

MY DAUGHTER BECAME A PROFESSIONAL MUSICIAN AT 9

- IF ONLY FOR A DAY



"My daughter has always been passionate about music. I remember her using the furniture in our house as her drums until we bought her a real set. Old Mutual took her to the Johannesburg Philharmonic Orchestra, where she got to play with them and experience her dream, 15 years before it comes true. I'm glad we started saving for her education early."

We all have dreams. The only way of making them real is by planning for them. Contact your Old Mutual Financial Adviser or your broker, or call **0860 60 60 60** for advice on an education plan for your child. Let us join you through every stage of your life journey, from today.

Magauta Mokoena – Old Mutual Customer

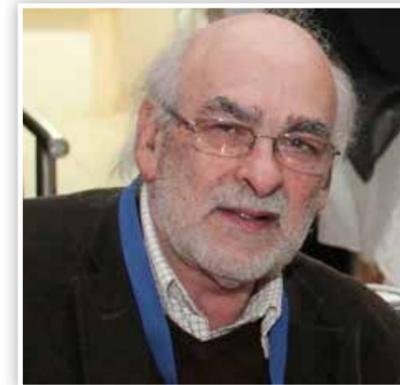
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DO GREAT THINGS



From the President's Desk...



In this little column I highlight two new professional development opportunities ...

Our country needs that more learners should be recruited into the fields of science, technology, engineering and mathematics. The vehicle to try to achieve an increase in tertiary mathematical sciences enrolment is to develop *quality mathematics teachers*, with the assumption that they will then

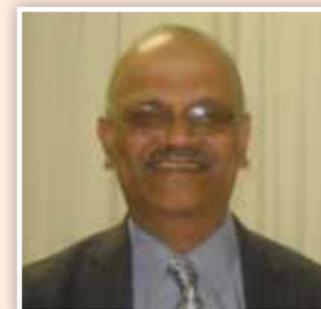
"produce" quality students entering the mathematical sciences.

Everybody propagates that the solution to our country's mathematics education problems is that teachers' *content knowledge* must be improved. I argue that we should not blame teachers. The new mathematics curricula require different mathematical content and different teaching and learning styles. The problem is that this vision not only challenges teachers' assumptions about mathematics and mathematics teaching and learning, but also asks them to teach a mathematics that they may never have experienced themselves. Teachers are themselves victims of their own previous education and are likely to continue to teach the way they were taught, unless a way is found to interrupt this self-perpetuating cycle.

To escape this self-perpetuating cycle, just more "content training" is not

Continued on page 2

Editor's Comment...



It is a great pleasure for me to serve as your AMESA Editor. We went wide in this issue, focussing on the many AMESA and AMESA-linked activities for the past six months. 2013 sees the 20th anniversary of AMESA. It is my intention to publish some of our history, with pictures, in the next AMESA News.

Enjoy reading!
VG Govender

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enough and cannot be successful. Any teacher training should give teachers “mathematical power”:

Mathematical power includes the ability to explore, conjecture, and reason logically; to solve non-routine problems; to communicate about and through mathematics; and to connect ideas within mathematics and between mathematics and other intellectual activity. Mathematical power also involves the development of personal self-confidence and a disposition to seek, evaluate, and use quantitative and spatial information in solving problems and in making decisions. (The National Council of Teachers of Mathematics, 1989)

I believe that a focus on *problem solving* will empower our teachers, especially those of us who have been exposed to only rote learning in our own schooling and teacher training, to enrich our own experiences of what it means to “do mathematics”, and that has the potential to fundamentally change our classroom culture, because this enriched experience will empower us to meaningfully engage with children and mathematics at a different level, that is a necessary prerequisite for a career in the mathematical sciences.

AMESA members are a special breed of teachers who are truly professionals - they do not wait for the Education Department to provide them with “in-service training”, but take responsibility for their own professional development. They do this through formal studies, reading, attending congresses and workshops,

AMESA’s role in this is that we should provide the *opportunities* for members to participate in activities that can contribute to their own personal growth. I want to “sell” two such

opportunities: AMESA and SAMF (the South African Mathematics Foundation) are collaborating to bring teachers quality, free short courses on problem solving at both the primary and the secondary level. I want to encourage individuals and branches and regions to actively embrace these opportunities.

The Grade 4-7 *Problem Solving Course* will be presented as two 6-hour live (Saturday) workshops, with course materials developed by AMESA, presented by two facilitators trained and supported by AMESA. The workshops are free to AMESA members, i.e. there is no registration fee, refreshment will be provided, but no travel and accommodation will be covered. It therefore makes sense for AMESA branches and regions to organise workshops in several venues in the province, so that travel of participants is minimised. All that we need is 15-20 teachers in close geographic proximity and a workshop will be arranged. Any members may request such a problem solving workshop by contacting your AMESA regional officers, or you can contact SAMF.

There is a similar Problem Solving Course for secondary teachers offered by SAMF as either a live taught course, or as a DVD course. Full details of both the primary and secondary courses are available on the SAMF website at <http://www.samf.ac.za>.

I very strongly recommend these courses to teachers, and I encourage our branches and regions to take the initiative to organise groups of 15-20 teachers in different areas of your province, and to contact SAMF or AMESA, so that a problem solving course can be arranged for them.

You will not look back!

Alwyn Olivier

AMESA OFFICE NEWS

Appointment of a second office administrator

The National Council is pleased to announce the appointment of **Nombulelo Mandindi** to the administrative staff at the AMESA office in Johannesburg. Nombulelo has been appointed to assist the National Council Members with their duties as well as to assist **Noeline Tomsett** with membership administration. It is hoped that with her appointment, we will be able to offer our members an even better service.



Nombulelo Mandindi



Noeline Tomsett

The AMESA office

The AMESA office is situated at the RADMASTE Centre, University of the Witwatersrand, Education Campus, Parktown, Johannesburg.

We would like to take this opportunity to thank Prof John Bradley (RADMASTE Director) and Jackie Scheiber (Mathematics Coordinator) for very kindly hosting the AMESA office, Noeline and Nombulelo, as well as for their support in assisting us with setting up the extended office.

AMESA recognises the RADMASTE Centre as one of our major sponsors, and would like to promote and support RADMASTE wherever possible.



Contact us

Nombulelo and Noeline may be contacted at:

P O Box 54	Tel :	011 484 8917
WITS	Fax:	011 484-2706 or 086 553-5042
2050	E-mail :	membership@amesa.org.za Nombulelo.mandindi@wits.ac.za

AMESA Congress 2013

In 2013 our National Congress will be held at the University of the Western Cape, in Belville, Cape Town. 2013 also marks the 20th anniversary of the launch of AMESA in Bloemfontein. The LOC for Congress 2013 is working very hard to ensure a very successful 2013 Congress as well as activities to celebrate our 20 years of existence as a non-racial transformed subject association serving the needs of all mathematics teachers in South Africa.

Theme: Mathematics vs. the Curriculum: What’s the score?

Date: 24 - 28 June 2013

For further news about Congress 2013 see the congress website at <http://www.amesa.org.za/AMESA2013/index.htm>.



Mathematics in the Spotlight

National Mathematics Week and the South African Mathematics Challenge

Two important events for learners and teachers from both primary and high schools took place during August, namely the National Mathematics Week and the South African Mathematics Challenge, organised by the South African Mathematics Foundation (SAMF) and the Association for Mathematics Education of Southern Africa (AMESA).

30 July - 3 August 2012 was National Mathematics Week and a variety of activities hosted by AMESA and funded by SAMF took place in three provinces:

- Eastern Cape: Walter Sisulu University, Butterworth. Twenty three schools participated in these celebrations (eight primary schools, 12 junior secondary schools and three primary schools). Activities for teachers included presentations on a wide range of topics while learner activities consisted of mathematics talks, mathematics poetry, quiz competitions, learner presentations, poster displays and the display of models.
- North West: North West University, Mafikeng campus. Attended by 117 teachers and 399 learners.
- Western Cape: 394 learners participated in Mathematics Relays at different schools in the province.

The SAMF received funding from the Department of Science and Technology to host the national event at the University of Pretoria's Mamelodi campus. The event was attended by 941 learners and 86 teachers. Activities included games such as Morabaraba, exhibitions, workshops for teachers and Mathematics Relays for learners.

The main purpose of the Mathematics Week is to highlight the beauty, utility and applicability of mathematics and also to dispel the myth that this subject is difficult, cold, abstract, and only accessible to a few.

During the same week, more than 70 000 Grade 4 to 7 learners from more than 300 schools participated in the First Round of the annual South African Mathematics Challenge. The Challenge consists of two rounds and participants can take part as individuals or in pairs.

More than 8 000 learners from more than 200 schools advanced to the Final Round, which was written on 5 September.

The Challenge is not an end in itself, but is intended as a vehicle to enhance the quality of the teaching and learning of mathematics. More specifically, the Challenge aims:

- to generate an interest in mathematics (to popularise mathematics);
- to promote a broader perspective on the nature of mathematical activity, including that mathematical activity is more than calculating;
- to promote problem solving in mathematics education;
- to promote the perspective that the calculator is a useful and necessary tool in mathematical activity (the calculator cannot solve problems for learners);
- to emphasise the importance of reading in mathematical activity;
- to provide a diagnostic tool to enable teachers to identify learners' misconceptions;
- to develop and disseminate materials that may contribute to meaningful mathematical activity in classrooms.



Mathematics relays: National Mathematics Week



SAMF exhibition: National Mathematics Week





Learners working with tangrams in Pretoria



Learners writing the Final Round of the South African Mathematics Challenge on 5 September 2012 at the University of Fort Hare



Celebrations in the Eastern Cape: Trophies lined up for the prize winners



Mathematics Week Message

Message included in the programme of the National Mathematics Week national event, University of Pretoria, Mamelodi Campus, 30 July to 3 August 2012.

What is mathematics all about? Why do we allocate so much time to mathematics in the school curriculum?

The different reasons for studying mathematics are often summarised like this:

- A study of mathematics *develops logical thinking*, which is not only important in mathematics and in the study of other subjects, but is especially important in our everyday interaction with people and issues involving decision making. Mathematics teaches us how to think!
- Mathematics is a *useful tool* to solve a variety of problems. Mathematics provides us with models (e.g. equations and formulas) to describe and analyse situations.
- Mathematics has an *aesthetical appeal*. Mathematics is beautiful! We can appreciate and love mathematics just like we can appreciate and love art, ballet, music, ...
- Mathematics is an essential part of human culture which the education system is designed to transmit. This includes understanding and appreciating the role of mathematics in the evolution of our civilization. Without mathematics we could not land a man on the moon, and there would be no traffic lights ...

Unfortunately, our school curriculum seldomly manages to address all these different aspects of mathematics. For example, at the moment we (over)emphasise mathematics as a useful tool to solve everyday problems, maybe to the detriment of the other aspects that are also important.

Unfortunately we do not often get it right – too many people are or were not successful in school mathematics, and (therefore?) do not like mathematics. Their experience of school mathematics leave many people with a negative perspective of mathematics and negative beliefs about their ability to do mathematics.

What is Mathematics Week all about? Mathematics Week is an effort to *popularise* mathematics. On the one hand it is in general about developing an increased public awareness, understanding and appreciation of the place and role of mathematics in our society, for example by highlighting the impact of mathematics on our daily lives and stressing the importance of mathematics as a foundation for careers in science, technology and managerial jobs. On the other hand it is specifically aimed at experiencing mathematics as more interesting, relevant, challenging, rewarding and engaging to learners and the community at large.

I hope everybody enjoys the activities on offer. But remember: *mathematics is not a spectator sport!* You will only learn and benefit from the activities if you actively engage, and get involved in the activities!

I trust that all participants, not only here at the main National Mathematics Week event, but all over the country, will enjoy the available mathematics activities during this week. But much more than that - I hope that you will not only enjoy it this week, but also next week and next year, and forever!

My biggest wish is that these activities will not be limited to Mathematics Week, but that these kind of activities will find their way into regular mathematics classrooms and become accepted as not just extra, fun activities, but as real mathematics that we do every day.

Alwyn Olivier
AMESA President



AMESA REGIONAL ACTIVITIES

Introduction

The level of AMESA activity in regions is increasing all the time. We are pleased to report on a number of Regional Conferences which took place in the last six months. In two regions (Gauteng and Northern Cape), elections at the Annual General Meeting (AGM) were part of the conference programme. We encourage members to participate and attend these conferences.

KwaZulu-Natal Regional Conference

On 3 May 2012, the KZN region organised its Regional Conference which was attended by more than 300 participants. The KZN region, under the dynamic Chairperson, Busi Goba, has worked very hard to raise the profile and image of AMESA in KZN. This has resulted in a surge of membership of AMESA in KZN.

Gauteng Regional Conference

The Gauteng Regional Conference was also held on 3 May 2012. This conference catered for a wide range of mathematical interests. More than 100 teachers attended the conference. The Gauteng Regional AGM was also held on this day. The new executive of the Gauteng region is:

Chairperson: Khangelani Mdakane
 Vice-Chairperson: Rencia Lourens
 Secretary: Betty Mrwebi
 Treasurer: Lerato Mathenjwa

Gauteng also ran a very successful workshop on 10 November to review the matric Mathematics and Mathematical Literacy examination papers.



CN Ncube, Thembelani Guma, Yasmin Wadee and John Chimange engaged in the matric papers review activity



Northern Cape Regional Conference

The Northern Cape conference and elections took place on Saturday 25 August 2012. This conference also catered for a wide range of mathematical interests. It was attended by about 50 teachers. Our President, Alwyn Olivier presented the conference plenary address.

We also congratulate the new office bearers in the Northern Cape region:

Chairperson: Jeffrey Thomas
 Vice-Chairperson: Peter Manzana
 Treasurer: Rebecca Maduo
 Secretary: Debby Langford

We take this opportunity of wishing the new executive in the Northern Cape all the best in their efforts to boost the image of AMESA and also raise its profile in the Northern Cape. This is indeed very timely as the Northern Cape will be hosting the 2014 AMESA Annual National Congress.



Alwyn Olivier in action in the Northern Cape



A Northern Cape session

Western Cape

The Western Cape held its regional conference on 15 September 2012 at the Cape Teaching and Leadership Institute. It was attended by 210 teachers. The theme was "Learning and Doing Mathematics".

By all accounts it was a very successful trial run for the National Congress which is due to be held in Cape Town in 2013.



Minutes of the Annual General Meeting of AMESA held on 26 June 2012 at North-West University (Potchefstroom) during the 18th Annual National Congress of AMESA.

Present:

Elspeth Mmatladi Khembo (President)
Harry Govender (Vice-President)
Lorraine Burgess (Treasurer)
Isaiah Ronald Shabangu (Secretary)
Region delegates as per register

Apologies:

No apologies were received.

AGENDA

- 1. Welcome:**
The meeting commenced at 12:15.
The President thanked and welcomed all present to the 18th Annual General Meeting of the Association.
- 2. Finalising the Agenda:**
No new matters were added to the agenda.
- 3. Minutes of the previous AGM:**
The minutes were adopted as a true reflection of the previous AGM happenings. The minutes were adopted as proposed by Phora Makoana and seconded by Mzwake Sokutu.
- 4. National Council Report on the Activities of the Association:**
The Secretary, Isaiah Ronald Shabangu, presented the National Council report on the activities of the Association. The report was adopted as proposed by Zodwa Mashaba and seconded by Phillip Mokoena.
- 5. Financial Report and Budget:**
The Treasurer, Lorraine Burgess, presented the 2011 financial statement as prepared by the auditors. The financial report was adopted as proposed by Nkosinathi Nkambule and seconded by Sphiwe Maseko.
- 6. President's Report:**
The President, Elspeth Khembo, presented her President's report.
- 7. Motions**
There were no motions presented at the Annual General Meeting.
- 8. Elections**
The following persons were elected unopposed to the respective Offices for the period June 2012 to June 2014:

President:	Alwyn Olivier
Vice-President:	Vasuthavan Govender
Secretary:	Isaiah Ronald Shabangu

9. Any Other Business

No other business was raised.

10. Closure

The outgoing President, Elspeth Mmatladi Khembo, declared the meeting closed and thanked all members present for their valuable contributions.

The next AGM will be held during the 19th Annual National Congress, 24 - 28 June 2013 at the University of the Western Cape. Notices will be posted to members in due course.

The meeting closed at 13:15.



Isaiah Ronald Shabangu
SECRETARY



Alwyn Olivier
PRESIDENT

Thanks to our outgoing Office Bearers



We bid farewell to our former President **Elspeth Khembo** who has stepped down after two terms as President. Elspeth has been President of AMESA since 2008 and has served AMESA with great distinction. Read her final President's report later in this edition of AMESA News.

The National Council of AMESA has taken the decision to make use of her vast expertise to the benefit of AMESA and its members by co-opting her onto the National Council.



We also say goodbye to **Harry Govender** (our former Vice-President) who took over from Prince Jaca at short notice in November 2010.

Harry played an important role in the transformation of AMESA in KwaZulu-Natal. We wish him well in his future endeavours.



AMESA National Council Report: Presented by the AMESA Secretary, Isaiah Ronald Shabangu at the 18th AMESA National Congress at North-West University, Potchefstroom on 26 June 2012

This report provides a summary of the activities of AMESA for the past year, i.e. June 2011 to June 2012.

