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FOREWORD

Welcome to the Eastern Cape and to the 23rd Annual National Congress of the Association for Mathematics Education of South Africa.

The theme of the 2017 congress "Restoring the dignity of mathematics learners through quality teaching and learning" is enshrined in Section 10 of the South African Constitution, which states that "everyone has inherent dignity and the right to have their dignity respected, and protested." As such every learner in our schools has the right to be respected, protected and fulfilled in every mathematics class. We need to ask ourselves if every learner has inherent dignity and if not, why this needs to be restored.

One key aspect of dignity is quality and in South Africa we have seen over the years the sense of mediocrity which seems to be perpetuating through policy that determines that if learners obtain a mere 30% for a subject, they still pass. There appears to be a lack of urgency to give effect to dignity through quality education, which implies that teachers and learners are not given sufficient equal respect and equal worth.

For mathematics teachers, provision of quality mathematics education should be driven by the quest to restore the dignity of learners and learning. Over two decades post-apartheid, inequalities are still evident in the education system and the question is who is perpetuating such in a democratic era? We need to provide quality mathematics education in every school in South Africa and this implies-training teacher to teach competently and to cover the curriculum adequately. Mathematics teachers need to inculcate in learners confidence, logical reasoning, understanding and the ability to deal constructively with various situations that arises in diverse South Africa and the constantly changing world.

A large number of long and short papers were submitted for AMESA Congress 2017. The papers cover topics ranging from numeracy in the early years to designing a professional development intervention at tertiary level.

The papers in the two volumes were grouped according to the presentations: plenary and long papers in volume 1 and short papers, workshops and how I teach papers in volume 2. In addition, a CD-rom with activities and worksheets from workshops are included here.

All long and short papers were reviewed by at least two reviewers. In some cases papers were sent to a third reviewer if consensus could not be reached regarding the quality of the paper. Papers that required major revisions were reviewed by the academic review committee and authors were asked to re-submit them as short papers once revisions had been made.

Our thanks go to Nico Govender and Tulsi Morar for their assistance in the preparation of the proceedings.

Tom Penlington and Clemence Chikiwa

June 2017

ACKNOWLEDGEMENTS

All papers submitted to the congress were sent for blind reviewing.

Many thanks to our review team who reviewed the papers in a constructive way. Stanley Adendorff

Sarah Bansilal

Dudley Bester

Piera Biccard

Deonarain Brijllall

Benard Chigonga

Clemence Chikiwa

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Zain Davis

Michael de Villiers

Joseph Dhlamini

Anthony Essien

Faaiz Gierdien

Nico Govender

Rajendran Govender

Thomas Haywood

Thandiwe Hlam

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Zanele Ndlovu

Tom Penlington

Craig Pournara

Gary Powell

Maoto Satsope

Jackie Scheiber

Menare Setati

Sibawu Siyepu

Debbie Stott

Cosmas Tambara

Roy Venketsamy

Review Process

Each of the submissions accepted for publication in this volume of the Proceedings (Short, Workshops, How I teach papers and posters) were subject to blind peer review by an experienced mathematics educator. The academic committee considered the reviews and made a final decision on the acceptance or rejection of each submission, as well as changing the status of submissions.

Number of submissions: 105

Number of plenary paper submissions: 5

Number of long paper submissions: 34

Number of short paper submissions: 14

Number of workshop submissions: 31

Number of 'How I teach' paper submissions: 10

Number of poster submissions: 2

Number of submissions accepted: 98

Number of submissions rejected: 5

Number of submissions withdrawn by authors: 2

We thank the reviewers for giving their time and expertise to reviewing the submissions.

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CULTURAL GAMES AND THEIR RELEVANCY IN TEACHING NUMBER CONCEPTS IN GRADE R

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There is a need for integrating skills and knowledge gained outside the classroom that stimulates and nurtures mathematics learning in South Africa. Literature indicates poor foundational knowledge in the early years whereas a lot of activities and mathematics games are experienced by young learner's prior and during their early years of learning. Hence, this paper explores traditional games for mathematics development at Grade R level. Sixteen teachers, learners and community workers participated in an IKS/NRF study that listed all traditional games played by learners outside the classroom. This list was then explored through classroom lessons and taped for reflections and thematic analysis. The findings of this exploration indicate that most of the games have potential for mathematics development, however some of them are gender biased and developmentally inappropriate.

INTRODUCTION

Mathematics Education is a big challenge in the South African context. The country battles to address this challenge as indicated by international studies (TIMSS, 2011, Spaull, 2011). Ethnomathematics has been proposed as the relevant cultural pedagogy for teaching Mathematics (Gerdes, 1998, Laridon, Mosimege & Mogari 2005). However, a number of shortfalls have been observed when it is not planned thoroughly (Feza, 2012, Mosimege and Onwu, 2004). Some of the cultural artefacts used bring stereotypical attitudes to the fore. Some favour girls over boys and vice versa (Feza, 2012), such as wire cars, string games etc. Poor planning results in one to focus more on cultural artefacts than the mathematics learning (Mosimege and Onwu, 2004). Research has indicated that the use of games in the teaching of mathematics stimulates learners' thinking and captures their interest (Nabie, 2012). One of the major reasons learners enjoy the use of games is that they involve play that comes natural to children. A structured curriculum for Grade R started after 2011 when the new CAPS curriculum was introduced. CAPS (2011) emphasises that young learners learn through play, using their senses, imitation, storytelling and mimicking adults. Games give both teachers and learners an opportunity to play and learn; hence our paper revisits the use of ethno-mathematics in teaching for learner gains by addressing the shortfalls. This paper aims to introduce isiXhosa cultural games to Grade R for developing number sense with the aim of discovering their role and relevancy in learning. In achieving this, this paper will address the following questions: (1) Which cultural games are relevant for Grade R learners? (2) If relevant, what counting concepts can be mediated with these cultural games? (3) Are these cultural games accessible to all learners?

RESEARCH DESIGN

The study is qualitative in its approach for its strength lies in unravelling and exploring phenomena for better understanding. A list of traditional games were collected amongst Grade R teachers, learners and community members with the aim of exploring their role in mathematics and their appropriateness for Grade R learners. The exploration was done through 16 classrooms participating in an IKS/NRF study to capture games that have potential for learning mathematics and also those that are appropriately developmental to Grade R learners. All 16 teachers, learners and community workers were video-taped playing the games and a reflection workshop was used to tease out challenges experienced on the appropriateness of the games. The notes collected from the workshop were then taken away and validated by three researchers who took the taped lessons and analysed the games presented responding to the research questions separately. The three individual analyses were compared with the workshop notes using written memos and this was done to reach consensus on different items. Themes emerged and these are reported as findings.

LITERATURE REVIEW

South Africa is continuously challenged by the mathematics performance of its learners. African learners in South Africa seem to be the poorest performers in the country as indicated in national and international studies (TIMSS, 2011). Researchers are tirelessly exploring the best approaches in tackling this problem. However, most factors that influence this status quo are beyond educational research such as scarcity of resources and scarcity of mathematics qualified teachers. This forces researchers to establish strategies that would improve this situation with what the system provides which is a learner from a poor home and a teacher with poor or no mathematics education qualification. The socio-cultural theory lead by Vygotsky (1978) suggests that learners' experiences and observations can be used to empower their abilities in facing new situations. In this case games carry potential to become tools that could be used as strong foundational tools that can be developed further for learning gains.

Ethno-mathematics cultural artefacts

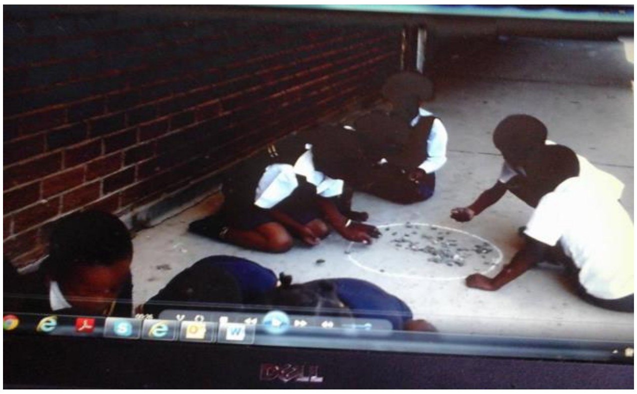
This paper focuses on cultural games as the artefacts that are used as models for mathematical ideas. Ethno-mathematics pedagogy has revealed successful learning when cultural artefacts such as wire cars, and games were used by enthusiastic teachers who practiced a learner centred approach (Nkopodi and Mosimege, 2009; Laridon et al., 2005). The challenge of these studies as highlighted by Feza (2012) is that they bring in gender stereotypical views through use of cultural artefacts that favour a particular gender. This paper therefore aims to guide against such practices as they take away a mathematics activity and nurture a cultural activity. Therefore, this paper uses games that are known by all children and not gender based; hence, exploration lead to this data collection. Two approaches that have been suggested are teaching about culture or using cultural artefacts or ideas to mediate mathematical ideas (Moses-Snipes, 2005). This paper explores the second one of using cultural artefacts to develop mathematical ideas of learners. In cultivating learners' self-esteem in a mathematics classroom, it should be noted that familiarity of ideas and practices develops a positive relationship between a learner and mathematics (Feza, 2012). This notion has been supported by the empirical evidence of studies that exposed learners to ethno-mathematics pedagogy (Moses-Snipes, 2005; Mosimege and Onwu, 2004). The success of this pedagogy has been claimed to go hand in hand with teacher's sound mathematical knowledge as well as the power to implement this pedagogy lies in teacher confidence (Laridon et al., 2005).

FINDINGS

Two themes emerged from the data. These were developmentally appropriate games as determined by motor skills and games that captured the interest of all learners.

DEVELOPMENTALLY APPROPRIATE GAMES AS DETERMINED BY MOTOR SKILLS

Generally, learners showed love and interest in all games regardless of challenges. For example, in one class teachers were exploring a game called "**upuca"** as shown in the pictures below:



Upuca

It is a traditional game among Xhosas as well as other Africans in Southern African countries that use pebbles and can be played by a number of children. There was a belief in Xhosa culture that if children play upuca, it will rain. A circle is drawn on the ground where these pebbles are placed. A player has to have one bigger stone, a spherical stone called ingqanda in his/her hand. A player throws ingqanda up into the air, removing as many stones from a circle and puts them back leaving only one stone outside the circle if the focus of the game is playing in ones. If you play two's only two stones will be left outside a circle. The game continues until you fail to leave one or your stone that you throw up falls down.

Mathematical concepts and skills that can be learned

The game develops skip counting in one's, two's, three's, four's, five's … to tens. Subitising develops automatically. It is a good game for developing eye- hand coordination, time control and motor skills. Number sense knowledge from "upuca" starts with grouping of objects into numbers. This game is developmental as once the stones are finished from the circle the number increases from one to two etc. The main counting part is at the end of each game as the winner is determined by the number of stones s/he managed to pull out properly. The game promotes skip counting; if the game was in ones then the counting will be in one's, if it was in twos then the counting will be in two's. This is where multiples develop from skip counting.
As much as learners wanted to play this game, they struggled as they were not able to co-ordinate their eye movement together with hand movement. This game had a lot of mathematics knowledge to learn from including eye-hand coordination skills.

This game is not developmentally appropriate for Grade R learners in the beginning of the year as their hand-eye-co-ordination skills are still developing. Mathematically there are many possibilities as some learners come to school already counting and even skip counting small numbers.

Ugqaphu (rope skipping)

Ugqaphu is a cultural game which uses a minimum of six learners; two learners who swing the rope and the other learners that skip in turns.

Mathematical concepts and skills that can be learned

This game allows the learners to have good sequential verbal counting skills, one to one correspondence and cardinality. At this level, it builds gross motor skills and develops interpersonal skills. This game requires supervision since learners can be a problem when playing it. Rewards for good behaviour must be done to encourage them to behave.

This game is developmentally appropriate as it does not demand co-ordination of many limbs at the same time. For example, those who are skipping need to jump watching the rope that is controlled by others. The complexity of using more than one sense is not there.

Itreyini / Train

This game requires two learners to stand facing each other and act as a train tunnel. The two learners use both arms and use the tips of their hands to touch each other to form the train tunnel. A minimum of 22 learners, standing in a line, one after the other, touching each other's back, act as the train carriage. The two members who act as a train tunnel are given names, for example chocolate team and biscuit team. The train carriage (for example 22 learners) moves slowly passing through the train tunnel. Each time the train carriage passes through the train tunnel, the two train tunnel members randomly pick up a learner using their elbows. The learner who has been caught by the train tunnel members will be asked to decide which team she/he wants to be part of. Then the learner will become a team member of the chosen train tunnel and will stand behind the team leader. In this way the number of train tunnel members will increase and the train carriage number of learners will decrease. The game will continue until all the learners are either part of the chocolate or biscuit team. Each team will count the number of its members. Then the two teams will participate in a tug of war where learners from each team standing in a line will start to pull the other team. This will continue until the members of one team collapse into the learners of the other team. The team that pulls all the members of the other team is the winning team.

Mathematical concepts and skills that can be learned

This game develops addition, subtraction, equality and measurement. The skills learnt are sorting, and balancing. All learners enjoy this game and this game is developmentally appropriate for Grade R learners. It also introduces team work as each side will only win if there is co-operation when they are pulling.

GAMES THAT CAPTURE THE INTEREST OF ALL LEARNERS

These games do not demand more co-ordination but are fun and competitive like 'how fast can you run'.

Umlilo phezu kwentaba

This is a group game where learners group themselves in one's, two's, three's etc. and then count themselves. If the number is more or less than the stipulated number they are out of the game. There is a song that is used in this game where the teacher sings the song and the learners respond to it.

Mathematical concepts and skills that can be learned

This game develops composition of number, counting and subitising. The game develops gross motor skills, listening skills, and collaborative skills. The game is developmental as the groups increase their mastery of certain numbers and therefore can be played throughout the year in Grade R and can be extended to an addition game or counting on game.

Black Toti

The game is played by 3 players and 3 opponents with ten cans of different sizes and a ball. A square is drawn on the ground and in the middle of the square you put the ten cans in a pyramid style. The three players stand on one border of the square and opposite that border stand the opponents. One player carries a ball, which will be used to hit the cans. When the ball hits the cans, they will be scattered all over the playground. The players run back and forth along the borders of the square, counting each time they reach the border. One of the opponents runs to pick up the ball while the other two opponents rearrange the cans again in a pyramid form. When the ball is picked up, the opponent throws the ball back to hit one of the players as they are trying to count to 24 to win. If they cannot count to 24 before the balance of the cans has been restored, their game is over.

Mathematical concepts and skills that can be learned

Black toti develops sorting, counting, motor skills, balance and timing. The game is developmental. There is lot of logic that is practised. Each other's ability is recognised. In higher classes, more cans are used and more mathematical skills are practised. Learners can continue playing and recording can be done.

ACCESSIBILITY OF GAMES

Of all the games explored here only one game is gender biased "ugqaphu." This game seems to favour girls. Teachers need to be aware of this limitation and develop strategies that will be inclusive to all or eliminate the game if such cannot be developed. "Upuca" one of the popular games needs careful planning and timing as young learners could be frustrated when they are not ready for the complexity of hand and eye co-ordination. Therefore, teachers need to only use this game towards the end of the year to avoid alienating learners from the fondness of the game. All other games explored here carry potential for mathematics development in the classroom.

CONCLUSION

Games carry power for learning and for a number of other reasons. Learners participate voluntarily when playing games. They learn while having fun and develop not only their cognitive skills but also their physical strength. Games can be used as a vehicle to encourage purposeful play in the classrooms of Grade R learners. In addition, games allow for the extension of skills and changing of rules making games more complex and demanding which is an asset for developing higher order thinking skills. Number sense development which is a broad topic in mathematics is covered by playing these games. It accommodates gifted learners by allowing them to extend the rules of the game themselves while other learners are gaining practice and mastery.

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DESIGNING A PROFESSIONAL DEVELOPMENT INTERVENTION TO SUPPORT MATHEMATICS TEACHERS IN THE TEACHING OF PROBLEM SOLVING

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This paper reports the design and findings of the first iteration of a large classroom based design research project which endeavors to design a professional development intervention for mathematical problem-solving pedagogy. This paper details the lessons learned from the study and how design-based research facilitated the development of a professional development (PD) intervention in a localised context. Two experienced grade 9 mathematics female teachers and their learners at a public secondary school in Gauteng, South Africa, participated in a six-month intervention. Findings from the data are discussed in light of their implications for the next cycle and other PD researches.

INTRODUCTION

Many curricula (National Council of Teachers of Mathematics (NCTM), 2000; Singapore Mathematics Curriculum (SMC), 2013; The South African Department of Education's Curriculum and Assessment Policy Statement (CAPS), 2011) consider problem solving as an essential aspect of mathematical teaching and learning. The CAPS states that problem solving and cognitive development should be fundamental to all mathematics teaching. However, this is not what a number of researchers have found in South African mathematics classrooms. Traditional methods of teaching mathematics are still prevalent in the South African context (Adler & Ronda, 2014; Mji & Makgato, 2006; Mullis, Martin, Foy & Arora, 2012). In response to this identified problem, we proposed to design a PD intervention that can be used to support mathematics teachers in teaching problem solving.

Traditionally, PD is normally delivered in the form of workshops, college courses, seminars or conferences (Villegas-Reimers, 2003) but these approaches have been fiercely criticized for their ineffectiveness since they are not directly related to an individual teacher's practice (Hawley & Valli, 1999). For this reason, we engaged the process of design-based research (DBR) to tailor a PD intervention that would be appropriate for teachers in a particular local context. We formulated and sought to answer the following research questions:

- What is the impact of the PD intervention on learners' learning processes and teachers' teaching of problem solving?
- What factors facilitated learners' learning and teachers' development of pedagogy?

- What are the possible design principles required to generate a professional development intervention on mathematical problem solving pedagogy for Grade 9 teachers in a particular local context?

LITERATURE REVIEW

Problem Solving

Problem solving is the whole process of dealing with a problem (Wessels & Kwari, 2003). NCTM (2000, p.52) defined it as "engaging in a task for which the solution method is not known in advance", and Polya (1957) saw it as finding a path around a challenge or an obstacle and finding a solution to a problem that is not known. Polya (1945), Krulik and Rudnick (1980) and Barmby, Bolden and Thompson (2014) have identified steps in the problem solving process. We chose Polya's steps to base our PD intervention on because they encapsulate the key aspects of mathematical problem solving that comprises this study.

Polya (1957) proposed a four-phase problem solving process, with identifiable strategies:

- understanding the problem;
- devising a plan or deciding on a strategy for attacking the problem;
- carrying out the plan; that is learners follow through with the strategy selected, carefully taking each step along the away and;
- looking back at the problem, the answer and what you have done to get there.

Effective professional development for teachers

Our objective for the main study is to design an effective PD intervention that can be used to support mathematics teachers in the teaching of problem solving. Wei, Darling-Hammond, Andree, Richardson, and Orphanos (2009) define professional development as the processes and activities designed to improve teachers' knowledge, the practice of instruction, and the learning outcomes of students. Villegas-Reimers (2003) advocates that for professional development to be effective it must be perceived as a process that takes place within a particular context and must be situated in classroom practice. Villegas-Reimers further accentuates that professional development is not similar in different settings, that is, what works for teachers in a certain area may not work for other teachers in a different area. Consequently, we adopted classroom-based design research in order to design a PD intervention that would work specifically for teachers in our context of study.

THEORETICAL PERSPECTIVE

Social constructivism informed this study. The social constructivist perspective to learning mainly originated from the work of Vygotsky (1986). It emphasizes that culture and social contexts are important in understanding what occurs in society and acquiring knowledge based on this understanding. This study took place in an under-resourced African context, where the teachers were working with large classes, with learners with English as a second language. The teachers had a wealth of experience of teaching mathematics based on their own cultural and social experiences. Therefore, we began with the teachers' experiences and how they were teaching problem solving, and we employed the DBR approach to develop the PD intervention within this context.

METHODOLOGY

Research design

Design-based research (DBR) (Kelly, 2003) is a relatively new research approach in the field of educational inquiry. DBR aims to solve real-world problems by designing, enacting and sustaining interventions (Van den Akker, Gravemeijer, McKenny & Nieveen, 2006). A number of researchers have attempted to give a definition of DBR and there is a discussion underway of what constitutes DBR (Van den Akker *et al*., 2006). We found Wang and Hannafin (2005)'s definition very persuasive. Wang and Hannafin (2005, p. 6) define DBR as

> a systematic but flexible methodology aimed to improve educational practices through iterative analysis, design, development, and implementation, based on collaboration among researchers and practitioners in real-world settings, and leading to contextually-sensitive design principles and theories.

There is a general agreement that DBR should generate valuable educational interventions and useful theory (Van den Akker *et al*., 2006). DBR uses the terminology of 'intervention' to refer to the object, output, and an activity or process that is designed as a possible solution to the identified problem. This study generated and examined a professional development intervention for mathematical problem solving pedagogy for grade 9 mathematics teachers in a certain district in Gauteng.

Context and the design process

The larger classroom-based design project focuses on designing a professional development intervention on mathematical problem solving that can be further modified and used with schools in South African contexts. The project has three iterative cycles. In March 2016, we conducted a baseline investigation with 31 teachers at 20 schools in the district of interest. The baseline investigation examined how grade 9 mathematics teachers in this district were using problem solving in their teaching of mathematics. We 'purposefully' selected three schools A, B and C out of the initial 20 schools. These schools were chosen because they could be conveniently accessed by the researchers and the grade 9 mathematics teachers reported on the open-ended questionnaire that they were using traditional methods of teaching. This paper reports on the first iteration of the larger project, which is the phase of the intervention in which we worked with two grade 9 female teachers in school A. The baseline investigation and a literature review that was conducted from May 2016 to December 2016 informed the design guidelines and specifications for this study.

Design of the professional development intervention

The PD intervention is designed to take place within a period of six months. The goals we set for the PD intervention were to improve learners' performance in mathematics and support teachers' mathematical problem solving pedagogy. We also aimed to explain and agree with participant teachers what mathematical problem solving pedagogy is and what it is not. We conducted three professional development workshops with teachers on the last Wednesday of each month. The workshops were from 12 p.m. to 3 p.m., three hours per workshop. This resulted in a total of nine hours of training for participant teachers. We carried out the training at a community centre in this district. These workshops were different from the traditional 'one-shot' workshops in that the teachers attended the workshop three times, collaboratively learned from each other during the intervention, were actively engaged in meaningful discussion, planning and practice (Loucks-Horsley, Hewson, Love & Stiles, 1998) and we observed and supported the teachers during the implementation. We selected PD activities that offered teachers the opportunity to become actively engaged in the meaningful analysis of teaching and learning.

In the first workshop, we initially presented the workshop's contents to the participant teachers who then watched two short videos on mathematical problem-solving pedagogy in action. The baseline investigation unearthed that participant teachers were uncertain what mathematical problem solving pedagogy involves. Therefore, these videos were to show the teaching of mathematical problem solving in action. We discussed the videos, focusing on what genuine mathematical problem solving pedagogy entails and how to apply Polya's (1957) four steps of problem solving as a teaching process. Teachers expounded on ways of introducing or posing the problems in such a way that learners understand the given problems. Teachers collaboratively solved at least two 'rich' and open-ended mathematical tasks relating to the work they were teaching, and with our guidance discussed how to teach problem solving as a process.

After attending the first professional development workshop, teachers were encouraged to go and implement the new ideas in their lessons for a month. During the implementation stage, we observed, supported and guided the participant teachers as was necessary and audio taped the lessons. Throughout the implementation phase participant teachers were encouraged to continuously reflect on their experiences as learners of the professional development programme and on their classroom practices.

After the first implementation, we conducted the second workshop where the aim was for teachers to further collaboratively reflect on their teaching experiences and to review the audio tapes of the observed lessons. We selected crucial and relevant audio recordings that foregrounded participant teachers' use of problem solving in their teaching. Participant teachers analysed how they had taught mathematical problem solving and they watched two further videos showing mathematical problem solving pedagogy in action. Once again teachers collaboratively solved mathematical tasks relating to what they were teaching and planned on how to teach similar tasks to their learners. After the second workshop the teachers implemented new ideas for a month whilst being observed, supported, audio-taped and interviewed by the researchers. We also encouraged the teachers to continuously reflect on their experiences and classroom practices. The third workshop and the implementation process were similar to the second stage

Data sources

Teacher data was collected through classroom observations and semi-structured reflective interviews. Classroom observations, which were audio-taped, were conducted before, during and after the PD intervention. In total, we had 13 observations with each teacher. Semi-structured interviews with the teachers were conducted with each teacher once a month. In total, we had 5 interviews with each teacher. In terms of assessing the possible impact of the PD intervention on participant learners, we gave them pre- and post- mathematics attainment tests and a self-reporting mathematical problem solving skills inventory (MPSSI) questionnaire at the beginning and at the end of the intervention. We used teachers' pre- and post- mathematics attainment tests that covered the topics that the learners were doing (geometry and data handling). Both tests had 15 questions and the marks were converted to a percentage. The attainment tests were useful in evaluating learners' ability to use mathematical problem solving skills because they required learners to supply the answers thereby avoiding guesses.

Data analysis

Our raw data included recorded notes on the observation comment card, audio-tapes from the classroom observations, semi-structured interview recordings, test marks and responses from the MPSSI. Audio tapes from the classroom observations and semi-structured interviews were transcribed verbatim into written notes in order to be able to identify common patterns and experiences. We employed inductive data analysis to analyse the teacher qualitative data from the classroom observations and the semi-structured interviews (Hatch, 2002). The statistical software package SPSS was used to analyse the student quantitative data from pre and post mathematical tests and the MPSSI.

FINDINGS AND DISCUSSION

The pre/post-test and pre/post-MPSSI demonstrated that the PD intervention implementation had a positive impact on learners' performance. However in the next cycle, we are going to conduct task-based interviews with the learners; we felt that there was a need to explain in detail why learners did well in their post-tests and why they gave better responses in the post MPSSI.

The classroom observations we conducted before we implemented the intervention unearthed that teachers relied solely on the DBE prescribed textbook. They introduced the topic by demonstrating the examples in the textbook on the board and there was no clarification of the task at hand. Classroom observations confirmed that after we implemented the PD intervention, teachers were working in a different way with learners and emphasizing problem solving approaches. This finding resonates with Barber and Mourshed (2007)'s results on the research of 25 national school systems, that there is a positive correlation between teachers' professional development and learners' achievement.

The pre-intervention observations revealed that the lessons were teacher-led and teachers implemented traditional methods of teaching. This finding aligns with what Adler and Ronda (2014) have exposed that South African classrooms have remained teacher-centered. As the intervention progressed we noticed that the participant teachers became more of facilitators than teachers; giving learners the opportunity to grapple with the given problems; and encouraging group work and discussion among learners. Collaborative learning was beneficial to participant teachers and learners. When learners grappled with the problems in pairs or groups, it allowed for richer and worthwhile whole class discussion. This outcome concurs with Cordingley et al. (2005)'s finding from systematically reviewing 17 studies of collaborative and/ or sustained continuous PD in diverse contexts. They discovered that when teachers engaged in collaborative PD there were vast improvements in learners' learning and behaviour, and in teachers' practices.

Classroom observations revealed that English language, the language of instruction, proved to be an obstruction to students' understanding. Polya (1957)'s first step in mathematical problem solving is to understand the problem, and if learners struggle with language, it means that they do not even understand the given problems. It is impossible to separate the language of instruction and the problem solving process (Chirinda & Barmby, 2017). The finding on language as an obstacle to learning in South African mathematics classrooms is in agreement with what a number of researchers have disclosed in the past (Adler, 2001; Webb & Webb, 2008). We recommend that a PD programme should comprise a segment that supports teachers on how to appropriately conduct code-switching during lesson delivery.

Initially, teachers were reluctant to participate in the intervention and implementing the mathematical problem solving pedagogy; they indicated that they were worried that they would fail to cover the CAPS syllabus within the prescribed time. This finding aligns with what Slattery (2013) uncovered, that syllabus and task completion places pressure on teachers. We had discussions with teachers so that they could realize that in problem solving, it is not compulsory to complete a task on the same day but the task can still be completed the next day. We agreed that the important issue was to enhance mathematics understanding. Teachers were missing the point that the knowledge gained and the procedures used in solving a specific problem may be applied to other problems and thereby addressing the time factor. As the intervention progressed, teachers began to understand mathematical problem solving pedagogy and started to implement the new ideas.

The semi-structured reflective interviews with participant teachers were imperative to the research process. As teachers looked back on classroom events during the interviews and made critical judgments about them, they modified their teaching behavior and this resulted in them constructing knowledge about themselves, their teaching practices and their students (Schunk, 2012). As participant teachers knowingly and systematically reflected on their teaching experiences (Farrell, 2007), we realised that they were consciously able to improve their own teaching. This procedure by teachers thinking about what they were doing and why they were doing it turned their experiences into meaningful learning. In this case, learning by teachers did not just happen but was derived from them constructing sense from their experiences and particular contexts.

A Follow up on teachers in the classrooms to check if they were correctly implementing the mathematical problem solving pedagogy, was advantageous as we were able to support teachers as was necessary. This is different from the traditional 'one-shot' workshops and we recommend that professional development practitioners should support the teachers in the classroom during the implementation stage.

Design principles

Design principles are one of the major outputs of DBR and McKenney and Reeves (2012) term them theoretical insights into a phenomenon in question, which recommend how to address specific issues in a range of settings. While we acknowledge that we have gone through only one cycle of the DBR, the study generated a number of design principles that will inform the next cycle of the study and are imperative to mathematics education and PD practitioners who are in the process of designing professional development interventions. The design principles include:

- A baseline investigation must be conducted to establish teachers' perceptions and practices on the teaching of mathematical problem solving before implementing the PD intervention.
- Facilitators of PD must create a positive relationship with participant teachers before implementing the intervention.
- PD should be built from teachers' experiences and current knowledge of mathematical problem solving.
- Facilitators of PD must observe teachers practically implementing the mathematical problem solving pedagogy and support them as necessary.
- PD should support teachers on how to implement mathematical problem solving pedagogy in a multi-lingual context.

Implications of the findings for the next cycle

One of the most significant outcomes from this cycle is that language stands so much in the way of learners' learning of mathematics. Therefore, in the next cycle, we will do a lot more with teachers on how we can support learners' difficulties with problem solving, English language and code-switching. In this cycle we used teachers' own tests to act as our pre- and post-intervention tests but in the next cycles we are going to design our own tests so that we can easily check on the reliability of these tests. In the next cycle we will also introduce learner task-based interviews to help us explain why learners were doing well in the post-intervention test and why they were giving better responses in the post-intervention MPSSI. We anticipate that the task-based interviews will unearth participant learners' problem solving processes and if after participating in the intervention learners will be genuinely good at problem solving or doing it differently. In the next cycle participant teachers will also be observed delivering lessons by other participant teachers and given feedback. This development emerged from the usefulness of teachers observing practice (in the first cycle from videos) and the usefulness of working on and reflecting collaboratively on lessons.

CONCLUSION

The findings of the study may prompt other researchers to develop PD interventions in local contexts. The study, having been done at one school, means that the transferability of the findings to larger contexts can be challenging. However, we believe that the above design principles can be used by other researchers as starting points for developing PD interventions for mathematical problem solving pedagogy for localised contexts.

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AN ALTERNATIVE WAY OF SOLVING GEOMETRY RIDERS IN GRADE 12: BACK TO SYNTHESIS AND ANALYSIS

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The way we teach Geometry has been debated over many years. Van Hiele (1986) and his wife developed a systematic way to approach geometry. Their approach involved various interconnected, interdependent thinking levels where a higher level could not be achieved unless all the lower levels were achieved. One important point that has been missed by the literature is all proofs use an inductive approach. But to use such an approach implies the person knows the path to follow. The shortfall in Van Hiele's idea was the assumption that if one follows the prescribed approach the learner would be able to solve riders. However, an analysis of answers of learners to riders points to a lack of strategies to solve riders. Nevertheless, geometry has been recognised as a difficult part of mathematics to teach and learn. The reason could be that procedural and conceptual knowledge appear simultaneously. This paper aims at guiding the teacher and the learner with declarative knowledge (theorems) towards solving the rider. The idea is based on Giannakopoulos's (2012) mathematics structures combined with analysis and synthesis.

SOLVING GEOMETRIC RIDERS AT SENIOR SECONDARY HIGH SCHOOL

Geometry, although a part of mathematics, might be considered as more complex even though it might appear less abstract than the rest of mathematics (Hoyles, 1998; Adolphus, 2011), at least at secondary education level. The problem of the teaching and learning of mathematics has been researched widely for decades and to a lesser extent the teaching and learning of geometry. A literature review has shown that although many factors have been identified and some progress has been made, the teaching and learning of Mathematics remains problematic even at the foundation phase let alone at grade 12 level. In fact, the performance in geometry tends to be much worse than performance in algebra, analytical geometry and trigonometry (Adolphus, 2011).

Geometry, being part of mathematics, comprises about 30% of the final NCS grade 12 paper in South Africa. A number of authors (Ali, Bhagawati & Sarmah, 2014; Hoyles, 1996, Ozarem, 2012) agree that geometry and the teaching and learning of geometry is complex as it requires the existence of many cognitive functions simultaneously. It is a subject that is used by other fields (Adolphus, 2011; Biber, Tuna & Korkmaz; 2013). Adolphus (2011) sees geometry as "the bedrock of engineering and technological development." It requires abstract thinking, logic, ability to visualize abstract concepts, argue backwards and forwards (synthesis and analysis), justify every step when solving a problem and above all it requires critical thinking (Ozarem, 2012, Ali et al., 2014; Gloria, 2015). In fact, Ozarem (2012) goes as far as to say that the reason for fear in mathematics is mainly due to poor geometrical conception.

A brief literature review on the problem has identified a number of factors that can be classified as, learner factors, teacher factors and institutional or environmental factors.

