AMESA 2015

Deepening the quality of mathematics teaching and learning

PROCEEDINGS OF THE 21st ANNUAL NATIONAL CONGRESS OF THE ASSOCIATION FOR MATHEMATICS EDUCATION OF SOUTH AFRICA

VOLUME 2

29 June - 3 July 2015 • Polokwane - Limpopo Province

Editors: Satsope Maoto, Benard Chigonga & Kwena Masha

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University of the Limpopo
Polokwane

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Association for Mathematics Education of South Africa (AMESA)

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FOREWORD

The 21st century challenge for South Africa as a Nation is to provide quality teaching and learning for all its young citizens. The teaching and learning of mathematics is at the core of this challenge. It thus makes sense that during our 21st Annual National Congress, we reflect on what AMESA can do to contribute to this national goal. The current theme, '*deepening the quality of mathematics teaching and learning*' was conceptualised against this background.

South Africa has, at a system level, introduced Annual National Assessment whose purpose is, among others, to monitor the quality of teaching and learning at different points in the system. A number of presenters are engaging with these assessments picking up issues that arise from those and strategies that can be used to address them. Similarly, we have papers that address issues that arise out of our National Senior Certificate examinations. Our panellists and plenary speakers were also invited to address issues affecting quality mathematics teaching and learning from various perspectives that include higher mathematics learning, teacher education, international perspective, curricular issues and learners support material. The workshops that form part of these proceedings offer hands-on experiences through which facilitators share successful strategies and activities that they have successfully developed to improve the quality of mathematics teaching and learning in different phases and context.

This congress, like any other, will hopefully inspire you to take up the challenges of quality mathematics teaching and learning beyond the five days in which the activities unfold. You will continuously use the two volumes as a critical resource material in your classrooms, seminars, workshops, planning sessions, and so on.

Lastly, we remain thankful for the presenters and their reviewers for their immense contribution to the 21st Annual National Congress of the Association for Mathematics Education of South Africa. We are forever inspired! Enjoy!

The Academic Coordination Team

Satsope Maoto (Coordinator),
Benard Chigonga, and Kwena Masha
June 2015

REVIEW PROCESS

The papers accepted for publication for the 2015 AMESA Congress were subjected to blind peer review by at least two experienced mathematics education reviewers. The academic committee considered the reviews and made a final decision on the acceptance or rejection of each submission, as well as changing the status of submission. Authors of accepted submission were given the option of *not* to have their accepted long papers published in the AMESA 2015 Proceedings, to keep open the possibility to submit it to a journal. They were requested to submit an extended abstract rather than their full submission, and this extended abstract will be published in the Proceedings for publication.

Number of submissions:	112
Number of plenary paper submissions:	5
Number of long paper submissions:	44
Number of short paper submissions:	2
Number of workshop submissions:	27
Number of *'How I teach'* paper submissions:	14
Number of poster submissions:	0
Number of Maths Market	20
Number of submissions accepted:	103
Number of submissions rejected:	9
Number of submissions withdrawn by authors:	8

We thank the reviewers for giving their time and expertise to reviewing the submissions.

Reviewers:

Stanley Adendorff	Mark Jacobs	Kwena Masha
Benadette Aineamani	Yusuf Johnson	Mike Mhlolo
Olivier Alwyn	Alex Jogymol	Mogege Mosimege
Margot Berger	Kerstin Jordaan	Themba Mthethwa
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Vasuthavan Govender	Satsope Maoto	Sheena Rughubar-Reddy
Belinda Huntley	Simons Marius	Jojo Zingiswa
Shaheeda Jaffer		

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Special interest group

SHARING AND GROUPING IN FOUNDATION PHASE

Tracey Bester & Melissa Mentoor
Capricorn Primary School, Capricorn Primary School

In the aptly named Foundation Phase, we lay the foundation for many mathematical concepts, including division and multiplication. The foundations for division and multiplication are the processes of grouping and sharing. Come and spend time with Tracey and Melissa as they share their experiences of teaching grouping and sharing to grade 1 learners using a concrete and practical approach.

TARGET AUDIENCE: Foundation Phase

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 30

MOTIVATION

Grouping and sharing are foundational models for multiplicative reasoning (multiplication, division, fractions, ratio and rate) in mathematics. Yet in Foundation Phase both teaching and learning is often dominated by sharing models. Ten divided by 2 is commonly approached as share 10 things between 2 people. Very seldom do we see teachers invoking problems such as there are 10 eyes, how many cats are there? Seeing what is the same and what is different about these two problems – and why both are necessary in the Foundation years, greatly supports teachers in their approach to these mathematical topics.

CONTENT OF THE WORKSHOP:

In this session Tracey and Melissa will share lessons they have taught with Grade 1 learners where the ideas of grouping and sharing were in focus. In these lessons the learners were

- Engaged in the lesson by answering practically (counter+ hoolla -hoops), orally (explain) and (written) be able to complete a written exercise.
- Making use of repeated addition/addition.
- Encouraged to connect grouping and counting in 2’s and 5’s etc.
- Learning to work co-operatively within in their groups as they will be able to solve and explain solutions to practical problems that involve grouping with whole numbers to their peers. They will also be able to converse to each other by using the related vocabulary.
- Identifying, counting, drawing and explaining the concept.
- Using resources such as number chart, hoola-hoop, counters, laminated sheet and white board markers, DBE books and worksheets.

In the second half of the workshop, Tracey and Melissa will support participants to make use of sharing and grouping ideas when multiplying and dividing using a higher number range. An approach that makes use of grouping (or chunking) to support long division processes. This approach has been used effectively in Grade 3 and 4 will be modelled. Participants will have opportunity to attempt this approach to division for themselves.

USING GEOGEBRA TO TEACH DIFFERENTIATION

George Chirume, Ingrid Mostert
West Coast TVET College, AIMSSEC and Kelollo Consulting

Differentiation is a very important mathematical concept and yet many learners don't find it interesting. One way to increase learners' interest in differentiation is by using technology, as modern learners are more technologically aware. In this workshop we will explore how a particular piece of software, GeoGebra, can be used to teach differentiation. GeoGebra is a free dynamic software that brings together different aspects of mathematics such as geometry, algebra, spreadsheets and calculus. GeoGebra can be used to sketch graphs of functions and the graphs of their derivatives. Because it is so easy to sketch graphs of functions, it becomes possible for learners to see for themselves that there is a relationship between the x-intercepts of the derivative function and the turning points of the original function. Without the use of the graph sketching, it would take much longer to see this relationship. GeoGebra can also be used to verify whether the rules of differentiation have been applied correctly and can be used to draw graphs for test and exams.

TARGET AUDIENCE: FET Phase

DURATION: 1 hour

MAXIMUM NUMBER OF PARTICIPANTS: 30

MOTIVATION

Differentiation is a very important mathematical tool concerned with the rate at which a function is changing. The rules of differentiation are used to determine this rate of change, also known as the gradient. Differentiation has daily applications ranging from determining the spread of AIDS to investigating variations in stock market prices. One way to increase learners' interest in differentiation is by using technology such as GeoGebra.

GeoGebra allows learners to easily sketch graphs and their derivatives. It also allows learners to verify whether their calculations are correct and accurate. GeoGebra is not only limited to being used in class to teach differentiation, it can also be used to set up test, exams and worksheets about differentiation. As such, it is a useful tool for teachers to have.

CONTENT

In this workshop teachers will be introduced to GeoGebra. They will learn how to use GeoGebra to sketch graphs, to calculate the derivative of a function and how to sketch this derivative.

Participants will also be shown simple functionality in GeoGebra (such as changing the colour of a graph) that can be effectively used to teach differentiation in class using a data projector as well as how to construct worksheets and exam questions about differentiation.

DEVELOPING PEDAGOGICAL CONTENT KNOWLEDGE FOR TEACHING MATHEMATICS: CONTENT REPRESENTATION MATRIX

Kabelo Chuene

University of Limpopo

Knowing to teach mathematics, despite knowledge of content, is recognised as a necessary trait for best mathematics teachers (Turnukly & Yisildere, 2007). Knowing to teach mathematics is, according to Shulman (1987), the coming together of mathematics content and its pedagogy into an understanding that benefit learners interests and abilities. He coined this Pedagogical Content Knowledge, PCK. 'Knowing to teach mathematics' also include knowledge of mathematics representations, knowledge of students, and knowledge of teaching and decision making (Fennema and Franke, 1992).

While studies on finding PCK that teachers possess proliferates related studies, a few articulate how such knowledge is developed and what planning goes into nurturing this development. The workshop session will assist teachers to design content representation matrices that can be used in planning lessons, implementing lessons, and developing PCK. The workshop will borrow from ideas on Content Representation matrices, CORE that have been used successfully for development of science teachers PCK.

Proposed duration for the workshop:	2 hrs
Targeted participants:	Intermediate and Senior phase teachers
Required materials:	Flip chards, coloured markers
Number of participants:	Maximum of 50
Hall specification	Movable tables and chairs. If fixed, movement between the chairs should be possible

TEACHING 2D SHAPES & 3D OBJECTS

Shereen Corker & Ingrid Mostert
Capricorn Primary School, Kelello Consulting and AIMSSEC

2D and 3D shapes form a central part of the geometry (shape and space) content area in CAPS. Do you know the difference between an edge, a surface and vertex? What about a line, a point and a side? You probably know about the four operations: adding, subtracting multiplying and dividing for number work. But how do we describe and make changes on shapes? Join this workshop to extend your own mathematical thinking about 2D objects, 3D shapes and the relationships between them. Find out how shapes and objects connect with patterns (both shape and number patterns). This is a hands-on workshop making use of resources that allow you to quickly construct and change 3D shapes. Learn about these shapes and objects for yourself so that you can confidently explore them with your Foundation Phase learners in future.

TARGET AUDIENCE: FOUNDATION PHASE

DURATION: 2 HOURS

MAXIMUM NUMBER OF PARTICIPANTS: 30

MOTIVATION:

The CAPS curriculum on shape and space tends to focus on 2D shapes and 3D objects independently. The way in which the naming of 2D shapes is extended to the naming of 3D objects is not made explicit. The connections with these classifying and organizing processes with other areas of the curriculum are hard to establish. The four transformations: sliding (translating), turning (rotating), enlarging/ reducing and flipping (reflecting) are only brought into focus at Intermediate phase. However Foundation Phase teachers should be aware of these as geometric operations/actions that are used to generate and describe shape patterns. In the absence of this geometric awareness we see the same shape patterns being repeated from Grade 1 to Grade 3 with no progression in difficulty.

CONTENT

In this workshop participants will generate and reflect on both number and shape patterns. The mathematical ideas underlying these patterns will be identified and experimented with. Participants will make use of specialist Geometro equipment to quickly build 3D objects from 2D shapes. They will then be able approach the teaching of shape and space in the Foundation phase with more confidence. By having their eye on the horizon (of where this content goes in intermediate phase and beyond) they will be better equipped to support the young learners in their classrooms.

LET'S NOT START WITH "X"

Neil Eddy
Wynberg Boys' High School

Unless we establish a basic grounding of algebra for our grade 8s, we doom them to mediocre performances in mathematics through the remainder of their school years and beyond. This workshop will establish a route through the introduction of algebra that sidelines concepts such as fractions, negative integers and roots so that the basic concept of generalization can be set in place without pupils losing confidence with arithmetic difficulties. The more complex arithmetic concepts are then introduced into the framework later in the course in a cyclical method that reinforces the algebraic concepts.

The workshop will run through 'think of a number' problems, flow diagrams and patterns leading to a concept of terms for order of operations, leading to formalization of algebra with addition and subtraction of like terms and expansion of brackets. Algebraic multiplication and division are only set in place later in the year after equation work has been established.

Material will include lessons written for a tablet-enabled environment. These can, however, be adapted to more traditional classroom environments. Much of the material requires prior investigation of concepts by pupils before they walk into a formal lesson on the ideas. Material is grounded in research of members of ATM in the United Kingdom

AN EXPLORATION OF DIRECT PROPORTION CALCULATION METHODS

Andrew Gilfillan
St. Anne's Diocesan College

The ratio is often the single most powerful calculation tool in everyday life, finding use in calculations of rate, scale, percentage change, conversion and several other applications. Yet learners continue to struggle to apply it correctly. This workshop seeks to explore several calculation methods which have worked very well with junior maths classes, Mathematical Literacy classes and even Core Mathematics students. Several methods will be explored as each learner is unique and they will often use different paths to find the same solution. If a teacher is looking for a fresh way to teach this vitally important section, then this workshop should give much practical food for thought.

TARGET AUDIENCE: FET and Senior GET

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 30

MOTIVATION

It could be successfully argued that the single most useful calculation method that anyone should know in daily life is a method for dealing with situations involving direct proportion. Direct proportion occurs in a host of different contexts: rates, proportional increase/decrease, scale, etc. During this workshop we will explore a variety of accessible methods for teaching and understanding situations involving direct proportion. We will use these methods to solve problems in contexts as varied as: scale, conversions, percentage increase, original price, VAT, Rate, Multi-rate calculations and a few more besides.

Strategies to be explored:

1. The "Pirate Method" (Divide up) – Ratio-based method.
2. The "Unitary Method" (Get to 1) – Ratio-based method.
3. The "Fraction Method" (Divide by what you have and times by what you need) – Fraction-based.
4. The "Equation Method" (Multiply by the opposite unit) – Algebraic Method
5. The "Cross-multiply Method" (Times across, divide backwards) – Ratio-based method.

As we shall see, each of the methods involves a central unifying theme:

"Identify the scaling factor and multiply by it".

STUDENT SURVIVAL SKILLS IN E-LEARNING

Belinda Huntley & Sarah Jane Johnston
Department of Mathematical Sciences
Unisa

ABSTRACT

This workshop is intended to create awareness amongst mathematics educators of the potential and possibility of teaching Mathematics in an online environment. The workshop will introduce the necessary skills that learners require and need to develop to ensure a positive learning experience, as well as the skills that educators need to acquire to promote teaching and learning in an online e-learning environment. The workshop will provide an opportunity for mathematics educators to discuss and reflect on some of the most important and necessary skills for both learners and mathematics educators alike in the online environment. This workshop is a hands-on, BYOD (bring your own device), guided online workshop.

Presenters

Dr Belinda Huntley and Prof Sarah Jane Johnston, Department of Mathematical Sciences, Unisa.

Level

This workshop is aimed at Secondary and Tertiary Mathematics educators.

Nature

The workshop is a hands-on, BYOD (bring your own device), guided online workshop.

Content

The workshop will provide an opportunity for mathematics educators to discuss and reflect on some of the most important and necessary skills for learners to learn and educators to teach mathematics successfully in the online environment.

Motivation

This workshop is intended to create awareness amongst mathematics educators of the potential and possibility of teaching Mathematics in an online environment. The workshop will introduce the necessary skills that learners require and need to develop to ensure a positive learning experience, as well as the skills that educators need to acquire to promote teaching and learning in an online e-learning environment.

Activities

- Discussion in groups
- Reflection in groups
- Online completion of questionnaire individually

Description of content

10 minutes Introduction

Background

Setting the scene

Exploring the objectives

Explaining the Instrument - questionnaire

Putting the participants into groups

60 minutes Discussion of questions and reflection in groups (10 minutes per question).

Key concepts to be displayed on a projector screen.

40 minutes Time for individuals to complete the questionnaire online

10 minutes Conclusion and wrapping up.

Total time 2 hours

PRACTICAL WAYS OF TEACHING VOLUME AND SURFACE AREA TO SENIOR PHASE LEARNERS

Sekgana Motimele
Masedibu High School

The workshop is aimed at senior phase where the different containers will be used by participants in their respective groups to find the nets of different prisms, which will hence be used for surface area of the respective 3D shapes

HOW THE WORKSHOP WILL BE RUN

Participants will be divided into groups of 5-6 members each, where they will use the given containers to calculate surface area and volume of each container

Each participant will use a given worksheet to record the findings as we investigate the different prisms as to their faces, the shapes of the different faces, and areas of those shapes.

What to investigate in the workshop

The other important thing that will be done will be where we investigate; using the given container, how many items can we pack inside, based on the volume of that container, and also the volumes of the smaller items. E.g In the case of weet-bix containers, we have to investigate and estimate how many biscuits can be packed into each size container, basing our calculation on the measurements of each individual item.

The other important things that will be investigated in the workshop will be where the volume is linked to the capacity in the case of liquids, where cold drink containers and juice/milk containers will be used as examples.