National Council Members

National Executive Committee members:

Elsbeth Mmatladi Khembo	President
Harry Govender	Vice-President
Isaiah Ronald Shabangu	Secretary
Lorraine Burges	Treasurer
Alwyn Olivier	Co-opted member

Council regional representatives:

Khangalani Mdakane	Gauteng
Danile Semelo	Free State
Busisiwe Goba	KwaZulu-Natal
Sipho Vilakazi	North West
Vasuthavan Govender	Eastern Cape
Jeffrey Thomas	Northern Cape
Nkhensani Duba	Limpopo
Rajen Govender	Western Cape
Phillip Mokoena	Mpumalanga

National Council Meetings

The Council had two meetings in the past year. The meetings were held as follows:
 in Gauteng (Kempton Park), 27 – 28 January 2012
 in North-West (Potchefstroom), 23 – 24 June 2012.

AMESA Congress

More than 1000 AMESA members attended the 2011 AMESA Annual National Congress at the University of the Witwatersrand. We thank the Congress Director, Lerato Mathenjwa and the LOC of Gauteng for organizing congress 2011 on behalf of the National Council.

The National Council would like to thank Sipho Vilakazi (regional representative), Hercules Nieuwoudt (Congress Director) and the LOC of the North West region for organizing the 2012 National Congress.

AMESA Branches

Each region is divided into a number of branches which report to the region. At the AGM the number of branches and members were as follows:

Region	Number of Branches	Number of Members
Gauteng	10	260
Limpopo	11	172
Mpumalanga	20	318
Free State	11	167
Kwazulu Natal	11	456
Western Cape	4	171
North West	10	83
Eastern Cape	6	141
Northern Cape	1	24
Total	84	1810



There was an increase of 10 branches in 2012 compared to 2011.

AMESA Committees

Curriculum Committee

National Coordinators	Rajen Govender and VG Govender
Mathematics FET	Harry Govender
Mathematical Literacy	Marc North
Mathematics Senior Phase	Isaiah Ronald Shabangu
Mathematics Intermediate Phase	Heather Collins
Mathematics Foundation Phase	Shanba Govender

These conveners play a very vital role in their subjects (phases) and are responsible for coordinating input and comment from members in their phases, which is then passed onto the AMESA Curriculum Committee coordinators for further action.

Sponsorship Committee

Members of this committee are Alwyn Olivier, Neil Eddy and Lorraine Burgess.

We thank the Old Mutual Foundation for funding AMESA to the amount of R450 000 in 2011 - 2012.

Problem Solving Course for Teachers

Four facilitators from each region attended a problem solving workshop for Grade 4-7 teachers held in Kempton Park, 2-4 April 2012. Alwyn Olivier presented the workshop.

These facilitators will run problem solving workshops in their regions with the aim to support teachers' professional development and to improve participation and achievement in the South African Mathematics Challenge.

South African Mathematics Challenge

The South African Mathematics Challenge is now managed by SAMF. AMESA is represented on the *Management Committee* by Alwyn Olivier, Lindiwe Tsabalala and Nkhensani Duba. The papers are set by a *Problem Committee* led by Alwyn Olivier.

The Challenge is now sponsored by JET (EDCON) and renamed the JET South African Mathematics Challenge. We aim for mass participation this year. Nearly 60 000 learners from 279 schools participated in the First Round in 2011. More than 5 000 learners from 169 schools participated in the Final Round and more than 3 000 learners received Gold, Silver or Bronze Certificates of Achievement.

AMESA Education Fund

This fund is available to regions on application to the National Council to support professional development programmes. The closing date for applications is 15 February for use of the money before the annual congress and 15 August for the money to be used during the third or fourth term of the year.

Mathematics Olympiad

AMESA has a strong representation in SAMO: Marcus Bizony, Harry Govender and Sipho Vilakazi. AMESA encourages schools to participate in Olympiad activities at both local and national levels.

South African Mathematics Foundation (SAMF)

SAMF was initiated by SAMS (South African Mathematics Society) and AMESA, to bring their expertise together to enhance the teaching and learning of mathematics in South Africa.

Our President, Elspeth Mmatladi Khembo, is the Chairperson of the SAMF Board since March 2011 and Alwyn Olivier also serves in the Board.

Statistics South Africa

AMESA has a good working relationship with STATS SA. STATS SA has been fully involved in the following regions: Mpumalanga, Gauteng, Western Cape and the Eastern Cape.



South African National Committee for the IMU (International Mathematics Union)

Members representing AMESA in this committee are:

Renuka Vital (Chair)
 Alwyn Olivier
 Mellony Graven
 Busisiwe Goba
 Lindi Tshabalala (Capacity building representative)

Their term of office is 4 years (July 2009 - July 2013). The Council has encouraged the committee to try for another bid for ICME (International Congress on Mathematical Education).

Publications

Pythagoras

The Editor of *Pythagoras* is Alwyn Olivier. *Pythagoras* is since July 2011 published online and in print by African Online Scientific Information Systems (AOSIS) OpenJournals. *Pythagoras* is now an Open Access journal, freely available to anyone to read online, reflecting Council's view to make information about mathematics education universally available. Two issues were published in the last year.

Learning and Teaching Mathematics

Mellony Graven has retired as an Editor of *Learning and Teaching Mathematics*. Duncan Sampson has been appointed by the National Council as the new Chief Editor. Marcus Bizony and Lindiwe Tshabalala are Editors. Two issues were published in the last year.

AMESA News

Two issues of AMESA News were published in 2011. We thank regions for their contributions. AMESA News keeps members up to date with current issues of the Association. We encourage members to e-mail branch and regional articles to news@amesa.org.za.

AMESA website

Alwyn Olivier is our webmaster. The address of the AMESA website is <http://www.amesa.org.za>.

We are proud to announce that our journals are now published online. *Pythagoras* is freely available at www.pythagoras.org.za. *Learning and Teaching Mathematics* is available for members only from the SABINET website.

National Mathematics Week

In 2011 the National Mathematics Week was held during the week 2 - 6 August. Eastern Cape hosted the national event. Regions that received funding from SAMF were Eastern Cape, Western Cape, Northern Cape and Limpopo. All regions participated in this celebration.

Membership

Noeline Tomsett, our membership secretary is present here in this congress. All members with queries must see her. On Friday 22 June 2012 statistics on membership indicating that we have 1810 members, which is an increase of 88 members compared to this time last year. Well done KZN for being the leading region with 456 members, followed by Mpumalanga with 318 members. Keep up the good work at branch level. We thank all regions for keeping AMESA alive.

Conclusion

As I step down today I want to thank all of you for the opportunity you gave me. I have learned a lot.

I thank you all! Ngiyabonga!



Isaiah Ronald Shabungu
 SECRETARY



Income and Expenditure Statement of AMESA for the Period 1 January - 31 December 2011

1. CURRENT ACCOUNT

EXPENDITURE			R1 300 375.91
Salary			R85 091.80
SARS	PAYE	R4 952.40	
	UIF	R1 518.50	R6 470.90
Bank fees	Current account	R2 751.78	
	Internet fees	R1 750.95	
	Credit card facility	R7 994.87	R12 497.60
Telephone	Office - Landline	R3 192.85	
	President - Landline	R6 505.00	
	President - Cell	R13 655.00	R23 352.85
Council meeting	Jul 2011	Accom - meeting	R11 860.00
		Travel	R19 027.00
		Accom - congress	R7 265.00
	Jan 2012	Accommodation	R11 760.00
		Travel	R18 264.00
Old Mutual	Pythagoras		R382 049.54
	AMESA News		R161 693.78
	LTM		R69 202.56
	Postage and stationery		R116 051.76
	Congress 2011 - Registration		R21 717.00
	Congress 2011 - First call		R20 488.08
	Congress 2011 - Programme		R14 952.12
	Membership cards and renewals		R7 843.91
	Presidential Projects		R33 887.82
	Regional development		R13 384.40
			R841 270.97
	Congress 2011 refunds		R88 590.00
	Subsidies to regions		R109 951.70
	Sundry		R33 600.68
	Member fees - bad debts		R2 780.00
	Auditors' fees		R8 550.00
	Attorney		R14 7500.49
	Printing - Stationary		R5 292.92

INCOME			R955 259.66
Old Mutual			R450 000.00
Sales and advertising			R0.00
Members fees	Cash, cheques, internet transfer	R289 387.00	
	Credit card	R79 521.75	R368 908.75
Other income			R20 287.66
WITS (R11 975 to be transferred to Congress 2011)			R116 063.25
Interest received			R0.00

SURPLUS

-R345 116.25



**Income and Expenditure Statement of AMESA for the
Period 1 January - 31 December 2011**

2. EDUCATION FUND

INCOME		R1 526.71		
Interest received	R1 526.71			
EXPENDITURE		R0.00		
Regional development	R0.00			
SURPLUS FOR THE YEAR			R1 526.71	

3. CALL ACCOUNT

INCOME		R26 642.50		
Interest received	R26 642.50			
EXPENDITURE		R0.00		
SURPLUS FOR THE YEAR			R26 642.50	

4. RESERVE FUND

INCOME		R9 165.38		
Interest received	R9 165.38			
EXPENDITURE		R0.00		
SURPLUS FOR THE YEAR			R9 165.38	R37 334.59

TOTAL SURPLUS ON ALL ACCOUNTS

-R307 781.66



**Income and Expenditure Statement of AMESA for the
Period 1 January - 31 December 2011**

SUMMARY OF BANK ACCOUNTS UP TO 31 DECEMBER 2011

1. CURRENT ACCOUNT

Opening balance from 2011
Net transfer to call account
Net transfer to reserve fund
Surplus for year

		R89 756.75
	R434 873.00	
	R0.00	
	R0.00	
	-R345 116.25	

2. EDUCATION FUND

Opening balance from 2011
Surplus for year

		R30 762.55
	R29 235.84	
	R1 526.71	

3. CALL ACCOUNT

Opening balance from 2011
Net transfer to current account
Surplus for year

		R827 442.64
	R800 800.14	
	R0.00	
	R26 642.50	

4. RESERVE FUND

Opening balance from 2011
Surplus for year

		R204 114.63
	R194 949.25	
	R9 165.38	

5. CASH ON HAND

R2.96

TOTAL OF ALL ACCOUNTS

R1 152 079.53

NOTES TO SUMMARY STATEMENT

The Reserve Fund is made up of 4 fixed deposits and a 32-day notice account with balances:

Acc No	20-6641-6798	R	13 221.88
Acc No	20-6519-7783	R	72 624.74
Acc No	20-6245-3984	R	38 915.58
Acc No	20-6108-0279	R	30 987.99
Acc No	91-8606-5955	R	48 364.44
	Total		R204 114.63



EXPENDITURE	BUDGET
Salaries	R200 000.00
Sundries	R30 000.00
Printing and stationery	R20 000.00
Council meetings	R150 000.00
Subsidies to regions	R180 000.00
Telephone	R30 000.00
Bank fees	R15 000.00
Bad debts	R1 000.00
Old Mutual sponsored projects	R500 000.00
Auditor's fees	R12 000.00
Congress 2013 (Seed)	R10 000.00
Excess publication costs	R50 000.00
Continuous professional development	R100 000.00
TOTAL EXPENDITURE	R 1 298 000.00

INCOME	BUDGET
Balance from 2012	R1 400 000.00
Membership	R500 000.00
Other income	R500 000.00
Sales and advertisements	R10 000.00
Interest received	R30 000.00
Old Mutual	R500 000.00
Congress KZN and 2012	
TOTAL INCOME	R2 940 000.00
Balance	R1 642 000.00

President's Report



Elspeth Mmatladi Khembo
26 June 2012 at the AGM of AMESA

I would like to start by thanking Prof Paul Laridon, Jackie Scheiber, the management and whole staff of RADMASTE Centre for the incredible support they afforded me during my two terms as President of AMESA.

When I was elected president of AMESA four years ago, I was completely unprepared for the amount of work I was expected to do to keep the wheels of this association running. I remember being introduced for the first time as the President to the July 2008 Annual General Meeting (AGM), and one AMESA member blurted out loud: "How can we be led by a bookseller?"

The very first task I had to do was to submit a year plan and budget to secure funding from our main sponsor, Old Mutual which had just started operating formally as a Foundation and they required a number of documents and reports as part of the service level agreement (SLA). I found myself in the office of the Head of Old Mutual Foundation, Mr Andile Ncontsa. At the beginning of the meeting, he said that Old Mutual could not continue funding AMESA to host an annual congress - he was under the impression that that was all AMESA did with their money.

I explained the whole structure of AMESA, the teacher professional development projects, ESP, Problem Solving Course, the end of the year Grade 12 Mathematics and Mathematical Literacy Analysis that the AMESA Curriculum Committee submits to the Department of Basic Education. I went on to talk about our publications - the bulk of Old Mutual money is used to produce our academic journal, *Pythagoras*, our teacher journal, *Learning and Teaching Mathematics* (LTM) and our newsletter, *AMESA News*. I went further to explain our plans to go open access with *Pythagoras*. I was very relieved at the end of our meeting when he expressed shock at "the amount and quality of work that AMESA does with very little money".

The partnership with old Mutual Foundation has grown in leaps and bounds since 1993 when AMESA was founded. At the opening ceremony of Congress 2010, Mr Ncontsa announced the intention of Old Mutual to continue the "marriage" with AMESA. This was reiterated by Ms Liyanda Maseko at the Congress 2012 opening ceremony, when on behalf of the current head of the Old Mutual Foundation, Dr Clarence Tshitereke, she announced Old Mutual funding for 2012 of R500 000. She expressed appreciation for the work AMESA is doing, but also emphasised the importance of fulfilling the requirements of the Service Level Agreement (SLA).

The Old Mutual SLA requires of us to submit regular reports on our activities, including financial reports. This calls on all of us to really work hard to make sure that the regional and branch activities are well attended and that we submit reports to can keep our end of the partnership.

Our partnership with CASIO has grown and strengthened over the years. CASIO is our second largest sponsor and we acknowledge CASIO as a major sponsor of AMESA activities at all levels. We are most grateful for their continued support, especially for branch and regional workshops and conferences. A special thank you goes to Ken Finlay, our CASIO go-to person for so many years, who is now retiring. The National Council needs to seriously consider to formalise the partnership with CASIO.

In our attempt to register AMESA as a Non Profit Organisation as required by our sponsors, the Department of Social Development indicated that before we can be registered, we need to amend our Constitution to provide for stricter financial accountability, especially in regional and branch structures. The National Council has therefore appointed a Constitution Committee comprising of Alwyn Olivier, Elspeth Khembo and Vasuthavan Govender to formulate the necessary amendments, to be formally proposed and adopted at our AGM at Congress 2013.

My second term of office was spent dealing with tough situations involving our Kwa-Zulu Natal region. After all the challenges, I was very impressed with the achievements of the new KZN Regional Council when I attended their regional conference on 5 May 2012 and over 250 members attended. An audited financial statement was presented and I would like to congratulate the new chairperson, Busisiwe Goba for a job well done. The National Council will continue to support the KZN region with their exciting plans.

Most of our members are primary school teachers and it is our wish to have all the mathematics teachers present at our annual congresses. Provincial Education Departments like Gauteng and Free State are now, during our congress, training their Intermediate Phase teachers in preparation for the implementation of the Curriculum and Assessment Policy Statement (CAPS) in 2013.

It has thus become imperative that the collaboration with the DBE has to be formalised and strengthened in order to have AMESA activities included in the calendar of the Provincial Education Departments, so that the activities of AMESA and the departments may be coordinated.

The National Council has therefore conceptualised a strategic plan for the next five years, to streamline the activities in regions and branches to further support and enhance the mathematics education development activities of the DBE. We intend to continue with the analyses of the Mathematics and Mathematical Literacy matric examination paper. We also need to focus on the quality of the learners released into Science-related careers at institutions of higher learning. We plan to analyse the quality of the Annual National Assessments (ANA) mathematics questions in keeping with the intended curriculum and international benchmark standards.

In our attempt to both increase membership, and assist our members to strive towards a high standard of professionalism in the exercise of their profession, the Curriculum Committee intends to increase the number of workshops and seminars for training and supporting teachers in challenging topics in the CAPS in regions and branches. The members have to enjoy the benefits of being AMESA members by experiencing a high level of activity in their branches that will result in improved performance of the learners and thereby creating vibrant communities of practice for both teachers and learners.

In preparation of the International Mathematics Olympiad to be held in Cape Town in 2014, the Problem Solving Committee, through SAMF, is finalising DVDs and other material for training teachers in problem solving skills. Through the Mathematics Challenge, talented learners and teachers in rural and township schools have already been identified.

The Department of Science and Technology (DST) has opened new negotiations on the funding of the National Mathematics Week and the apparent continuation of the Education Support Project (ESP) in a modified form.

We continue to enjoy a good relationship with Statistics South Africa (STATS SA) in the regions, where they present Data Handling workshops under the banner of Maths4Stats.

In conclusion, let me remind us all that the office bearers in AMESA at national, regional and branch level, take upon themselves a lot of work in the interest of developing mathematics education in our country over and above their normal jobs, and with no pay. It requires people at all levels who are passionate about mathematics education. The only reward is personal professional development. One can never avail yourselves to serve in one of the structures of AMESA and come out the same person. "It is one of the most beautiful compensations of life, that no man can sincerely try to help another without helping himself" (Ralph Waldo Emerson) because "only a life lived for others is a life worthwhile" (Albert Einstein).

I thank you for the opportunity you afforded me and hope that I have served you well. It is my greatest pleasure to hand over the presidential baton to my mentor and friend, Alwyn Olivier, who taught me everything I know about AMESA and selfless leadership. Alwyn and I had our little and big fights during the past four years because, as you will all find out, he is a very principled person and I must say I have never met anyone as passionate about AMESA as Alwyn. I know he is going to take AMESA to even greater heights.