Factors that influence the teaching and learning of Geometry

It can be argued that there are three categories of factors that affect the teaching and learning of geometry. Learner factors, teacher's pedagogy and teaching methodologies. With respect to the learner, the first point that literature reveals is that the performance of learners in geometry is poor (Ali et al., 2014; Adolphus, 2011). This could be attributed to a number of factors such as poor foundation of basic knowledge from their primary stage (Ali et al., 2014; Amazigo, 2000), lack of willingness and readiness to learn geometry (Ali et al. 2014; Amazigo, 2000) with learning being mostly instrumental (lack of deep understanding) (Ali et al., 2014; Mayberry, 1983); the teaching environment is not conducive to learning (lack of properly trained teachers (Ali et al., 2014; Amazigo, 2000); imbalanced teacher-learner ratio (Ali et al., 2014); lack of physical models (Adolphus, 2011). Furthermore, errors or misconceptions of learners (Herholdt & Sapire, 2014; Luneta, 2015; Ball, Thames & Phelps, 2008; Clements & Battista, 1992) are other factors that have been extensively researched. Lim (2011) found that many of the misconceptions by the learners are because teachers and learners operate on different geometric levels (here the author refers to Van Hiele's (1986) levels).

With respect to the teacher, lack of Pedagogical Content Knowledge (PCK) by the teacher as defined by Shulman (1986) and that of Hill, Ball and Schilling (2008) is another important factor. Ellerhorst (2014) also identified years of experience, degree level, certification type and gender impact learners' performance in geometry.

Ellerhorst (2014) found these to be contrary to the assumption that these four factors possessed by teachers (though a small number) with a few years of experience, lower qualifications and certification affected learners' performance positively. Although Ellerhorst (2014) does not make any suggestion as to why this is the case, based on 40 years of experience in teaching and having held the head of department position a probable factor could be passion for teaching. Passion leads to motivation to improve one's teaching and one cannot teach passion to new teachers! Ellerhorst (2014) suggests that more research is necessary. Ackerman, Heafner and Bartz (2006) state that quality teaching in general impacts learners' performance in geometry.

With respect to the teaching of geometry, work by Van Hiele could be considered as an important breakthrough in the teaching of geometry. Combining Van Hiele's (1986) ideas with Bloom's (1979) taxonomy could be a partial solution into the teaching of geometry. However, as it will be shown, they are not sufficient to assist the learner solving geometric rider problems. How to teach geometry (Van Hiele, 1986) and how to assess learning does not assist the learner in solving problems. And if we accept Van Hiele's statement that higher levels of thinking in geometry can only be achieved through instruction, then it should be accompanied by a particular geometric method of solving geometric riders.

Geometric riders: Inductive or deductive approach?

A review of the existing literature has highlighted the difficulties that learners encounter in solving geometric riders. The teachers on the other hand follow the 'text book method' in a kind of sub-conscious way and they transfer that to the learners too and hope that if the learners understand the theorems and can apply them everything will be ok. However, it is not. What teachers do not realize is that all textbooks use the inductive approach, meaning the writer knows the beginning and the end. Nevertheless, to the reader is not obvious why one should start from X and not from the Y statement. For the reader it could be a 'hit or a miss'. What we need is a method that the problem solver knows exactly where to start, assuming he/ she knows where to end.

Before one suggests an alternative way of solving geometric riders, it is necessary to make two assumptions:

a) The learner knows and understands and can apply learned theorems. More than 90% of geometric riders are about theorems. If the learner does not know the theorems, he/she cannot do riders.

b) The teacher is competent in geometry (Ali et al., 2014). Geometry can only be learned by instruction (Van Hiele, 1986). If (a) it is true and the teacher is not competent, then the learner stands a little chance of understanding geometry.

To simplify the problem of solving riders, we can divide the information of a rider (assuming there is a diagram given) into:

a) The visual part (diagram – explicit). This is the most basic level of Van Hiele's theory.

b) The given information (for example, AB = 20m, <ABC = 30°) (explicit)

c) Information that theorems are involved (parallel lines, intersecting lines, cyclic quads and so on) and stated (for example, given ABCD is a cyclic quadrilateral) or not stated (for example if two lines intersect, or a triangle it is not necessary to state the lines intersect or ABC is a triangle). A lot of geometry involves making deductions from certain primary information. This is what Van Hiele calls formal or informal deduction. For example, we cannot prove two lines are parallel, a triangle is isosceles, or a quadrilateral is cyclic. These concepts are constructs. Constructs are indicators (variables) that measure something. These indicators depend on factors and it is the factors that are measured. In geometry, geometric constructs have attributes. It is the attributes that we try to determine whether they are possessed by a shape. Therefore, to prove that two lines are parallel (construct) we can try to prove that certain angles are equal (for example, corresponding); A triangle is isosceles (construct) if the base angles are equal; A quadrilateral is a cyclic quadrilateral (construct) if the exterior angle is equal to the interior opposite (implicit, information has to be deduced).

d) Information to be derived. That is, given all the information determine X, or prove Y. (implicit)

The (c) part is the most critical one, because the learner must be able to derive all possible information but at the same time use only what is necessary and choose the best path to a solution. The suggested method assists the learner to do just that: Use only necessary information and know the path to follow.

It is necessary though to define what is meant by analysis and synthesis in the context of geometry. Literally speaking analysis is the breaking down of something into its individual components, like 'dismantling' a car. One has to follow a certain sequence in doing so. Putting the parts back together is synthesis. Whatever sequence was followed in analysis the reverse sequence has to be followed. In a geometric sense, it is more than that; it is necessary to understand each construct and its attributes and justify why a certain statement is made. It is related to Van Hiele's (1986) levels of formal and informal deduction. Furthermore, solving a geometric rider can be considered to be a simultaneous application of all 5 levels of Van Hiele.

In solving Geometric riders, assumptions can be used. In the analysis part, we can start from the unknown (required to prove) statement and assume it to be true. A further assumption can be made to arrive at another unknown statement and so on, till we arrive at a known statement (a statement that can be proved, or given information). In the synthesis part, we write the solution by reversing the process. Knowing X we can prove Y. Knowing Y we can prove Z and so on. This method has the advantage that the path is constructed during the analysis. The problem solver knows exactly where to start the solution. But this method cannot be used if the problem solver does not understand the geometric structure. An important point here is that learners are not encouraged to construct their own accurate diagrams, which contribute to getting a better insight into the geometric structures (Hoyles, 1996).

The study of Geometric structures

The idea used by Giannakopoulos (2012, 2015) for differentiation could be applied here to develop the geometric structures. The following rules could be used. If we consider a rider as a geometric structure,

a) It could consist of points, lines and various shapes, special lines such as radius, tangents, perpendiculars, medians, bisectors, circles and so on (the bricks of the structure).

b) These could be fundamentally related (that is satisfy various theorems/ relationships) or by construction (that is Let line AB//CD) (the mortar that binds the bricks)

c) The sequence of constructing a rider could follow one and only one route (one way of solving the problem), or multiple routes (multiple ways of solving the problem).

d) It is the synthesis of (b) and (c) that could solve the problem.

In order to solve the problem we need to divide the extracted information into primary and secondary information. We can classify as primary information the one that is contained in Grade 11 and 12 theorems and given information, while secondary information is that derived from previous grades such as parallel lines, vertically opposite angles, angles in a triangle, supplementary or complementary angles, properties of an isosceles triangle and so on.

In order to illustrate this let us use the following two riders (see Figures 1 & 2). From Figure 1, how this rider could have been constructed (the most probable one based on logical argument; justifying why that is the sequence).

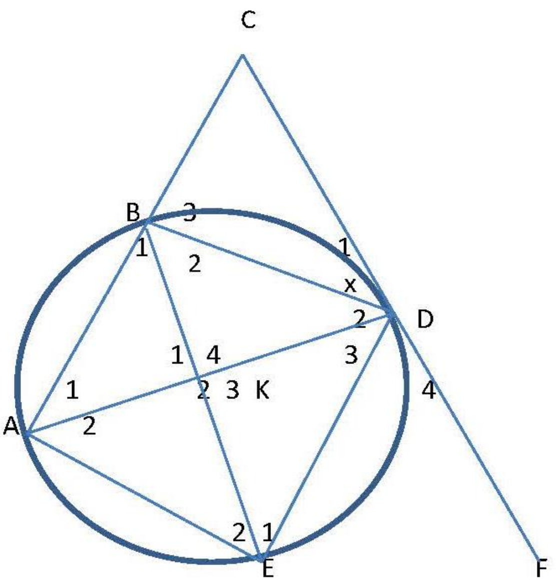

Figure 1: A Geometric rider A

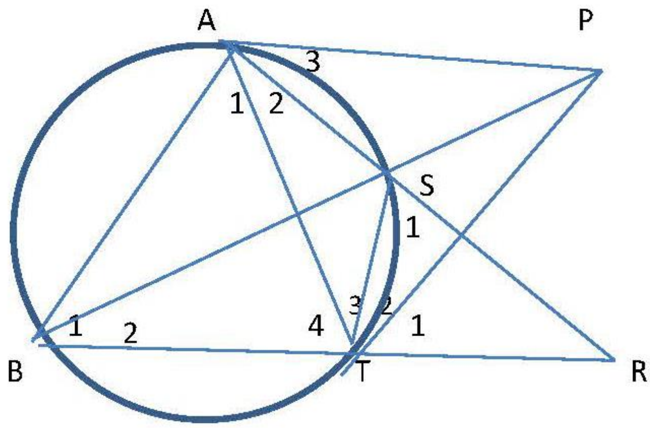

Figure 2: Geometric rider B (1985 NSC)

In Figure 1:

Given Cyclic ABDE CDF tangent to circle at D. diagonals AD and BE. AB=AE, DB=ED, D_1=x. Required to determine:

a) Angles E_1 , B_{12}, E_{12}, C, B_1 and E_2 in terms of x.

b) AKB = 90°.

In Figure 2:

Given tangents PT and PA. Sides AS and BT of the cyclic quadrilateral BAST extended meet at R and <A=<B

Prove: a) AB//ST

b) RS = RT

c) $T_1 = B_1$

d) $A_1 = T_1$

e) $P = B_1 - T_2$

Analysis1

There is a circle, a tangent, a random cyclic quadrilateral with pairs of adjacent sides equal, its diagonals and tangent (all primary information) meets extended side AB at C.

The most probable way to draw the diagram is:

1. Construct circle.
2. Draw cyclic quadrilateral by a) From A draw AB = AE (for example, with center A and some radius preferably less than the radius of the given circle, it will cut circle at B and E). b) From E (or B) draw ED = BD (here if you draw from mid-point of B a perpendicular (perpendicular bisector) cutting circle at D and from D join B and E)
3. Draw tangent FD (it will be perpendicular to the perpendicular bisector) and extend it.
4. Extend AB until it meets tangent at C.

Once we know the structure we can derive the possible primary and secondary information (whatever is necessary) to solve the rider.

Synthesis 1

The above four points contain primary information or conceptual knowledge. From (1) we know, radii are equal, radius at the point of tangent is perpendicular to the tangent. From (2), $B_{12}+E_{12} = A_{12}+ D_{23} = 180°$. Angles on same segment are equal, viz $A_1 = E_1$, $A_2 = B_2$ and so on. Also, isosceles triangles thus $E_2 = B_1$ and $E_1 = B_2$. From (3), $D_1 = E_1=A_1$ and $D_4 = B_2$. From (4) exterior angle B_3 is formed which is equal to E_{12}.

If we can extract this information we should be able to answer any question. The problem that could arise is answering the question: Where do we start? The answer is simple. Start from whatever you need to determine/ prove and assume it is true as stated earlier. E_1 is equal to what? From (2) is equal to A_1 and from (3) is equal to D_1 = x.

B_{12} is equal to what? $B_1 = E_2$ and B_3 (from (2)). $B_2 = A_2$ and D_4 which is also equal to $B_2 = x$ (from (3)). Thus, $B_{12} = B_2 + B_1 = E_2 + A_2 = E_2 + x$ but $2E_2 + 2A_2 = 180°$ that is, $E_2 = 90° - x$. Then $B_1 + B_2 = 90°$. Similarly $E_{12} = 90°$.

For <C? It is involved in triangle CBD and thus in triangles, CBD and CDA. We can choose either. Since CAD involves known angles, we can use it instead of CBD (which we could use). $C = 180° - A_1 - D_{12} = 180° - x - x - D_2 = 180° - x - x - 90° + x = 90° - x$. Alternatively, we could use CBD with $B_3 = E_{12} = 90°$ from (4) and thus $C = 90° - x$.

For E_2, it was determined to be $90° - x$ while B_2 is equal to E_2.

For angle BKA = 90°, there are many ways to prove that. For example in Δ ABD, $B_{12} = 90°$, Then $A_1 + D_2 = 90°$ but $D_2 = B_1$.

Analysis 2

For Figure 2, how could that figure have been constructed? 1) Given cyclic quadrilateral with two adjacent angles equal and the diagonals. 2) Since sides are extended we have two exterior angles. 3) Two tangents. 4) Given BAS = B

What information can we get? From (1) All angles on segments, $A_1 = B_2$, $B_1 = T_2$ etc. Opposite angles supplementary, B + AST=180°, B=BAS (given) and exterior angle equal to interior opposite. From (2), $T_2 = A_2$, $A_3 = B_1$. From (3) $S_1 = B$, $T_{12} = A_{12}$, $T_2 = B_2 = A_2$, $A_3 = B_1 = T_3$ and $A_{23} = B_{12}$. From (4) Since $B_2 = A_2$ then $A_2 = B_2$.

Synthesis 2

For AB//ST that is, prove corresponding /alternate <s equal, or interior angles add up to 180°.

Here corr.<s $S_1 = B = BAT$ (S_1 exter. <) from (1). For RS = RT, isosceles Δ STR.. $S_2 = T_{12}$? $T_{12} = B$ (corr. <s) = S_1, from (1) and (a). For $T_2 = B_1$, T_1 is involved in T_{12} and B_1 in B_{12}, where $B_{12} = T_{12}$ (corr. <s). But B_2 = ? $B_2 = T_2$ (from (3)). Then $B_1 + B_2 = T_1 + T_2$ or $T_1 = B_1$. For $A_1 = T_1$, A_1 = ? T_1 = ? From (4) $A_1 = B_1 = T_1$ from (c). For $P = B_1 - T_2$, P is included in Δ PTB, $P + T_{234} + B_2 = T_{34} + B_1 + B_2 = 180°$. Or $T_2 + P = B_1$. Then $P = B_1 - T_2$

These two examples use the same method which is dependent on the analysis (breaking down the rider into primary information and using deductive thinking to extract the information) and synthesis, where we begin from what is required to determine and trace it to something given that was extracted in the analysis.

Looking at this method, which is accompanied by understanding, it relies on the fact that the learner has understood the essence of all theorems and axioms and can use them with ease and confidence. Using constructions continuously reinforces concepts learned. This way the learner builds his/ her knowledge of geometry on solid bases. It also reinforces the argument of why learning geometry is not easy as it relies heavily on conceptual knowledge. As a rule, there will always be more information than is needed. Assuming one can extract all the information from the analysis then the question to be answered determines the path, tracing back the unknown to the known. What could be very interesting is to see how many learners can construct a diagram from given information as accurately as possible! This could be a research problem on its own.

Finally, it is necessary to keep in mind the two assumptions made (which themselves can form the basis for more research). One main point that cannot be disputed is that a lack of knowledge of theorems (declarative knowledge) and being able to make deductions from them which implies they understand the theorems (conceptual knowledge), it is impossible to solve a rider. Deconstructing a rider (analysis) and subsequent synthesizing the derived information could lead to a successful solution of the rider.

CONCLUSION

Although a lot of research has been done on identifying factors that affect the teaching and learning of geometry and how to teach geometry (for example, Van Hiele), the solutions of riders, as a rule, follows an inductive approach. If any analysis takes place that could be part of the thinking process. However, to solve a geometric rider and follow a specific path, it can only be done using the deductive approach. The deductive approach forces one to rephrase a question asked. For example I can prove A = B if I can prove A = X and B = X. So the question is not about proving A = B but about A and B being equal to X. In geometry, we cannot prove a concept (construct) is true but what the attributes are of a concept. For parallel lines, we cannot prove two lines are parallel, but if we could prove corresponding angles are equal (attribute) then we proved that the lines are parallel. Through analysis (deconstruction) and synthesis (deduction) it is possible for more learners to be able to solve geometric riders.

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DOES THE CURRENT CURRICULUM FRAMEWORK CATER FOR ALL STUDENTS' MATHEMATICAL NEEDS?

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Over the past four years, only 10% of the total number of Matric learners have both written Mathematics and have achieved a mark of at least 50%. This is a rather shocking statistic. This paper seeks to use the results for the Grade 12 NSC examinations for the past four years for Mathematics and Mathematical Literacy to question whether the current curriculum framework is sufficient to provide for all students' mathematical needs. Initially, the results are averaged and summarized as a proportion of the total number of candidates who wrote the two subjects from 2013 to 2016. This summary is then analysed to interrogate whether the current offering of only two forms of mathematics education is meeting the needs of matric learners.

INTRODUCTION

Prior to 2008, learners could choose to take Mathematics as a subject to Matric or not. Since then, learners have been obligated to take some form of mathematics, either the subject Mathematics or Mathematical Literacy. The aim of Mathematical Literacy was to "allow individuals to make sense of, participate in and contribute to the twenty-first century world" (DBE, 2011:9). By contrast Mathematics is identified as "the study of quantity, structure, space and change" (DBE, 2011:6) which "enables us to understand the world… around us and to teach us to think creatively" (DBE, 2011:6).

While these are both necessary and noble aims, for the ordinary South African the success of the subjects are demonstrated quantitatively through results obtained in the Matric examinations. According to the National Policy pertaining to the Programme and Promotion requirements, the requirement for a "pass" in both Mathematics and Mathematical Literacy is defined as any mark above 30% (DBE, 2011:45).

In this paper, I perform a secondary analysis of the Matric results for Mathematics and Mathematical Literacy and discuss the implications thereof for the current educational framework which offers Mathematics and Mathematical Literacy as the only two options that a learner can take in Matric.

STATISTICAL ANALYSIS

In order to draw conclusions with respect to the suitability of the current curriculum framework, where only two forms of mathematics are available for students (Mathematics and Mathematical Literacy), I first explore the results obtained by the Grade 12 learners in the NSC examinations. To simplify the analysis, the overall results in both subjects are first averaged over the last four years. These averages are then applied to the total number of learners using the relative proportions of enrolment in the two subjects by the past Matric learners. This is done so that the implications of the achievement of the learners in each level of each subject can be seen as a proportion of the total number of matric learners.

MATHEMATICS

In the 2016 NSC Diagnostic Report (DOE, 2016:156), the results achieved by the Matric candidates in Mathematics over the past 4 years are represented in the following graph:

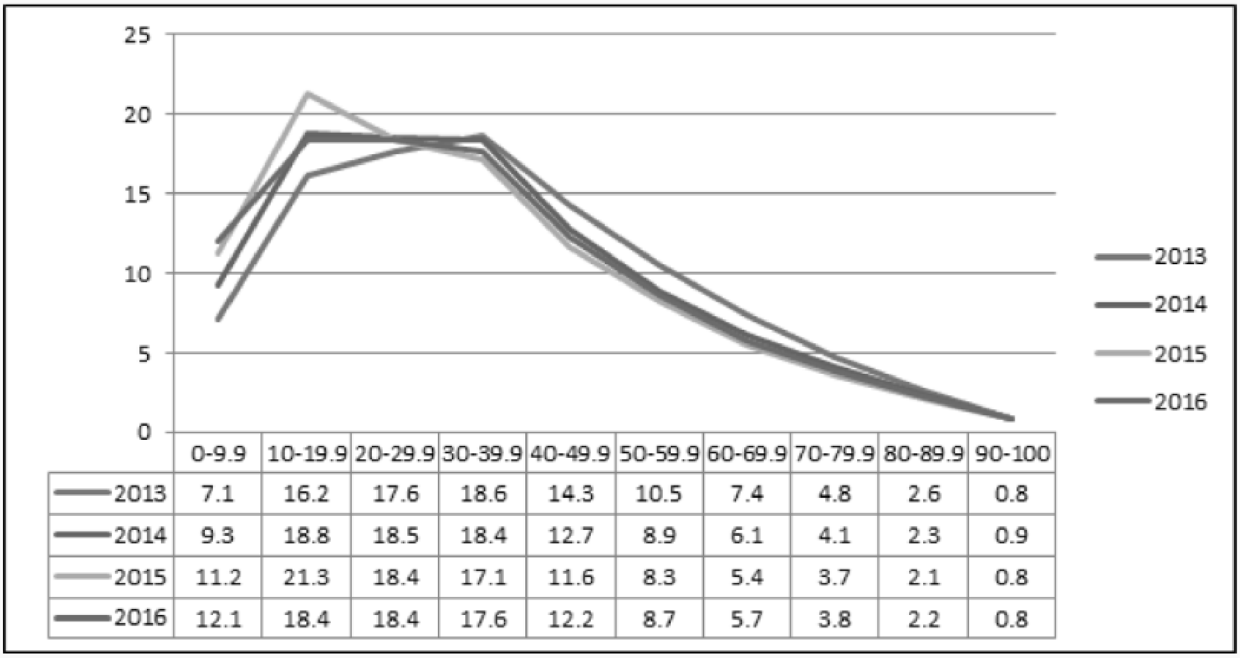


Figure 1: Overall achievement rates in Mathematics (2013 – 2016)

The values in the table below the graph show the percentage of each year's Mathematics candidates who achieved a result in the respective grouping (e.g. in 2016, 17,6% of the candidates achieved a final result between 30,0 and 39,9%). By averaging these results over the 4 years, the following table of averages is obtained:

Table 1: Distribution of Mathematics results from 2013 - 2016

Result	0-9,9	10-19,9	20-29,9	30-39,9	40-49,9	50-59,9	60-69,9	70-79,9	80-89,9	90-100
Average	9.9	18.7	18.2	17.9	12.7	9.1	6.2	4.1	2.3	0.8

When results are reported on Matric certificates, any result from 0 – 29% is represented as a Level 1, with subsequent deciles representing consecutively increasing Levels (30 – 39% = Level 2, 40 – 49% = Level 3, etc.). Using the values in Table 1 and rearranging them according to Level indicators, we obtain the following graph:

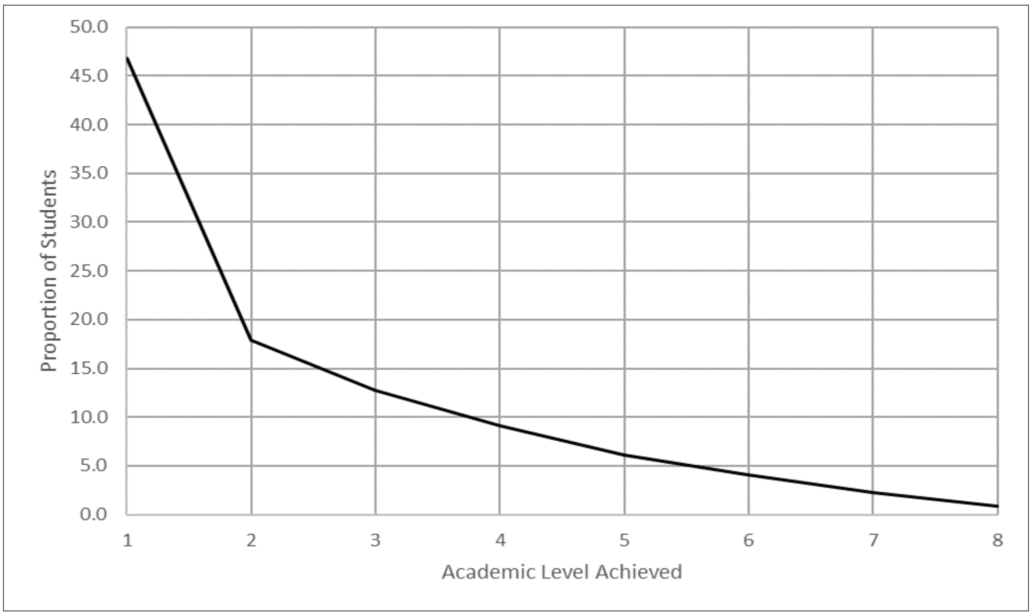
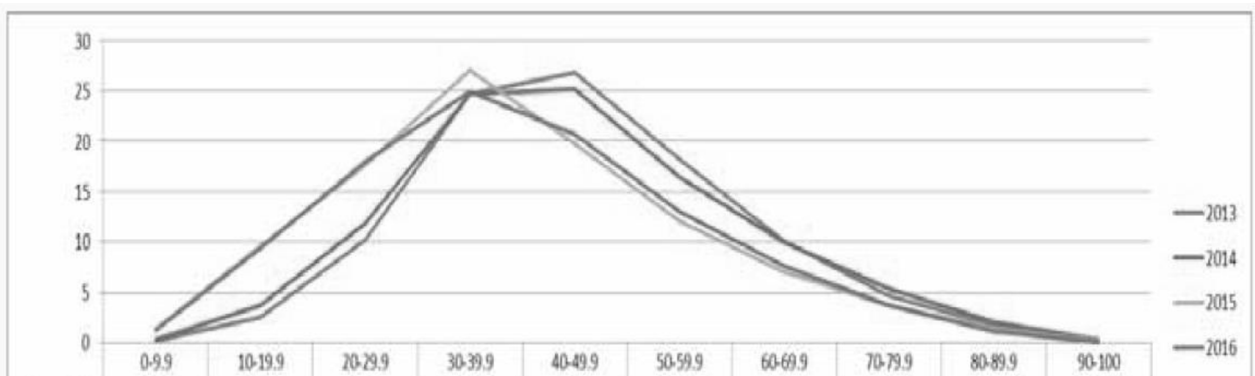


Figure 2: Overall average achievement rates in Mathematics per Level achieved (2013 – 2016)

The graph (Figure 2) shows that over 46% of Mathematics candidates failed to achieve a result of at least 30% (more on this later).

MATHEMATICAL LITERACY

In terms of Mathematical Literacy, the published graph in the 2016 Diagnostic Report (DBE, 2016:138) takes this form:



	0-9.9	10-19.9	20-29.9	30-39.9	40-49.9	50-59.9	60-69.9	70-79.9	80-89.9	90-100
2013	0,2	2,5	10,2	24,7	26,8	18,2	10,1	4,7	1,7	0,1
2014	0,3	3,8	11,8	24,7	25,2	16,5	10,0	5,4	2,1	0,3
2015	1,4	9,6	17,6	27,1	19,8	12,1	7,1	3,7	1,4	0,2
2016	1,3	9,4	18,0	24,9	20,8	13,1	7,6	3,7	1,1	0,1

Figure 3: Overall achievement rates in Mathematical Literacy (2013 – 2016)

By performing the same analysis as was done for Mathematics, where the achievement rates are averaged and then consolidated into Levels, the graph then transforms to the following:

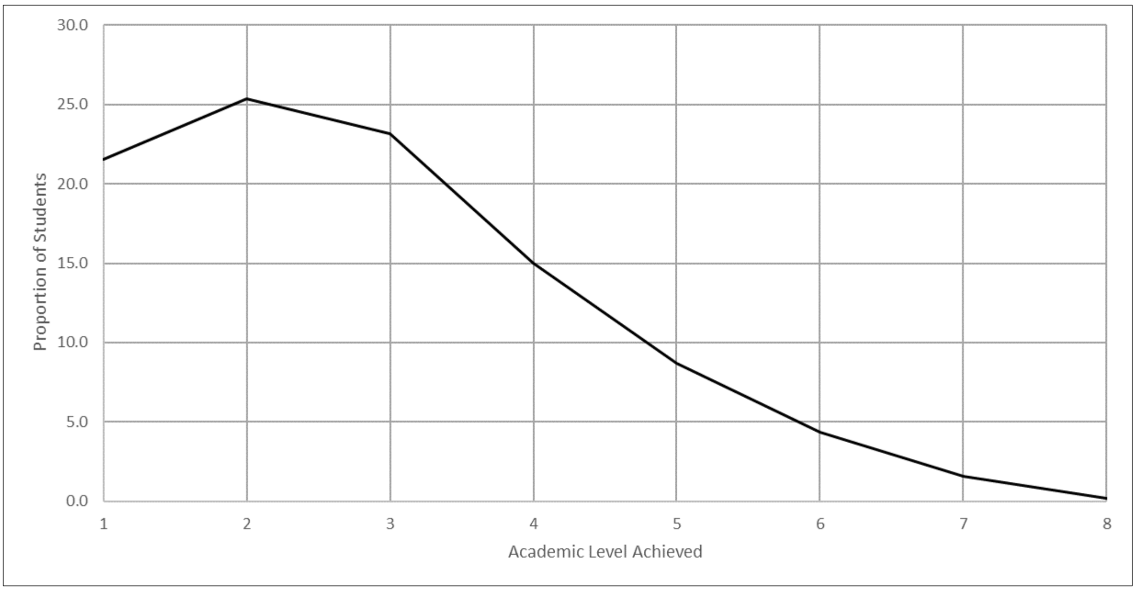


Figure 4: Overall average achievement rates per level achieved in Mathematical Literacy (2013 – 2016)

COMBINING THE ANALYSES FOR MATHEMATICS AND MATHEMATICAL LITERACY

Referring only to the two graphs which summarize the distribution of Levels achieved in both subjects, the contents of both graphs can be summarized as follows:

Table 2: Distribution of levels attained within each subject separately

	Wrote Mathematics	**Wrote Math Literacy**
Achieved at least 50%	22,5 %	29,9 %
Achieved 40 – 49%	12,7 %	23,2 %
Achieved 30 – 39%	17,9 %	25,4 %
Achieved 0 – 29%	46,8 %	21,5 %
Totals	**99,9 %**	**100 %**

The 2016 Diagnostic Report (DBE, 2016), reports that the total numbers of students taking mathematics and Mathematical Literacy in the years 2013 to 2016 were as follows:

Table 3: Numbers of candidates who wrote each subject and the total candidates

	Wrote Mathematics	Wrote Math Literacy	TOTAL
2013	241 509	324 097	565 606
2014	225 458	312 054	537 512
2015	263 903	388 845	652 748
2016	265 810	361 948	627 758
AVERAGE	**249 170**	**346 736**	**595 906**

Applying the proportions from Table 2 to the averaged totals from Table 3, the levels achieved can be seen as a proportion **of the total Matric candidates** over the past four years as follows:

Table 4: Distribution of levels attained within the **Matric candidates as a whole**

	Wrote Mathematics	Wrote Math Literacy
Achieved at least 50%	10 %	18 %
Achieved 40 – 49%	5 %	14 %
Achieved 30 – 39%	8 %	15 %
Achieved 0 – 29%	20 %	12 %
Totals	**42 %***	**58 %***

*(*adding anomalies in totals due to rounding)*

ANALYSIS OF STATISTICS

Referring to the summary in Table 4, a few key observations can be made:

- Only 10% of all Grade 12 candidates both took Mathematics and were able to achieve a result of at least 50%.
- A little under half of all Grade 12 candidates who took Mathematics failed to achieve a result of at least 30%. This is regarded as failing the subject.
- Less than a third of all candidates who took Mathematical Literacy achieved a result of at least 50%.
- One in five Mathematical Literacy students were unable to achieve at least 30% in the subject. This is regarded as failing the subject.

The education department (DBE, 2011:45) stipulates that a pass in Mathematics or Mathematical Literacy, requires at least 30%, which is a very low benchmark. Hence, a learner needs to score only 30% of the available marks, to meet this requirement. A more significant measure of competence could be the same percentage applied by Universities, namely 50%. Considering Mathematics initially, if 50% is to be a more appropriate benchmark for a pass, then only 10% of all Matric learners over the past four years (2013 to 2016) who have both written Mathematics and have achieved it. This is a very low proportion of the entire Matric cohort.

Even more concerning is the fact that twice that number (20%) of the Matric cohort attempted Mathematics and failed it (achieved a mark lower than 30%). A concern is that many of those learners were not even given the option of taking Mathematical Literacy. The 2016 NSC School Subject Report (DBE, 2016), identified several schools where students achieved above 30% in Mathematics while simultaneously did not have any students writing Mathematical Literacy. It is noteworthy that most of these schools are Quintile 1 schools which cater for some of the most economically needy learners. Overall, the 20% of the Matric learners who failed amount to approximately 120 000 young people per year (20% of 600 000 Matric candidates) will leave school with very theoretical, non-contextualised knowledge of Mathematics after at least 12 years of study. A glance through any Grade 12 final Mathematics examination highlights that Mathematics is assessed through context-independent, mostly algebraic questions. Thus the urging of the Mathematics CAPS document to contextualise the work through “Real life problems… in all sections whenever appropriate” (DBE, 2011:6) does not seem to be adhered to.

Switching the focus to those Grade 12 learners who took Mathematical Literacy, only 18% of all the Grade 12 learners have taken this subject and can achieve at least 50%. This is less than a third of the total number that take Mathematical Literacy. While Mathematical Literacy results are better than that in Mathematics, it cannot be understandable that 12% of Matric learners write the subject and fail it. A subject which was initially intended to empower “a citizen … to be a productive independent member in the society” (Christensen, 2006:10) cannot be seen as meeting these goals with results such as these.

IS THERE ROOM FOR A “MIDDLE GROUND” SUBJECT?

Considering the results achieved across the past four years, it can perhaps be understood that only a minority of Matric candidates should be able to achieve an acceptable level of competence in Mathematics. After all, it is unrealistic to expect the average person in society to be highly competent in such a technical, abstract version of mathematics. However, for twice that number to fail must surely indicate that they took the wrong subject (or perhaps it was not offered to them). In terms of Mathematics, the exam papers are attempting to accommodate both the highly able student and the average student in the same paper, meaning that they have to be made more challenging and not less. This alienates the struggling average learner with abstract concepts that, at a more basic level, could be applied, grasped and understood, but at an abstract level, they cannot.

Considering Mathematical Literacy, there are 11 official languages, but only English and Afrikaans are used for exam papers. This is the same for resources such as textbooks. This system favours the first language speaker of that language while simultaneously disadvantaging the second- or third-language speaker of those languages. When this cognitive disadvantage is applied to high-stakes, pressurised final examinations, the student inevitably struggles to understand the question, let alone correctly apply the mathematical skill. While language alone cannot account for the poor performance of learners in Matric examinations, it can significantly affect learners' performance. Debba (2011), found a statistically significant difference between the results attained by first and second language English speakers in their Grade 12 Mathematical Literacy preparatory examination

So the majority of learners seem to be caught between two unenviable choices. On the one hand, they have the abstract nature of Mathematics curriculum content which is seemingly targeted at an above-average level. On the other hand, the language of the final assessments in Mathematical Literacy make interpretation and expression of solutions challenging for the non-first language English or Afrikaans speaker.