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EUCLIDEAN GEOMETRY WORKSHOP: ABSTRACTION OF CONSTRUCTIONS

Gabriel Mphuthi
University of South Africa

TARGET AUDIENCE: SP & FET Band Mathematics Teachers

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 30

The workshop is designed with a view to reinforce SP & FET educators' approach to teaching Euclidean Geometry. In view of the fact that Euclidean geometry is the core of high school geometry, the compass and straightedge constructions are still an important part of the geometry curriculum. For example, CAPS document for Senior Phase (DBE, 2011, p. 10) expresses the content knowledge for geometry as follows:

- *Drawing and constructing a wide range of geometric figures and solids using appropriate geometric instruments*
- *Developing an appreciation for the use of constructions to investigate the properties of geometric figures and solids*

Visualizing in geometry learning is reinforced through constructions. Constructions help students to understand geometric relationships. Furthermore, appreciation of formal geometric concepts is developed by geometric constructions (NCTM, 2000). Emphasis will be placed on establishing Geometric properties and formulation of conjectures. Participants will individually do constructions then share, compare and discuss the established facts in groups.

The workshop activities are interconnected, interrelated and interdependent, in other words, activities are developmental.

CONTENT OF THE WORKSHOP

In the activities that follow, geometric shapes, that is, triangles and quadrilaterals will be made through construction and measurement using only two construction tools – a straight edge (for drawing lines) and a compass (for drawing circles and arcs and marking off distances). Discussions will be held at the end of each construction activity. The flow of activities is as follows:

Activity	**Description**
0	Introduction
1	Construct a congruent copy of a given line segment
2	Construct a triangle given three sides (SSS)
3	Construct an angle equal to a given angle
4	Construct a triangle given an included angle
6	Use a compass to bisect a line
7	Use a compass to bisect a given angle
8	Construct diagonals and determine the type of Quadrilateral

MOTIVATION

Geometry is taught in different ways from primary school through to high school. The known popular methods of teaching Geometry are informal and formal approaches. The challenge is to integrate the informal way and formal way for effective and efficient results in the classroom.

The informal approach of teaching geometry means that certain basic concepts such as point, line, angle, perimeter, area, triangle and quadrilateral are taught by using experimentation, measurement, and folding, cutting, drawing and similar practical methods. Basically some facts are discovered using this approach.

The formal approach of teaching geometry means that the well-known theorems are treated within a mathematical system of logical reasoning. One theorem is derived from and based upon one or more previous theorems.

Teaching Geometry firstly by allowing learners to construct and investigate has many advantages, among others, learners readily participate actively in constructing and developing the content knowledge, and that the meaning of the content is gradually highlighted as constructions and investigations are done. Activities/Investigations are designed in such way that learners construct, explore or investigate, report the observed characteristics and then generalise. It is envisaged that as they do the investigations, meaningful knowledge is constructed and understanding is well cemented. This workshop aims to assist mathematics teachers to engage learners in the activity of establishing geometric facts and the space for learners to choose own definitions.

ACKNOWLEDGEMENTS

The workshop material is drawn from the material developed by Dr Piet Human for The Ukuqonda Institute for the Learning of Maths and Science.

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USING GEOGEBRA TO TEACH TRIGONOMETRIC GRAPHS

Alfred Mvunyelwa Msomi
Department of Mathematical Sciences,
Mangosuthu University of Technology

TARGET AUDIENCE: FET educators

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 30

The chapter of trigonometry is an essential section in school mathematics that links algebraic, geometric and graphical reasoning. Learners in Grades 10 – 12 struggle with this kind of reasoning. Trigonometric graphs are dealt extensively in the FET band for the reason that learners must understand the effect of parameters in a trigonometric graph. This exercise seems to be difficult to learners since the traditional approach of chalk and talk in a classroom does not encourage them to explore and discover on their own the effect of parameters involved in their trigonometric graphs. During the workshop, the focus areas of behavior of graphs, interpretation of graphs, modelling and overall performance of the learner will be adequately addressed. Trigonometry provides learners with valuable mathematical skills that they need to further their studies in the fields of science and engineering. The dynamic software, GeoGebra can be used to create activities where learners can explore and investigate the effect of parameters. It can be used extensively to explore and visualize trigonometric graphs by using sliders to transform graphs in ways beyond the scope of traditional pen and pencil method. The workshop will assist educators to be able to learn how the software can be used in the classroom situation. It will also empower participating educators with the skills that will make their learners improve significantly when working within the dynamic software.

MOTIVATION

There seem to be a great need to align basic school education with rapidly changing societal needs. Currently, education appears to be predominantly characterized by individualistic learning of isolated facts and skills for examinations. The education officials suggest the introduction of Information and Communication Technology (ICT) in education without giving educators a platform to be exposed on different available technologies as well as giving them an understanding of when and how to use technology in the classroom. This, therefore suggest that there is a need to develop a community of educators who collaborate in the use and integration of ICT in education. Technology plays a vital role in the teaching of Mathematics in our school according to the new curriculum (CAPS). Educators are therefore expected to equip themselves with new technological gadgets available in their school. The dynamic software called GeoGebra, is a freely available software from internet, which can assists educators a great deal to explore the effect of parameters on trigonometric functions with great success.

CONTENT OF WORKSHOP

Trigonometric Graphs

Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology GeoGebra, to make and test conjectures and hence generalize the effects of the parameter which results in a horizontal and vertical shift and that which results in a horizontal and vertical stretch and reflection about the both axis.

Content Clarifications

The trigonometric functions can be graphed to show the variations of magnitude of each. The graphs give a picture of each trigonometric function, which will help learners to visualize and therefore increase their understanding of the character of each trigonometric function. The graphs of trigonometric function each have a lyrical flow and rhythm. For this reason, their curves have many practical applications, especially for monitoring natural patterns and motions. In the workshop, participants will explore the effect of the parameters, a, k and investigate the effect of and on the graphs of the trigonometric functions defined by:

$$y=asinkx, y= asin\ kx; \quad y=asink(x-p)+q$$
$$y=acoskx; \quad y=acos\ kx; \quad y=acosk(x-p)+q$$
$$y=atankx; \quad y=atan\ kx; \quad y=atank(x-p)+q$$

Comment

Once the effect of the parameters has been established, various problems need to set such as: drawing sketch graphs, determining the defining equations of trigonometric functions from the sketch and making deductions from graphs.

Proposed Time Allocation for Workshop Activities

Activity	Time
• The process of learning and history of GeoGebra.	10 minutes
• Participants to familiarize themselves with the icons in GeoGebra that they will use during this session.	15 minutes
• Create activities in GeoGebra, explore, visualize and investigate the effect of parameters in a graph. 1h15minutes	
• Related discussions and installation of software	20 minutes

Workshop Objectives

- Download and install a Dynamic Mathematics Software, GeoGebra
- Explore and use Menus and basic tools of GeoGebra to construct mathematical objects
- Design activities and objects to demonstrate integration of GeoGebra in school mathematics
- Evaluate the potential of GeoGebra as a tool for integrating ICT in mathematics education, and its potential towards stimulating higher order thinking.

MAKING REGRESSION ANALYSIS EASY USING A CASIO SCIENTIFIC CALCULATOR

Astrid Scheiber
Casio

Adequate knowledge of calculator skills makes the teaching of Statistics to Grade 12 learners easier and enables the educator to assist their learners more efficiently. This workshop will guide you through Linear Regression Analysis, including finding relationships between variables, the line of best fit and making projections, using the CASIO Scientific calculator.

TARGET AUDIENCE: FET (Mathematics Educators)

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 30

Equipment Required: Casio Fx-82ZA Plus Scientific Calculator.

MOTIVATION

As of 2014, the Grade 12 Statistics syllabus involves the learners' making use of available technology to: **[12.10.1 (b)] calculate the linear regression line which best fits a given set of bivariate numerical data** and **[12.10.1 (c)] calculate the correlation co-efficient of a set of bivariate numerical data**. As stated by the current Maths CAPS document. This workshop serves to increase educators' understanding of the CASIO Scientific calculator. In turn, it will foster self-confidence and a positive attitude towards Statistics, enhancing both the educators' and learners' understanding of the topic.

CONTENT

This workshop will cover: Identifying the relationship between bivariate numerical data, inputting bivariate data into the CASIO scientific calculator, calculating the correlation co-efficient, finding the equation of the regression line, calculating projected values - Interpolation & Extrapolation, using TABLE MODE to find the co-ordinates of the line of best fit & selecting random samples.

Introduction	5 mins
Identifying the relationship between bivariate numerical data	10 mins
Inputting bivariate data into the CASIO scientific calculator	10 mins
Calculating the correlation co-efficient	10 mins
Finding the equation of the regression line	15 mins
Calculating projected values - Interpolation & Extrapolation	15 mins
Using TABLE MODE to find the co-ordinates of the line of best fit	15 mins
Selecting random samples	10 mins
Discussion & worked example	30 mins

HOW A SCIENTIFIC CALCULATOR CAN ASSIST MATHS LITERACY LEARNERS WHEN WRITING EXAMS

Jackie Scheiber

In this workshop teachers will be shown how using a scientific calculator can assist the learners when they write Paper 1 and Paper 2 of the matric exam. Topics that will be covered are: Converting between fractions, decimals and mixed numbers; Fraction calculations; Rounding off numbers; Square and square roots; Simplifying complex arithmetic calculations; Simplifying ratios; Time calculations; using the DEL key to assist with multiple calculations; Interpreting error messages

TARGET AUDIENCE: FET Mathematical Literacy teachers

DURATION: 2 hour workshop

PARTICIPANTS: 30- 40

Equipment: Teachers must bring their own calculators to the workshop

MOTIVATION

The Maths Literacy CAPS (p8) states: As a rule of thumb, if the required calculations cannot be performed using a basic four-function calculator, then the calculation is, in all likelihood, not appropriate for Maths Literacy.

Contrary to what is stated in CAPS, the presenter feels that Maths Literacy learners should be using a scientific calculator as it will make the calculations that they are required to do in both Paper 1 and Paper 2 of the matric exam much easier to do.

Content of the workshop

In this workshop we will be exploring how to use the scientific calculator to do the following:

- Convert between fractions, decimals and mixed numbers
- Fraction calculations
- Round off numbers
- Square and square roots
- Simplify complex arithmetic calculations
- Simplify ratios
- Time calculations
- Use the DEL key to assist with multiple calculations
- Interpret error messages

MENTAL MATHS IS AS IMPORTANT TO LEARNING MATHS AS PHONEMICS IS TO LEARNING TO READ

Connie Skelton
Data Mind

TARGET AUDIENCE: Foundation and Intermediate Phase

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 30

MOTIVATION

The importance of mental mathematics to learning mathematics is like phonemics is to learning reading. Although the analogy of phonemic awareness and number sense is not perfect, it does appear to be useful in many ways. Early reading involves a high degree of auditory skills, whereas number sense is less dependent on the auditory component. However both reading and mathematics are dependent on the development of fluency and comprehension. Fluency can be enhanced with practice if it is paired with conceptual skills.

Mental mathematics is a very good method for teaching mathematics facts. It develops automaticity and allows learners to solve problems more quickly and with greater confidence. The skills that are taught in mental mathematics will build learner's abilities to develop their own strategies to solve more complex problems.

This workshop aims to look at some of the strategies that can be reinforced to improve both automaticity and conceptual skills.

Comparing reading and mental mathematics

Most reading experts agree that there are five main components to the teaching of reading:

- Phonemic awareness (Analogy: number sense)
- Word recognition (sight words and phonics) (Analogy: bonds and tables)
- Comprehension (Analogy: problem solving)
- Vocabulary (Analogy: strategies)
- Fluency (Analogy: speed and ease)

Each of these components needs to be taught explicitly and practised on a daily basis. (Curriculum and Assessment policy statement Grades R-3 English Home Language). These five components were also found in all the CAPS documents for all the home languages.
(20 minutes)

The following are some activities that can be used for practice:

Activity 1 (times tables to at least 10 × 10) (10 minutes)

Activity 2 (Multiples of 1-digit numbers to at least 100) (10 minutes)

Key Strategies

Each strategy is like a tool in a learner's toolkit. They need time to master the strategy and once they have a good understanding of how each strategy works, they can choose the most effective ones.

The key strategies below are a selection of the strategies recommended in CAPS

- Counting
- Using a number line
- Ordering a given set of numbers using place value
- Doubling and halving
- Using the commutative, associative and distributive properties

(15 minutes)

Discussion and Activity 3: Counting (10 minutes)

Discussion and Activity 4: Mental number line (10 minutes)

Discussion and Activity 5: Using place value to order numbers (10 minutes)

Discussion and Activity 6: Doubling and halving (10 minutes)

Discussion and Activity 7: Pairs that make multiples of 10 (10 minutes)

CONCLUSION

It is important to realise that mere drill and practice in isolation will not necessarily promote competent learners in mathematics. Learners need to have a combination of approaches, because conceptual and procedural knowledge may develop interactively.

Having a command of number facts empowers learners with confidence and independence in mathematics. In much the same way as reading liberates learners to access knowledge, mental mathematics liberates learners to access mathematics. (5 minutes)

Questions and wrap up (10 minutes)

MEMORIES! ALL THE FORGOTTEN FUNCTIONS ON THE CALCULATOR

Connie Skelton
Data Mind

Activity 1 How diameter of a can affects volume (height constant)

Investigate how changing the diameter of a can of height 11 cm will affect the volume. Use the formula $V = h \times d^2 \times \frac{1}{4\pi}$ to calculate the volumes for cans with the following diameters. (Hint: Find a value for $\frac{h}{4\pi}$ that can be entered into the calculator memory and re-used for each calculation.)

diameter, *d*	**volume, *V***
9	
18	
27	

Table 1: Calculate the volume for the diameters given

Activity 2 Recursion

The estimate for the annual population growth rates for 2013 to 2014 is 1,71 for males, 1,45 for females and 1,58 for the total population. If the total population was 54 001 953 in 2014, what will the population be in 2020? (From Statistical release P0302, Mid-year population estimates 2014

http://beta2.statssa.gov.za/publications/P0302/P03022014.pdf)

Year	**Males**	**Females**	**Total population**
2013			
2014	26 366 008	27 635 944	54 001 952
2015			
2016			
2017			
2018			
2019			
2020			

Activity 3 Constant ratios

We often have to scale assessments to different values. For example, if the test was actually out of 34 and it needs to be scaled to a mark out of 25, we would be dividing the mark by 34 and multiplying by 25. If we then store this value in one of the memories, we don't have to retype the ratio each time.

Scale these marks:

Test total: 34	**Cass total: 25**
32	
24	
17	

Table 2: Marks to be scaled using a constant ratio

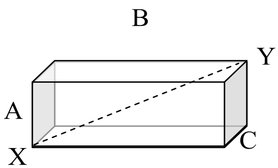
Activity 4 Evaluating functions

1 Store the value 0, 34 in the X memory. Find the value of

$f(x) = x^3 - 3x^2 + 4x - 2 =$ ______________

2 Store the value 0, 24262 in the X memory. Find the value of f (0,24262). ______

3



You can calculate the value of the diagonal XY by using the expression $\sqrt{A^2 + B^2 + C^2}$. Use the A, B and C memories to find the diagonal for:

a A = 3,2 m; B = 4,5 m and C = 2,1 m ___________________

b A = 6,4 m; B = 9,0 m and C = 4,2 m ___________________

4 Give memory variables D, E, and F the values of 8; 7; and 6 respectively. Evaluate DE^2F.

Calculating the date of Easter

Easter Sunday, from 326 A.D., is always one of the 35 dates from March 22 to April 25. It falls on the first Sunday after the first full moon on or after the autumnal (in the southern hemisphere) Equinox.

The first correct mathematical procedure was given by an anonymous American in 1876 in the scientific journal *Nature*. The procedure that we will use was described by Thomas H. O'Beirne in 1965 in his book *Puzzles and paradoxes*.

Activity 5 Calculating the date of Easter

We will use the year 2015 to calculate the date of Easter, but the process should work for most years since 326 A.D.

1 Divide 2015 by 19 to get a quotient and a remainder. Subtract the quotient. Multiply the remainder by 19. Store the answer in memory A. ((2015 ÷ 19) – 106) × 19

`2015P19=qnp106=O19=qJz`

(Answer: 1)

A lunar month is about 29.53 days long, and the solar year is about 365.24 days long. This gives 12.37 lunar months per year, which is not an integer. However, 235 lunar months are very close to 19 solar years, so we divide by 19.

2 Divide 2015 by 100. Store the quotient in memory B and the remainder in memory C.

`2015P100=qnp3a20=qJx`

(Answer: 20)

`2015P100=qnp20=O100=qJc`

(Ans: 15)

This gives the leap year rule for the Gregorian calendar. B increases by 1 for every century.

3 Divide B by 4 to get a quotient D and a remainder E.

`JxP4=qJj0=qJk`

(Answer: D = 5; E = 0)

D increases in century years that are leap years and E gives the number of century years that have not been leap years, e.g. 1900.

