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- IF ONLY FOR A DAY



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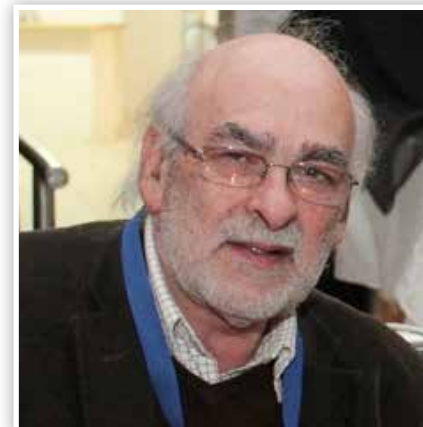
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DO GREAT THINGS



From the President's Desk...



AMESA's annual national congress is one of the highlights on the academic calendar. This year it was indeed a highlight, when over 900 AMESA members attended the 19th congress in Cape Town. Thank you, and congratulations to the Congress Director, Rajendran Govender and his local organising committee (LOC) for offering us such a rich congress, in all senses - academically, culturally, socially, ...

Many of the participants were new AMESA member. One of the objectives of the National Council for 2013 is to try to increase our membership, so this is very pleasing. Given our policy that all

participants must be AMESA members, the annual national congress is an important vehicle to grow our membership. Provided of course that teachers find the congress enriching, so that they will want to also attend future congresses.

Our current membership at the end of 2013 stands at 2027 (1525 individual and 502 institutional members), which is about the same as at the end of 2012. So we maintained our member numbers, although we did not increase it. Interestingly, when I further analysed the data I found that of our current 2027 members

751 joined in 2013
367 joined in 2012
279 joined in 2011

Continued on page 2

Editor's Comment...



This issue of AMESA News gives you a detailed account of the activities of AMESA and some other mathematics related activities for the past few months. AMESA is the voice of Mathematics Education in South Africa and we do all in our power to uphold this important position. Our regions have been very busy and so has our National team. We proudly celebrated our 20th anniversary at our National Congress in Cape Town. Read about our Congress and other key happenings in this bumper issue. Should you like to comment or make suggestions, please contact me at news@amesa.org.za.

Enjoy reading!
VG Govender

Contents

From the President's Desk...	1
Editor's Comments...	2
AMESA National Office Bearers	2
Changes in the National Council	4
Mathematics in the Spotlight	5
Report on the AMESA Congress 2013	7
Congress 2013 Welcome Message	14
AMESA 20th Annual National Congress	15
Regional Activities	16
19th Annual General Meeting (AGM)	17
The National Council Report	19
Income and Expenditure Statement	25
President's Report	31
AMESA Curriculum Matters	33
Mathematics Teacher Education Special Interest Group	40
Our international guest reflects on his visit to South Africa	41
Reflections on the 2013 AMESA Congress	45
Report on academic visit to Curtis Math Centre, UCLA, USA	48
International Seminar at the Park City Mathematics Institute (PCMI) in the USA	52
The Old Mutual Foundation Advertorials	55
Approaching trigonometry from different angles	55
Working with functions: Understanding length relationally	60
Mathematics Relays	65
Obituary	66
AMESA National and Regional Officers	67

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I would claim that 751 new members out of 2027 is a substantial increase, it is excellent! But of course if the membership is about the same as in 2012, it also means that we *lost* about 750 members, i.e. about 750 people who were members in 2012 did not renew their membership in 2013, and that is sad!

One must come to the conclusion that we are good at drawing new members, but we are not managing to keep our members. Indeed, the data I analysed show that people do not remain members of AMESA for very long. Of the current 2027 members

2027 have been a member for at least 1 year
1232 have been a member for at least 2 years
761 have been a member for at least 5 years
402 have been a member for at least 10 years
146 have been a member for 20 years

This realisation has many implication, and Council will have to make plans how best to serve and service new members so as to keep them. First, it is clear that we continually are having new members, so we have to cater for them as new members by providing them with more information about AMESA as an organisation, which is of course the purpose that AMESA NEWS serves.

Second, we must assume that members not renewing their membership are dissatisfied with the service or benefits they received from membership.

I spoke above about congresses and about membership. Now put the two together. The fact is that in the past few years, half of our members are attending our national congresses.

So our annual congress is a good place to start to address the issues. We should be organising better and better congresses that teachers find rewarding! I can share that the National Council has decided to try to improve administrative aspects of congresses, by centralising or 'nationalising' the registration for the national congress. So from 2014 the national office, and not the presenting province, will be handling congress registration. We hope that we will be giving (new) members a better service and overcome many of the registration-membership-payment problems we have encountered in the recent past. Furthermore, the National Council has established a sub-committee (the national organising committee or NOC) who will implement national tasks/functions relating to the national congress, initiate and drive aspects of other Council responsibilities (e.g. plenary speakers, theme, ...), and advise, support and oversee the LOC on all aspects of congress organisation. I have high expectations that this will improve our experience of attending congress.

So I will see you in Kimberley for AMESA's Annual National Congress 2014, 7–11 July 2014!

Alwyn Olivier

AMESA National Office Bearers

We would like to introduce our national office bearers.



PRESIDENT:

Alwyn Olivier recently retired as senior lecturer in Mathematics Education and Director of the Research Unit for Mathematics Education at the University of Stellenbosch (RUMEUS).

Alwyn played a key role in establishing AMESA. As the then President of the Mathematical Association of Southern Africa (MASA), he initiated the talks and facilitated the process that led to the birth of AMESA in 1993. He was a member of the first AMESA National Council, and then again intermittently from 2004 to the present. He has served AMESA in many different capacities, including as AMESA representative on the Board of the South African Mathematics Foundation (SAMF) and the South African National Committee for the International Mathematical Union (SANCIMU), AMESA webmaster, organiser of the AMESA Grade 4-7 Mathematics Challenge, coordinator of the AMESA-DST Educator Support Project (ESP), and coordinator of the AMESA-SAMF-Sasol Primary Teacher Problem Solving initiative. Alwyn is the current Editor-in-Chief of AMESA's accredited research journal *Pythagoras*, which he guided through the golden route to an online Open Access journal.

Alwyn is one of five Honorary members of AMESA, honoured in 2002 for his contribution to Mathematics Education.



VICE-PRESIDENT:

Vasuthavan (also known as Nico) Govender has been a member of AMESA since 1993. He has served in several positions within AMESA:

- Member of Port Elizabeth Branch Executive Committee 1994 - 2009
- Secretary: AMESA PE 1996 - 1997
- Vice-Chairperson: AMESA PE 1997 - 2003
- Chairperson: AMESA PE 2003 - 2009
- Chairperson AMESA Eastern Cape: 2009 - 2013
- AMESA Coordinator of the Curriculum Committee since 2003
- AMESA Coordinator of Mathematics Week 2010 - 2012
- Congress Director: Congress 1999 and Congress 2008

Nico completed his doctorate in Mathematics Education in 2007 and serves on two DBE committees for Mathematics. His various positions include high school mathematics teacher; College of Education lecturer; Subject Advisor (Mathematics FET); Lecturer mathematics in the Faculty of Science (NMMU); Senior Lecturer: Mathematics & Science Education, (University of Fort Hare) and now Deputy Chief Education Specialist in the Port Elizabeth District. Nico gathered information on the history of AMESA and chaired the Panel discussion "Twenty years of AMESA" at the AMESA National Congress in Cape Town. He currently serves as the Chairperson of the SAMF Advisory Committee on Mathematics (ACM).



SECRETARY:

Isaiah Ronald Shabangu has been serving as the National Secretary since 2010. He is also serving the second term as the Mpumalanga Regional Chairperson. Isaiah has over the years actively participated at various levels in the Mpumalanga region:

- National Representative for Mpumalanga 2008 to 2010
- National Advertising Manager 2009 - 2011
- Malelane Branch Secretary 1994 - 1997
- Malelane Branch Chairperson 1998 - 2001
- Mpumalanga Regional Deputy Chairperson 2006 - 2008
- Mpumalanga Regional Chairperson 2009 - 2013
- Presenter in Regional and National AMESA Conferences

Isaiah is currently the principal of Sincobile Primary School in Mpumalanga. He holds a Primary Teachers Diploma, ACE in Mathematics and B.Ed. Honours in Mathematics Education. He is currently studying towards a Master's in Education degree at the North-West University.



TREASURER:

Rajendran Govender is currently a lecturer in Mathematics Education in the School of Science and Mathematics Education in the Faculty of Education at the University of the Western Cape. Rajendran started teaching school mathematics in 1985 and was promoted to a college mathematics lecturer in 1995. In 2002 he became a subject advisor of Mathematics in KwaZulu-Natal. In 2003 he joined the University of Limpopo as a mathematics education lecturer, and in 2008 took up the post of mathematics education lecturer at UWC.

Rajendran is a member of SAARMSTE, Congress Director of AMESA 2013, UMALUSI external moderator for Mathematics, and he represents AMESA on the SAMF Advisory Committee on Mathematics. He is also an associate editor of the *Pythagoras* journal. He recently completed his doctorate in Mathematics Education.



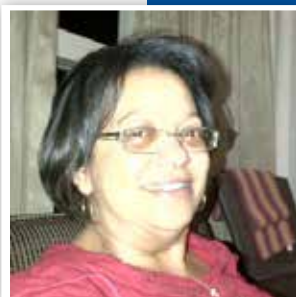
Changes in the National Council

TREASURER

At our recent National Congress Lorraine Burgess, treasurer since 2009, stood down due to pressing work commitments at the Oprah Winfrey Academy for Girls in Henly-on-Klip.

We would like to place on record our heartfelt thanks to Lorraine for the outstanding work she did for AMESA over the past four years.

We are pleased to announce that National Council member, Rajen Govender was elected as the new national treasurer of AMESA for the two-year term 2013-2015.



Siphos Vilakazi



Makhalanyane Moeti

NORTH-WEST REGIONAL REPRESENTATIVE

One of our stalwart National Council members, Siphos Vilakazi, recently stepped down as the National Council representative for the North-West province. We would like to thank Siphos for his outstanding organisational AMESA work in the North-West and wish him well in his future endeavours. We trust that he will continue with his good work in the North-West. Makhalanyane Moeti is the new chairperson of AMESA North-West, and also the new North-West regional representative on the National Council.



Jeffrey Thomas



Kagiso Peace
Mocumi Mojaki

NORTHERN CAPE REGIONAL REPRESENTATIVE

We say goodbye to our National Council member, Jeffrey Thomas from the Northern Cape who has left Kimberly to assume a position at the South African Mathematics Foundation (SAMF) in Pretoria.

We would like to thank Jeffrey for keeping the work of AMESA going in the Northern Cape and for doing the spadework for AMESA 2014. We would like to wish him all the best in his new job and trust that he will continue with his AMESA work in Gauteng.

We would like to welcome Kagiso Peace Mocumi Mojaki as the new Northern Cape representative on the National Council.

WESTERN CAPE REGIONAL REPRESENTATIVE

The Western Cape regional representative of the past few years, Rajen Govender, become national Treasurer.

The new Western Cape regional representative on Council is Gary Powel, who has just recently retired as AMESA Western Cape Chair.



Gary Powel (left) and Rajen Govender

Mathematics in the Spotlight

Report by the South African Mathematics Foundation (SAMF)

The third quarter of the year was a busy one with many projects taking place concurrently. The Pan African Mathematics Olympiad, the Bulgaria International Mathematical Competition, the International Mathematical Olympiad, the First Round of the South African Mathematics Challenge and the Second Round of the South African Mathematics Olympiad. Our individual teams all achieved good results in these competitions and their participation is valuable in the preparation for the International Mathematical Olympiad that will be hosted in Cape Town next year.

The South African Mathematics Foundation (SAMF) in association with Harmony Gold Mining Company of South Africa announced the appointment of the first qualified mathematics teachers at JC Motumi School in Kutlwanong near Odendaalsrus in the Free State and the AB Phokompe Secondary in Randfontein (Gauteng). Enos Makhubu and Olatunde Oluwasan will be teaching mathematics to Grade 8 and 9 learners at the respective schools. These two schools were identified by the SAMF and Harmony Gold Mining to become part of the Harmony Adopted Schools project.

The Harmony Teacher Development (HTD) project is a short course for teachers to improve their skills in solving Olympiad type mathematical problems. It is available as a live taught (GET and FET phases) or DVD course (Level 1 for GET and FET phases and Level 2 for GET phase) and certificates are issued which may be used for continuous professional development. A total of 185 teachers have completed the course since January 2013 with an overall pass rate of more than 75%.

On 14 May a total of 11 847 learners from 786 schools around the country took part in the second round of the South African Mathematics Olympiad (SAMO). A total of 199 learners from 107 schools qualified for the Final Round that took place on 9 September. The overall winners will be announced at the Annual Gala Awards function that will be held on 12 October.

Between 25 June and 2 July four high school learners, one from Kwa-Zulu Natal and the others from the Western Cape, took part in the Pan African Mathematics Olympiad (PAMO) that was held in Abaju, Nigeria. Over the past six years South Africa was ranked under the top three countries and for three consecutive years from 2006 until 2008, ranked first. This year's results are no different.

Andrew McGregor from Rondebosch Boys' High School in the Western Cape - Silver
Bronson Rudner from the South African College School in the Western Cape - Silver
Nashnlen Govindasamy from Star College in Kwa-Zulu Natal - Silver
Yaseen Mowzer from Fairbairn College in the Western Cape -Silver
The top three countries in this year's PAMO is: Tunisia, South Africa and Nigeria.

From 30 June to 5 July four South African youths, three from Kwa-Zulu Natal and one from the Western Cape, participated in the Bulgaria International Mathematics Competition (BIMC) that was held in the city of Burgas. The team, who are all part of the Siyanqoba Regional Olympiad Training Programme sponsored by the Department of Science and Technology, was one of eight South African teams that participated under the watchful eye of Kitty Phillips, team leader and country coordinator. Arthur Qampi from Star College in Durban, Mbonmuhle Mncwabe and Nkanyiso Mthembu from Maritzburg College in Pietermaritzburg and Shandukani Malaudzi from Herschel Girls' High School in Cape Town all returned home with certificates and motivated to participate in their next competition.

On 30 July, a group of six high school learners from across the country returned from Colombia where they took part in the 54th International Mathematical Olympiad (IMO).



Kitty Phillips, team leader and country coordinator of the Bulgaria International Mathematics Competition, with her award for her contributions to the SAMO Committee, presented at the SAMF Award Function in October.



Learners during the SAIPO (Interprovincial Olympiad) on 7 September 2013 during the team round

This year, as with each year since South Africa's first participation in the IMO back in 1992, the team returns with a number of awards.

Shaylan Lalloo	(Pearson High School, Eastern Cape) - Bronze
Robin Visser	(St George's Grammar School, Western Cape) - Bronze
Nashlen Govindasamy	(Star College, Kwa-Zulu Natal) - Honourable mention
Lauren Denny	(Rustenburg Girls High School, Western Cape) - Honourable mention
Andrew McGregor	(Rondebosch Boys' High School, Western Cape) - Honourable mention
Bronson Rudner	(South African College High School, Western Cape) - Certificate of participation

A total of 97 countries participated in this year's IMO. This equates to 528 learners from all over the world. In total, 45 gold medals, 92 silver medals, 141 bronze medals and 141 honourable mentions were awarded to participants. The winning country is the People's Republic of China, with the Republic of Korea and the United States of America in second and third place respectively. The host country, Colombia, is ranked 48th with South Africa in 58th position.

Next year the IMO will be taking place in Cape Town from 3-13 July 2014. Selection for the South African team will commence at the first Olympiad Training Camp that will be taking place in December. Learners are chosen for the selection process based on their performance in the SAMO and other SAMF Olympiad programmes.

On 4 September, 6 398 learners in Grades 4 to 7 from 240 Primary Schools across the country participated in the Final Round of the South African Mathematics Challenge. Coordinated by the South African Mathematics Foundation (SAMF) and AMESA and sponsored by Sasol, the Challenge saw more than 71 000 learners from 323 schools participating in the First Round which took place from 29 July to 2 August. The Challenge, which originated in 1977 as a project of AMESA, aims to improve problem solving skills in young learners thus preparing them for High School and tertiary education as well as equipping them with important life skills. Learners' achievements are acknowledged by awarding Gold, Silver and Bronze certificates based on their results.