While this study does not explore the reasons for the poor overall results, it does recognise that the current situation has not changed in the past four years and is unlikely to change at all unless something radically different occurs in terms of the curriculum provision. It would seem sensible that a subject which is both less rigorous than Mathematics, but less language-based than Mathematical Literacy is a step in the right direction. Perhaps a course which has a rigorous financial maths section, graphing section, elementary trigonometry as well as volume, area, data handling and probability. All in a context-based framework. Effectively, a balance between the life-skill rich Mathematical Literacy and the more rigorous Mathematics. Perhaps it could be called **Key Mathematics**?

CONCLUSION

This year, 2017, represents the tenth year that the NSC curriculum is being assessed at Grade 12 level. It has been 10 years since the move to two subjects, Mathematics and Mathematical Literacy. The above statistics are consistent with the results of earlier years and so conclusions drawn from these statistics can be reasonably extrapolated to years before these and can be expected to be consistent with future results. Hundreds of thousands of Grade 12 learners every year leave the FET phase with inadequate mathematics skill.

In conclusion, I draw from an article written for a national newspaper (Gilfillan, 2016):

> "It seems sadly coincidental that we are commemorating 100 years since the first World War battle of the Somme, where so many young people died because the generals felt that they could conquer if they only had enough bodies committed to the

> battle. This seems to be the same strategy being applied by the Department of Basic Education. Just push enough people into Mathematics and we'll get the number of passes up. However, what if the majority of those bodies were committed to a battle that they could win and that would ultimately make us a more informed and empowered society? That would be better surely?"

A last sobering thought from that same article:

> "If a committee were to meet today and make the decision to approve and implement this Key Maths subject, the first Matric exam in it would only be written in 5 years' time (2 years for policy and resource development and then 3 years of phased introduction). No quick fixes available here, but at least the majority of the country would be on a wiser path."

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EXPLORING GRADE ELEVEN USE OF GEOMETER'S SKETCHPAD IN CONDUCTING PROOFS IN EUCLIDEAN GEOMETRY

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The study seeks to explore the use of Geometer's Sketchpad by Grade 11 learners in learning proofs in Euclidean Geometry. It aims to explore the impact of the incorporation of Geometer's Sketchpad and e-learning. E-learning refers to learning utilizing electronic technologies. In this context the electronic technology used is the Geometer's Sketchpad.

The current study is embedded in social constructivism, a theoretical framework which believes that human beings generate knowledge and meaning from their experiences. It employed a qualitative action research enquiry approach underpinned within the interpretive paradigm. This study confirmed that Geometer's Sketchpad illuminates mathematics and enhances learners' comprehension. Furthermore, the incorporation of Geometer's Sketchpad instills the love of conducting mathematical proofs by eradicating abstractness the learners experienced and provides them with visualization. This leads to meaningful learning where learners are motivated by working practically which they enjoy. Learners are actively engaged in problem-solving, work easily with enthusiasm and faster as they measure sizes of angles using the Geometer's Sketchpad built-in facilities. Then learners develop geometrical ideas as they formulate conjectures which lead to appropriate proof. Thus, the Geometer's Sketchpad has proved to be a scaffolding and motivational tool which nurtures learners' mathematical perceptions.

INTRODUCTION

There have been changes made in Education by policy makers after the first democratic elections which were held in South Africa on 27 April 1994. There is a lack of technology tools in most schools. Some schools do not even have electricity. Technology tools incur more costs in schools and in some areas crime is rife. In case of financial constraints in schools, teachers may purchase laptops for themselves and the school buys the data projector. This will expose learners to technology even if they do not use computers individually. Awe (2007) affirmed that learners can still understand the visual aspects of geometry even if the teacher has a few computers. There are some schools with computers, but with no software to solve Euclidean Geometry problems. However, there are also schools with computers, laptops, tablets and software used by learners to solve all types of mathematics problems.

Mathematics teachers ought to be developed in the use of technology based tools so that they are competent in their attempts to explain to learners how they integrate technology and mathematics (Ndlovu, Wessels & De Villiers, 2013). Ndlovu, Wessels & De Villiers (2011**),** made use of *Geometer's Sketchpad* to conjecture and implement a hypothetical learning trajectory (HLT). Mudaly (2004) used the *Geometer's Sketchpad* for learners to prove that the perpendicular bisectors of a cyclic quadrilateral are concurrent. Govender (2013) used the *Geometer's Sketchpad Program* (GSP) to prove Viviani's theorem.

The current study was conducted in a peri-urban area (township) about 25 km north of Durban city centre, in South Africa. It seemed important to conduct the study as it would attempt to add to the knowledge on the teaching of proofs in Grade eleven using Geometer's Sketchpad.

AIMS AND OBJECTIVES OF THE STUDY

The aim of the study was to investigate Grade eleven learners' use of the Dynamic Geometry Software, the Geometer's Sketchpad in conducting proofs in Euclidean geometry. It was also to explore how the Geometer's Sketchpad can be used to teach proofs in Euclidean geometry. The current study intends to answer the following research question: How can the Geometer's Sketchpad effectively be used to teach Grade Eleven mathematics learners' proofs in Euclidean Geometry?

METHODOLOGY

This paper employed a qualitative action research approach underpinned within the interpretive paradigm. The study aimed to seek an in-depth understanding of using the Geometer's Sketchpad in conducting proofs in Euclidean geometry. The final sample comprised eight learners who were chosen at random.

RESEARCH INSTRUMENTS

The study used an observation schedule and a semi-structured interview schedule. I observed participants while they were using the *Geometry Sketchpad* to prove that the angle subtended by the arc at the centre of a circle equals double the angle subtended by the same arc on the circumference. The participants measured the sizes of angles and formulated conjectures based on their mathematics experiences. Finally, for all eight participants, face-to-face, individual semi-structured interviews were conducted, and all their responses were audio recorded. In instances where more clarity was required, I probed using additional questions (Patton, 1990).

The interview schedule comprised four questions. The first question focused on how learners felt about using *Geometer's Sketchpad.* The second one focused on why the learners felt the way they did. The third question aimed at finding out if the learners encountered any problems while using *Geometer's Sketchpad.* The fourth question attempted to extract learners' perceptions about the use of the *Geometer's Sketchpad Program.*

DATA GENERATION PROCEDURES

As the research was qualitative in nature, the participants were observed while they were engaged in a problem-based task (an investigation) using an observation schedule. I compiled open-ended questions with the aim of allowing participants to respond to them. After compiling the questions, I ensured the validity and reliability of each instrument. I ensured that the procedures to generate data were the same for all participants by asking them the same questions in the same manner. However, I rephrased the questions whenever the participants did not understand the question(s). I also probed to obtain more in-depth data.

DATA GENERATION METHOD

After compiling the interview questions, I conducted a pilot study with eighteen grade eleven learners who were non-participants, to ensure the validity and reliability of each instrument. I ensured that the procedures to generate data were the same for all participants during the pilot phase. I also evaluated the questions for clarity and intention (McMillan & Schumacher, 2006).

THEORETICAL FRAMEWORK

The theoretical framework within which this study is embedded is social constructivism. Von Glasersfeld (1989) described constructivism as a psychological theory of knowledge or epistemology which believes that human beings generate knowledge and meaning from their experiences. Furthermore, one of the principles this framework asserts is that knowledge, when received is actively constructed by the perceiving subject.

The formalisation of theory of constructivism is generally attributed to Piaget, a Swiss developmental psychologist and philosopher who explained mechanisms by which knowledge is internalised by learners. Cognitive constructivism is based on his work and he is known for his epistemological studies with the development of thought in children. Piaget's theory of cognitive development proposed that human beings cannot be given information which they immediately understand and use. He believed that human beings must construct their own knowledge and build it through experience.

The Curriculum and Assessment Policy Statement (CAPS) Grades 10-12 Mathematics policy document encourages an active and critical approach to learning as opposed to rote and uncritical learning of given truths (DoE, 2010, p.4). Constructivism therefore, adheres to the (CAPS) policy document as it encourages active engagement of the learners in learning Mathematics.

OBSERVATIONS

In the current research, the learners formulated conjectures after measuring the sizes of angles in the sketches that I provided on the Geometer's Sketchpad. The aim was that learners prove the angle at the centre theorem on their own. I created an environment where learners discovered and constructed their own knowledge (Gagnon & Collay, 1999) with the help of the Geometer's Sketchpad. I only guided them where necessary. This addressed the following critical question: How effectively can the Geometer's Sketchpad be used to teach Grade Eleven mathematics learners' proofs in Euclidean Geometry? The (CAPS) Grades 10-12 Mathematics policy document encourages an active and critical approach to learning as opposed to rote and uncritical learning of given truths (Department of Basic Education, 2010, p.4).

I observed that fifty percent of the participants could not prove this theorem appropriately even after extensive revision, remedial work, application of the theorem, and conducting the same proof three times. Each of the eight participants then proved the angle at the centre theorem using Geometer's Sketchpad. TO3.3's work, on Geometer's Sketchpad, is shown in Figure 1. (TO3.3 represents learner Thula's observation on question 3.3).

What is the size of angle *BCE* in relation to angle *BAC*? What is the size of angle *ECD* in relation to angle *DAC?* Determine the relationship between angles *BCD* and *BAD* and give a full explanation of the relationship. These questions are based on the diagram below shown as Figure

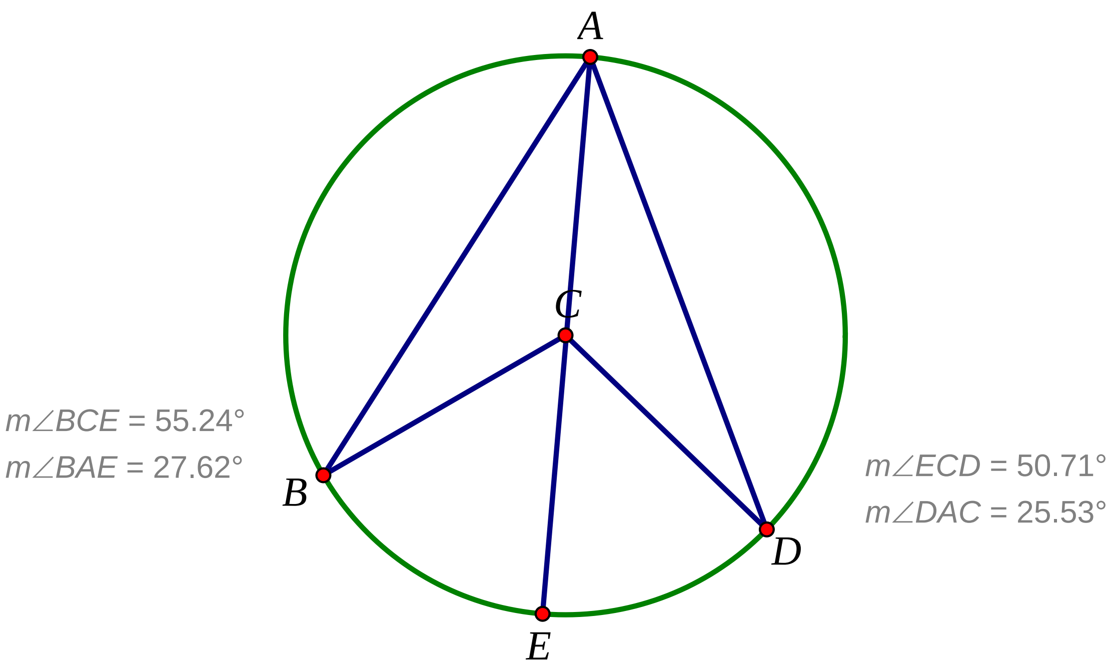

Figure 1: A diagram showing TO3.3's work on Geometer's Sketchpad

FINDINGS AND ANALYSIS

Meaningful learning became effective as all eight learners were actively engaged in problem solving whilst using the Geometer's Sketchpad. The empirical evidence revealed that the learners found it easier and faster to conduct proofs using the Geometer's Sketchpad which has built-in measuring facilities, than using the paper and pencil method. The use of the Geometer's Sketchpad eradicated the abstractness the learners experienced initially. Having measured the sizes of the angles in Question 3.3 above, all the learners managed to prove empirically, the angle at the centre theorem. They also understood the term ‘subtended by’.

CONCLUSION

The current study confirmed that the Geometer’s Sketchpad illuminates mathematics and enhances learners’ comprehension (Mays, 2003). It also concurred with Stols, Mji & Wessels (2008) who highlighted that Geometer’s Sketchpad is a powerful teaching tool. The incorporation of Geometer’s Sketchpad in mathematics teaching, motivates learners to be actively engaged in learning while the teacher scaffolds (guides and supports) them.

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PRELIMINARY FINDINGS FROM A STUDY BASED ON GAGNE'S 10 COMMANDMENTS FOR THE EDUCATION OF MATHEMATICALLY GIFTED LEARNERS (MGL'S)

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The purpose of this study was to investigate strategies that high school teachers use when they identify mathematically gifted learners in their inclusive classrooms. The study was grounded in Gagné's Ten Commandments for the education of mathematically gifted learners. A total of 19 grade 10 high school teachers from Bloemfontein, South Africa participated in the study. Preliminary results have shown that a large number of the respondents (47.4%) were not sure about how a gifted learner can be defined. The most widely used method by the teachers to identify gifted learners in mathematics classrooms centered upon nomination and assessment results. A recommendation coming from this study as well as similar studies is that in deciding who is gifted and in need of support, we need to cast the net widely and rely on qualitative as well as quantitative indicators of potential.

Key words: Mathematically gifted leaners, teachers' strategies, inclusive classrooms, commandments

INTRODUCTION

There is global consensus on the view that progress of human civilization is based on scientific, technological, educational, moral, political, and commercial achievements of the minds of its most talented individuals (Shavinina, 2009). Evidence of this view comes in statements like "the gifted and talented children are our best resource available," "highly able kids are the most precious asset that any country possesses," "individuals with extraordinary developed intelligence and creativity are the most valuable gift that humankind has" (Kholodnaya, 2007). In post-apartheid South Africa education, Kokot (2011) argued that reforms seem to have lost sight of this important observation because stakeholders have been hostile to and resentful of gifted education programs which are critical for academic talent development.

The controversies over gifted education start with how they are defined and consequently how they are selected and supported (Subotnik et al., 2011; Borland, 2014). Yet according to Gagné (2011) early identification of giftedness is an important mechanism which provides a pathway to the full realisation of a gifted student's potential. In South Africa few studies if any have been carried out to understand how teachers are identifying and supporting the gifted students. Given this gap, the purpose of this study was to investigate strategies used by high school teachers to identify mathematically gifted learners in their inclusive classrooms and the grouping strategies they design to support such learners.

Gagné's 10 commandments as a theoretical framework

According to Renzulli (2012) theory guides research and organizes its ideas. Similarly, in this paper we also needed a theory through which to organize our ideas and in which to ground our arguments on giftedness in education in South Africa. While there are a number of theories and models in the field of gifted education, Gagné's work is among those that have been considered dominant in affecting international classroom practice. His work has received worldwide recognition because it is generally viewed as resolving the controversies that the gifted field has struggled with for years (Pfeiffer, 2013). Of particular interest in this paper are Gagné's ten commandments which grew out of the need to formulate a coherent set of basic statements, or rationale, concerning the nature of human abilities, gifts, and talents (Gagné, 2007). It is not the intention for this paper to go into much detail about each of the ten commandments for talent development but suffice to say that the 10 commandments can be subdivided into two sets (Gagné, 2007). The first four commandments target identification procedures, the "who" of talent development. In this sense Gagné says thou shalt distinguish, discriminate, identify and select. The next five pertain to intervention modalities, the "how" of talent development. In this sense Gagné says thou shalt intervene, condense, accelerate, enrich and group. The tenth commandment (thou shalt dream eyes wide openly) cautions talented youth, as well as their educators and parents against dreaming of fame and eminence with their eyes shut. For the purposes of this paper, it was felt that this tenth commandment may not be applicable in a classroom situation so the focus of this paper was on the remaining nine commandments. Although the study examined all the nine commandments, this paper presents preliminary findings on identification and grouping strategies used by teachers in an attempt to support gifted learners in their regular classrooms.

MATERIALS AND METHODS

The study used a qualitative-quantitative approach in examining strategies used by teachers for supporting mathematically gifted learners. A total of 19 grade 10 high school teachers from Bloemfontein, South Africa participated in the study. All the participants were mathematics teachers. Permission was obtained from the Free State Department of Basic Education as well as the principals of the schools prior to conducting the study. Questionnaires were hand delivered to the participants at 10 schools and were collected after 2 days. Moreover, the participants completed the questionnaires at their schools, and in case of the absent teachers, the questionnaires were left to school administrators.

RESULTS AND DISCUSSION

The first aspect examined in the study is the methods which teachers use to identify gifted learners in the inclusive classrooms.

Table 1: Methods used by the teachers to identify gifted learners (n=19).

Method	Yes		No		Not Sure		Total	
	Freq	%	Freq	%	Freq	%	Freq	%
Nomination	13	68.4	5	26.3	1	5.3	19	100
Assessment results	16	84.2	2	10.5	1	5.3	19	100
Identification by other teachers or previous school	2	10.5	12	63.2	5	26.3	19	100
Identification by parents	2	10.5	14	73.7	3	15.8	19	100
Other methods	2	10.5	12	63.2	5	10.5	19	100

The results show that the most widely used method by the teachers to identify gifted learners in mathematics classrooms centered upon nomination and assessment results. About 84% of the participants cited assessment results and approximately 68% mentioned nomination as observing the learner's performance during question and answer sessions. A smaller proportion just above 10% mentioned identification by other teachers or previous school, identification by parents and other methods. These results run contra to Gagné's recommendations as stipulated in both the first and the third commandment. The first commandment enjoins educators to acknowledge the large diversity of gifts and talents, as they manifest themselves in domains and sub-domains of giftedness, as well as numerous fields and sub-fields of talent. The third commandment invites gifted program coordinators to expand their list of identification criteria beyond intellectual giftedness and academic talent, and look especially for the presence of three intrapersonal catalysts: motivation, willpower, and self-management.

Similarly, Terman (1926) warned against total reliance on tests arguing that: "We must guard against defining intelligence solely in terms of ability to pass the tests of a given intelligence scale" (p. 131). Similarly, Thorndike (1921) had earlier stated; "to assume that we have measured some general power which resides in [the person being tested] and determines his ability in every variety of intellectual task in its entirety is to fly directly in the face of all that is known about the organization of the intellect" (p. 126). Other studies have also shown that underrepresentation of learners from previously disadvantaged backgrounds results in large part from improper identification practices based on mostly invalid definitions of the key concepts of giftedness and talent, but especially what Borland (1997) has labelled the “socially constructed” giftedness concept. For her part, Ford (2003) puts the blame on the educational system, more specifically (a) on “the pervasive deficit orientation that prevails in society and our schools,” (b) on “low referral rates of diverse students” by teachers, (c) on an almost exclusive reliance “on tests that inadequately capture the strengths and cultural orientations of these students,” and (d) on “educators” lack of understanding of cultural diversity” (p. 507).

In line with Gagné’s second set of commandments, this study was also interested in identifying the grouping methods used by teachers in the regular classrooms. Research has shown that grouping arrangement for gifted children should be of concern when establishing good practice for these children.

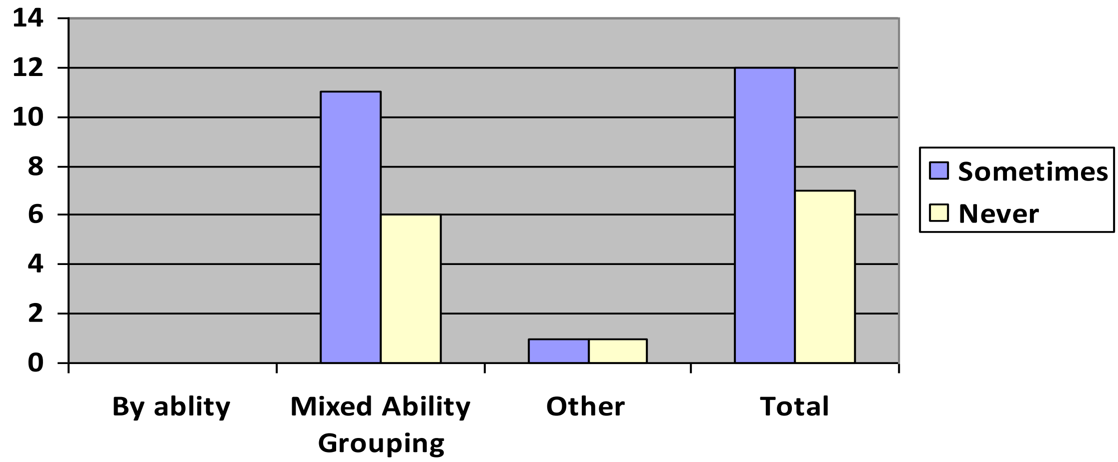

Figure 1: Grouping strategies used by teachers (n=19).

The results show that the majority of the teachers (17) appear to use mixed ability grouping in the classrooms in two ways. Most of the teachers (11) which use the mixed ability grouping indicated that they sometimes change the groupings by swapping the learners between different groups. Fewer teachers (6) in the same mixed ability grouping however prefer not to change groups. None of the teachers chose ‘grouping by ability’ as one of the grouping strategies. The 2 teachers who cited other grouping strategies explained that they use the merit list to group the learners. Table 3 shows teachers’ explanations as their reasons for re-arranging the groups in the classrooms.

Table 2: Reasons for rearranging the learners' groups in the classroom

Reasons for rearranging	Frequency
Learners get motivated when they interact with different peers	2
To help underperforming learners	4
Learners enjoy different topics, they are grouped according to the classroom activity	3
According to their performance in the given test	2
Not answered	2
Total	13

As both Figure 1 and Table 2 show, most teachers rearrange their grouping based on learners who act as peer tutors and the classroom activity. There are a few teachers who swap their learners either for learners to get motivated when they interact with different peers or based on the test result. These results are in contrast to what Gagné suggests in his ninth commandment. Gagné considers full-time grouping as the only way to create appropriate conditions for an enriched curriculum for the gifted learners. Although results from ability grouping studies might not be conclusive, the weight of evidence indicates that differentiation of the curriculum which was afforded through some grouping by ability level is important in catering for the needs of gifted children. Research has indicated that these manipulated grouping practices allow teachers to be more responsive to the needs of all their students, engaging in practices that reflect positive achievement outcomes for those of all abilities (Brulles, 2005 as cited in Winebrenner & Brulles, 2008). Other researchers have also confirmed that students will benefit when curriculum and instruction is adjusted to their individual level of achievement and skill (Gentry & MacDougall, 2007). As such, there must be provision for this methodology in our schools (Biddick, 2009). Moreover, gifted students are afforded a sense of belonging, in an environment where daily and consistent interactions with intellectual peers can be maintained. They can also expect to be supported by a teacher who acknowledges and actively addresses their unique academic and affective needs (Peters, 2005). A group of researchers examined more than a century of data on the subject and came to the conclusion that putting students of similar skills and abilities together in the same class is a highly effective, low-cost method to increase educational achievement (Steenbergen-Hu, Makel, & Olszewski-Kubilius, 2016). Proponents for ability grouping therefore argue that these techniques greatly benefit students who are insufficiently challenged in their grade-level classroom. When classes have more students of the same ability level, it's easier for teachers to teach at a level that matches a student's needs.

CONCLUSION

This study aimed at investigating the different methods used by teachers to identify gifted learners and how they support talent development for these learners. Results show that current teacher practices may not be creating an environment suitable for gifted learners to grow to their full potential. Since these are preliminary results more studies need to be done to further understand how teachers are handling gifted students in their classes.

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WHEN TEACHERS' MATHEMATICAL KNOWLEDGE IS WOBBLY: WHAT IS THERE TO LEARN?

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The paper reports on an ongoing research study that explores mathematics teachers' mathematical knowledge for teaching Algebra. While the larger study focuses on 45 teachers, this paper will focus on one of the teachers. Data will be sourced from a video-recorded lesson where the teacher will be teaching quadratic equations to Grade 10 learners. Great interest will be on the enacted object of learning and so what is made possible for learners to learn. To analyze the video-taped lesson we will recruit two theoretical perspectives from which we will move to develop our analytical tools. A grounded theory approach will be conducted using phenomenological tools to tease out what is made available for learners to discern. The Theory of Variation and the notion of ***mathematical reality*** *and* ***mathematical appearance*** *will provide additional theoretical resources that will be used to describe the enactment per se.*

INTRODUCTION

All learners in the Republic of South Africa- irrespective of where they live or who they are- deserve qualified and trained mathematics teachers in their classrooms. Subsequently, all learners deserve access to teachers who possess profound understanding of mathematics in/and for teaching (MfT). Sadly, many learners do not have access to the kind of teachers we desire for them – and there is an assortment of studies (e.g. Carnoy & Chisholm, 2008) that support this claim. There are factors that might be associated with the situation we have in South Africa. Some of the factors that are common in the South African education research literature include the following: Firstly, teacher education under the apartheid era is often blamed for the crisis in the South African education system. By way of example, some authors often associate education under the apartheid regime (particularly its uneven distribution of educational opportunities) with the current education crisis in South Africa. In one of their seminal papers Adler & Davis observed that:

> the majority of black secondary school teachers, trained under apartheid only had access to 3 year College of Education diploma, and the quality of training in general and in mathematics in particular was by and large poor. Hence, many current secondary mathematics teachers have not had an adequate opportunity to learn further mathematics (2006, p. 277).

For the above authors, mathematics teacher education in the apartheid era did not equip teachers with deeper understanding of the content they later taught in school. Suffice it here to mention that some of the teachers who received teaching qualifications in that era are still offering mathematics in our schools. As such, perhaps Adler and Davis' (2006) observation might suggest that South Africa's current educational crises with particular focus on its outcomes, might have links with education under the apartheid rule. The authors stressed the endemic limited teacher knowledge when they maintained that the problem of level and depth of teachers content knowledge was abound in the literature of research on mathematical knowledge for teaching

Secondly, there are reports (Moalosi, 2015; Simkins, 2013; Spaull, 2013) that link South Africa's education crisis to the presence of under-qualified and untrained mathematics teachers in the education system. The nature and origin of under-qualified teachers is exemplified below in an extract from an interview between a teacher and a professional development facilitator in one of the teacher professional development initiatives in South Africa. The goal of the interview was to obtain information that would enable facilitators to develop professional development strategies that would help teachers to close gaps in their mathematical knowledge for teaching. In the interview, the under-qualified teacher was a qualified Primary School teacher trained to offer Natural Sciences, Biology and South Sotho. However, the teacher was offering mathematics at Grade 8 -10 level. The extract is not part of the data that will be analyzed for this paper, but is only used to exemplify what we mean by an under-qualified teacher.

Interviewer: So how did you come to be a mathematics teacher teaching in a secondary school?

Teacher: Because when I came here, the school was starting from grade one to grade twelve. So the primary section was taken away and I was left and then I took on the maths…the principal motivated that I do the Maths ACE course, that's why I'm here. Then I did my ACE and then my Honours.

Interviewer And in terms of how you were learning how to teach mathematics?

Teacher: I didn't do maths at the college.

Interviewer: You didn't do maths at the college. What did you do?

Teacher: I did Biology and Physical Science and South Sotho. Those three. I didn't do maths at the college

Interviewer: So when did you start maths?

Teacher: Maths. With the ACE. When I did the maths ACE.

While under- qualified teachers consist of teachers who teach mathematics at Grade levels beyond their certification, untrained teachers are those who hold a Bachelor's Degree or Diploma Qualification in areas other than teaching. To elaborate on these two types of teachers further, suffice it to mention that the under-qualified bracket largely consists of primary school teachers offering secondary school mathematics or General Education and Training (GET) Senior Phase qualified teachers offering Further Education and Training (FET) mathematics. Untrained teachers consist of teachers who do not hold any teaching qualification (Simkins, 2013). The presence of these types of teachers was facilitated by an increase in the number of secondary schools post 1994. The expansion in the number of schools surpassed the number of qualified mathematics teachers and thus ushered under-qualified and untrained teachers into the South African teaching corps.

Gaps in teacher knowledge have been mentioned quite often among the factors since it is believed that teachers can only teach what they know and nothing more. So what teachers do not know is not taught. However, in this paper we will present evidence of a teacher who appears to have taught the mathematics he did not know and the outcomes aroused interest to write this paper. As such, the study seeks to answer the following research questions:

1. What is the nature of the teacher's mathematical knowledge for teaching?

2. What is the nature of the enacted object of learning?

3. What is possible for learners to learn?

LITERATURE REVIEW

The problem of low mathematical knowledge for teaching is a characteristic of small and large scale research done in South Africa (Carnoy & Chisholm, 2008). These studies have consistently documented gaps in teachers' subject matter knowledge of the mathematics they taught. While previous research sought to describe the gaps, this paper moves beyond previous research to evidence the gaps and articulate their implications for learning. The concern for this paper is founded on research that revealed that teacher knowledge and learner academic achievement were strongly related. Hill and her colleagues found that among factors associated with learner achievement, teacher knowledge had a larger effect size (Hill, Rowan, & Ball, 2005). Surely, unrelenting gaps in teacher knowledge and declining learner academic achievement are twin challenges facing the South African education system. It is surprising that while both government and non-governmental organizations are making efforts to redress the status quo through funding of professional development for teachers, to this end, not much has come forth. It is indeed demoralizing to funders when efforts put in place to improve academic achievement fail to yield results that correspond with the investment. Most teachers in South Africa have attended at least one professional development effort or initiative funded by the Department of Basic Education or non-governmental organizations. New reforms that are promoted in professional development fail to see the light of day in the classroom and reasons underpinning this phenomenon are abound in literature of research.

Problems associated with teacher knowledge are indeed historical and are global. In the late 1980s Shulman and his colleagues articulated the absence of content knowledge in research and the need to conceptualize content knowledge for teaching. As a result the 1990s saw an upsurge in studies (e.g. Ball, 1990; Carpenter, Fennema, & Franke, 1996; Even, 1990; Even & Tirosh, 1995; Fennema & Romberg, 1999; Leinhardt, Zaslavsky, & Stein, 1990; Ma, 1999) that sought to describe teachers' mathematical knowledge. By way of example, Leinhart et.al's (1990) study of teachers' subject matter knowledge of functions revealed that teachers had limited knowledge of arbitrariness and univalence in functions. Teachers in their study talked about functions in algebraic terms and appeared not to know that functions existed in different representations other than in algebraic formation.

THEORETICAL UNDERPINNINGS OF THE PAPER

As already highlighted in the abstract, the paper is undergirded by three theoretical perspectives. Phenomenology will provide tools that include (i) *meaning clusters* (ii) *horizonalization* (iii) *intentionality* and (iv) *bracketing*. These tools are critical for gaining insight into the nature of the enactment. From Theory of Variation we will recruit the notions of (i) *patterns of variation* that include *contrast* and *generalization* to study how the object of learning was 'visibilized'(bringing mathematical objects into focus). While there are three patterns of variation, we have selected to work with only two. In Variation Theory terms, *contrast* refers to awareness brought about by discernment of difference between the *critical features* and *generalization* is awareness brought about by discernment of similarity between *critical features*. A critical feature is that which is important to learn or what the teacher has privileged for learners to learn. The notions of *mathematical reality* and *appearance* developed from a larger study (see Moalosi, 2015) and have since been used to classify the kinds of mathematical knowledge teachers possess or make available for learners to discern. This notion developed as an extension on Adler and Davis'(Adler, 2009) methodology for studying mathematics teacher education.

METHODOLOGY

Data for this paper will be obtained from video-taped mathematics lessons of one mathematics teacher. Video data will be organized and cleaned by first transcribing the lessons and thereafter transcriptions will be loaded into Nvivo to be coded. The analysis process will be conducted using theoretical resources which will be recruited from Phenomenology (Stumpf, 1994), Theory of Variation (Lo, 2012; Runesson, 2005) and the notion of *mathematical reality* and *mathematical appearance* (Moalosi, 2015). Phenomenological approach to data analysis is necessitated by our interest on theorizing the enacted object of learning. We will call upon the Theory of Variation to help us describe the enactment process while the notions of mathematical reality and appearance will be needed to classify the enacted object. The analysis will be constituted by both descriptive and interpretive analysis. In the descriptive analysis we will describe verbatim, how and what happened in the lesson. Our descriptions will be supported by data which will be presented as conversations between teacher and learners to support and provide evidence to enrich our descriptions. In addition, pictures from video clips will be used to support and provide further evidence what was said and done in the lessons. Interpretive analysis will be done using the notion of patterns of variation and dimension of variation.

FINDINGS

The findings of the study will be informed by the answers to the research questions and the interpretive analysis of the answers. Findings will move further to theorize the teacher's mathematical knowledge for teaching (MKT), the nature of the enacted object and that which was made available to learn. Recommendations will be articulated and further research will be proposed.

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REFLECTING ON THE IMPLEMENTATION OF TWO EDUCATORS' LEVEL OF INSTRUCTIONAL KNOWLEDGE AND LEARNERS' PERFORMANCE: A CASE STUDY

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This study was aimed at reflecting the impact of linking educators' level of instructional knowledge and learners' performance. A case study was carried out in two South African schools which indicated that challenges associated with mathematical misconceptions and errors among educators and learners in mathematics lessons is due to a lack of pedagogical content knowledge (PCK). This form of an instructional knowledge was found to be associated with effective teaching strategies characterised by efficient planning and preparation. High levels of PCK were also associated with effective application of teaching approaches such as the Concrete -Representation–Abstract "CRA" in the context of the constructivist's theoretical perspective. It is therefore recommended to implement practical staff developmental strategies that would produce an educator who is well equipped with pedagogical content knowledge to improve the quality of teaching in mathematics.