4 Complete the expression (8B + 13) ÷ 25 and store the quotient as F.

`(8OJx+13)P25=qnp23a25=qJl`

(Answer: F = 6)

F gives the month correction to the epact.

5 Divide (19A + B – D – F + 15) by 30 to get a quotient (which we ignore) and a remainder. Store the remainder as X.

`(19OJz+JxpJjpJl+15)P30=p1=O30= qJ)`

(Answer: X = 13)

X is equivalent to the epact (which is 23 – X or 53 – X whichever >0).

6 Divide (A + 11X) by 319. Store the quotient as Y.

`(Jz+11OJ))P319=0=qJn`

(Answer: Y = 0)

Y deals with an exceptional case related to the epact.

7 We will have used all but the constant memory M, so we will be re-using the memories from A onwards. Divide C by 4 to get a quotient A' and a remainder B'.

```
JcP4=nqnp3a4=qJz
JcP4=qnp3=O4=qJx
```

(Answer: A' = 3; B' = 3)

The beginning of the calculation for the day of week for the Easter full moon. It deals with the ordinary leap years.

8 Complete (2E + 2A' – B' – X + Y + 32) ÷ 7. Subtract the integer and store the remainder as C'.

```
(2OJk+2OJzpJxpJ)+Jn+32)P7=p3=O7=qJc
```

(Answer: C' = 1)

This algorithm calculates the date of the full Moon from the epact.

9 Complete (X – Y + C' + 90) ÷ 25. Store the quotient as D'.

```
(J)pJn+Jc+90)P25=qnp4a25=qJj
```

(Answer: D' = 4)

Gives the month of Easter.

10 Complete (X – Y + C + D + 19) ÷ 32. Store the remainder as E'.

```
(J)pJn+Jc+Jj+19)P32=qJk
```

Gives the day of the month for Easter Sunday.

(Answer: E' = 5)

Easter Sunday is the on the E'th day of the D'th month. So in 2015, Easter Sunday is on 5 April.

This is quite a challenging activity on the calculator because it does not have the Quotient and Mod functions that MsExcel has. It is useful to write the answers down for each step, because the memories have to be re-used, so checking will not be easy. Although it is easier to use MsExcel to do this calculation, it is also useful and challenging to do some multi-step activities on the calculator using all the memories.

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MAKING SENSE OF ADDITION AND SUBTRACTION PROBLEMS THROUGH STORIES AND DIAGRAMS

Hamsa Venkat, Nicky Roberts and Marie Weitz
Marang Centre, University of the Witwatersrand

This workshop is focused on supporting teachers to associate stories and diagrams with additive relation number sentences. This focus is driven by two motivations: first, there is evidence of poor learner performance on more complex additive relation problems (i.e. those involving missing addends and subtrahends, or missing start values; secondly, there is evidence that teaching presents 'rules' for transforming number sentences to calculate missing values without communicating the rationales for deriving these rules.

TARGET AUDIENCE: Foundation Phase

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 30

Evidence points to difficulties for learners in dealing with additive relations word problems (DBE, 2012). Allied with this, there is also evidence that teachers have difficulties in supporting learners to solve missing number additive relations problems, and particularly so, in missing addend/subtrahend and missing start value problems (Venkat & Askew, 2012). This suggests that it would be useful to investigate ways of supporting teachers to help learners to solve a variety of addition and subtraction problems. The focus on stories and diagrams builds on an approach developed within Nicky Roberts' doctoral study (Roberts, in process), where substantial gains in learner performance were seen in the context of teaching that placed emphasis on supporting children to develop narratives – made up of stories and diagrams – that could be associated with specific number sentences.

In this workshop, we will focus on developing a range of story situations and diagrams linked to a wide range of additive relation statements, that can help learners to make sense of these situations and solve missing number problems.

Activity 1: 25 minutes

The body of the workshop will begin with an activity in which participants develop stories to go with key additive relation statements. These statements will include 'result unknown' situations like: 5 + 7 = __ and 22 – 6 = __, but will expand to include the kinds of statements that research tells us that children find harder to solve, such as those shown below:

__ = 6 + 5	9 – 7 = __	6 + __ = 14
__ - 8 = 10	14 = __ + 12	4 = 15 - __

The aim in this section is to encourage the development of a range of stories that can be associated with particular number relations. For example, for 14 = __ + 12:

> 'I have R14 now that Thandeka has paid the R12 that she owed me. How much money did I have before she paid me back?'

Activity 2: 30 mins

In the second activity, we share descriptions of key categories of additive relations situations and variations within these situations. Participants will then sort their stories on the basis of these categories. We will also focus on developing stories to go with any categories that are under-represented in the stories offered by teachers.

Change increase situations	Change decrease situations	Collection situations	Compare situations

Activity 3: 25 minutes

Our focus in this activity is on sharing key representations that are described in the literature as particularly useful for additive relation problem-solving – number tracks/lines and part-part-whole diagrams. For example, the problem: 8 - __ = 3 can be associated with the following diagrams:

<table>
<tr><td>3</td><td>?</td></tr>
<tr><td colspan="2">8</td></tr>
</table>

Activity 4: 30 mins

Teachers will have the opportunity to develop and share stories and diagrams that they can use in classrooms in their work on additive relations problems.

REFERENCES

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FRACTIONS AND FRACTION WALLS

Shereen Corker & Melissa Mentoor
Capricorn Primary School

Fractions are a 'hard to teach' topic in primary mathematics - filled with misconceptions and difficulties with the symbolic notion. Do you know that Foundation Phase learners are expected to write '1 half' and not − or 3 quarters and not − ? And if so, why? Joint this workshop to learn how to make fraction walls and put them to good effect for your lessons. Share and reflect on real fractions lessons tried with Foundation learners, and make your own resources to use in your classroom.

Target audience: Foundation Phase

Duration: 2 hours

Maximum number of participants: 30

Motivation for workshop:

In CAPS fractions are experienced right from Grade 1 with the grouping and sharing problems. They are formally introduced as 'sharing leading to fractions' through problem solving involving practical problems. Yet many learners (and teachers) have very shaky fraction concepts. Do you know how to divide six by a half? Or how many halves make six? Do you know how to make your own fraction wall, situate it in meaningful contexts and use it in your lessons to ask effective questions?

In this workshop teachers will work on fraction tasks together with teachers who have tried these fractions ideas in their own township school context. Participants will be supported to make two fraction walls for teaching fractions in Foundation Phase classrooms which they will be able to take home with them. These ideas can then be used with the children in their classes

Content:

The workshop will begin with a discussion on some of the common misconceptions learners have about fractions. This will be followed by a brief discussion of what learners are expected to know according to CAPS.

Shereen will then guide the participants through a lesson she has developed in which learners build their own fraction wall. The lesson uses the context of baking bread to help learners make meaning of fractions. Learners make their own fraction walls and use these as a tool to answer questions about fractions.

Shereen will also share some of the lessons that she has learnt from teaching this lesson and some of the benefits she has seen for her learners.

The fraction wall built by the learners in Shereen's lesson relies on the context to help learners make meaning of fractions. As they become more comfortable with fractions, learners will be able to work with fractions without referring directly to a context. In the second part of the workshop Melissa will help participants make a fraction wall that is not context specific and can therefore be used in a variety of contexts.

Throughout the workshop participants will be exposed to the types of questions that can be asked and the concepts that can be developed by using fraction walls.

The workshop will end with participants playing a number of games which are designed to help learners consolidate their understanding of fractions.

TEACHING SOMEONE ELSE'S LESSON: ADOPT OR ADAPT

Marie Joubert, Ingrid Mostert
AIMSSEC, AIMSSEC

In FaSMEd, a design research project which aims to develop mathematics lessons to support the use of formative assessment in the mathematics classroom, teachers are asked to teach lessons designed by someone else, namely by the Mathematics Assessment Project (MAP) at the University of Nottingham in the UK. The lessons are then revised in the light of the South African teachers' experiences.

The thought of teaching a lesson designed by a team of mathematics education researchers, with engaging activities and a detailed lesson plan describing what to do at each stage of the lesson, can be very appealing. However, as the South African teachers discovered, and as is generally accepted, teachers find it challenging to teach someone else's lesson, no matter how well the lesson is designed. This workshop will explore some of these challenges and look at ways in which they can be addressed.

In the workshop participants will be divided into small groups to work through a model lesson for Senior Phase on exponents that the South African teachers were asked to teach. After attempting the lesson's main activity participants will work through the detailed lesson plan and other information provided. They will then have an opportunity to discuss which parts of the lesson they would adopt and which parts they would adapt. These ideas will feed into the discussion about the ways in which the teachers involved in the research project adapted the lessons for their classes. The session will end by drawing together the lessons learnt by the teachers about how to prepare.

MATH LAB PRESENTATION

Maduke MM
Aryans School Kolkata India

NIIT NGURU math lab was launched in the Aryans School Kolkata India to make mathematics easier and fun to learn for school students

Mathematics help children make sense of the world around them and find meaning in the physical world. Through mathematics, children learn to understand their world in terms of numbers and shapes. They learn to reason, to connect ideas, and to think logically, and independently. Mathematics is more than the rules and operations we learned in school. It is about connections and seeing relationships in everything we do.

Mathematics has always been an important subject to understand the physical phenomena around us. So mathematics as a tool is indispensable in our daily lives. This means that children learn best when they are interested and even excited about what they are doing. So give your children many opportunities to see and hear different things and move about and play with things they can touch and feel.

Objectives of Math Labs

The students/Learners will be able to:

- Explore, reflect and interpret ideas presented through a variety of media.
- Display intellectual curiosity and initiative to acquire process and interpret information.
- Use variety of strategies and perspectives in solving problems individually and collaboratively.
- Frame and test hypothesis; and identify, describe, formulate and reformulate problems.
- Describe and interpret difficult point of views, and distinguish facts and opinions.

Math Lab Manipulative

There are different kinds of manipulative in the math lab that can be used to enhance teaching and learning. Examples are: Interlocking cubes, frameworks, Pentablocks, plastic mirror, Drawing templates, Dice, Playing cards, Dummy currency notes, Rubber bands, Tangrams, Zome struts and Nodes, Transparency Geoboards etc.

Programmes

The Lab consisted of three main programmes, including:

1. Sketchpad (GSP 5)
2. Shell

3. Manipulatives

Target

The programme has been designed to cater learners of Grade R-9. The learners supposed to visit the lab once per week. The timetable ought to be drawn to help in the smooth running or to regulate the visitation.

MULTIPLICATION OF WHOLE NUMBERS BY POWERS OF 10 AND SOME APPLICATIONS THEREOF

Phatisizwe T. Mahlabela
KZN Department of Education

INTRODUCTION

Participants will do the activities under the content below. I will then facilitate the discussion of their responses. We will then together explore applications of these skills in performing calculations. We will also identify topics within the curriculum that require the skills learnt from the activities.

I am focusing on this topic because teachers strongly assert that learners find very easy to understand. Teachers often agree that learners find the topic easy because after working out a few calculations, a pattern that is easy to observe emerges. Once learners pick up the pattern, teachers then assume that they do understand. I often candidly ask them questions such as: Do they really understand? What does it mean to understand?

The activities that will be done during the session aim at ensuring that Intermediate Phase learners really do have an understanding of the multiplication by powers of 10. Van de Walle (2007, p. 25) refers to understanding as “a measure of the quality and quantity of connections that a new idea has with existing ideas”. Skemp (1976) also distinguishes between instrumental and relational understanding. He refers to the later as knowing “both what to do and why” (p. 2). He further maintains that people with relational understanding are characterized by a rich interconnected web of ideas. The activities intend to do so by taking into consideration knowledge of multiplication learners have from the Foundation Phase.

The activities also aim at ensuring that learners are proficient in this multiplication. Therefore, conceptual understanding, procedural fluency and adaptive reasoning will also be taken into consideration. By the end of the activities, learners should have an “integrated and functional grasp” of multiplication by powers of 10 “resulting in the ability to see connections between ideas and the big picture of procedures” (Kilpatrick, 2001).

Lastly, the activities will not just engage learners in inductive and deductive reasoning, but learners will also be engaged in transformational reasoning. That is, learners will not be drawing conclusions using observed patterns or known laws of logic; they will also be assisted to “mentally visualizing actions and the results of actions” (Rubinstein-Avila and McGraw, p. 5).

CONTENT

Activity 1

In the flow diagram below, some outputs are given.

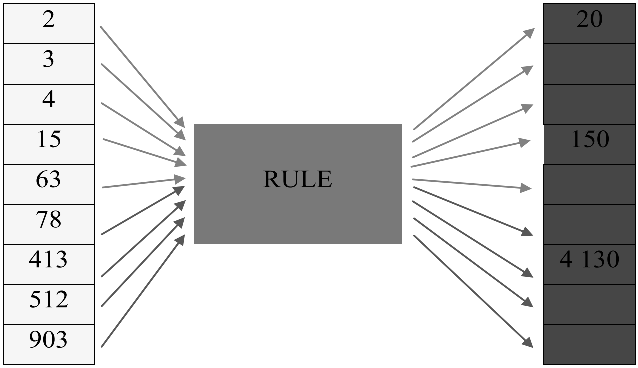

Fig. 1: Flow diagram

1.1 .Write down the rule for this flow diagram

1.2 Write an explanation that justifies your rule

1.3 Fill in the missing outputs

1.4 Now complete the table below using the first, fourth and seventh inputs and their outputs

	Thousands	Hundreds	Tens	Units
Input 1				
Output 1				
Input 4				
Output 4				
Input 9				
Output 9				

Table 1: Flow Diagram inputs and outputs

1.5 Does the table confirm the answers you provided above?

1.6 Does it reveal anything new? If yes, what?

Activity 2

In the flow diagram below, some outputs are given.

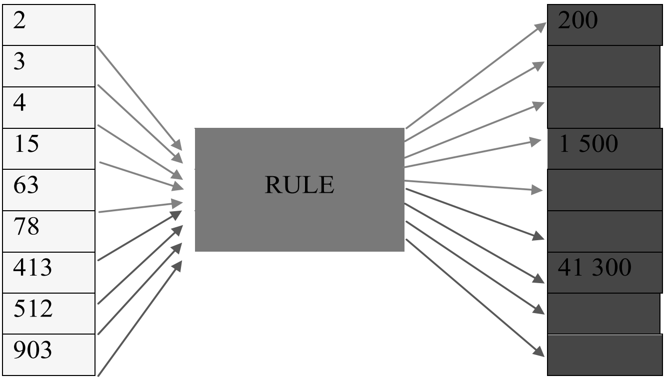

Fig. 2: Flow diagram

2.1 Write down the rule for this flow diagram

2.2 Write an explanation that justifies your rule

2.3 Fill in the missing outputs

Now complete the table below using the first, fourth and seventh inputs and their outputs

	Thousands	Hundreds	Tens	Units
Input 1				
Output 1				
Input 4				
Output 4				
Input 9				
Output 9				

Table 2: Flow Diagram inputs and outputs

2.4 Does the table confirm the answers you provided above?

2.5 Does it reveal anything new? If yes, what?

Activity 3

3.1 Draw and complete your own flow diagram for the rule: × 1 000.

3.2 Write short notes on your observations

3.3 What conclusion have you arrived at?

Note that knowledge of multiplication by 10 could be used to do multiplication by 5, 20, 11, 9, etc. Similarly, knowledge of multiplication by 100 could also be used to multiply by 50, 200, etc. This requires applications of skills such as halving numbers, doubling numbers and writing numbers as a sum or difference of 10 or 100 and some number. For example: 5 = half of 10, so multiplying by 5 requires multiplication by 10 and then halving the product. Apply this knowledge in the next activity.

Activity 4

Apply the skills learnt above to do the following calculations:

4.1 73×5

4.2 132×20

4.3 $2\,315 \times 15$

4.4 87×11

4.5 734×9

4.6 13×50

4.7 42×200

4.8 295×99

CONCLUSION

The assumption is that learners know that multiplication by a whole number is repeated addition. From doing the activities, learners should ultimately see that the above multiplications result in place value change. They should be able to relate this to repeated addition, a number added 10, 100 or 1 000 times.

In the Foundation Phase they do a lot of work on bonds, doubling and halving. These skills could be used to do activity 4. Probable solutions are:

4.1 73×5 = half of 73×10

4.2 132×20 = Double 132×10

4.3 $2\,315 \times 15$ = half of $2\,315 \times 10 + 2\,315 \times 10$

4.4 $87 \times 11 = 87 \times 10 + 87$

4.5 $734 \times 9 = 734 \times 10 - 734$

4.6 13×50 = half of 13×100

4.7 42×200 = Double 42×100

4.8 $295 \times 99 = 295 \times 100 - 295$

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THE DEVELOPMENT OF MATHEMATICAL CONCEPTS FROM GET THROUGH FET IN THE TEACHING AND LEARNING OF MATHEMATICS WITH SPECIAL REFERENCE TO PATTERNS, FUNCTIONS AND ALGEBRA.