On 7 September twenty eight teams took part in the Southern African Interprovincial Mathematics Olympiad (SAIPMO) that is organised by the South African Mathematics Foundation in association with the Actuarial Society of South Africa and Casio. The winning teams both hail from the Western Cape.

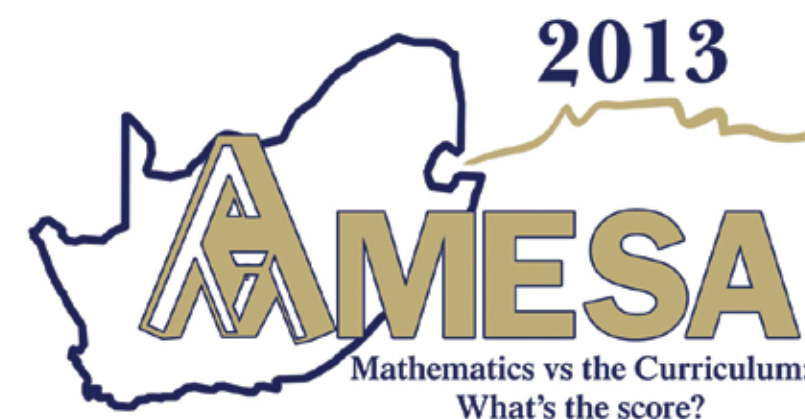


Learners writing the Final Round of the South African Mathematics Challenge in Port Elizabeth on 4 September 2013



Report on the AMESA National Congress 2013

held at the University of the Western Cape, Bellville, Cape Town



Opening ceremony:

The **Opening Ceremony** was a formal session to a packed hall with over 900 participants.

The UWC Rector and Vice-Chancellor, Prof Brian O'Connell, delivered an emotional call-to-arms, calling on participants to seize the present and work to raise the level of education for all South African students significantly.

The Head: Western Cape Education Department, Penny Vinjevold, brought a message of cooperation between the Department of Education and AMESA. She stated that the Department of Education valued and respected AMESA, and called on AMESA as an association to take the lead in improving mathematics education.

The congress was structured around different types of presentations:

Plenary lectures:

There were 5 plenary lectures, selected by the organisers to address important issues in mathematics education.

Zalman Usiskin (University of Chicago) analysed what it means to understand some mathematics, offering both academics and classroom teachers a framework to think about what is worthwhile teaching, with a direct message for curriculum construction.

Hamsa Venkat (University of the Witwatersrand) showed how important teachers' mathematical knowledge is for the teacher to structure activities and to adequately respond to children's thinking.

Phadiela Cooper (Principal of the Centre of Science and Technology) described her classroom research on learners' understanding of Geometry.

Peter Dankelmann (a Professor of Mathematics at the University of Johannesburg) unpacked some non-school mathematics (graph theory), enriching teachers' views of the nature of mathematics and the applications of mathematics.

Zain Davis (University of Cape Town) used the idea of coherence to describe and analyse school mathematics curricula, textbooks and pedagogy.

Plenary panel discussions:

Panel 1 on the theme of the congress (Mathematics vs the curriculum: What's the score?) unpacked different views of how teachers could implement the curriculum for learners to develop the necessary mathematics.

Panel 2 presented a view on the 20-year history of AMESA. It gave some interesting insights into the formation of AMESA and its gradual transformation into the organisation it finds itself today.

Panel 3 (An overview of mathematics teaching and learning in the Western Cape) gave participants some insights in how mathematics education is organised in the Western Cape, with valuable inputs on specific projects.





Dignitaries singing the National Anthem at the opening ceremony

Parallel sessions:

The different types of parallel sessions were presented in simultaneous strands aimed at the Foundation Phase, Intermediate Phase, Senior Phase, FET, and Mathematics Teacher Education:

- 20 long paper presentations of an hour each reporting on on-going research.
- 15 short papers of 30 minutes each.
- 35 workshops of one or two hours duration offering teachers hands-on mathematical activities.
- 10 'How I teach' presentations of 30 minutes each, mostly involving teachers illustrating their classroom practices.
- 9 Poster sessions displaying the work of projects.

AMESA business sessions:

The Annual General Meeting offered members reports of AMESA activities and procedures.

The Curriculum Phase Committees offered teachers the opportunity to directly discuss issues about the implementation of the mathematics curriculum.

The Special Interest Groups on writing and on problem-solving gave teachers the opportunity to engage with their interests.

Social activities:

Three evening social functions and an excursion gave teachers the opportunity to interact with each other socially.

Some feedback:

We quote some teacher feedback (these are from the South African Numeracy Chair Project in Grahamstown, taken from <http://www.ru.ac.za/sanc/numeracybuzz/spreadingthebuzz/#AMESA-June2013>):

"Using dice and card games in mental maths sessions: Fantastic session! Participants enjoyed it and gave some very positive feedback."



"The conference on its own was wonderful. The plenaries, presentations, audience discussions, I learnt a lot. I wish to share and plough back everything I have [learnt], not forgetting the challenges & gaps we still have to overcome."

"I found the conference to be most informative and enriching. I feel I have developed professionally. I have been able to network with other teachers and mathematics advisors from around the country. I really had a wonderful experience at the conference and have come home feeling inspired and committed to teaching Mathematics."

"To me this whole experience is not about personal growth or professional development only, it is about the development of the learners and what I can take back to implement in allowing them to become the best at what they are doing."



The Head of the WCED, Ms Penny Vinjevold at the opening ceremony



Zalman Usiskin at his opening plenary presentation

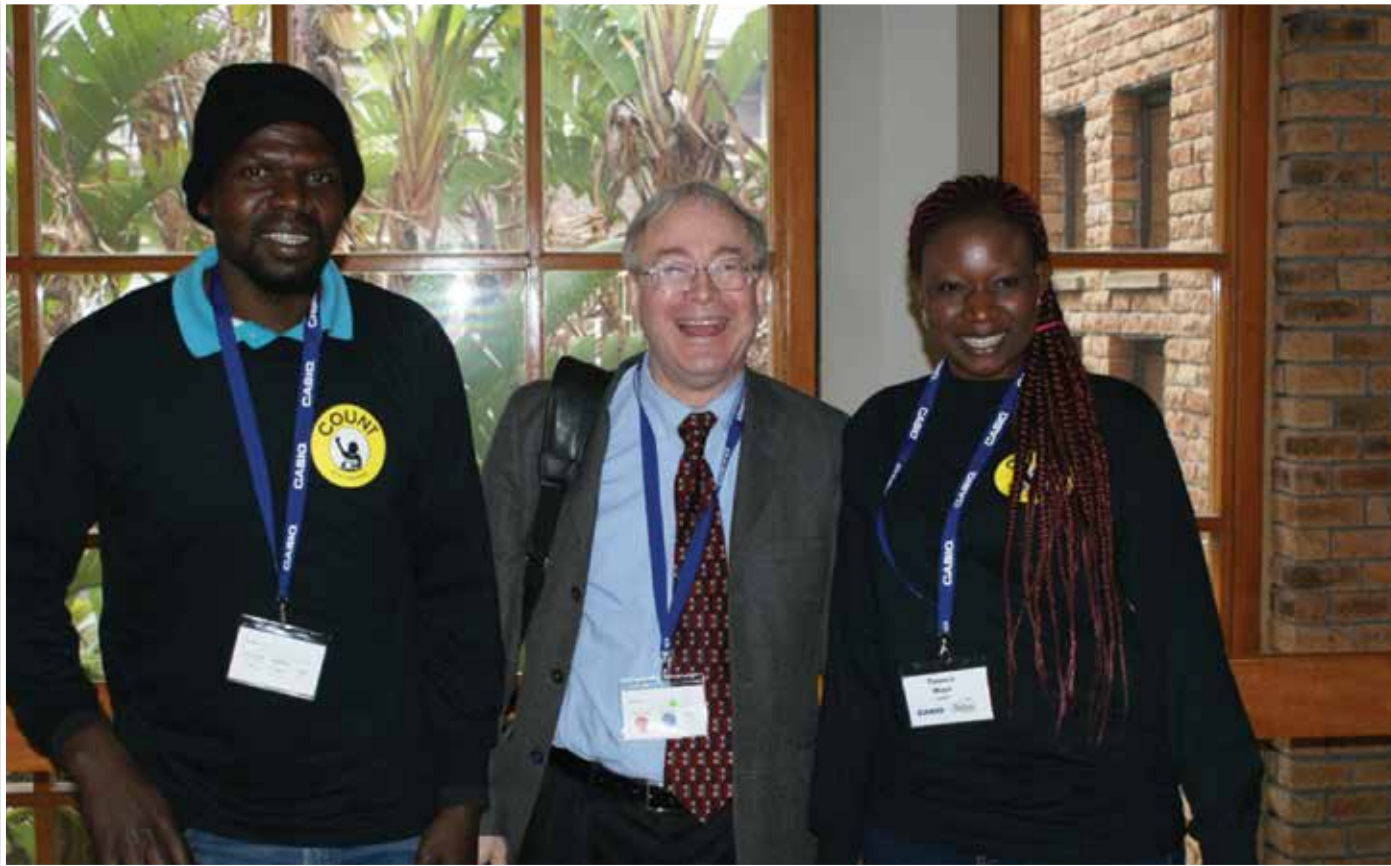


Louise Jones and Liyanda Maseko from Old Mutual with Elspeth Khembo (centre)



During a workshop session





Zalman Usiskin with some participants from COUNT



Congress Director Rajendran Govender with two LOC members



Prince Jaca asks a question during one of the panel discussion sessions



The panellists for panel 3: The History of AMESA





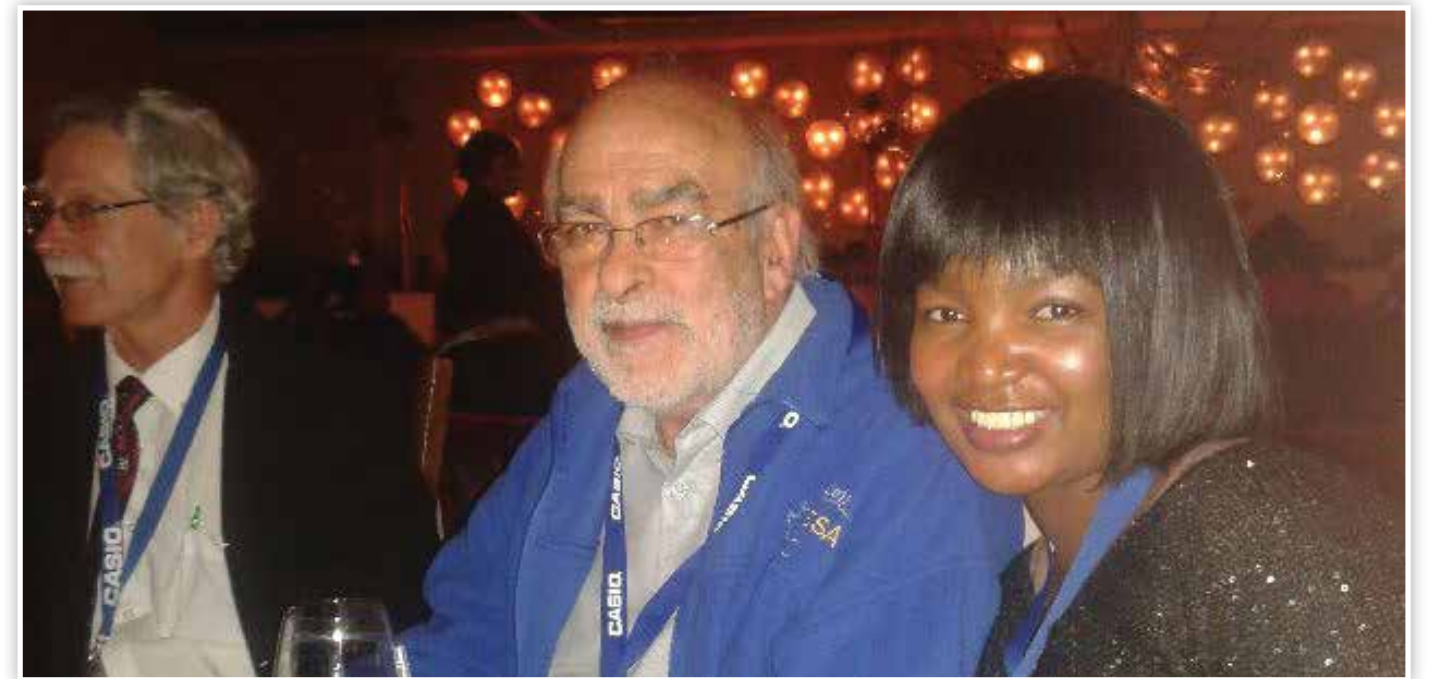
On top of Table Mountain during one of the excursions



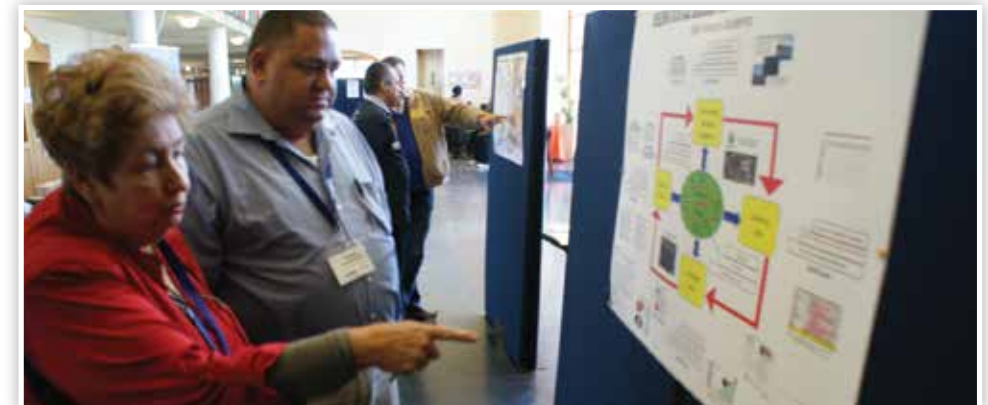
The choir in action (at the cultural evening)



A section of the audience at the AGM



Michael de Villiers, Alwyn Olivier and Merriam Mafojane at the gala dinner



Discussing a poster

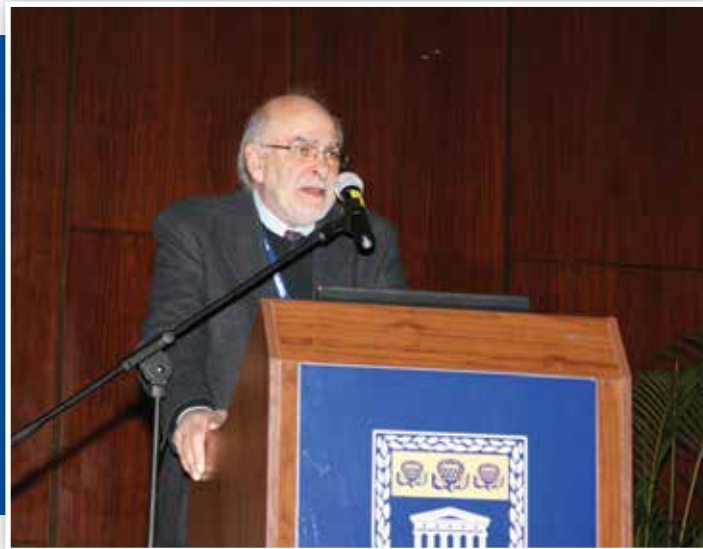


Susie Groves from Australia presenting a workshop



A hands-on workshop





Congress 2013 Welcome Message

by Alwyn Olivier, AMESA President

Dear distinguished dignitaries, colleagues

In my official capacity as AMESA President, I would like to officially, and very heartily welcome you all on behalf of AMESA at Congress 2013.

First a hearty welcome to the most important people here, namely our presenters - without our presenter there would be no congress. Thank you for sharing your expert knowledge with us.

