stories abound of mathematicians getting ideas for solutions of problems in strange ways and places. Of course, the contexts in these cases are the problems that the mathematicians have been grappling with for some time.

The inspiration for considering the development of some model for teaching school mathematics emerged from my involvement in LEDIMTALI. I observed some 40 lessons in grades 10 to 12 in 10 different schools in 2012. From these observations a baseline teaching model for the teaching of procedures was derived. This model was depicted as follows (Julie 2012):

> The style of teaching when procedures are introduced is a demonstrative exposition of the steps of the procedures with its unfolding written on the chalkboard as the demonstration proceeds. This is interspersed with questions directed at learners which are either answered by the teacher him/herself or learners. Upon completion of the demonstration of a few key examples by the teacher, the learners are presented with similar or near-similar problems to work through. During this phase the teacher monitors learners' progress and understanding and provides assistance to learners who are struggling or request further assistance. Near the end of the period the learners are given similar problems for homework. At the start of the subsequent session, the given homework is checked and selected learners are requested to work through the solutions of the homework problems on the chalkboard. During this process, the teacher checks the work of the learners at the chalkboard whilst also checking whether learners did complete the assigned homework.

Some the questions that arose from this observation were: How can this established way of teaching school mathematics be exploited to move teaching in a direction of more work being done by learners? How can it be somewhat ensured that a resulting model will in some way conform to the principle of immediacy of applicability in classrooms which systematic reviews of continuous development initiatives found was a crucial factor for teacher buy-in of the goods distributed by such initiatives (Cordingley, Bell, Rundell & Evans 2003)? What should be included to bring and keep the disciplinary objects of school mathematics in the foreground? Must the model have some connectivity with those which teachers have encountered in their own initial and in-service teacher education programmes?

Since teachers were encouraged to share ideas and insights at workshops (±1½ hours after-school professional development activities at the university) and teacher institutes (1½ to 2½ days with overnight stays by all project participants), one teacher requested to share the insights from a course on Assessment for Learning (AfL) organised by the South African Democratic Teachers' Union. It was agreed that she would be offered a slot on the programme for the next institute. During her presentation my thinking was drawn to intentional teaching of which I was reading on and off for the design of a degree programme for foundation phase teacher education. I thought that there are areas of overlap between AfL and intentional teaching but the idea was parked since it was near closure of schools and institutions for the 2012 academic year.

The revisiting of the consideration of a possible teaching model linked to intentional teaching and AfL was again ignited during the first workshop in 2013. Part of the 2012 activities of the LEDIMTALI project was the design of a common end-of-year examination for grade 10 of the participating schools. Eight of the schools wrote the examination in 2012 and the first workshop focused on the analysis of the achievement results. The item difficulties for the first paper as obtained via Rasch analysis for one school were presented. The item difficulties were divided into four bands of difficulty. These were presented to teachers as a task given in Figure 1.

TASK

Consider the classification of learners' performance in paper 1 of the common examination we have set for 2012.

Select a topic/area in the "difficult" or "most difficult" band you will be teaching during this quarter and develop some strategies and actions you will embark on and use in your teaching of this topic/area to improve learners' performance on these questions.

Keep in mind the following:

- The ideas *** shared with us about purposeful teaching with assessment of student learning
- The value of regular/spiral revision of completed work

Difficulty	Items of 2012 Paper 1 examination	Number of items
Least difficult	1.1.1; 1.1.2; 1.1.3; 1.2.1; 1.2.2; 1.3.1; 1.3.2; 1.4.2; 4.3.1; 4.3.3; 4.3.4; 5.1	12
Slightly difficult	8.1.1; 2.2.1; 6.1.2; 1.1.4; 4.3.2; 5.2; 2.1; 3.1	8
Difficult	1.4.1; 3.2.2; 3.4; 4.1.1; 4.1.2; 4.2; 5.3; 6.1.1; 6.2.1; 6.2.2; 6.2.3; 7.1.2; 7.2; 7.5; 8.1.2; 8.2; 8.3.1; 8.3.2	18
Most difficult	2.2.2; 2.3; 3.2.1; 3.3; 7.1.1; 7.3; 7.4; 8.1.3; 8.3.3	9

Figure 1: Task based on item difficulties presented to teachers.