Nomathamsanqa Mahlobo & Themba Ndaba
Centre for Advancement of Science and Mathematics Education (CASME).
KwaZulu Natal

TARGET AUDIENCE: FET Phase

DURATION: 1 hour

MAXIMUM NUMBER OF PARTICIPANTS: 30

MOTIVATION

The National Research Council, Mathematics Science Education Board, defines, "Mathematics as a science of pattern and order." An awareness of the patterns found on number tables is important for developing a deeper understanding of number relationships. The ability to find and describe patterns found in numbers, charts, operations, geometric figures, and graphs are important for the development of a deep understanding of mathematics in general and of algebra in particular. Patterns are an effective way to encourage students to explore variables and functions, which are important ideas in the study of algebra. In this workshop we will explore the development of patterns from General Education and Training (GET) to Further Education and Training (FET). The GET teachers will explore different skills on how to teach patterns in preparation for FET; and the FET will also have an understanding on how to close the learners' gaps in patterns to link GET patterns to FET.

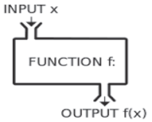
The aims of the workshop:

- To share fun and easy strategies with the teachers in teaching Pattern, functions and Algebra.
- To expose GET and FET mathematics educators to what they need to share with each other when teaching mathematics.
- To alleviate fears of GET mathematics teachers that they might have about FET mathematics.
- To develop teachers' love of the subject that could be transferred to learners.

Patterns are repetitive sequences and can be found in nature, shapes, events, sets of numbers and almost everywhere you care to look.

For example, seeds in a sunflower, snowflakes, geometric designs on quilts or tiles, and the number sequence. A Pattern constitutes a set of numbers or objects in which all the members are related with each other by a specific rule.

In **mathematics**, a **function** is a relation between a set of inputs and a set of permissible outputs with the property that each input is related to exactly one output. An example is the **function** that relates each real number x to its square x^2.



Algebra is a branch of **mathematics** that uses **mathematical** statements to describe relationships between things that vary over time. These variables include things like the relationship between supply of an object and its price.

CONTENT

The participants will be introduced to Patterns, Functions and Algebra; and be guided on how to do the hands on activities. The development of the concepts will be gradually introduced to participants with activities to unfold the concepts in different phase levels. Clear explanation by the teachers will result to better understanding of the concepts which in turn will boost the love for mathematics by the learners. The love of the subject will improve learner attendance at school. The strategy of linking the concepts through phases will assist the educators in bridging the learners' knowledge gaps that seem to exist at FET level.

CONCLUSION

This workshop aims at addressing the concern of the FET mathematics educators about learners who are alleged to lack content knowledge background of some concepts in mathematics. It will also encourage team teaching, consultation and networking among teachers at different grades and phases. It will open space and rapport for respect among GET and FET mathematics educators.

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THE USE OF RESOURCES AS A STRATEGY IN THE TEACHING OF MATHS IN GRADE 1

Machaka Matebo

The current situation regarding the performance of learners in Mathematics across the country indicates that Foundation phase learners are not performing as expected. Studies conducted by the Department of Basic Education through systemic evaluation and ANA confirm this. This paper emanates from the fact that teachers are not able to choose and use relevant resources to support effective mathematics teaching and learning. In primary schools, teaching and learning mathematics is easier when children play and sing.

Types of mathematical resources to introduce numbers (after this presentation, chance will be given for clarity seeking questions).

1. Manipulative (improvised material/ resources)

- These are objects designed to represent mathematical ideas that are abstract- Moyer, 2001:176.

Example 1: Train and its numbered coaches. Participants / learners will be shown how it works.

Example 2: Matching numbers using participants themselves.

2. Commercial resources

- Expensive but useful resources such as stop watches, number balance, scale, 100 beads learner abacus, cubes, dice, high chart, clocks.

Example 1:100beads learner abacus. The presenter explains how they work.

Example 2: Dice, The presenter explains how they work.

Lastly, participants are taken for a gallery walk where different types of resources are displayed. Explanations are given for any of the displayed resources.

THE USES OF PRIME FACTORS

Lerato Mathenjwa
TEACH South Africa Ongoing Support and Training Manager.

The purpose of this session is to share ideas on Factorizing Big Numbers Using the Fishbone Method. Factorisation of numbers starts in grade 4 in the Curriculum Assessment Policy Statement CAPS. The manner in which the concept is staggered in the document will be discussed. Factorisation of whole numbers ends at grade 9 focusing mainly on the use of prime factorization for finding the HCF (Highest Common Factor) and the LCM (Lowest Common Multiple).The use of factorisation continues in higher grades in many sections such as fractions, algebra, functions, quadratic equations etc. The necessity to master factorization at the lower grades therefore cannot be adequately emphasized. In the workshop, the fishbone method will be used to find prime factors of a number. Consequently, it will be shown how the HCF of two or more numbers can be identified.

TARGET AUDIENCE: Intermediate and Senior Phase

DURATION: 1 hour

NUMBER: Not more than 50.

MOTIVATION

Most learners get to Grade 8, 9 and 10 still lacking the basics of finding factors of numbers results in their inability to access higher concepts that requires them to use this concept in other sections of the mathematics curriculum.

Finding factors 2 and 3 digit numbers is done from grade 5 and the finding the HCF and LCM start in grade 7. It is at grade 9 where they use prime factorization to find the HCF and the LCM. Should the learners be taught to apply the use of prime factorization in other sections of mathematics such as addition and subtraction of fractions, they would deepen their understanding even more. The activities shown below will be used to encourage the use of prime factorization to apply in finding the HCF and LCM for the purpose of applying in addition and subtraction of fractions.

The table below shows the stated outcomes for factors in Grade 5 to Grade 9.

DEFINITIONS

Prime Number

A **base** is a method of expressing numbers using place value which is the value of a digit based on its specific position. With the use of columns and a number pattern that will help in the understanding of the concept of bases can be formed.

Firstly the column should always be read from the right to left; hence the first furthest

Grade 5	Grade 6	Grade 7	Grade 8	Grade 9
Number range for multiples and factors	**Number range for multiples and factors**	**Number range for multiples and factors**	**Number range for multiples and factors**	**Number range for multiples and factors**
Factors of 2-digit whole numbers to at least 100	Factors of 2-digit and 3-digit whole numbers	Revise the following done in Grade 6:factors of 2-digit and 3-digit whole numbers		
Prime factors of numbers to at least 100	prime factors of numbers to at least 100 • List prime factors of numbers to at least3-digit whole numbers	Prime factors of numbers to at least 3-digit whole numbers		
		Find the LCM and HCF of numbers to at least 3-digit whole numbers, **by inspection or factorisation**	Revise: LCM and HCF of numbers to at least 3-digit whole numbers, **by inspection or factorisation**	
				Use prime factorisation of numbers to find LCM and HCF

What is a prime number? – a prime number can be divided by one number and itself and is greater than 1.

- List 5 prime numbers and 5 numbers which are not prime numbers.

Prime Numbers

2, 3, 5, 7, 11

Not Prime Numbers

1, 4, 6, 8, 9

What are a **factor**?- factors are numbers that can be multiplied together to give another number.

2 x 3 = 6

Factor Factor

- Give an example of two factors that give a number.
- Are the factors of these numbers prime numbers? Write the prime factors in exponential notation.

Using the fishbone method to find the **factors** of the following numbers:

Please note that one is included for the sake of not leaving it out. Emphasis should be made that it not a prime number.

225	
5	225
5	45
3	9
3	3
	1

1 048	
2	1 048
2	524
2	262
131	131
	1

45 628	
2	45 628
2	22 814
11	11 407
17	1 037
61	61
	1

So, the other factors can be found by making different combinations:

Factors for 225 are:	Factors for 1048 are:
$\underbrace{1}_{1} \times \underbrace{3 \times 3 \times 5 \times 5}_{225}$ $\underbrace{1 \times 3}_{3} \times \underbrace{3 \times 5 \times 5}_{75}$ $\underbrace{1 \times 3 \times 3}_{9} \times \underbrace{5 \times 5}_{25}$ $\underbrace{1 \times 5 \times 3}_{15} \times \underbrace{3 \times 5}_{15}$	$\underbrace{1}_{1} \times \underbrace{2 \times 2 \times 2 \times 131}_{1048}$ $\underbrace{1 \times 2}_{2} \times \underbrace{2 \times 2 \times 131}_{524}$ $\underbrace{1 \times 2 \times 2}_{4} \times \underbrace{2 \times 131}_{262}$ $\underbrace{1 \times 2 \times 2 \times 2}_{8} \times \underbrace{131}_{131}$
The factors are: 1, 3, 9, 15, 25, 75, 225.	The factors are: 1, 2, 4, 8, 131, 262, 524, 1048.
Prime factors are: 5 x 5 x 3 x 3 = 225	Prime factors are: 131 X 2 X 2 X 2 = 1048
15 is repeated for the sake of the young ones. You can also mention that 225 is a **SQUARE NUMBER** and 2 of its factors are the same.	

Finding the factors of a number with 5 digits is done for purposes of expanded opportunities. It is not compulsory for all learners to do it.

Factors for 45 628 are:	
$\underbrace{1}_{1} \times \underbrace{2 \times 2 \times 11 \times 17 \times 61}_{45\,628}$ $\underbrace{1 \times 2}_{2} \times \underbrace{2 \times 11 \times 17 \times 61}_{22\,814}$ $\underbrace{1 \times 2 \times 2}_{} \times \underbrace{11 \times 17 \times 61}_{}$	$\underbrace{1 \times 2 \times 2 \times 17}_{68} \times \underbrace{11 \times 61}_{671}$ $\underbrace{1 \times 2 \times 11 \times 17}_{374} \times \underbrace{2 \times 61}_{122}$ $\underbrace{1 \times 17 \times 11}_{} \times \underbrace{2 \times 2 \times 61}_{}$

<table>
<tr><td>4 X 11 407
1 X 2 X 2 X 11 X 17 X 61
44 X 1 037
1 X 2 X 2 X 11 X 17 X 61
748 X 61</td><td>187 X 244
1 X 2 X 11 X 17 X 2 X 61
22 x 2074</td></tr>
<tr><td colspan="2">The factors are: 1, 2, 4, 22, 44, 61, 68, 122, 187, 244, 374, 671, 748, 1037, 2074, 11407, 22814, 45628.</td></tr>
<tr><td colspan="2">Prime factors are: 2 x 2 x 11 x 17 x 61 = 45 628</td></tr>
<tr><td colspan="2"></td></tr>
</table>

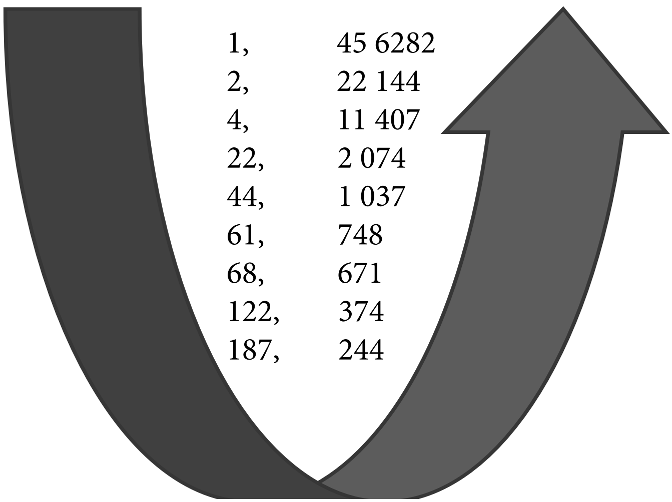
Learners need to mark the PAIRS to ensure that all factors are listed. There must always be a pair even if the other pair has **same numbers**.

NB: REMEMBER to explain PRIME FACTORS (Using a Mathematics DICTIONARY). These materials can be used as posters to reinforce their understanding until the concept kids are familiar to them.
The pairing of the factors is shown below with the arrow indicating their ascending order.to check that they are all listed:

Expanded Opportunity: Let them find if a numbers is SQUARE NUMBERS. (If two of its factors are the same)

Use the ladder method to find the LCM.

Find the LCM OF 12, 18 and 30. The smallest common prime factor is used to divide into ALL the given numbers. The process is continued until ALL numbers in the yellow column are **prime numbers**. These numbers are multiplied to give the LCM.



	12	18	30
2	6	9	15
3	2	3	5

LCM is 2.2.3.3.5 = 180

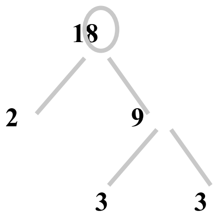
HCF is 2.3 = 6

The HCF is all the numbers on the vertical column and LCM all the numbers highlighted.

1. Find the LCM of the following numbers:
   a) 16, 48
   b) 12, 30
   c) 24, 36
   d) 14, 20, 30, 21
2. Find the HCF of the following numbers:
   a) 16, 48
   b) 12, 30
   c) 24, 36
   d) 14, 20, 30, 21

The tree diagram method can also be used to find prime factors.

Factors of 18:



So the prime factors of 18 are **2 . 3 . 3**

Prime Factorization

"Prime Factorization" is finding **which prime numbers** multiply together to make the original number. From the example above:

The prime factorization of 18 is 2. 3. 3

This can be written in **Exponential Form** or **Exponential Notation.**

$$\mathbf{18 = 2 \times 3^2}$$

Adding and Subtracting Fractions

Add the following fraction:

$$\frac{5}{24} + \frac{7}{36}$$

$$\underbrace{2.\ 2.\ 3.\ 2.}_{24}\quad \underbrace{3}_{3}\qquad\qquad \underbrace{2.}_{\mathbf{2}}\quad \underbrace{2.3.2.3}_{36}$$

Step 1: Find the LCM of 24 and 36

	24	36
2	12	18
2	6	9
3	2	3

LCM is 2. 2. 3. 2. 3 = 72

$$\frac{5 \times 3}{24 \times 3} + \frac{7 \times 2}{36 \times 2}$$

$$\frac{15}{72} + \frac{14}{72}$$

$$\frac{19}{72} \text{ ¤}$$

3. Add or Subtract the fractions given below:

a) $\frac{4}{15} + \frac{3}{48}$

b) $\frac{11}{21} - \frac{23}{63}$

c) $\frac{3}{58} + \frac{17}{29}$

d) $\frac{2}{24} + \frac{5}{48} - \frac{8}{72}$

CONCLUSION

Prime factors are very important as they are like basic building blocks for numbers. They can also be used to find factors of a large number as demonstrated earlier. They are also used in finding the HCF and the LCM.

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https://www.youtube.com/watch?v=3W8SeYgZcMohttp://www.mathsisfun.com/prime-factorization.html

THE DETERMINATION OF COGNITIVE LEVEL VERSUS ALLOCATION OF MARKS – SENIOR PHASE EUCLIDEAN GEOMETRY

Themba Leslie Ndaba & Nomathamsanqa Mahlobo
Centre for Advancement of Science and Mathematics Education (CASME)
KwaZulu Natal

The Annual National Assessment (ANA) (2014) results have shown that there is poor learner performance especially in Senior Phase mathematics. According to ANA report the average performance of Grade 9 learners in 2012; 2013 and 2014 was; 13%; 14% and 11% respectively. du Plooy (2014) states that, "if teaching, learning and assessment remain on the factual recall and operational efficiency levels, progression to the more advanced levels of mathematical activity is severely restricted." This workshop will address the determination of the cognitive levels versus assessment which is perceived as the main course for low performance among learners in this phase.

CONTENT

The Curriculum and Assessment Policy Statement (CAPS) (DBE, 2012, p. 296) describes four cognitive levels at which assessment have to be conducted. These levels are: knowledge (25%), routine procedures (45%), complex procedures (20%) and problem solving (10%). These cognitive levels used in CAPS, correspond directly with the Subject Assessment Guidelines of the 2007 TIMSS taxonomy of categories of mathematical demand (Stols, 2013, p. 13). The cognitive categories used in this international test warrant careful interpretation for productive application in the classroom assessment.

The participant in this workshop will be carefully introduced to the cognitive levels as spelled out in the CAPS document. Some exercises will be performed:

(a) To prepare suggested solutions,

(b) Marks allocation will be determined, and

(c) The cognitive levels will then be determined based on the strength of the exercise.