My sincere gratitude goes to the National Council for the support from the time I was first elected in 2008, as the naive "bookseller"...

I would never have managed all the hard work without the support of my son Mpho.

Ke a leboga

Elsbeth Mmatladi Khembo



Address by Ms Louisa Mabe, MPL (North-West Province) the MEC for Education and Training, at the opening ceremony of the 18th Annual National Congress of the Association for Mathematics Education of South Africa (AMESA) at North-West University, Potchefstroom Campus



Programme Director, Teachers, Distinguished Guests, Ladies and Gentlemen

I am honoured to be part of the 18th Annual National Congress of the Association for Mathematics Education of South Africa (AMESA). Events like this one are a significant step towards improving the standard of Mathematics, Science, and Technology in our country. They also assist to celebrate that which is good about our education system, and encourage our move forward to improve and promote the attainment of excellence in the different facets of our system.

There is no doubt that the sustenance of our economic growth and the development of our scientific facilities are dependent on the skills base that we have as a country. It is our view that for the country to also address the challenges of poverty and attack underdevelopment in its different forms, we must empower our people with the requisite skills and knowledge. We must as a matter of urgency ensure that we provide our people with skills that will make them active role players in the broader economic development agenda. Our country is not static in terms of infrastructure development.

Mathematics as a discipline is enabling us to acquire skills required to improve our economy and infrastructure. It is for that reason that the Department of Basic Education has a very high regard for AMESA conferences. The Department of Basic Education has noted that AMESA is, amongst others, promoting Mathematics Education to enhance the quality of the teaching and learning of Mathematics in South Africa. It provides a forum for all concerned with the teaching of Mathematics

at all levels of education; encourages research related to Mathematics Education and brings the results of such research to the attention of its members.

The Department also acknowledged the analysis made by AMESA on Grade 12 final papers for both Mathematics and Mathematical Literacy before teachers could commence marking the Grade scripts.

Ladies and gentlemen, I want you to know that the Department is working cooperatively with AMESA, either directly or through their mother body SAMF (the South African Mathematics Foundation), to achieve its goals. I am pleased to inform you that our relationship is cordial.

I want to acknowledge the current National President of AMESA, Mrs Elspeth Mmatladi Khembo, for serving in the Ministerial Project Committee that was established to implement the recommendations of the Task Team that reviewed the implementation of the National Curriculum Statement. She subsequently served as the Project Committee coordinator for the development of the Curriculum and Assessment Policy Statement (CAPS) for Mathematical Literacy and Mathematics for both GET and FET.

It is imperative to inform you that the Department of Basic Education supports organisations that add value to the quality of education in the country. That is the reason why we acknowledge the contributions that AMESA is making in improving Mathematics Education. I also need to commend the way in which the AMESA conference in 2011 at Wits University engaged its members on CAPS. Your valuable analysis and comments on CAPS are warmly welcomed.

We acknowledge the great role that teachers and schools have played in ensuring that we introduce the CAPS in Foundation Phase and Grade 10 this year. I am pleased to inform you that all subject advisors have been trained responsible for the Intermediate Phase and Grade 11. We look forward to all Intermediate Phase and Grade 11 teachers being orientated during the Winter and Spring vacations.



There is still a challenge on a number of issues regarding Mathematics. These include the number of learners passing Mathematics in Grade 12 and the ratio of learners offering Mathematics against learners offering Mathematical Literacy and learner performance in Mathematics from Grades R to 12. There are still huge gender disparities in both participation and success rate of boy versus girl learners.

The Department is concerned about the continuous drop of the number of learners sitting for Grade 12 final examinations. We have observed the continuous imbalance of learners doing Mathematics against Mathematical Literacy. The number of learners registering Mathematics is decreasing while the number of learners offering Mathematical Literacy is increasing. We need to shift this trend together to achieve the goals we have set-out in the *Action Plan to 2014: Towards the Realization of Schooling 2025*.

The ANA results demonstrated that learners are not failing Mathematics only in Grade 12, but failing it from the lower Grades. Learners are lacking basic computational skills in the Foundation, Intermediate and Senior Phases. Teachers who are teaching a particular grade must make sure that learners acquire relevant knowledge and skills in that grade. There is no place to hide for both learners and teachers as the focus is not only on learner performance at Grade 12 level. Each grade is now assessed thoroughly with a uniform and proper standardized assessment instrument.

We appreciate the modern technology which our learners are exposed to although not without challenges. For instance, some learners in Grade 9 cannot respond to the quotient of negative 6 divided by 2 without using a calculator. When you pose such a simple question, it's like you are requesting learners to take out their calculators from their school bags. The remedy for this needs to be explored in forums of this nature.

Many learners in Grade 12 are still struggling to manipulate fractions. Mathematics is a two way system, backward and forward mechanism. Teachers should teach learners how to sketch a graph, demonstrate the key aspects from a graph and how to derive its equation when it is sketched. Teachers should introduce and expose learners to graphical interpretation questions. I therefore urge you to continue sharing best practices in teaching Mathematics as you have been always doing. Sharing of good practices should not only happen at such conferences, sharing should happen also at cluster level and school level.

Together we shall make it.

I thank you.



The National Anthem at the opening ceremony, with the Seiphemelo High School Choir



Congress opening address

Liyanda Maseko
Education Programme Manager, Old Mutual Foundation



Honourable North West Minister of Education and Training, Ms Louisa Mabe
AMESA National President, Ms Mmatladi Khembo
Congress Director, Professor Hercules Nieuwoudt
Academic Coordinator, Dr Susan Nieuwoudt
North-West AMESA representative, and deputy director of Congress, Mr Sipho Vilakazi
North-West provincial chair, Ms Patricia Walaza
North-West University acting Dean, Faculty of Education Sciences, Professor Kobus Mentz
International guests, esteemed Panelists and Presenters
Members of the Local Organising Committee
Members of the Provincial Management Board from Old Mutual, Mutual and Federal and Nedbank
Old Mutual colleagues present
Distinguished guests, Congress delegates
Seiphemelo High from school learners and their teachers. Thank you for your inspiration
In the interest of time, allow me to say "All protocols observed"

I stand here today representing Old Mutual as an expression of our commitment to the value of our partnership with the Association for Mathematical Education of South Africa. I wish to congratulate AMESA for hosting this 18th Annual National Congress and for their commitment to encourage mathematics educators and recognition of the teaching profession.

Mathematics, Science and English at school are vital to higher education, skilled jobs, and the national economic growth. Often, the emphasis is on Mathematics and Science - but learners' command of formulas may not be sufficient if they cannot articulate themselves in English eloquently. For this reason, Old Mutual has an interest in the performance of the South African schooling system in these subjects.

Our schooling system continues to produce fewer passes in Mathematics and Science - particularly in the higher grade - than our economy requires. Many university degrees, professional and technical careers require a solid grounding in Mathematics and or Science, and the critical shortfall in learners leaving the schooling system with HG Maths and Science is a significant constraint on economic growth and long term development objectives.

It also impedes the development of state capacity, and undermines both public and private programmes for black economic empowerment. There are many anecdotal evidence for economic development inspired by or associated with good quality output in Mathematics and Science.

There is widespread agreement that a radical improvement is necessary. Our government spends proportionately more on education than many other developing countries, yet our learners perform worse than those of other developing countries - including African countries - in international tests.

Performance across schools is very uneven; most HG Maths passes (about 70%) are produced by a very small number (about 11%) of schools. Further, about 80 per cent of secondary schools produce only 16% of HG Maths passes - an average of one each.

The potential for doubling the number of HG Maths passes exists. To realize this potential, more learners must be persuaded to take this subject, and if it is not offered at schools close to where they live, learners with aptitude for Maths must be given the opportunity to attend schools where they exist. There is also much scope for improvement in other subjects relevant to the economy, such as Technology and Commerce.

Data on teachers is inadequate, but indications are that:

- There is a shortage of teachers, particularly in specialized subjects such as Maths and Science;
- That teachers are unevenly distributed throughout the education system, with rural areas in particular being starved of specialized teachers.



As Old Mutual, we place a premium on education in our corporate social responsibilities. We invest in the education of our brightest young minds—thereby enhancing our economic future. Our investment in Education creates shared value, between the learner, the teacher, the public and corporate.

There are compelling business reasons for making substantial commitments to education.

Old Mutual Foundation contributed about R4.5m (2007 - 2010) to 10 of the Dinaledi schools (2 from North-West - Letsatsing and Batswana High schools). We support the Department of Basic Education's Action Plan to 2014 - which seeks to achieve the following:

- Increase the number of Grade 12 learners who pass Mathematics and Science;
- Ensure availability of teachers to avoid excessively large classes;
- Improve the professionalism, teaching skills, subject knowledge and computer literacy of teachers throughout their entire careers;
- Ensure that the basic annual management process occurs across all schools in the country in a way that contributes towards a functional school environment.

Addressing this deficit will increase our nation's competitiveness. In addition, there has been a specific call by the National Planning Commission to increase the focus on Mathematics and Science. We are committed partners with government towards addressing our challenges in education.

We also remain committed to assisting our education system, particularly Mathematics and Science through funding various organizations working with these subjects. We have supported AMESA since 2001 with a total commitment of at least R3.8 million by funding the publication of mathematics education journals for educators.

We recognize the current challenges with Numeracy and as Old Mutual we are strengthening our interventions at school level. We are currently working on a comprehensive proposal to maximize our interventions at school to assist with:

- **Instructional functionality**, by improving the quality of learning and teaching of Mathematics and Science;
- **Regulatory functionality**, by assisting with the development of leadership and management of schools.

We have had several meetings with the Department of Basic Education to work out the form and extent of our envisaged intervention. Examples of projects we support as Old Mutual include:

- Bursaries for Actuarial Science and Accounting students,
- Bursaries for children of members of eight Trade Unions members with whom we do business as Old Mutual, including SADTU.

We also support other organizations such as Rapport Education Fund (for the training of students to become teachers) and the African Institute for Mathematical Science Schools Enrichment Centre.

With our increasing funding for Education in South Africa, we remain committed to our partnership with AMESA well into the future - a partnership of mutual interest for the common good.

In 2011, the Old Mutual Foundation requested AMESA to write Teaching of Geometry articles for teachers which are published in the Teacher newspaper (Mail and Guardian educational supplement). We have received excellent feedback from teachers who are using these articles and finding them beneficial. A special thank, you to Jacques du Plessis and outgoing AMESA president Elspeth Khembo in this regard.

I wish to thank AMESA for allowing us to be your partner of choice. While professional development of AMESA members is important - your financial wellbeing is equally important, hence Old Mutual has made Financial Advisors available for consultation at your own leisure throughout the duration of this Congress.

To all the delegates, have a great week ahead.

I thank you



AMESA Congress 2012 report

The 2012 AMESA Congress took place at the Potchefstroom Campus of North-West University from 24 to 28 June. The theme of the Congress was "Mathematics as an educational task". Over 900 members attended the Congress.

The Congress had a very diverse, comprehensive and informative academic programme, consisting of the following:

- Two pre-congress workshops
- 18 other workshops
- 4 plenary lectures
- 4 discussion sessions with the plenary speakers
- 2 panel discussions
- 17 long papers
- 5 "How I teach" presentations
- 8 interest group sessions
- A vibrant Activity Centre
- Maths Market
- AMESA regional meetings
- The social programme catered for all social tastes and included the following:
 - Welcome dinner with delicious food and relaxing music
 - Jazz evening
 - Cultural evening

Three excursions were arranged.

- For the more geographically-inclined, the Vredefort Dome offered a geological guided walking tour of the dome, the biggest meteorite impact that geologists have yet found on earth.
- Letsatsi la Africa game farm provided for the animal lovers among the participants and offered different activities to embark upon.
- The Ikageng tour offered delegates the opportunity to embark on a historical tour through the township as well as the newly built Mooi Rivier Mall.

We take this opportunity of thanking the North-West region of AMESA and the LOC for organising an exciting, interesting and well planned AMESA Congress.

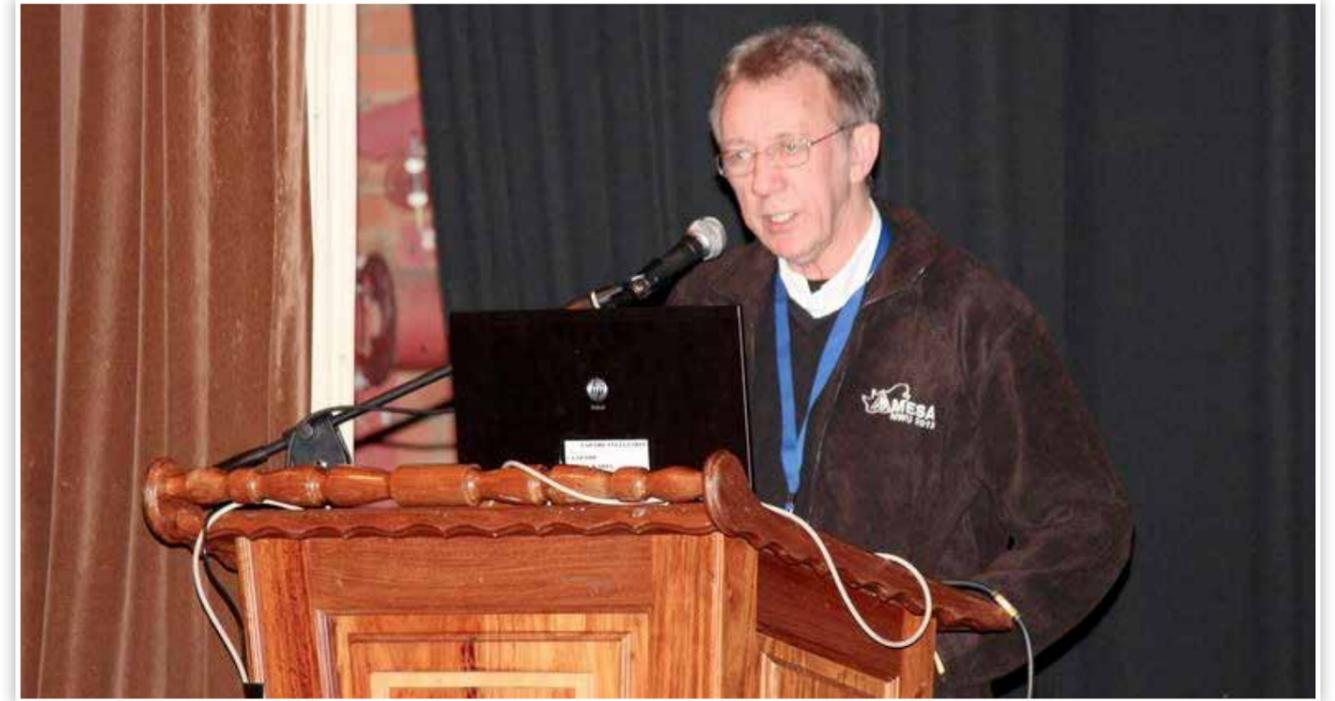


Plenary speaker Tim Rowland from the UK, with Congress Director Hercules Nieuwoudt





AMESA Congress 2012 participants during lunch



Johann Engelbrecht, plenary speaker



Tea time ...



Members in attendance at the Jazz evening





A performance at one of the evening social functions



Registered participants ...



Corin Mathews working with Foundation Phase teachers.



Sipho Vilakazi, Master of Ceremonies at the Opening Ceremony.



AMESA Curriculum Matters

AMESA Curriculum Committee

Coordinators: R. Govender (westerncape@amesa.org.za)
VG Govender (vicepresident@amesa.or.za)

The AMESA National Council notes that our mathematics teachers face a number of issues in their roles as teachers. We would like our members to inform us of successes and challenges in mathematics teaching and learning. In this regard, our phase convenors could be used to channel your input. The phase coordinators (appointed by National Council on 23 June 2012) are:

Phase	Convenor	E-mail
Foundation phase	Shanba Govender	shanbasg@discoverymail.co.za
Intermediate phase	Isaiha Shabungu	irs1@vodamail.co.za
Senior phase	Wandile Hlaleleni	lelomso@gmail.com
FET Mathematics	Stephen Muthige	stevenmuthige@yahoo.com

Training of AMESA members on the analysis of Grade 12 Mathematics examination papers Report by VG Govender

Introduction

Since 2009, AMESA regions have been very active in analysing Mathematics and Mathematical Literacy matric examination papers. These analyses have been well received by the Department of Basic Education.

In order to make the process more inclusive, it was decided to have two workshops during Congress 2012 in June on this important AMESA activity. The workshops were conducted by VG Govender and he was assisted by R. Govender.

Thirty one teachers attended the first workshop (on Paper 1) and 20 teachers attended the second workshop (Paper 2). It was pleasing to note that members from nearly all provinces participated, as shown in the following table:

Province	Number of participants	
	Workshop 1	Workshop 2
Eastern Cape	4	1
Free State	4	4
Gauteng	5	2
KwaZulu-Natal	4	3
Limpopo	5	3
Mpumalanga	3	3
Northern Cape	3	1
North West	3	2
Western Cape	-	1



Prior to the analysis

Each workshop started with a discussion on what should happen prior to the actual analysis. This discussion consisted of the following key points:

- Check the paper at first glance for initial comment
- Ask a cross section of Grade 12 mathematics learners for their views
- Ask other mathematics teachers from your own school
- Ask mathematics teachers from neighbouring schools
- Make initial comments on the paper
- Substantiate comments by working on a possible memorandum

After this discussion, teachers were placed into groups for the analysis of the question papers. There were six groups for Paper 1 and four groups for Paper 2.