What was observable from the designs of the teachers was that the strategies they came up with were basically descriptions of the solutions procedures as is evident from Figure 2.

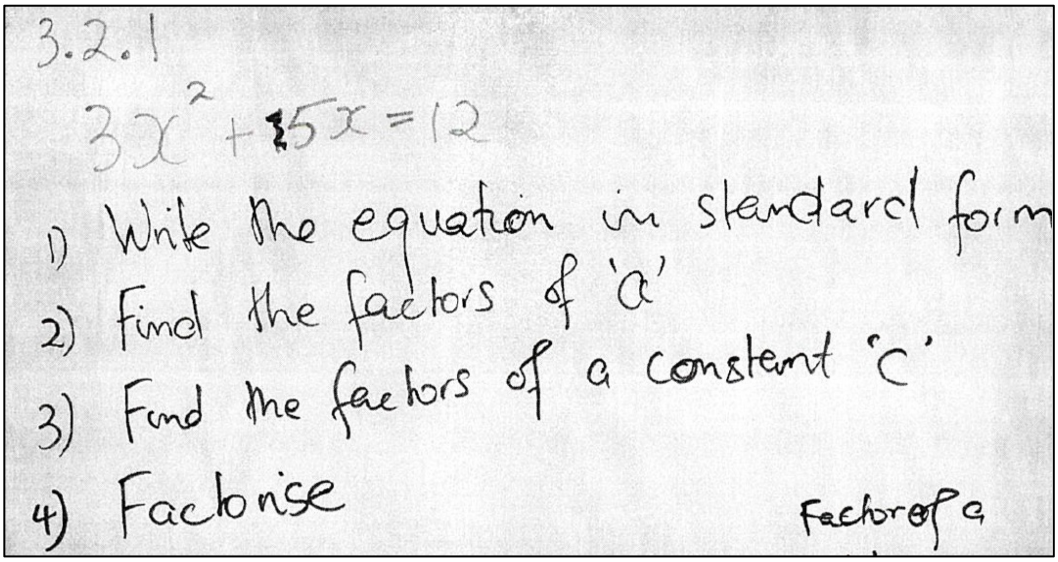

Figure 2: Except of a response to task.

This gave the final impetus to design some model of teaching based on intentional teaching and some of the principles of AfL.

TOWARDS A MODEL

Slavin (2000: 7) asserts that although "there is no formula for good teaching, the one attribute that seems to be characteristic of outstanding teachers is *intentionality*, doing things on purpose". Epstein (2007) sheds further light on intentional teaching and contends that "Intentional teaching does not happen by chance; it is planned, thoughtful, and purposeful" (op.cit.: 1). The purposeful aspect of intentional teaching is linked to learners' acquiring knowledge and skills. Thus, despite Morrow's trepidation about the weight of the word intention, intentional teaching does contain the elements of his "organising of systematic learning". For the model that is being proposed intentional teaching is seen as the purposes and goals of teaching being clear to all participants. It forms the base of the model. An aspect of this clarity of goals is that both teachers and learners must be aware of which of the objects mathematical knowledge is the focus of teaching. A general classification of the objects of mathematical knowledge taught in school mathematics is:

Conventions, representations and notations: The way things are written, the symbols that are used. Examples: "+" is used to indicate "addition"; "√" is used as the "square root" sign.

Concepts and definitions: A mathematical "thing" with the property that there are certain elements which cannot be changed so that it remains the "thing" that it is. Examples: Solution; equation; midpoint; area and real number.

Procedures and techniques: Step-by-step actions to obtain a desired result. Example: Factorisation of quadratic trinomials; constructing the altitudes of a triangle and simplifying algebraic fractions.

> *Relationships between concepts:* Links between different concepts and combinations, mostly operationally (addition, subtraction, multiplication, division, etc.) linked, of concepts. Theorems are the most commonly known relationships.