CONCLUSION

There will be some engagement of discussions for the participant to justify the allocation of ticks and the assignment of the cognitive level. Participant should then convince themselves that cognitive levels should not be judged just by mere looking at the mathematical problem. Levels need to be tried and tested. The balanced question papers during internal assessment at schools should be ascertained at all levels. This endeavor might improve the performance in the ANA results.

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SUBITISING: A MEANS TO HELP SHIFT LEARNERS FROM COUNTING IN ONES TO CALCULATING?

Ursula Röntsch
The Grove Primary

We know that many of our Foundation Phase learners continue to make use of inefficient counting in ones strategies (Schollar, 2008, and Hoadley 2012). How can we support young children in their shift from counting in ones, to being flexible in their calculations strategies, and building on and making use of known facts? In this session participants work on several tasks designed to make use of flexible calculation strategies (and not routine written procedures). The importance of subitising as facilitating actions on groups, and not ones is in focus.

INTRODUCTION

This 1 hour workshop offers a hands-on experience of games and tasks related to supporting early number development for Foundation Phase children.

CONTENT

Subitising is the ability to visualize and know the quantity of an arrangement of a small number of items, without counting in ones. By arranging objects into groups and then visualizing these patterns, children can be supported to quantify collections (without counting each object in the collection). In this session where work within modes of representation to interpret how learners are approaching problems, and shift them to using more group-wise representations and actions. This is intended to work with what they are able to do (count in ones) towards more efficient and flexible calculation strategies.

The presenter is an experienced Foundation phase teacher who couples years of teaching in a high attaining suburban context with practical experience in afternoon maths clubs in a township school.

CONCLUSION

In this session participants will engage in tasks and games building on the subitising. These tasks develop the teachers own mathematical thinking about basic arithmetic processes, and provide stimulus for suitable tasks with Foundation Phase learners.

MEASURING TIME AND ALL THINGS DIVINE

Ian Schleckter
Free State Department of Education Motheo District Office

The workshop is aimed at Intermediate Phase teachers as audience in discovering a different approach in the teaching of the concept of measurement.

THE NEED FOR SHARING GOOD PRACTICES

The Ministerial Report on the Annual National Assessment of 2014 clearly highlights the following areas as problematic since no improvement was shown since 2013:

- Calculation of time intervals;
- Conversion of units in measurement;
- Problem solving.

Research findings about the relationship between opportunity to learn and student achievement have important implications for teachers. In particular, it seems imperative to allocate sufficient time for mathematics instruction at every level. Short class periods in mathematics, instituted for whatever practical or philosophical reason, should be seriously questioned. Of special concern are the 30–35 minute class periods for mathematics being implemented in some primary schools.

Education standards not only specify the content a student needs to know, but also the level of understanding expected for the mastery of the content of that standard. In addition to needing to reflect the content of the standards (CAPS), assessments need items that match, where possible, the level of understanding required to assure that what is being assessed reflects what is being taught. A major challenge to teachers, is to "cover the curriculum" in a specific time, using "Pace Setters". Unfortunately, it becomes practice to teach each concept in an isolated partition without allowing learners to see the world as an integrated system where all concepts of mathematics are evident to a lesser or greater extent.

Integrating a variety of mathematical topics while teaching Measurement, could be of great help to teachers in their effort to "cover the curriculum" and to follow a more inductive approach in their instructional methods.

Integrating Content through Activities.

The content introduces us to the phenomena of time and length. Teachers will engage in fun-filled practical activities that can be brought into the classroom, allowing for scaffolding learners' skills and ultimately deepening the quality of teaching and learning.

ACTIVITIES	
Activity 1	When we think of Time.
Activity 2	How many days, how many hours?
Activity 3	Geometry and Time
Activity 4	Fractions in Time
Activity 5	Numbers in the Clock, Ticking
Activity 6	Time Zones
Activity 7	From seconds to metres
Activity 8	Converting Units

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PROBABILITY IN THE INTERMEDIATE PHASE

Henda Smith & Lilian Mahlangu
Sand du Plessis and Botlehadi Primary Schools.
Free State Department of Education.

Probability is the measure of the likeliness that an event will occur. An event is a collection of possible outcomes of an experiment. It occurs as a result of and experiment if it contains the actual outcome of that experiment. The measure of certainty assigned to an event is called the probability of an event.

AIMS AND ACTIVITIES

The aim of this presentation is to provide teachers with alternative activities and exercises to present the topic of Probability to their learners apart from the suggested 'tossing a coin', 'throwing a die' and 'spinning a spinner'.

Learners have come to associate Probability with the abovementioned activities, because they are suggested in the CAPS document for gr 4, gr 5 and gr 6 and therefor also contained in almost every text book we have had the pleasure of using in our class rooms. Once the lesson is introduced in gr 6, they already anticipate that they are going to toss a coin or roll a die. To surprise them with a different activity is unexpected by them – even though the aim of our lesson is once again: Probability.

Activities to be done by or explained to groups are the following:

- Marbles in a bag
- Buttons and blindfold
- Tossing drawing pins

Tally tables, fractions, percentages and P (3) notations will be used to describe the possible outcomes. The rationale behind these group work activities will be to provide teachers with first-hand experience on how to present the activities to their learners. Groups should discuss their outcomes with each other, explain their activity to the other groups and evaluate their group's activity mentioning possible advantages or disadvantages.

Teachers could also prepare their learners before presenting an activity by asking interesting and relative questions regarding Probability. For example:

- Every child in your school buys a ticket in the raffle at the bazaar. What are your chances of winning the prize?

- If the Springboks have a 0,6 chance of winning the Rugby World Cup, what is their chance of NOT winning the Cup?

Teachers are often restricted by time. Our intention is normally to make teaching more interesting and hands-on, but when our time is consumed by the many responsibilities we have we revert back to easily accessible activities for the teaching of, among others, Probability. With this presentation we shall endeavor to provide teachers with new, fun activities for the teaching thereof.

TERMINOLOGY IN PROBABILITY

TERM USED	DEFINITION
Frequency	How many times and event takes place
Random	Happening without a chosen method/ decision
Non-random	Happening by chosen method/conscious decision
Probability	*Probability is the* measure *of the likeliness that an event will occur.*
Experiment	A situation involving a chance or probability that leads to results or outcomes
Event	A collection of possible outcomes of an experiment. Occurs as a result of an experiment if it contains the actual outcome of that experiment. The measure of certainty assigned to an event is called the probability of an event.

PROBABILITY SCALE

Impossible poor chance even chance good chance certain

0 0,25 0,5 0,75 1

0% 25% 50% 75% 100%

1 – – – 1

not likely likely

MENTAL MATHS: (10 minutes)

Complete the table below:

Common fraction	Decimal fraction	Percentage
–		
	0,25	
		75 %
–		
	0,6	
		62– %

	0,375	
		125 %
	0,40	
–		

[1] ^"Probability. Webster's Revised Unabridged Dictionary". G & C Merriam, 1913.

[2] www.cut-the-knot.org/Probability/Dictionar.shtml

TEACHING VOCABULARY IN MATHEMATICS CLASSROOMS

Lindiwe Tshuma
Institution: African Institute for Mathematical Sciences Schools Enrichment Centre (AIMSSEC)

TARGET AUDIENCE: Intermediate and senior phase

DURATION: 1 HOUR

MAXIMUM NUMBER OF PARTICIPANTS: 30

MOTIVATION

During the workshop, different strategies for teaching vocabulary in primary school mathematics classrooms will be discussed.

Description of content of workshop

30 minutes practical activities, 20 minutes PowerPoint presentation, 10 minutes question and answer session.

Activities and worksheets to be used during the workshop: Please refer to annexure attached as pdf document.

The majority of teachers and learners in South African schools are not first language speakers of English, and according to Tshabalala (2012:22), "... many teachers and learners are not fluent in English". However, from grade 4 up to tertiary level, the language of teaching and learning in the majority of schools in all subjects, including mathematics is English, therefore teaching and learning as well as assessment are compromised by the poor mastery of the English language. If learners do not master basic as well as specialized vocabulary used in mathematics classrooms performance in mathematics may be affected negatively. Gunning 2003, believes that: "... when effective vocabulary instruction is built into a mathematics curriculum, learner achievement is likely to improve". Vacca et al. (2009) reaffirm this notion by stating that "... direct teaching of vocabulary builds essential prerequisite knowledge". A mathematics classroom may be one of the few places where learners engage with mathematics vocabulary therefore teachers must create opportunities for mathematical vocabulary learning. Some strategies that can be useful in teaching vocabulary in mathematics classrooms include creative writing, finding word origins, crossword puzzles, word searches, creating communicative word walls and the use of contrasting meanings.

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Activity 1: Quiz

Instruction: Choose words from the word bank below to complete the following sentences:

1. The state of being equal -..........
2. An argument that establishes a result -..........
3. The state of not being equal -..........
4. A rule that gives a single output for a given input -..........
5. To suggest a conclusion based upon observation -..........
6. Exactly the same shape and size -..........
7. A quantity that can take different values -..........
8. When the order of an operation does not matter :**(a + b = b + a)** - ....
9. A function whose graph is a straight line -..........
10. The number 5 in **5x** or the 'a' in $\mathbf{ax^2}$ -..........
11. An operation that takes you back where you started -..........
12. A measure of steepness -..........
13. The measure of how likely an event is in increments from 0% to 100% -...
14. A different arrangement of the same set of objects -..........

WORD BANK:

permutation, system, probability, coefficient, congruence, commutative, slope, inverse, equality, proof, linear function, inference, function, variable, inequality

Activity 2: Word Search - Multiplication table of 2

Instruction: Calculate the multiplication sums and shade the answers on the word search below:

A	M	C	N	R	H	J	N	L	T	E	S
D	I	O	E	S	L	U	E	W	D	I	J
N	O	W	T	Y	T	N	E	W	T	G	P
D	E	O	W	T	D	N	T	O	R	H	F
T	Z	E	F	W	T	K	H	K	U	T	N
U	W	O	T	Y	W	N	G	Z	X	E	W
Y	U	E	F	R	L	B	I	J	E	I	O
R	P	O	N	U	U	V	E	T	N	X	S
L	U	G	K	T	Z	O	X	J	I	U	I
R	Y	W	S	G	Y	I	F	M	R	I	N
Q	N	X	P	B	S	T	W	E	L	V	E
K	B	C	C	B	C	H	Y	S	V	B	X

1 X 2	**5 X 2**	**9 X 2**
2 X 2	**6 X 2**	**10 X 2**
3 X 2	**7 X 2**	**11 X 2**
4 X 2	**8 X 2**	**12 X 2**

Activity 3: Word Search - Addition up to 20

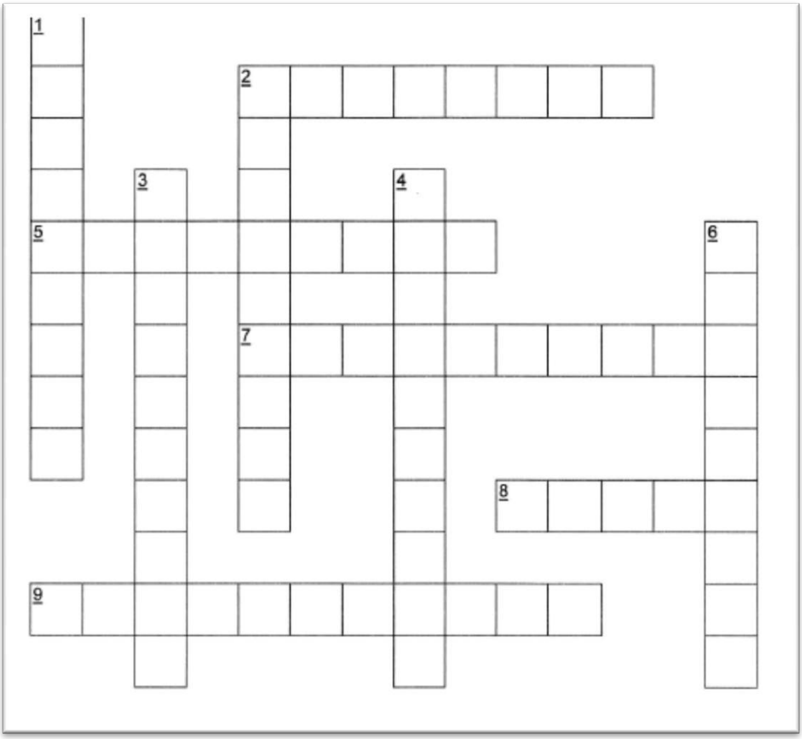
Instruction: Calculate the addition sums and shade the answers on the word search below:

T	W	E	L	V	E	N	D	F	J	L	N	K	T	N
U	F	G	K	Q	K	I	E	N	V	P	M	H	W	E
S	T	H	Z	G	F	N	E	E	B	L	I	Z	E	E
Y	E	A	H	M	M	E	X	I	T	R	J	C	N	T
D	O	V	H	S	T	T	K	U	T	R	R	P	T	F
B	D	Z	E	X	U	E	J	E	V	J	U	E	Y	I
L	E	H	I	N	J	E	E	N	X	M	I	O	Y	F
U	O	S	M	Q	T	N	B	C	G	G	G	M	F	F
W	N	L	E	S	E	E	K	X	H	A	C	D	J	X
Z	E	E	H	E	E	B	E	T	M	B	X	Z	M	T
W	I	G	N	Q	L	J	W	N	E	V	E	S	U	R
J	N	N	Z	I	Q	E	V	H	M	B	G	N	U	P
L	E	K	U	X	N	U	V	U	N	H	K	F	X	Z
T	N	E	E	T	H	G	I	E	X	R	C	L	I	H
H	H	Y	U	E	W	V	O	U	N	D	U	U	S	X

4 + 4	**10 + 8**	**7 + 4**
11 + 4	**8 + 6**	**1 + 8**
12 + 7	**2 + 5**	**15 + 2**
4 + 2	**3 + 3**	**5 + 5**
11 + 2	**11 + 1**	**10 + 10**

Activity 4: Cross word puzzle - Double digit addition up to 100

Instruction: Calculate the addition sums and use the answers to fill in the cross word puzzle below:



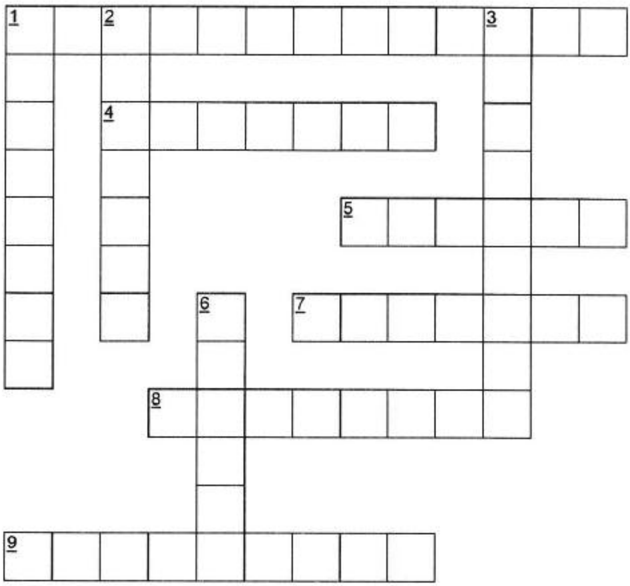
Across		Down	
2	53 + 8	**1**	57 + 25
5	13 + 13	**2**	28 + 36
7	10 + 37	**3**	37 + 39
8	23 + 17	**4**	41 + 12
9	20 + 17	**6**	29 + 63

WORD BANK:

eightytwo, fiftythree, forty, fortyseven, ninetytwo, seventysix, sixtyfour, sixtyone, thrityseven, twentysix,

Activity 5: Cross word puzzle - Geometric shapes

Instruction: Use the given clues to fill in the cross word puzzle below:



Across		Down	
1	A four-sided polygon with two pairs of parallel sides.	1	A five sided polygon.
4	An eight sided polygon.	2	A four sided polygon with all four sides of equal length.
5	A quadrilateral with four equal sides and four right angles.	3	A quadrilateral with four right angles and two sets of parallel opposite sides that are equal.
7	A six-sided polygon.	6	A collection of points in a plane that are the same distance from a centre point.

8	A three-sided polygon, the sum of interior angles equals 180^0		
9	A four-sided polygon with one pair of parallel sides.		

WORD BANK:

circle, hexagon, octagon, parallelogram, pentagon, rectangle, rhombus, square, trapezoid, triangle,

Activity 6: Creative writing in mathematics; poem

Creative writing is a form of writing that can strengthen vocabulary learning in mathematics. Creative writing strategies appropriate for mathematics teaching and learning include short stories, songs and poems. A poem based on a mathematical concept can be easily composed by learners of varying abilities: once learners formulate the first two sentences, they build up more impressive lines and they begin to play with the rhythms created by the lengthening sentences. The poetry session can be done impromptu, within specified time and without warning, or it can be done over a longer period of time to allow learners to find out more information on the content.

