HOW I TEACH
“SOLVING 3-DIMENSIONAL PROBLEMS IN TRIGONOMETRY”
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INTRODUCTION
Three dimensional problems are usually solved by taking a succession of triangles in different planes and applying to each separately the results which are already established (Area, sine and cosine rules).

But in cases of right angle triangles definitions of sine \( \frac{O}{H} \); cosine \( \frac{A}{H} \); tangent \( \frac{O}{A} \) are used. It is always necessary and sufficient to start with the triangle with the most information (like 2 sides and an angle, etc.)

SOLUTION OF TRIANGLES IN THREE DIMENSIONAL DIMENSIONS:
Whereas two-dimensional space occupies a single plane, three-dimensional space occupies three planes. The three planes are horizontal, vertical and inclined. The sine, cosine and area rules can also be used to solve problems in three dimensional space. The diagram below illustrates the three different planes for an object in three dimensions.
A rectangular birthday card is tied with a ribbon at the midpoints, G and H, of the longer sides. The card is opened to read the message inside and then placed on a table in such a way that the angle $\angle AFE$ between the front cover and the back cover of the card is $90^\circ$. The points G and H are joined by straight lines to the point C inside the card, as shown in the sketch.

Let the shorter side of the card, $BC = x$, and the longer side, $CF = 2y$.

![Diagram of the card with points G, H, and C labeled]

Prove that $\cos \angle GCH = \frac{y^2}{x^2 + y^2}$. [8]
In the diagram below, RS is the height of a vertical tower. T and Q are two points in the same horizontal plane as the foot S of the tower. From point T the angle of elevation to the top of the tower is 60°. RTQ = 20°, RQT = 60° and TQ = k metres.

10.1 Express TR in terms of θ and k. (3)

10.2 Show that \( RS = \frac{3k}{2(\sqrt{3} \cos \theta + \sin \theta)} \). (7)
Question 3: Dec 2013

The Great Pyramid at Giza in Egypt was built around 2 500 BC. The pyramid has a square base (ABCD) with sides 232.6 metres long. The distance from each corner of the base to the apex (E) was originally 221.2 metres.

13.1 Calculate the size of the angle at the apex of a face of the pyramid (for example CEB).

13.2 Calculate the angle each face makes with the base (for example EFG, where EF ⊥ AB in ΔAEB).

TRIAL EXERCISES
EXERCISE 1

In the following sketch, ΔBCD is in a horizontal plane. A is directly above B (AB is a vertical line).

AB = h, BC = CD = x, B̂AD = α and B̂CD = θ.

(a) Show that \( BD^2 = 2x^2(1 - \cos \theta) \)

(b) Hence show that \( h = \frac{x\sqrt{2(1 - \cos \theta)}}{\tan \alpha} \)

(c) Calculate the value of \( h \) if \( x = 100 \), \( \theta = 60^\circ \) and \( \alpha = 40^\circ \) (two decimal places)
EXERCISE 2

Refer to the figure. B, C and D are three points in the same horizontal plane so that \( BD = CD = d \) and \( \angle BD = \theta \). AB is perpendicular to the plane. From C the angle of elevation of A is \( \alpha \).
(a) Express \( \theta \) in terms of \( \theta \).
(b) Prove that: \( AB = 2d \cos \theta \tan \alpha \)
(c) If \( d = \sqrt{2} \) units; \( \alpha = 30^\circ \) and \( \theta = 75^\circ \) calculate AB without the use of a calculator.

EXERCISE 3

AC represents a vertical tower which is perpendicular to the horizontal plane BCD at C. AB is 2 units. \( \angle BCD = 90^\circ - \theta \), \( \angle BDC = 2\theta \) and \( \angle BAC = \theta \).
(a) Determine BC in terms of \( \theta \).
(b) Show that BD = 1 unit.
(c) If \( AD = \sqrt{3} \) units, calculate the size of \( \angle ADB \) without using a calculator.

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