The cognitive levels

The cognitive levels for mathematics were discussed. Participants had to make sense of these cognitive levels. The following points emerged from this discussion:

Knowledge (Level 1)

- Recall
- Straight forward
- Seen before
- No struggle in working out the answer

Routine procedures (Level 2)

- Extension of what is known already
- Calculations without a lot of reasoning
- Recalling the procedure that needs to be followed

Complex procedures (Level 3)

- Route to the answer not evident from the information given
- Combination of different rules and procedures
- Knowledge and routine procedures implicit in problem

Problem solving (Level 4)

- Involves a high level of thinking
- Involves exact reasoning and application with thorough understanding of concept in question
- Order of procedures - hidden; not immediately able to put down on paper and solve

This input from the participants was compared to the description in the Subject Assessment Guidelines for Mathematics and there appeared to be a great deal of synergy between the two.

The analyses of papers in the workshop(s)

The papers are analysed according to the criteria below:

- Technical aspects
- Language used
- Coverage of syllabus
- Standard of paper
- Compliance with cognitive levels
- Comparison with previous year's paper
- Overall verdict

There was also a question-by-question analysis for each paper. The workshops were very interactive and all participants contributed positively to the discussions.

Conclusion

The facilitators reminded participants of their important role during the review of the Grade 12 Mathematics and Mathematical Literacy papers in November/December 2012.

Participants were given guidance on procedures to be followed during the review sessions. This is summarised below:



- There should be at least five teachers from different schools who should be involved in the review at a particular site. There is no limit on the maximum number of teachers.
- Each site should decide on a strategy, work through all questions as a site or combine questions and ask groups to work on sets of questions.
- All input should be recorded on the specific AMESA template.
- Each site leader should check the final input and then send it to the regional Chairperson.
- The AMESA regional Chairperson will consolidate all the input and submit it to the Curriculum Committee coordinators (vicepresident@amesa.org.za or fax 086 241 2733).

We look forward to a great inclusive AMESA review of both Mathematics and Mathematical Literacy papers in future.



Participants at the two workshops



Advisory Committee on Mathematics

Report by VG Govender

AMESA is represented on the SAMF Advisory Committee on Mathematics (ACM). A meeting was held on 30 July 2012 at the SAMF office in Pretoria and was attended by the following members of the ACM:

VG Govender (AMESA representative)
 Rajen Govender (AMESA representative)
 Kerstin Jordaan (SAMS representative)
 Werner Olivier (SAMS representative)

A range of issues were discussed across the following broad categories:

- Nature of the NCS curriculum and the challenges linked to the recent CAPS amendments.
- Alignment of pre-service teacher training in response to the new curriculum.
- Wide ranging support needs of in-service mathematics teachers.
- Content gaps of learners at FET level.
- Learner progression policies including pass requirement at Grade 9 and 10 levels.
- Dissemination of critical information about the new CAPS curriculum amongst stakeholders at Higher Education level.
- Academic support mechanisms for students at Higher Education level.

A scribe took notes during the meeting and it was agreed that a concept report would be circulated amongst members for consolidation before reporting back to the SAMF.

To assist in this regard further information was provided to the team. This information, compiled by Nico Govender, came directly from AMESA members during the curriculum phase committee meetings at the 2012 Congress in Potchefstroom. The trends emerging from the AMESA curriculum phase reports were the following

- On-going support and monitoring should be a feature of teaching and learning; subject advisors are key persons in this process and should have the requisite skills to do so; they need to be empowered to do their jobs.
- Need for teacher qualifications to be upgraded.
- Training for the new curriculum should be more intensive; a two or three day session is orientation, not training.
- Not all teachers in the Foundation Phase and Intermediate Phase were trained.
- In the Foundation Phase it was found that there was no correlation to what was being taught to what was stipulated to be assessed; there were mistakes in the textbooks.
- There is a general fear that the expectations in CAPS are too high and time frames have not been considered. Lessons are planned such that the learner seems to have all this prior knowledge and moves at a very quick pace; this is not what is happening at schools
- Foundation Phase and Grade 10 teachers have complained about the rigidity of the pace-setters; they find they are behind with their week; Grade 10 teachers were up to 5 weeks behind.
- For CAPS training still to take place in 2013, for the Senior Phase and Grade 12, the DBE should consult with other stakeholders (e.g. AMESA) to plan and implement the training. The DBE should re-look the cascade model as it leads to misinterpretations of the curriculum.
- Content training for Geometry and Data Handling should be made a priority for the FET; promises were made in 2008 that this training would occur, but this has not happened. Geometry and Data Handling are now "compulsory" and should receive priority when teachers are trained.
- Senior Phase teachers decried the emphasis on Grade 12 learners; they believed that more should be done for Senior Phase teachers in terms of support and monitoring.
- The pass mark for mathematics needs to be re-examined; it is 40% in Grade 9 and 30% in Grade 10; this is not improving learners. In fact, learners go into subsequent grades with 'huge gaps' in their knowledge.
- Assessment is a challenge amongst teachers. With a plethora of common examinations and tests, especially at FET level, teachers are not given the opportunity to set their own assessment tasks. As a result their creativity in assessment is being eroded. Most teachers reproduce exemplar or previous question papers for their tests. Teachers need training on all forms of assessments (examinations, tests, projects, investigations, assignment).

It is clear that AMESA has a very important role to play in the implementation of the Mathematics curriculum across all phases in South Africa. We await the next step in this important process.



Interview with a first-time participant in an AMESA Congress



Interview with Tshabagae Mosiwao
Senzile Secondary School
Hertzogville, Free State

Tshabagae is in his third year of teaching.

What made you decide to come to Congress 2012?

I have been reading some articles that AMESA has published and became curious to see what the Congress is all about. Other than that I am a curious person and wanted to develop myself around the world of mathematics. My aim is to equip myself to help my learners. I wanted to learn some way of overcoming the problems that I am encountering from my learners and I had the belief that I would meet people who could assist me at the AMESA congress.

What for you has been some of the highlights of Congress 2012?

I have learnt that mathematics teachers must understand algebraic concepts; the whole idea is to know what you are going to teach and do your utmost not to misinterpret the concepts as this might lead to confusion at a later stage. The congress has been unique in a sense that some of the mistakes that teachers have been making are being addressed, although it has not only been about teaching methods but more on rectifying errors that contribute to the high failure rate in the country. I also learnt that South Africa is not alone with problems in mathematics with other countries having similar problems. At the end of the day, it is not about blaming teachers but examining ways of helping learners. I think it is also important to explain and show mathematics in real life contexts. Learners should not be taught mathematics in isolation but should see mathematics when rondavels and dams are being built. They should use their knowledge of financial mathematics to advise family members on investments.

What message would you take back to your colleagues at school?

My colleagues need to know that the poor learners do not know the importance of mathematics and they should be sympathetic to their plight and help them. Learners must be made to realise that mathematics is not a beast but just a subject like the others. One of the key issues why learners fail mathematics is that concepts are taught in an abstract manner and the language barrier does not help matters. Teaching needs love and passion and no matter how challenging the subject is, we must always do our best to motivate our learners to adopt a positive mind-set to mathematics. If you want to be happy as a teacher, then you must ensure that you are able to instil positive attitudes in learners by teaching them in a developmental and caring way.

Any other comments

I would like to call on all young mathematics teachers to join AMESA as early as now and they will definitely not regret it. The association is about showing professionals how important mathematics is to the communities and schools. Remember that a teacher is the never-ending learner and therefore development must be considered if ever you want to excel in your job.

Above all, I would like to thank AMESA for giving us the opportunity of going beyond the boundaries. The Department of Basic and Higher Education must be wholly engaged and see how best we can make mathematics a success in South Africa.

VIVA AMESA!



Our IMO Team which made us proud

Medals for South African IMO Team

The South African Mathematics Foundation (SAMF) is proud to announce the results of the International Mathematical Olympiad (IMO) that took place in Argentina on 10 and 11 July 2012. The South African team was awarded two bronze medals and three honourable mentions at the closing ceremony.

Since 1992 South Africa has been an active participating country in the largest and oldest Olympiad for the exact sciences. The South African team is made up by Dylan Nelson (Benoni High School, Gauteng), Robert Spencer (Westerford High School, Western Cape), Dalian Sunder (Star College, Kwa-Zulu Natal), Robin Visser (St George's Grammar School, Western Cape), Junho Son (St Alban's College, Gauteng) and Mickey Chew (Elkanah House, Western Cape). They have been training under the watchful eyes of team leaders Maciek Stankiewicz and Dr Jay van Zyl.

"This year a total of 548 contestants took part in the IMO and the jury voted to present extra medals which resulted in 277 contestants being awarded a medal for obtaining a score of 14 or above," explains Maciek Stankiewicz. "An honourable mention is given to contestants who solved one problem perfectly."

Bronze medals were awarded to Dalian Sunder, who scored a total of 18 and Dylan Nelson with a score of 15. Mickey Chew, Junho Son and Robert Spencer all received honourable mentions. With these results team South Africa is ranked 56th in the world. The top countries, in this order, are South Korea, China, the United States of America and Russia. Coming out on top with a perfect score of 42 was Jeck Lim from Singapore.

The SAMF, as the organizers of the South African team, and the Harmony Gold Mining Company, who sponsored the team's travel expenses, would like congratulate the team on their performance.



EXPLAINING EXPLAINING

Tim Rowland

University of Cambridge, UK

Explanation is central to the task of teaching. Gaea Leinhardt has written that “Instructional explanations are recognizable as being a part of the instructional landscape by teachers, students and observers”. It is hard to conceive of mathematics teaching without it. One can surmise whether an explanation has been successful in achieving its intended purpose, or otherwise, by the response of the intended audience – usually a student, or students. Yet analysing our own explanations, or teaching someone else ‘how to explain’, is problematic. This paper examines some examples of explanation, from my own teaching and drawing on videotapes of other teachers, with the aim of identifying some key components of what might be called a mathematics explanation repertoire.

ORIENTATION

In most countries, primary school children are taught, at some point, a written algorithm for subtraction. The actual algorithm taught changes over time, and from place to place, but basically it will be one of two types, with minor variations in the way it is recorded.

Figure 1: Two written subtraction algorithms

In most of the UK, the first of these algorithms has come into fashion since the 1960s, and been predominant since the 1980s. It is called the *decomposition* algorithm, because the minuend (85 here) is decomposed into 70+15, so that both 30 and 7 can be subtracted to leave 40+8. The text books usually demonstrate this with Dienes multibase apparatus, beginning with 8 longs (‘tens’) and 5 units. One of the longs is then traded for – in effect, decomposed into – 10 units.

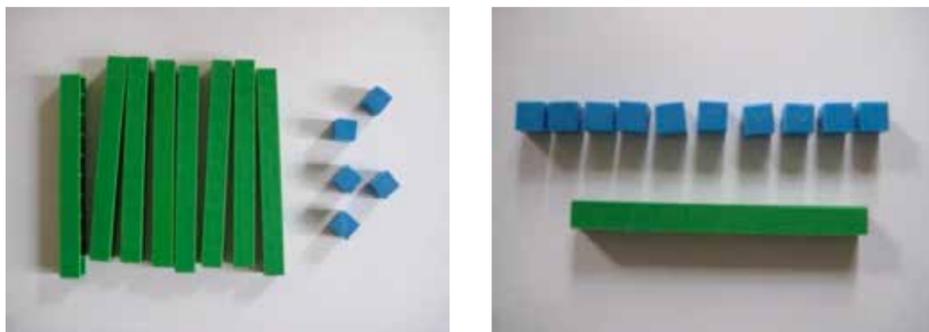


Figure 2: 8 tens and 5 units: a ‘long’ is decomposed into 10 units

Since the 1970s, this subtraction algorithm has gradually displaced the second one (shown on the right in Figure 1) and for some time has been taught in most primary schools in the



UK. For this reason, teacher educators include it in their methods courses at university departments of education, so that prospective teachers understand *why* it works. But what I most enjoy is when someone in the class – usually, but not always, an older student, or one from overseas – says that they do it differently. I invite them to the board, and they demonstrate the second method, invariably adding that they don’t know why it works. In effect, they ask me to *explain* it.

Now, I know why the second method works, in the sense that I am satisfied that it will always give the right answer. The subtraction 85-37 has been replaced by (80+15)-(40+7). That is, the minuend (85) and the subtrahend (37) have both been increased by 10, so the difference remains the same. In fact, this algorithm is named the *equal-addition* algorithm. Formally, (85+10)-(37+10) = 85+10-37-10 = (85-37)+(10-10) = 85-37. But my explanation looks a bit different. I ask two students to come to the front of the class.

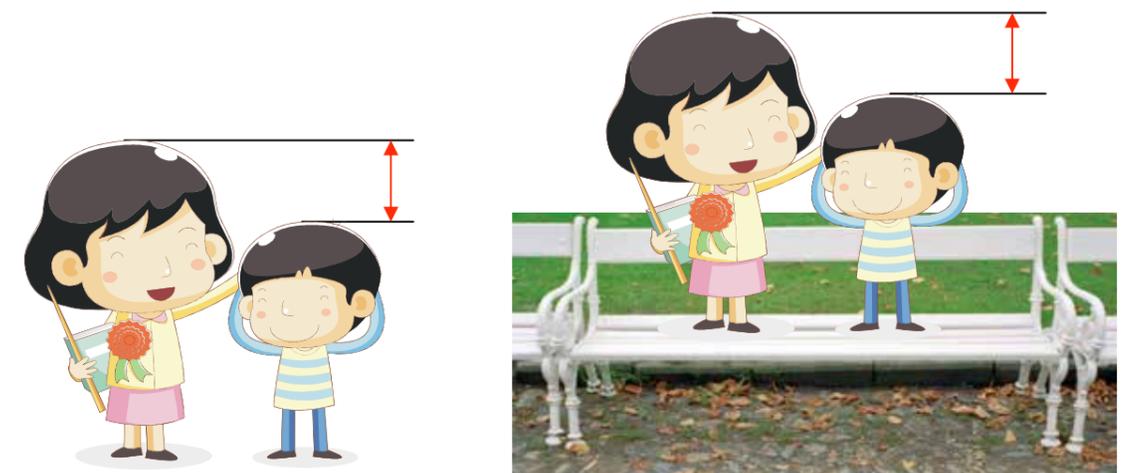


Figure 3: Alison and Bob – difference unchanged by equal addition

I’ll call them Alison and Bob. I ask them to stand next to each other (Figure 3, left) and for the students to estimate the difference in their heights. I then place two identical chairs next to each other and ask Alison and Bob to step up onto them (in Figure 3, right, they are standing on the same bench). I ask the class to estimate the difference in their heights now. The ‘explanation’ draws on this analogy: the difference in the two heights is unchanged when they are both ‘taller’ by the same amount – the height of the bench. In the same way, the difference between two numbers remains the same when the same number is added to both.

More recently, I have drawn on an idea of Ian Sugarman’s (2007), which begins with my asking (usually written on the board): “Write down a subtraction that is different from 643-267, but which has the same answer”. The idea is that they propose things like 743-367, 646-270 and so on, and then once again the equal-addition, equal-difference connection is made and related back to the subtraction algorithm. To deviate for a moment, two common responses to this activity are of interest. First, there are always some students who actually perform the subtraction to get the difference 376, and then proceed to invent subtractions like 380-4 with the desired difference. This is an interesting case, I think, of what Kevin Collis (1975) referred to as difficulty with “lack of closure”; that is a preference for finding ‘the



observe mathematics teaching, we are bound to witness some explanation? In her classic treatise on explanation, Gaea Leinhardt (2001, p. 338) expressed it like this: “Instructional explanations are recognizable as being a part of the instructional landscape by teachers, students and observers”. Part of the instructional landscape ... so how could it be that none of us made claim to ‘explanation’ as a code when we analysed the 24 mathematics lessons? In fact, my puzzlement in the face of this conundrum began the train of thinking that led to this paper.

Ten years later, I have an answer to this conundrum, or perhaps a partial answer. I will reveal it later. The clue lies in examples of explanation above, and others, and the codes that we grouped (Table 1) under the Knowledge Quartet dimension ‘Transformation’.

REFLECTING ON EXPLANATION

You might have sensed, in my account of my explanations of the subtraction algorithms, is that there is no reluctance to explain, and no hardship in the act of explanation. What teacher can resist an opportunity to explain something to an attentive student or class? Candidates for pre-service teacher education sometimes talk at interview about wanting to “pass on” their knowledge to students. To expound something that we know to someone who does not know it, in a way that gives it meaning, to assist another person to understand it, somehow meets a very basic, fundamental need in the pedagogue. The sense of reward at the response “Oh, I see!”, and the light of insight in the eyes of the student, more than rewards our efforts, and the sense of satisfaction never dims.

When I started out as a mathematics teacher, although I never expressed it like this, I believed that the task ahead of me was to find the perfect explanation of every topic in mathematics, or at least of the topics that I would teach. Once I had found these perfect explanations, I would be properly equipped to do my job, my students *would* learn, and they *would* understand: I would have succeeded in sharing what I knew, and also the pleasure of knowing. Of course, it didn’t turn out like that. Over time I fine-honed my explanations, in the style of a one-man lesson study group, but the process turned out to be one of infinite regress. More often than not, there were students for whom my supposedly-perfect explanation misfired. Worse, an explanation that was a total success one year would go down like a lead balloon the next. My self-explanation for this, now, is that sometimes, if not most of the time, we cannot ‘pass on’ what we know and understand for ourselves ready-made and gift-wrapped. Surprise, surprise, students have to construct knowledge for themselves, and their constructions sometimes look different from our own; or even if they look the same, they only ‘work’ for the student because the edifice was their work, not the teacher’s. In a nutshell, as von Glasersfeld (1983, p. 66) says, “it appears that knowledge is not a transferable commodity and communication not a conveyance”. In fact this is very clearly exemplified by my experience (described earlier) of implementing Ian Sugarman’s “Write down a different subtraction with the same answer” approach to equal-addition. It seems to me that Sugarman’s approach is constructivist in its intent – the teacher does not ‘tell’ the students why equal-addition works, not even by analogy. S/he offers the student a



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task, and asks them to make sense of it. As I remarked, this approach misfires when the student desires closure, so that their meaning-making, their cognitive construction, is not at all 'wrong', but is nevertheless incompatible with the one intended.