A further feature related to the objects of knowledge is the particular mathematical knowledge that should be foregrounded. It seems obvious that the mathematical knowledge de- and prescribed in the curriculum should be the foregrounded knowledge. However, the various mutations an intended curriculum goes through renders that it manifests itself differently in its implemented and examined morphings. Based on the finding that examinations dictate what desired and valued knowledge is (Kvale: 1993), I contend that for a model for intentional teaching, the knowledge assessed in high-stakes examinations should be the privileged knowledge. High-stakes examinations are examinations where the major beneficiaries of the outcomes of levels of success are the examinees. These levels of success provide them with various forms of access to certificates of worth with which they can trade. The National Senior Certificate examination is one such high-stakes examination. It has tremendous consequences for learners in terms the passages of rites that are accorded to them. It is on par with examinations such as those accounting graduates must write to become chartered accountants. This is about it being entirely set, moderated and marked externally.

As alluded to above AfL is an inspirational constituent of the model being proposed. According to Black et al. (2004: 10)

> Assessment for learning is any assessment for which the first priority in its design and practice is to serve the purpose of promoting students' learning. It thus differs from assessment designed primarily to serve the purposes of accountability, or of ranking, or of certifying competence… it provides information that teachers and their students can use as feedback in assessing themselves and one another and in modifying the teaching and learning activities in which they are engaged.

In AfL four issues must be attended to and addressed. Firstly, the learning intentions (LIs) for the unit of work must be clearly specified and everyone involved in the teaching and learning endeavour should know and be able to articulate them. Here it is crucial that learners should also be able to articulate the LIs. From the Northern Ireland Curriculum (n.d.) support documents, an LI answers the question "What are we learning to do?" for which WALT (We Are Learning To) is the hooking acronym. The second component of AfL is the clear specification of the success criteria (SCs) to ascertain whether the LIs have been achieved. It is called WILF (What I'm Looking For) and is concerned the recognition of success and mastery of the learning intentions. (Northern Ireland Curriculum n.d.).

Once the WALTs and WILFs are specified, the teachers and learners engage in activities related to the knowledge objects of import for the lesson.

The final component of AfL is concerned with assessment and feedback. The SCs are used for assessment and feedback is provided based on both the LIs and SCs. The assessment can be self-, peer- or teacher assessment.

Early in the continuous professional initiative, the notions of spiral revision and productive practice (Selter 1996) were developed. This was based on concerns about learners not doing homework. Spiral revision was the strategy devised to address the aspect of practising of skills and processes in class. Spiral revision is the repeated practising of work previously covered. It is underpinned by the notion that through repeated practice learners will develop familiarity with solution strategies of mathematical problem types that they will come across in high-stakes examinations. Productive practising has to do with allowing learners to develop general ways of working in school mathematics through "deepening thinking"-like problems whilst practicing previously covered work. An example of such a problems is: "Factorise $ak-(k+a)+a^2$ in more than one way."