But the really most important people here are our participants, because without you there is no congress. You are very special teachers who take responsibility for your own professional development, by spending your vacation at the congress. I am sure that you will learn much from attending the lectures and workshops at congress.

But I want to emphasise that to really make congress an experience, it is not about receiving, but by interaction with peers. So talk to each other about the presentations and talk mathematics education. You should feel a sense of urgency and excitement - there are so many interesting people here, so many great ideas - you must pursue the people and pursue the ideas. If you enjoyed a presentation, and feel you would like to exchange some ideas with the presenter, do not be shy! Go up to him or her during teatime, and do not talk about the weather, talk about mathematics education. And this applies also to the plenary speakers.

And may I say that everything does not happen in the lecture rooms, or during teatime. I suggest when you want to chat to someone, take out the timetable, compare and make an appointment, decide to miss a session, and meet for coffee in the coffee shop - I am serious, I especially asked the LOC to make sure that there are good cafeteria areas available so that people can be social! And the social functions have also been designed for you to meet great people and exchange great ideas.

But the most important people here today are our sponsors, because without their financial backing, there would be no congress. But it is much more than financial backing, we have a relationship with our funders in which we pursue shared objectives and support each other. So I would like to welcome them and introduce you to these very important people, and I ask that you applaud them as I introduce them to show AMESA's appreciation:

There is CASIO - I would like to introduce Astrid Scheiber - Astrid, if you would please stand up.

Then there is Ruth Rakosa from the Zenex Foundation.

Next I would like to introduce Allison Philander of Macmillan.

AMESA's principal sponsor is the Old Mutual Foundation, and we have a very special relationship with Old Mutual not just this year, or the past decade, but since the founding of AMESA in 1993. So as we this year at congress celebrate 20 years of AMESA, Old Mutual is very much part of that history and part of the celebrations. I would like to welcome and introduce you to my right-hand person, my go-to-person at Old Mutual, the person who is always there for us, and that is Liyanda Maseko.

Now I want to introduce Louise Jones, who is the Old Mutual Foundation's representative for the Western. I had dinner with Louise last night and she personifies the attitude of the Foundation's social investment programme of being involved with and supporting worthy projects, and I realised anew, how privileged we are to have Old Mutual as a partner. Louise, we very much appreciate you and our relationship with Old Mutual. Louise will now say a few words on behalf of the Old Mutual Foundation ...



AMESA 20th Annual National Congress



In terms of AMESA's rotation policy, the next AMESA National Congress will be take place in Kimberly, Northern Cape.

Venue: Diamandveld High School and Kimberley Boys' High School

Date: 7 - 11 July 2014

Theme: Demystifying Mathematics

Congress Director: Lawrence van Heerden, lawrence@amesa.org.za

Congress Co-Director: Nina Scheepers, nina@amesa.org.za

Academic Coordinator: Mandisa Lebitso, mandisa@amesa.org.za

Congress website

See the congress website for updated relevant information: <http://www.amesa.org.za/AMESA2014/index.htm>

Important dates:

Offer to review papers:	31 January 2014
Submission of presentation proposal manuscripts:	20 February 2014
Notification of proposal review results:	20 April
Application for financial support:	30 April
Early registration at a reduced fee:	by 30 April
Normal registration at normal fee:	1 May to 31 May
Late registration at an increased fee:	after 31 May

Future AMESA National Congresses will take place as follows:

Year	Province
2015	Limpopo
2016	Mpumalanga
2017	Eastern Cape
2018	Free State
2019	Kwa-Zulu Natal
2020	Gauteng



Regional Activities

AMESA regions have been involved in various regional activities. Amongst other activities such as workshops and seminars, two key regional activities are the regional conferences and AGMs and participation in the Mathematics Challenge (sponsored by SASOL). The latter has been included in the report by SAMF elsewhere in this publication.

All regional conferences and AGM details are published in the AMESA website and it is very clear from the programmes of regional conferences and reports from delegates that AMESA is a very important and significant player in Mathematics Education both at regional level and nationally.

The table below shows a summary of the dates, venues and attendance at regional conferences.

Date	Region (Province)	Venue	Attendance
2 March 2013	Limpopo	Floropark Comprehensive School, Polokwane	264
11 May 2013	North West	Reebone Secondary School, Swartruggens	400
11 May 2013	Gauteng	WITS School of Education, Johannesburg	120
18 May 2013	Free State	New Education Building, UFS, Bloemfontein	150
25/26 May 2013	Mpumalanga	Kamhlushwa Primary School, Ehlanzeni District	250
20 July 2013	KwaZulu-Natal	UKZN, Edgewood Campus, Pinetown	150
21 September 2013	Northern Cape	NIHE, Galeshewe Campus, Kimberly	60
5 October 2013	Western Cape	Cape Teaching and Leadership Institute, Nooiensfontein Road, Kuilsriver, Cape Town	140
19 October 2013	Eastern Cape	Algoa College, Struanway, Struandale, Port Elizabeth	120

At each of the Regional Conferences, there have been both plenary sessions and parallel sessions. Thus, each regional conference had been structured to accommodate teachers from both primary and high schools. University mathematics education lecturers have also been involved in the conferences. At the Gauteng conference in May, there was a special focus group on mathematics lecturers from FET colleges.

A close study of the various Regional Conference programmes show common trends in the academic programme. Some of these common trends are highlighted below:

- **Euclidean Geometry and the use of programmes such as Sketchpad and GeoGebra in its teaching:** This is significant since Euclidean Geometry is no longer optional in the FET and teachers (as in the past) require further development when it comes to teaching Euclidean Geometry. Regional programmes also included Geometry (or Space and Shape) in activities for Foundation, Intermediate and Senior Phase teachers.
- **Probability:** This is a new section in the curriculum and teachers have not had adequate preparation or training for this section. Probability is covered mainly in the FET Mathematics parallel session although one region included it as part of Mathematical Literacy.
- **How I teach:** This has long been an important outlet for teachers to share their knowledge about teaching certain topics (from across the grades) with delegates at AMESA National Congresses. It is very significant that teachers can now do this at a Regional level as well. One of the key issues in teaching mathematics is the “classroom lesson delivery”. “How I teach” sessions can help teachers in this regard by keeping them updated with “best practices” in teaching specific parts of the curriculum.
- **The use of the calculator:** The calculator is an important tool for teaching and learning mathematics. Our long-term partnership with CASIO and recently, SHARP, has made it possible for AMESA members to be kept up to date with the latest calculator technology. In this regard, a CASIO Mathematics Market and a SHARP Mathematics feature strongly as part of our Regional Conference programmes, either as plenary sessions or as part of the parallel sessions.
- **Fractions:** One of the challenges of primary school mathematics is the teaching of fractions and it is significant that a number of regions included “fractions” as a topic for its regional conference. It is hoped teachers would have found new and innovative ways of teaching fractions in the primary school. The basics of fractions are very important for understanding and working with algebraic fractions in the high school.



- **Problem solving:** Problem solving competitions (Mathematics Olympiads) offers teachers an opportunity to expose their learners to “non-routine” mathematics problems which may be solved using very simple strategies. The SASOL Mathematics Challenge (for Grade 4-7 learners) and the South African Mathematics Olympiad (SAMO) for Grade 8-9 learners (Junior) and for Grade 10-12 learners (Senior) are examples of such problem solving competitions. It is clearly evident from regional conferences that regions would like to promote problem-solving competitions among primary school teachers. However, it is also important to promote these competitions amongst high school learners.

AMESA Regional Conferences are growing in stature and offer AMESA members (or potential members) a high level of academic engagement. These conferences can also provide a fertile ground for participants to submit their work for presentation at the AMESA National Congresses.

Minutes of the 19th Annual General Meeting (AGM) of the Association for Mathematics Education of South Africa, held on Tuesday 25 June 2013 during the 19th Annual National Congress in the Auditorium at the University of the Western Cape Bellville, Cape Town

Present:

Alwyn Olivier (President)
Vasuthavan Govender (Vice President)
Lorraine Burgess (Treasurer)
Isaiah Ronald Shabangu (Secretary)
Regional delegates as per register

Apologies:

Gary Powell, AMESA Western Cape Chair

AGENDA

1. **Welcome:**
The meeting commenced at 16:35
The President thanked and welcomed all present to the 19th Annual General Meeting of the Association. He outlined the meeting procedures, highlighting that by the Constitution all AMESA members have the right to speak, but only the National Office Bearers and the 98 regional representatives (calculated by the number of AMESA members per region) may vote.
2. **Finalising the Agenda:**
No new matters were added to the agenda.
3. **Minutes of the previous AGM:**
The minutes were adopted as a true reflection of the previous AGM happenings as proposed by Vasuthavan Govender and seconded by Rajendran Govender.
4. **National Council Report on the Activities of the Association:**
The Secretary, Isaiah Ronald Shabangu presented the National Council report on the activities of the Association. The report was adopted as proposed by Peter Manzana and seconded by Bridget Cameron.
5. **Financial Report and Budget:**
The Treasurer, Lorraine Burgess presented the 2012 financial statement as prepared by the auditors. The financial report was adopted as proposed by Prince Jaka and seconded by Patricia Walaza. Vice-President Vasuthavan Govender then thanked Lorraine Burgess for the work well done during her two term (2009-2013) as a treasurer.



6. President's Report:

The President, Alwyn Olivier presented his President's report emphasising the importance of teacher development in all the regions.

7. Motions:

The following motions for constitutional amendments proposed by the AMESA Constitutional Committee (Alwyn Olivier, Elspeth Khembo and Busisiwe Goba) were presented to the AGM by Alwyn Olivier. Each motion was adopted unanimously by the AGM.

Motion 1:

Add to Clause 35.4: "The Secretary shall be responsible for safe keeping of the minutes of all meetings so that they are always on hand for members to consult."

Insert Clause 51.12: "The Association's financial transactions will be conducted by means of a banking account with a registered commercial bank."

Replace Clause 58 with: "The Association shall continue to exist notwithstanding changes in the composition of its membership or Office Bearers."

Motion 2:

In Clause 49.6, replace "journal" with "journals".

Motion 3:

In Clause 53, replace "the President, the National Secretary" with "the President, Vice-President and Treasurer".

8. Elections:

For the vacant position of Treasurer, Rajendran Govender was elected unopposed for the period June 2013 to June 2015.

9. Any Other Business:

M. Mabalela raised a concern that the Department of Basic Education was introducing the subject Technical Mathematics in the Curriculum. Alwyn Olivier mentioned that AMESA was represented on the relevant committee and was making inputs concerning the matter.

10. Closure:

The President, Alwyn Olivier declared the meeting closed and thanked all members present for their valuable contributions.

The next AGM will be held during the 20th Annual National Congress of AMESA, 7-11 July 2014 in Kimberley in the Northern Cape.

The meeting closed at 18:00



Isaiah Ronald Shabangu
SECRETARY



Alwyn Olivier
PRESIDENT



The National Council Report: Presented by the Secretary of AMESA, Isaiah Ronald Shabangu at the AGM on 25 June 2013 at the University of the Western Cape

This is a factual report, presented annually to the AGM covering its activities, finances and other relevant issues.

The National Council

The National Council is annually constituted as follows:

The National Office Bearers

One Representative from each Region, appointed by the Region

Any persons co-opted by the National Council

The National Executive Committee (2012 - 2013)

Name	Designation
Alwyn Olivier	President
Vasuthavan Govender	Vice-President
Isaiah Ronald Shabangu	Secretary
Lorraine Burgess	Treasurer
Busisiwe Goba	Elected Member
Elspeth Mmatladi Khembo	Co-opted member

National Council Regional Representatives (2012 - 2013)

Region	National Council Representative
Eastern Cape	Wandile Hlaleleni
Free State	Vuyani Pop
Gauteng	Khangalani Mdakane
KwaZulu Natal	Busisiwe Goba
Limpopo	Steven Muthige
Mpumalanga	Philip Mokoena
North West	Sipho Vilakazi
Northern Cape	Jeffrey Thomas
Western Cape	Rajendran Govender

National Council Meetings

The National Council met twice between consecutive Annual General Meetings. The dates and venues of Meetings were:

1 - 3 February 2013 at African Tribes in Kempton Park, Gauteng

22 -23 June 2013 at The Protea Hotel in Durbanville, Western Cape

AMESA Annual National Congresses

More than 1000 members attended the 2012 Congress at the NWU, Potchefstroom Campus. Our heartfelt thanks to Hercules Niewoudt (Congress Director) and the LOC of Northwest for organizing Congress 2012 on behalf of the National Council.

The National Council thanked Rajendran Govender (Congress Director) and the LOC of Western Cape for organizing Congress 2013. The number of participants at the time of the AGM stood at 915.



Branches in the Regions

Each region is divided into a number of branches which report to the Region. At the AGM the number of branches and members was as follows:

Region	Number of branches	Members
Western Cape	4	257
Limpopo	11	238
Gauteng	9	231
KwaZulu-Natal	11	227
Mpumalanga	20	218
Free State	8	211
Eastern Cape	3	140
North West	8	126
Northern Cape	1	68
Total	75	1740

There was a decrease of 9 branches in the past year.

AMESA Committees

The National Council appoints committees to promote the aims and functioning of the Association. These committees are:

- Curriculum Committee
- Advertising Committee
- Sponsorship Committee
- Education Fund Committee
- SANCIMU
- Website

The Curriculum Committee

The Curriculum Committee coordinator is Vasuthavan Govender, assisted by Rajendran Govender. During Congress, for the Special Interest Group sessions, the National Council nominates members to chair the individual phase committees. During the 2012 Congress the chairpersons were as follows:

- FET Mathematics - Steven Muthige
- Senior Phase - Wandile Hlalaleni
- Intermediate Phase - Isaiah Shabangu
- Foundation Phase - Shanba Govender

Key curriculum issues were discussed during these phase committee meetings. These included CAPS training, teachers' content knowledge, assessment and teacher development.

During the past year, the following submissions were made by AMESA to the Department of Basic Education (DBE):

- Grade 12 Analyses on Mathematics and Mathematical Literacy Papers 1 & 2 (in November 2012)
- Grade 9 Annual National Assessment Analyses (in December 2012)
- A critique of the draft Technical Mathematics curriculum for technical high schools (in May 2013)

Advertising Committee

The coordinator of the Advertising Committee is Isaiah Ronald Shabangu. Other members of the Advertising Committee who served during 2012 - 2013 are:

- Lorraine Burgess (Treasurer)
- Alwyn Olivier (President)

AMESA published advertisements for the main sponsor, Old Mutual and in the journals (hardcopy and online).



Sponsorship committees

The sponsorship committee is made up of the following members:

- Alwyn Olivier (President)
- Neil Eddy
- Lorraine Burgess (Treasurer)

AMESA's Main Sponsor of publications and President's projects as per service level agreement is Old Mutual. The following allocations were made to AMESA by Old Mutual:

- R500 000 for 2012 - 2013
- R550 000 for 2013- 2014

Other major sponsors of AMESA in 2012 - 2013 were:

- CASIO (National, Regional & Branch Activities)
- Zenex Foundation (National Congress Speakers & Teachers)
- MacMillan (National, Regional and Branch Activities)

South African National Committee for the International Mathematics Union (SANCIMU)

The members representing AMESA in SANCIMU are:

- Renuka Vithal (Chair)
- Jayaluxmi Naidoo
- Busisiwe Goba
- Lindiwe Tshabalala
- Elspeth Khembo

The next meeting will be held on the 26 July 2013 at UKZN where the following will be discussed:

- 2020 ICME 14 Bid
- 2015 PME in South Africa
- ICSU Workshop (3 October 2013, Pretoria)
- South African representation in IMU- Busisiwe Goba

The AMESA website

The AMESA website is managed by Alwyn Olivier. Over the past year there have been more than 4000 visits to the AMESA website (amesa.org.za). The AMESA website contains very important information for those involved in mathematics. In the past year the following key activities were published on the website.

1. The South African Mathematics Challenge
2. Grade 9 ANA Analysis
3. Congress Announcements (First & Final)
4. Membership Form
5. Grade 12 Paper 1 & 2 Review for both Mathematics and Mathematical Literacy.
6. Regional Conference and AGM - details of programmes and agenda
7. Other important mathematics news

The May 2013 issue of The Teacher is now available online. It includes an article that was commissioned by Old Mutual and written for AMESA by Jacques du Plessis of the University of the Witwatersrand. This article also appears on AMESA's website.