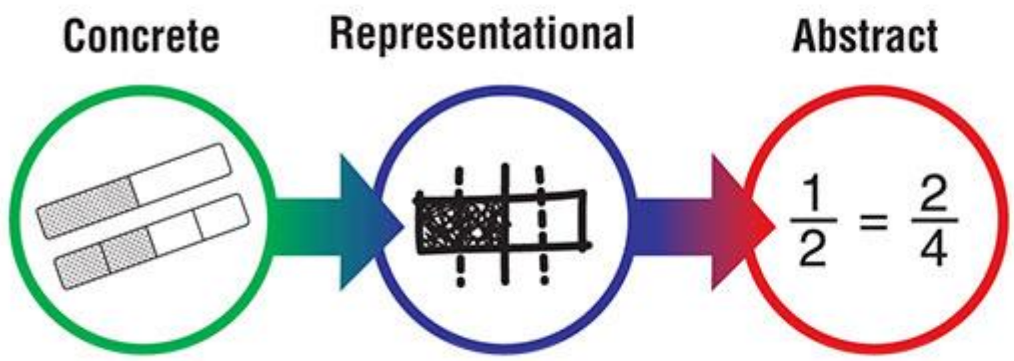
INTRODUCTION AND BACKGROUND TO THE STUDY

Mathematical misconceptions and errors are sometimes associated with poor teaching strategies and the application of irrelevant theoretical approaches of learning (**Philipp,** 2007; Burton, 1997). Considering that learners achieve highly when directly involved in the construction of knowledge, the Concrete-Representational- Abstract (CRA) approach (Kosko and Wilkins, 2010:79) in the context of the constructivism theoretical perspective has been adopted and used in this study as one of the important aspects of

PCK to examine and establish the link between PCK, planning and lesson preparation and learner performance. A more detailed account of the above term will be highlighted under the literature review. Although many researchers have concentrated on developmental strategies of basic mathematical numeracy and literacy, very few empirical studies have been conducted in holistic approaches of embracing the teachers' instructional knowledge with special reference to pedagogical content knowledge (PCK) (Philipp, 2007; Mishara & Koehler, 2006) in South African schools. Research findings claim that the quality of the instructional knowledge provided by the teacher is the "most important variable" which determines the learners' level of performance (De Clercq, 2008:7; Stewart, 2011). Although there is a belief that mathematical errors that occur in the classroom are seen as an opportunity to enhance the learning process, there is no substance to validate this view if the teachers' level of PCK is not established for evaluating teaching competency and addressing shortcomings. This is probably the reason why many South African schools are still facing challenges of poor performance in mathematics.

LITERATURE REVIEW

Pedagogical content knowledge (PCK) results from a combination of the educators' content knowledge (CK), knowledge of students, as well as the different ways in which content (CK) can be used in teaching and learning in the classroom (Helms & Stokes, 2013; Gess-Newsome & Carlson, 2013). It is believed that a competent educator is one who is able to see learners as being active rather than passive so, they will be the centre of the learning situation as advocated by the proponents of the constructivism theoretical perspective. From a mathematical point of view, the Concrete Representational Abstract (CRA) is a three step instructional approach that has been found to be highly effective in teaching mathematical concepts.



According to Kosko and Wilkins, (2010: 79), the first step in the diagram above is called the concrete stage. It is known as the "doing" stage and involves physically manipulating objects to solve a mathematics problem. The representational (semi-concrete) stage is the next step. It is known as the "seeing" stage and involves using images to represent objects to solve a mathematics problem. The final step in this approach is called the abstract stage. It is known as the “symbolic” stage and involves using only numbers and symbols to solve a mathematics problem. CRA is a gradual systematic approach. Each stage builds on the previous stage and therefore must be taught in sequence. The use of the CRA approach is also associated with high level of PCK which demands effective planning and preparation.

THEORETICAL FRAMEWORK

CONSTRUCTIVISM

This study is underpinned by the constructivist theoretical approach which broadly covers a wide spectrum of current research that overlaps with cognitivism. This theory argues that learners independently construct knowledge according to their own context and build new ideas and concepts (Piaget. 1991. Vygotsky, 1980). In the context of the constructivist theory, the CRA approach as highlighted above enables learners to be actively involved in concept development and direct their own learning as much as possible while the educator acts as a mediator, facilitator or coach. This however seems to contrast with what is happening in teaching in many South African classrooms, where the emphasis has been on the educator transmitting facts to learners who are expected to memorise these by rote learning. There also seems to be little research conducted on establishing the educators’ level of PCK and learners’ performance as related to planning and preparation and concept development effective enough to eradicate mathematical errors and misconceptions. It is, however, against this background that the research question is stated as follows:

What are the implications of the educators’ level of instructional knowledge and learners’ performance?

METHODOLOGY

This study used a case study in two conveniently selected South African primary schools based on minimising travelling costs, time and also considering availability of respondents. The case study gave room for the observer to be part of the lesson in terms of teacher support basing it on the expertise and experience as a mathematics coach/mentor. During the lesson observations this study was aimed at establishing the teachers’ pedagogical content knowledge as linked to lesson planning and preparation as well as application of the CRA instructional approach in the context of the constructivism theoretical perspective. The study applied the problem solving strategy, the Critical Incidence of Change (CIC) as advocated by Leon Burton. According to Burton, (1997) problem solving using the Critical Incident of Change approach involves the starting point, the entry, the attack and the review phase which will be illustrated during and after lesson observation. In choosing face-to face and focus group interviews, the researcher amongst others, also considered their subjective analysis and descriptive abilities including a high level of accuracy in exploring feelings, experiences and perceptions of mathematics educators regarding reflecting the impact of linking teachers' level of instructional knowledge and learners' performance (Lauer, 2006: 76; Hatch, 2002; Groenewald, 2004: 6, Musundire, 2015).

FINDINGS AND DISCUSSION

A grade 4 lesson on fractions was observed in school A.

Subtopic: Understanding Concepts:

- Ordering and comparing common fractions.
- Equivalent fractions
- Date: 28/08/16
- No. of learners: 38

Errors identified:

- The teacher had no sound knowledge of developmental teaching strategies of describing, ordering and comparing common fractions.
- The teacher did not know the relationship between fractions in terms of equivalence.

Starting point: The teacher asked three learners to compare fractions using <, > or =

The answers were as follows:

Learner A: $\frac{1}{3} > \frac{1}{2}$

Learner B: $\frac{1}{8} > \frac{1}{4}$

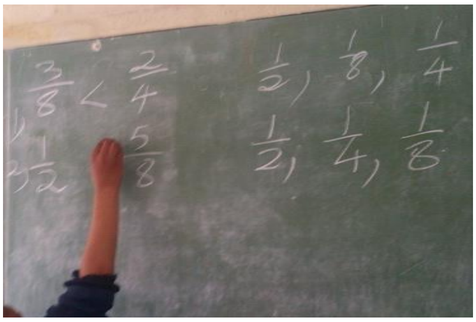
Learner C: $\frac{1}{4} < \frac{1}{8}$

When asked to explain how they got the answers, they showed that their answers were based on the size of the denominators.

Entry: Although the educator realised the learners were making a mistake; the challenge was on how to correct the error. The educator requested the observer to taken over the lesson for support after admitting a lack of conceptual development skills. This was done on the grounds that the teacher already knew that the observer was a qualified mathematics coach.

Attack: The observer took ownership of the challenge after being granted authority by the educator. The starting point was to introduce a fraction wall chart where learners could compare and order fractions with different denominators such as (halves; thirds; quarters; fifths; sixths; sevenths; eighths). This was later followed by learners using fraction strips to compare and order fractions as illustrated below:

Critical incident of change: In terms of equivalent fractions, the above pictures show that the learners were able to identify that $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$



Critical incident of change: By means of a fraction chart and fraction strips, learners were able to order and compare fractions as illustrated by the above picture.

Lesson Review phase:

This stage involved a review of the lesson including an intervention strategy by way of responding to interview questions. The educator discovered that key steps for the learner within the describing, ordering and comparing of fractions will start with the use of concrete fractional objects or pictures. This can also include practical activities such as the use of fraction strips. By so doing, learners are in a position to correctly describe order and compare fractions. This will also have enabled them to tackle addition and subtraction of common fractions.

The implication from the observed lesson was a low level of PCK on the part of the educator. This affected the planning and preparation part of the lesson. This is the reason why learners perform poorly under such circumstances. Comparatively, the observer effectively applied the CRA approach in such a way that learners were able to grasp the concept. In other words, the misconceptions and errors were eradicated.

A grade 7 lesson on capacity was observed in school B

Errors identified:

- The educator had no sound knowledge of the difference between capacity and volume.
- The educator was failing to relate units of measuring volume to those of measuring capacity

CONCEPT DEVELOPMENT:

Capacity

Starting point: From the introduction, the teacher mistakenly introduces capacity as volume. The educator tells the learners that a jug of water that was in the classroom holds a volume of 2 litres of water. She went on to explain that the volume of a drum that was shown on the wall chart holds 20 litres of water.

Entry: The educator realises that there was a bit of confusion during lesson delivery. The educator grants authority for the observer to take over the lesson.

Attack:

By way of demonstration, the observer asked one of the learners to hold two containers, a 1 litre and a 2 litre containers in front of the class as show by the picture below:

Observer: What is the quantity of the drink that goes in the small bottle?

Learner A: I litre

Observer: What is the quantity of the drink that goes in the big bottle?

Learner B: 2 litres

Observer: From your previous grade 6 knowledge, what is the quantity of the space covered by the 1 litre bottle?

Learner C: 1000 cubic centimetres

Observer: Why do you say so?

Learner B: Because 1000 cubic centimetre is equal to I litre.

Observer: What is the space covered by the big bottle?

Learner D: 2 000 cubic centimetres

From the above answers, which is the volume of each of the containers?

Learner A: 1000 cubic centimetres and 2000 cubic centimetres?

Which of the above answers is the capacity of the two containers?

Learner E: 1 litre and 2 litres

Observer: What is the difference between volume and capacity?

Learner A: The capacity is the drink that goes in the bottles.

Observer F: By way of demonstration, the learner pours water into the small bottle and the big bottle.

“The water in the bottle is 1 litre. That is its capacity. The water in the big bottle is 2 litres and that is its capacity”

Learner F empties the two bottles, “The space occupied by each of the bottles is the volume.”

The review phase:

Just like in school A, the educator and the learners in surprise acknowledge that they now know that volume is the space an object occupies and capacity is the amount of substance that an object can hold or the amount of space inside the object. It shows that some of them had an idea from their previous grade, only that the educator was confusing them. They also came to remember the different units of measuring volume and capacity. The post lesson observation involving face to face interviews had similar outcomes like the first school.

The implications from this lesson were a lack of instructional knowledge and lesson planning and preparation by the educator.

CONCLUSION AND RECOMMENDATIONS

It was therefore concluded that teachers' level of instructional knowledge determines the effectiveness of planning and preparation in order to be in a position to eradicate mathematical errors and misconceptions among both learners and educators. It is therefore recommended to implement practical staff developmental strategies that would produce an educator who is well equipped with pedagogical content knowledge to improve the quality of teaching in mathematics. It is also recommended that educators attempt to apply the Critical Incidence of Change (CIC) and the Concrete-Representational –Abstract (CRA) instructional approaches in a mathematics lesson.

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THE USE OF MATHEMATICS TEXT BOOKS TO ENHANCE THE QUALITY OF TEACHING AND LEARNING: REFLECTING ON A GRADE 7 LESSON

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The purpose of the study was to reflect on how grade 7 teachers in a rural setting use mathematics textbooks effectively. Six teachers in rural schools in one district of Mashonaland East Province in Zimbabwe constituted the sample. Random sampling was used. Data was collected through lesson observations and semi-structured interviews with the teachers. The findings of the study showed that teachers still rely heavily on mathematics textbooks, and they generally use the textbooks in creative and effective ways. This they do through the use of learner-centred teaching approaches, developing correct mathematical language and using real life examples in their mathematics lessons. There is, however, a need for teachers to encourage the learners to solve textbook problems in their own ways, as long as they arrive at the same correct answers and can explain their methods.

INTRODUCTION

Textbooks are crucial in the teaching and learning of mathematics. Teachers use mathematics textbooks to decide what to teach, how to teach it and the nature of tasks assigned to the learners. Hence this study was considered worthwhile in order to reflect on the quality of textbook use by grade 7 teachers in rural schools of Mashonaland East in Zimbabwe. Remillard (2005) observed that mathematics is a subject that has historically been driven by textbooks; therefore the field of mathematics offers a fruitful opportunity to examine how teachers use textbooks in teaching and learning. It is necessary to examine how well mathematics is taught in the primary school, and this includes looking into the use of mathematics textbooks, and where possible to recommend improvements. Without knowledge of how textbooks are used in the classrooms, it is difficult to come up with appropriate teacher support programmes that will empower them to meet the challenges they face in their day-to-day work and in their efforts to help students to learn mathematics meaningfully.

PURPOSE OF THE STUDY

The study was guided by the following research question:
To what extent do grade 7 teachers in rural schools of Mashonaland East Province in Zimbabwe use mathematics textbooks to enhance the quality of teaching and learning?

THEORETICAL FRAMEWORK

This paper on grade 7 teachers' use of mathematics textbooks used social constructivism as a theoretical framework. Social constructivism emphasises the importance of culture and context in our understanding of what occurs in society and in the construction of knowledge. Shuford, Howard & Facundo (2006) highlighted three important assumptions of social constructivism. Firstly, reality is constructed through human activity. In other words, members of a society together invent the properties of the world. Secondly, knowledge is a human product, and is socially and culturally constructed. This means individuals create meaning through their interactions with each other and with artefacts in their environment. Thus the textbook is an example of a socio-cultural artefact that teachers use in a classroom situation. Thirdly, learning is a social process, and meaningful learning takes place when individuals are engaged in social activities. In this paper it was important to describe these social activities in the context of using the *Step in New Primary Maths Grade* 7 textbooks in teaching and learning.

Implications of the constructivist theory for mathematics teaching

Paulsen (2008) summarised the implications of constructivism by suggesting that meaningful learning is likely to occur when teachers -

- use activities which build on learners' prior experiences, what they already know.
- use activities which learners find interesting.
- provide immediate feedback to learners, not only on the correct answer but also on the process of arriving at that answer.
- use and develop correct mathematical language.
- encourage learners to work together and to make decisions.
- are more learner centred in their teaching (p.39).

In other words, teachers should be able to plan to teach mathematics effectively, and for this to happen teachers need to have a good understanding of the use of text books and how students learn mathematics.

METHODOLOGY

Design

A qualitative design consisting of lesson observations and semi-structured interviews was used. It was possible to obtain alternative explanations and clarification through the observations and interviews, something not possible when using, say, questionnaires.

Ethical considerations

All the ethical considerations were taken into account in conducting this study. These included, among others, obtaining clearance to conduct the research, informed consent, anonymity and confidentiality of participants.

Participants

Six grade 7 teachers at six randomly identified rural schools in a district of Mashonaland East Province participated in the study. All the six teachers in the sample held at least a Diploma in Education (the minimum teacher qualification in Zimbabwean schools) and had between 5 and 15 years of teaching experience.

Data collection

Both the interviews and lesson observations were audio-recorded with the consent of the teachers. The audio-recordings were complemented with note-taking. Semi-structured teacher interviews were held at their respective schools. Lesson observation schedules were used to record lesson proceedings.

Quality criteria

In order to ensure trustworthiness of the interviews and lesson observations procedures for attaining credibility (through member checks), dependability and confirmability (Stringer, 2008) were followed.

Data analysis

In data analysis, the 'verbatim principle' (Stringer, 2008: 99) was used. This principle encourages the use of the exact words that the respondents actually said in the conversations. The data were then analysed in relation to the teachers' use of the mathematics textbook in teaching.

FINDINGS AND DISCUSSION

The findings and discussion presented here are for only two of the teachers that participated in this study. The reason is these teachers' lessons fitted in with the theme of restoring the dignity of mathematics learners through quality teaching and learning by using a mathematics textbook.

Teacher A: Lesson on percentage increase and decrease:

The teacher used question 4(a) (Find the percentage decrease from 16 to 12) on page 157 to demonstrate how to calculate percentage decrease as follows:

16-12=4

$={}^{4}/_{16} \times {}^{100}/_{1}$

=25% decrease

The teacher told the learners that: when you decrease you subtract, then the common fraction must be changed to a percentage; divide using the highest common factors…

One volunteer learner was asked to come to the front and work out question 4(b) on the chalkboard.

The working out was as follows:

75 to 70

=75-70=5

$={}^{5}/_{75} \times {}^{100}/_{1} \quad ={}^{20}/_{3}$

=6,66%

Teacher: Our denominator is the original number before the decrease.
The final answer of 6,66% was not rounded off to the first or second decimal.
The teacher accepted this answer but went on to explain how to round off to the 1st, 2nd or 3rd decimal; and to talk about recurring decimal numbers. The textbook question is silent on whether the answers should be rounded off to the 2nd decimal or not. However, the teacher could have shown the learners that $6\ ^{2}/_{3}\%$ is also an acceptable answer to this question. Also, the teacher should have related percentage decrease or increase to socio-economic contexts that the learners could easily identify with.

Teacher B: Lesson on discount

Discount: There was a question and answer session on the meaning of the term 'discount', and when it applies.
Teacher B: What does the term 'discount' mean? Even in the vernacular, tell us what it means.
Learner: Mari inobviswa kana watenga chinhu (in Shona). **(**This means the money deducted from the sale price when you buy an article).
Teacher B: Yes, it is deducted when you buy in cash, and this discount is expressed as a percentage, for example 10% discount when there is a sale; no mention of the amount of money, only the percentage.

Use of pictures/diagrams in a mathematics textbook to develop concepts

Find the sale price (after the 10% discount) for these items whose original prices are given.

a) men's long sleeved shirts @ \$125,50 each
b) men's short sleeved shirts @ \$99,00 each
c) men's T shirts @ \$57,50 each
d) ladies' cotton shirts @ \$36.60 each
e) men's trousers @ \$180,60 per pair
f) ladies' dresses @ \$179.90 each
g) ladies' skirts @ \$93,60 each
h) ladies' blouses @ \$ 73.50 each

The teacher spent time looking at the picture of Zama Zama Outfitters on page 157 in the textbook together with the learners, and asking them a variety of questions based on the picture, as listed here:

- Who are Zama Zama Outfitters?
- What goods are on sale?
- What is a short-sleeved shirt? A long-sleeved shirt?
- Do you see we have both long and short sleeved shirts in the picture?
- What percentage discount is being offered? On which products?
- What is a discount?
- When and why do shops give a discount?
- In what ways do shop owners benefit from offering a discount?
- How do customers benefit from receiving discounts?

As can be seen from this list, both low and high order questions were asked. This manner of questioning is good, and consistent with that advocated in Bloom's taxonomy of educational objectives (Lawton, 1982; Stenhouse, 1979).

Teacher B:

10% off selected goods, meaning they will make you pay 10% less on the sale price. Question 5(a), men's long sleeved shirts @ \$125,50 each. So first calculate 10% discount, then subtract this from the original price, this gives you the sale price.

$^{10}/_{100} \times {}^{125,50}/_{1}$

=\$12,55 discount.

So we must now go on to answer the question. Therefore the sale price =\$125,50 - \$12,55=\$112,95. This means you have saved \$12,55.

Group work was given at this stage. The activities that occurred during the group activities are summarised below.

- The group members read the question from the textbook, discussed it and wrote down the answers.
- The teacher walked from one group to the other, to check on the learners' progress and to give assistance where necessary.
- Most of the discussions were conducted in the learners' vernacular language, although the final answers were presented in English. This practice is an example of language-switching.
- A group representative wrote the answers on the chalkboard, and explained to the rest of the class how they obtained the answer.
- The groups worked on different questions, but from the same exercise.

One group's answer was incorrect. The group had presented their solution to question 5(d) on page 157 as follows:

10% of \$36,60
$={}^{10}/_{100} \times {}^{\$36,60}/_{1}$
=36,66-36,60
=\$0,06

Teacher B had to work out the solution correctly on the chalkboard, with the participation of the learners.

Requiring the groups to report back on their deliberations helps, especially in cases like the above where they worked it out incorrectly. Here Teacher B used small groups to discuss mathematics textbook tasks. Group work helps in the sense that those learners who are timid to ask the teacher some questions or who have language problems can ask their group mates. During group work the learners read from the textbook, interpreted/clarified the questions and solved them. The report back sessions enabled further discussion by the whole class and gave the teacher an opportunity to confirm the correctness of each group's answers.

CONCLUSION

The terminology used in parts of the mathematics textbook requires teachers to explain and clarify for the benefit of the learners. Teachers can make use of real life examples from the socio-cultural or economic backgrounds of the learners to explain these textbook concepts. This is because nowadays it is widely accepted that mathematics should be taught in context, in order to bring real-life situations into the classrooms (Department of Basic Education, 2013). Teacher B made clear links between the topic in the mathematics textbook and real life objects and practices by referring the learners to the picture of Zama-Zama Outfitters in the textbook (page 157) when discussing discounts on short and long-sleeved shirts, and blouses. This practice helps the learners to understand concepts better, and as Nickson (2001) argues, they can link the mathematics they meet at school with what they already experience in their natural and social environment. Thus it fulfils the expectation to teach in context, by not confining mathematics to the four walls of the classroom.

This to some extent reflects a rich teacher's subject content knowledge which, according to Hoadley and Jansen (2009), enables a teacher to enrich the learning process with a broad range of illustrations, and to integrate prescribed content with the wider world, the learners' lives, and other learning areas. Even after working out a question on discount the teacher went on to interpret the answer in terms of how much the actual discount is, how much the customer will finally pay, and how much the customer will save. This approach helped the learners to understand the underlying reasons for offering discounts, as it went beyond the purely mechanical process of working out an answer. The teacher also wanted each group's answer to be written on the chalkboard, so that the whole class could see what the other groups did and also to check the correctness of the group answers.

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USING VIDEOS AS A TOOL TO INVESTIGATE TEACHERS' REFLECTION IN AN IN-HOUSE PROFESSIONAL DEVELOPMENT INITIATIVE

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A teacher PD initiative with six FET Mathematics teachers is investigated where the use of video-stimulated reflection by teachers is used. Mathematics teacher development practitioners have been doing interventions away and detached from the hands-on classroom analysis and are therefore not customized according to the specific needs of teachers. However, it is essential to go into the classroom, analyse the needs and develop school teachers' Mathematical knowledge for teaching (MKfT) in a relevant research-based mode. If there is to be improvement in teachers' conduct in the classroom, emphasis should be placed on teachers' learning and self-reflection on classroom practice as means of PD (Muir & Beswick, 2007). In this context then, self-reflection, where “deliberate and consistent examination of (one’s) teaching practice” (Pellegrino & Gerber, 2012, 1) takes place, is a critical component for developing teachers’ appropriate instructional practices and decisions on “what to teach and how to teach it” (ibid). This contribution will explore the usage of the technique called, Video-stimulated recall (VSR). VSR “is an introspection procedure in which videotaped passages of behavior are replayed to individuals to stimulate recall of their concurrent cognitive activity” (Lyle, 2003, 861). This reflection is done in the natural setting of the teacher’s classroom – thus attached to their work place and therefore customized for specific individuals and their development. Video-stimulated recall is based on observing the current practice of teachers, while striving to develop suitable pedagogical practices (Muir, 2010). As a technique it gives an opportunity for immediate and specific feedback which can lead to the effective development of an individual with the support of the teacher educator/researcher/ HOD. This is done where the researcher supports the teachers to develop their Mathematical Knowledge for Teaching (MKfT).

This qualitative study is guided by the following research questions:

1. In what ways, if any, does video-stimulated recall support reflection by Mathematics teachers on their classroom practice?
2. What changes, if any, occurred in teachers’ MKfT that can be associated with video-stimulated recall interviews?

The data collection will include video-recorded lesson observation cycles, semi-structured interviews after a time lapse and then follow-up video-recorded lesson observations. The interview serves as the intervention where the researcher will pose critical questions to allow the teacher to reflect on his/her practice and decisions. The significance of this study is that video-stimulated recall can be adopted by HODs and their subject teams for in-house PD, which is customized to both the teachers’ needs and unique classroom contexts.

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BLENDED LEARNING AT MATHEMATICS ENTRY LEVEL FOR THROUGHPUT AUGMENTATION OF THE BACHELOR OF SCIENCE DEGREES: SMU CASE

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University first-year math is a common prerequisite for many majors in the degree. The module's pass rates were conservatively below 50%, a concern to the University management and a new lecturer was appointed in 2011. This paper explains the tasks she undertook and the results she produced. The students of her module in the four years (2007-2010) before and four years after incorporation of blended learning (2011-2014) are compared. Blended learning in this study showed to produce significantly improved results in student passes and a higher retention rate. It offered to achieve the included higher access to information, student support and assessment. The result was more successes and higher throughput.

INTRODUCTION

Admission to higher learning is often done with the belief that the qualification can be, and will be completed. It is done on the bases of the minimum requirements set, which are believed to be the minimum capability to complete the qualification (Learning Resources, 2011). A qualification at tertiary level is usually failed or passed at first year level, its entry level. Experience shows that most failures occur at first year level, and the dropout rate at first year level seems also to be the barrier for many tertiary qualifications. Higher percentages of students who pass the first year at first attempt, end up completing the degrees (Johnston, 1997). More dropouts at higher year levels seem to be those who repeated the first year. The problem at first year is not inability in terms of intelligence, but the complexity of the way teaching and learning are conducted at tertiary level. This is totally different from the way schools conduct their teaching and learning. Thus teaching at first year should be done to transform the thoughts of first years from school approach to the demands of universities. This approach is intended to increase pass rates at first year level, hoping that it would increase pass rates at higher levels. These will be assisting in completion of the qualifications and therefore, increasing the throughput rate. The Faculties of the various Sciences have a history of not producing enough scientists for the country. In South Africa, in the earlier years of democracy, the South African government put pressure on the higher learning institutions' science faculties to develop means to increase the student throughput rate. Some institutions devised means to achieve the call, while others did little to improve the situation.

At the SMU, there were a few courses that were known to be blockades to student throughput rates increase. Math, due to large numbers of students enrolments because as a prerequisite of many science courses. There was also a math access course offered as a bridge from matric (for those with low grading of passes in the subject) to first-year math, which in the years before 2011, was failed in high numbers. This access course was in fact one of the modules having the highest number of failures. Moreover, the ones who had passed this access course at the time were generally also not able to match students who came directly from matric. Hence, they contributed to the failure rate at first-year math. A new lecturer was sought to replace the previous one who was not seen to be willing to participate in improving the plight of the students in this access courses. She was found and replaced the previous lecturer. Soon after coming, together with statistical colleagues in the same environment, she investigated what was happening and what could have caused the high failure rates in that module. Changes were made regarding the teaching approaches in this module, together with the first year Statistics (stat-I) and a third year module in Statistics (stat) as an intervention and experiment or pilot to improve the pass rates with more understanding and appreciation from the students.

Prior to the interventions, summative assessment used to take place through two formal tests in a semester, which was the case in first year and third year stat modules. In the access math module which is done over one academic year, assessment was in the form of four formal tests for the year. The tests contribute 60% of the final marks in a module and the examination account for the remaining 40%. Thus, in addition to the tests being important for preparing students in understanding the module and the examination, it contributes more in the final mark in a course/module. In the year 2013, the second-year math and stat consisted of students from the first group of former first years who were in the experimental access math group. Hence, at the time of writing this paper (middle of 2014), there were no results for the third year from that first group involved in the blended learning described in this paper.

The context is that since mathematical and physical science majors depend on Mathematics I (math-1) as a prerequisite, high passes in these modules in this subject would enable mode admissions and progress into higher levels of the other subjects. More students of math and stat courses would benefit tremendously as well. Also, understanding of first-year calculus and matrix methods would also enhance more forceful understanding of the concepts used in Physics and Chemistry at higher levels. Thus, this would give a higher chance for improved performances in these subjects as well.

The paper discusses the specific research questions in the teaching model developed, the research methodology used for the study, the results obtained in the experimentations, the increases in pass rates in the modules, resultant intake in the stat major, resultant passes in first-year math, majors in the math, and the implication for throughput rates. In addition, the paper isolates the results of the students from the access math group to determine how they performed at second and third year levels of math and Statistics (stat 1), separately from the rest.

RESEARCH QUESTIONS

Math and stat training for the BSc at SMU basically start in the majors at undergraduate level. These require a pass with 50% in matric math at matric level. Lower passes in matric maths, where a student applying for entry into a BSc study, are usually allowed to take other majors, but in some cases also required to complete a module at a level between access and first-year math. There are students staying on campus, and others in Tshwane (i.e. Pretoria city), in accommodations arranged by the university campus**.** Traveling by buses enables commuting daily between campus and residences. The buses are sometimes delayed. The students are often late for lectures. If the lectures take place in their absence, then the purpose of assisting them to grab the concept and pass well and qualify for entry into mainstream courses is compromised. This paper attempts to estimate the end of 2014 passes in the third year of math and stats from the trends of the student groups.

First the study wants to determine if there was a significant change in the results of the students in math access and third-years stat due to the interventions made. Focus was to determine if the pass rates increased in the years after the interventions. Another interest was to determine if there are changes in intake of first-year math, as well as increased pass rates as a result of the interventions. Then the study checked if the students in the pilot group matched the others in the first- and second-year modules involved, and also if their dropout rate had decreased. Based on these changes the paper also estimates the pass rates in the 2014 math and stat majors. Five research questions of interests to the study were:

- Was there a significant improvement in the results of students in the math access and third year stat at Medunsa after the interventions were implemented?
- Were there significant improvements in intake of first year math from the access module after intervention?
- Did the pass rates in access and first year math increase significantly as a result of the interventions?
- Did the students in the pilot group significantly match the others in the first and second year modules?
- Did the dropout rate decrease significantly?
- What are the predictions for the pass rates in the 2015 math and stat majors?

THROUGHPUT AND ITS ENHANCERS

The assumption of this paper is that it is default that good or suitable planning is the starting point. Thus, in addition to the enhancers, skillful planning would have served as a starting point.

Students enrolled for tertiary level courses or modules were admitted due to their perceived capabilities demonstrated by past performances in the prerequisites levels. Ideally, therefore, when enrolling at first level for the first time, they are all potential future graduates. Experience shows that they do not always all complete and graduate. In some cases those who fail exceed those who become successful. The number graduating at the end of the duration for a course/module explains throughput rate. Throughput rate is a measure of efficiency (Computer Science Curricula, 2013) defined in education formally as the fraction of the students who graduate at the end of the minimum duration of the course of study, based on the number who were registered for the course (Trowler, 2010). However, since interest is more on completion of a course of study, institutions tend to calculate throughput as all the students who end up graduating within the maximum and the extended period permissible for the study in the institution.

Throughput

NASULGC (1998) states that for students, the problem is one of "getting in, hanging on, and getting out," as one of them has put it. As it stands, it does not mention timing of getting out after hanging on. Hence, it implies that throughput can be measured by also looking at lags. That is, as throughput is measured as students graduate after the first possible chance for a group that started a course together, the ones who keep completing are added to the previous group and a new throughput rate is updated. Those who do not complete the course are the ones who lower the throughput rate. The principal enhancers of throughput are student access to the necessary resources, the capability to persist in a course of study as students, student support programmes, mode and role of assessments, passing the examinations and the way study methods are mixed or blended by the lecturers.

Student access

Access for a student who is already pursuing a course refers to having resources necessary to perform the necessary exercises to acquire the prescribed knowledge in order to pass (Wirth & Perkins, 2008). NASULGC (1998**) insinuates that** institutions need to pay attention to issues of "access *to* the institution," "access *through* the institution," and "access *from* the institution," and also implies that access is required not for its sake, but for academic success. When access is lacking, then academic success cannot happen.

Student retention

Writings on student retention suggest that early warning programs designed to identify students at risk of dropping out can also be effective tools to improve persistence and graduation rates (College Board, 2009). When students are not retained in larger proportions, then it implies that higher proportions are dropping out. This simply implies that throughput rate is being lowered. Student throughput therefore is highly dependent on student persistence in a course of study.

Academic support

Student support includes all efforts and resources put in place to support student to perform well, and to identify students who can be identified as at-risk, which is an indication of high propensity to perform poorly in studies (OECD, 2012). The student support at SMU is a program consisting of academics, financial support officials, psychologists, student mentors, and co-opted members for helping with any problem the students face that threaten progress in learning. It serves mainly the undergraduate students, but will be extended to postgraduate students should it become necessary. Its design is that it is a reactive program waiting for identification of students who show to be heading for failure. In universities that do not support students, those from disadvantaged backgrounds tend to fail in large numbers. Also, without student support, even the ones from highly privileged backgrounds can find studies to be difficult and unmanageable. In the past era before the concept of student support was introduced, throughput rate was usually very low in institutions (OECD, 2012). Academic support for students was then introduced after extensive research and piloting exercises.

Assessment

Students are always assessed at the end of lessons or programme to determine if they have achieved the learning outcomes (Cartwright, Weiner & Streamer-Veneruso, 2009). In the past summative assessment was the main form of examining if the students have learned as required. That form of assessment used to carry a lot of mass of content expected to be completed or remembered within a short three-hour examination. Experience showed that shorter portions of content are easier to remember, especially when they occur immediately, or not too long after the lesson. The courses were then semesterised in an attempt to enable shorter pieces of work to be assessed and to ease the workload at every examination stage. Furthermore, many short and high frequencies of pieces of assessment increase reinforcement of content. When a summative assessment at end of semester is given after many shorter formative assessments, there is usually a higher number of passes compared to when fewer were given.

Success rate of students

Success rate simply refers to when more students enrolled for a course of study are able to perform well (i.e. pass the assessments) in larger proportions (Kuh, Kinze, Shuh & Whitt, 2005**)**. It is easier to achieve high success with fewer students because the needed special attention by teachers and tutors can easily be affected for student at-risk. In traditional teaching, higher failure rates are usually experienced when the student numbers are too large, because there is often limited time to give individual students attention. It requires exceptional teaching aptitude and attitude for a teacher in a lecture- or classroom to effect high success rates when the numbers of students are excessively high.

Blended learning

Blended learning is described as a formal education program in which recipients of education learn at least partially through online delivery of content and instruction with some element of student control over time, place, path or pace

(Bonk & Graham, 2006). Emphasis of blended learning is the contact or face-to-face integration with computer assisted information and instruction. The method has been seen to benefit learning in terms of time, because it is usually quicker; and also provide more content because of its ability to search for information from various sources. Apart from the resources placed on blended learning, Alexander (2010) points out that blended learning is flexible, and allows the learners to decide on pathways to gain learning.

Rumble and Litto (2005) defines blended learning is a fruitful effort in integrating live classroom activities including face-to-face instructions along with online learning and instructions so as to reap the maximum benefits by utilising the best elements of all through effective planning by an ideal facilitator.