If I were a shape, I would be a *circle*
I'll have no end or beginning
I'll be *infinite*
You see me everywhere
You see me everyday
On your watch, the shape of the sun
Even when you eat
My *circumference* loves *pi*
But my *radius* loves pi half as much as my *diameter*
I wouldn't like to be a *square*,
Everything will always be the same
Everyone knows me wherever I go
Even when I am stretched,
I'm still just a *special square*
You can find me in any *area*...how boring!

Althea Boartman (SP teacher, Aloe Junior High School, 2015), Cape Town

Now it is your turn; write your own poem in the space below:

If I were a shape, I would be a

....
....
....

HOW I TEACH ROUNDING OFF TO THE NEAREST 10

Celeste Abrahams, Ingrid Mostert
Capricorn Primary School, AIMSSEC and Kelello Consulting

Rounding off is an important lifelong skill, which all learners should master and which can help with calculation techniques. Do some of your learners struggle to with this skill? Celeste Abrahams will share practical strategies for teaching rounding off which she has used in her Grade 3 class.

INTRODUCTION

Rounding off is an important skill that is used in many aspects of life such as when telling time or adding up monetary values. According to CAPS, rounding off is one of the techniques learners are expected to use when doing context free calculations. This presentation will share some ideas for teaching rounding off to Foundation Phase learners. The approach allows learners to make sense of the concept and to relate it to concrete contexts through the use of their bodies, storytelling and number lines.

CONTENT

The presentation will begin by exploring why rounding off is important and what mathematical skills and concepts are prerequisites this skill. This will be followed by illustrating a variety of ways of helping learners remember how to round off and particularly what to do with the 5. The kind of activities that I will share includes:

- A warm up counting activity to practice the prerequisite skills;
- Ways to ascertain learner's initial "rounding off" knowledge;
- Establishing and/or extending on the learner's current knowledge;
- Ways to help the learner remember what to do with the 5 (applying assimilation and accommodation- *Piaget J [1896-1980]*)

A discussion about ways of representing rounding off concretely and abstractly will follow:

- Using a number line;
- Using a rounding off mountain;
- Using a rounding off card;
- Introducing the rounding off symbol.

The presentation will also look at how to support learners with weak conceptual development and how to give learners enough practice with rounding off by using manipulatives such as dice and cards.

CONCLUSION

Making sense of rounding off can be difficult for some learners. Presenting them with stories and contexts that they can relate to helps them make sense of the concept; while allowing them to practice rounding off through the use of card and dice games reinforces the concept. The combination of these two approaches has helped the learners in Celeste's class to be comfortable with rounding off and this in turn has improved their number concept and their ability to do calculations.

HOW I TEACH PROBABILTY: A FOCUS ON TERMINOLOGY

Benadette Aineamani
Pearson Holdings South Africa

INTRODUCTION

Probability is one of the topics in Further Education and Training (FET) phase that requires attention in the process of teaching it. The attention required is due to the terminology and the different strategies that are involved in the topic (Cañizares & Batanero, 1998).

WHAT IS PROBABILITY?

Probability refers to the extent to which an event is likely to occur, and this is measured by the ratio of the favourable outcomes to the whole number of possible outcomes. Research has shown that over the years, mathematics learners appear to have difficulties developing correct intuition about fundamental ideas of probability for at least three reasons. First, many learners struggle with rational number concepts, set theory and proportional reasoning, which are used in calculating, reporting, and interpreting probabilities (Li, 2000). Second, probability ideas often appear to conflict with learners' experiences and how they view the world (Green, 1982). Third, difficulties in translating word problem statements, as they do the rest of school mathematics (Fischbein & Schnarch, 1997).

In the presentation of how I teach probability, I will focus on how to help learners understand the difference between experimental probability and theoretical probability, and how to explain the terms 'at least', 'at most' and 'exactly' to learners.

Defining Experimental probability

Experimental probability is the ratio of the number of times an event occurs to the total number of trials the experiment is performed. Experimental probability is the result of an experiment and it is used to obtain the relative frequency with which an event will occur.

Defining theoretical probability

Probability describes the likelihood of an event to occur.

The probability of an event denoted as P(E) is defined as a ratio of number of outcomes in an event and the number of all possible outcomes in the sample space.

P(E) = ——

It is important to note that theoretical probability cannot predict what the actual results will be, but can give us an estimate of what is likely to happen.

When teaching probability, learners need to be guided and nurtured into using the appropriate terminology with understanding (Fischbein, Nello & Marino, 1991), for example, when a learner is told to determine the experimental probability and the theoretical probability of an experiment, he/she must be able to identify the difference between the two types of probability. The table below shows some ways of simplifying terminology for learners.

Term	Definition	Example
Experiment	Any process that is uncertain is called an experiment.	Tossing a coin is an experiment
Outcome	An outcome refers to a single result of an experiment.	A head (H) is an outcome of tossing a coin, a tail (T) is another outcome of tossing a coin
Sample space	Sample space is the set of all possible outcomes of an experiment. It is denoted by the symbol S and the size of the sample space is denoted by n(S).	The sample space of tossing a coin is (H; T); n(S) = 2.
Event	An event, also known as favourable outcome comprises a specific set of outcomes that we are interested in.	The event of getting the same face when a coin is tossed twice is (H;H) or (T;T)
At most	This refers to highest or largest number possible -At most is the number given or less than it, in other words, not more than the number given	The event of getting at most three heads when a coin is tossed three time means that: the maximum number of heads should be three, but minimum can be zero
At least	This refers to the lowest number possible -At least is the number given or more than it, in otherwords, not less than the	The event of getting at least three heads when a coin is tossed three times means that: the minimum number of heads should be three, but

	number given	maximum can be more than three if possible, in this example more than three heads is not possible
Exactly	This refers to the actual number required	The event of getting exactly two heads when a coin is tossed three time: means that we have to get two heads and not more or less

Table 1: Some terminology to emphasise when teaching probability

Some tips for teaching Probability

- Introduce probability through activities and simulations, not abstractions
- Try to help learner develop the feeling that mathematics relates usefully to reality and is not just symbols, rules, and conventions
- Use visual illustration and emphasise exploratory methods
- point out to learners common misuses of probability terminology and provide them with a detailed table of common misconceptions when using terminology and how to avoid them
- Use strategies to learners' rational number concepts set theory before approaching proportional reasoning
- Recognise and confront common errors in students' probabilistic thinking
- Provide learners with situations requiring probabilistic reasoning that correspond to their views of the world (Fischbein, Nello & Marino, 1991).

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USING INNOVATIVE RESOURCES TO TEACH EXPONENTS

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This talk is about the use of a 'formative assessment lesson' in the context of the teaching and learning of exponents. The lesson was designed by a team at the University of Nottingham in the United Kingdom, and it is meant to provide many different opportunities for teachers to use formative assessment in their lesson.

ABOUT THE LESSON

Memory used a 'formative assessment' lesson to assess her learners' understandings of exponents so that she would be able to tailor her teaching to the learners' needs. The lesson was taught in the first term to a Grade 9 class after Memory had revised the exponential laws taught in Grade 8 and introduced integer exponents. By teaching this lesson after the main concepts had been taught, it provided opportunities to consolidate the key concepts involved in the topic.

This lesson takes an 'active learning' approach. The learners work in small groups on a joint activity involving matching cards from two sets. In one set, the cards take the form of expressions, such as $2^2 \quad 3^2$ or $2^2 \div 2^3$. In the other set, the cards are single exponents, such as 6^2 or 2^{-1}. The challenges for the learners are first to decide whether a specific exponent law can be applied and then to work out which law to apply. The cards are carefully designed so that learners' misconceptions are challenged. For example, if they make an incorrect match, they may realize this because other cards are left unmatched. They then have to re-think their original match.

In the process of matching cards, the learners discuss their thinking and move cards around accordingly. The teacher is able to gather information about what they are thinking which informs her questioning. The learners find their conceptions challenged by both the actual card matching and their peers, which means they need to re-think their previous ideas.

Memory is taking part in a research project, FaSMEd, which looks at the use of formative assessment in the mathematics classroom. The researchers, Marie and Ingrid, asked her to use this lesson with her Grade 9 class. We think our joint experience is worth sharing. In the talk we will introduce the lesson and the classroom activity, and we will provide some details about what happened in Memory's classroom.

The two sets of cards are provided below.

E1 $2^2 \square 3^2$	E2 $3^2 \square 2^3$
E3 $2^2 + 2^3$	E4 $2^2 \square 2^3$
E5 $6^8 \square 6^4$	E6 $2^2 \square 2^2$
E7 $3^2 + 3^3$	E8 $4^2 \square 2^3$
E9 $2^3 \square 2^{\square 2}$	E10 $(2^3)^2$
E11 $3 \square 2^2$	E12 $2^3 \square 2^3$
E13 $5^2 \square 3^3$	E14 $(3^2 \square 2^2)^2$

S1 2^{1}	S2 2^{5}
S3 $(\square 2)^{1}$	S4 $2^{\square 1}$
S5 2^{0}	S6 2^{6}
S7 6^{4}	S8 6^{2}
S9 0^{2}	S10 4^{3}

CONCLUSION

On many levels, using this lesson was challenging for Memory. In terms of the small-group work, she was challenged because there is not much spare space in her classroom, which accommodates sixty learners. In terms of the mathematics, it was a challenge to move the learners beyond simply applying the laws of exponents.

However, for both Memory and the learners, it was a positive experience and Memory fully intends to use this activity with different classes and also to use different activities which are designed using similar ideas.

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The lesson used in this research can be found on the website of the Mathematics Assessment Project: http://map.mathshell.org/materials/lessons.php

DEMYSTIFYING AGE WORD PROBLEMS THROUGH STRATEGIC COMPETENCY (KILPATRICK'S STRAND OF MATHEMATICAL PROFICIENCY)

Wandile Hlaleleni
Butterworth High School

The paper is intended to share the presenter's approach to word problems based on ages. The presenter hated the word problems based on ages because he did not understand the language until the presenter gained an insight into word problems through strategic competency. According to Kilpatrick etal (2001) strategic competency is the ability to formulate, present and solve mathematical problems. It has not been an easy task to grasp or understand word problems; the presenter undertook many investigations, converting numbers to words and words to numbers. The presenter wanted to master the skill. Hence he is competent enough today to make this presentation. The rationale for word problem study was to transfer the skill to the clients i.e learners. Hopping that this workshop will help his participants to see word problems with different lenses. The presenter wishes to advise the workshop participants to encourage learners to use multiple representations of concepts namely 3x +4 means three times a certain number plus four.

TARGET AUDIENCE: Senior Phase teachers

DURATION: 1 hour

MAXIMUM NUMBER OF PARTICIPANTS: 30

MOTIVATION

Let us suppose that you are 40 years old and your child is 10years old. How would you write your age in terms of your child's age? How would you write your age 5 years ago in terms of child's age?

Solution

I am four times as old as my child because 40 years is 4×10years.5 years ago I was 35years old and my child was 5 years old therefore 5years ago I was seven times as old as my child because 35years is 7×5years.

PRESENTATION

According to the introduction I am 40 years old and the child is 10years.

<u>Possible word problems from the above data</u>

- I am four times as old as my child. 5 years ago I was seven times as old as my child. How old are we?
- I am thirty years older than my child. In five years to come I will be three times as old as my child. How old are we?

- I am thirty years older than my child .Eight years ago I was sixteen times as old as my child. How old are we?
- I am four times as old as my child. Fours ago I was six times as old as my child. How old are we?
- I am four times as old as my child. If the sum of our ages is fifty years, how old are we?

  How the above problems formulated?

  40 years = 4×10years hence I am four times as old as my child. If I am 40 years old, 5years ago I was 40years – 5years = 35 years and the child was 10years – 5years = 5years. 35years is 7×5years hence I was seven times as old as my child.

  40years is bigger than 10years by 30 years hence I am 30years older than my child. If I am 40 years now I will be 40years + 5years in 5years to come and the child will be 10years + 5years = 15years. 45years is 3×15years hence I will be three times as old as my child.

  Problem solving

- I am four times as old as my child. Five years ago I was seven times as old as my child. How old are we?

  Solution

  Let the child's age be represented by d

  That means my age 4d

  5years ago

  The child's age was (d- 5) years

  I was (4d – 5) years.

  $7(d-5) = 4d - 5$ (7times is balancing the ages 5years ago)

  $7d - 35 = 4d - 5$

  $7d - 4d - 35 = 4d - 4d - 5$

  $3d - 35 = -5$

  $3d - 35 + 35 = -5 + 35$

  $3d = 30$

  $\therefore d = 10$

  My child is 10 years old and I am 4×10 years = 40 years old

- I am thirty years older than my child. In five years to come I will be three times as old as my child. How old are we?

Solution

Let my child's age be (v) years

That means that my age is (v + 30) years

In 5years to come

My child's age will be (v + 5) years

My age will be (v +30 + 5) years = (v + 35) years

3(v + 5) = v + 35 [3times is for balancing the ages in five years to come)

3v + 15 = v + 35

3v – v +15= v –v +35

2v + 15 = 35

2v + 15 -15 = 35- 15

2v =20

— =—

My child is 10 years old and I am 10 years + 30years = 40years old

- I am thirty years older than my child. Eight years ago I was sixteen times as old as my child. How old are we?

Solution

Let the child's age be x

That means my age is (x+ 30) years

8 years ago

My child was (x – 8) years and I was (x +30 -8) years = (x + 22) years

16(x – 8) = (x + 22)

16x -128 =x +22

16x –x -128= x-x +22

15x -128 = 22

15x -128 +128 = 22+128

15x =150

— = —

.' . x = 10

My child is 10 years old and I am 10years + 30years = 40years old.

Worksheet1

If father is 75years old and son is 15years old. Formulate and present four word problems using the above given data.

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1.2…………………………………………………………………………

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1.3…………………………………………………………………………

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1.4…………………………………………………………………………

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Worksheet2

2.1 Temba is three times as old as Tembisa. Four years ago he was five times as old as Tembisa. How old is Tembisa?

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2.3 The combined age of the father and the son is 84 years. In six years' time father will be twice as the son was three years ago. How old are they?

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2.4 The combined age of a brother and a sister is thirty two years. Six years ago the brother was nine times as old as the sister. How old were they six years ago?

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2.5 The brother is sixteen years older the sister. Seven years ago, the brother was seven times as old as the sister. How old are they?

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HOW I TEACH QUADRATIC PATTERNS IN A GRADE 10 CLASS

Wandile Hlaleleni
Butterworth High School

TARGET AUDIENCE: FET Phase

DURATION: 1 hour

MAXIMUM NUMBER OF PARTICIPANTS: 30

This paper is intended to share my teaching with other teachers. In the paper there are various types of patterns, representations which will be used to enhance conceptual understanding. According to Kilpatrick et al. (2001) learners with conceptual understanding know why a certain concept is learned and when it is used. Through diagrammatical representation, verbal representation and symbolic representation. This paper is intended to share with other teachers how I promote conceptual understanding.

MOTIVATION

Quadratic pattern representation is a good approach to solve quadratic equations. Learners learn from quadratic patterns that quadratic equations are from cultural or social contexts. For instance $n^2+2n+1=4$ means that the diagrams whose area is $4cm^2$.

PRESENTATION

I usually give the learners diagrams of squares and the areas of squares show in worksheet 1.I usually ask them to list areas in ascending order and ask them to find a common difference e.g 4;9;16;25;36;49, but they usually found it in the second level of subtraction .For instance in the case of 4;9;16;25;36;49 it is 2. Common difference of means:

$2a=2$ and $a=1$

Quadratic pattern is n^2

Patterns generated by n^2

1 ; 4 ; 9 ; 16 ;25 ; 36

P_1 = 4 ; 9 ; 16 ; 25 ; 36 ; 49

$\underline{-P_2 = \ 1 ; 4 ; 9 ; 16 ; 25 ; 36}$

P_3 = 3 ; 5 ; 7 ; 9 ; 11 ; 13

Common difference =2

$T_n = a + (n - 1)d$

$T_n = 3 + (n - 1)2$

$= 3 + 2n - 2$

$= 2n + 1$

T_n of 4; 9 ; 16 ; 25 ; 36 ; 49 = $n^2 + 2n + 1$

Diagrams

Pattern 1	**Pattern 2**	**Pattern 3**	**Pattern 4**
D D D	D D D D	D D D D D	D D D D D D
D D D	D D D D	D D D D D	D D D D D D
D D D	D D D D	D D D D D	D D D D D D
	D D D D	D D D D D	D D D D D D
		D D D D D	D D D D D D
			D D D D D D

1.1.Change the D patterns to numeric patterns

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1.2. Determine the nth term for finding the number of D's in any pattern .