AMESA Education Fund

This fund is available to regions on application to support professional development programmes. However, the amounts are not significantly high and no regions applied for funding. There are usually two dates. In the first semester, the closing date is 15 February and in the second semester, the date is 15 February.

Partnerships

AMESA is in partnership with the following organisations/government departments:

South African Mathematics Foundation (SAMF)

AMESA is one of the two affiliates of SAMF. SAMF arranges the following activities in partnership with AMESA:

- The South African Mathematics Challenge (Grade 4-7)
- South African Mathematics Olympiad (Grade 8-12)
- Problem Solving Course (Teachers)
- National Mathematics Week (Teachers & Learners)



The South African Mathematics Challenge papers for Grades 4-7 was set by the Problem Committee, which includes these AMESA representatives: Mark Rushby, Leon Roode, Nkhensani Duba, Steven Muthige, Nico Govender and Alwyn Olivier (Coordinator).

Some facts about the South African Mathematics Challenge:

- Nearly 60 000 learners from 279 schools participated in the First Round in 2012
- More than 5 000 learners from 169 schools participated in the Final Round
- More than 3 000 learners received Gold, Silver or Bronze Certificates of Achievement

The Challenge was sponsored by JET (Edcon) in 2012 but will be sponsored by SASOL from 2013. We express our thanks to SAMF for securing the funding.

South African Mathematics Olympiad (SAMO)

The SAMO falls under the SAMF banner and AMESA is represented in the SAMO committee by Marcus Bizony, Duncan Samson and Harry Govender. The following was noted about the participation of learners in SAMO over the past year:

There was a significant growth in the numbers writing in both Gauteng (senior) and in KwaZulu Natal (senior and junior). However, there was a slight drop in numbers writing in the Eastern Cape and Limpopo.

We would like more high schools to participate in competitions such as SAMO and other such competitions in the country. Both teachers and children can learn a lot from these competitions. To quote from the SAMF website about SAMO:

“... we continue to emphasize that Mathematics is about thinking and the discovery, and validation, of problem solving methods.”

National Mathematics Week (NMW)

National Mathematics Week also falls under the SAMF banner. The 2012 National Mathematics Week was celebrated from 30 July till 3 August 2012. Gauteng hosted the National Event held at the Tshwane Showground.

The following regions received funding from SAMF towards their NMW activities in 2012:

North West; Western Cape and Eastern Cape

Other AMESA regions participated in this celebration through various learner and teacher development activities. Unfortunately, from 2013 National Mathematics Week has become part of National Science Week and funding for this event is sourced through SAASTA (South African Agency for Science and Technology Advancement). AMESA will be attempting to find out more from SAMF and SAASTA about the way forward for Mathematics activities during this week

AMESA's other partners are:

1. Statistics South Africa (STATS SA)

AMESA enjoys a good working relationship with STATS SA's teacher development initiative, Maths4Stats. STATS SA provincial coordinators have arranged successful Data handling and Probability in the following regions of South Africa (in collaboration with AMESA): Mpumalanga; Gauteng; Western Cape; Free State; North-West and Eastern Cape

2. National Science and Technology Forum (NSTF)

NSTF is a broadly representative organisation which plays a powerful and consultative role in SETI (Science Engineering & Technology initiatives). AMESA is a member of the NSTF and is invited to its meetings. AMESA also receives regular updates on SETI initiatives in South Africa via the NASTEF eNewsletters.

3. Department of Basic Education (DBE)

AMESA has a good working relationship with the DBE. We believe that working together is the best way of developing learners and teachers. Thus, AMESA is always more likely to assist the DBE in its initiatives, rather than being critical.

AMESA is regarded by the DBE as key player in Mathematics Education in South Africa. It is consulted by the DBE on a number of issues regarding the teaching and learning of mathematics in South African schools. The DBE always appreciates AMESA's constructive submissions on various mathematics issues.

4. Department of Science and Technology (DST)

There has been concern about the numbers of top mathematics learners coming from the South African school system. Mathematics is a key school subject and success in school mathematics is likely to lead school learners taking up Science oriented programmes at Universities. The DST regards AMESA as a key role-player in helping with improving the success rate of mathematics learners. AMESA's very successful ESP (Educator Support Programme) for mathematics teachers was sponsored by the DST for 4 years.



AMESA Publications

Pythagoras

We would like to remind our AMESA fraternity about the important role that AMESA plays both nationally and internationally with the creation of new knowledge through its research journal, Pythagoras. This is captured apt in the Pythagoras website as follows: *“Pythagoras is an influential, frequently cited, accredited, peer reviewed and Open Access Journal which has been published since 1980. It provides a forum for the presentation and critical discussion of current research and developments in Mathematics Education at national and international level”.*

The evidence of the popularity of Pythagoras is noted by the article downloads and monthly online visitors as indicated in the next two tables.

Article downloads from Pythagoras from June 2011 until March 2013.

ABSTRACT	PDF	XML	HTML	EPUP
65 977	59945	14 514	4377	7017

Pythagoras: Monthly online visitor from June 2011 till March 2013.

Month	2011	2012	2013
January		1218	2091
February		1582	2401
March		1500	2382
April		1376	
May		1483	
June	8	1448	
July	276	1423	
August	375	1452	
September	472	1633	
October	814	2019	
November	774	1953	
December	1095	1997	
Total	3814	19084	6784



The Editor-in-Chief of Pythagoras is Alwyn Olivier. He is assisted by the following Associate Editors:
 Michael de Villiers (University of KwaZulu-Natal)
 Anthony Essien (University of the Witwatersrand)
 Rajendran Govender (University of the Western Cape) (He is also AMESA's treasurer)
 Dirk Wessels (Stellenbosch University)

Learning and Teaching Mathematics Journal (LTM)

Another of our important publications is the Learning and Teaching Mathematics Journal. This publication provides a medium for stimulating and challenging ideas relating to mathematics teaching and learning at all levels. It does not publish conventional research reports; rather it presents articles that describe or discuss mathematics teaching and learning from the practitioner's perspective. LTM is a peer-reviewed journal, published twice a year and distributed to all members of AMESA.

The Chief Editor of LTM is Duncan Samson from Rhodes University. He is assisted by Marcus Bizony (Bishops) and Lindiwe Tshabalala (Gauteng Department of Education).

Two issues were published in 2012. In 2013, LTM issues will be published in June 2013 and December 2013.

Learning and Teaching Mathematics is also available online to subscribers and to AMESA members through Sabinet's SA ePublications at http://reference.sabinet.co.za/sa_epublication/amesal

To access the journal online, paid-up members may request the necessary login details by e-mail from the AMESA secretariat at info@amesa.org.za.

AMESA News

AMESA News is published three times a year and keeps members informed about issues and events in AMESA. The current vice-president of AMESA (Vasuthavan Govender) is the editor of AMESA News. We thank all regions and other stakeholders such as SAMF and the ACM for contributing to articles for AMESA News. We encourage members to email branch and regional articles to news@amesa.org.za. Due to organizational challenges, we are behind with issues of AMESA News. However, we promise to address these backlogs by the end of 2013.

Membership

One of the challenges with regard to membership of AMESA is that members tend to be very tardy when it comes to renewing their AMESA membership. We appeal to our members to respond timeously to all requests for membership renewal. Noeline Tomsett, the membership secretary is present here at the Congress and those with membership queries should see her during this week. On Monday, 24 June 2013 AMESA membership stood at 1740 members. This represented a decrease of 70 members, when compared to 2012. We attribute this decrease to incomplete forms and 81 untraceable deposits (institutional/individual) between 08/02/2013 and 29/05/2013. We appeal to members to please fax/email/post both the deposit slip and membership form to our membership secretary. We would like to congratulate the Western Cape for being the leading Region with 257 members followed by Limpopo with 238 members. Keep up the good work at branch level.

Conclusion

As you can see ladies and gentlemen, AMESA is a very powerful and thriving organization. We would not be where we are at present if it was not for our membership body. We would like to thank all our members for keeping AMESA alive and vibrant in their regions. We also appeal to each and everyone of you to actively recruit more members for AMESA.

Thank you



Isaiah Ronald Shabungu
SECRETARY



Income and Expenditure Statement of AMESA for the Period 1 January - 31 December 2012

1. CURRENT ACCOUNT

EXPENDITURE				R1 299 926.68
Salary			R145 239.10	
SARS	PAYE		R8 258.80	
	UIF		R743.80	R9 002.60
Bank fees	Current account		R6 760.70	
	Internet fees		R2 361.57	
	Credit card facility		R4 627.80	R13 750.07
Telephone	Office - Landline		R4 351.14	
	President - Landline		R2 303.00	
	President - Cell		R7 804.00	R14 458.14
Council meeting	Jan 2012	Accom - meeting	R4 299.00	
		Travel	R10 357.00	
		Accom - meeting		R14 656.00
	Congress 2012	Accommodation	R45 730.00	
		Travel	R66 800.00	
		Registration	R14 560.00	R127 090.00
Old Mutual	Pythagoras		R182 317.92	
	AMESA News		R156 518.72	
	LTM		R62 506.20	
	Postage and stationery		R35 286.00	
	Congress 2012 - Registration		R0.00	
	Congress 2012 - First call		R19 861.08	
	Congress 2012 - Programme		R19 889.00	
	Membership cards and renewals		R13 882.54	
	Presidential Projects		R32 069.93	
	Regional development		R0.00	R522 332.11
Congress 2011				R84 376.00
Subsidies to regions				R82 381.08
Sundry Khembo (ESP + Nombulelo)				R172 773.95
JET Expenses				R1 230.00
Member fees - bad debts				R10 089.00
Education fund				R5 000.00
Congress 2012				R84 992.52
Printing - Stationary				R12 556.11

INCOME				R1 752 735.69
Old Mutual				R500 000.00
Sales and advertising				R14 642.30
Members fees	Cash, cheques, internet transfer	R430 783.53		
	Credit card	R1 920.00		R432 703.53
Congress KZN & North-West				R501 559.47
SAMF - JET				R236 000.00
Sabinet, Dalco and others				R67 830.39

SURPLUS

R452 809.01



Income and Expenditure Statement of AMESA for the Period 1 January - 31 December 2012

2. EDUCATION FUND

INCOME		R857.80		
Interest received	R857.80			
EXPENDITURE		R5 000.00		
Regional development	R5 000.00			
SURPLUS FOR THE YEAR			-R4 142.20	

3. CALL ACCOUNT

INCOME		R27 849.39		
Interest received	R27 849.39			
EXPENDITURE		R0.00		
SURPLUS FOR THE YEAR			R27 849.39	

4. RESERVE FUND

INCOME		R9 308.53		
Interest received	R9 308.53			
EXPENDITURE		R0.00		
SURPLUS FOR THE YEAR			R9 308.53	R33 015.72

TOTAL SURPLUS ON ALL ACCOUNTS

R485 824.73



Income and Expenditure Statement of AMESA for the Period 1 January - 31 December 2012

SUMMARY OF BANK ACCOUNTS UP TO 31 DECEMBER 2012

1. CURRENT ACCOUNT

Opening balance from 2012
Net transfer to call account
Net transfer to reserve fund
Surplus for year

R542 565.76

R89 756.75
R0.00
R0.00
R452 809.01

2. EDUCATION FUND

Opening balance from 2012
Surplus for year

R31 602.35

R30 762.55
R857.80

3. CALL ACCOUNT

Opening balance from 2012
Net transfer to current account
Surplus for year

R855 2920.03

R827 442.64
R0.00
R27 849.39

4. RESERVE FUND

Opening balance from 2012
Surplus for year

R213 423.16

R204 114.83
R9 308.53

TOTAL OF ALL ACCOUNTS

R1 642 901.30

NOTES TO SUMMARY STATEMENT

The Reserve Fund is made up of 4 fixed deposits and a 32-day notice account with balances:

Acc No	20-6641-6798	R 13 761.65
Acc No	20-6519-7783	R 76 540.05
Acc No	20-6245-3984	R 40 874.46
Acc No	20-6108-0279	R 32 653.83
Acc No	91-8606-5955	R 49 593.17
Total		R 213 423.16



AMESA NATIONAL BUDGET 2012

EXPENDITURE	BUDGET	ACTUAL
Salaries	R106 000.00	R154 241.70
Sundries	R30 000.00	R82 381.08
Printing and stationery	R15 000.00	R12 558.11
Council meetings and Congress 2012	R120 000.00	R141 746.00
Subsidies to regions	R150 000.00	R84 376.00
Telephone	R30 000.00	R14 458.14
Bank fees	R10 000.00	R13 750.07
Bad debts	R1 000.00	R1 230.00
Old Mutual sponsored projects	R450 000.00	R522 332.11
Auditor's fees	R10 000.00	R10 089.00
Congress 2012 (Seed)	R7 500.00	R7 500.00
Excess publication costs	R50 000.00	-
Continuous prof development JET	R100 000.00	R172 773.95
Education Fund		R5 000.00
Congress 2011 (R15 400) and 2012		R77 492.52
TOTAL EXPENDITURE	R 1 079 500.00	R1 299 926.68

INCOME		
Balance from 2011	R1 600 000.00	R 1 152 076.57
Membership	R450 000.00	R432 703.53
Other income (SAMF - JET - R236 00)	R600 000.00	R303 830.39
Sales and advertisements	R5 000.00	R14 642.30
Interest received	R40 000.00	R38 015.72
Old Mutual	R450 000.00	R500 000.00
Congress KZN and 2012		R501 559.47
TOTAL INCOME	R3 145 000.00	R2 942 827.98
Balance	R2 065 500.00	R1 642 901.30



AMESA NATIONAL BUDGET 2013

EXPENDITURE	BUDGET
Salaries	R200 000.00
Sundries	R30 000.00
Printing and stationery	R20 000.00
Council meetings	R150 000.00
Subsidies to regions	R180 000.00
Telephone	R30 000.00
Bank fees	R15 000.00
Bad debts	R1 000.00
Old Mutual sponsored projects	R500 000.00
Auditor's fees	R12 000.00
Congress 2013 (Seed)	R10 000.00
Excess publication costs	R50 000.00
Continuous professional development	R100 000.00
TOTAL EXPENDITURE	R 1 298 000.00

INCOME	BUDGET
Balance from 2012	R1 400 000.00
Membership	R500 000.00
Other income	R500 000.00
Sales and advertisements	R10 000.00
Interest received	R30 000.00
Old Mutual	R500 000.00
Congress KZN and 2012	
TOTAL INCOME	R2 940 000.00
Balance	R1 642 000.00



AMESA NATIONAL BUDGET 2014

EXPENDITURE	BUDGET
Salaries	R200 000.00
Sundries	R30 000.00
Printing and stationery	R20 000.00
Council meetings	R150 000.00
Subsidies to regions	R100 000.00
Telephone	R20 000.00
Bank fees	R15 000.00
Bad debts	R1 000.00
Old Mutual sponsored projects	R500 000.00
Auditor's fees	R12 000.00
Congress 2013 (Seed)	R10 000.00
Excess publication costs	R50 000.00
Continuous professional development	R100 000.00
TOTAL EXPENDITURE	R 1 208 000.00

INCOME	BUDGET
Balance from 2013	R1 600 000.00
Membership	R500 000.00
Other income	R500 000.00
Sales and advertisements	R10 000.00
Interest received	R30 000.00
Old Mutual	R500 000.00
Congress KZN and 2012	
TOTAL INCOME	R3 140 000.00
Balance	R1 932 000.00

President's Report



The Annual General Meeting (AGM) is about accountability - for the National Council to report to members about the governance of AMESA. The Treasurer's report is *financial*, the Secretary's report is *administrative*, and the President's report is characterised as *political*. With political I understand and here mean issues about policy making - the methods or tactics involved in managing AMESA, its relationships with other organisations, and visions about the direction we should go... It is supposed to be visionary, providing leadership - what are we all about, where are we going, and how do we get there?

Of course I and the Council have strong views on this, but it is impossible to unpack it here in a few minutes. I highlight and focus on some aspects for you.