STUDY PERSPECTIVE

The teaching in math access module was radically changed in year 2011 when blended learning methods were introduced by a newly appointed lecturer. The student numbers had been increasing in this module, but the pass rates were disturbingly declining. Blended methods included the use of blackboard, which is an information and communication technology (ICT) approach in which students can be reached even outside contact sessions. Access was increased by the flexibility of the lecturer's tendency to wait for students when some lecturers were not releasing them in time for the next lecture, and when students staying in town were having transport problems. It was also increased by holding after hour sessions for those who could not follow instructions of blackboard on their own. Assessment frequency was increased in multiples, with almost fortnightly tests and weekly quizzes for reinforcement. Moreover, at-risk students were identified and encouraged to agree to be admitted to the student support program on campus, which was equipped to identify any form of problem from personal, financial, psychological and others which were not easily identifiable without intense and qualified proficiency. Academic support, financial support and social support are the core offerings of the student support program at Medunsa. More tutors were used in this module, who were selected from past student mentors and best performing students in third-year and Honours levels math. A trend analysis of performance of these students is made in this study, as well as a comparison with the previous groups of the same module before the radical changes.

METHODOLOGY

The students of math access in the four years (2007-2010) before and four years after incorporation of blended learning (2011-2014) are compared. These results were supplied by the examination department on campus, and verified also in the Department of Mathematics.

RESULTS

Student numbers increased in maths from first to third year levels. The first blended learning group also encouraged the introduction of postgraduate applied maths in 2015 for the first time in SMU's history. The students who started with the ECP courses performances were tracked from year 2007 when the new math curriculum was introduced after harmonisation for the two UL campuses. Firstly, the performances were captured for annual average pass marks and on the annual pass rates. Thereafter the comparison was made on the BSc students who had gone through the blended approach with those who had been on traditional teaching methods. Statistical tests that were originally planned have been halted because the results from comparisons were clear and did not warrant further investigation. The study showed that there were significant improvements in both the pass rates and student.

Quantitative

Table 1: ECP student performances

	Pre-blending period				**Blend learning era**			
	2007	2008	2009	2010	2011	2012	2013	2014
Average marks	49	54	46	49	85	81	83	81
Pass rates	48	51	44	47	92	98	95	97

Figure 1a: ECP annual average marks

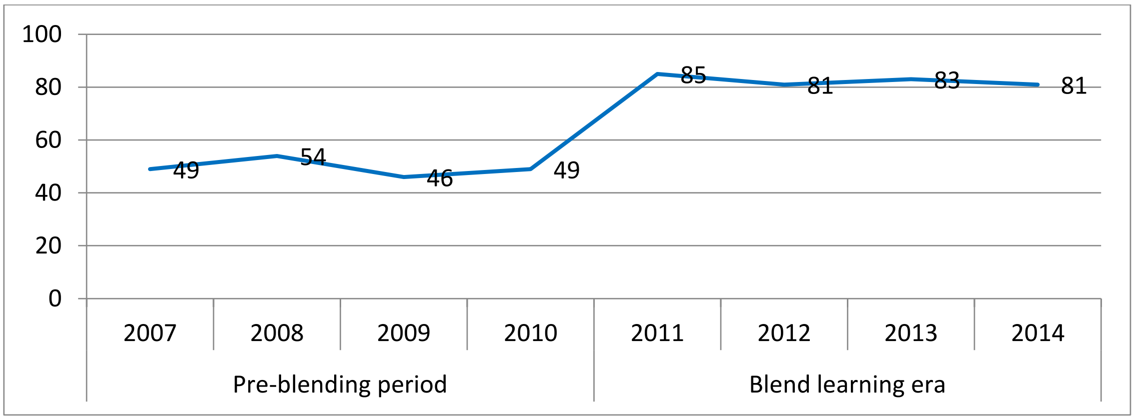

Figure 1b: ECP annual pass rates

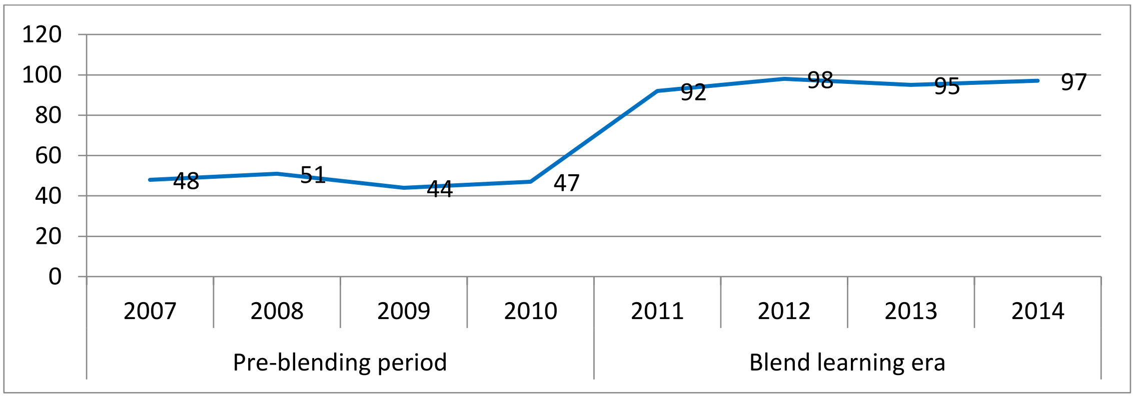

Table 2: Pass rates at Math 1: Blended students vs straight BSc

	2012	2013	2014
Blended	95	93	93
Non-blended	65	68	66

Figure 2: Comparing Math-1 groups during BSc level

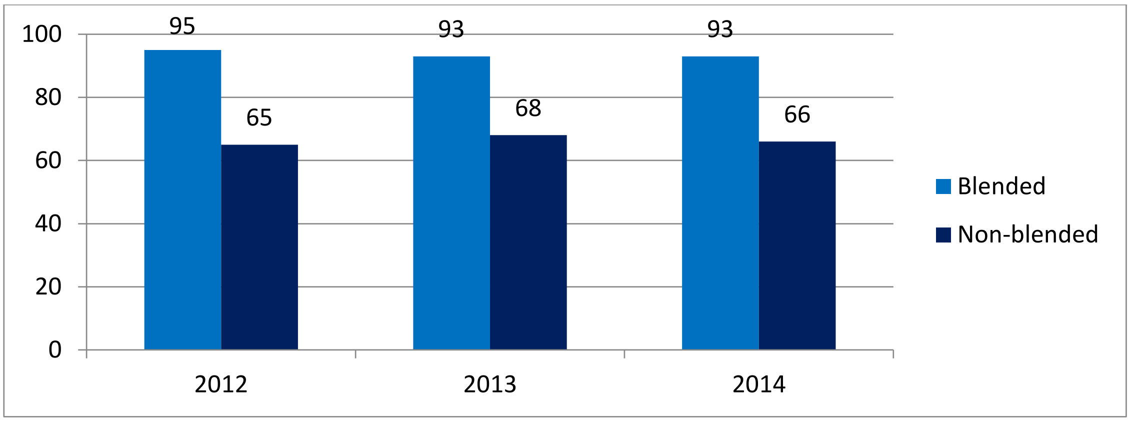

Table 3: Pass rates at Math 2: Blended students vs straight BSc

	2013	2014
Blended	95	92
Non-blended	60	63

Figure 2: Comparing Math-2 groups during BSc level

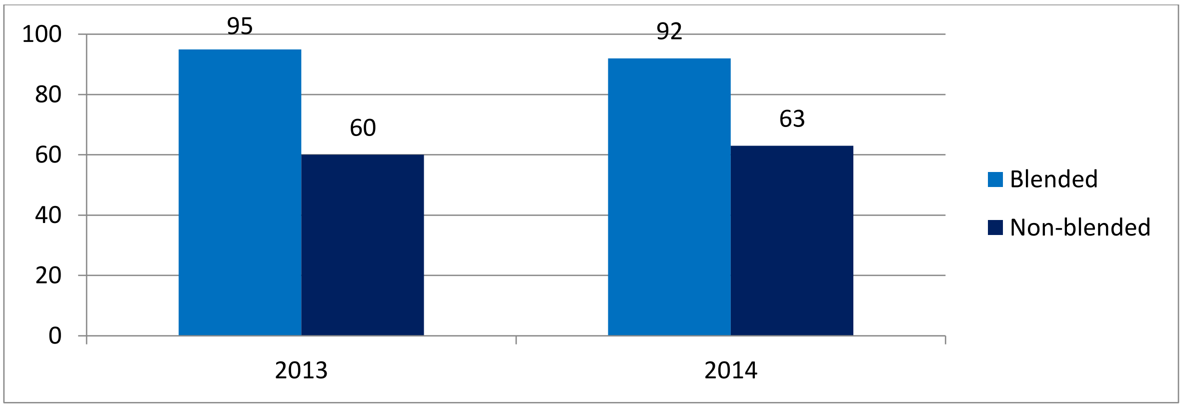

Table 4: Pass rates at Math 3: Blended students vs straight BSc

	2014
Blended	100
Non-blended	58

No graphical display accompanies this table since the observation that blended group far outperformed the other is noticeable.

Qualitative

The math lecturers indicated that since blended learning was introduced, there has been student number increases in percentages of over 75% in the subject. Literature attests that blended learning is effective at all levels of education facilitation. Evidence exists where postgraduate students during coursework period were exposed to blended learning. In the assignments of these students it was manifest that they could source more relevant information. During research, these students managed to complete their dissertations on time, and others also managed to produce journal publication.

Empirically from this study, the tutors who assisted in tracking performances of the students attested that students who came from ECP during blend learning era were able to study, find knowledge using many means than the other group, and showed more passion in learning. They also suggested that the blended group spoke well about the subject while the others were found to be complaining about some lecturers and/or the course. The unblended group was more difficult to work with because during tutorials and other subject activities, they would be observed leaving when the lecturer was late or absent while the other groups would use the opportunity to discuss some topics and exchange ideas on solving some exercises. The blended group had several interesting methods they used in studying and finding information. On the other hand, the other groups only followed lecture materials and lecture instructions, which were not even easy for them to use effectively.

DISCUSSION

Blended learning enables students to gather more information, and much quicker. The methods of sourcing information, as well as the sources of information are various in blended learning. Many of them are interesting and preferred by the students. They also differ in level of difficulty as well as in their accessibility. However, technology is leading in blended learning, and there is plenty of student support as well. Research also benefits more with blended learning approaches than in traditional modes. Pass rates increased, and learning was more enjoyable for those who used blended learning. Dropout from math was experienced only for those who did not go through blended learning. The students in the blended mode showed to match and in many instances outperform the traditional ones when they in the first and second year modules. The predictions for 2014 year end results were expected to be very high. This was true since the pass rates for the blended group was 100%.

RECOMMENDATIONS

The paper recommends that:

- Blended learning should be extended to all the modules/courses on SMU campus;

- Training should be rolled out in blended learning methods to SMU teaching staff and the assistants (i.e. tutors);
- SMU academics should be trained in various blended learning methods;
- SMU should adopt blended learning methods in lecture halls; and
- The SMU research directorate should stock blende learning materials for part time researchers.

CONCLUSION

Blended learning is both effective in teaching and in conducting research. Those exposed to it early often have a much faster pace for completing work, and superior to the slower traditional methods users. Researchers can determine the extent to which their study interests have been explored in past researches. Hence, plagiarism can be avoided while study foundation is cemented.

FURTHER RESEARCH

Blended learning was introduced in other modules on campus, but mainly for modules that were not showing serious problems in pass rates. These modules include first-year and third-year stat, some modules in physiotherapy and others in psychology. However, these were not part of this study. This study wanted to expose the occurrence in the math courses.

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THE LANGUAGE PRACTICES OF GRADE R EDUCATORS: ALIGNMENTS AND MISALIGNMENTS WITH LANGUAGE POLICY EXPECTATIONS/DIRECTIVES

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Many international studies document South Africa's poor performance related challenges in mathematics. There have also been concerns raised in relation to the South African policy that stipulates the compulsory use of home language for teaching mathematics, as to whether or not it is suitable for the proficient development of Grade R to three learners' mathematics abilities. This paper aims to report on an investigation into the nature of Grade R educators' language practices during mathematics teaching to find out whether they are supportive of developing learners to be proficient in mathematics. It further explores the educators' views of their language practices and questions whether these are aligned or misaligned to policy expectations in South Africa, in order to inform professional development and shape possible future intervention. Preliminary findings point to a complex struggle where educators expect and rely on rich isiXhosa cultural language capitals learners bring to the classroom to handle numeracy concepts, but find competing English language vocabulary that has become a culture in the home of modern families. The situation of cultures and languages at cross roads becomes complex to navigate for educators who find they have to improvise and complement the language prescribed by policy with the one learners have already picked up.

BRIEF OVERVIEW

South Africa's poor performance related challenges in mathematics have been documented in international studies such as the Progress in International Reading Literacy Study (PIRLS) and the Trends in International Mathematics and Science study (TIMSS) by Mullis (2012) as well as in regional studies like the Southern Africa Consortium for Monitoring Education Quality (SAQMEQ) by Spaull (2011). Annual National Assessments (ANAs) also reveal the same poor performance. The poor performance has been attributed to multiple factors including poor foundational knowledge in the Grade R (Feza, 2013), as well as educator practices considered by many stakeholders to be questionable or out of step with policy expectations (Excell & Linington, 2011; Artmore, Van Niekerk & Ashley-Cooper, 2012). There are also concerns raised in relation to the mathematics language policy in South Africa as to whether or not it is suitable for the proficient development of foundation phase learners' numeracy abilities. This is a critical question in research because there is compelling evidence that the years from birth to 3 years are decisive in moulding learners because cognitive development relating to the intellect, astuteness, personality, emotions and self-control takes form during this window in a human beings' life. This paper aims to report on the investigation of the nature of Grade R educators' language practices to find out whether they are supportive of developing learners to be proficient in mathematics. It further explores the educators' views of their language practices and questions whether these are aligned or misaligned to policy expectations in South Africa, in order to inform professional development and shape possible future intervention.

Competing Language Dynamics

While the language policy for education in South Africa as detailed in the Curriculum and Assessment Policy Statements (CAPS) (2011) document states that learners in the foundation phase (which includes Grade R) should learn in their home language (Mullis et al. 2012). However, this view has many other stakeholders who support it, while others contest it (Setati & Planas, 2012). Stakeholders such as parents, documented as preferring English to be the instructional language for all learning from the foundation phase hold a contesting view. They hold the perspective that English as the language of higher education, the language of business and commerce as well as governance should be the language of instruction. Their view is because they consider it able to open doors for employment (Setati, 2005; 2008; Moschkovich, 2012). Conversely, political and civic groups as well as student movements have viewed use of English as the language of instruction as a hegemonic practice that allows minority groups of foreign origin dominate African people with various home languages in the majority in South Africa somewhat unfairly (Setati-Phakeng, 2014; Planas & Civil, 2013). Planas & Setati (2014) label this as over-valuing of English and an under-valuing of African languages resulting in inequity.

The challenge of teaching learners is complex and multifaceted for Grade R educators. Grade R educators, for example in the Eastern Cape region of South Africa, where this study is situated are largely not professionally trained (Feza, 2013). Research also has compelling evidence that it is only when instructors are proficient in the language of instruction that they become effective in supporting academic achievement (Moschkovich, 2002; 2012; Setati, 2008). The Grade R educators teaching in some schools in the Eastern Cape are isiXhosa first language speakers. English is not their first language. Their low academic qualifications do not make them to be proficient in English. However, the concepts they have to teach as well as the curriculum documents they have to read, comprehend, reflect and interpret are in isiXhosa as well as English. This arises because the learners come in with English mathematics competencies.

The language policy in South Africa

The language policy in South African schools (including Grade R) makes it imperative for learners to learn mathematics in their home language. This policy was a politically driven decision contrived to ensure social inclusion for all learners in South Africa (Setati, 2008; Setati & Planas, 2012). This is because the cries to end inequality of use of instructional language through many platforms such as civic groups, students in universities fighting the uneven ground where some students learn in home languages such as Afrikaans while others such as blacks with multiple indigenous languages learn in English. The Minister of basic Education launched the CAPS mathematics policy for Grade R on a political platform that is available in the CAPS document (DBE, 2011). Research supports that use of the child's home language at this age as protection of inherent human rights as well as creation of opportunities to learn mathematics in an enabling environment (Klass & Trudell, 2011; Planas & Phakeng, 2012). In addition, leading research considers home language a resource that deepens the comprehension of concepts in mathematics when it complements the English mathematics vocabulary (Adler, 2001). The use of English to complement the home language, for this population of learners, helps them to pick up numeracy skills. When only the home language is used, the knowledge learners bring to class captured in English is lost. The dual use of the two languages also widens the choice of activities from which educators select. It also widens access to participation in classroom activities (Planas, 2014; Planas & Setati-Phakeng, 2014; Setati-Phakeng, 2014). Participation is important because even if a child attends class, if he or she does not participate, he or she cannot benefit from the activities and concepts taught. Only active engagement such as answering questions or practicing counting improves learners' knowledge and skills (Feza, 2016).

Curriculum Assessment Policy Statement (CAPS) Language and Mathematics prescriptions

The CAPS 2011 document (DBE, 2011) details the mathematics as well as the kind of teaching approaches prescribed for Grade R learners. Daily lessons provided guide educators. Detailed complimentary materials such as the integrated literacy, numeracy and life skills Grade R workbook (DBE, 2011) are availed to educators. This prescription formalises the learning and teaching in Grade R. It however, conflicts with researched based rationale, which urges learning at this level to be informal and mediated with play. What complicates this scenario is that educators tend to struggle to make sense of the curriculum. The reason for this is a lack of formal training on their part. Their challenge is failing to aim their teaching to match the learners' developmental stages. This is compounded, by the fact that learners arrive in Grade R with advanced knowledge in numeracy (Ginsburg, Lee, & Boyd, 2008). Educators with inadequate skills evaluate the entry-level point for this advanced knowledge but struggle to build on it. The study observed this to be the case in the Eastern Cape.

Educators' Competencies

Research has revealed that there are educators currently teaching Grade R who have not undergone formal professional training. There are also inadequately trained educators who are only inducted by way of in-service programmes, as is the case in some provinces in South Africa like the Eastern Cape. The educators struggle to teach the Grade R learners (Department of Performance Monitoring and Evaluation (DPME), 2012; Banard & Braund, 2016). Such educators are not equipped with adequate pedagogic skills. Educators are also not academically specialised in a subject like mathematics. They have gaps in their delivery of subject matter as well as in their foundational content knowledge (Ndlovu, 2011; Feza, 2013, 2015; Siyepu, 2013.

The mathematical trajectory as outlined by Feza (2015) involves foundational knowledge of the key landmarks in numeracy. This involves counting verbally by memorising; counting of concrete objects by placing a number to each object also referred to as one-to-one correspondence; answering to the how many questions, which brings out cardinals; reverse counting; forward counting; and skip counting where a learner makes a pattern by omitting a number in between two others as they count. This is also closely tied to a sensing skill of spotting how many objects there are viewed in a quick sweep known as subitising (Feza, 2016).This is subject knowledge which only those who have specialised in numeracy can handle accurately in teaching Grade R learners and is clearly out of scope of those who are inadequately trained.

Since the educators in question are not trained, they are not equipped to utilise alternative methods such as scaffolding children's learning, stimulating and extending their thinking through questioning or enfolding mathematical concepts in play (Feza, 2013). While some resources for teaching mathematics are available in centres where schools can access them, not all educators are availed such resources. At times educators only know of these possibilities without actually getting them. There may for example be a policy document in the school, kept in the files of the principal or head of department. The educators, who at times are not assertive, will rarely ask for it. The problem of material being short is also true (Feza, 2016).There is for example no mathematics dictionary in isiXhosa currently, which would be a rich resource for teaching concepts and vocabulary for both educators and learners.

Professionally trained educators can certainly use alternative ways of teaching mathematics. However, in the case where they are not, that becomes a challenge as they cannot fall back on such competencies due to a lack of training .This was the case in the study.

Scholars like Vygotsky (1978) also urge that first the educator must be an expert able to locate the learner's level of operation. Next, the educator has to establish the learner's potential which enables the educator to aim the lesson at the child's Zone of Proximal Development. The ZPD is a point where the learner's potential is established; the educator assists the learner with cues as well as scaffolding of relevant information. The learner then makes associations and connections he or she would otherwise fail to make without the stimulation. The argument buttressed by Clements and Sarama (2014) traces the developmental pathways children follow in order to develop mathematical competencies in line with their class level of learning. Thus, an expert educator would not fail to aim at the levels that connect with the child's developmental stages. However, currently, competencies of Grade R educators in the non-affluent, low socio economic sector schools falls far short of such expertise (Feza, 2012, 2013; Siyepu, 2013).

Because of skills and subject matter deficiency, educators sometimes struggle to engage and fully stimulate the learners at this level of learning. They resort to formal approaches used during their school years such as teaching by telling and chorusing mimicry based on rote learning (Banard & Braund, 2016). This is in contrast to recommended approaches of participatory engagement where the learner has an environment that is enabling created then gets to be guided to explore and solve problems by manipulating the world presented (Ginsburg et. al 2008; Sarama & Clements, 2014; Montague-Smith & Price, 2012). This also leaves out such strategies as posing questions that stimulate the learner to engage in practical play with manipulatives, mathematical reasoning and spontaneous deductions. The development of the learner occurs when he or she justifies his or her line of reasoning and this leads to learning of new meanings (Feza, 2016).

There is also a challenge of educators getting learners to play endlessly without any concepts tucked into the play (Banard & Braund, 2016). When play has no scaffolding of mathematical knowledge, the exercises learners engage in do not develop math concepts and thus become a waste of learning opportunity (Banard & Braund, 2016).

THEORETICAL FRAMEWORK

This study adopts the oral-situational approach of language learning which is based on the behaviorist learning theory (Ellis, 2005).The oral-situational approach views language as habit formation based on repeated practice in response to stimuli. The stimuli could be questioning, requests for repeated rhymes, poems or songs (Feza, 2016). The language gets absorbed without need for complex interrogation and explanation of grammatical structures (Ellis, 2005). Focus in the oral-situational approaches is on explaining the mathematical reasoning and clarification by way of examples. The educators' role becomes that of explaining the concept learnt as well as explaining the learners' contributions in the mother language. The theory is attractive because it uses context-based scenarios enriched by learner experiences making them relevant and easy to comprehend.

RESEARCH DESIGN

Seventeen educators were included in the study although only fourteen attended the workshop and responded to the data collection instrument. These educators have no formal standardized teacher training. As described in the literature they received diverse, fragmented training from non-governmental organisations (NGOs). All educators have a level 4 qualification from these NGO's. All educators have at least 3 years and above experience as Grade R teachers. The tool used focusses on practices of educators in terms of mathematics activities taught in isiXhosa. This would capture data on vocabulary and the competencies learners bring to the classroom, as well as numeracy skills achieved as reflected by assessments when Xhosa language is used.

The educators critically reflected on their practices when teaching numeracy in the home language. They wrote brief memos of their impressions to indicate their perceptions in response to questions on the tool. This was complimented by interviews of five purposively selected educators who were asked to give details about how they teach mathematics in isiXhosa; focusing on their own practices. The educators did the exercise in the backdrop of principles taught in professional development workshops on the isiXhosa language usage in numeracy activities, foundational numeracy concepts and numeracy assessment aspects. The two researchers separately annotated the two data sets separately. The two sets of annotations were triangulated. They grouped similar patterns that came out of this triangulation. They also grouped contrasting patterns separately. The two group sets put on Venn diagrams yielded themes that emerged and consolidated into a thematic report. The thematic report highlighted issues on the key objectives such as the proficiency of learners in isiXhosa, the activities used to develop numeracy as well as concepts educators held in numeracy.

FINDINGS

Proficiency in numeracy advanced in English but very low in isiXhosa.

Findings indicate learners arrive with advanced numeracy skills expressed in English. Scholars have long argued that learners arrive in school with mathematical knowledge. What is unique in the study is that the numeracy skills exist tied to English. The numeracy skills are in English and not in the learner's home language isiXhosa. As educators first introduce learners to counting, they find they bring a rich and advanced knowledge of counting from home in English. Most learners in the classes of the educators included in the study could count efficiently without hitches to over a hundred. They began to mix numbers and repeat them after the hundred mark. However even after undergoing the activities and learning in class in isiXhosa, they counted up to ten. When the educators tried to advance the learners in home language, the learners struggled. This was despite the fact that both learners and educators shared the same language background. In response to the question of how proficient learners were in counting by memory recall, the following excerpts indicate that learners were very advanced in recall counting as long as they did it in English. They counted only up to ten in isiXhosa and struggled to go on after ten. This shows that there is a wide gap of competencies when expressed in English compared to the home language. Excerpts from educators A and D notes reveal this.

Educator A

> *Learners many cases come to class able to count from one to a hundred in English. They then start mixing numbers or repeating them thereafter. They however struggle to get to ten in isiXhosa, their home language. It appears there is a lot of effort from parents to teach children at home to count but in English. Learners also appear to enjoy learning to count in English where they display a lot of knowledge and prowess. They however take learning to count in isiXhosa as a task that is difficult as they are on unfamiliar ground.*

Educator D

> *The learners seem to count fluently in English but fail to do so in isiXhosa. There are even some who do not even know any numbers in isiXhosa. They seem to hear them for the first time in class .I expected them to know how to count in isiXhosa than in English.*

IsiXhosa and English activities used by educators to enhance numeracy only partly aligned to the language policy.

The study found that educators use games, songs and action poems to enhance numeracy. These activities are in both English and isiXhosa. In this practice, educators are partly in line with education policy because they include use of isiXhosa poems, songs and games. However, educators by their own admission also use English activities to complement the isiXhosa ones. The educators insisted they do so since the learners already come in with that underlying knowledge. This is not full compliance with the language policy of teaching learners in their home language. Excerpts from educators C and E attest to this:

Educator C

> *There are concepts for which I readily have a game or action poem in isiXhosa. In such situations, for example the song "inkau ezilitshumi esihlahleni" is a Xhosa* song that teaches counting *from one to ten. To repeat the counting I can then switch to the English song "ten little ducks went out one day" That not only creates fun and dance but also makes the concept and skills stick as the learners are repeat them. Therefore, I do use both isiXhosa and English activities. This is not strictly teaching in isiXhosa as I complement the home language with English.*

Educator E

> *I use isiXhosa games such as "ubona amagama amangaki" translated to mean how many letters you see. I use this game where letters are hidden on a card and the player flips quickly and the group identifies accurately how many items are on the card. However at times I use skipping to make them count in English. I use both languages because learners come to school more able to count in English than in isiXhosa and not developing this knowledge further makes them lose the knowledge only for it to be re-taught.*

Low vocabulary levels of Grade R learners in the home language

Educators expressed the view that while learners had advanced levels of mathematics vocabulary on arrival, it was highly developed in the English language compared to the home language. Educators indicated that they had to work hard to build the learners' isiXhosa mathematical vocabulary. Educator B insisted that it took many lessons to develop basic isiXhosa vocabulary for learners in isiXhosa. The following direct quotes from the educator's notes confirm this

Educator B

> *It is very hard to make the learners count in isiXhosa. The isiXhosa mathematics vocabulary is often new to them. Introducing "inye, zimbini, zintathu, zine, zintlanu…up to itshumi" takes many lessons, activities and games. Some of the learners cry or keep quiet when asked to count in isiXhosa. One has to be very patient. There are times the learners lose interest in the activities because they know how to count to advanced numbers in English but we teach in isiXhosa, as it is a requirement. If I insist, they keep counting in isiXhosa; at times, they are frustrated and burst out crying.*

Educators' lack of knowledge in mathematical concepts also contributed to the struggle to teach numeracy.

Educators encounter multiple challenges in trying to teach mathematics at grade R level. Their lack of foundational knowledge curtails their abilities to match the activities that build on the knowledge already there to make it develop further. Educators admitted that learners appeared to know more on arrival than they did at the end of their first class. Educators appeared puzzled and observed that learners appeared to lose some of the knowledge and skills they brought on arrival. The subject knowledge plays a big part in learning and educator weakness in this aspect contributed to the overall challenges they faced. There were concepts like rote or verbal counting that educators were good clear about. Educators eagerly displayed their abilities in this concept. However, object-counting showing the one to one correspondence, subitising and cardinality were concepts they appeared unable to mediate, as they were not clear of this knowledge themselves. The following direct quotes from the educators' F and Gs' notes confirm this:

Educator F

> *I have an idea of teaching counting. I am not too clear about teaching subitising, as I never had exposure to it. I use many activities to teach counting. I can for example teach counting by use of a song such as Kwakunye, kwaba kubili, kwaba kuthathu …and numbers build until the targeted number.*

Educator G:

> *I am not sure what the concept of cardinality is. I teach reverse counting in many activities. I can use a game song such as Inkawu ezisibozo ezisileyo, Zixhuma xhuma ebedini. Enye kuzo yawa, yabetheka ngenhloko; umama ubizu gqhira, Waze u gqhira wathi, inkawu akufuneki, zixhuma xhume ebedini.*
>
> *The song continues with a descending line up of numbers from inkawu ezisixhenxe, ezntandathu, ezitlanu, ezine, ezintathu, ezimbini, enye where the game song ends. The learners jump around pretending to be the monkeys and sing along with actions.*

DISCUSSION AND CONCLUSION

Several scholars confirm that learners bring to class high levels of mathematical knowledge (Ginsburg, 2008; Feza, 2016). The educators who find this knowledge in the English language and not in isiXhosa find it challenging to build on it. They have to teach learners mathematics in isiXhosa in terms of policy at this level. Based on views of the educators they expect to find learners mathematic skills in isiXhosa more than in English but the reverse is true. The advanced numeracy expressed in English from learners whose cultural language is isiXhosa points to factors in the home. This may point to a changing language in the modern South African homes. In addition, the artefacts in the home such as the television may also have an input on shaping the language capitals of learners before the come into school. Educators expose the learners to both English and isiXhosa activities but learners' abilities expressed in isiXhosa always appear to trail behind the same competencies expressed in English.

It is clear from this study that educators are faced with a situation where the choice to parallel the two languages in their teaching is a practical one. While the use of two languages at this level is not compliant with the policy to use the home language for this level of learners, it is not avoidable. Educators have to deal with the situation of developing mathematics in isiXhosa, which is a requirement, but also in English because that is where the learners already are. The educators and learners cannot avoid the use of English because they have to deal with learners already advanced in numeracy in English. They cannot backtrack on that knowledge. They have to build on it. That is why they run activities in both English and isiXhosa. This has implications on the use of the home language policy that calls for more discussion by various stakeholders. It may be that the solution to an apparent policy and practical classroom contradiction that entangles educators may lie elsewhere. Researchers such as Moschkovich (2007b) argue for using two languages together to complement each other. It may on the other hand lie in doubling efforts to start numeracy in the home language in such programmes as family mathematics. Some scholars have argued family mathematics can narrow the gap in the two language based learner competencies because what is taught at home will be coordinated with school driven programmes (Sheldon & Epstein, 2005; Feza, 2016).

Finally, the fact that educators are not fully equipped to teach all the mathematical concepts in a subject like mathematics is a situation, which calls for in-service training as a priority. Scholars like Feza (2013) and Ginsburg (2008) have long advocated for in-servicing and giving educators specialised training. What remains is for stakeholders to heed the call.

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WORKSHOPS

PROVOKING DISCUSSION OF PROBABILITY

Barrie Barnard & Christine Hopkins

AIMSSEC African Institute for Mathematical Sciences School Enrichment Centre

Learners often find probability difficult and confusing. We will start with interesting practical probability activities designed to draw out all the ideas about probability that the learners already have. If their school football team played Kaizer Chiefs who do they think would win? They know Kaizer Chiefs is massively more likely to win but may have muddled ideas about how they find the probability. We will argue amongst ourselves and find really useful ways to explain to learners. Once learners master the formal language they will be able to cope well with probability questions. This is a great opportunity to see communication in action, to see how we can better understand learning by actively listening to our learners and how to adjust our teaching to meet the needs of the learners.

TARGET AUDIENCE: Senior Phase

DURATION OF WORKSHOP: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 35

MOTIVATION

Learners often find probability difficult and confusing. We will start with interesting practical probability activities designed to draw out all the ideas about probability that the learners already have. We will discuss amongst ourselves and find really useful ways to explain to learners. Once learners master the formal language they will be able to cope well with probability questions. This is a great opportunity to see communication in action, and how to adjust our teaching to meet the needs of the learners.

CONTENT OF THE WORKSHOP

Activity 1: What's the chance of these events?	40 minutes
Activity 2: What's the probability?	40 minutes
Activity 3 and Conclusion	**40 minutes**

CONCLUSION

The purpose of the workshop was to encourage talking about probability and to get clarity about words like highly likely, likely and possible, etc. and we hope that this was achieved by the first two activities.

We also demonstrated ideas for teaching with two classroom activities and hope that you as teachers will implement these ideas in future.

SURFACE AREA AND VOLUME

Barrie Barnard & Sinobia Kenny

AIMSSEC African Institute for Mathematical Sciences School Enrichment Centre

When working on problems of surface area and volume learners are often confused about which formula to use or how to work out a^3 or 2HL. We will look at practical activities which help the learners to visualise the link between the letters in the formula and the actual measurements of a shape. ***Visualising*** *is an important mathematical skill. Providing pictures and objects for learners to look at is the obvious way of helping learners to visualise but we are all also capable of making images in our heads. Many learners will be able to describe exactly how a goal was scored at a football match or how a runner won a race. You can capture this ability by asking the learners to shut their eyes and think about mathematical shapes.*

TARGET AUDIENCE: Senior Phase

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 30

MOTIVATION:

Many grade 7, 8 & 9 learners are confused when working with Surface Area and Volume. They often choose the wrong formula or do not know what the letters represent. In this workshop we will look at practical activities which can help the learners to visualise the link between the letters in the formula and the actual measurements of a shape.

CONTENT OF THE WORKSHOP

Activity 1: 60 minutes

Activity 2: 40 minutes

Discussion

CONCLUSION

This hands-on practical workshop will enhance the visualisation skills of teachers in order for them to see the significance of linking the measurements of a shape to the formula.