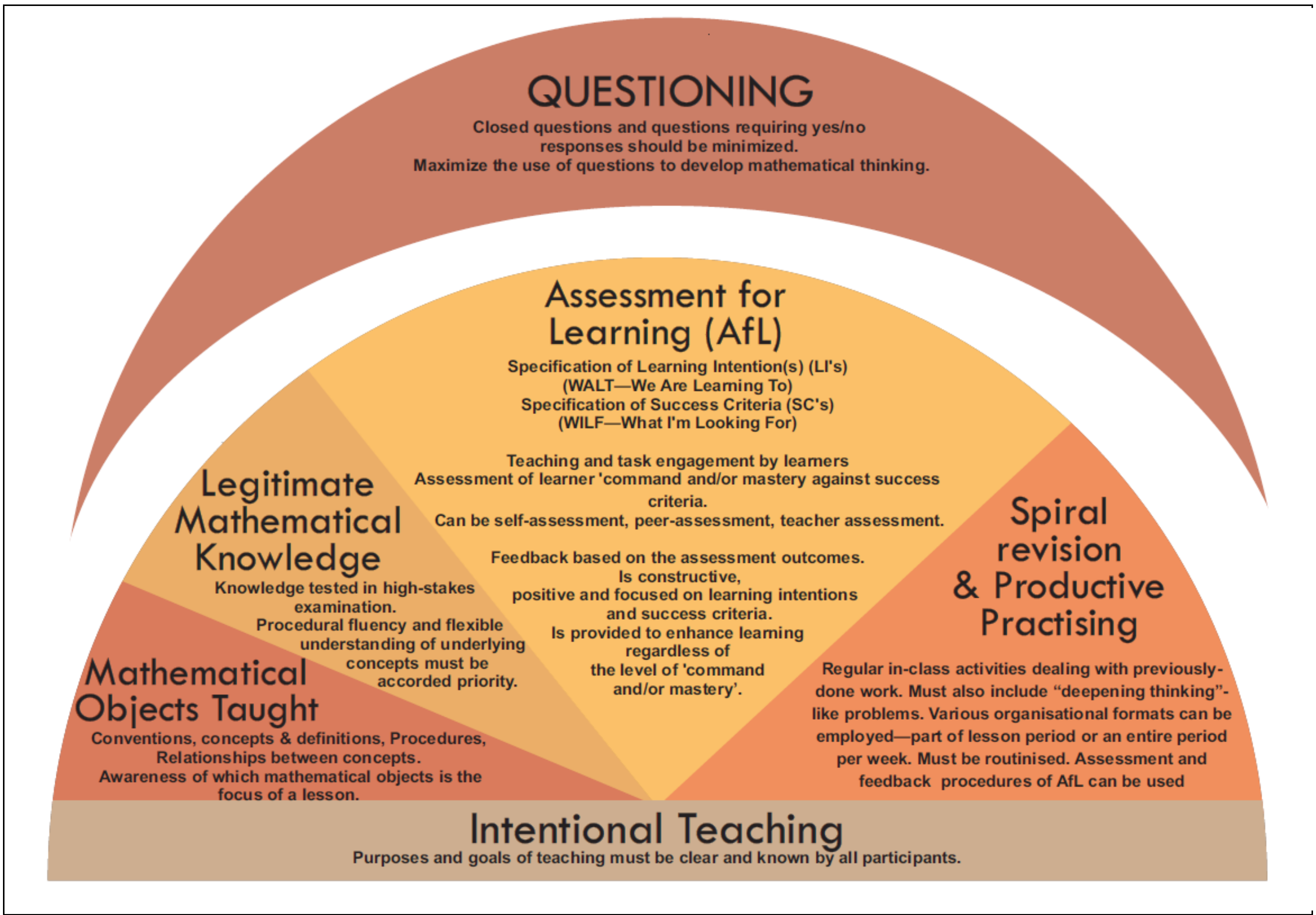

Figure 3: A model for intentional teaching of school mathematics.

The procedure suggested for implementing and sustaining spiral revision and productive practising is that at the start of a period 2 or 3 three problems on work previously done are presented to learners. This should preferably not take more than 10 minutes of the time allocated for the lesson period. Some teachers in the project reported that they preferred to set aside a period a week for spiral revision and productive practice. Given the somewhat home-grown nature of this strategy to address a specific need expressed by teachers, it is included in the model.

Lastly, an overarching component of the model is the notion of questioning to foster mathematical thinking. The major resource teachers have for this is *Questions and prompts for mathematical thinking* (Watson & Mason: 1998) and the focus is on questioning and prompting related to concepts and procedures.

Overall the model that is proposed and in varying degrees being implemented is presented in Figure 3.

CONCLUSION

I conclude by giving a sense how the elements of the proposed model manifested itself in the observed lessons referred to above. A crude organisational scheme is used to do this. This scheme was not used for the observation of lessons during classroom. It emerged during this deliberation and construction of the above model and is merely an organisational mechanism for bringing order to the large number of lessons observed in 2012. The links with the model can be clearly discerned.

Table 1: Describing teaching in terms of the model.

Element of model	Present	Slightly Present	Not Present
Purposes and goals of lesson are made clear to learners		X	
Mathematical objects that were the focus of the lesson are made clear			X
Learning intentions are made clear and there is evidence that learners understood them		X	
Success criteria are made clear and there is evidence that learners understood them			X
Assessment and feedback used		X	
Spiral revision and productive practising activities are used		X	
Fostering mathematical thinking through appropriate questioning strategies.		X	

The scheme gives some picture of where the teachers participating in the LEDIMTALI project are with respect to the development of their teaching in relation to the proposed model. This does not mean and should not be construed that the current ways of teaching that are being employed do not have positives. Indeed, as Julie (2012) reports, there are many identifiable positive aspects in the LEDIMTALI teachers' current teaching practices. Through constructive engagement with teachers, the challenge is to ascertain how and in which manners teachers further appropriate and adapt this model for the development of their teaching. Black et al. (2004) draws attention to one of the obstacles teachers will encounter when they implement a model of this nature. They assert that, "Many teachers who have tried to develop their students' self-assessment [as part of the AfL component] have found that the first and most difficult task is to get their students to think of their work in terms of a set of goals." (Black et al. 2004: 14). Thus, as most of us who have taught and are teaching in schools in socio-economically deprived environments are aware, the development of our teaching is hard but the first fruits of experiencing learner success in terms of achievement in high-stakes examinations are sweet.