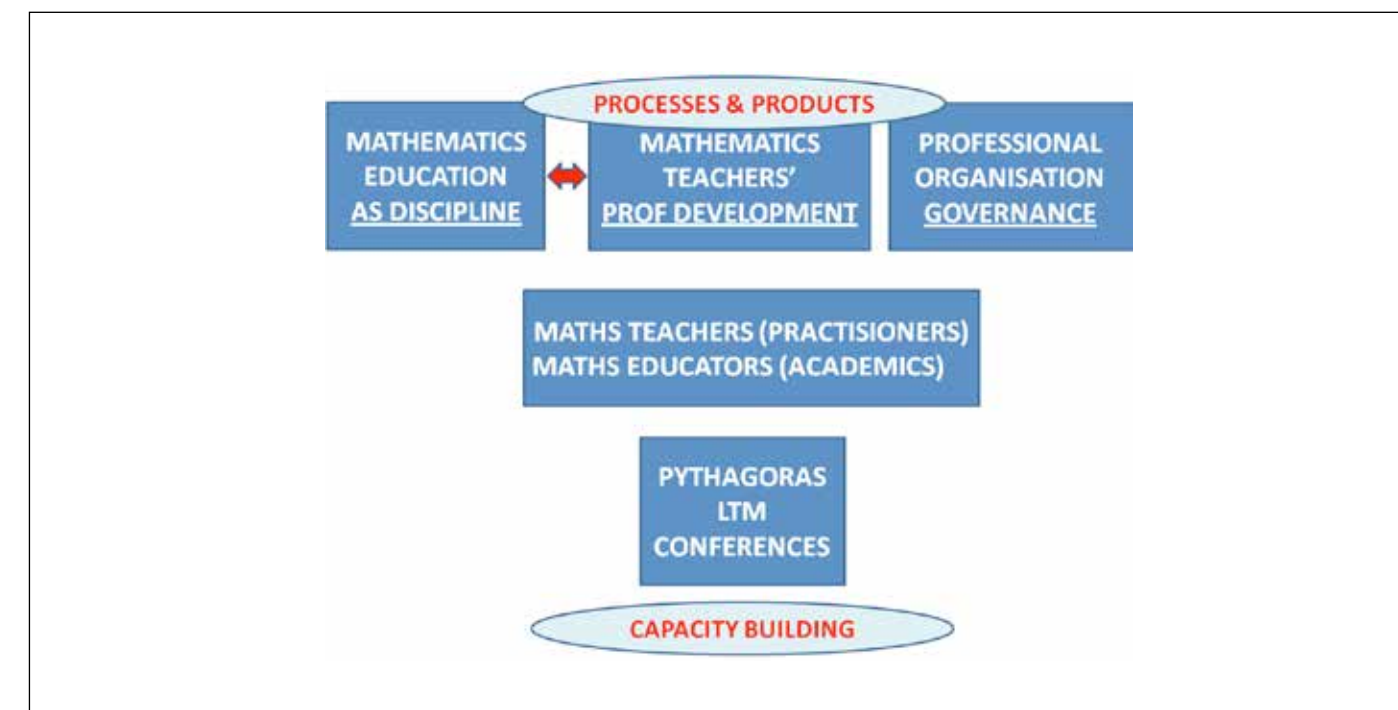
Of course our Constitution spells out exactly what we are about and how to achieve it:

The aims of the Association shall be, in general, to promote Mathematics education and, in particular, to enhance the quality of the teaching and learning of Mathematics in South Africa by providing a forum for all concerned with the teaching of Mathematics at all levels of education.

The Associations envisages attaining the main aims by the following strategies:

- encouraging **research** related to Mathematics education and bringing the results of such research to the attention of its members;
- formulating **policy statements** on matters regarding Mathematics education and promoting such perspectives among its members, policy-making bodies and organs in civil society involved in education;
- actively engaging in **mathematics education projects** which will result in the social, economic, political and cultural development of society.
- encouraging and assisting its members to strive towards a **high standard of professionalism** in the exercise of their profession.

My own view on our task is captured in the graphic below:



I emphasise that our main task, the necessary *prerequisite* to enhance the quality of the teaching and learning of mathematics, is to support mathematics teachers in their **professional development**. This we may do through launching special projects like the Educator Support Project (ESP) which we ran in 2006-2009. Our current focus is on a Problem Solving Course for Grade 4-7



teachers, a project funded by Sasol, in cooperation with the South African Mathematics Foundation (SAMF). We have developed course materials and workshop activities and we are developing 40 problem solving facilitators, who will be able to present workshops on problem solving to teachers in all the provinces in the next few years.

Our journals must support teacher professional development – *Pythagoras* by providing a research base for teacher professional development, and *Learning and Teaching Mathematics* by sharing teachers’ work at the level of the classroom, what I call the *wisdom of practise*.

But the route to teacher professional development that I want to emphasise, is through our annual national congress and regional congresses. We must organise rich congresses and workshops attracting large number of members, coming to congress eager to learn. We should create the kind of sessions through which they can learn - presentations made by academics, and presentations by teachers.

I now focus on this last point. Council is eager to facilitate more and better presentations by both academics and practitioners.

Firstly, it is absolutely essential that we get more practising mathematics *teachers* at all levels to share their wisdom of practice by presenting at AMESA congresses. That means that their ideas must be in writing, to be published in the Congress Proceedings and on the web, so that it can be accessed by others who did not attend their session at congress. To make this possible, AMESA must support our teachers! Firstly, we want our regions to identify and encourage teachers who could/should present at congress, e.g. teachers who presented something good at a Regional Conference. Regions should not only try to get their members to *attend* our national congress, but to *present* at congress! One idea for support is to let two or more teachers work together so they can support each other in writing up their presentation. A next idea is to arrange a mentor/coach for the teacher(s), e.g. a local “academic” or someone who has experience of writing, who can give the teacher(s) feedback on their efforts, or they can be co-authors ... Also, regions should offer teachers who present some financial support from the region - it has always been AMESA policy to give preference for funding to members who contribute to the congress programme. Please note that we have now officially changed our policy so that Congress Programme Committees will explicitly follow a “developmental” approach to reviewing Long and Short and How I Teach Papers. Papers will not merely be accepted/rejected, but reviewers will offer constructive feedback to enable authors to improve their papers and (if necessary) to go through more than one cycle of writing and reviewing. To manage this, papers must be submitted earlier than in the past: We have now made the date for submission of papers 20 February.

Also, Council is eager to build out the poster session format to make it a worthwhile and valued presentation! I call on everyone to please encourage and support teachers to submit poster proposals to congress!

Secondly, we need more mathematics *educators* (“academics” involved in pre-service and in-service teacher education) to attend and to present at congress! It is vital that we build out the Mathematics Teacher Education (MTE) strand, running concurrently with the FET, SP, ... strands in the congress programme - we need to fill all sessions in the MTE strand. I believe the way to achieve this is to build out participation in the Mathematics Teacher Education Special Interest Group. People involved in the “training” of mathematics teachers should present their ideas on teacher development, and discuss and influence each other towards becoming a community of practice with a shared view on good training courses for mathematics teachers. We all know that teacher training and teacher professional develop is key to the improvement of mathematics education in our country. And AMESA as the voice of mathematics education in our country should do something about it! The recent criticism of pre-service mathematics teacher education by the Ministerial Task Team for Mathematics, Science and Technology emphasises the urgency of the matter.

I briefly describe two initiatives by the National Council aimed at support and capacity building of our members regarding writing.

As a first initiative we have established a Special Interest Group on *Writing for AMESA*, covering writing for *Pythagoras*, *Learning and Teaching Mathematics* and for the *Congress Proceedings*. We have had at least one workshop session on writing at our national congresses since 2011, but there is definitely a need for us to build this out.

Secondly, we want congress Long Papers to be submitted to *Pythagoras* for possible publication! However, *Pythagoras* cannot re-publish a presentation that has already been published in a Congress Proceedings (that is called double-dipping, getting two rewards for the same article, and is deemed as unethical). Therefore, to keep open the possibility to later submit their presentation to a journal, we have introduced an option to authors: authors will be given the choice to publish accepted papers in the Congress Proceedings or not. If not, we will not publish the full paper, but only an extended abstract in the Congress Proceedings. In this way AMESA therefore offers authors support through the congress review process and further peer feedback at congress, to improve their papers towards eventual possible publication in an accredited journal like *Pythagoras*.

That is enough vision for now ...

Thank you



AMESA Curriculum Matters

AMESA Curriculum Committee

Coordinators: VG Govender (vicepresident@amesa.or.za) assisted by R. Govender (treasurer@amesa.org.za)

The AMESA National Council notes that our mathematics teachers face a number of issues in their roles as teachers. We would like our members to inform us of successes and challenges in mathematics teaching and learning. In this regard, our phase convenors could be used to channel your input. The phase coordinators (appointed by National Council in June 2013) are listed in the table below:

Phase	Convenor	E-mail
Foundation Phase	Busi Goba	gobab@ukzn.ac.za
Intermediate Phase	Isaiah Shabungu	irs1@vodamail.co.za
Senior Phase	Wandile Hlaleleni	lelomso@gmail.com
FET Mathematics	Jeffrey Thomas	jthomas@mailbox.co.za
FET Mathematical Literacy	Marc North	mnorth@telkomsa.net

Review of Grade 12 Mathematics examination papers: Report compiled by VG Govender

In 2012, more than 50 teachers were trained on how to review mathematics examination papers. These teachers and other AMESA members were responsible for giving input into AMESA’s review of both Grade 12 Mathematics and Mathematical Literacy papers. This review has been published on AMESA’s website.

To remind AMESA members on what goes on when analysing mathematics examination papers, I summarise the key points in this process:

Prior to the analysis

- Check paper at first glance for initial comment
- Ask a cross section of grade 12 mathematics learners for their views
- Ask other mathematics teachers from own school
- Ask mathematics teachers from neighbouring schools
- Make initial comments on the paper
- Substantiate comments by working on a possible memo

The cognitive levels

When analysing examination papers, one should have a clear understanding of the cognitive levels which are highlighted below:

Knowledge (Level 1)

- Recall
- Straight forward
- Seen before
- No struggle in working out the answer

Routine procedures (Level 2)

- Extension of what is known already
- Calculations without a lot of reasoning
- Recalling the procedure that needs to be followed

Complex procedures (Level 3)

- Route to the answer not evident from the information given
- Combination of different rules and procedures
- Knowledge and routine procedures implicit in problem



Problem solving (Level 4)

- Involves a high level of thinking
- Involves exact reasoning and application with thorough understanding of concept in question
- Order of procedures - hidden; not immediately able to put down on paper and solve

The analyses of papers

The papers are analysed according to the criteria below:

- Technical aspects
- Language used
- Coverage of syllabus
- Standard of paper
- Compliance with cognitive levels
- Comparison with previous year's paper
- Overall verdict

This is followed by a question-by-question analysis for each paper.

Procedures to be followed

A summary of the procedures to be followed is listed below:

- There should be at least 5 teachers from different schools who should be involved in the review at a particular site. There is no limit on the maximum number of teachers.
- Each site should decide on a strategy, work through all questions as a site or combine questions and ask groups to work on sets of questions.
- All input should be recorded on the specific AMESA template.
- Each site leader should check final input and then send to the regional chairperson.
- The AMESA regional chairperson will consolidate all the input and submit to AMESA (vicepresident@amesa.org.za) or fax to 041-4093723.

SAMF Advisory Committee on Mathematics (ACM)

AMESA is represented on the Advisory Committee on Mathematics which operates under the auspices of SAMF. In November 2012, the ACM was formally constituted with two members from AMESA and two members from SAMS. At present the ACM is made up of the following persons (serving up to November 2014):

Vasuthavan Govender (PE District) (AMESA Representative) - Chairperson
Rajen Govender (UWC) (AMESA Representative)
Werner Olivier (NMMU) (SAMS Representative)
Kosie Smit (US) (SAMS Representative) - Scribe

The ACM has commissioned three reports, two of which have been accepted and has already been published on the SAMF website. These are:

- A survey of selected schools on the 2012 Grade 9 Mathematics ANA results: Implications for teaching and learning
- Reflection on the training of teachers for the CAPS mathematics curriculum - a brief report

Four further reports have been commissioned. These reports would cover the following broad themes:

- Coping with the teaching of CAPS mathematics in classrooms (sequencing and pacing)
- Geometry readiness for the Grade 12 2014 examinations
- Mathematics teacher education in the 21st century
- Is there a need for a Grade 9 national examination in Mathematics?

The Phase Committee Meetings

Phase committee meetings form part of the AMESA National Congress Special Interest Group (SIG) sessions. In these sessions participants attend meetings appropriate to the phase they are teaching or are interested in. This year (2013) each phase committee chairperson were given a set of issues to discuss with participants.

These issues included the following:

- CAPS training, implementation and monitoring
- New content in the CAPS curriculum
- Assessment



- Teacher development
- Lesson planning and preparation (FET Mathematics only)
- The under-representation of Mathematical Literacy at the AMESA Congress (FET Mathematical Literacy only)

The Phase Committee chairpersons supplied the information. This has been edited to fit in with the format of AMESA News.

Foundation Phase (FP) (Meeting chaired by Busi Goba)

CAPS training, implementation and monitoring

- There has been a variation in the number of days for CAPS training (from one to two weeks)
- Only general curriculum matters were discussed during the training.
- No support from DBE on how to use workbooks yet these are compulsory.
- Some vagueness in the policy such as counting in 3s. What does this mean? Does it mean 3, 6, 9, ... or does it mean counting objects?
- Inclusivity not taken into account in the curriculum policy. Teachers in special schools feel that CAPS is not responsive to the needs of these teachers or learners. Support and guidance for these teachers is necessary.
- CAPS does not consider weak learners. There is no guidance on how to allow weaker learners to catch up.
- FP CAPS appears to be very fragmented. A 'big picture' scenario progression map is needed to outline the trajectory.
- Group teaching should be encouraged in the FP.

New content in the CAPS curriculum

- There was no content covered during CAPS training; SADTU trained teachers on content.
- Teachers are only covering certain aspects of the content omitting others, for example problem solving is avoided and easier areas (like data) are given more focus.
- There is a need to take account of the weighting prescribed by CAPS to design learning activities.
- SAMF is an organisation that can be approached to give support to teachers with problem solving courses.

Assessment

- CAPS does allow for differentiation, but the assessment is the same and no accommodation of learners of different abilities/ learning styles.
- Some teachers were not sure on how to grade learners according to prescribed levels and observing learners from one level to the next as they progress and required support from the DBE in this regard.
- ANA translations are an issue. There are inconsistencies and variations with the English versions.
- Slow learners need concession when writing ANAs. White paper 6 indicates that the slow learners need to be considered and translations are responsive to their language needs.
- Concession/exemption for slow learners in the Western Cape is given if necessary supporting documents are submitted to DBE. In this case, concession may mean that someone translates for the special needs child.
- FP teachers should be given feedback on the quality of the ANA, including language inconsistencies - communicated by AMESA whose input is valued by the DBE.
- Teachers' experiences regarding ANA: Teachers questioned why the use of manipulatives is encouraged in the CAPS document, but learners are not allowed to use manipulatives in the ANAs. Fractions, in written form, is NOT part of the CAPS curriculum, but this is included in the ANAs. They stated that according to the DBE, the ANAs are intended to be for the benefit of learners, but they complain that the ANAs are not being used this way.
- There was a complaint that FP teachers are training or coaching their learners to write the ANAs and NOT covering content or applying formative assessments. They were afraid of their learners doing badly in the ANAs.
- FP teachers complained about having Intermediate Phase (IP) invigilators for the FP ANAs. They believe that this was not appropriate for FP learners who are used to their own classroom teacher. Thus, the classroom environment, which should be as normal as possible for the ANA, is compromised.
- Is the language of the ANAs accessible to the learners? There was a general feeling is that it is not. Learners often cannot read the instructions of the ANA papers. Simple language is needed to ensure that the translations are 100% correct. Perhaps papers should be in both English and home language or allow teachers to read instructions to the learners as the mathematics paper is not meant to teach language.
- Are teachers supported to use ANA results to improve classroom practice? This is complicated by the time frame in that ANAs are written at the end of the third term and timing makes



it difficult for feedback to be relayed back to teachers and used where there is enough time to change practice. General finding is that teachers are teaching for success in the ANAs and not teaching for learning.