INTEGRATING TECHNOLOGY TO TEACH FUNCTIONS IN GRADES 10 AND 11

Lorraine Burgess & Elaine Van der Merwe

Mathematics Subject Advisors – WCED

This workshop aims at presenting the participants with different manipulatives that can be used effectively in the classroom to enhance the teaching and learning of these concepts. The focus is on using different manipulatives in a classroom situation to simplify normally abstract concepts. The participants will be empowered to use various technologies, which include the computer (Geogebra), cell phone and paper folding to strengthen and illustrate these concepts. A strong focus on changing classroom practices to re-align itself to today's modern day lifestyle where the use of technology is at the forefront.

TARGET AUDIENCE: FET educators

DURATION: 2 hours

NUMBER OF PARTICIPANTS: 30 educators (Computer lab)

MOTIVATION

Functions have always been one of the challenging sections in the final NCS examinations. The diagnostic report of the NCS examinations suggests that teachers must ensure that learners know basic shape and properties of each type of graph before they can attempt questions of a higher cognitive and difficulty level.

The need exists to adapt the teaching methodology in order to strengthen the conceptual understanding of learners with regards to Functions and their graphs. An improved understanding of concepts in Grades 10 and 11 can only benefit the learner in his final NCS examination.

The diagnostic report (2016) mentions the following as the main problematic areas:

- Conceptual understanding
- Inequalities associated with functions and its notation.
- Transformation of Functions

CONTENT OF THE WORKSHOP

Activity 1: Introduction	15 minutes
Activity 2: The parabola	20 minutes
Activity 3: The effect of variables on the parabola	20 minutes
Activity 4: Domain and range and inequalities	15 minutes
Activity 5: The hyperbola and exponential graph	15 minutes

Activity 5: Application of a typical exam question	20 minutes
Overall discussion	15 minutes

CONCLUSION

The context of the workshop covers all the functions in Grades 10 and 11as mentioned in the CAPS document. Concepts like inequalities, transformations and reflections will also receive special attention throughout this workshop. The ultimate goal is to empower the participants to be able to use and create dynamic tools to explain and demonstrate mathematical concepts.

ACTIVITIES: INTEGRATING TECHNOLOGY TO TEACH FUNCTIONS IN GRADES 10 AND 11

Introduction: [15 minutes]

Curriculum across the phase.

Cell phone – Task to consolidate previous knowledge of Grade 10 learners. (15 min)

TASK 1: [20 minutes]

The parabola:

Investigation with companied Geogebra file.

Focus: General properties and vocabulary used to introduce a Gr 10 learner to the parabola.

TASK 2: [20 minutes]

The effect of variables on the Parabola.

Participants use the sliders to create the given functions and create a summary that can be given to learners.

TASK 3: [15 minutes]

Domain and Range and Inequalities:

Paper folding manipulative to simplify the finding of the Domain and Range

The same concept is used for inequalities.

DISCUSSION:

The participants are encouraged to express their opinions about the manipulative and its benefits.

TASK 4: [15 minutes]

The hyperbola and exponential graph

Task 5: [20 minutes]

Application of a typical exam question

Overall discussion [15 minutes]

DOING MATHEMATICALCONSTRUCTIONS BY USING A PAPER CLIP

Liezel du Toit

Oxford University Press South Africa

This practical, hands-on fun workshop will provide any teacher with a solution to the lack of compasses amongst the learners. Participants in the workshop will learn how to use a paper clip instead of a compass when doing geometric constructions. Participants will draw circles, bisect line segments, bisect angles, construct perpendicular lines and draw special angles without using a protractor.

INTRODUCTION

According to the CAPS document constructions provide a useful context to consolidate knowledge of angles and shapes. Learners in the Senior Phase should use constructions to explore properties of triangles and quadrilaterals.

The problem is that this approach is only practical if ALL the learners have geometric instruments (protractors and compasses) AND if they remember to bring their geometric instruments to the classroom. On the other hand, in a classroom with a large number of learners it can be challenging to manage the situation if there are so many compasses available.

By replacing compasses by paper clips, many of the above-mentioned challenges are addressed. Furthermore, paper clips are easily available and it is fun to use them for constructions.

CONTENT OF THE WORKSHOP

Introduction 5 minutes

- **Constructions of** Concentric circles and quarter circles
- Bisecting line segments
- Bisecting angles
- Perpendicular lines from given points to a line

(5 minutes)

Materials needed:(Oxford will provide for the workshop)

- copy of worksheets
- two paper clips
- two pencils (or a pen and a pencil)
- ruler and eraser

- Concentric circles and quarter circles

- Bisecting line segments
- Bisecting angles
- Perpendicular lines from given points to a line
- Construct special angles without protractors
- Conclusion

CONCLUSION

It is no longer necessary to skip constructions as part of the curriculum because of the lack of compasses in the classroom. Teachers should look forward to teaching constructions once they know how to introduce the paper clip to their classroom. Using this simple tool will lead to lessons where all the learners are actively involved and can experience Mathematics through their fingers?

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AN EXPLORATION OF DIRECT PROPORTION CALCULATION METHODS

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St. Anne's Diocesan College

It could be successfully argued that the single most useful calculation method that anyone should know in daily life is a method for dealing with situations involving direct proportion. During this workshop we will explore a variety of accessible methods for teaching and understanding situations involving direct proportion. Direct proportion occurs in a host of different contexts: rates, proportional increase/decrease, scale, etc. We will use these methods to solve problems in contexts as varied as: scale, conversions, percentage increase, original price, VAT, Rate, Multi-rate calculations and a few more besides.

TARGET AUDIENCE: GET Senior Phase & FET Mathematical Literacy Teachers

DURATION: 1 Hour

MAXIMUM NUMBER OF PARTICIPANTS: 30 – 40

MOTIVATION

It is often surprising to find that students regularly struggle with such a rudimentary, and widely-used, direct proportion calculations. Over the years I have worked with teachers and students from a variety of backgrounds and have collected together a variety of simple and successful calculation tools that can easily deal with these kinds of calculations. This allows the student to "get beyond" the calculation and explore the numbers and outcomes. This also allows the teacher to confidently support the student's learning. All of the methods could, at first, be seen as routine procedures but, once confidence is established, the underlying principles of the methods can be readily explored for the more able student.

CONTENT OF THE WORKSHOP

5 mins: Introduction

10 mins: Exploration and explanation of 3 calculation methods

35 mins: Application of the methods in several situations

10 mins: Discussion

CONCLUSION

Having more than one practical strategy for solving situations involving direct proportion has proven to be 'key' to students' success. This workshop will enable teachers in this regard.

PENTOMINOES

Thomas Haywood

Rhodes University Mathematics Education Project (RUMEP)

Learners might be familiar with the family games played with dominoes. They however might not be aware that a domino is a group of shapes known as polyominoes. Polyominoes are 2-D shapes formed by congruent squares, which are named according to the number of squares. For example, a shape consisting of one square is called a monomino, a shape consisting of three squares is called a trinomino and a shape consisting of five squares is called a pentomino. While rotations and reflections are not considered to be distinct shapes, there are 12 different free pentominoes. When reflections are considered there are 18 one-sided pentominoes. During the workshop participants will explore how pentominoes can actively be used during the teaching and learning of area and patterns. Participants will use a given number of pentominoes to build different rectangles. They will then use the number of pentominoes in relation to the area of the different rectangles to develop a pattern. They will find the general rule of the pattern and discuss how this rule could be used to find areas of rectangles by using different numbers of pentominoes. During the activities participants will need to apply their tessellation and transformational geometry skills.

TARGET AUDIENCE: Senior Phase (SP)

DURATION: 1 hour

MAXIMUM NUMBER OF PARTICIPANTS: 30

MOTIVATION

Patterns, tessellations, transformational geometry and area of shapes form an integral part of the senior phase curriculum. It is envisaged that the teaching of the above mentioned topics using pentominoes will support teachers in the integration of the above mentioned topics.

CONTENT OF THE WORKSHOP

Activity 1 (10 minutes)

Activity 2 (25 minutes)

Activity 3 (15 minutes)

Activity 4 (10 minutes)

CONCLUSION

The name pentomino was coined by Solomon Golomb in the 1900's. Although it ends in the same suffix as dominoes, pentominoes are significantly different in its purpose and goal. Rather than relying on the number of dots as in dominoes, pentominoes describe the number of blocks that are joined. Pentominoes are a unique way of thinking about shapes, because when creating shapes using pentominoes, learners need to think geometrically as well as critically (Goodger, 2016). As the shapes are created, they can be manipulated to form puzzles, rectangles and squares. Logic can be utilized and developed by arranging the shapes in such a manner as to cover the least possible area. Manipulating and moving the shapes also allows for concrete learning about principles of connecting a variety of shapes, which is beneficial for life events such as the arrangement of furniture within a room (Universal Class Inc., 2017).

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BASIC COUNTING PRINCIPLES FROM AN INTUITIVE PERSPECTIVE

Lindy Hearne

Fellowship College (Melkbosstrand)

Garry Knight

College of Southern Nevada (USA), Fellowship College (Melkbosstrand)

TARGET AUDIENCE: Further Education & Training – Mathematics

Teachers wanting more intuitive teaching approaches and greater personal understanding

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 50

Counting Principles are extremely important in many areas of Mathematics, especially in Probability. In addition, the logic used in counting principles is applicable to many types of problem-solving: in Mathematics and in life. Sometimes manipulatives, such as coloured blocks, can assist students in discovering the principles themselves. Blocks and worksheets can present principles which are quite *abstract in fairly concrete ways: notably the "counter-intuitive" definition of 0! and the logical development of combinations. Teaching counting principles gives instructors an opportunity to model a problem-solving strategy; start small and look for patterns.*

MOTIVATION

Often the principles are taught as a series of formulas, rather than teaching the reasoning behind them. It is important that we, as educators, impart to our learners the thinking process and not merely memorizing skills. The "intuitive" approach is probably harder to teach than pure memorization, but once learners understand, they can apply the material more effectively. It also sets them up to think more logically in others area of academia and life.

CONTENT OF THE WORKSHOP

The workshop is designed to give teachers new ideas in exploring this subject with students – a subject which many learners seem to struggle with. The basic areas covered include:

- Factorial notation, including 0!
- Fundamental Counting Principle
- Permutations
- Combinations
- "Specialty counting" – # ways to rearrange letters in word: "statistics"

CONCLUSION

On a logical, concrete foundation you can build the abstract walls of understanding. Using manipulatives, discovery and guided logical discourse, learners will comprehend the concepts and not merely memorize formulas. This is especially true in Mathematical counting principles and this workshop will present a model to teachers which they can use for their own students.

WHY ARE $f(x) = \frac{a}{x} + q$, $g(x) = ax^2 + c$, $h(x) = a^x + b$ and $k(x) = mx + c$ CALLED FUNCTIONS?

Wandile Hlaleleni

Butterworth High School

The presenter seeks to unpack the meaning of the word "function" in the context of graphs as he has observed that some learners are able to solve mathematical problems which involve functions but cannot tell you what a function is. The presenter intends using verbal representations, tabular representations, symbolic representations and graphical representations to guide the participants to conceptualise functions. The presenter seeks to enhance conceptual understanding. According to Kilpatrick,Swafford & Findell (2001) conceptual understanding is the functional grasp of an idea and the learners who have conceptual understanding are able to use multiple representations of a concept and know why a certain concept is learned .The presenter intends defining a function as a relation in which one element of x is associated with one element of y. Or a relation in which many elements of x are associated with one element of y. The presenter will guide the participants to identify functions, tabulated relations or graphical relations.

TARGET AUDIENCE: FET teachers

DURATION: 1 hour

MOTIVATION

This is an important workshop because it seeks to enhance conceptual understanding of functions. It must be remembered that conceptual understanding is one of the strands of mathematics proficiency. The workshop will afford participants an opportunity to verbally represent functions , to represent functions using tables and decide whether tables represent functions or not. This workshop will help some of the participants to understand why teaching of hyperbolas, parabolas, exponential graphs and straight line graphs is the teaching of functions. The participants will go home having understood that hyperbolas , parabolas, straight line graphs and exponential graphs are all functions because they have one to one mappings.

CONTENT OF THE WORKSHOP

Activity 1:

In activity one the presenter will ask the participants to define a function in a mathematical context and the presenter will guide the participants to a verbal representation of the concept.

Activity 2:

In activity two the presenter will give the participants the tables to identify which tables represent functions and which do not represent functions and the presenter will move from group to group trying to help group members to identify functions.

Activity 3:

The presenter will give the participants grid papers and ask them to sketch tabulated data of activity 2.

Activity 4:

The presenter will ask the participants to draw vertical lines on the graphs drawn in activity 3. The presenter will explain that if the line has touched the graph at one point the graph is a function, if the line touches the graph in more than one point the graph is not a function because it is a one to many mapping.

CONCLUSION

Conceptualisation is key to mathematics teaching because it helps learners to visualise abstract mathematical ideas. Conceptualisation through its multiple representations helps learners to understand. Hence the presenter is pleading with mathematics teachers to try it in their mathematics classrooms.

All many to one mappings are functions .When many to one mappings are graphically represented, they form curves for example, $y = ax^2$. All one to one mappings are graphically represented and they form lines which are cut once by the vertical line tests. Functions can be defined as relations where n element is associated with one element y or can be defined as relations where n element is associated with one element y or can be defined as relations where many elements of n are associated with one element of y. As teachers we need to emphasize the meaning of a concept even if the textbooks have not emphasised it.

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PLACE VALUE AND DECIMAL FRACTIONS

Christine Hopkins, & Barrie Barnard

AIMSSEC African Institute for Mathematical Sciences School Enrichment Centre

This workshop is for teachers of grades 6, 7 and 8. Games are used to practice important skills with decimal fractions. Competition is motivating for learners. Whilst they are playing you will hear them discussing and talking about the mathematics. If they can enjoy building up their skill, their learning will be better. If learners keep a record of their scores they can compete against their own previous best work. With any game it is important that at the end of the game the teacher makes sure that all the learners have understood the concept or mastered the skill that the game was designed for. Encouraging communication in the classroom is a great way to judge what your learners understand and then adjust your teaching to help the learners to develop their understanding.

TARGET AUDIENCE: Senior Phase (Grade 6 teachers will also find it useful)

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 30

MOTIVATION

If you are teaching at grades 6, 7 or 8 you need to be sure that all your learners really understand whether 0,8 or 0,67 is bigger. Learners need practice at calculating with decimals but if they are not motivated then practice sessions can be slow and wasteful of time. In this session, you will find out about some simple activities in which learners place random digits. If they choose skillfully then when they have completed their calculation they will be closest to the target and win the game. A classroom full of learners concentrating to finish a calculation as quickly as possible has a great atmosphere.

CONTENT OF THE WORKSHOP

Activity 1:	Wipe Out and variations of the game	45 minutes
Activity 2:	Place the digits	60 minutes
Discussion		

CONCLUSION

The use of the calculator when dealing with decimal fractions is a good resource to use. Learners will come to see how the size and place value of digits change.

CALCULATOR BASICS –DESIGNING WORKSHEETS USING THE EMULATOR

Merrick James

CASIO

In our fast-moving world, very few people read the product manuals when using their electronic gadgets. Some basic functionality is often overlooked or unknown. No assumptions are made as we start with the basics; from switching on the calculator leading up to using the table function and statistical analysis. In conjunction with the CASIO calculator there is the emulator – a useful teaching aid and an invaluable tool when developing worksheets and instructional materials.

TARGET AUDIENCE: Senior Phase & FET

DURATION: 1 hour

MAXIMUM NUMBER OF PARTICIPANTS: 50

MOTIVATION

We will cover the set-up of the CASIO fx82ZA PLUS calculator and the emulator. I will explain several important functions of the calculator i.e. rounding-off, factorising, time calculations etc.

A demonstration of how the emulator can be effectively used as a teaching tool will be demonstrated.

NB All those attending will need to have their own calculator as this is a hands-on workshop.

CONTENT OF THE WORKSHOP

Introduction	5 minutes
Different functions	10 minutes
Demonstration of the emulator	10 minutes
Activities	35 minutes

CONCLUSION

From the first-time user to those familiar with the CASIO calculator there is something for everyone.

INTRODUCING ANGLES

Sinobia Kenny & Christine Hopkins

AIMSSEC (African Institute for Mathematical Sciences School Enrichment Centre)

Understanding of angles underpins the whole geometry curriculum so if you want your learners to get more interested and confident in geometry this is a great place to start. It is much easier to see that one line is longer than another than to judge the size of angles. Simple practical activities will get your learners brilliant at estimating angles. Kinaesthetic learning means learning though moving around. Kinaesthetic activities will be used which will help your learners understand about the angles of polygons. An activity tearing paper will show why the angles of a triangle add up to 180°. You will be introduced to some general teaching approaches which can transform your teaching of all the geometry curriculum.

TARGET AUDIENCE: Senior Phase (but Intermediate Phase will benefit as well)

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 30

MOTIVATION

The activities are hands-on and practical which can be used to introduce or to review angles. The teaching and learning strategy is visual and people maths. The session will help you with new ways to teach the properties of angles on parallel lines, triangles and polygons outside of the classroom.

CONTENT OF THE WORKSHOP

Activity 1: Estimating angles	30 minutes
Activity 2: Identifying and naming angles on parallel lines	20 minutes
Activity 3: Angles in a triangle	30 minutes
Discussion	

CONCLUSION

This workshop will help you in showing learners new techniques of teaching the properties of angles on parallel lines, triangles and polygons.

AREA AND PERIMETER

Sinobia Kenny & Christine Hopkins

AIMSSEC (African Institute for Mathematical Sciences School Enrichment Centre)

ACTIVITIES AND WORKSHEETS: AREA AND PERIMETER

Activity 1: Same perimeter different area, same area different perimeter

Resources Needed: Centimetre square paper. Organisation: Pairs, whole group. Time: 30 minutes

Activity 2: Twice as big

Resources Needed: Centimetre square paper.

Organisation: Pairs, whole group. Time: 20 minutes

Activity 3: Perimeter of circle

Resources Needed: String; compasses. Organisation: Pairs, whole group. Time: 20 minutes

CLASSROOM ACTIVITIES FOR LEARNERS

Activity 1: Twice as big

Resources Needed: Centimetre square paper. Organisation: Pairs / Whole group. Time: 45 minutes

Activity 2: Circumference of circle

Resources Needed: Long piece of string for each pair (around 1.5 m); compasses. Organisation: Pairs, whole group. Time: 45 minutes

MULTIPLES, FACTORS AND PRIMES

Sinobia Kenny & Barrie Barnard

AIMSSEC African Institute for Mathematical Sciences School Enrichment Centre

Do you like to learn by reading, by listening to someone or by doing something? Most people (and children!) have a preference. In this workshop we will explore factors, multiples and prime numbers, by doing different kinds of activities – those that work with sight, hearing and movement – around the same learning outcome we are offering all learners an opportunity to learn in the way which is most natural to them. These teaching approaches which are formally referred to as Visual, Auditory and Kinaesthetic (meaning movement) can be used for all topics in the curriculum. Do you think it would be useful to use these approaches more often in your teaching? Come and try it out using multiples, factors and primes!

TARGET AUDIENCE: Senior Phase (Grade 6 teachers will also find it useful)

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 30

MOTIVATION

The activities will help with the understanding of different representations of multiples, factors and primes by using beans, a 100-grid and string. The teaching and learning strategy is visual and all will experience the activities as visual, auditory and kinesthetic. The activities are useful because it helps to reinforce understanding of multiples, factors and primes and their relationships in different ways.

CONTENT OF THE WORKSHOP

Activity 1: 20 minutes

Activity 2: 20 minutes

Activity 3: 40 minutes

Discussion

CONCLUSION

Teachers will come to realize that all learners need to be catered for in their classes and this workshop will help to reinforce multiples, factors and prime numbers using all the senses.

USING A SCIENTIFIC CALCULATOR FOR FACTORISATION OF TRINOMIALS IN GRADE 10

Rencia Lourens

Hoërskool Birchleigh

INTRODUCTION

Factorisation is a very important part of the Grade 10 curriculum. Factors continue to play an important role right through the curriculum, not only in Grade 10 but also in Grades 11 and 12.

The scientific calculator assists learners when they are doing numerical calculation with fractions, but without the understanding of the LCM and HCF, when learners get confronted with algebraic fractions, they struggle. Factorising trinomials – especially when the leading coefficient is not 1 – remains a struggle for a lot of learners.

The aim of the workshop is to use the scientific calculator as a tool to improve the understanding of trinomials, so that learners can do the work and improve their understanding of the work. A better, not only procedural but also conceptual understanding of factorising will increase the understanding of the algebraic concepts.

CONTENT OF THE WORKSHOP

Activity 1:	Introduction to the Scientific Calculator	15 minutes
Activity 2:	Finding factors	30 minutes
Activity 3:	Table mode	30 minutes
Activity 4:	Finding zeros (roots)	30 minutes
Activity 5:	How can this help when I do Algebra?	15 minutes

CONCLUSION

The scientific calculator as a tool will assist educators to ensure that learners are able to successfully factorise trinomials, which is so important in the FET phase.

CALCULATION OF INCOME TAX IN THE MATHS LITERACY CLASSROOM

Salmina Letsoalo

Ngwana-Mohube High School

One of the topics that learners struggle to read and interpret is an income tax table and the tax rebates for individuals, which contains a lot of information and which they fail to comprehend when questions like these are asked in a test or an examination.

TARGET AUDIENCE: Maths Literacy

DURATION: 1hour

MAXIMUM NUMBER OF PARTICIPANTS: 45

MOTIVATION

The workshop is very important to educators, business owners and all workers who have to contribute income tax to the Receiver of Revenue in the sense that it teaches them to verify the amount they have to contribute as indicated on their IRP 5 forms and detect if they are under or over taxed.

CONTENT OF WORKSHOP

Educators will be grouped and will be reminded of a few of the terms that need to be understood in order to solve such problems. These include salary, income tax, income tax table rebates.

I will go through an example with them which has a scenario, a tax table and detailed steps to be followed in order to arrive at the final answer.

BREAKDOWN OF TIME SLOT

Grouping of participants - 5 Minutes

Introduction - 5 Minutes

Revision of terminologies - 5 Minutes

Explanation of example - 5 Minutes

Participants working time - 30 Minutes

Discussion - 10 Minutes

CONCLUSION

This workshop will assist those educators who are finding it difficult to get their learners to understand how income tax works in the South African context.

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WORKSHOP: PROBLEMS WITH DIGITS

Florian Luca
University of Witwatersrand

*In this workshop, we will solve a few olympiad type problems concerning positive integers whose digits exhibit some interesting pattern***.**

TARGET AUDIENCE: Students and teachers interested in the number theoretical part of Mathematical Competitions.

DURATION: 2 hours.

NUMBER OF PARTICIPANTS: 30.

MOTIVATION

Mathematics Competitions are important since it exposes students to Mathematical problems which are easy to state yet in order to solve them they need to think out of the box as the answer or the solution do not follow some recipe taught in the classroom. In this workshop we will look at a few such problems. In searching for their solutions, we will see that factorization, divisibility tests and inequalities come together in unexpected ways.

CONTENT OF THE WORKSHOP

We will solve the following problems. The first five are easier and the last five are harder. As a time frame, we aim to spend 5-10 minutes with each of the first five problems and 15-20 minutes with each of the last 5 problems.

Problem 1. Find all three digit N such that 11 divides N and N/11 is the sum of the squares of the digits of N.

Problem 2. If 10^n+1 is prime, then show that n is a power of 2.

Problem 3. Show that 10^n+1 is not a perfect power of exponent >1 of some other integer.

Problem 4. Let S(n) be the sum of digits of n. Show that $S(8n) \geq S(n)/8$.

Problem 5. Let A be the sum of digits of 4444^{4444} and B be the sum of digits of A. Find the sum of digits of B.

Problem 6. Find the least n such that n! ends in 2017 zeros.

Problem 7. Show that each positive integer k has a multiple less than k^4 which has at most four distinct digits.

Problem 8. Find the smallest positive integer N consisting of the digits 0,1,…,9 once each which is a multiple of each of 2,3,4,…,9.

Problem 9. Let f(n) be the first digit of n!. Show that there is no n such that f((n+k)!)=k for each k=1,2,3,…,9.

Problem 10. Note that (216,630,666) is a Pythagorean triple: $216^2+630^2=666^2$.

Are there any other such Pythagorean triples (d^k,b,dd…d), where d is in {1,2,…,9} and the hypothenuse has k digits all equal to d?

CONCLUSION

The solutions of the above problems will give the participants the occasion to brush up on some known facts like divisibility tests with 9 and factorization but also review or learn maybe less known facts like the exponent of a prime in a factorial, the pigeon hole principle and the formula for Pythagorean triples.

MAKING MATHEMATICS FUN THROUGH MUSIC AND DRAMA

Banele Lukhele

Director of Luk Arts

As educators our prime objective is to teach our students the content of our subjects to the best of our ability. Ideally, we would like them to leave our spaces feeling more empowered and able to participate in everyday life. This workshop offers a small and simple step towards assisting educators, in the Foundation and Intermediate Phase, to reach their learners and accomplish this objective. Through music and drama educators are able to combine various learning styles, making their space more inclusive; they are also able to make their learning space attractive and engaging for their learners; most importantly, they are able to teach their content in a memorable way. Though it may seem like extra work, this workshop offers experiential evidence of the simplicity and success of this approach. Join us and expand your teaching and learning tool kit.

TARGET AUDIENCE: Foundation and Intermediate Phase

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 30

MOTIVATION

Over the past 23 years of South Africa's democratic education system we have seen great struggles in the classroom from infrastructure to discipline to resources. These struggles have affected the level of learning and this has been reflected in assessments done by organisations such as Statssa, Department of Education and Read Educational Trust, to name a few. The curriculum has attempted to respond to this problem in various ways including being revised and in some cases being changed completely. However, we still see very slow results. I argue that this may be because one of the most stagnant elements in our education space is the adaptation of teaching methods to the growing diversity of learners entering the classroom.

CONTENT OF THE WORKSHOP

This workshop offers an alternative approach to teaching Mathematics using two art forms, Music and Drama. Not only will the participants gain insight on how to make the classroom space more inclusive but they will begin to see the possibilities of collaboration across subjects, particularly with languages. The workshop will follow the following process[1]:

1. The participants will be divided into groups of 3-5 people. **(5min)**
2. Each group will be given an element of either music or drama. For example rhythm, melody, poetry, stage design or storytelling. **(5min)**
3. They will then be asked to study a short informative piece about that particular element and make a list of as many connections to their subject as they can. **(10min)**
4. The group will then present 2-3 topics on their list **(30min)**
5. As a workshop group we will select one topic from each group to continue the process with **(5 min)**
6. The facilitator will then lead a practical unpacking of how to get the idea into a CAPS appropriate lesson plan **(45min)**
7. The workshop will end with a discussion around best possible ways to implement the plan within the various learning spaces **(20min)**

CONCLUSION

Mathematics spaces are generally very intimidating due to the pressures that surround it in the school context. It is important that they become less intimidating to encourage further study in related fields and this method affords that opportunity. When learners are placed in an environment that allows them to play they become more open to giving their time and full attention to the reflection. In most cases it means they, the learners, are more likely to absorb and retain the information they receive in that time. It then becomes clearer to the learner that they can grasp the topic and thus mathematics becomes less intimidating resulting in more engagement in the subject.

THE BEAUTY OF MODELLING FACTORISATION

Robert Tlou Mabotja

Limpopo Department of Education

Capricorn District

TARGET AUDIENCE : Grade 10 educators

DURATION : 2 hours

MAXIMUM NUMBER OF PARTICIPANTS : 30

MOTIVATION

Factorization is still a challenge to secondary school learners at most of them are unable to factorize and solve mathematical problems that requires factorization. In most cases we assess educators' content knowledge on learners' performance. I believe that this workshop will help participants to become more familiar with ways in which factorization can be modeled and contextualized in making mathematics meaningful to them and enable them to teach learners with confidence. By modeling in classroom environment learners find mathematics exciting and relevant to their personal interest when solving mathematical problems.

CONTENT OF THE WORKSHOP

By modeling mathematical content participants will acquire the following attitude and abilities:

- Reason inductively to construct concepts as well as discover relationships.
- Gain knowledge of conversions and facts as well as develop and polish skills.
- Communicate with and about mathematics.
- Reason deductively to devise solutions to real world problems.

DESCRIPTION OF WORKSHOP CONTENT

ACTIVITY	CONTENT	TIME FRAME
1	Introduction and Background	20 minutes
2	Factorize binomials using paper folding modeling $a^2 - b^2 = (a-b)(a+b)$ $(a+b)^2 = (a+b)(a+b)$ $(a-b)^2 = (a-b)(a-b)$	40 minutes
3	Factorize trinomials using paper folding modeling	30 minutes

	$a^2 + 2a + 1 = (a + 1)(a + 1)$ $2a^2 + 3a + 1 = (2a + 1)(a + 1)$ $a^2 + 5a + 6 = (a + 2)(a + 3)$	
4	Questions and Clarifications	20 minutes

CONCLUSION

Participants should be encouraged to use deductive and inductive reasoning to validate a mathematical assertion. This more formal practice, traditionally addressed in geometry should be expanded to other mathematical topics. Generalizing from patterns observed (inductive reasoning) can be tested using logical verification (deductive reasoning). The study provides an opportunity to make conclusions so that learners can:

- Draw logical conclusions about mathematics.
- Use models, known facts, properties and relationships to explain their thinking.
- Link conceptual and procedural knowledge.
- Relate various representations of concepts or procedures to one another.
- Use mathematics on their daily lives.
- Believe that mathematics makes sense.

TEACHING REASONING IN GEOMETRY THROUGH THE GET PHASE TO RESTORE TEACHERS' DIGNITY IN MATHEMATICS LEARNING

Nomathamsanqa Mahlobo[1], Patisizwe Mahlabela[2] and Themba Ndaba[3]

[1,2&3]Centre for Advancement of Science and Mathematics Education (KZN)

"The ability to reason systematically and carefully develops when students are encouraged to make conjectures, are given time to search for evidence to prove or disprove them, and are expected to explain and justify their ideas" (NCTM, 2000, p. 122). The workshop will engage participants in activities that seek to enable them to facilitate the development, in the learners that they teach, "the ability to reason systematically". Participants will, through paper folding and paper cutting, explore properties of triangles and other polygons. They will be given time to "search for evidence" required to classify polygons. They will be expected to explain their observations and make conjectures.

TARGET AUDIENCE:	GET Phase
DURATION:	2 hours
MAXIMUM NUMBER OF PARTICIPANTS:	40

MOTIVATION

Introduction

The South African Curriculum and Assessment Policy Statement (CAPS) (2011) define mathematics as:

> a language that makes use of symbols and notations to describe numerical, geometrical and graphical relationships. It is a human activity that involves observing, representing and investigating patterns and quantitative relationships in physical and social phenomena and between mathematical objects themselves. It helps to develop mental processes that enhance logical and critical thinking, accuracy and problem-solving that will contribute in decision-making (p. 8).

The definition implies that in order to understand and communicate better in mathematics, one requires to have a better command of the mathematical language. The definition further refers to mathematics as "a human activity that involves observing" (DoE, 2011). This implies that learners should be engaged in activities in order to learn mathematics better, especially geometry.

In our experience, the teaching of geometry is the most challenging section of the curriculum. This is generally evident from student responses to geometry questions.

Most students are able to observe and identify the components of the geometric figure that are equal, but they are unable to provide the reasons for their conclusions. Learners seem to experience difficulties with communication using mathematical language. The intention of the workshop is to suggest interventions that educators may use to address the observed problems.

Why focus on geometry

Sahin (2008) asserts that geometry teaching provide students with the ability of thinking critically, and solving problems. This in turn enables them to have a better understanding of the other subjects by making students maintain a high level of thinking skills. This is supported by Hizarci (2004) when he maintains that geometry helps an individual gain vision, ease thinking and reach a solution by realizing the shape before their eyes. In general, geometry is a significant tool for a student to give meaning to his/her surroundings (NCTM, 2000).

Geometry enhances reasoning. Reasoning is the use of logical thinking to make sense of a situation or idea. Reasoning is essential for understanding mathematics. Reasoning is a way to use mathematical knowledge to generate and solidify mathematical ideas that are new to us (NCTM, 2000). Students should be assisted in developing reasoning skills in much earlier grades by giving them many opportunities to explain their thinking in a variety of ways – through drawing pictures, explaining with words, and listening to their classmates' explanations. Through reasoning, students develop their own sense of the mathematical coherence of the things they are learning (NCTM, 2000). The workshop intends to ensure that the above qualities are attained through the teaching of geometry.

CONTENT OF THE WORKSHOP

Activity	Focus/content	Duration
1	• Explore, through paper folding and cutting, properties of triangles across General Education and Training (GET) phases. • Use the properties to classify triangles into different types.	**35 min**
2	• Explore, through partitioning into triangles, the sum of the interior angles of other polygons. • Explore the relationship between the number of sides in a polygon and the number of triangles. • Use relationships identified to determine the sum of the interior angles of a polygon.	**35 min**
3	• Explore, through paper cutting and folding, the number of lines of symmetry in regular polygons.	**35 min**

	• Establish a relationship between the number of sides in a polygon and the number of lines of symmetry.	
4	• Conclusion of the workshop will be reflection, feedback and questions from the participants.	**15 min**

CONCLUSION

The focus of the workshop will be on exposing the participants into activities that allow them opportunities to communicate their observations and to enhance their logical and critical thinking. The participants will be exposed to the activities from Foundation Phase to Senior Phase that enhances the understanding of what is prescribed in the CAPS. They will lastly be expected to give feedback on how each activity approach will impact on their teaching.

The workshop will equip participants with some strategies of alleviating fear of geometry in our students. Sharing strategies across the band will expose the lower grade educators into developing reasoning skills among children early and also assist the upper grade educators with practical approaches to help learners reason with understanding when doing their geometry. It will also encourage team teaching, consultation and networking of South African teachers across the band.