What is explanation?

In one of the most-cited articles in the whole of mathematics education, Richard Skemp (1976) points out that the word 'understanding' is used with at least two different meanings: whereas one entails "knowing both what to do and why", the other mere familiarity with "rules without reasons". My pejorative 'merely' serves to emphasise that the first meaning, which Skemp calls 'relational' understanding, is the one that educationists would usually associate with it. Yet the second, 'inferior' kind of understanding underlies the pupil's response "Oh, I see" to the clear exposition of a rule, an algorithm perhaps, for finding the answer to a standard type of problem.

I believe that the same potential confusion surrounds the verb "to explain", which has multiple meanings. In a book written for teachers, Wragg & Brown (1993) acknowledge this diversity of meanings in the classroom, but offer as an operational definition:

Explaining is giving understanding to another (p. 3).

The harnessing of understanding to the definition is interesting, given Skemp's dichotomy, but Wragg and Brown proceed to indicate a variety of things that might be understood, such as concepts, procedures, purposes, cause and effect, processes and relationships. Skemp's distinction between relational and instrumental understanding is especially relevant to consideration of what it might mean to explain a *procedure*. In the case of fraction division, for example, "invert and multiply" is an explanation of sorts, but one in the category of 'rules without reason'. The distinction here is between explaining *how* and explaining *why*. In the case of a procedure (such as a subtraction algorithm), both kinds of explanation are necessary. Rather, the how explanation is essential to effective application of the procedure, and the *why* explanation supports meaningful learning and flexible application.

Chloe: subtracting near-multiples of 10

I mention here one of the lessons that we observed in the research study mentioned earlier. Chloe was teaching a class of 5 to 7-year-olds a particular strategy for subtracting near-multiples of 10, such as 9, 11, 19 and 20. The strategy was to subtract 10 or 20, and then adjust (one more or one less). The teachers' handbook presented this symbolically, for instance "70-11=59 because it is the same as 70-10-1". However, just as the lesson was about to begin, Chloe noticed a large 1– 100 square lying on a table in the classroom. She mounted it on a metallic vertical board at the front of the class, and proceeded to demonstrate the compensation strategies *spatially*, by reference to horizontal and vertical 'moves' on the grid. In particular, she demonstrated 70-19 by placing a magnetic counter on 70, at the right end of one of the rows, moving up two squares to 50, and then 'adding 1', necessitating a move to the beginning (51) of the next row. It would be fair to suggest, I think, that Chloe was aiming to explain *how* to perform this type of strategy, and also *why* it



works, by reference to the moves on the 100 square. In the event, the children found it difficult to choose the appropriate variant from the four possibilities (up/down, then left/right).

Instructional explanations and other explanations

I find the writing of Gaea Leinhardt on this topic, within and beyond the realm of mathematics, especially illuminating. In order to understand explanation in the context of teaching, Leinhardt says that we need first to distinguish it from other kinds of explanation. Leinhardt (2001) identifies four types, or families, of explanation: common explanations, disciplinary explanations, self-explanations, and instructional explanations. Drawing on Leinhardt, my account of these four types is as follows.

Common explanations are answers to why-questions, usually about everyday matters, such as: Why are you wearing a suit today? Why must I go to school? Common explanations are usually given orally, and build on a particular relationship. The success of the answer depends on the asker's evaluation, not on a set of externally-imposed 'rules'. By contrast, *disciplinary explanations* arise from enquiries embedded in a discipline, such as: Why does it get dark at night? What were the causes of the American Civil War? Why are the diagonals of a rectangle the same length? The purpose of these explanations is to establish normative truths in established domains subject to their rules of evidence and argument. The one giving the disciplinary explanation needs a good command of the relevant *syntactic* knowledge (Schwab, 1978). The third type identified by Leinhardt (2001), a *self-explanation*, is, as the name suggests, given to the self, often in response to inconsistency, puzzlement or seeming contradiction. The purpose of this explanation is to make sense of experience, and to give meaning to claims from outside. Self-explanations may be fragmentary, idiosyncratic, contextual; the criteria for success are self-determined.

In contrast to common, disciplinary and self- explanations, *instructional explanations are designed to teach*. They are pedagogical actions in response to questions (sometimes questions posed by the teacher him/herself). Leinhardt (2001, p. 340) states that they are intended to help students learn, understand, and use knowledge "in flexible and creative ways", and that "Such an explanation is complete when coherence exists among the critical components". Instructional explanations tend to be more extended, less formal, and with more redundant content, than disciplinary explanations, and to conform to general discourse conventions, with awareness of an 'audience'.

In the earlier discussion of subtraction algorithms, I gave one-line justifications for the two procedures, and these were disciplinary explanations, whereby the mathematical community can be confident in the efficacy of each procedure. Since (at some time, long-forgotten) I concocted these disciplinary explanations, they also served for me as self-explanations. The explanations with Dienes apparatus, and with students standing on chairs, are intended as instructional: they draw respectively upon a concrete *representation* and a carefully devised *analogy*, in order to render the explanations more accessible and more palatable. Both of these explanations are prepared in advance, are part of an instructional repertoire, and have been tried and tested many times. In the next section, I describe a more spontaneous explanation-response to an unanticipated, contingent question-situation.



as Jennifer was concerned. Whether it would be helpful to her Year 5 class was another question, but I believe that sometimes explaining for others is preceded by explaining for ourselves, and that the two can be different. In fact, mensuration of cones and spheres only enters the curriculum very much later, around Year 10.

There remained one final demand for *self*-explanation on my part, resolved during a sleepless night soon afterwards. The sum $(1^2+2^2+ \dots+10^2)$ can be found using the formula $\frac{1}{6}n(n+1)(2n+1)$ for the sum of the first n squares, and the process previously applied to 4, 10 slices readily applied and extended to any number of slices – 100, 1000 ... one would expect that even with 100, the process would yield an approximation to the volume of the cone extremely close to a third that of the corresponding cylinder.

In, fact, taking n slices, the quotient [volume of slices / volume of corresponding cylinder] is $\frac{\frac{1}{6}n(n+1)(2n+1)}{n^3} = \frac{1}{6}(1+\frac{1}{n})(2+\frac{1}{n})$, which can be seen to approach $\frac{1}{3}$ as n increases². This self-explanation was important for me, but not for Year 5. I don't know about Jennifer. Only later did I recall that my 'slices' approach to Jennifer's problem recapitulated early Greek hints at what was to become integral calculus, and the limiting approach with n slices is more or less a Riemann integral.

KATE: EXPLAINING BY MEANS OF A GENERIC EXAMPLE

Earlier, I discussed the close relationship between explanation and understanding. In mathematics, it can also be difficult to distinguish between explanation and proof. The theorem, after all, states what *is* the case: the proof demonstrates why it must be true. De Villiers (1990) and others have drawn attention to the multiple purposes of proof, including verification and explanation. Verification relates to the function of proof which lends certainty to the truth of a mathematical claim. (This begs the meaning of 'truth', but this is not the place to discuss it.) The explanation role, on the other hand, is about understanding why something must be true, and giving insight of some kind. These two roles are easily matched, respectively, to Leinhart's categories of disciplinary explanation and instructional explanation. The first kind of proof is subservient to the edifice of mathematics, perhaps to the editor and reviewers of the mathematics research journal. The second kind recognises an obligation to the learner. Anyone who has studied mathematics at university will instantly recognise the distinction. One fruitful approach to the second kind of proof is by means of the generic example (Balacheff, 1988). Essentially, it is about 'seeing' *why* something is true in general, but through the medium of a *particular* example of it. Explanatory insight, or illumination, can sometimes be achieved by 'talking through' a *single* example, drawing attention to the *structure* of that example. If such a demonstration succeeds, the audience – colleagues, students, whoever – see why the same, or analogous, reasoning would work just as well in other cases. In a video made for the National Numeracy Strategy in England in the late 1990s, a group of 10 to 11-year-old children investigate the well-known 'Jailer Problem'³ in small groups, before being brought together by Kate, their teacher. The solution turns out to hinge



on the fact that every square number has an odd number of factors. In fact, Kate explains this to the class by reference to a generic example, although no reference is made in the commentary to this aspect of her teaching and proof strategy. She points out that every factor of 36 has a distinct co-factor (i.e. $1 \leftrightarrow 36$, $2 \leftrightarrow 18$ and so on) – with the exception of 6, which remains alone, un-paired. So it must follow that 36 has an odd number of factors. Kate's choice of 36 is interesting – small enough to be contemplated with mental arithmetic but with sufficient factors to be non-trivial. Presented in algebraic generality (as a disciplinary explanation) the same argument would not be accessible to Kate's audience. Instead, her instructional explanation is, indeed, a proof that explains. For more about generic examples, see Rowland (2001); NRICH (2012).

THE MISSING CODE: DENOUEMENT AND CONCLUSION

As I wrote earlier, my colleagues and I were surprised, on reflection, that our list of codes relating to teachers mathematics-related classroom actions did not include a code for explanation. Now, I have laid the ground, or as much ground as space allows, for my own account – my explanation, in fact – of how this could have come about. In this paper, I have described some examples of explanation – my own, Chloe's, and Kate's. Looking back at these explanations, one can extract from them four characteristic ingredients or components, pedagogical devices that identify their instructional purposes and delineate what might be called an 'explanation repertoire'.

The synthesis is as follows: *instructional explanations in mathematics are characterised by one or more of:*

The use of representations: for example, my use of Dienes' multibase apparatus and Chloe's use of the 100 square, as different embodiments of the place value number system.

The use of examples: this is more-or-less universal in instructional explanations (e.g. Bills & Watson, 2008) and evident in all the example-explanations given earlier.

Reference to analogies: for example, in my explanation of the equal-addition algorithm.

Inductive, plausible reasoning: for example, in my 'volume of a cone' explanation which stood in for an otherwise-inaccessible proof by calculus.

Proof via generic examples: for example, Kate's justification of the jailer problem solution. In this case, the explanation moves beyond induction to demonstrating how a general proof would be structured.

It is also essential to my argument that (a) I could cite numerous additional examples from classroom practice, of each type; (b) I am confident that readers will recognise these same components in their own instructional-explanatory practices. At the same time, the five-item list is provisional, and open to enlargement and revision.



It should come as no surprise that the first three of these explanation strategies can be matched directly to Shulman's (1987) description of teachers' *pedagogical content knowledge* resources. Moreover, Leinhardt, Zaslavsky & Stein (1990) observed that:

Explanations consist of the orchestrations of demonstrations, analogical representations, and examples. [...]. A primary feature of explanations is the use of well-constructed examples, examples that make the point but limit the generalization, examples that are balanced by non- or counter-cases. (p. 6).

I would add that knowledge of the relevant *subject matter* is a *sine qua non*; in order to explain, for example, a procedure or to justify it, first one must know the procedure, or the justification, for oneself. This claim is inherent in what Shulman (1987) called 'comprehension', being the first stage of a six-point cycle of pedagogical reasoning.

In contrast with instructional explanations, given the rules of evidence for truth in the field of mathematics, *disciplinary* explanations are necessarily characterised by deductive reasoning. That is not to say that the above five characteristics are entirely absent in written discourse in mathematical communities, only that they are an embellishment in this context, and not a necessity. This can have well-recognised consequences for students as they make the transition from school-mathematics to lectures in university mathematics departments (e.g. Tall, 2008).

To conclude: once the five ingredients of mathematical explanation are disentangled from the narratives in which they are necessarily set, and brought into the light, the reason for the absence of 'explanation' as a code in our lesson analysis becomes apparent in the presence of the components of the Transformation dimension of the Knowledge Quartet, notably the choice and use of representations and of examples. It is also fair to remark that explaining *how* was commonplace in our 24-lesson database (corresponding to the 'teacher demonstration' code). However, explaining *why* was almost absent, although we have subsequently found it more often in our observations in secondary mathematics classes. In any case, our inadvertent deconstruction of explanation has almost certainly turned out to be more helpful and more useful to teachers planning and evaluating their own classroom explanations. The focus on representations and examples in particular has lent some transparency to this very familiar, but otherwise inscrutable, "part of the instructional landscape".

Acknowledgement and thanks

This paper is the text of a plenary lecture given at the 18th Annual National Congress of the Association for Mathematics Education of South Africa (AMESA), held at the Faculty of Education Sciences, North-West University, Potchefstroom: 25 June to 28 June 2012. I am very grateful to the President of AMESA and the Congress 2012 Local Organising Committee for their kind and generous invitation.

Notes

1. In 2002 there were 18 in fact: two more were subsequently added in the light of new data.



2. This particular argument, for a cone with equal base-radius and height, is easily adapted to the general case, but the notation is then more 'messy' and outwardly complex – something I was deliberately avoiding when responding to Jennifer. I also note that the formula for the sum of consecutive squares is amenable to visual proof (Nelson, 1993): by generic example, in fact.

3. A certain prison has 100 prisoners in 100 cells, and 100 jailers. One night, when the prisoners are all locked away, the first jailer unlocks all the cells. Then the second jailer locks all the cells whose numbers are multiples of 2. Next, the third jailer changes the state of all the cells that are multiples of 3, and so on through to the 100th jailer. The jailers then fall asleep. Which prisoners were able to escape?

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THE PRIMACY OF TEACHING PROCEDURES IN SCHOOL MATHEMATICS

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Currently, it is unpopular to propose that procedural fluency should be accorded high priority in the teaching of school mathematics. In this paper a case is made for teaching for procedural fluency. It is argued that procedural knowledge is the major component of the legitimate school mathematics knowledge as constituted by high-stakes school mathematics examinations such as the National Senior Certificate Mathematics examination. Furthermore it is argued that the teaching of procedures is not only about the step-by-step actions to accomplish whatever results are to be accomplished. Rather, examination of mathematics teachers' activity shows that there is evidence of seeds for exploitation to enhance the mathematicalness of learners by focusing on the teaching of procedures.

INTRODUCTION

"Mathematics as an educational task", the title of Fredunethal's seminal work, conjures up many images and begs many questions. For example, what is at play? An education in Mathematics? The role of Mathematics in school education? Both? It is without doubt that Freudenthal through the Realistic Mathematics Education (RME) programme had a major influence on both the direction of the development of school mathematics curricula and research in Mathematics Education.

RME a comprehensive mathematics research and development programme, with its core "reality is the source and domain of applications of Mathematics." It spread from Netherlands to various parts of the world. In South Africa there was the Realistic Mathematics Education in South Africa (REMESA) project (Julie, et al, 1999), the USA had the Mathematics in Context (MiC) project (Romberg and Shafer, 2002) and in Indonesia there is the still operative Pendidikan Matematika Realistik Indonesia (PMRI) project (Sembring, Hadi & Dolk, 2008).

In a sense RME was initially a dissident development against the prevailing direction of school mathematics developments in particularly the then Western countries. These nations were, to put it mildly, shocked when the Soviet Union launched Sputnik in 1957 and they sought renewal and modernisation of their school mathematics enterprise. A seminar was held at Royaumont in 1959 (Organisation for European Economic Cooperation (OEEC), 1961) to discuss the direction of this renewal and modernisation. Vigorous debate and interchange characterised this seminar with Freudenthal and Dieudonné apparently at each other's throats. Dieudonné was advocating a school mathematics curriculum founded on the



formal structures of Mathematics. His quest is characterised by the famous statement “Euclid must go!” Freudenthal was agitating for a different starting point: school mathematics embedded in the experiential world of the child. During the interchange it is reported that Freudenthal shouted at Dieudonné by stating “Don’t shout at me, for I can shout louder than you—and in more languages.” (Howson, 2004). However, the outcome of the seminar was that school mathematics curricula should be firmly based in the formal structures of Mathematics. This culminated in the “New Maths” school curricula characterised by set theory as the underpinning for school mathematics. Emblematic of the content of school mathematics textbooks during the era was as illustrated in Figure 1 below.

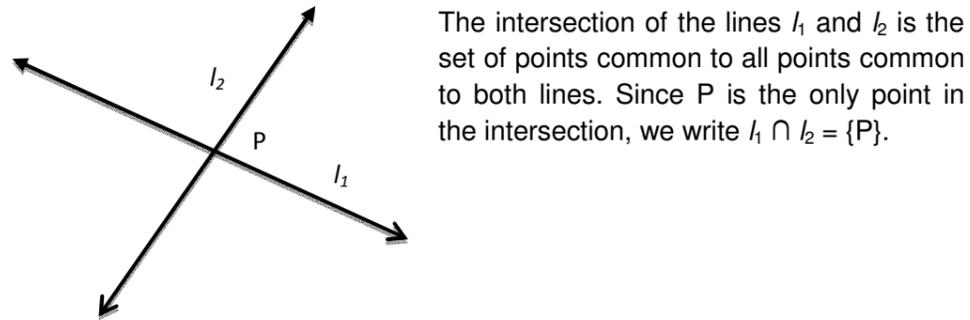


Figure 1: Excerpt from a "New Maths" textbook (School Mathematics Study Group, 1968, p 11)

South Africa also adopted the “New Maths” programme and an exemplar of the textbook entries of the day was as presented in Figure 2.