- What is the purpose of the ANAs and how can they be used effectively?
There is a need to do an item analysis so teachers have an idea of where learners need support.
The previous year's ANA results could be used to address issues that point to where learners are falling behind.
Each school should compile its own list and use these in an attempt to address learners' shortcomings.

Teacher Development

- It was felt that teachers needed support in certain areas of the work. This could happen if the DBE gives feedback early enough so the necessary interventions could be made.
- They should also 'own up' to areas where their learners are performing poorly and reflect on what they can do to change things.
- They could support each other by developing communities of practice where they take charge of the process of analyses themselves and devise programmes of support and remediation.

Intermediate Phase (IP) (Meeting chaired by Isaiah Shabangu).

CAPS training, implementation and monitoring

CAPS training for the Intermediate Phase had already been completed in 2012, although due to union action some teachers were not trained.

- The training occurred for 3 to 5 days but teachers complained that this training was not enough.
- Teachers wanted demonstration lessons to be included as part of their training but this was not done.
- The workbooks were delivered on time and these were to be used in combination with the prescribed textbook.
- They required more support and assistance from their subject/curriculum advisors.

New content in the CAPS curriculum

- The content in the intermediate phase was largely the same as the previous curriculum.
- However, there was a concerted effort by the DBE to streamline the curriculum to adequately prepare learners for the Senior Phase.

Assessment

- The assessment tasks which have to be done are largely the same although the weightings of tasks have changed.
- As in the Foundation Phase, teachers were concerned about the ANA in Grade 6. They were worried about the poor performance of their learners being attributed to them and were using departmental exemplars to revise with learners.
- Teachers also complained about large classes, making it difficult to support those who were performing poorly in their assessment tasks.

Teacher Development

- Teachers claimed that they were not receiving adequate support from their subject advisors. However, in some provinces subject advisors in the Intermediate Phase were required to also service the Senior Phase and this made it difficult for them to give their teachers individual support.
- Teachers believed that NGOs, Publishers and Universities may have a vital role to play in their professional development and should be encouraged to become partners of the DBE in assisting with teacher development.

Senior Phase (SP) (Meeting chaired by Wandile Hlalaleni)

CAPS training, implementation and monitoring

- CAPS training has taken place or has been scheduled to take place. The number of days for training varied from province to province with a minimum of 3 days and a maximum of 5 days allocated.
- Although the training which had taken place was largely satisfactory, some participants complained that their facilitators were ill-equipped to deal with the training and in fact were very poor.



New content in the CAPS curriculum

- There was not much new content for Grades 7-9. However, the emphasis on constructions was a concern as it was difficult to give individual attention in overcrowded classes; also these children do not have mathematics sets so teachers will have to improvise.
- The beginning of Algebra has now been moved to Grade 7, giving teachers more time to develop their learners.
- Teachers tend to use textbooks uncritically and need help on how to use the textbook judiciously.
- With the removal of Transformation Geometry from the FET, teachers noted that there was an emphasis of this section in Grades 7-9.
- The curriculum is too full so in order to cover all the topics teachers will be forced to use traditional teacher-dominated teaching methods rather than learner-centred ones.
- The language issue is always a problem in the senior phase as learners are still developing their skills in the LOLT. In this regard, teachers were advised to be more sympathetic to learners and encourage them.

Assessment

- The analysis of the ANA for Grade 9 shows an urgent need for teachers to improve their teaching of mathematics in these grades. In this regard, teachers should work with others in their clusters to share ideas and learn new teaching strategies.
- There is a need to teach for both procedural and conceptual understanding; the ANA exemplars provide typical examples for teachers to discuss with their learners. These example types should be incorporated into their lessons.
- In some areas of the country there are shortages of mathematics teachers due to redeployment. Classes are left without teachers and this affects learners performance in assessments such as ANA.

Teacher Development

- The teachers stated that they needed more support in teaching mathematics in the senior phase and would like assistance from organisations such as AMESA, Higher Education Institutions and the Department of Basic Education (DBE).
- They would like more hands-on workshops and teacher development programmes.
- They also wanted continuous support and monitoring from their subject advisors in the District offices.

FET Mathematics (Meeting chaired by Jeffrey Thomas)

CAPS training, implementation and monitoring

- The training was scheduled for the winter holidays and comprised 2 to 4 days. Two days were regarded as inadequate.
- Some training was disrupted by teacher unions.
- Facilitators at previous training sessions (2011 and 2012) were fairly competent.
- The training materials were good.
- Not all teachers remained for the full duration of the training.
- The training covered both general and content specific matters.
- There was a debate on the scheduling of such training (school holidays or school time?).

New content in the CAPS curriculum

- There was quite a bit of new content in the curriculum, especially those topics which were optional.
- This included Euclidean Geometry where teachers had to become reacquainted with Geometry theorems and concepts and how to teach these.
- The other new topic, Probability, was covered very well during the training. Trigonometry was also included in the training.

Assessment

- The CAPS document spells out how many assessment tasks should be completed each term but teachers wanted to do more with their learners. However, they were not sure how to make these additional tasks count.
- There was agreement that more shorter tests and assignments (although not counting) will help improve learners understanding of key concepts.
- There was a concern about the lack of correlation between the SBA marks and those of the final examinations.
- There was a concern about the time used for examinations and its effect on teaching time.



- Teachers complained about the quality of the common exam papers and asked that the DBE ensures that the papers are moderated thoroughly.
- Due to the plethora of common tests and examinations across provinces, teachers asked for assistance in developing their own papers for use at their school. There should be a reasonable split between what teachers set and what the DBE sets in Grades 10 and 11.

Lesson planning and preparation

- The teachers reported that they were not doing lesson plans anymore.
- They believed that one cannot predict how a lesson will unfold and finding out where learners make mistakes is more important than the lesson plan.

Teacher Development

- Teachers were happy with their own development and their attendance at the AMESA Congress was proof that they were willing to improve themselves.
- They believed that they should get incentives for attending the CAPS training.
- They also asked for additional support in content areas such as Trigonometry, Euclidean Geometry and Probability.
- Those of them who attended the GeoGebra workshops claimed they learnt a lot but required further support.

FET Mathematical Literacy (Meeting chaired by Marc North)

CAPS training, implementation and monitoring

- Not all Grade 12 training had taken place but plans were in place for this to happen in the second half of 2013.
- The quality of training (for Grades 10 & 11) had significantly improved from previous years.
- Curriculum content was also included in the training manual.
- The number of days for training was 2-4 days.

New content in the CAPS curriculum

- The content for Mathematical Literacy was largely unchanged.

Assessment

- There was no notable changes to the assessment in Mathematical Literacy.
- However, participants believed there was a need to change the structure of Mathematical Literacy examinations.
- The first option discussed was doing away with two papers and rather having only *one examination paper* in which each broader question or context includes questions posed at each of the four levels of the taxonomy. In this way the weaker learners would be provided with increased access to the more difficult application questions through the presence of lower-level / scaffolding questions.
- The second option was to continue with the two examination papers, but with each examination paper comprising questions drawn from all four levels of the taxonomy. Importantly, however, the papers cannot be distinguished according to content - this is due to the fact that in contextualised problem solving scenarios the use of integrated content and skills is essential.
- A further alternative is to publish beforehand the contexts that will be dealt with in the examination papers. This will, perhaps, limit the effect of the ‘unfamiliarity’ of a context on the learners’ abilities to engage successfully with a contextualised problem situation. This would also, perhaps, reduce the need for increased scaffolding in the Paper 2 examination.

Teacher Development

- Mathematical Literacy teachers have made very good progress as teachers over the past 7 years.
- Although the Mathematical Literacy pass rates were very high, teachers believed that they could become even better with additional support from their Subject Advisors and other stakeholders such as AMESA and Universities.



The under-representation of Mathematical Literacy at the AMESA Congress

- A concern was expressed regarding the perceived ‘under-representation’ of Mathematical Literacy at the AMESA Congress. The limited numbers of workshops / papers / HIT sessions that deal with issues relating to the subject Mathematical Literacy, together with a complete absence of a plenary session or panel discussion that deals with a topic relating to the subject, were cited as evidence of this under-representation.
- This scenario is becoming particularly prevalent and concerning given that - at least at FET level - more learners are now enrolled in Mathematical Literacy than in Core Mathematics.
- Participants further discussed whether the under-representation of workshops / papers / HIT sessions reflected a general disinterest in the subject or a possible low level of confidence amongst practitioners in the subject due to the relative ‘newness’ of the subject.
- To remedy this situation all present were encouraged to send through presentation proposals on Mathematical Literacy topics for the 2014 AMESA Congress, and Jackie Scheiber and Marc North volunteered to provide assistance with the proposal process for those who are uncertain of what to do.

Trends emerging from the phase committee meetings

I would like to highlight the following trends which emerge from the AMESA Phase committee meetings:

- CAPS training for all grades/phases had taken place or was scheduled to take place but the concern was the number of days allocated for training.
- Content was covered in all training except for the Foundation phase. In the FET, there was an emphasis on Euclidean Geometry and Probability.
- Although the number of SBA (School Based Assessment) tasks were now at a more acceptable level, teachers could give their learners additional tasks (in the form of shorter tests & assignments) to improve learners’ understanding of the work.
- There seem to be an emphasis on preparing well for the ANAs (FP; IP and SP) with teachers being afraid of their learners performing poorly. However, some teachers tended to focus only on the ANAs at the exclusion of teaching mathematics content.
- High school teachers were concerned at the high number of common assessment tasks/examinations set at provincial or district level which used quite a bit of teaching time.
- Although teachers, across all levels, tended to be largely satisfied with their own performance, they felt they could do with more support from key stakeholders in education.
- There was a need for collaboration amongst teachers in learning community forums where they could share ideas and learn from each other.
- The inclusion of constructions in the Senior Phase curriculum was a step in the right direction for conceptual understanding of basic geometry. However, its teaching could be difficult in large classes and where learners do not have mathematical instruments.
- Concern was expressed at the under-representation of Mathematical Literacy workshop/HIT/ paper sessions at the AMESA Congress and this seemed to feed into the so-called “lack of interest or confidence” in Mathematical Literacy as a school subject. This was ironical since more learners were doing Mathematical Literacy when compared to Mathematics.

The trends emerging from the reports of the AMESA Phase committees gives us a “birds-eye” view on the state of mathematics at our schools. Embedded in these reports are possible solutions to some of the challenges mentioned. AMESA has a powerful role to play in this regard and is a key partner of the DBE in its efforts to support mathematics teachers at local and regional level.



Mathematics Teacher Education Special Interest Group

Report by Prince Jaca

The AMESA Mathematics Teacher Education Special Interest Group met during the 19th AMESA annual National Congress on 27 June 2013. There were 58 members present at the meeting.

Chairperson's opening remarks

The Chairperson, Caroline Long, outlined that the aim of the meeting, and the aim of the Special Interest Group in general, is for AMESA to participate and make contributions to the Mathematics Teacher Education (pre-service as well as in-service) discussions in South Africa. She gave a brief background detailing what has happened in the past. The group met at every national congress since 2009, but she expressed concern that the Special Interest Group seems to be coming together only during national congresses. Between congresses there were no activities in the form of discussions and/or other activities to take forward any resolutions taken during the meetings. We would like to change that ...

The Chairperson suggested the following questions to guide discussions at the meeting:

- What are the concerns of mathematics teacher education?
- What is to be done?
- What and where are the success stories?
- Any other?

What are the concerns of teacher education?

A number of issues and concerns were raised during the discussion. These included:

- Why 'we' were not attracting to mathematics teacher education students with a strong base in mathematics?
- Why students would become teachers if they are always bombarded with 'messages' that constantly tell them that teachers/teaching are bad? (Negative perception about teachers/teaching)
- Do we have shared principles as to what content and pedagogical knowledge is relevant for teacher education across institutions? (Pedagogical Content Knowledge and Mathematics Content Knowledge)
- What were the minimum admission requirements for Mathematical Literacy teacher education?
- What were strategies to develop Mathematical Literacy teachers? (Training of Mathematical Literacy teachers)

What is to be done?

Some suggestions were made during discussion to guide attempts to address some of the concerns:

- Good stories need to be told about mathematics education. There may be a need to design some strategy to use the same media to tell good stories about mathematics education.
- In our teacher education programmes, we need to teach more school mathematics and how to teach it.
- Classrooms are for *learning* - we need to go beyond preparing material for effective teaching and design material for effective learning.

The meeting further resolved:

- We should advocate for a panel session on Mathematics Teacher Education at the next AMESA national congress.
- We should have a special call for papers on Mathematics Teacher Education for the next AMESA national congress to have more Higher Education Institutions colleagues attend congress and have more presentations and interaction in the Mathematics Teacher Education strand at congress.

During the discussion on this point and guided by interest raised on certain issues three special subgroups were proposed:

- *Mathematics Literacy Teacher Education subgroup*: This subgroup is coordinated by Marie Weitz. Anybody who is interested in discussing issues related to Mathematical Literacy teacher education is welcome to engage with Marie Weitz (maria.weitz@wits.ac.za).
- *Primary Teacher Education subgroup*: This subgroup is coordinated by Sharon McAuliffe. Sharon is currently working on re-curriculum of their BEd programme (Foundation and Intermediate Phase). Anybody who is interested in discussing issues related to primary teacher education is welcome to contact Sharon (mcauliffes@cput.ac.za)
- *Secondary Teacher Education subgroup*: Anybody who is interested in discussing issues related to secondary teacher education is welcome to contact Caroline Long (caroline.long@up.ac.za) or Prince Jaca (pjaca@wsu.ac.za).



Our international guest reflects on his visit to South Africa



How SA can find safety in numbers: Reflection of a United States Mathematics Educator in South Africa for the first time. - Zalman Usiskin

This report was first published in the Mail & Guardian and is now reproduced in AMESA News with kind permission of the newspaper. Zalman Usiskin is an emeritus professor at the University of Chicago and director of the university's Chicago School Mathematics Project, a position he has held for 26 years. He was an invited speaker at our recent congress at the University of the Western Cape.

Teaching is a middle-class job. A teacher can earn a living at teaching but no one gets rich on a teacher's salary. Teaching is also a job that looks to the future, that helps to prepare young people for the rest of their lives. And so one might say that the more than 900 teachers who gathered at the 19th annual national congress of the Association for Mathematics Education of South Africa represented South Africa's middle class, important players in South Africa's future.

They came to the University of the Western Cape during their school holidays, and almost all were assembled in the hall for the opening of the congress, waiting for the association's officers, university and government officials, plenary session speakers, and other dignitaries to march in. As the opening plenary session speaker, I was among the marchers, and immediately on entering the hall was startled to hear a group of about 15 or so young children singing Gaudeamus Igitur.

"It's South Africa in 2013," I thought, "yet I am hearing a famous European university song that is hundreds of years old, one quite familiar to me." I started singing with the children, harmonising as I always do when a part song is being sung. It comes naturally to me to do this, because my mother was a professional singer and through high school and college I spent much of my spare time singing in choirs and directing them.

When I was certified to teach maths, music – choral music – was the subject that I used to fulfil the requirement of having a second subject to teach. A full hour was allocated for the speeches and another ceremony that took place before I was to speak. This is a long time and a speaker learns to sit stoically through boring welcomes and other rituals. But almost immediately there was a disruption in my stoicism. The national anthem was not just being sung; it was being sung in four parts. The audience was singing like a well-trained church choir, pulsating the verses in a manner that was unlike any I had ever heard before. People of all races all singing together – this was the new South Africa. I took out my handkerchief from my pocket.