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INTEGRATING DYNAMIC GEOMETRY SOFTWARE IN MATHEMATICS CLASSROOMS

Beena Deepak Mavani & Deepak Pravin Mavani

Khanyisa High School Mthatha / PG Students @Rhodes University

This workshop is conceptualised to develop the appropriate ICT skills to use GeoGebra effectively and strategically as both a teaching and a learning tool of mathematics. In this workshop the basic tools available in GeoGebra to construct geometric shapes like polygons will be introduced. The workshop will also expose the participants to explore and develop understanding of horizontal and vertical shift of a quadratic polynomial. The worksheets are made simple so that the beginners of GeoGebra can learn easily. The significance of this workshop is to empower and equip teachers with the dynamic mathematics software GeoGebra and enable them to incorporate its powerful capacity in their teaching of mathematics.

TARGET AUDIENCE: FET Mathematics Teachers

DURATION: 1 hour

MAXIMUM NUMBER OF PARTICIPANTS: 30

MOTIVATION

The ever-growing use of technology in all realms of life has also affected teaching styles in many parts of the globe. Dynamic Geometric Software (DGS) is a technological tool which provides an opportunity for discovery and exploration, thus opening new possibilities for visual experiences in mathematics teaching and learning.

The National Strategy for implementing Mathematics, Science and Technology (MST) acknowledged the significance of ICT in education as a teaching and learning resource, but the ministerial committee report of 2013 found that the technology equipment such as computers and tablets provided are under-utilised because many teachers are not sufficiently equipped for effective use of this technology in the classroom (Dept of Basic Education, 2013). There are many educational software resources available on the market, but many of them are expensive and require annual license fees. We actively support open source software community like GeoGebra that offers an 'excellent classroom-teaching tool' (Dept of Basic Education, 2013) for Mathematics.

GeoGebra, an open source dynamic geometry software package, is a powerful visualisation tool for the exploration of mathematical concepts, properties and ideas. GeoGebra is a multiplatform mathematics software package that gives "everyone the chance to experience the extraordinary insights that maths makes possible"

("GeoGebra," n.d.). With GeoGebra, abstract ideas may be reified not only by making connections between the real and abstract world, but also between different mathematical concepts. The software runs virtually on any operating system as it requires only a Java plug-in and, unlike commercial products, students and teachers are not constrained by licenses.

Tasks in GeoGebra are useful when learners are encouraged to uncover the hidden relationships (Holzl & Schafer, 2013). The interactive nature of dynamic applets has the potential to promote student's understanding, which would have been difficult in a static environment. With the help of GeoGebra, teachers can ask open-ended questions like what-if and what-if-not, thereby supporting and guiding students to discover properties themselves.

CONTENT OF THE WORKSHOP

An introduction of dynamic mathematics software GeoGebra to the participants can take 5 minutes. There are two activities envisaged in this program.

Activity 1 – Rectangle 20 minutes

Many students have difficulty in understanding geometric figures by themselves because of the perceptual recognition of shapes and figures is so predominant and spontaneous that it impedes mathematical ways of visualising the figures (Duval, 2013). Many students, for example, fail to recognise a square when it is tilted. Geometrical properties correspond to what remains invariant when the drawing is changed, by moving either one of its points or its segments.

In the activity, participants will be taken to various tools in GeoGebra and construct a dynamic rectangle. The worksheet for this activity has been adapted from those of Hohenwarter's (2009), and his consent has been sought to use for our training purposes.

Activity 2 –Parabola Function of the form $f(x) = a(x + p)^2 + q$ 25 minutes

Here GeoGebra offers an opportunity to visualise functions and its transformation. Though all forms of transformation can be visualised using this software, we are confining our activities to horizontal and vertical shift. As teachers, more often than not, we explain without convincing the students that the horizontal shift is in the opposite direction of the value of 'p', thereby encouraging them to memorise the rules of transformation. But with the help of dynamic worksheets we can easily convince our learners.

Activity 3: Conclusion 10 minutes

CONCLUSION

After completion of the activities, the participants reflect on their experiences and engagement with the software. We may lead them to a discussion on the pedagogical strategies to be adopted when technological tools are introduced as a teaching and learning material. The first activity of a dynamic rectangle can help us to realise that a rectangle is not only an image but is a shape controlled by its properties.

The second activity allows us to conclude that for a given function or graph, f(x+p) represents a horizontal shift of the original graph of f(x), and move in the opposite direction of ‘p’, i.e. when ‘p’ moves to the right of the number line, the graph shifts to the left and similarly when ‘p’ moves to the left of number line, the graph shifts to the right of the original graph of f(x). Also, when q is positive f(x) + q represents a vertical shift upward for the original graph of f(x), and when q is negative, f(x) + q represents a vertical shift downward for the original graph of f(x).

An hour or two of engagement with the tools may not be sufficient to fully understand the capabilities of a DGS, we strongly encourage participants and readers to construct the following in GeoGebra:

1) Construction of a Square (without using regular polygon tool);

2) Construction of an equilateral triangle (without using regular polygon tool);

3) Explore and discover the changes of parameters of a quadratic polynomial

$f(x) = ax^2 + bx + c$ (Hint: use ‘Trace’)

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FRACTIONS IN THE INTERMEDIATE PHASE: PRACTICAL IDEAS AND FUN ACTIVITIES

Ingrid Mostert

Axium Education and University of Johannesburg

Learners often struggle to make sense of fractions. Because fractions play such a foundational role in mathematics in higher grades, if learners do not develop a good understanding of fractions in the Intermediate Phase they are at a disadvantage in later phases. This workshop will give you practical ideas for introducing fractions, doing calculations with fractions and linking fractions to division and measurement.

TARGET AUDIENCE: Intermediate Phase

DURATION: 1 hour

MAXIMUM NO. OF PARTICIPANTS: 50

MOTIVATION

Learners often struggle to make sense of fractions. By using a range of approaches and appropriate contexts, learners can be lead to have a robust understanding of fractions. This is important as learners are required to be proficient in doing calculations with fractions in higher grades.

Activity 1: Sharing and Grouping [15 min]

The workshop will begin by exploring the different ways in which division can be seen (both as sharing and as grouping). Participants will work through a number of problems which highlight the difference and which demonstrate the difference between contexts that result in remainders and contexts that result in fractions.

Activity 2: Fractions and division [10 min]

The second set of activities will demonstrate the link between fractions and division by allowing the participants to solve further problems related to specific contexts.

Activity 3: Naming fractions [20 min]

The third set of activities will discuss the importance of learning to name fractions (for example the difference between 'the whole is divided into fifth**s**' and 'if a whole is divided into 5 equal parts, each part is a fifth') and why the short notation for fractions is only introduced after learners are comfortable with using the words (i.e. one fifth instead of $\frac{1}{5}$). Participants will also be shown how to make a movable fraction wall using strips of carton paper (poster paper).

Activity 4: Fractions greater than one [15 min]

Finally, by joining a number of fraction walls, participants will be introduced to the idea that a fraction can be a number bigger than one. This concept will be extended through the use of chanting and the use of measurement (2,5 liters of milk is the same as 5 half litres of milk).

CONCLUSION

This workshop will provide teachers with concrete ideas that they can use in their classes to teach fractions – a topic that many learners struggle with. By experiencing the activities themselves and by having time to discuss with other teachers what adaptations they might make for their own classes, teachers will be equip to implement the activities when they return to their classes.

GAMES FOR FOUNDATION PHASE MATHEMATICS: HOW TO KEEP A BIG CLASS ENGAGED

Ingrid Mostert

Axium Education and University of Johannesburg

Do you have more than 70 learners in your foundation phase class? Do you feel like there are always only a few learners who answer your questions? Are you looking for new ideas to engage your whole class? This workshop will give you ideas for games, songs and stories to keep your whole class interested and doing mathematics.

TARGET AUDIENCE: Foundation Phase

DURATION: 1 hour

MAXIMUM NUMBER OF PARTICIPANTS: 131

MOTIVATION

Participants will find this workshop useful because it will provide them with ideas for engaging large foundation phase classes. The strategies and games that will be shared have been used successfully in large classes in the rural Eastern Cape (the largest being a grade 2 class of 131).

CONTENT OF THE WORKSHOP

The workshop will model a large foundation phase class with the participants acting as learners and the facilitator acting as the teacher. Therefore the maximum number of participants is 131 and participants will be sitting in rows without the option of moving around or working in groups. The venue will be decorated with simple resources made from carton paper (poster paper) typically available in schools.

Games, songs and stories (in English and isiXhosa) will be demonstrated. Where possible, participants will have the opportunity to lead the games or songs with the large group.

Introduction: Why 131? [5 min]

Activity 1: How many fingers? [5 min]

Activity 2: Fizz pop [10 min]

Activity 3: Number chart [10 min]

Activity 4: Songs [10 min]

Activity 5: Stories [15 min]

Conclusion: What can you do in your class? [5 min]

ACTIVITIES AND WORKSHEETS TO BE USED

Games include fizz pop (the teacher describes a rule, e.g. 10 more, when the teacher says a number, the class responds with the number 10 more than the teacher's number) and

Songs include both English and isiXhosa songs to help learners practice the ordinality of numbers (e.g. 5 little speckled frogs).

Stories include simple 'maths stories' as well as stories from Xhosa folklore that involve numbers.

CONCLUSION

This session aims to address the need for interactive approaches to teaching mathematics that cater for very large classes. In the session teachers will experience the use of the activities with a large group and will be in a position to judge their effectiveness and make adaptations for their own classes.

USING AN APPROPRIATE LANGUAGE IN THE TEACHING AND LEARNING OF EUCLIDEAN GEOMETRY.

Themba Ndaba, Thami Mahlobo & Patisizwe Mahlabela

Centre for the Advancement of Science and Mathematics Education (CASME)

This workshop serves to make teachers aware of the importance of communication and proper usage of language in the teaching and learning of Euclidean Geometry. Participants will be given an opportunity to derive and come up with some definitions of concepts which will be identified by the presenters. These will then be debated and discussed, for all participants to come to a consensus.

There will be some build-up activities on the nature of theorems and participants will be provided with more theorems to work on, which will be followed by extensive discussions.

The participants will be further given riders to do where they will show the importance of using language in geometry. This should be coupled with evidence of some reasoning applied whenever they engage in geometry problem solving.

TARGET AUDIENCE: Further Education and Training (FET) Teachers

MAXIMUM NUMBER OF PARTICIPANTS: 30

DURATION: 2 hours.

MOTIVATION

Geometry is one of the major sections in mathematics where learners do not perform well. This problem is not only a South African dilemma, but worldwide. Research and South African matric results have proven that this failure is due to the lack of mathematical language proficiency among learners. According to the National Senior Certificate (2015) diagnostic report, the achievement of mathematics questions from question 8 to 11 which were based on Euclidean Geometry was thirty eight percent (38%).

Performance Trends (2012-2015) in Mathematics in general were as follows:

Year	No. wrote	No. achieved at 30% and above	% achieved at 30% and above	No. achieved at 40% and above	% achieved at 40% and above
2012	225 874	121 970	54.0	80 716	35.7
2013	241 509	142 666	59.1	97 790	40.5
2014	225 458	120 523	53.5	79 050	35.1
2015	263 903	129 481	49.1	84 297	31.9

The report shows that the number of candidates in 2015 increased by 38445 in comparison to that of 2014. The general performance of candidates declined in 2015 as indicated by 49.1% of candidates achieving 30% and above, with 31.9% achieving 40% and above.

According to the report, there are suggestions made, one of which is the one below:

> In the classroom, teachers and learners should use terminology that is expected of them in the examinations" The report further cites the example that "Instead of saying a line cuts another in half, they should say that the one line bisects the other. (NSC, 2015)

The Importance of Teaching and Learning Mathematical Vocabulary

The National Council of Teachers of Mathematics (NCTM, 2000) states that,

> Mathematics vocabulary refers to written words that express mathematical concepts or procedures and mathematics vocabulary is necessary for demonstration of mathematics proficiency.

According to Riccomini, Smith, Hughes and Fries (2015),

> Vocabulary understanding is a major contributor to overall comprehension in many content areas including mathematics. Effective methods for teaching vocabulary in all content areas are diverse and long standing. The importance of teaching and learning the language of mathematics is vital for the development of mathematical proficiency.

Halliday (1978) as cited by Ndaba (1997) asserts that knowing a language is a mastery of three interlocking systems, namely, the forms, the meaning and the functions. Ndaba (1997) says the first two are self-explanatory and the last category involves correct use of language to do specific things, for instance, solving geometry problems. Pimm (1987), on the other hand gives three aspects by which he proposes that mathematics is a language, namely meaning, symbols and things symbolised. Barnard (1990) agrees with Pimm(1987) that mathematics is a language in its own right.

> In Wiskunde is daar komplekse verbandhoudende reëls. Die bemeestering van 'n wiskunde woordeskat is indirek 'n bemeestering van hierdie reěls. en die gevolglike bemeestering van begrippe om Wiskunde te verstaan. Hierdie wiskundige woordeskat maak deel uit van die onderrigtaal van Wiskunde, dit is die wyse, waarop oordrag van kennis plaasvind. (Barnard, 1990:87)

Barnard (1990) states that in mathematics there is a complex connectedness of rules. He further states that the mastery of mathematics vocabulary is indirectly the mastery of these rules which is followed by mathematics comprehension and understanding. (As translated by authors). Barnard (1990) further asserts that a student who does not understand the mathematical language does not easily cope with mathematical instruction. Different authors comment on the relationship between language usage and the impact on the learning of the student.

CONTENT OF THE WORKSHOP

No.	**Activity**	**Duration**
1.	Participants will be given 10 important basic geometry concepts to define in groups of four or five.	15 mins
2.	Group presentations and consolidation will be embarked on	15 mins
3.	Participants will be given ten statements of theorems and their converses to state and explain.	20 mins
4.	Participants will be given five theorems to prove using appropriate language	25 mins
5.	Four different riders will be performed in groups. Each group will be given a specific exercise. Reporting and discussions will be embarked on.	35 mins
6.	Reflection, feedback and comments from the participants.	15 mins

CONCLUSION

In the light of what has been highlighted above, the authors decided to have this workshop. It is designed in order to draw the attention of teachers to the fact that the language of teaching and learning in geometry should be appropriate. Teachers should also be aware that the words and symbols of geometry are used to describe specific spatial ideas and relationships accurately and succinctly. It should be realised that for many students, geometry concepts are either new to them or are familiar words used in unfamiliar ways.

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THE IMPLEMENTATION OF RESTORATIVE PRACTICES TO BUILD AND RESTORE THE DIGNITY OF MATHEMATICS LEARNERS

Mariëtte Reyneke & Roelf Reyneke

University of the Free State

In addition to the challenges posed by the content of the curriculum other issues such as ill-discipline and bulling contributes towards the anxiety many learners experience at school. Anxiety often impacts negatively on children's ability to cope with the curriculum. Experiencing a sense of belonging and safety reduces anxiety in children. Restorative practices provide a model to build and repair relationships between teachers and learners and among learners themselves. This will inevitably influence the classroom climate positively and ultimately contribute to address mathematics anxiety and general discipline problems in the classroom. The way in which teachers deal with the shortcomings of learners impacts the classroom climate. In this workshop participants will be exposed to restorative practices as an alternative model to deal with learners and to instill a positive classroom climate. The impact of fear on the brain will be discussed, practical examples of how to move children from their emotional brain to their cognitive brain will be discussed, the restorative approach to classroom management will be explained with reference to the social discipline window and participants will be taught a few restorative practice techniques to instill a positive classroom climate conducive to teaching.

TARGET AUDIENCE: Appropriate for any phase

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 50

MOTIVATION

To build and restore the dignity of mathematics learners requires more than just finding creative ways to teach the mathematics curriculum. In general mathematics and science teachers are very analytical and task driven people who often find it difficult to build personal relationships with learners. Many children experience anxiety in the mathematics class and cannot perform optimally. Some learners also misbehave more in the mathematics classes in an effort to try to distract the teacher not to continue with the work because they cannot cope with the work. The aim of this workshop is to provide teachers with an alternative approach to discipline, the creation of a positive classroom climate and to assist teachers in the process of building personal relationships with mathematics learners. Recent research indicates that classroom climate and the way teachers discipline learners have an impact on mathematics anxiety and the overall performance of mathematics learners.

CONTENT OF THE WORKSHOP

Activity 1: 30 minutes:

The functioning of the brain and fear will be discussed – show a video

The need to rethink the mathematics classroom climate and disciplinary measures will be discussed.

Provide practical activities that can assist teachers to help learners to move out of their emotional brain (anxiety/fear) and back into their cognitive brain. (Being able to learn)

Activity 2: 30 minutes

Explain the social discipline window and its link to classroom climate and mathematics anxiety.

Participants do a case study in groups to understand the social discipline window better.

We bring the case studies.

Activity 3: 15 minutes

Explain the difference between a punitive and restorative approach to discipline and the impact on classroom discipline and the restoration of the dignity of learners and teachers.

Group discussion on the potential of this approach in a mathematics classes.

Activity 4: 40 minutes

Practical work on how to implement restorative practices in a class and how to build relationships with learners. These include the application of:

- Affective statements
- Restorative reminders
- A restorative chat

CONCLUSION 5 minutes

Revisit the principles of the restorative approach to discipline

Answer questions of participants.

The activities and work sheets to be used in the workshop:

Participant will get an opportunity to practice affective statements and will be taught to do a restorative chat. They will do this in groups. Participants will also be introduced to the use of restorative reminders.

THE APPLICATION OF THE CIRCLE OF COURAGE TO UNDERSTAND
THE NEEDS OF MATHEMATICS LEARNERS

Roelf Reyneke & Mariëtte Reyneke

University of the Free State

Teachers could contribute to the improvement of resilience in learners. When resilience is improved, learners develop the ability to manage difficulties in their lives. In the school environment this helps them to manage personal and academic problems more effectively, leading not only to the improvement of interpersonal relationships but also improving their self-concept. When a learner sees him or herself and their abilities in a positive light, it can have an enormous impact on their academic achievements. In this workshop participants will be introduced to the Circle of Courage model. This model provides some guidelines on how to reclaim at risk learners. The four universal needs of belonging, mastery, independence and generosity will be explained. A happy child needs to experience that these needs are fulfilled in the classroom environment. If any of these needs are not attended to, disciplinary problems may be experienced. During this interactive workshop participants will also receive some practical guidelines on how to improve learner's sense of belonging, mastery, independence and generosity. This will help to create a supportive environment where discipline problems could be prevented and academic learning supported.

TARGET AUDIENCE: Appropriate for any phase

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 50

MOTIVATION

Knowledge of the Circle of Courage provides teachers with information of the fundamental needs of learners that provide the foundation for the development of resilience and self-worth. Happy learners excel in their work but only if the environment contributes to their psychological well-being. This workshop will provide some ideas on how this could be achieved in the mathematics class.

CONTENT OF THE WORKSHOP

After completing the workshop participants will be able to:

- Understand troubled behavior through the lens of the Circle of Courage.
- Explain the components of the Circle of Courage model namely belonging, mastery, independence and generosity.

- Use some practical ideas of how these fundamental needs in learners could be strengthened and developed, thus creating an environment that will reclaim learners.
- Improve relationships in the classroom with the Circle of Courage as background that relates to:
  - ✓ Disengaging from the conflict cycle
  - ✓ Connection before correction
  - ✓ Respect begets respect

CONCLUSION

Every teacher should want their learners to excel. Not just in their subject but also in life. Although this workshop will primarily focus on the latter, it has the positive outcome that it can also improve their focus and motivation in the classroom. This workshop will guide participants on how to do this.

The activities and work sheets to be used in the workshop

Videos and group discussions will be used to illustrate important concepts.

Group activities will be done using different techniques and props.

TEACHING MATHEMATICS DURING PHYSICAL EDUCATION LESSONS

Vanessa Ruiters, Occupational Therapist

Educanda CC.

This workshop builds capacity for educators to teach across subjects.

Too often educators are only teaching the subject at hand and they forget the valuable opportunities that arise during a Physical Education Lesson to include the mathematical concepts underlying the understanding of mathematics.

We share the basic mathematical concepts of size, length, weight and time and how they normally develop in the young learner. We guide the educators to understand that learning that takes place while the learner is actively involved is a very powerful teaching tool. We guide them to grasp how kinaesthetic learning of the prepositions will lead to better understanding of the language of mathematics.

This is a practical workshop that allows the attendees to experience the benefits of movement during a lesson. While engaged in the tasks of the workshop, they come to realise how important action is during learning and how implementation in the classroom makes teaching maths practical and fun.

TARGET AUDIENCE: Foundation Phase; Grades R to 3

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 50

MOTIVATION

Educators often forget the valuable opportunities that arise during a Physical Education Lesson where mathematical concepts underlying the understanding of mathematics concepts can be included.

This workshop builds capacity among educators to use the Physical Education Lesson to add value to the lesson by including mathematics.

CONTENT OF THE WORKSHOP

For the first thirty minutes, we share the basic mathematical concepts of size, length, weight and time and how they normally develop in the young learner. We guide the educators to understand that learning that takes place while the learner is actively involved is a very powerful teaching tool. We guide them to grasp how kinaesthetic learning of the prepositions will lead to better understanding of the language of mathematics.

The next thirty minutes teachers are invited to do the obstacle course set up for them. They experience first-hand the kinaesthetic learning that takes place. Teachers will be divided into groups to the "circuit", similar to how they would structure their learners.

The teachers can now return to their seats and discuss what they experienced. Comments from the floor usually include practical questions on how to structure an obstacle course and how to use the material. The obstacle course is constructed from "The Complete Motor Skills Set" and other grass motor equipment from the Educanda range. We allow twenty minutes for this.

There will be time in the next thirty minutes for teachers to plan and explain to the group the obstacle course they would construct to achieve the learning goals of a Physical Education lesson while at the same time establishing kinaesthetic learning of mathematical concepts. Peers will comment on these.

The workshop will be concluded with fifteen minutes to spare for question time.

The Power Point Presentation will be e-mailed to attendees who hand in their e-mail addresses. They will each receive a certificate of attendance stating they attended the two-hour workshop.

MAKING REGRESSION ANALYSIS EASY USING A CASIO SCIENTIFIC CALCULATOR

Astrid Scheiber
CASIO

Adequate knowledge of calculator skills makes the teaching of Statistics to Grade 12 learners easier and enables the educator to assist their learners more efficiently. This workshop will guide you through Linear Regression Analysis, including finding relationships between variables, the line of best fit and making projections, using the CASIO Scientific calculator.
Equipment required: ***CASIO FX-82ZA PLUS*** *Scientific Calculator.*

TARGET AUDIENCE: Further Education & Training – Mathematics

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 30

MOTIVATION

As of 2014, the Grade 12 Statistics syllabus involves the learners' making use of available technology to: **[12.10.1 (b)] calculate the linear regression line which best fits a given set of bivariate numerical data** and **[12.10.1 (c)] calculate the correlation co-efficient of a set of bivariate numerical data**. As stated by the current Maths CAPS document. This workshop serves to increase educators' understanding of the CASIO Scientific calculator. In turn, it will foster self-confidence and a positive attitude towards Statistics, enhancing both the educators' and learners' understanding of the topic.

CONTENT OF THE WORKSHOP

This workshop will cover: Identifying the relationship between bivariate numerical data, inputting bivariate data into the CASIO scientific calculator, calculating the correlation co-efficient, finding the equation of the regression line, calculating projected values - Interpolation & Extrapolation, using TABLE MODE to find the co-ordinates of the line of best fit & selecting random samples.

Introduction	5 mins
Identifying the relationship between bivariate numerical data	10 ins
Inputting bivariate data into the CASIO scientific calculator	10 mins
Calculating the correlation co-efficient	10 mins
Finding the equation of the regression line	15 mins
Calculating projected values - Interpolation & Extrapolation	15 mins

Using TABLE MODE to find the co-ordinates of the line of best fit	15 mins
Selecting random samples	10 mins
Discussion & worked example	30 mins

CONCLUSION

It is imperative that educators have a thorough understanding of the CASIO scientific calculator in order to build their confidence when working with learners in their classrooms.

UNIT FRACTION-BY-UNIT FRACTION

Connie Skelton

Iconic Maths

This workshop gives an overview of how fractions can be taught based on the principles and concepts in CAPS for the Foundation Phase and Intermediate Phase. The workshop looks at some of the issues that can arise if exercises and examples are not specific about defining the whole at the outset. It is also very important that learners are aware that when using 2D shapes to represent fractions, it is the area of the shape and not the form of the shape that is considered. The workshop then traces the progression towards using unit fractions on a number line which maintains a constancy of concept.

TARGET AUDIENCE: Foundation Phase and Intermediate Phase
DURATION: 2 hours
MAXIMUM NUMBER OF PARTICIPANTS: 50

MOTIVATION

Learning fractions can be divided into two stages. In the initial stage, learners are introduced to fractions visually and intuitively. They learn simple calculations using word problems. They use drawing and various manipulatives like fraction strips, rectangles, number lines and so on.

It is important that these young learners form good habits with respect to fractions. This includes making sure that the whole is clearly defined. It is also important that when using rectangles or circles or other 2D shapes to represent fractions that learners clearly understand that it is the area of the shape and not the form of the shape that is important.

In the second stage, the more formal mathematical development of fractions begins. The emphasis moves towards fractions being represented by lengths on a number line. So, just like 1 is the building block of whole numbers, so the unit fraction is the building block of fractions. Just as counting whole numbers is adding or subtracting a multiple of the unit, so adding and subtracting fractions can be taught with a constancy of concepts.

Teachers will have the opportunity to try out various exercises highlighting the different stages as well as considering the implications of each step.

CONTENT OF THE WORKSHOP

What area is shaded? 10 minutes

Activity 1	What fraction of the area is shaded?	10 minutes
The basic building blocks		5 minutes
Activity 2	Building blocks	5 minutes
Two stages of learning fractions		5 minutes
Activity 3	Fractions represented visually	5 minutes
Define the whole		5 minutes
Activity 4	What is the whole? What is the fraction?	10 minutes
Geometric shapes that define the whole		5 minutes
Activity 5	Discussion on pros & cons of various shapes	10 minutes
Clear cut model or definition of a fraction		5 minutes
Shape versus size		5 minutes
More than the unit fraction		5 minutes
Activity 6 Counting in fractions		
Fractions on the number line		10 minutes
Activity 7	Fractions on the number line	5 minutes
Conclusion		5 minutes

* Note that the time estimate includes the discussion of strategies.

The activities and worksheets to be used in the workshop (maximum 8 pages)

CONCLUSION

In conclusion, the unit fraction is a certain part of the number 1. The parts are defined by the denominator. Once learners have mastered unit fractions for several different denominators, it becomes a logical step to start counting more of these parts. Counting whole numbers on a number line is a visual way for learners to begin the operations of addition, subtraction, multiplication and division. Similarly, the number line can be used for learners to visually begin the basic operations with fractions.

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MAKING AN ICONIC DIFFERENCE IN ALGEBRA

Connie Skelton

Iconic Maths

Mental Mathematics deals with the properties and manipulation of particular numbers; whereas Algebra uses learners' understanding of numbers and progresses to generalised numbers, variables and functions. The transition from number to algebra can lead towards tension for learners. This transition can be facilitated by making sure that learners' number sense is very well grounded.

Although Mental Mathematics is recommended in CAPS for Grades 7 and 8, it is often not harnessed as fully as it could be in the Senior Phase. Mental Mathematics is a very good method for teaching mathematical facts and developing number sense.

Teachers will have the opportunity to try out various strategies that could be used to improve SP learners' number sense during their transition to formal Algebra.

TARGET AUDIENCE: Senior Phase
DURATION: 2 hours
MAXIMUM NUMBER OF PARTICIPANTS: 50

MOTIVATION

Mental Mathematics is the branch of mathematics that deals with the properties and manipulation of numbers. Algebra uses learners' understanding of Mental Mathematics to express their understanding using variable notation. Algebra deals with generalised numbers, variables and functions.

In Grade 8 (and Grade 9 although not in the curriculum) learners consolidate number knowledge and calculation techniques for whole numbers, developed in the Intermediate Phase and Grade 7.

The following list of rapid recall facts and rapid calculations has been summarized from the CAPS document. This workshop will highlight some of the main strategies.

CONTENT OF THE WORKSHOP

Introduction			10 minutes
Summary of mental maths for SP from the CAPS document			10 minutes
1	Number bonds and times tables		10 minutes
	Activity 1	Adding quickly	5 minutes
	Activity 2	Algebraic thinking	5 minutes
2	Multiplication of whole numbers to at least 12 x 12		
	Activity 3	Practising tables	5 minutes

	Activity 4	Practising bonds & tables	10 minutes
	Why is number sense important?		5 minutes
3	Inverse operations		10 minutes
4	Ordering		5 minutes
	Activity 5	Arrange in ascending order	5 minutes
5	Counting		
	Activity 6	Dots to rules	5 minutes
6	Prime numbers		10 minutes
7	Prime factors and total number of factors		
	Activity 7	Total number of factors	10 minutes
8	HCF and LCM		
	Activity 8	HCF and LCM	10 minutes
	Conclusion		5 minutes

* Note that the time estimate includes the discussion of strategies.

CONCLUSION

Mental mathematics is at the heart of all mathematics taught and as such needs to be taken seriously by all educators. It will assist in learners being able to recall basic facts easily and flexibly.

Table 1: Summary of mental maths for SP from the CAPS document

Rapid recall facts	**Rapid calculations**
Number facts (number bonds and times tables);	Multiplying and dividing by multiples of 10, 100, 1 000, 10 000
Multiplication of whole numbers to at least 12×12;	Checking answers using inverse operations;
Squares to at least 12^2 and their square roots;	Place value;
Cubes to at least 6^3 and cube roots;	Order and compare numbers up to 1 000 000 000;
Odd and even numbers;	Rounding off to the nearest 5, 10, 100, 1 000, 10 000, 100 000 or 1 000 000.;
Prime numbers to at least 100;	Estimation;
Multiples and factors;	All four operations with integers;
Identity elements;	

EXPLORING INTEGER MEANING IN CONTEXTS

K. (Ravi) Subramaniam

Homi Bhabha Centre for Science Education

In learning about the abstract objects of mathematics and ways to deal with them, it is best not to start directly with them, but to start with real situations. This is the approach that is advocated by many mathematics educators, for e.g., by the school of Realistic Mathematics Education founded by Freudenthal. For teachers to design appropriate contexts for learning mathematics, they need to explore how abstract mathematical objects are interpreted or modelled in contexts.

TARGET AUDIENCE: Intermediate Phase and Senior Phase

DURATION: 1 hour

MAXIMUM NUMBER OF PARTICIPANTS: 50

MOTIVATION & CONTENT

In this workshop, the mathematical object of focus is “integers” (or more generally “signed numbers”) and I present a framework to systematically explore the meanings that integers and integer meanings take on in contexts. The framework is derived from the mathematics education research literature and is useful in deepening teachers’ knowledge about the topic of integers.

CONCLUSION

This workshop is suitable for teachers teaching grades where integers and integer operations are introduced (typically Grade 6 or 7).

THE ORDER OF OPERATIONS: PUTTING THE 'TERM' BACK INTO BODMAS

Sheila Wood

Rhodes University Mathematics Education Project (RUMEP)

Enjoy participating in this introductory workshop, where the reasons behind 'BODMAS' are explored. The concepts of 'terms' and 'factors' are used to present the order of operations in such a way that the logic and structure of using numbers and the four basic operations for quantifying is made clear.

All exercises are at an introductory level, using only the four basic operations, finishing always with whole value answers. There is some simplification of fraction multiples, but its use is restricted to whole value answers.

The punch comes when you realize that the terminology and approach used here can take pupils all the way through their high school mathematics, including the challenges of more complex notation (fractions, decimal fractions, exponential notation, roots and integers) and more complex applications (trigonometry, financial mathematics, ...)!

FET teachers in both Mathematics and Mathematical Literacy may find this workshop of use, as it gives a very visual approach for pupils still struggling to find logic in the order of operations and to fit their mathematics lessons to the reality of life around them!

TARGET AUDIENCE: IP and SP; FET Mathematics and Mathematical Literacy, especially those who deal with pupils in support groups.

DURATION: 2hours

MAXIMUM NUMBER OF PARTICIPANTS: 30 - 40

MOTIVATION

The work we present in younger grades should have logic and structure, and be consistent to the vocabulary and needs of more complex numerical and algebraic applications. It should not contain a blind application of a set of 'guidelines' whose usefulness stops as the pupil moves from one level of complexity to another in their mathematical thinking and experience.

It is hoped this particular approach to the order of operations will provide language and visual structure that can be used with consistency from simple applications, to meeting more complex numerical and algebraic needs.

CONTENT OF THE WORKSHOP

1. Addition and Subtraction (20 minutes) – quantities are represented by terms.
2. Multiplication and Division (40 minutes) – terms can present in factor form.
3. Brackets (40 minutes) – brackets indicate complex factors within a term.

CONCLUSION:

As one extends mathematics skills through the grades, ***ALWAYS*** keep in mind the idea of the **‘*term rule*’**.

When pupils can identify terms, recognize terms in factored form, and interpret various forms of notation, they will have a rationale for ***the Order of Operations*** that will serve them well in both numeric and algebraic applications.

HOW I TEACH

MEDIATION OF COUNTING CONCEPTS TO GRADE R LEARNERS

Feza, N N, Mavaleliso, N., Mangcu, N. & Ntwana, N.
University of South Africa

TARGET AUDIENCE: Foundation Phase

INTRODUCTION

Foundational knowledge has been highlighted as the major factor that contributes to South African poor learner perfomance in mathematics (Reddy et al., 2013, Spaull, 2011, Spaull and Kotze, 2014, Venkat and Spaull, 2015). On the other hand Feza (2015) highlights a lack of educator knowledge of counting concepts for teaching in early years.This leads to rote teaching of meaningless counting. In addressing this challenge a professional development programme funded by NRF has been conducted for Grade R educators in 6 primary school in one township in the Eastern Cape. This project has 16 Grade R educators who participated in the project. Three educators from this group volunteer to share own lessons on developing counting concepts by sharing their video lessons.

COUNTING CONCEPTS

Our approach in presenting these lessons will first begin by engaging the audience on eliciting some counting concepts from the following scenario:

Scenario 1

Dolly was walking with her five and a half year old daughter (Ana) to a Grade R class for the first time. They were both so excited. As they climb the stairs to Anna's classroom. Dolly started counting each step they climbed. She said, "Ana let us count together, 1 one, 2, two, threeeeee," as they finish the third step. Then Dolly asked, "How many steps did we climb Ana?" Ana responded, "1one, (climbing up), 2 two, 3 threeeee" Then Dolly asked again, "How many steps are they Ana?" Again Ana climbed up and counted, "1 one, 2 two, 3 threeeee". Climbing up counting each step she climbs. Then again Dolly asked, "How many steps did you climb my angel?" Again she moved up and started counting. Then Dolly stops her and said, "Remember you counted them already. How many steps did you climb?"