Example.—Draw the diagram of the relation is double on the set {2, 3, 4, 6, 8, 9} (Figure 6). This example suggests that one or more elements may not appear in the relation. In the above case, the element 9 is not used.

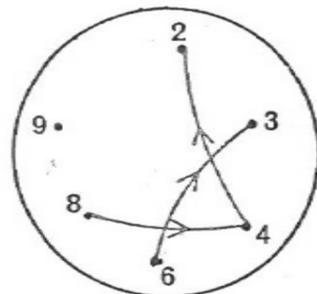


FIG. 6

Figure 2: Excerpt of "New Math" South African textbook example (Malan and Smith, 1967, p 16)

Freudenthal, however, continued with his project and this sense he was swimming against the stream of formalistic structural mathematics as the starting point for school mathematics.

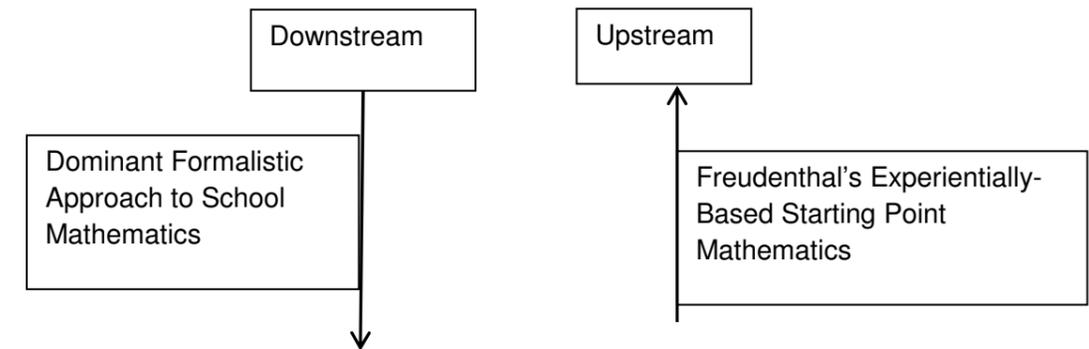


Figure 3: "New Maths" and Freudenthal's 'oppositional' approach

Freudenthal's curriculum project for school mathematics was not the only one. There were others such as Papert's Logo-programming with its defining “Teaching children to be mathematicians vs teaching them mathematics” (Papert, 1972) and Davis's Madison project (Davis, 1984) with its “assimilation paradigm”. Variants of these school curriculum mathematics orientations eventually replaced the “New Maths” and today are the dominant paradigmatic orientations for school mathematics curricula more or less across the world. These orientations favour the development of the processes of doing mathematics with the formal aspects of mathematics coming into play only later. This is diagrammatically depicted in Figure 4.

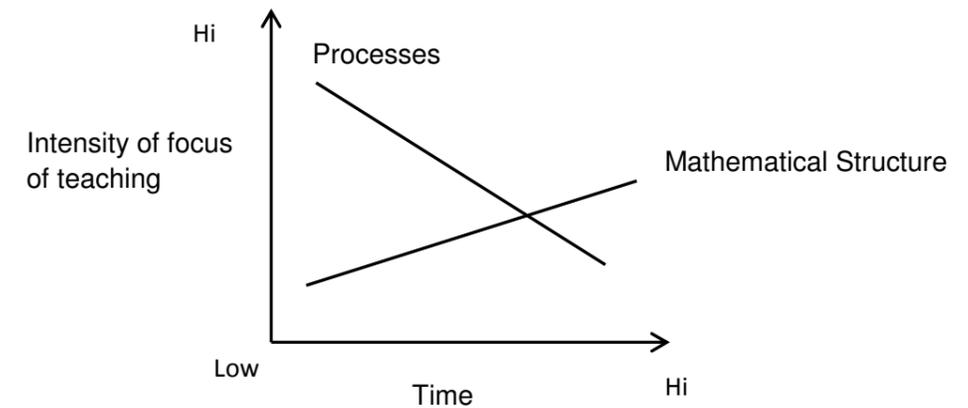


Figure 4: Current dominant focus of school mathematics approaches

The issue pursued in this paper is “Is it time, in South Africa with her highly unsatisfactory performance in school mathematics, to swim against this stream and accord procedural fluency a higher priority?” This question is considered by firstly discussing what legitimate school mathematics knowledge is followed by an exposition of the content of this knowledge. Thirdly a model for the focus for teaching for the proposed legitimate school mathematical knowledge is presented. The penultimate section provides examples on how teachers are near to realizing this model in their activity of teaching procedures and it is concluded that

this might lead to ways of mathematics classroom pedagogies to arrest the unsatisfactory performance in high-stakes school mathematics examinations.

LEGITIMATE SCHOOL MATHEMATICS KNOWLEDGE

The choices of the content to be included in school mathematics and what to make its focus for teaching are not always clear. Mathematical content other than the current selected ones with different foci for teaching could as well have been chosen. School mathematics is a particular kind of mathematics. (Julie, 2002). It is a hybrid of pure mathematics, mathematical modelling, the application of mathematics and probability and statistics, the traditional disciplines in higher education institutions. A further characteristic of school mathematics is that it is wholly situated on the established knowledge side of a knowledge continuum as illustrated in the Table 1. The traditional disciplines in higher education institutions are more towards the frontier side in as far as its research and research training concern.

Established	Frontier
Stable (current state of intransitivity). Vetted by the knowledge custodial community. State of intransitivity has a long existence. Generally publicly visible and recognisable Is teachable having gone through didactical transposition processes	Transitive (Unstable and fragile). Prone to refutation and falsification. Visibility and recognisability restricted to that sector of the custodial knowledge community who concerns themselves with the particular area of knowledge. Is not teachable but only co-explorable by the immediate community.
School Knowledge	

Table 1: A Knowledge Continuum

School mathematics is in essence the mathematics that is legitimized by tests and examinations. This notion of valid disciplinary knowledge in general is convincingly argued by Kvale (1993, pp. 219-220) who states that "...the examination itself contributes to establish the valid knowledge of a discipline...The examination committee defines what is valid and legitimate knowledge in a field." The contention in this paper is thus that legitimate school mathematics knowledge is the knowledge that is tested in high-stakes examinations such as the National Senior Certificate (NSC) Mathematics examination.

This notion of legitimate knowledge as the knowledge constituted by high-stakes examinations for obtaining a certificate of worth is common in many other practices. The

knowledge necessary to obtain a learner's driver's license is a less esoteric common example. Of all the knowledge around about driving a motor vehicle, what counts and focused on to obtain this initial certificate of worth for driving a motor vehicle legally, is the knowledge that will be tested as encapsulated in the K53 manual. A less esoteric example is the knowledge required to be admitted to various professions such as a certified financial risk analyst or a chartered accountant.

The above constitution of legitimate school mathematics has long been realized by schools serving learners from high socio-economic status (HSES) environments. These schools are characterized by high achievement and an aspect of their practice to attain this high achievement is portrayed as follows for the Australian situation.

Private establishments...exploit the socially discriminating potential of school subjects to the full. [They do this] by interpreting and formalizing its cognitive demands of [a curriculum] into a programme of group work in which concepts and operations are mastered pragmatically, that is, as a set of practices that represent ends in themselves, the "business" of the classroom and the exam.

... These classes can be worked to a high standard of competitive performance, following the bureaucratic procedures of syllabus-stripping (reduction to examinable topics), chalk and talk, worked examples, worksheets, past papers and exam rehearsals. To this participation in national and university mathematics contests add luster and encouragement, as does the annual release of exam results and the media parade of top students. (Teese, 2000, pp 186 - 188).

This pragmatic approach to reform in curricula is also evident in South Africa. With the introduction of Outcomes-Based Education, schools serving learners from HSES environments re-interpreted of the curriculum to not lose the focus on the high-stakes examinations. Harley and Wedekind (2004, p 202) captures this re-interpretation as follows:

But there is another imperative, no less powerful than in all other secondary schools: the Senior Certificate 'exit' qualification known as 'matric'. Accordingly, the OBE programme has a time allocation of only of only three periods per week, and the responsibility rests with the school librarian. For over 90 percent of the available time, learners are engaged in traditional subject lessons.

Clearly, the focus is on getting the learners to succeed in high-stakes examinations. This situation generally holds in high-performing schools serving learners from HSES environments as is evident from these schools finishing the syllabus during the second quarter of the grade 12-year and focusing attention on preparing students to succeed in the NSC examination by, in a structured way, working through past and imagined papers drawn up by teachers. Of course, it has to be borne in mind that there are academic, cultural and economic capital in HSES environments which account for learners to acquiring the necessary wherewithal to participate in the in the "national and university mathematics contests" mentioned Teese. The so-called progressive ways of doing mathematics is so near to what happens in their extra-school environments that the school environment need not to be organized to spend much time on these issues. Thus in designing teaching for success in schools in LSES environments the practice of examination-focused organization should not be looked upon as retrogressive since success in high-stakes examinations, as Muller (2000: 62) argues, provide learners from LSES backgrounds with a necessary "article of universally recognized cultural capital such as a school diploma [which] confers symbolic power on the holder."



By claiming that the legitimate knowledge is the knowledge that is prioritized and constituted by the examination, does not imply that the examination questions are the knowledge. It is the knowledge inherent in such questions that are at stake.

THE CONTENT IN HIGH-STAKES SCHOOL MATHEMATICS EXAMINATIONS

Consideration of the two lists, with some exemplar examination questions, in Table 2 reveals that different “objects” of mathematical knowledge are privileged in the two situations. With privileging viewed as question types having more than 50% of the questions of the kind given in the lists, in the two examinations, then different kinds of knowledge are accorded high priority in the two situations. Consequently, the intensity of the focus of teaching will differ for the two situations.

List A	List B
Take any three-digit number whose digits increase as you read from left to right. Make another number by reversing the order of the digits, and subtract the smaller from the larger. Reverse the order of the digits of the difference, and add it to the difference. What is the answer and why?	Find the difference between 732 and the number formed by reversing the digits.
Take any fraction. Make a new fraction with the new numerator the difference between the numerator and denominator and the new denominator the sum of numerator and the denominator. Do the same to your new fraction. What do you notice?	Simplify $\frac{54}{104}$
Is “OTTO” always divisible by 11?	Show that 5665 is divisible by 11.
<p>Comment on the method below for the factorisation of quadratic expressions.</p> <p>$2x^2 - 7x + 3$</p> <p>$x^2 - 7x + 6$...Divide 2 by 2, multiply 3 by 2</p> <p>$x^2 - x - 6x + 6$... Split middle term so that there are common factors for first and last term</p> <p>$x(x - 1) - 6(x - 1)$...common factor</p> <p>$(x - 1)(x - 6)$...common factor</p> <p>$(x - \frac{1}{2})(x - \frac{6}{2})$...Divide both 1 and 2 by 2</p> <p>$(x - \frac{1}{2})(x - 3)$...Simplify</p> <p>$(2x - 1)(x - 3)$... Multiply the first factor by 2. (The second is multiplied by 1 since the denominator of 3 is 1)</p>	Factorise $2x^2 - 7x + 3$

Table 2: Examination Question Types



rationales, who are majority of learners in non-specialised schools, and contribute to enhancing their achievement. It is also possible that learners with these two rationales might change their rationales to the upper two ones. This results from the common notion of “success breeds success”. Accompanying this common sense notion is a current strong view that the relationship achievement \rightarrow self-esteem is stronger than the one self-esteem \rightarrow achievement. This counters the notion that the development of the self-esteem of learners from low socio-economic status (LSES) environments must first be developed before they can achieve at much higher levels. Catering for the needs of learners holding the two rationales does not necessarily jeopardise chances of learners in the two upper bands. Learners in these bands are normally self-directed and goal-focused as can be gleaned from the success stories of in the media of learners from LSES environments who perform above normal expectation. In fact, the chances are high that such learners will achieve even higher given the development of learners’ mathematicalness through a strong focus on the procedural aspects and their accompanying concepts. Given these considerations the possibility is high that an increase in the number of learners offering Mathematics as a NSC examination might result and so counter the unhealthy practice of shepherding learners away from Mathematics to boost the overall pass rates of schools.

A last potential benefit is that the focus on a particularly highly examination-prioritized aspect of mathematics more definitive guidance is provided for teaching in the sense of intentional teaching as teachers doing their business with “specific outcomes or goals in mind for children’s development and learning.” (Epstein, 2007, p. 1). Intentionality—doing things on purpose—having been identified as the singular attribute of outstanding teachers. (Slavin, 2000, p. 7).

WHAT HAPPENS IN CLASSROOMS?

This question is engaged with drawing on experiences of a project, the Local Evidence-Driven Improvement of Teaching And Learning Initiative (LEDIMTALI), which is based on the considerations mentioned above. Examples of how teachers participating in this project deal with teaching procedures are used to illustrate that there are traces of shifting the teaching of procedures towards one of incorporating the development of learners’ mathematicalness.

It is widely recognized that teachers accord a lot of time to the teaching of procedures. The normal style of teaching procedures when they are introduced is a demonstrative exposition of the steps of the procedures with its unfolding written on the chalkboard as the demonstration proceeds. This is interspersed with questions directed at learners which are either answered by the teacher him/herself or learners. Upon completion of the demonstration of a few key examples by the teacher, the learners are presented with similar or near-similar problems to work through. During this phase the teacher monitors learners’ progress and understanding and provides assistance to learners who are struggling or request further assistance. At the end of the period the learners are given similar problems for homework. At the start of the session following the introduction, the given homework is checked and selected learners are requested to work through the solutions of the homework problems on the chalkboard. During this process, the teacher checks the work of the learners at the chalkboard whilst also checking whether learners did complete the assigned homework. The responses provided are the used by the teacher to draw learners’ attention

to aspects of their ways of working. Some these that were observed during visiting more than 30 classrooms during the first and second quarters of this year are:

Drawing attention to the ‘action words’ in mathematical problem statements: It is a fairly normal occurrence that when learners are engaged with particular procedures they attend only to the relevant steps required to obtain a particular outcome—the final answer. They imitate these steps as explained by the teacher or in the textbook and do not attend to whatever ‘action words’ such as, for example, ‘find’, ‘simplify’, ‘solve’, ‘test’. This non-attendance might lead to learners experience problems later when the immediate cue of ‘we are busy with it’ is not available and they have to respond to the ‘action words’ in the formulation for executing the procedure. During classroom visitations, it was observed that there are slight indications of teachers drawing learners’ attention to such ‘action words’. For example, a teacher was dealing with the factorisation of trinomials in a grade 10 class. After her exposition of the procedure, the demonstration by a learner of a problem given as practice to the class, the teacher brought to the learners’ attention that the instruction given for these kinds of problems is “Factorise fully” or “Find the factors”. In another instance, dealing with exponents the teacher stressed that answers must be in their simplest form and the instruction to these problems will be “simplify”. A learner who did not “simplify fully” was reminded firmly that the answer must be in simplest form. It is contended that drawing learners’ attention to explicit ‘action words’ contributes towards a greater awareness of the language machinery inherent in working with procedures. For example when dealing with a

question such as “Evaluate $\frac{x^3 + 1}{x^2 - x + 1}$ if $x = 7,85$ without using a calculator. Show all your

work” [Thanks to Mr Raymond, chief curriculum advisor of the Western Cape Education Department for the problem], learners will probably be able to discern what procedural steps must be followed to arrive at the solution as per the prescriptions in the problem statement.

Reference to common errors/misconceptions: During the feedback on homework the attention of learners is also drawn to common errors/misconceptions they commit related to topic. It is surmised that these are errors/misconceptions which teachers have identified with learners’ work during their experience. In a lesson dealing with exponents, the simplification

of $\left[\frac{2x^{-1}}{(2x)^{-1}} \right]^{-3}$ was dealt with. The teacher drew the learners’ attention to “-3 has to do with

the whole bracket”. The “with the whole bracket” was illustrated with

$$\left(\frac{1}{2} \right)^{-3} = \frac{(1)^{-3}}{2^{-3}}$$

where it was stressed that the exponent, -3, has to do with the “top”

and “bottom” of the bracket. $\left(\frac{1}{2} \right)^{-3} = \left(\frac{2}{1} \right)^3$ was presented and learners were reminded that

the negative index indicates the “reciprocal”. They were then asked if the answer is “6”. The teacher was thus explicitly and, with the example at hand, drawing learners’ attention to an oft-repeated misconception.



Working from the general structural representational format of a problem type: Solution of exponential equations was the topic being dealt with and checking of homework related to $ab^{cx}+d=0$ and variants were completed. The teacher announced that lesson of the day will

be on equations of the type $ax^{\frac{m}{n}}-b=0$, which was written on the chalkboard. He explained

the features of this general representation and stressed that $ax^{\frac{m}{n}}$ means $a \cdot x^{\frac{m}{n}}$, highlighting the multiplication (\cdot). After the explanation of the general structural representational format of

the problem type an exemplar problem $8x^{\frac{3}{4}}-27=0$ was demonstratively solved with constant reference to how this specialized form was linked to the general structural representational format.

The three examples above are representative of ways teachers deal with procedures. Although the primary focus is on the demonstrative exposition of the steps pertinent for the procedures, there are, as alluded to above, traces of reference to elements of mathematical ways of working with procedures given in table 3 above. I content that it is these traces that can be fruitfully exploited to bring them to a level of explicitness. For example, with respect to the common errors and misconceptions alluded to above, Mason (2000, p 10) suggests, amongst others, that these are converted to exercises in which learners are asked to “explain what is wrong about” the particular way of working.