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1.3. Which pattern will be made by 324 D's?

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1.4. How many D's will make pattern 15?

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DISCUSSION

What did you learn from this presentation?

Have you noticed that diagrammatical representation of patterns promotes conceptual understanding?

How many ways of representing the concept have been used in the presentation?

CONCLUSION

Concepts need to be represented in many different ways to help the learners understand the concepts.

I JUST DO NOT SEE WHAT THEY SEE – DEVELOPING VISUAL CAPACITY FOR GENERATING MULTIPLE ALGEBRAIC REPRESENTATIONS FOR VISUAL PATTERNS

Rolene Liebenberg
Cape Peninsula University of Technology

INTRODUCTION

Approaches to introducing learners to algebra in senior phase, build on the early exploration of visual patterns using the patterns to generate algebraic expressions, with a focus on functional thinking, and thinking between two data sets (Bennett, 1988).

Samson & Schafer (2011) looked at learners' ability to generate multiple algebraic representations for visual patterns. A grade 9 high-ability learner in Samson & Schafer's (2011) study produced nine different visually mediated expressions for the *n*th term of a given pattern in which two consecutive terms of the figural pattern was given. There were ambiguities in some of the learners' algebraic expressions. Samson & Schafer's (2011) research points to the importance of teachers having both the visual and algebraic skills to interpret and validate learners' general expressions and to engage critically with the explanations for their generalisations. It is against this background that I focus on engaging my pre-service intermediate-phase mathematics teachers with figural pattern generalisation and encourage them to provide the visual explanations for different algebraic expressions of a figural pattern (See in Appendix1 a question from the final BED 1 Mathematics examination and a student's answer). In this paper I will present pre-service teachers responses to a visual approach to figural patterns and the challenge of disrupting the "quick and easy- I know how to" mind-set that poses a danger to bringing learners' creativity to the fore.

RESISTANCE TO VISUAL APPROACH

The following problem was given to BED 1 pre-service teachers who were asked to find the algebraic expression that described the *n*th term using a visual approach:

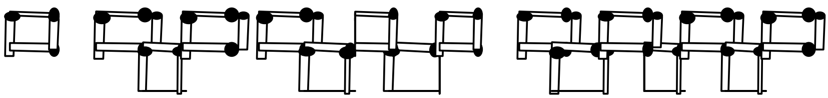

Figure 1 Figure 2 Figure 3 Figure 4

The vast majority of the students resisted using a visual strategy arguing that at school they were introduced to tables and know that for this pattern they could use the formula for an arithmetic sequence. I emphasise to my students that a range of equivalent algebraic expressions could be developed for this pattern through a visual approach which was important in developing both visual and algebraic skills of learners. Eisenberg and Dreyfus (1991) argue that the reluctance of students to follow a visual approach is due to their difficulty they have with visualisation.

CONCLUSION

The vast majority of the students had great difficult in generating visual explanations and acknowledged that they experienced anxiety about not being able "to see what their learners may see". I pointed out that they will have learners who may have much higher visual capacity than they and that they should be open to listen to learners' explanation of how the learner saw the pattern. We need to create a classroom culture in which teachers' anxiety not stunt the creative expression of learners.

There is perhaps not sufficient ongoing exploration of visual explanation for figural patterns in higher grades once learners have followed the numerical route with tables and work with the general formula of either arithmetic or geometric sequences.

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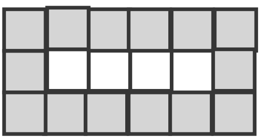
APPENDIX 1

Grey tiles are packed around white tiles as shown in the diagram below

Grade 7 learners are asked to solve the following problem: How many grey tiles are packed around a row of 100 white tiles?



Four grade 7 learners look for patterns in the diagram that is given to help them describe the relationship between the number of grey and white tiles. These four learners **did not draw** any additional diagrams. The learners saw different patterns in this specific diagram and described the number of grey tiles as follows:

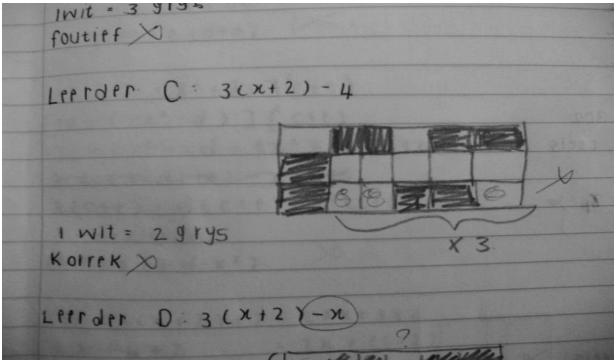
Learner A: $2 \times (x + 2) + 2$

Learner B: $2 \times x + 3 \times 2$

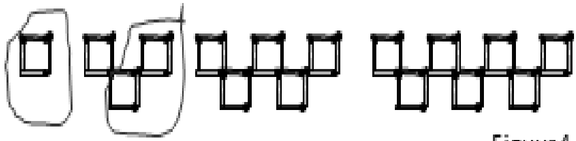
Learner C: $3\,(x + 2) - 4$

Learner D: $3\,(x + 2) - x$

Explain how the learners saw the pattern that is described by their formula. Use the sketch to support your explanations of how the learners saw the pattern. If a learners’ formula is not correct explain what the error is from a visual perspective.



APPENDIX 2



Figuur1 Figuur 2 Figuur 3 Figuur 4

YASSIER SE FORMULA

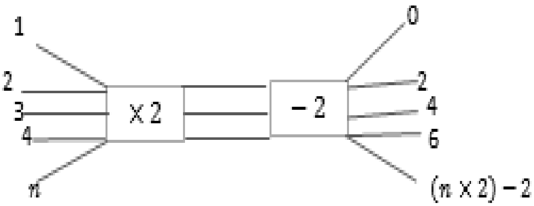
Aantal vuurhoutjies in figuur 1 = $4 \times (1 + 0)$

Aantal vuurhoutjies in figuur 2= $4 \times (1 + 2)$

Aantal vuurhoutjies in figuur 3= $4 \times (1 + 4)$

Aantal vuurhoutjies in figuur4 = $4 \times (1 + 6)$

Aantal vuurhoutjies in figuur n = $4 \times [1 + (2n - 2)]$



HOW I TEACH NUMBER BONDS AND ADDITIVE RELATIONS

Melissa Mentoor
Capricorn Primary School

Number bonds and additive relations are two basic but important concepts which form the foundation for understanding how numbers work. Understanding how numbers work, or having a good number sense, is essential for flexible calculation strategies and meaningful mathematical thinking. In this presentation I will share the shift in my teaching to a greater focus on these concepts for adding and subtracting (additive relations) through the use of whole-part-part diagrams. I will also share a variety of approaches that I have used successfully with my Grade 1 learners to introduce number bonds such as singing songs, playing games and mental maths exercises.

INTRODUCTION

The concept of number bonds is a basic but important foundation for understanding how numbers work. With a solid number bond foundation, learners are able to see the inverse relationship between addition and subtraction. Number bonds create a mental picture of the relationship between a number and the parts that combine to make it. This mental picture can be represented in a whole-part-part diagram.

CONTENT

I will begin with some explanation about the whole-part-part diagrams and the family of equivalent number sentences that can be generated by each diagram (see below for one example).

5	
3	2

3 + 2 = 5   5 - 2 = 3

2 + 3 = 5   5 - 3 = 2

I will then discuss some of the addition and subtraction facts to 20 which form the basis for adding and subtracting numbers to 100.

Finally I will share some of the strategies that I have used to teach number bonds. These strategies are designed to address following issues which I think should be taken into consideration when teaching number bonds:

- Learners should experience number bonds in real life situations
- Learners should see the connection between whole numbers and the parts that make up the whole number
- Learners should represent the number bond via families of number sentences.
- Learners should use concrete resources such as number lines and bead strings.

CONCLUSION

Number bonds and additive relations are an essential part of the foundation of mathematics that should be laid in the Foundation Phase. Shifting my teaching to a greater focus on these concepts has changed the way I think about mathematics and the way my learners engage with these concepts.

INVESTIGATING THE EFFECT OF PARAMETERS A, B, C AND D ON THE GRAPHS OF TRIGONOMETRIC FUNCTIONS USING GEOGEBRA SOFTWARE (FET).

James Mica
International School of South Africa

Computer has become a powerful and helpful tool in the teaching and learning of mathematics, in particular, in enhancing the understanding of various mathematical concepts. The teaching and learning process with the use of technology has proved to have so many advantages, such as providing greater learning opportunities for students (Roberts, 2012), enhancing student engagement (White, 2012) and encouraging discovery learning (Bennet, 1999). Investigating the effect of parameters on the graphs of trigonometric functions is one of the major concepts on functions of graphs for Grade 11 learners as well as Cambridge learners. Having introduced the concept of drawing basic graphs of $y = \sin x$, $y = \cos x$ *and* $y = \tan x$ *for* $x \in \left[-360^0, 360^0\right]$, *students will now have to investigate the effect of parameters* a, b, c *and* d *on the graphs of these trigonometric functions, say* $y = a\sin(bx + c) + d$. *Typically, the drawing of these various transformations using paper and pencil is confusing and unproductive and often students struggle to comprehend these transformations. In most cases, there is not sufficient time for learners to explore the nature and effect of the transformations. GeoGebra might play the role in filling this gap by assisting students to visualise and understand the behavior of trigonometric graphs through exploration, without repeated use of tables of values and manual plotting and drawing of graphs. In this article, GeoGebra is used to explore the behaviour of trigonometric graphs, especially the effect the parameters have on the period, position and amplitude of the trigonometric curves.*

How to Create a Geogebra File on Graphs of Trigonometric Functions

1. In **GeoGebra**, click on the **Algebra & Graphics**.
2. Create assigned values of the parameters to *a, b, c* and *d* which will be used as sliders. To assign a number to *a*, type $a = 1$ in the *Input bar*, then press the ENTER key. Repeat the same procedure for *b, c* and *d* (i.e. $b = 1$, ENTER; $c = 1$, ENTER, $d = 1$, ENTER).
3. Create slides *a, b, c* and *d* by right clicking on each of them in the *Algebraic View* and click on *Show Object* from the context menu. *See Figure 1*

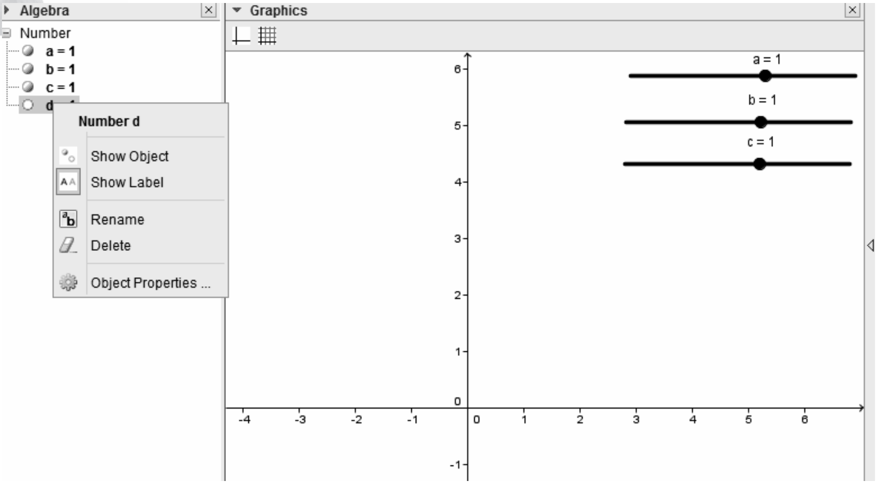


Figure 1

4. Change the interval of the x-axis from 1 to $\frac{\pi}{2}$ by right clicking and blank on the *Graphic View* and select *Graphics* from the context menu to display the *Settings* dialog box.

5. In the *Settings* dialog box, click the **xAxis** tab, check the *Distance* check box, select $\frac{\pi}{2}$ from the *Distance* drop down list box, and then close the dialog box. *See Figure 2.*

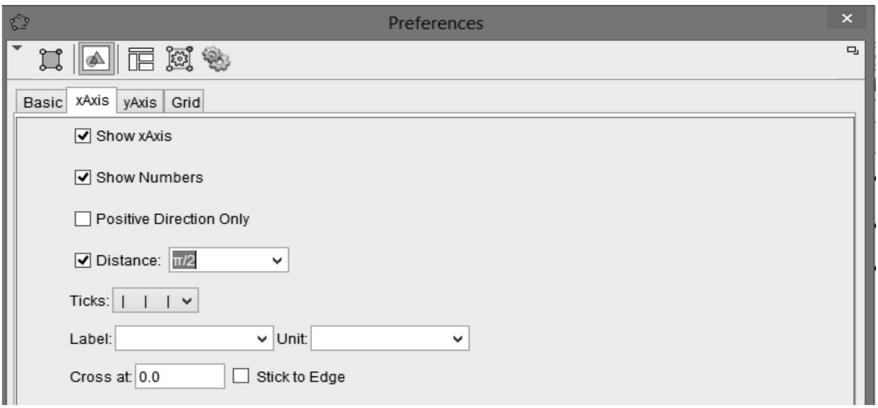


Figure 2

6. To graph the sine function, type $f(x) = a * \sin(b * x + c) + d$ in the *Input Bar*, then press the ENTER key.

7. To graph the cosine and tangent functions, type $\mathrm{g}(x) = a * \cos(b * x + c) + d$, then press the ENTER key; $\mathrm{h}(x) = a * \tan(b * x + c) + d$, then press the ENTER key. *See Figure 3*

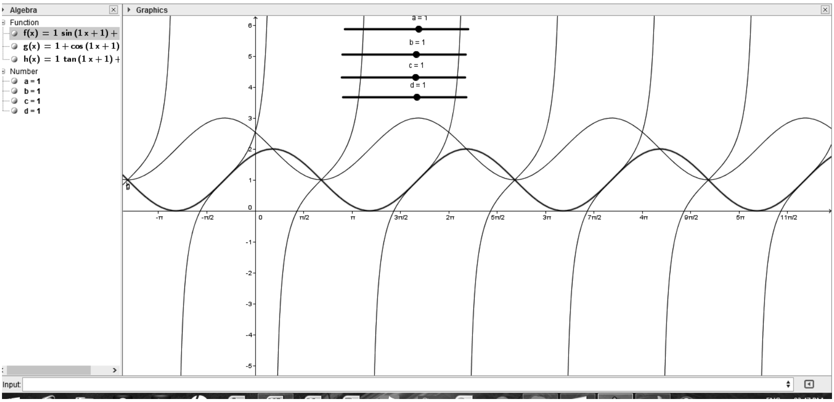


Figure 3

8. To hide/show the graphs of the other two functions as one works with a chosen function, right click on the function you want to hide/show and click on *Show object*.

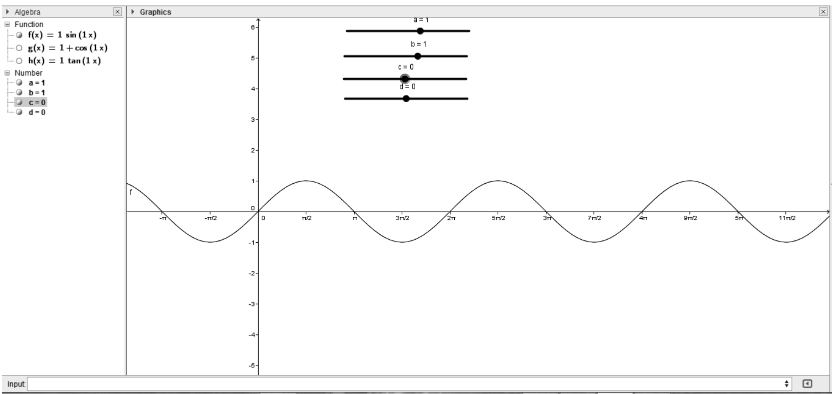


Figure 4

Figure 4 shows the basic graph of the function $y = \sin x$ with a maximum value of 1 and minimum value of – 1, and a period of 2π .

y = *a Sin x*

To explore the effect of parameter *a* on the graph, move slider *a*, on either side of 1, for example, move it to 2 which gives the function $y = 2\sin x$. *See Figure 5*

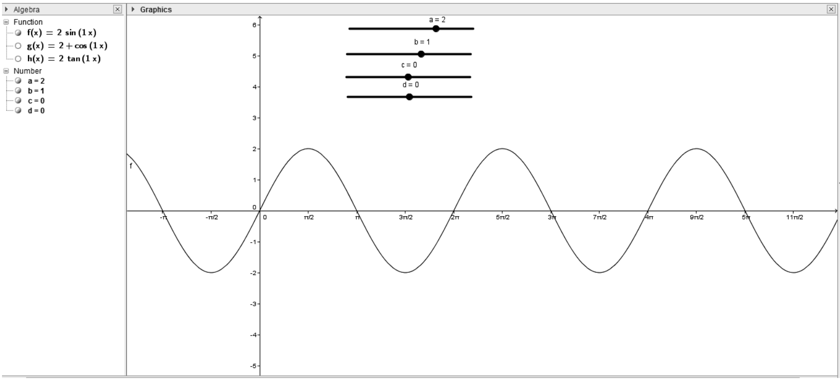


Figure 5

It can be noted that the amplitude changes to 2; that is, the maximum and minimum values becomes 2 and – 2 respectively. The period remains 2π .