ACKNOWLEDGEMENT

This research is supported by the national research foundation under grant number 77941. Any opinions, findings and conclusions or recommendations expressed in this material are those of the author and do not necessarily reflect the views of the National Research Foundation.

REFERENCES

Black, P., Harrison, C., Lee, C., Marshall, B. & William, D. (2004). Working inside the black box: Assessment for learning in the classroom. *Phi Delta Kappan,* 86(1), 8–21.

Cobb, P. (2007). Putting Philosophy to Work: Coping with Multiple Theoretical Perspectives. In Lester, F. K. (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning*. Charlotte, NC: Information Age Publishing Inc.

Cordingley P., Bell, M., Rundell, B. & Evans, D. (2003). The impact of collaborative CPD on classroom teaching and learning. In: Research Evidence in Education Library. Version 1.1*. London: EPPI-Centre, Social Science Research Unit, Institute of Education.

Davis, Z. (1997). Gender and mathematical ability: a semiotic reading of a South African primary school textbook. *Philosophy of Mathematics Education Journal*, 10, http://www.people.ex.ac.uk/PErnest.

Denny, T. (1978). Story telling and educational understanding. Paper#12 in Occasional Paper Series. Kalamazoo, Michigan, College of Education, Western Michigan University.

Epstein, A. S. (2007). *The intentional teacher: Choosing the best strategies for young children's learning*. Washington, DC: National Association for the Education of Young Children.

Freudenthal, H. (1991). *Revisiting Mathematics Education: China Lectures*. Dordrecht, the Netherlands: Kluwer.

Harley, K. & Wedekind, V. (2004). Political change, curriculum change and social formation, 1990 to 2002. In L. Chisholm (Ed.), *Changing class, education and social change in post-apartheid South Africa*. 195–220, Cape Town, South Africa: HSRC Press.

Julie, C. (2002). The Activity System of School-Teaching Mathematics and Mathematical Modelling. *For the Learning of Mathematics*, 22(3), 29 – 37.

Julie, C. (2012). The primacy of teaching procedures in school mathematics. In Nieuwoudt, S., Laubscher, D. & Dreyer, H. (Eds.), *Proceedings of the 18th Annual National Congress of the Association for Mathematics Association of South Africa.* Potchefstroom: North-West University.

Kvale, S. (1993). Examinations reexamined: Certification for students or certification of knowledge? In J. Lave & S. Chaiklin (Eds.), *Understanding practice: Perspectives on activity and context*, 215-240. Cambridge: Cambridge University Press.

Morrow, W. (2007). *Learning to teach in South Africa*. Cape Town: HSRC Press.

Northern Ireland Curriculum (n.d.). Assessment for Learning Guidance Booklet. http://www.nicurriculum.org.uk/foundation_stage/assessment/assessment_for_learning.asp

Shalem, Y. & Pendlebury, S. (Eds.), (2010). *Retrieving teaching: Critical issues in curriculum, pedagogy and learning*. Claremont: Juta.

Selter, C. (1996). Doing mathematics whilst practising skills. In van der Boer, C. & Dolk, M. (Eds.), *Modellen, meting en meetkunde: Paradigma's van adaptief onderwisj.* p. 31 – 43, Utrecht: Panama/HvU & Freudenthal Institute.

Slavin, R. (2000). *Educational Psychology: Theory and Practice*. 6th Edition, Englewood Cliffs, New Jersey: Allyn & Bacon.

Watorfsky, M. W. 1979. *Models: Representation and scientific understanding.* Dordrecht: D. Reidel Publishing Company.

Watson, A. & Mason, J. 1998. *Questions and prompts for mathematical thinking.* Derby, England: Association of Teachers of Mathematics.