I had already been in the country for a week, at the University of the Witwatersrand and the University of Pretoria, speaking and meeting with people, discussing curriculum, teaching and learning. But universities are special places, "ivory towers" some call them, shielded from everyday life. They tend to be liberal and they are filled with people with a common purpose of probing knowledge, learning and teaching. Race does not play as great a role. Although I had met many currently practising teachers, they were subdued in the university environment. Now I was seeing the teachers as they are, for the first time. At opening sessions of conferences at universities, it is almost always the case that a university official will greet those assembled. Accordingly, Brian O'Connell, the rector at UWC, stepped to the podium. He and I had chatted for 10 minutes or so at breakfast just a few hours earlier, and I expected the usual low-key welcome with a description of the university, both stated with pride. Instead, Brian delivered an emotional call to arms, recounting the role of UWC in dismantling apartheid, pointing out the poor performance of South African students in international maths assessments, and exhorting those assembled to seize the present and work to raise the level of education for all South African students significantly.

In that moment, I wished I had prepared a different talk. My talk, about what it means to understand some maths, was analytical, not the kind of talk that rouses a crowd. But I can be passionate, as I was on one occasion in Pretoria when I said: "What sense is there in manipulating symbols if you don't know a purpose for them, if they do not answer a question or solve a problem in which someone might be interested? If the teacher of algebra does not convince students of the utility of the subject, beyond its being on a high-stakes test, who will?"

But here, in the opening session of the congress, my talk had been prepared and could not be changed. How could I follow Brian O'Connell? Fortunately, there were a number of speakers in between us, and there was a short break while people left the stage. As an outsider who still has never visited one maths classroom in South Africa, I cannot presume to know what is going on in them, and my views come only from what I heard. It does seem that learning maths as symbol manipulation without meaning permeates the entire curriculum, and that the maths we use every day is not related to the maths needed for business, science, engineering, and all the other advanced areas that use maths.



How else can one explain that at grade 10 the curriculum divides into two distinct tracks, maths literacy and (pure) maths, where even a successful student in the former track cannot transfer into the latter?

How else can one explain that the (pure) maths track does not seem to include any real-world applications of maths? Taking hope of eventual success in maths away from the large numbers of students in the mathematical literacy track is not a way to increase their performance.

My impression is that the range of student performance in South Africa is about the same as the range in the United States, but the distribution is significantly different. In the US, perhaps 15% of students are very high-performing, as a group equal to students anywhere in the world. They form the upper end of a normal distribution with a mean about equal to the countries of Europe and a larger standard deviation than most. South Africa has its high performers — often from private schools — but does not seem to have as much of a middle-performing group and has a large group of low performers, some lower than any in the US. South Africa's low performers are generally found in the townships and rural areas, whereas the lowest performers in the US tend to be in the cities. But there are commonalities in our two countries. In both, these places of low performance have high unemployment, fewer families with two parents, and a much lower standard of living than places with high-performing schools. The schools are often under-equipped with books and support materials and, most seriously, understaffed and with an undereducated current staff. I heard of many places in South Africa where there are 45 or more pupils in a class, and of primary school teachers who know essentially no arithmetic.

In the US, a class size of about 25 is the norm, a class size of over 35 is uncommon, and even the poorest-prepared of teachers know some of their subject. My particular interest is in curriculum; that is, what maths is expected to be learnt, who is expected to learn that maths, and at what ages.

From what I heard, the following are great needs in South African school maths:

Disassociation from pure rote. A large body of research dating back to before World War II has shown that "meaningful" learning produces better skill performance than rote learning. This meaning can derive from showing why the maths works; that is, discussing general properties that underlie the needed processes. Alternatively, the meaning can derive from real-world applications familiar to the student or from concrete materials or pictures that mimic the processes. In contrast, it seems that everything from basic facts to complex algorithms are taught in some classes with no meaning whatsoever. If a child cannot check his or her work in some way other than to repeat it, then the child does not have command of that content;

Incorporation of applications. Maths is a language through which we attempt to describe and understand the physical, biological, economic and social world. A look at a daily newspaper demonstrates that in today's world numeracy is not separate from literacy but a part of it. Maths permeates the descriptions of current events, editorials, advertisements, sports, business and weather. Every physical object has a geometry. We cannot get along without computers, and computers run on maths. The ability to deal with data and statistics is needed by everyone, for everyday tasks such as where to shop and what to buy and for high-level governmental and business decisions that affect millions. Accordingly, all over the world today's students are being asked to learn more maths than their parents or grandparents encountered in school, and this maths tends to be peppered with real data and real applications. South Africa seems behind in this regard;

Technology. South Africa also seems behind most of the rest of the developed world in its use of calculators and computers to assist in learning and doing maths. For instance, calculators are required on 70% of the Singapore national test required of all students in grade six. In the US, graphing calculators have been ubiquitous in high schools for almost two decades and their use is required on college entrance tests. In Australia, parts of the new national curriculum include the use of computer algebra systems that can automatically do the manipulative skills that have been commonly taught in algebra through calculus, and the national tests assume students have this technology. These developments make it possible for students to deal with a host of interesting situations and problems that would be difficult to approach without the technology, but they are particularly threatening in classrooms and for people whose only experiences in maths at school have been with these manipulative skills; and

Teacher training and teacher recruiting. Even in today's computer age, in which — at least in theory — students can learn through instruction delivered electronically, the future of any country's educational system depends on the teachers it has to guide those students. Here is where a great strength of South Africa lies — in people like those I saw and met at the congress. I heard the horror stories of teachers sitting in teachers' lounges and not teaching, but I also spoke with many teachers who are teaching six classes a day with large numbers of young children in each class. It is obvious that to get the most learning from our children, conditions in schools must make it enjoyable to come to school, and there must be knowledgeable teachers guiding students everywhere along the way. In the countries that perform the best on international assessments, teachers are valued and, as a result, some of the best students enter the teaching profession. For South Africa, the creation of a large and well-educated teaching force in schools would seem perhaps to deserve the highest priority.



What does it mean to understand some mathematics?

Extended abstract of the opening plenary by Zalman Usiskin,
University of Chicago

To understand mathematics *as a whole* would entail a discussion of the roles mathematics plays in everyday personal affairs, in schooling (e.g., as a sorter), in occupations, in other fields such as physics, and in its existence as a discipline studied for its own sake. In contrast, this talk is primarily concerned with what it means to understand some mathematics, which generally means to begin with a bit of mathematics and to subject it to detailed analysis, usually from the perspective of the learning of that bit.

It is natural for mathematics educators to view mathematical understanding from the standpoint of the learner, whether that learning is for use in life, for use on a job, for personal enjoyment, or for a test. But the *full* or complete understanding of mathematics in schools requires more than the learning perspective. It includes the understanding of mathematics also from the standpoints of *educational policy* and the teaching of mathematics. For mathematicians, the understanding of mathematics includes an understanding from the standpoint of those who invent or *discover* new mathematics.

We in education act both as behaviourists and cognitivists. As behaviourists, we want students to answer questions correctly and sometimes do not care how they got their answers. As cognitivists, we want to know what students are thinking as they work with mathematics and we ask students to show their work. In the 1970s, Skemp (1976) used the phrases *instrumental understanding* and *relational understanding*, essentially meaning *procedural understanding* and *conceptual understanding*. I agree with Skemp that instrumental and relational understanding are different but I do not agree that they are different subjects. I view them as different aspects of understanding the same subject. Also, I believe there are more than two aspects or types of understanding, as different from each other as Skemp's two types, but all different aspects of understanding mathematics. I call these aspects *dimensions of understanding*.

Educational policy towards mathematics includes the selection of content to be covered in school, who should encounter that content, and when. The understanding of the invention or discovery of mathematics from the mathematician's perspective has been the subject of many books. The understanding of mathematical invention would not be complete without consideration also of the inventors, mathematicians themselves, through the many available biographies.

Mathematics deals with both concepts and problems. On problem solving, Polya's *How To Solve It* (1957) has long been a seminal work. It is significant that the first of his four steps of problem solving is *understanding the problem*.

To exemplify understanding of concepts, in this presentation two dissimilar concepts are discussed: *multiplication of fractions*, an arithmetic operation, and *congruence*, a geometric relation. For each, we discuss vocabulary and the dimensions of understanding. It is the ability to apply all the dimensions to these ideas that makes them concepts and not uni-dimensional entities.



TABLE 1: Dimensions of understanding.

Dimension of understanding	Range		
	From	Through	To
Skill-algorithm (calculators and computers included)	application of an algorithm	selection and comparison of algorithms	invention of new algorithms
Property-proof	justification of a property	derivation of properties	proofs of new results
Use-application	application of a concept	use of mathematical models	invention of new models
Representation-metaphor	representation of an idea	analyses of representations	Invention of new representations
History-culture	historical facts and demographics	analysis and comparison of mathematics in cultures	discovery of new connections or historical themes

The dimensions of understanding detailed in Table 1 have common qualities. Each has supporters for whom that dimension is preeminent, and who believe that the other dimensions do not convey the real essence of the understanding of mathematics. Each dimension has aspects that can be memorized yet the potential for the highest level of creative thinking. The dimensions of understanding are relatively independent in the sense that they can be, and are often, learned in isolation from each other, and no particular dimension need precede any of the others.

For over a quarter century, we have been using this multi-dimensional framework to help guide our development of the University of Chicago School Mathematics Project materials for secondary schools. By keeping a watchful eye on these dimensions, we feel that our materials become richer and reach more students than if we did not implement this broad perspective.

The teacher is an applied mathematician whose field of application involves the classroom and the student. Like other applied mathematicians, in order to apply the mathematics, the teacher needs to have a good deal of knowledge about the field itself - that is, the about the educational process - as well as about mathematics. Thus, the understandings that a teacher needs involve more than mathematical understandings. The teacher also must take into account students, classrooms, teaching materials, and the necessities of explaining, motivating, and reacting to students.

To understand the teaching of mathematics, teachers must have pedagogical content knowledge, an understanding of mathematical concepts, an understanding of mathematical problems, and be able to integrate and connect these ideas to other mathematics and the other subjects studied in school.

ABOUT MEMBERSHIP RENEWALS

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We call on members to **renew your membership as soon as possible** to ensure that you continue to receive AMESA services and benefits.

When making the payment, please **ensure that you enter your name and/or membership number in the reference section of the transaction**. Then fax or e-mail the form and proof of payment to the AMESA office. This will ensure that your membership renewal will be recorded.

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Reflections on the 2013 AMESA Congress by Melissa Tweedie, a teacher at Port Alfred High School in the Eastern Cape, attending her first AMESA Congress

One of the features of a post-congress AMESA News is to get the views of first-time attendees and to give their views or impressions of the Congress. We hope that these views or impressions will encourage other mathematics teachers to attend AMESA Congresses.

For this edition of AMESA News, we are pleased to have the reflections of Melissa Tweedie from the Eastern Cape. She teaches Mathematics (Numeracy) to pre-primary learners. She attended the AMESA Congress as a result of her association with the South African Numeracy Chair (under Melony Graven) at Rhodes University. This was her *first* AMESA Congress and she also presented a "How I teach" paper.



Here are her reflections:

First impression when we arrived: Overwhelmed!!!! As I have never had the privilege of attending an AMESA conference before, I had no expectations.

Transport: Brilliant! I was comfortable and felt very safe. Thanks again to Debbie and Varonique!!

Accommodation: Comfortable, warm, even though I was sharing a bathroom with the whole floor I always had a hot shower!



Catering: Well organised in Student Centre but took some time in the Residence dining hall - stood in a queue for over an hour on Monday morning for breakfast but decided to turn this experience into something positive and learned a lot about patience! I enjoyed the food very much.

Social Programme: Functions themselves were well organised, but unfortunately the admin side very poor - to have one list for all those people? Some colleagues had booked and paid for functions but were not on lists, excursions were over booked etc.

Memorabilia: There were not enough sizes or they ran out of various items, they did say they would post any outstanding orders?

Programme: Lovely variety of presentations, but so many that I wanted to attend were presenting at the same time. Some sessions were in small venues so there was a limited number of people that could attend.

Brief summaries of presentations and discussions I attended:

Monday:

- **Refining a learning network for 'doubling and halving' and 'odd and even numbers' - Nicky Roberts and Heather Mills.** Thinking of different ways of teaching these concepts. Use the sharing model (quotative) and grouping model (partitive) for doubling and halving. Using definitions of doubling and halving to be specific about parity creates at least 6 conceptual nodes. Basically this topic shows the connections between the 3 concepts. Some nice ideas of how to use different equipment and representations and children as manipulatives.
- **I am thinking of number - Ursula Rontsch.** I loved this game. It is not a silent lesson, develops vocabulary, whole class activity, develops social skills and cognitive skills. Teacher is able to assess a number of areas while children are playing. Using an empty number line is a quick way for children to work out an answer.
- **Children's muscle skill development and mathematical concepts. How are they linked? - Heleen Johnson.** Very interesting. Shows how reaching certain milestones as a child can affect the development of certain mathematical concepts. Age 9-12 months, Crawling (most important phase to develop spatial concepts, spatial concepts that underpin mathematics understanding - vocabulary e.g. in, under, over etc.). Age 12-18 months, turning corners etc. (underpins problem solving). It is important to create a space where children can internalise maths concepts.

Tuesday:

- **Panel 1- Mathematics vs the curriculum: What's the score? Panellists Piet Human, Karin Brodie and Zalman Usiskin.** The point of the first speaker was not very clear to me: "What is Mathematics anyway?" The second speaker: "Building mathematics and the curriculum". My understanding of what was conveyed is that Mathematics and the Curriculum are linked and not against each other. Teachers need to reflect on lessons, with a focus on errors. Teachers need to learn to link meaning, bring in previous knowledge and link arithmetic to algebra. Enjoyed the third speaker: The discipline vs the school subject". The speaker used examples that I could relate to. He discussed the different dimensions of understanding. Teachers must ask meaningful questions to find out whether children have made the connection with mathematical concepts.
- **Plenary 2- Curriculum development minus teacher development = mathematics education - Hamsa Venkat.** Very insightful. The reality is how well do our teachers know mathematics? It is important for teachers to co-knowledge and we should teach mathematics we can communicate. There seems to be a lack of awareness of how givens and unknowns figure within the problem solving process, systematic and completeness as important aspects of mathematical working. Knowing mathematics means being able to use mathematical concepts mathematically therefore the two cannot be separated.
- **Geometry in art and design - Susie Groves.** Interested in Art so really enjoyed this presentation. We were active and were encouraged to work with the MATHOMAT. I feel I have a better understanding and appreciation of pattern, the MATHOMAT, Tessellation, Polya - founder of problem solving in mathematics. Just love this quote: 'The essence of mathematics is not about making simple things complicated, but making complicated things simple.'



Thursday:

- **Foundation Curriculum Phase Committee - ran out of time so we didn't finish what was on the agenda.** General feeling was to take guidance from the CAPS but also take guidance from your class. We need a 'birds eye' view of the curriculum of foundation phase. CAPS training doesn't cater for children with disabilities. Experiences with ANAS - doesn't always follow the policy, curriculum lets you use manipulatives but ANAS doesn't. ANAS should be supportive and not just to see performance. Grade 1-3 struggle to read exam papers, teachers feel we should be testing mathematics and not language (reading skills).
- **Using dice and card games in mental maths sessions - Debbie Stott, Nombulelo Yaleka.** Fantastic session! So privileged to be a team member! Participants enjoyed it and we had some very positive Feedback. Debbie is an absolute natural! She is an inspiration to all of us!

Friday:

- **How I teach shape using geometric solids - Melissa Tweedie.** I was quite nervous! Felt like such an amateur compared to all the other presentations I had attended! Had a fantastic lady Chairing our sessions, Gabeba Agherdien. She gave some positive feedback from my and Elvira's presentations and was impressed with what we are presenting to our Grade R learners. Really enjoyed presenting my lesson!
- **How I teach numeracy using 3D cubic building blocks - Elvira Milborrow.** Elvira presented very well and the participants enjoyed the practical activities.
- **Plenary 5 - Zain Davis.** An interesting talk on connections, continuity and coherence in Mathematics. Teachers need to develop an understanding of the hierarchy in mathematics in schools. Was very 'academic' at times so a lot went 'over my head'. Wish I was at a level at that I could have understood more.