CONCLUSION

There has been a long-standing debate about the prioritization of procedures as the objects of teaching in school mathematics. This debate is characterized by a view that the teaching of procedural fluency will degenerate into mindless drill-and-practice and hence mitigate against the development of the mathematicalness of learners. However, there have also been voices calling for a sense of balance. (Brownell, 1987 (first printed in 1956); Sfard, 1991). My contention is that the balance must be tilted in favour of procedural fluency with the enthusiasm for a focus on the process aspects of school mathematics replaced by a form of pragmatism. This pragmatism is not about moving forward to the past in the sense of ‘back to the basics’. It is, as argued above, about legitimate school mathematics. It is about what can reasonably work in schools in its current context of massive socio-political challenges that the majority of teachers face. It is also about affording more learners the opportunity to perform at higher levels of achievement in high-stakes mathematics examinations than is currently the case—a dire need in South Africa.

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Minding the gap between secondary and university mathematics¹

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The 2009 intake of university students were the first to have received complete school education within the recently implemented Outcomes-Based Education (OBE) system. A feature of the matriculation examination results of these students was the exceptionally high Grade 12 marks for Mathematics. This paper addresses the question of how the 2009-intake of students performed at university with respect to general performance, general attributes, mathematical attributes and content related attributes. It appears that these students are better prepared with respect to personal attributes such as confidence. However, in many instances they are weaker than their predecessors with respect to mathematical and content related attributes. Yet, there are positive indications that these students adapt and improve over a semester. We make some suggestions on how to make the transition from secondary to university mathematics somewhat smoother.

One of the concerns of the Department of Education (DOE) after 1994, was that many learners did not develop the required problem solving skills and the critical reasoning ability during the learning process (DOE, 2000). A new education system became a priority and in 1998 a new education system called Curriculum 2005 was implemented.

The key principles of Curriculum 2005 include integration, holistic development, relevance, participation and ownership, accountability and transparency, learner centredness, flexibility, critical and creative reasoning, quality standards and international competitiveness (DOE, 1997). Not only did the OBE system bring about changes in the approach but also a new curriculum. This meant a change at the very heart of the education system.

One of the concerns regarding the new education system was to what extent these learners would be prepared for university mathematics. Therefore the university preparedness of the 2009 intake of students received much attention. The first-year intake of the previous three years all had had partial exposure to OBE – these students experienced OBE for a few years but returned to the old curriculum in their final three years of schooling. The 2009 intake was the first group of students who had followed the OBE curriculum for their entire school career.

Mathematical university preparedness

In a recent study in Ireland investigating students' inability to cope successfully with the transition between secondary and university mathematics, Hourigan and O'Donoghue (2007) found that essentially there is a big difference between the nature of first-year students' mathematics experience at pre-university level and that which they experience at university in mathematics intensive courses. They also found that the unpreparedness of students caused permanent damage to students' further mathematics careers at university.

Several researchers have reported on this problem. Craig (2007) and Hoyles, Newman and Noss (2001) reported on a number of studies dealing with the difference between university expectations and the wide spectrum of mathematical abilities of the new students. De La Paz (2005) and Hoyles et al. (2001) name, amongst others, the changing school curricula as one of the reasons for these changes.

¹ The contents of this presentation was developed jointly with Ansie Harding and Patrick Phiri and has been published as Engelbrecht, Harding, & Phiri (2010). Are OBE-trained learners ready for university mathematics? *Pythagoras*; 72, 3-13. <http://dx.doi.org/10.4102/pythagoras.v0i72.16>



The focus in this paper is specifically on this facet. Changes in the school curriculum bring challenges for university lecturers and may result in the development and implementation of bridging courses as well as the need for changes to university curricula and assessment strategies (Craig, 2007; Wood, 2001).

We have reported on the first stage of our project (Engelbrecht & Harding, 2008), in which we investigated the preparedness of the transition group (the 2008 intake of students, who had had partial exposure to OBE). We found that the transition group was on par in most skills categories and performed even better in geometry, but was lacking in modelling skills. However, the results of the 2008 paper and results presented here are not directly comparable as the instrument used in 2008, namely the Alternative Admissions Research Project (AARP) tests then used for entrance purposes, is no longer used.

We have reported preliminary results of the 2009 intake of students in Engelbrecht, Harding and Phiri (2009). This paper expands on this report² by including additional data and re-interpreting the findings.

The 2009 intake of students

The 2009 intake of students at university were the first to have written a Grade 12 paper in the OBE system. The matriculation Mathematics examination results were surprisingly positive. More than 47% of those who wrote the examination passed Mathematics compared to about 43% in 2008. Of the learners who passed Mathematics, almost half (63 000) obtained a mark of more than 50% in contrast to the typical 25 000 of previous years whose achievement was similar to that of Mathematics Higher Grade (Keeton, 2009).

Among the concerns raised were that the papers were too ‘easy’, and that too many learners achieved distinctions. The Concerned Mathematics Educators (2009) group claimed that

the final examination in Mathematics was watered down and has therefore widened the gap between school and university for the top learner. The type of questioning was unchallenging for talented and competent learners and if this standard is going to be used as a benchmark for future examinations it will not adequately prepare young learners to study Mathematics related courses at university level. (para. 2)

The Department of Education appointed a panel of experts to evaluate the papers. The Ministerial Panel (2009) reported that those learners who scored 50% or more on the 2008 National Senior Certificate Mathematics papers would historically have passed mathematics on the higher grade in the previous system at 40%. It was also found that there was a lack of differentiation between the 70 – 79% level and the above 80% level and that the paper did not provide enough questions at the “knowledge” level of the taxonomy.

Although the Grade 12 results look good on paper there are reasons for concern. The question remains whether the OBE system is sufficiently preparing the learners for university studies.

In the National Benchmark Tests Project, which assesses how much of the school Mathematics curriculum has been mastered “a staggeringly low 7 percent of the students [2009 university entrants] who wrote the maths tests were found to be proficient. In other words, they would not need extra help to pass their exams. About 73 percent had ‘intermediate’ skills. The rest, 20 percent, had only the most basic skills and would need long-term, consistent attention” (Smith, 2009, para. 7). The challenge faced by universities is clearly enormous, and the need for curriculum responsiveness is evident.

Several lecturers who taught first-year mathematics in 2009 reported on under preparedness of students. For example, Huntley (2009) reported that 71 percent of first-year engineering students had passed in 2008 compared with just 35 percent in the 2009 mid-year examination. In addition, the fact that more Grade 12 learners than ever before had passed in 2008, caused record enrolments in university mathematics courses. This had led to enormous strain on the university’s ability to teach.

In short, although learners may have achieved the minimum requirements to allow them to study for a bachelor’s degree, this does not necessarily mean that they will be able to cope at university (Roodt, 2009).

² We re-publish here some of the initial data with the permission of the editors of *Suid-Afrikaanse Tydskrif vir Natuurskap en Tegnologie* and *Pythagoras*.



Mathematics in die OBE system

The OBE system resulted in differences in the curriculum and approach. In Mathematics some of the more difficult topics, including geometry, were moved to a third paper, an optional examination paper written by only 11 000 students in 2008 nationwide (Keaton, 2009). Certain topics were excluded from the new curriculum, for instance absolute value and some parts of trigonometry and logarithms.

It is easy to determine the differences in content but much more difficult to determine the level of mathematical skills of the learners who passed Grade 12 in 2008. We refer here to mathematical skills such as algebraic manipulation and graphical interpretation. In addition to these mathematical skills there are also a number of personal attributes, such as confidence and work ethics, that may have been either improved or compromised because of the different approach of the OBE system. These attributes could also have an influence on success in mathematics at university.

The research question addressed in this paper is: To what extent was the 2009 intake of students prepared for university mathematics with regard to performance, general attributes, mathematical skills, and content related skills?

The sample

This study is firstly based on a questionnaire completed by a number of experienced lecturers involved with the 2009 intake of mathematics students. It is secondly based on the results of two tests in a first semester calculus course, written by a group of first-year engineering students at a large university in Gauteng, after five and ten weeks of lectures, respectively, hereafter referred to as Semester Test 1 and Semester Test 2. The course had an enrolment of 1282 students. We experienced some administrative problems, but managed to gather and analyse the data of 924 students for Semester Test 1 and 820 students for Semester Test 2, with an overlap of about 750 students. In this calculus course students attended four lectures of one hour each and one tutorial of two hours per week. The entrance requirement in mathematics for these students was level 6 (a mark of at least 70%) in Grade 12 mathematics.

We used the questionnaire to determine the experience of the lecturers with the 2009 intake and used these opinions as guidelines for further investigation in analysing Semester Tests 1 and 2.

Grade 12 marks distribution

In 2007, 22% of students enrolling for this course achieved a distinction (more than 80%) in Grade 12. In 2008 the corresponding figure was 24% and in 2009 it was 55% – dramatically higher than for the previous two years. Note that these percentages include students repeating this course and that the actual percentages for new students were somewhat higher. In fact, in 2009, 72% of the new students enrolling for this course in 2009 achieved a distinction in Mathematics in Grade 12.

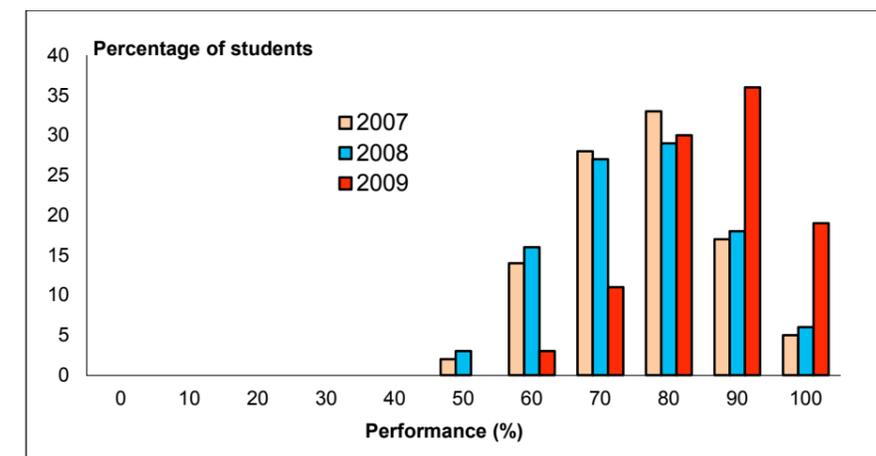


Figure 1: Percentage frequency distribution of Grade 12 performance



Figure 1 shows the percentage frequency distribution of the 2009 first-year-students' Grade 12 marks. It is clear that the Grade 12 marks of the 2009 intake were exceptionally high relative to the previous two years.

Note that all the graphs (Figures 1 – 10) are percentage frequency distributions, with the performance intervals as a percentage (0-100) on the horizontal axis, and the percentage of students in the relevant category on the vertical axis. On the horizontal axis the category 50, for example, refers to performance scores of 41 – 50 and 100 to scores of 91 – 100. Note that the category 0 then refers to scores of exactly 0.

General performance in university mathematics

In Figure 2 we compare the Grade 12 marks of the 2009 group of students with their performance in Semester Test 1 and Semester Test 2.

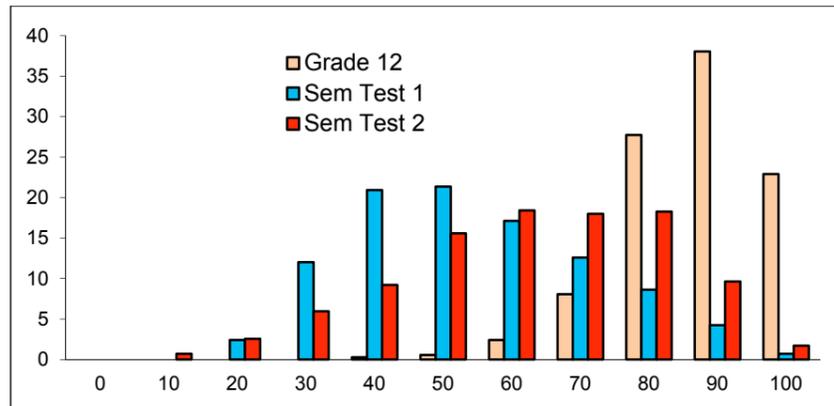


Figure 2: Student performance in Grade 12 and in Semester Tests 1 and 2

From Figure 2 it is clear that there was a strong shift to the left from the Grade 12 marks to marks in Semester Test 1. The marks for Semester Test 2 does not display the same strong shift to the left. In fact, the marks distribution for Semester Test 2, as well as for the June examination (see Figure 3) show a slight shift to the right and is more similar to the Grade 12 marks distribution.

The Pearson correlation coefficient between Grade 12 performance and performance in Semester Test 1 is 0,37 and for Semester Test 2 it is 0,35, and are both significant ($p < 0,001$). This implies that Grade 12 can still be considered as a good predictor of university success in mathematics but that students have to realise that their Grade 12 performance will not easily be repeated in university mathematics.

The percentage frequency distribution of marks in the June examination is given in Figure 3. From this figure it is clear that students have recovered somewhat. However, we should not conclude too much from this graph, because students with a semester mark of less than 30% did not comply with examination entrance requirements and are excluded, reducing the group size for the June examination to 1144.

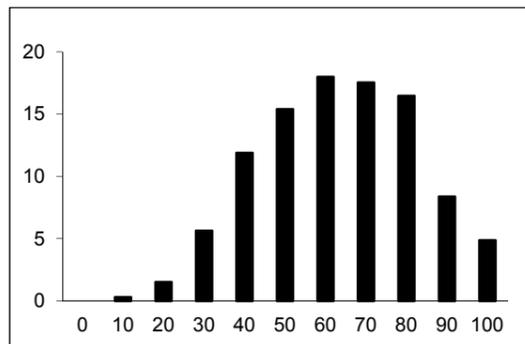


Figure 3: Student performance in the June examination



Questionnaire to lecturers

A questionnaire was completed by ten experienced first-year lecturers – some involved in first-year mathematics for engineering students, some in teaching the mainstream first-year courses including mathematics, actuarial sciences, financial mathematics and computer sciences and others in the pre-calculus extended programme. Lecturers were asked to express an opinion on aspects in which the 2009 students were better off, were worse off and in which there was no change with respect to the following three facets: general personal attributes, general mathematical attributes and content related skills.

General personal attributes

Lecturers were almost unanimous in their opinion that the 2009 intake of students had more confidence and were more willing to try. Students had a positive outlook and had confidence in their abilities. It was noticeable that students were not prepared to blindly follow the method suggested by the lecturer, they wanted to “experiment” and do things “their” way. Unfortunately this way of doing goes along with a lack of mathematical rigour and a way of writing that often makes sense only to the student him/herself. Some lecturers felt that students had too much self-confidence and spoke about mathematics too loosely rather than writing it out carefully. When marking tests this personal characteristic of (excessive) confidence was also noticeable. As one lecturer remarked: “Students write down everything that they can think about, without any coherence, and hope that somewhere there will be something that can earn them a mark.” This way of doing was not entirely new for 2009 students but probably more so than in the past. It is also possible that the noticeable confidence of the new students can be ascribed to their high Grade 12 symbols for Mathematics and not only to the influence of OBE.

Our findings of increased confidence agree with that of Adler, quoted by Smith (2009, para. 23): “the kids are much better than they were, they are more confident, they are more aware of what they do and don’t know and they are more willing to try.”

General mathematical skills

There was agreement amongst lecturers regarding the deterioration in general mathematical skills. Lecturers were unanimous regarding decrease in the specific skills of factual knowledge, algebraic manipulation and mathematical formulation. Most lecturers felt strongly that there was a decrease in all these skills while the odd lecturer felt that there was an improvement in graphical manipulation and mathematical intuition. It appears that the self-confidence with which students started out with in the course was not justified and that this was not supported by the necessary mathematical skills. For example, in Semester Test 1 many students made mathematical errors such as

$$\ln x^3 = 4 + 4 \ln x \Rightarrow e^{\ln x^3} = e^4 + 4^{4 \ln x}$$

$$\ln x - \ln 3 = \frac{\ln x}{\ln 3}$$

$$\ln(x - 4) = \ln x - \ln 4$$

$$\ln x = 4 \Rightarrow x = 4^e$$

According to lecturers there was not only a noticeable weakening of knowledge regarding the properties of logarithms and logarithm manipulation, but there was also an increase in elementary errors such as

$$\frac{a+b}{a} = b$$

$$(1+4x)^{\frac{1}{2}} = 1^{\frac{1}{2}} + 4x^{\frac{1}{2}}$$

$$\sqrt{x} = 2 \Rightarrow x = \pm 4$$

$$\sqrt{x} = 2 \Rightarrow x = |2|$$



Furthermore, there was a particular lack of knowledge and a lack of skills in trigonometry, leading to errors such as

$$\sin x > -\frac{1}{\sqrt{2}} \Rightarrow x > -\frac{\pi}{4}$$

There was particular concern regarding the poor ability of students to ‘write’ mathematics. Students wrote little and appeared to be uncertain. Often something that made sense was obscured between nonsensical writing.

We distinguished between the following general mathematical attributes: algebraic manipulation, graphical manipulation and interpretation, concept application and interpretation, and basic factual knowledge. We grouped questions in the two semester tests into these categories and investigated students’ performance in each of these components.

The performance of students in questions that mainly required algebraic manipulation in Semester Tests 1 and 2 are shown in Figure 4.