The use of the above scenario in unpacking Ana's ZPD the educator needs to respond to the following questions:

1. Does Ana know the number of steps she climbed
2. What response did Dolly expect from her daughter?
3. Can Ana count to three?
4. From the scenario can you list about two skills that Ana showed when she was counting with her mother?
5. Do you think Ana has attained the two skills you listed completely?
6. Would you be able to assess Ana's counting skills to know her level of counting?
7. What assessment activity would you use to assess Ana's level of counting?

The above scenario is one of the scenarios used in the professional development programme to address counting concepts through discussions with educators.

Educator 1 will present her video and share her challenges and lessons learnt.

Educator 2 will present her video and share her challenges and lessons learnt.

Educator 3 will present her video and share her challenges and lessons learnt.

CONCLUSION

Educators will engage in discussion on the counting skills of learners by reflecting on the three video presentations.

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HOW I TEACH A LESSON ON GROUPING FOR GRADES 000 – 00

Susan Green
Yellowwoods Preparatory School

TARGET AUDIENCE: Foundation Phase
DURATION: 30 minutes

INTRODUCTION

Bruner's theory of representation states that, each stage of representation builds on the foundation of the previous stage (McLeod, 2008). While working with young learners (age 3-5) it is important therefore that we consider the fact that they are still in the early stages of this development. The first stage is the enactive (action based) stage, otherwise referred to as the concrete stage. For this reason I use large manipulatives in this lesson. The second stage is the iconic (or image based) stage. I have included this stage by using a follow up 2D worksheet, which uses pictures of the objects used in the previous activity and allows the learners to begin moving towards the iconic stage (Culatta, 2012).

This lesson also includes the executive functioning skill of task switching or cognitive control, which enables the children to exercise their ability to switch tasks without losing focus, or to be able to apply different rules in different settings. This has been achieved by asking the learners to group the objects using different criteria.

CONTENT

Resource material used

Blocks of different shape, size, pattern and colour.

Learners are asked to describe differences and similarities between a collection of wooden blocks. The learners look at the objects which are initially mixed together.

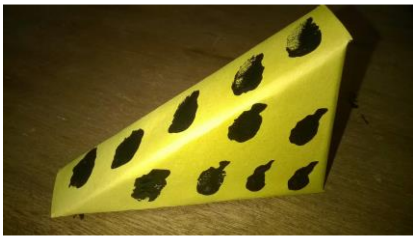
First they are asked to group the objects according to colour. They then discuss what the differences are within each colour set. Learners are then asked to group the objects according to different criteria (shape, pattern, number of sides etc.)
How many different ways could you find to group these objects?

CONCLUSION
A discussion including questions such as the following will take place:
Can we only group things one way?
How could you group yourselves?
What does it mean to put things into a group?

FOLLOW UP ACTIVITY (optional)

Learners may complete a 2D - perceptual worksheet on what they have learnt in the above.
Match the pictures with the same shape:

Now match the pictures that are the same colour:



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HOW I TEACH LOWEST COMMON MULTIPLES (LCM) USING PRIME FACTORS

Wandile Hlaleleni

Butterworth High School

TARGET AUDIENCE: Senior Phase

DURATION: 30 Minutes

INTRODUCTION

As teachers, we are supposed to have many methods of solving mathematical problems as we have learners who have different capabilities. I have decided to share the method of finding the lowest common multiplies by prime factors in order to afford my participants an opportunity to learn from my presentation. I hope that my participants will learn from this presentation and will use this method in their mathematics classrooms.

CONTENT

I usually explain to learners that lowest common multiple means least common multiple when multiples of the given numbers are listed. For instance, find the lowest common multiple (LCM) of 8 and 12 by using prime factors solution.

$8= 2x2x2$

$12=2x2x3$

LCM $=2\times2\times2\times3$

LCM = 24

Example 2

Find the LCM of 250 and 200 by using prime factors solution

$250= 2\times5\times5\times5$

$200=2\times2\times2\times5\times5$

LCM = $2\times5\times5\times5\times2\times2$

LCM= 1000

Example 3

Find the LCM of 162 and 243

$162= 2\times3\times3\times3\times3$

$243=3\times3\times3\times3\times3$

LCM= $3\times3\times3\times3\times2\times3$

$=486$

LCM is lowest common multiple.

-To get the L.C.M, I first prime factorise the given numbers.

-I identify the common factors and multiply them.

-I also multiply the common factors by the remaining factors.

In 200 and 250 above the common factors = $2\times5\times5$, remains in factors of 200 after taking $2\times5\times5$ are 5.

Remains in factors of 250 after taking out $2\times5\times5$ remains are 2×2.

LCM = Common factors × remains in factors of 200 × remains in factors of 250.

$= 2 \times 5\times5\times5\times2\times2$

$= 1000$

In 162 and 243

$162= 2\times3\times3\times3\times3$

$243=3\times3\times3\times3\times3$

LCM = Common factors x remains x remains.

$=3\times3\times3\times3\times2\times3$

$=486$

CONCLUSION

Finding the lowest common multiple using prime factors is a simple strategy for learners to use when required to.

HOW I TEACH CIRCUMFERENCE AND AREA OF CIRCLES

Delmerie Kara-Addison

Mary Waters Secondary School

TARGET AUDIENCE: SP and Gr 10 Mathematical Literacy

DURATION: 30 minutes

INTRODUCTION

Measurement is very important for the development of mathematical concepts and skills. As a life skill, measurement is used by everyone in their daily lives. The how I teach session focuses on how learners need to be able to reason, explore and understand the formulae for the circumference and area of circles.

Research in the field of mathematics education, locally and internationally, often reveals poor understanding of the concepts of area and perimeter (Gough, 2004; Helen & Monicca, 2005; Tirosh & Stavy, 1999). It was found that the concepts of area and perimeter are a continual source of confusion for learners. Van de Walle, Karp and Bay-Williams (2014) suggest that it is perhaps because both area and perimeter involve measurements, or because students are taught formulae for both concepts at about the same time, that they tend to get the formulae confused. The confusion between these two concepts results in learners developing misconceptions and misunderstandings.

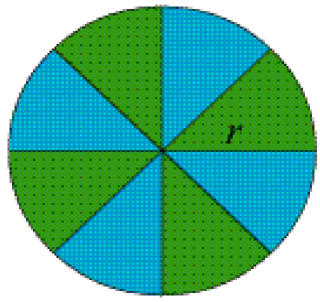
As recommended by van de Walle (2014), students will measure and explore relationships among radius, diameter, and circumference, which will lead them to discover that pi represents the ratio of the circumference of a circle to the diameter. Students need to develop an awareness that pi is an irrational value which must always be approximated when making calculations. Using a rounded value of 3.14 for pi means all values calculated using the rounded value are approximations. In addition, reporting values where an exact value for pi is used to calculate (e.g., pi key on a calculator) requires rounding, and therefore those values are approximations as well.

As many teachers and parents know, learning the different names and formulae connected to circles can be difficult. However, by using creative ways instead of merely presenting a formula, where learners investigate and make use of technology and visual models not only improves learners' understanding of circles, but it cements their conceptual framework.

CONTENT

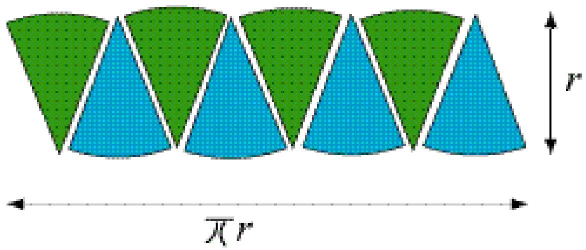
Before teaching circles, I assess my learners' existing prior knowledge and understanding. I use a written pre-test to ascertain learners' conceptual understanding of the topic.

After having written the pre-test, discussions are held to investigate learners' concerns and challenges. As an introduction circle terminology related to the parts of the circle are explored. Various flashcards are used consolidating the parts of a circle. Then using various circular objects such as a medium sized bucket, standard measuring cups and different bottle tops we investigate how pi is derived. By doing this, learners get a better understanding of the relationship between the diameter and the circumference. Learners deduce that:



Circumference ÷ Diameter = π and that π × Diameter = π D.

We resolve any conflict through discussion and changing the subject of the formula**.** In finding the area, learners draw circles with a radius of 5, 5 cm and draw sectors. Learners deconstruct each circle into a figure resembling a parallelogram. The circle sectors are cut out, arranged and pasted into a figure resembling a parallelogram or a rectangular shape. See figure below:



C = 2 ×r × π

Area = base · height

Area = $\frac{1}{2}$ circumference · radius

Area = $\frac{1}{2}$ (2·π·radius) ·radius

Area = $\frac{1}{2}$ (2·π·r) ·r

A=π· r · r A=π·r2

Breadth is the radius

Length is 1/2 the Circumference

The one half of the circumference sectors are pasted at the base of the parallelogram and the radii are pasted and substituted at the height of the parallelogram and the other half on top. Different colours can be used to differentiate the different parts.

I scaffold learners through this process one-step at a time, referring students back to the figures they created, and previous discussions about circumference.

Area = base × height of the parallelogram

Area = $\frac{1}{2}$ C × radius

Area = $\frac{1}{2}$ C (2 × π × radius) × radius

Area = $\frac{1}{2}$ C (2 × π × r) × radius

Area = π × r × r

Area = π r^2

A worksheet with questions that guides learners is a good idea. The formula for the area is found through investigating by substituting and finding it through a trial and error method. Allow learners to find it for themselves and guide or steer them through the process. Learners with impediments to fractions and algebra need assistance on how to multiply with the half and showing them practically can aid their understanding.

CONCLUSION

Always use creative ways to encourage the learners to do better. Investigating the circumference and area of circles through short cuts leads to misconceptions, misunderstandings and errors. Be enthusiastic in teaching measurement and in particular circles and all other mathematical concepts in a practical and interesting way to produce sound understanding and positive results.

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HOW I TEACH MATHEMATICS TO LEARNERS WITH DYSCALCULIA

Diau Ledimo

Kgato Primary School

TARGET AUDIENCE: Intermediate Phase

DURATION: 30 minutes

INTRODUCTION

Dyscalculia is a learning disability that involves the inability to work with numbers and lack of understanding of numbers. It is often known as dyslexia of numbers. It is a confusion of numbers that exists in the brain. Dyscalculia is not a life sentence - children can be taught mathematics even with dyscalculia but a different approach needs to be used.

Learners with dyscalculia often struggle with the following: Understanding word problems, working out the change when buying at the shop, lack of number sense, computations and interpreting and the transferring of information.

CONTENT

Application of intervention strategies

To manage learners anxiety, let the learners play by counting 1;2;1;1;2;3;2;1;1;3;4;3;2;1;1;2;3;4;5;5;4;3;2;1

They first count slowly and then secondly they count as quickly as they can.

Make them work on concrete material

- Equivalent fractions like filling of egg boxes
- Addition of numbers - they use manipulatives
- Addition by adding 9 mentally
- Example 47 + 9= 56 on the left what digit comes just after 4, and on the right what digit comes just before 7

- Play using multiplication cards.
- Flashcards.
- The one learner flashes a 7 on the card and the other learner flashes a 2.
- Play multiplication tables (video)

CONCLUSION

Active learning strategies engage learners instead of learners passively listening to the teacher. The use of co-operative learning, peer teaching and the use of technology helps learners to be actively involved and motivated throughout the lesson. Active learning is the opposite of traditional teaching where learners listen to the teacher and are never engaged.

HOW TO INTRODUCE DECIMAL FRACTIONS

Thotobolo Mdladlamba

Department of Education, Sterkspruit District

TARGET AUDIENCE: Intermediate Phase

DURATION: 30 Minutes

INTRODUCTION

Decimal fractions are types of fractions which are unique in the way that they follow the base 10 number system. It is a fraction where the denominator is a power of 10, and can be written without the denominator but with a decimal point, which makes it easier to calculate.

CONTENT

Decimals are an extension of the place value system. Like whole number place value representation of numbers, the position of each digit determines its value. A comma is used to separate the units from the decimal part. For example:

1. 10,01
2. 325, 67
3. 0,5

TH	H	T	U	,	t	h	th
		1	0	,	0	1	
	3	2	5	,	6	7	
			0	,	5		

Converting common fractions to decimal fractions

$\frac{1}{5} \rightarrow \frac{1\times2}{5\times2} = \frac{2}{10} = 0,2$

$\frac{1}{4} \rightarrow \frac{1\times25}{4\times25} = \frac{25}{100} = 0,25$

Ice breaker:

I am less than 10, I am 10 when rounded to the nearest whole, my tenth digit is odd and I have no hundredths digit. Who am I?

CONCLUSION

The building of the conceptual understanding of decimal fractions has to be done thoroughly. Decimals are used in everyday situations, such as counting money, looking at price tags and in measurement. The decimal system is a way to express large and small numbers by utilizing a decimal point. Using the decimal system is less cumbersome than writing, adding, subtracting or multiplying mixed numbers with fractions.

HELPING INTERMEDIATE AND SENIOR PHASE LEARNERS CONSTRUCT THEIR OWN CONCEPTUALIZED MULTIPLICATION OF TWO MULTI-DIGIT NUMBERS USING THE INTERSECTION OF LINES

Julius Olubodun

ORT South Africa

TARGET AUDIENCE: Intermediate and Senior Phase

DURATION: 30 Minutes

INTRODUCTION

This session tries to create opportunities for teachers to explore the use of intersection of lines as a platform for learners to develop conceptual knowledge of multiplication. The ideas of ‘breaking down and building up’; as well as ‘carry over’ has been difficult to present to the learners such that they can apply it in a meaningful way (DOE, 2011). This idea was adapted from a YouTube video I watched in my quest for how to create meaningful learning experiences for learners in the classroom (Novak, 2002).

The CAPS document requires grade 7 learners to do “multiplication of at least whole 4-digit by 2-digit numbers” (DoE, 2011). It goes further to state that “multiplication of whole numbers to at least 12 × 12” be revised with grade 8 learners. The grade 9 learners are expected to “use a range of techniques to perform and check written and mental calculations of whole numbers including: - long division - adding, subtracting and multiplying in columns”. During class visits of grade 9 mathematics lessons that we undertake, as part of our project in ORT South Africa, it is common to find learners who are unable to do multiplication of two numbers, especially 2-digits by 2-digits or more.

One of the learning barriers we identified in this area is the inability of learners to regurgitate the multiplication table. We have seen instances of learning deadlocked due to the fact that the individual learners of a group of learners were unable to rehearse the multiplication table.

We try to encourage teachers to minimize how much they “tell” while the learners are presented with the following activities that help them construct their own learning.

CONTENT

Modelling multiplication problem with intersecting lines

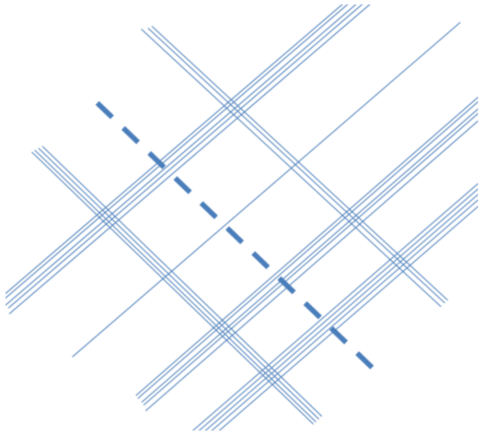

Figure 1

In figure 1 above the learner is required to multiply 5155 by 403. If the learner has difficulty with breaking down and building up; place values; and the multiplication table, then he would shy away from the activity, but we have to bring them on board. We build up their confidence and skills through the following steps.

Representing numbers with lines

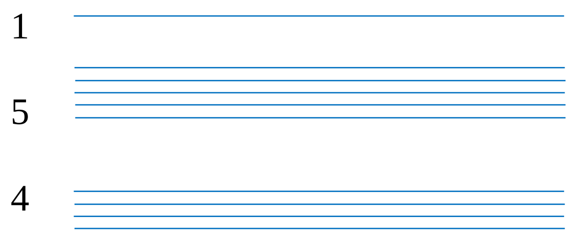

Figure 2

Each group of lines depicts a particular number. 1 line depicts number 1; 5 lines depicts number 0; and 4 lines depicts number 4.

Intersection of lines depicting multiples

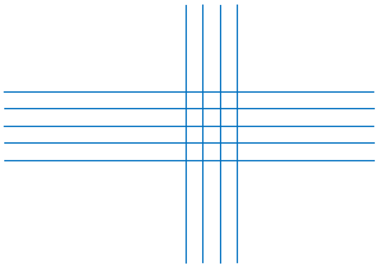

Figure 3

The 20 intersections in figure 3 depicts the multiple of 5 and 4. The learners can count (at least when they are new to this activity).

Representing multi – digit numbers

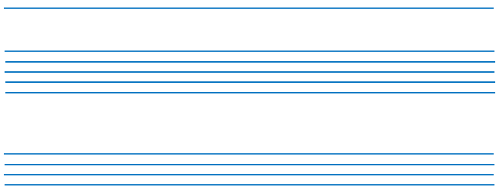

Figure 4

Figure 4 could be used to represent the number ‘one hundred and fifty four’ (154) or ‘four hundred and fifty one’ (451).

Directions matters

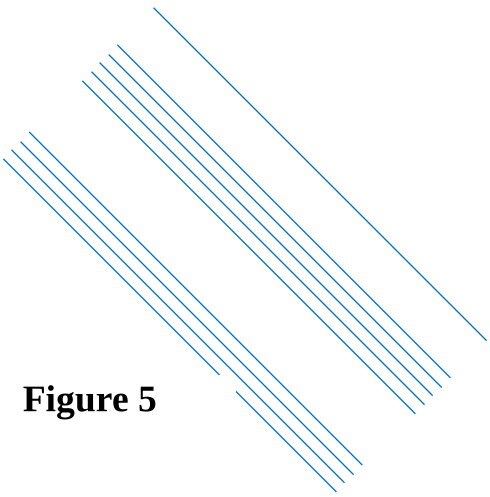

Figure 5

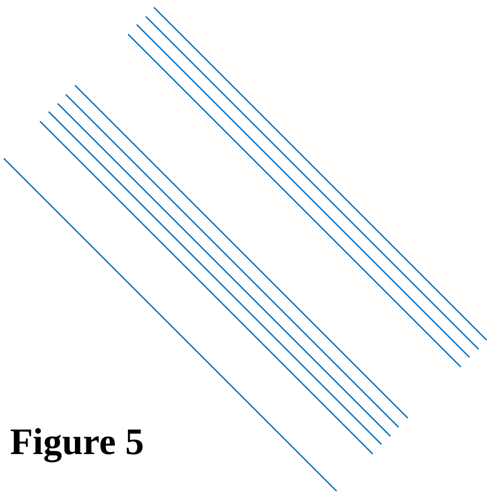

Figure 5

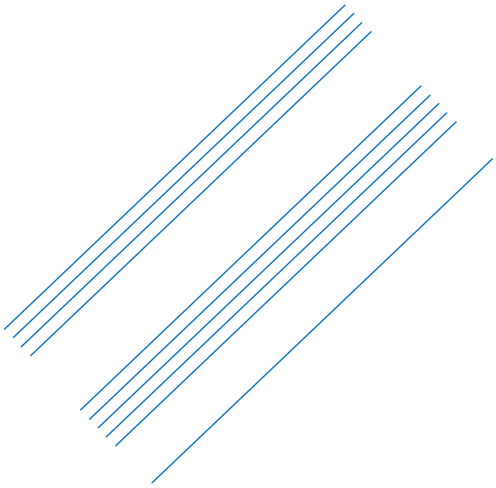

Figure 6

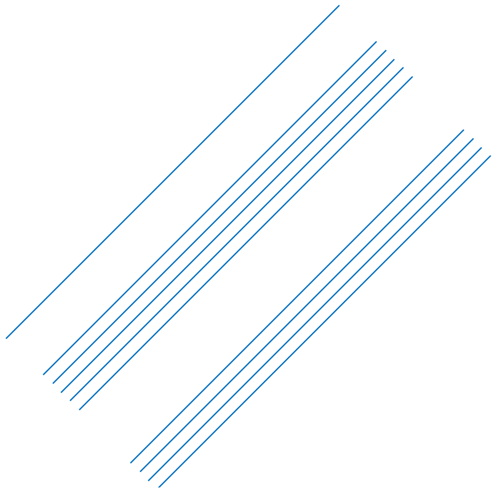

Figure 7

We adopt a top-down; and right-left model so figure 5 represents 451; figure 6 represents 154; and figure 7 represents 451while figure 8 represents 154.

Alignment of intersections clarify grouping

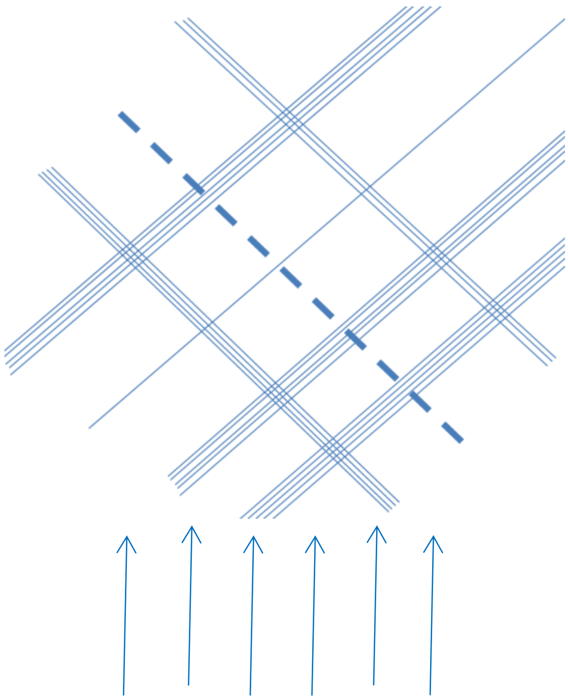

Figure 8

The alignments of the intersections make the groupings of values 'visible' to the learners, all intersections within the groups are added together (or counted together depending on the level of the learner).

What we have experienced is that while going through the steps above, learners started by counting the intersections one by one, but then realized the necessity of group counting. The idea of a multiple table became meaningful (Novak, 2002). Also while adding all the intersections together they realized why a group of ten ‘units’ become a ‘ten’; a group of ten ‘tens’ become a ‘hundred’; and so on. As the pupils progressed with the activity, patterns such as 1, 2, 1;   1, 2, 2, 1;   1, 2, 3, 3, 2, 1 and so on evolved.

CONCLUSION

We were amazed at how this activity helped grade 9 learners improve how they engage with mathematics activities. They were able to make meaning of their prior learning using a ‘breaking down and building up’ method.

This activity became a learning tool for multiplication of polynomials.

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INTERACTIVE LEARNING AND TEACHING

K.G. Sekano, P.S. Lebabo & T. Diale

TARGET AUDIENCE: Senior Phase

DURATION: 30 minutes

INTRODUCTION

Technology is becoming more prominent in today's classrooms. Students use IPads, computers, tablets, and smart boards while learning. In mathematics, these tools can be very useful for teachers to increase students' access to information, ideas, and interactions that can support and enhance sense making, which is central to the process of taking ownership of knowledge. As such, our talk is based on how teachers can use IPads, tablets and computers in a classroom in order to engage students to identify mathematical concepts and relationships.

CONTENT

As a demonstration of how we teach, a maths problem on solving cubic equations (Topic: Calculus) will be posed to the audience, thereafter, randomly selected teachers will write their solutions on the tablet or IPad (provided). Participants' solutions will be projected on the screen while the instructor (teacher) and the audience (learners) simultaneously analyse the thinking behind their approach. As such, this method of instruction helps the teacher in knowing how learners think and solve problems, which is essential in teaching and learning of mathematics. Moreover, the projection is wireless so it allows for more engagement of learners while saving time.

The disadvantages which can be encountered during lesson presentation would be, learners being unable to write their mathematical solutions using a stylus pen, however, they become better with time.

CONCLUSION

Mathematics is a conceptual subject and learners will have misconceptions, of course. Noticing the misconceptions of learners during the lesson is tremendously essential, and every teacher would like to correct them as early as possible and not only after an assessment. Wireless projector connectivity using a tablet or an IPad has enormous benefits during class: it allows you to witness the approach your learners use and to clear any misconceptions during the lesson while saving time for trips to the writing board in front.

The talk will end with a demonstration of how teachers can make short videos for their learners and send them to their smart phones.

HOW I TEACH NUMBER SENSE IN THE GRADE R CLASSROOM

Melissa Tweedie

Port Alfred High School (Pre-primary)

TARGET AUDIENCE: Reception Year – Grade R (5-6 year olds)

DURATION: 30 minutes

INTRODUCTION

Number sense refers to an organised structure of number information that helps a person understand numbers, number relationships, and number representations and use them in real world situations.

These skills are important because they lay the foundation for more advanced skills. In Grade R lots of attention needs to be given to the development of number sense. Learning to count with understanding is a crucial number skill and learners need to be able to understand the meaning of different kinds of numbers, the relationship between different kinds of numbers, the relative size of different numbers, the representation of numbers in various ways and the effects of working with numbers.

In the early grades children should be exposed to Mathematical experiences that give them opportunities to 'do, talk and record.' (S.A. Department of Education, 2012)

CONTENT

The lesson that I share in this 'How I teach' session focuses on number and number name recognition, one-to-one correspondence and number sequences. This lesson will hopefully help learners make the connection between numerals, number names and the various representations. Below I unpack the resources, methods and activities I use in my lesson and which I will use in my presentation at the conference.

Resources:

Flash Cards with number symbols and number names
Flash cards of Pictures and dots
Bamboo blocks; paper plates; coloured pegs; bead-strings; unifix blocks; prestick

Method:

Learners sit in groups of 4-6 on the mat. We begin the lesson with a number rhyme. I first introduce the number symbol and name for example 1, 2 and 3 and then number names one, two and three. I then demonstrate one-to-one correspondence using the bamboo blocks. I repeat the process three times. I then remove the number symbols and give the learners a chance to place the number symbols with the corresponding names and blocks. I then remove the number names and ask the learners to place the names with the corresponding number symbols and blocks. Finally I remove the bamboo blocks and ask the learners to place the blocks with the corresponding number names and blocks.

Activity 1
Each learner is given a set of number symbols, number names and 6 bamboo blocks. Learners are asked to turn the number symbol and name cards over and play "Knock, knock". The learner knocks on any card and says "Who's there?" The learner turns it over and identifies the card.

Activity 2
Each learner is asked to sequence their number symbols and number names 1-3. They then have to demonstrate one-to- one correspondence using the bamboo blocks.

Activity 3
Learners are given different kinds of manipulatives such as bead –strings and unifix blocks to demonstrate their understanding of numbers 1-3. I show them numbers 1-3 in any order and the learners must show me how many beads or unifix blocks each number represents.

Activity 4
Learners are given small paper plates, coloured pegs, number symbols and name cards. I say a number from 1-3 in any order and the learners must stick the corresponding number symbol, name and correct number of pegs on the plate.

Extension activities

Introduce the number line with some numbers missing. Ask the learners to fill in the missing number symbols using the number symbol cards.

Role playdough balls to match number symbols and names.

CONCLUSION

I believe that through repetition and using these activities it really helps the learners discover and understand number concepts. The advantage of these types of activities is that the learners are actively engaged in the learning process. This lesson also helps develop Mathematical language as the learners are asked questions like "What number comes before/after/ in between etc." I have found that this lesson is enjoyable and keeps the learners focussed. It is an interesting and exciting concept that attracts the learner's attention.

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EXTENDED ABSTRACTS

TEACHING AND LEARNING MATHEMATICS USING VISUAL ARTS IN THE EARLY YEARS

Olusola Ariba & Kakoma Luneta

University of Johannesburg

This paper reports on an intervention designed by infusing the visual arts into the curriculum of mathematics in the early years with the aim of improving on the teaching and learning of mathematics. It discusses the effect of the intervention after the administration of a pre-and post-test. The study gives some insight into gender related issues and the intervention.

Mathematics, the Queen of science is peculiar to all societies and cultures and highly relevant in every human endeavour. All aspects of human existence is directly or indirectly affected by mathematics. It is required for effective functioning in the society. Distressingly, mathematics is one of the subjects where there is a high record of failures in many countries. e.g. South Africa, Namibia, Australia, etc. The poor performance of students in mathematics generally in most parts of the world is not encouraging and cannot be overlooked considering the relevance of mathematics in the previous, present and future dispensations. Various attempts have been made to identify the reasons and causes for failure in mathematics. (Chinn, 2012) argued that learners have a phobia for maths while Brunkalla (2009) affirmed that many students dislike classes in mathematics because it is considered as boring, hard and irrelevant. Coupled with learners' problems in understanding mathematics is the diverse teaching style which has also been queried. Some blamed teachers for the use of rule based instructions which is to the detriment of learners. The combination of all these factors with the abstract and rigid nature of mathematics makes it more complicated.

However, it is unlike this in the arts, as all children have a positive disposition to the arts generally. The arts which entail the fine, visual, performing arts, etc., can create rich experiences that can promote comprehension which are actually critical for knowledge construction in mathematics. It is a medium through which children give air to their feelings (Fox & Schirrmacher, 2012). Children appear to enjoy and use the arts to convey their feelings, thoughts and emotions. (Scaptura et al., 2007) allege that the arts can assist the learners to describe their mathematical ideas in a way that makes meaning to them. This paper seeks to aid teaching and learning of mathematics by infusing skill and the contents of visual art into mathematics in early years to provide meaningful experiences that will enhance learners' performance. As a result four hypotheses were tested namely.

1. There will be no significant difference between the test scores of learners exposed to art infused mathematics lesson plans (AIMLP) and their counter parts that are exposed to the traditional teaching method (TTM).

2. There will be no significant difference between the mean scores of female learners exposed to art infused mathematics lesson plans (AIMLP) and their female counter parts that are exposed to the traditional teaching method (TTM).

3. There will be no significant difference between the mean score of male learners exposed to art infused mathematics lesson plans (AIMLP) and their male counter parts that are exposed to the traditional teaching method (TTM).

4. There will be no significant difference between the performance of female learners and male learners exposed to art infused mathematics lesson plans (AIMLP).

This study is underpinned by the Howard Gardner theory of multiple intelligences and the constructivism theory. The constructivism theory canvasses strongly for interactive learning and also a curriculum that is integrated, (Brewer, 2007). The multiple intelligence theory supports the ideas of creating and formulating instructions that can assist learners to develop their strength which can have a ripple effect on areas that they are weak in. The multiple intelligence theory emphasises that the human brain is like a set of multiple computers with some functions working appropriately and some not. Due to the multi-dimensional nature of the brain all learners are likely to grasp some mathematics skill and concepts using various methods and techniques which supports the different capacity the individual brain is endowed with.

A random sampling technique was used to select two private elementary schools in Abeokuta south local government area of Ogun state, Nigeria. One served as the experimental group (school A with twelve participants), and one control group (school B with nine registered learners). An intervention programme was designed in which the skills and content of art was infused into topics in mathematics and termed art infused mathematics lesson plan (AIMLP). Topics like the identification of geometric shapes, addition and subtraction etc. in mathematics were taught infusing topics in arts, - colour identification and mixing, and painting respectively.

The infusion was done based on literature. In identifying basic geometric shape for example, different colours were introduced and mixed. Learners applied their own colour to the basic shapes. The intervention lasted for twelve weeks i.e. 3 hours per week (A third of a session) with the classes taught by the researcher. A pre-test- post-test non-equivalent, quasi-experimental design was used for the study in order to identify the effects of AIMLP (art infused mathematics lesson plan) and traditional teaching method (TTM) on a learner's performance/ achievement in Maths. The quasi-experimental design was preferred above other designs because intact classes were used. Data was analysed using inferential statistics (T test and Anova).

The outcome of the study revealed that art infused mathematics lesson plans had positive effects on learners' performance in mathematics. The analysis showed a significant difference between the post-test scores of the experimental group and the control group, $t(19) = 3.84$, $p < 0.05$. However, the results further reflected that art infused mathematics lesson plans had no effect on being feminine. On the contrary, the intervention appears to have positive effects on the male learners' performance in mathematics. The analysis revealed that there is a significant difference between the mean of the post-test scores of the male learners in the experimental group and the control group, t (11) = 2.98, $p < 0.05$.

In conclusion, the learning of mathematics can be improved upon by meaningful subject integration that can promote the desire and interest of learners in mathematics. Further research on the integration of the arts into subject domains must be sponsored to bring Africa into the limelight as regards technological advancement which depends deeply on mathematics.

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LEARNERS' PERSPECTIVES ON SERVICE PARTNERSHIP AND ALTERNATING COLLABORATIVE TEACHING OF GRADE 12 MATHEMATICS: A PILOT STUDY

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This presentation focuses on understanding the perceptions of learners using an alternating collaborative teaching approach

Mathematics is one of the key gate keepers to higher education in South Africa while its pass rates remain low. A diagnostic test administered at a Grade 12 upgrade facility revealed that 72% of the registered learners were underprepared to study mathematics at grade 12 level, yet their immediate future careers are informed by their performance in the subject. A partnership was shaped between the neighbouring University's community service mathematics education lecturers and the facility's mathematics teachers. An alternating collaborative teaching method, where the learners are taught by the lecturers and the teachers was implemented. The body of collaborative teaching research rarely provides an indication into the perspectives of learners.

This study aimed to understand the perceptions, experiences and attitudes of learners in a Grade 12 upgrading facility in relation to the alternating collaboration and parallel teaching methods they were subjected to. A mixed methods study gathered data from a stratified random sample of fourteen participants selected from a population of 204 registered learners across five classes. Participants responded to a structured five point Likert scale questionnaire consisting of ten items, with open spaces for additional comments. Furthermore, a focus group interview of seven participants was conducted for follow up and in depth understanding of learners' experiences. A content analysis approach informed reflections on the personal experiences lived by the learners in the context of parallel teaching.

Study findings revealed participants' perspectives of the paradox of parallel teaching methods. Learners find the two instructional styles to be beneficial on the one hand yet paradoxical insofar as they feel they are caught between two paradigms. Furthermore, this study guides future facility recommendations.