The effect of – *a* can also be explored, say $y = -2\sin x$. *See Figure 6*

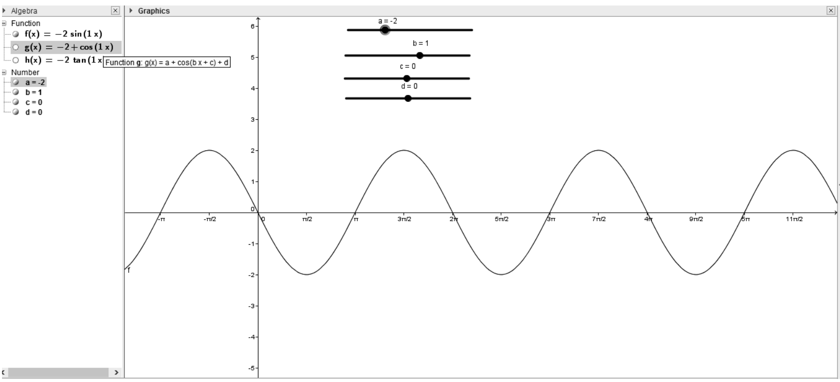


Figure 6

As one drags the slider slowly towards – 2, the graph is flipped. Further explorations can be done with other values of *a*.

y = *Sin bx*

The effect of parameter b can be shown by moving slider b, for example, to 2, which gives the function $y = \sin 2x$. *See Figure 7.*

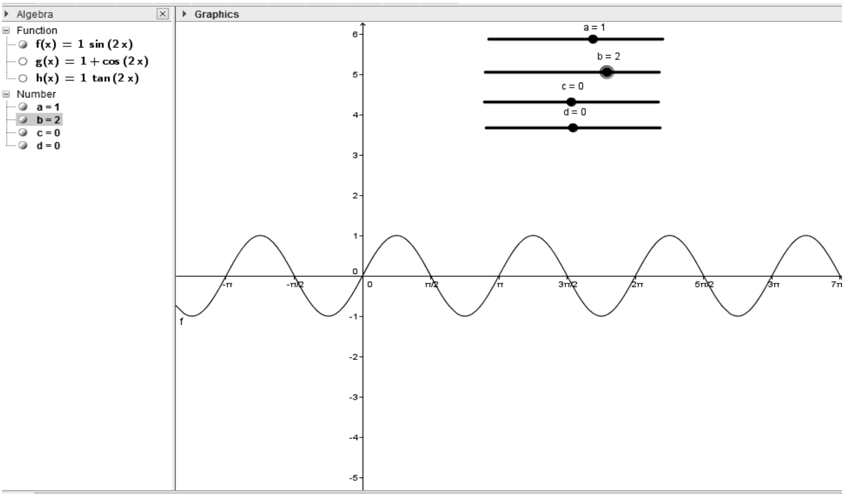


Figure 7

The period changes to π , that is, the period = $\dfrac{2\pi}{b}$. There are now two complete revolutions between 0 and 2π . The amplitude is not affected by the movement of slider b. Further explorations can be done by varying the values of b, for example, $y = \sin 3x$, $y = \sin \frac{1}{2} x , \ldots$

y = *Sin* (*x* + c)

Consider the function $y = \sin\left(x + \dfrac{\pi}{6}\right)$, which can be written approximately as

$y = \sin\ \ x + 0.5$.

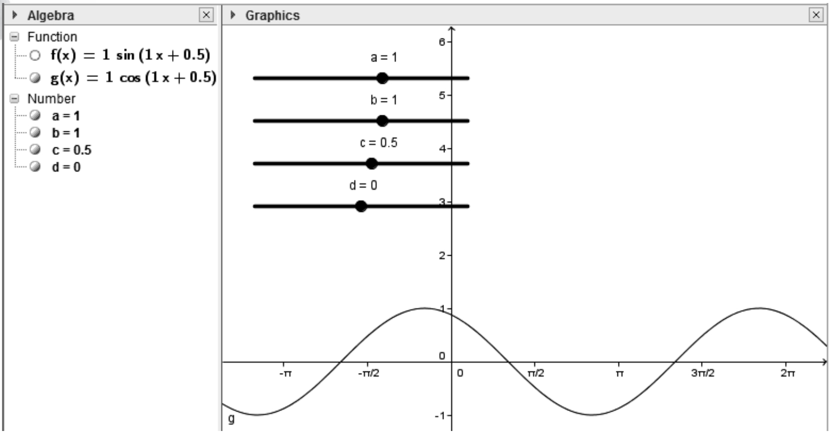


Figure 8

The graph has been shifted 0.5 units to the left (*c* units to the left). However, the period and the amplitude are not affected. Further explorations can be done on the effect of $y = \sin\ \ x - c$ which shows that the curve is shifted *c* units to the right.

y = d + *Sin x*

Consider the function $y = 2 + \sin x$. *See Figure 9*

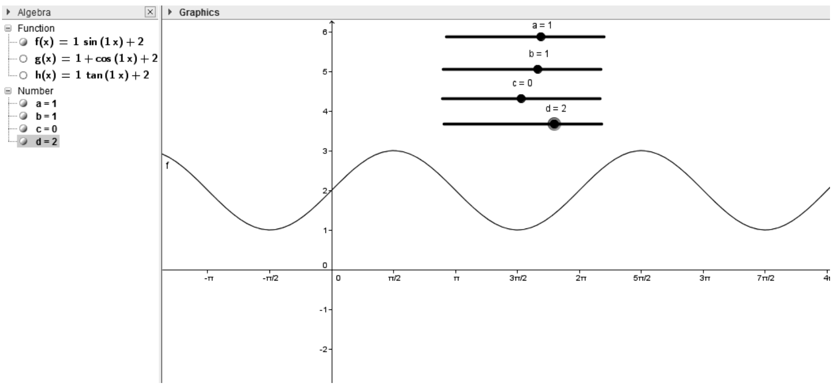


Figure 9

The graph is translated 2 steps up (*d* units up). Both the period and amplitude remains the same.

Similarly, for the function $y = -2 + \sin x$, the curve is translated 2 steps or units down.

A combination of parameters can be explored with students making valid conclusions from discussions. For example, the function $y = 3\sin 2x$. *See Figure 10.*

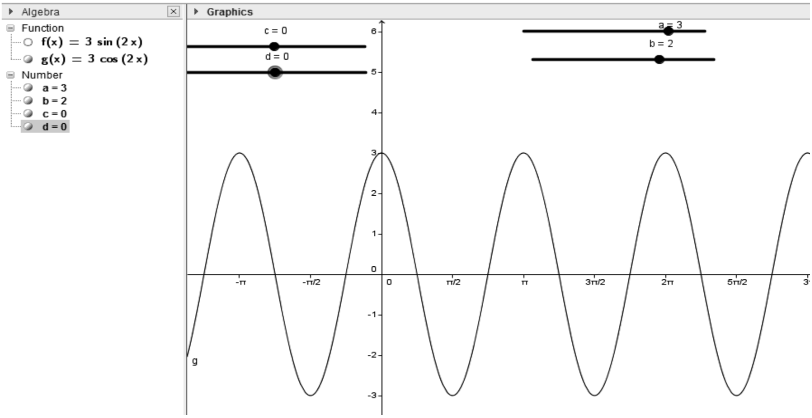


Figure 10

The same activities are also done on the functions $y = \cos x$ and $y = \tan x$.

In conclusion, GeoGebra has proven to be an effective tool in enhancing teaching and learning of graphs of trigonometric functions. Students are able to experience a hands-on, visualised method which has a positive effect in enabling them to understand the concepts better rather than just being passive recipients of information as advocated by traditional teaching methods.

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HOW I TEACH PLACE VALUE

Ivy November & Suzan Mojahi

INTRODUCTION

A place value is a place or position where a digit stands in a number, when referring to a position this indicates Hundreds, Tens and Units/Ones. It is often where educators in primary schools are experiencing challenges where learners are unable to identify place values in numbers. This proposal will focus on the discussion of learning aids to be used to address place values. The following aspects will be tabled: definition of place value, identifying the place values, use of creativity to promote understanding, golden rule of placing digits, number range in CAPS document, use of place value cards from grade 4-6 and the conclusion.

DEFINITION

As indicated above a place value is a place or position where a digit stands in a number, when referring to a position this indicates Hundreds, Tens and Units/Ones for example 786 where 7 will have to stand under Hundreds, 8 under Tens and 5 under Units/Ones.

IDENTIFYING THE PLACE VALUES

It is important to allow learners to read aloud the number e.g. 784 this means that they will read it as seven hundred and eighty-four. By reading aloud a teacher will enable learners to identify the place value of each digit.

USE OF CREATIVITY TO PROMOTE UNDERSTANDING

It is important to take note that using creative diagrams does help in stimulating the minds of young learners. Use of bright colours (robot colours) and creative images will assist learners to envision the place/homes of digits. For example one can make use of the place value house (a picture of a house divided in three robot colours where each block represents the place value: hundreds, tens and units).

GOLDEN RULE OF PLACING DIGITS

This is the most important rule in order to assist learner in placing the numbers in the correct place values. The rule is that in placing the number one must start from the right side to the left side but learners need to be aware that when constructing a sentence one must start from the left side to the right side in order to prevent misunderstandings.

NUMBER RANGE IN CAPS DOCUMENT

This policy states the required number digits that should be introduced in a grade, for example, in grade 4 the policy states that 3-digit number should be used in term 1, then 4-5 digit numbers in term 2. In grade 5 the policy states that 4-6 digit numbers must introduced. In grade 6 the policy states that 6-9 digit numbers must be introduced.

SUPPORTING MATHEMATICS LEANING IN HOME LANGUAGE CLASSROOMS THROUGH THE USE OF WORD PROBLEMS

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Teaching and learning of mathematics in home language in the context of South Africa demands for a shift in teaching so that learners in contexts where African languages are used can access mathematics learning without any prejudice. Some research argues for mathematics as a human activity which must make sense and must be close to the children. In this paper I share insights from an intervention lesson that was conducted in an isiZulu classroom where word problems were used in order to make the mathematics accessible to Foundation Phase Grade 2 learners.

INTRODUCTION

My presentation is about supporting additive relations word problem learning in Home Language classrooms. I chose to talk about this topic because the South African Curriculum Assessment Policy Statement (CAPS) document points to teaching and learning of mathematics in home language for all foundation phase learners. Whist some provinces have introduced strategies such as the Gauteng Province Language and Mathematics Strategy (GPLMS) to assist with this transition, there are still challenges with teaching and learning in home language. The talk will be based on a lesson that is from an intervention that I conducted in an isiZulu classroom as part of my PhD study. The approach that I used during the intervention is based on the Big Books of Word Problems (Askew, 2004) which promotes the sense making of additive relation word problems (Carpenter, Moser, & Bebout, 1988) by using themes, pictures and the context relevant to the children being taught. The theory that underpins the notion of use of word problems is The Dutch Realistic Mathematics Education (RME) and their view of mathematics as a human activity (Gravemeijer, 1997) which must make sense and be close to children. This activity can be made possible through the use of everyday word problems.

CONTENT

During the presentation I will talk about the structure of the lesson:

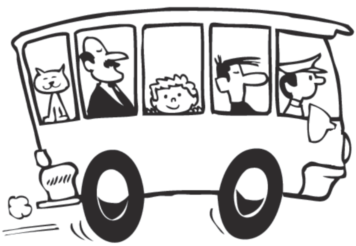
- ***Activity 1:*** Fill in the missing numbers on the 'Line' (rope)
  (Here artefacts - wool and number cards will be presented to the audience)
  In this activity two learners were asked to hold the 'line' in front of the classroom. The rest of the learners were called individually to pick up a number card from number cards scattered on the floor and place it on the

'line'. These numbers were called randomly to ascertain if the learners can identify and order the numbers.

- ***Activity 2:*** Jumping on the number track (from 8 to 12)
  (A picture of a bus and a number track will be presented to the audience)

8	**9**	**10**	**11**	**12**

  In this activity the context of a bus moving from one stop to another picking up people was discussed with the learners, a number track was put on the floor to represent the stops. Learners were asked individually to pretend to be the bus driver picking up people. They had to first show how they would jump from one number to another. The aim of this activity was so that the jumping could translate or connect to a use of a number line later in the lesson.
- **Activity 3:** Jumping on a number line
  (An artefact will be presented to the audience)
  In this activity the learners were asked to demonstrate exactly what happened on the number track. They were expected to draw an empty number line and start with 8 then make jumps of '4' and stop at '12'
- ***Activity 3:*** Word Problem Discussion and Problem Solving – Context: Bus
  In this activity a word problem written in isiZulu only on an A3 paper was pasted on the board next to the picture of the bus (see examples below). (These will be presented to the audience during the presentation in their A3 forms)



The problem was discussed with the learners. Learners were asked questions such as: What is a bus used for? What kind of places do people travel to on a bus? How many people can a bus take? After the discussion of the context, the learners were asked to describe the story presented to them in their own words to ascertain if they understand it. A worksheet was then given to the learners

- ***Activity:*** Wrap Up
  In this activity a discussion was meant to take place about what they have learnt during the lesson, but due time limitations, we did not get to do it.

CONCLUSION

Since this lesson was the first lesson of my intervention, I found that the learners were not familiar with discussing the mathematics because the tradition has been that the teacher does all the talking and explanations, while the learners listen and write down what they have been told by the teacher. Also the learners were not familiar with the artefacts that were used in the class which resulted in the activities taking longer than planned.

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PUBLISHING IN PEER-REVIEWED JOURNALS

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Writing an academic paper can be overwhelming, hard to get down to and hard to finish. Yet, writing is a central aspect of our scholarly identity. In this special interest group on writing, we would explore what it entails to write for publication. In doing this, we would examine what to do when preparing a manuscript for publication; how to make arguments in one's writing; reasons why manuscripts are rejected and how to get one's manuscript accepted; and how to choose journals for one's manuscripts. Writing is hard work. But seeing one's work in print – in a good journal – makes it all worthwhile.

THE PROFESSIONAL DEVELOPMENT OF MATHEMATICS TEACHERS: PERSPECTIVES ON SOCIO-POLITICAL SCENARIOS, POLICY INITIATIVES AND TEACHER AGENCY

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University of Pretoria & University of Stellenbosch

The quote “We can't solve problems by using the same kind of thinking we used when we created them” is attributed to Einstein. Many of the problems confronting teacher education and the solutions presented may not be progressive or productive simply because the solutions presented are on the same plane. Circular reasoning, such as our teachers don’t have content knowledge, as indicated by a test purportedly of content knowledge, and therefore they require an injection of “content” knowledge, circumvents the critical questions of knowledge development and teacher agency and the interrelationship between them. These critical questions we believe, are prior questions to be addressed at the level of teacher education.

Along with Batra (2014) we interrogate the intricate relationship between teacher education and classroom practice, by raising the following questions about views on the epistemological underpinnings of teacher education programmes.

- How is **knowledge** “presented” to prospective teachers? How successful are professional development courses in fostering mathematics as a human endeavour? Are the historical roots, for example of mathematical conventions shared? Is the mathematics curriculum “covered” or is the mathematical thinking prioritised? Are complex topics such as proportional reasoning (e.g. in ratio and percentage) given enough weight? Do we acknowledge prospective teachers existing concepts and promote the deepening and generalising of this knowledge?
- Are prospective mathematics **teachers** narrowly perceived as implementers of current curriculum policy? Or is there some leeway for developing advanced knowledge of mathematics, and adapting to the particular contexts in which they practise as educators?
- Who is “**the student**” in mathematics teacher education courses? How do we envisage their personal growth and development as professionals?
- How are the **socio-political realities** of the South African schools engaged in courses to prepare aspirant teachers for their roles as agents of change, in the sense that they can make a difference in the lives of the children they take responsibility for? For example, how do we think about enabling teaching for

higher order reasoning in a computer era in schools without immediate access to technology?

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