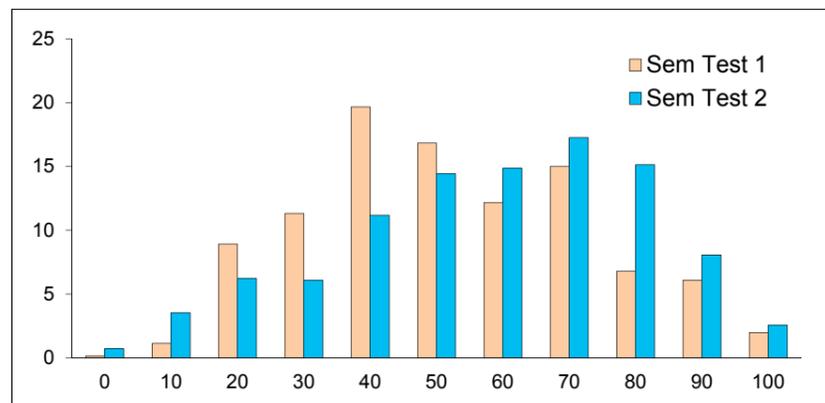


Figure 4: Performance in algebraic manipulation in Semester Test 1 and 2

In Semester Test 1 the distribution leans to the left. There are too many students that scored less than 50% for these questions. It is expected of students to be fluent in algebraic manipulation to ensure success in university mathematics. For Semester Test 2 there seems to have been a slight improvement in the skill of algebraic manipulation. The distribution leans somewhat to the right and although the marks are still poorer than the Grade 12 marks, there was less reason for concern. Whereas students seemed to be out of their depth in Semester Test 1 as far as algebraic manipulation was concerned, they seemed to have adapted and improved.

In Figure 5 we give the performance of students in questions in Semester Tests 1 and 2 that mainly required graphical interpretation.

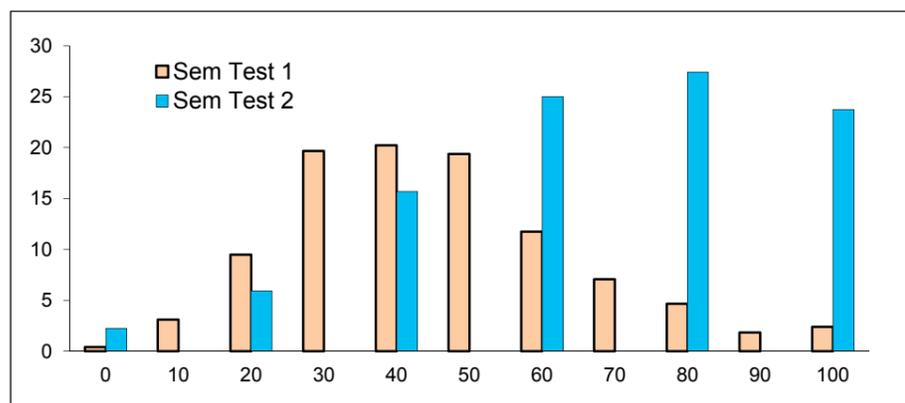


Figure 5: Performance in graphical interpretation in Semester Tests 1 and 2

It is clear that performance improved noticeably from Test 1 to Test 2. Graphical interpretation has always been difficult for students but although it appeared to still be the case in Semester Test 1, the pleasing finding was the remarkable improvement in this category in Semester Test 2.

Performance in the other mathematical skills categories for the two semester tests is given in Figure 6. Possibly the best of these categories is the concept application category, especially in Semester Test 2, a surprising finding. It appeared that students’ comprehension was not poor at all but that technical manipulation was the stumbling block. A disturbing observation was that basic factual knowledge was not on par, neither in Semester Test 1 nor in Semester Test 2. Is it perhaps a feature of the OBE generation that they prefer to do but are not so keen to spend the required time on studying?

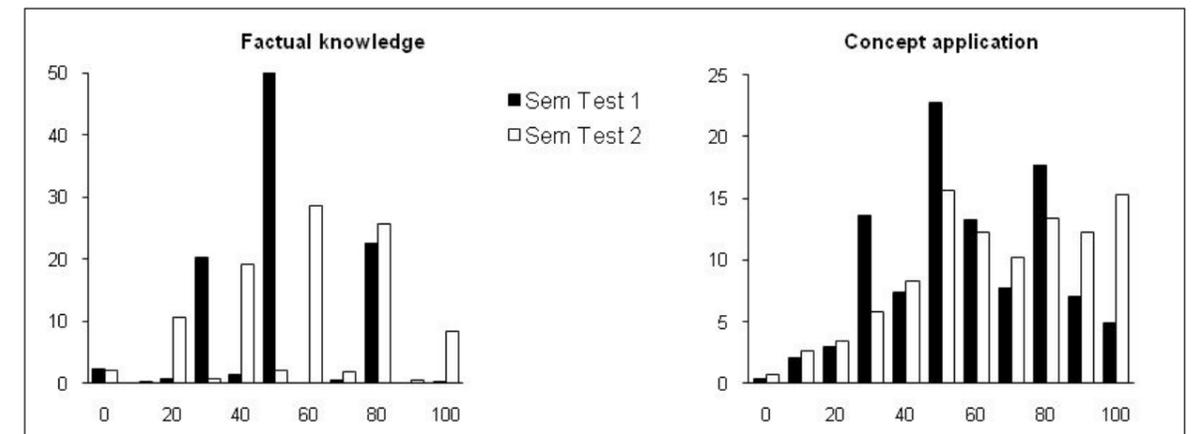


Figure 6: Student performance in factual knowledge and concept application in Semester Tests 1 and 2

The correlation coefficients between student performance in each of the components and their Grade 12 performance are given in Table 1.

Table 1: Correlation coefficients between university and Grade 12 performance

	Semester Test 1	Semester Test 2
General results	0,37	0,35
Algebraic manipulation	0,47	0,28
Graphical manipulation and interpretation	0,39	0,21
Concept application and interpretation	0,34	0,32
Basic factual knowledge	0,27	0,22

These correlation figures are all significant ($p < 0,001$). The poorer correlation in basic factual knowledge supports the notion that many of the students simply did not study enough, and that this was not only the case for poor students.

Content related skills

It was not surprising that a deterioration in preparedness was observed regarding topics such as absolute value, trigonometric functions and exponents and logarithms, as it was exactly in these topics that content decreased or was totally omitted in the new curriculum. It appears that students were not only inept with respect to general mathematical skills but also with respect to content related skills. In the empirical investigation we grouped questions that mainly concerned each of the content components and considered student performance in each of these components.



The correlation coefficients between student performance in Semester Test 1 in each of the components and their Grade 12 performance are given in Table 2.

Table 2: Correlation coefficients between performance in components of Semester Test 1 and Grade 12

General result of Semester Test 1	0,37
Inequalities and absolute value	0,33
Functions and graphs	0,36
Trigonometry	0,38
Exponents and logarithms	0,19
Limits and continuity	0,46

The correlation coefficients between student performance in Semester Test 2 in each of the components and their Grade 12 performance are given in Table 3.

Table 3: Correlation coefficients between performance in components of Semester Test 2 and Grade 12

General result of Semester Test 2	0,35
Differentiation – concepts	0,29
Differentiation – technical	0,24
Differentiation – applications	0,29
Limits and asymptotes	0,25
Continuity	0,25

All correlations are significant ($p < 0,001$) and of approximately the same size, except for the component of exponents and logarithms in Semester Test 1, which caught the lecturers slightly off guard. Exponents and logarithms are in the school curriculum and lecturers assumed that students were equipped with the same level of knowledge as in previous years. This was not the case. Students were initially inept in the properties of logarithms and manipulation with exponents and logarithms, more so than before. The result was that although this topic was included in the school curriculum, the teaching pace at university was such that students floundered with their shallow knowledge. On realising this, effort was made for allocating extra time and to giving more attention to logarithms and exponents.

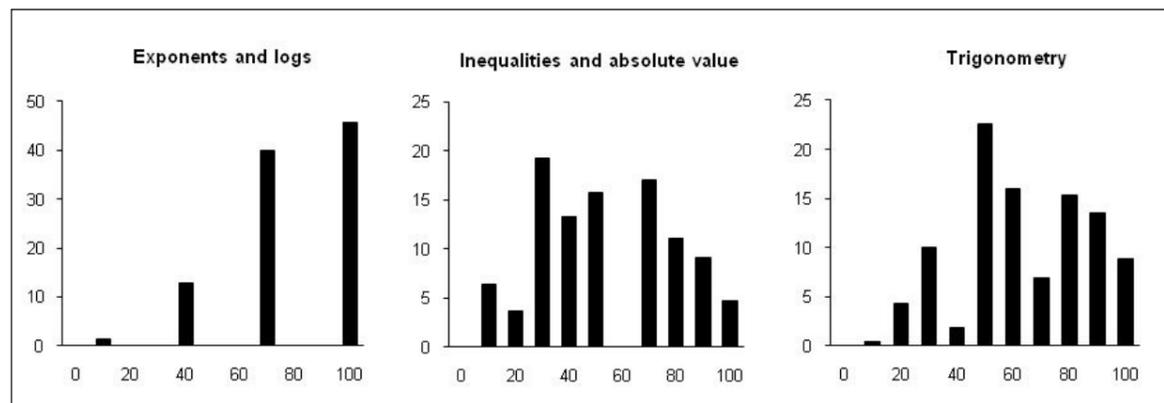


Figure 7: Student performance in inequalities and absolute values, logarithms and exponents and trigonometry in Semester Test 1

Inequalities and absolute value together form a topic to which more time needs to be devoted, as is clear from Figure 7. Too many students performed poorly in this topic.

As for trigonometry, although there is a group of students that has mastered the topic, there are too many that scored less than 50%. These students could probably be rescued through a slower presentation pace.

The main thread of the mathematics course runs through functions, limits, continuity and differentiation – from the concepts to techniques to applications. We now follow this thread with respect to student performance. For functions, limits and continuity (the start of the topic) the performance in Semester Test 1 is disappointing (see Figures 8 and 9). The level of knowledge in these topics was too low. As a result a decision was taken to spend considerably more time on this topic and to gain the time by moving some of the work to the second semester.

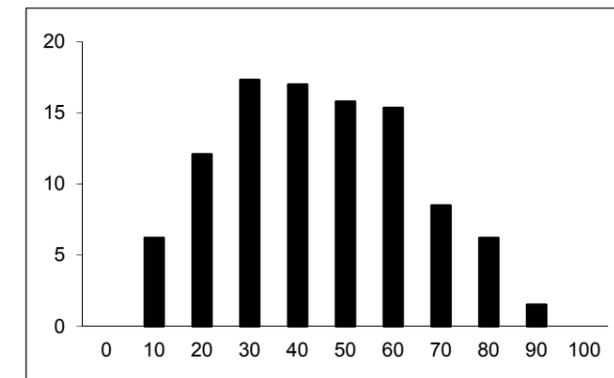


Figure 8: Student performance in functions in Semester Test 1

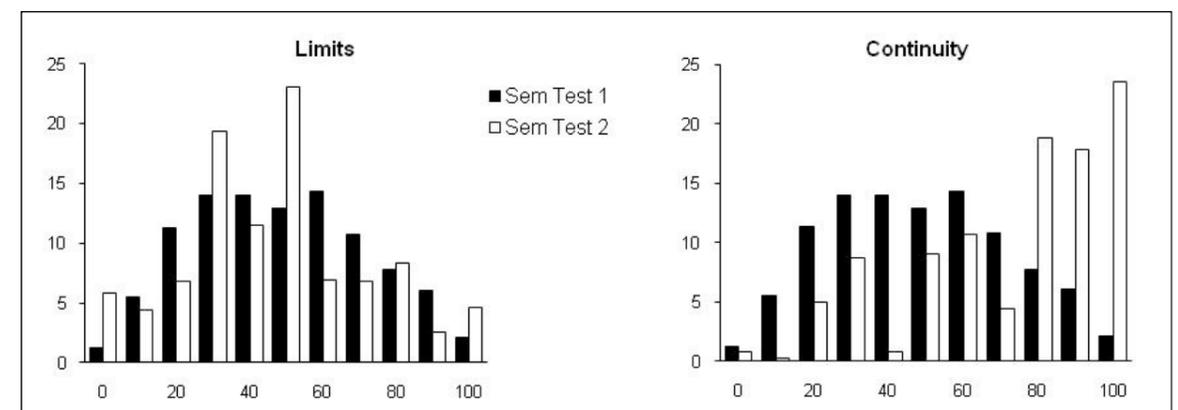


Figure 9: Student performance in limits and continuity in Semester Tests 1 and 2

There was no improvement from Semester Test 1 to Semester Test 2 in the limits topic (see Figure 9). It is clear that the topic is difficult for students. Yet, student performance increased in the continuity category. As was seen in other categories, students initially struggled to find their feet but then improved in performance.

The concepts involved in differentiation did not appear to be problematic for the majority of students (see Figure 10), although there was a tail of students for which this finding was not true. This tail seemed to be increasing as the topic was deployed. The performance decreased through technical differentiation to applications. Applications are by nature more difficult but the relatively poor performance in technical differentiation did not bode well for the follow-up topic of integration, which is far more complex. Technical differentiation requires the rigour of applying rules, secured by repeated practice. These students did not seem to be geared for practice, and more practice, until it becomes second nature to do differentiation.

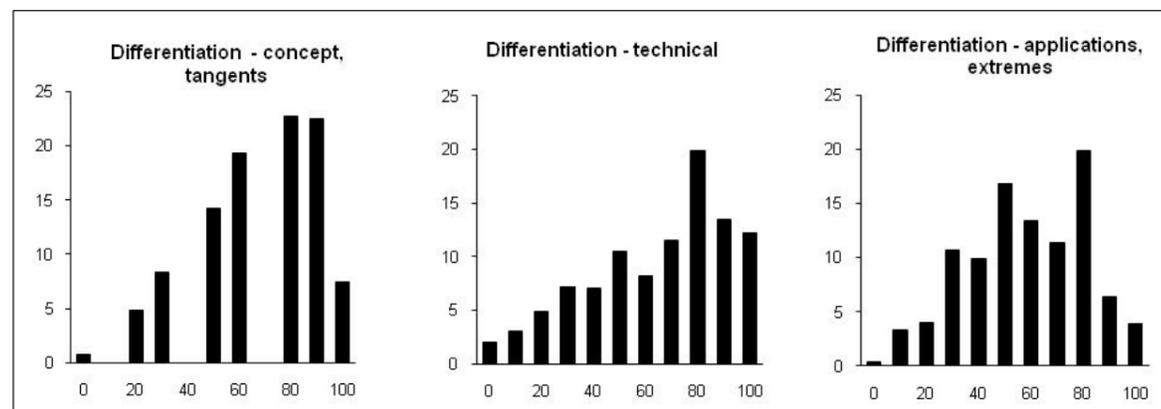


Figure 10: Student performance in differentiation in Semester Test 2

Discussion

The change to a new education system at school level has been the most significant of the past few decades and it was to be expected that certain difficulties and discrepancies would be experienced. It was important to identify problem areas and address eminent issues. The interface between secondary school and university has to provide for a smooth transition and it would have been fortuitous if the new system decreased the existing gap. This study unfortunately shows that for mathematics, the new system has enlarged rather than decreased the gap.

We do not put blame on anyone in this study. The national Department of Education involved higher education continually during the curriculum development process and the problems experienced at the moment are not necessarily someone's "fault". We are in a transition phase and need to address unexpected problems as they emerge.

The secondary education structures as well as universities should devote attention to addressing the widening gap between secondary school and university mathematics. One cannot accept students for university courses if we know in advance that they are under prepared and will most likely experience difficulties.

Possible solutions to smoothen the discontinuity between secondary and university mathematics include

- Further research to make a detailed analysis of the problem
- Continued close cooperation between university and secondary education authorities
- Possible changes in the school curriculum
- The possibility of an additional mathematics subject as another of the seven school subjects
- Raising of admission requirements at universities
- Placing more students in extended programmes at universities
- Extension of support programmes at universities
- Possible changes in the university mathematics curriculum

At university level there is a realisation that the teaching pace should be slowed down and that the curricula in first-year mathematics courses need to be revised. Content in first-year courses has gradually increased with time and this resulted in a quickening of the teaching pace. This trend should be halted and reversed.

In some university mathematics courses the emphasis has also shifted to a more theoretical approach for which the new intake of students is clearly not ready.

The 2009 intake of students thus forced both universities and schools to pause for stock taking. An outreach should be done from both sides for successful education systems at both levels.

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OBITUARY

We say goodbye to the following members of our Mathematics Education community who passed away over the past year. We are so much poorer without them. We will miss them!



Hanlie Murray (Stellenbosch University) was an active member of AMESA and contributed a number of interesting articles to our *Pythagoras* journal.

Her series on “Windows on a child’s mind” was very popular within the mathematics community in the late 1990s and early 2000s.



Lovemore Nyaumwe (UNISA) passed away in 2012. His research interest was mathematics student teacher thinking and reflective practice.

Two of his research articles were published in our *Pythagoras* journal.



Humphrey Atebe moved to South Africa from Nigeria to read for a PhD in Mathematics Education at Rhodes University. In his research Humphrey used the van Hiele theory to understand and deconstruct why Grade 11 learners in his home country and South Africa struggled with Euclidean Geometry.

Before securing a lectureship at WITS University, Humphrey completed a post-doc at Rhodes University and assisted in setting up the FRF Mathematics Education Chair project in Grahamstown. Humphrey also worked at Nyaluza Secondary School in Grahamstown where he was a much loved and respected mathematics teacher.



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