Overall reflection: Despite the problems we had with registration, excursions, queues, etc., we were there for one important purpose: MATHEMATICS. The general feeling that Mathematics in Schools is in a dismal state is a reality but the fact that the AMESA conference was so well attended by teachers and practitioners proves that an effort is being made to rectify this problem. I found the conference to be most informative and enriching. I feel I have developed professionally. I have been able to network with other teachers and mathematics advisors from around the country. The people at the UWC who organised this event were friendly and helpful. I really had a wonderful experience at the conference and have come home feeling inspired and committed to teaching Mathematics.

Many thanks to Prof Graven, Debbie, Varonique and SANC (Rhodes) for all your support and financial assistance.

Our remarks

We would like to thank Melissa for her frank but informative reflections on our AMESA Congress in Cape Town. We hope that these reflections will serve as a motivation on three levels:

- Firstly, that AMESA Congresses offer a high level of academic engagement at all levels.
- Secondly, AMESA members can share their expertise with others through papers, workshops or "How I teach" sessions; and
- Finally, encourage those on the outside of AMESA to become members of a vibrant mathematics organisation (AMESA) and to participate at local, regional and national level where appropriate.

It is pleasing to note that Melissa will also be doing a presentation at the Eastern Cape Conference of AMESA on Saturday 19 October 2013 in Port Elizabeth.



Report on academic visit to Curtis Math Centre, UCLA, USA

by Michael de Villiers (e-mail: profmd@mweb.co.za)

Michael de Villiers is Professor of Mathematics Education at the University of KwaZulu-Natal, South Africa. He is an active AMESA member. He reports on his visit to the USA in March 2013

I recently had the privilege of visiting the Curtis Mathematics Centre at the University of California in Los Angeles (UCLA) in USA from 10 - 25 March 2013 and thought that sharing some of my observations from visits to schools, interaction with teachers and their enrichment programme for talented learners in mathematics, might be of interest to the broader AMESA community. I also wish at the outset to thank my university and the South African Mathematics Foundation (SAMF) for providing partial funding for this academic visit. I will report firstly on the two schools that I visited, then on a workshop I had with beginning teachers in the California Mathematics Project, and lastly on the Los Angeles Mathematics Circle, which is their enrichment programme.



School visits

The first school that I visited was the Sussman Middle School on the outskirts of Los Angeles where a group of 4 mathematics teachers were following and trying to implement the Japanese Lesson Study Method. This was being done under the supervision (and direction) of their 'subject advisor', who seemed very well read and knowledgeable about mathematics education generally¹. The Japanese Lesson Study Method involves teachers jointly planning lessons together and after observing each other, they would meet again and review their lesson plan and try to improve.

On this particular day they were busy designing a lesson using geoboards for Grade 6 learners to find the areas of rectangular shapes (such as the floor plan for a house). After the first meeting and discussion of the lesson plan, the first teacher gave the lesson with his class while the other teachers were observing and taking notes. After that lesson, they got together and discussed what worked well and what seemed not to work so well. After making some suggestions to improve the lesson the next teacher gave her lesson to her class, and the process was repeated until 4 teachers had taught the lesson.

The teachers used group-work and the geoboard as manipulative, and throughout all 4 lessons, they encouraged classroom discussion among learners, and elicited different responses from them regarding the worksheets. Instead of starting by given learners a definition for area, they used the useful teaching principle of Ausubel to first ascertain learners' understanding by asking what the word 'area' meant to them as they had already encountered the concept in the previous grades. It was noticeable that several students seemed to confuse area and perimeter, for example, writing down that area was the distance around some figure while some just wrote down the perimeter formula for a rectangle $2(l + b)$ or for a square as $4s$. Other learners just wrote down the area formulae for rectangles or squares respectively as $l \times b$ or s^2 , clearly indicating a rule-oriented and limited conceptual understanding of area.

After some discussion and argumentation between the learners, and some guidance from the teacher, but not much, each class arrived at a definition of area more or less as finding the number of squares in a figure. Learners were also asked to make a figure with an area of half a square unit on their geoboards, which they all did. However, in the last lesson right at the end, one learner strongly objected and argued with others in the class that the area of the figure shown in Figure 1 was not $5 \frac{1}{2}$, but only 4 as only 4 complete squares could fit into it, and that the 3 fractional parts (half-squares) should be discarded. Unfortunately the bell rang and the time was up to sort this out satisfactorily, but the teacher promised that they would discuss this further in the next lesson. This episode showed a limitation in this learner's personal definition of area, or rather in his personal interpretation of area, as just counting the 'complete' squares in a figure, which might not have surfaced in this classroom if discussion and dialogue was not encouraged as much as it was.

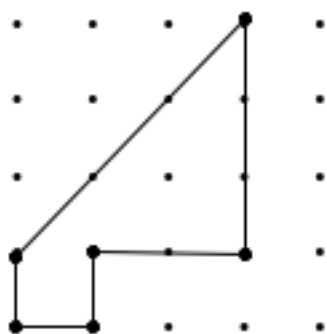
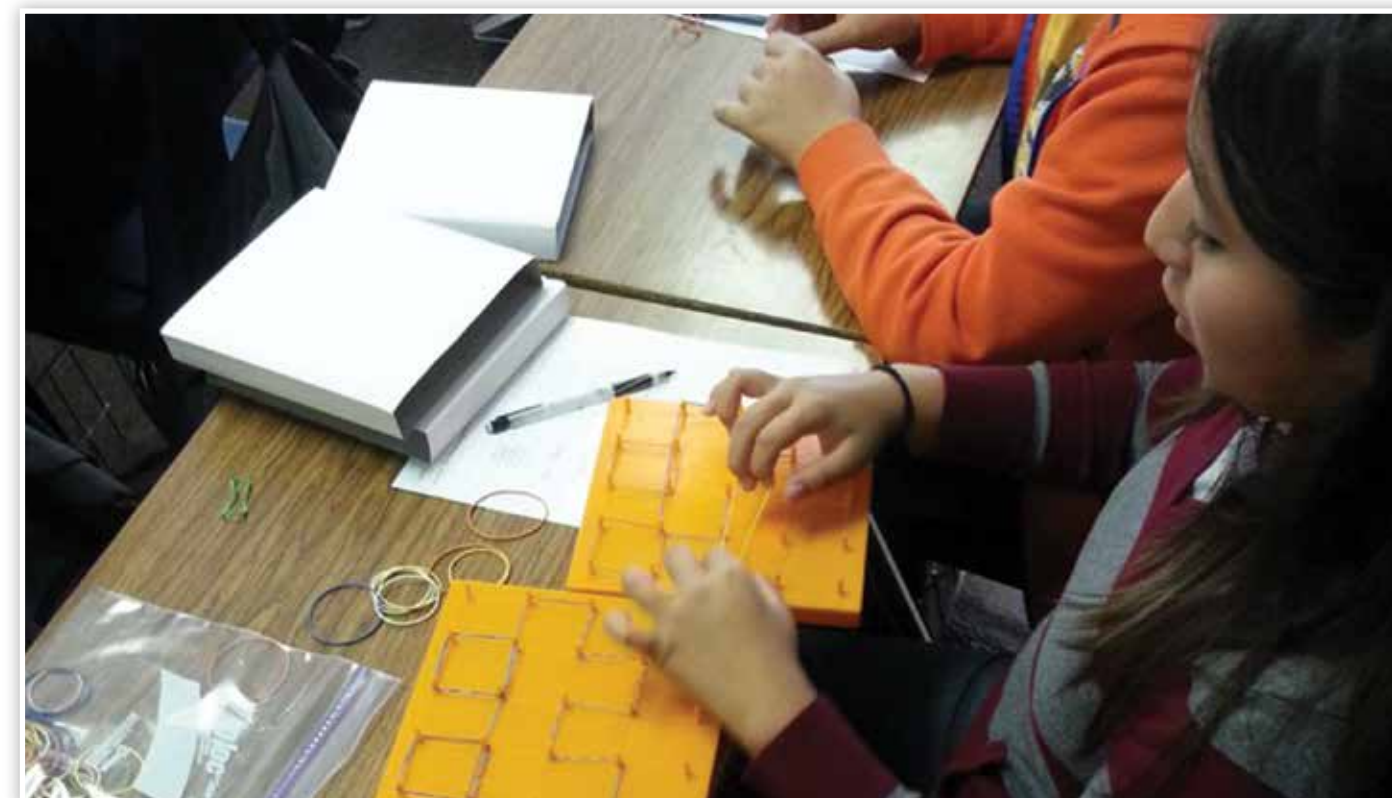


Figure 1

One might criticize the lesson by not first developing a more general concept of area and also not using a real world problem-context for finding area. This might have been

¹ Visit his website which has great lessons with video clips, photographs and worksheets which focus on modeling and solving contextual, real-world problems for Middle and High School ranging from estimating the number of the stars in the universe to determining the amount of paint necessary for painting a wall. Go to: <http://robertkaplinsky.com/lessons/>



achieved by perhaps starting with the problem of estimating the surface of an irregular, non-rectangular shape, for example, like a kidney-shaped swimming pool for which one wants to buy a safety net. Children could, for example, overlay square-grids on a scale drawing to estimate the total surface of the pool.

However, the interest and willingness of the teachers at this school to learn together and cooperate with each other, as well as professionally critique one another was exemplary. One can only hope they will continue developing professionally in this way. The children in all the classes were definitely actively engaged and the culture of open discussion between the children in groups and in the whole class context was excellent. Children corrected each other by these discussions and it helped teachers to identify misconceptions that needed attention. On the walls of the classroom there were also various mathematics poster, including one on tips about "Classroom Discussion Builders", grouped into three categories, namely "Presenting Alternative Ideas", "Expanding on Others' Ideas" and "Posing Additional Questions", and in the one lesson the teacher specifically directed the learners' attention to this poster during the class discussion.



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The other school I visited was a project school by UCLA in the city centre of Los Angeles: both a middle and high school section. Academic staff members from UCLA conduct research collaboratively with teachers at various grades in these schools, and they also provide an innovative practical context for their own prospective mathematics teachers to see theory, research and practice meet in class. At this school, I visited a Grade 4 class where the children were doing a problem worksheet on various aspects of fractions such as equivalence, conversion, scaling up and down, and several other applications. The teacher showed good sensitivity to learners’ methods and allowed children to explain on the whiteboard how they solved the problems, which in some cases varied quite a bit.

The other class I visited was a small class of Grade 11-12 learners grouped together doing some algebra and solving of equations involving square roots. Again the teacher did not ‘lecture’ to students formally, but interactively engaged with them asking for suggestions on how they thought an equation like the following could be solved:

$$\sqrt{2x+2} = x$$

to which they one learner immediately responded by saying they should square both sides, and then solve for x . The teacher then reminded them that they needed to check whether both solutions were valid, and they found that the one solution did not satisfy the original. Using a graphing calculator the learners in each group were then also asked to plot the graphs of $y = \sqrt{2x+2} = x$ and $y = x$ on the same axes to obtain the figure shown in Figure 2.

The graphical representation helped learners ‘visually’ see why only one of the algebraic solutions was valid. However, just as in South Africa, no mention was unfortunately made of also solving the problem purely numerically, for example, by guessing and checking (by hand and/or using a calculator or spread-sheet), or by using iteration or more advanced methods like Newton-Raphson (which is done in A-level mathematics in the UK and several other countries). The technological skills of the learners with the graphing calculator also varied quite considerably, and while some easily zoomed into the intersection of the graphs, others didn’t know how to do that and had to be shown either by the teacher or other learners.

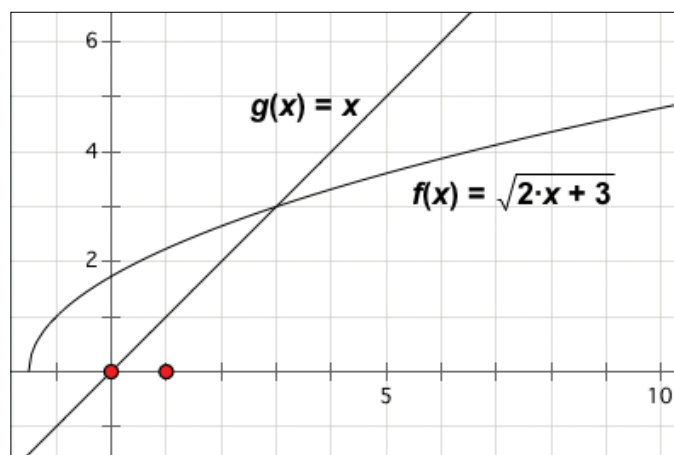


Figure 2

In summary, these two schools I visited had relatively small classes between about 18-25 learners, compared to a much higher average in South Africa. The schools were well resourced with good textbooks, computers, fast internet, and had teachers access to relevant manipulatives and computing technology. However, it should be noted that these two schools are not typical of the USA either, as there is a wide variation between schools in the USA as well. Much more than the resources and the small classes, the teachers’ willingness to adopt and implement, and reflect, on a learner- and problem-centered approach in their teaching, impressed me. The learners in all classes I attended seemed very motivated to learn and participated eagerly in the discussions.



Los Angeles Math Circle

I also attended one session of the UCLA Math Circle, which is free and open to elementary, middle school and high school students interested in mathematics and eager to learn. Due to lack of space and staff numbers are limited and some selection takes place. Students are divided into 3 groups: Junior circle (elementary school students, grades 1-4); Group A (students in grades 4-7) and Group B (students in grades 7-12). They meet weekly on Sundays and activities include problem-solving sessions, expository talks on various topics, and preparation for the American Mathematical Competitions (AMC8, AMC10, AMC12, AIME, USAMO).

This was a very interesting and informative experience for me. Each group had a graduate student in mathematics (or a mathematics dept. staff member) in charge and was supported by 3-4 undergraduate students who acted as ‘tutors’. Each group operated more or less independently from each other and some of them were further subdivided into smaller groups mostly according to grade, as well as ability.

In the first Group for more or less Grades 6-8, the students were given a Monthly Contest of 6 problems for which they were supposed to submit written solutions by 14 April 2013. They were given opportunity to start working on the problems right there, and the students worked individually and collaboratively in groups of about 3-4 on the problems, and the tutors only gave hints or pointed out errors in their arguments. The level of the problems were quite high compared to the 3rd Round of our Junior South African Olympiad and the first four are presented just to give a flavour of the level of problems:

1. Show that a number of the form $4k + 3$ is not a sum of two squares.
2. Find all digits a, b, c (so $a, b, c \in \{0, 1, 2, \dots, 9\}$) such that $ab^2 = cab$.
3. Show that all the numbers of the form 1007, 10017, 100117, 1001117, ... are divisible by 53.
4. Show that $\frac{1}{\sqrt{x} + \sqrt{1006}} + \frac{1}{\sqrt{2012 - x} + \sqrt{1006}} = \frac{2}{\sqrt{x} + \sqrt{2012 - x}}$ has exactly 2013 integer solutions.

The students were all given some (needed) direction in no. 2 to understand the problem, and it was explained to them that the symbols were place holders and not variables, for example, if $a = 1, b = 2, c = 3$, then $ab = 12$ and $cab = 312$.

There was only one geometry problem, namely no. 5, but it was unfortunately ill defined, which was noted by me and mentioned to the Group Leader who then pointed it out to the students. The problem was subsequently ‘removed’ from the contest, but not replaced by another one. It was interesting that one of the students, a girl, complained loudly when the only geometry problem was removed, because she said she liked geometry problems most. The last problem dealt with some set theory and use of Venn diagrams.

Though most of the students seemed very confident in tackling the problems, I noticed that there were a few who struggled and after a while they turned either to the children next to them or the tutors for some hint for a plan of attack. Some were weak in algebraic representation and manipulation, but given the age of the children I suppose they were all ‘above average’ relative to their grade level. Each student had a file with notes of previous sessions in the term and I noted that they had already done some probability, geometry (with proof) as well as an introduction to graph theory. Some attention had also been given to general problem solving heuristics such as looking at special cases, finding patterns, drawing diagrams, etc.

The next group I attended for about 30 minutes were the Grades 1-3 where they were doing a Quarterly Review, which dealt with questions among others, about properties of the Mobius strip, probability, logic, and some more or less traditional word problems that could be solved either by guessing and checking or by using algebra. Again the children were impressive with their confidence, and sometimes inventive, intuitive ways of solving problems, and especially not been afraid to hazard a guess, or make a mistake. The last 45 minutes I spent with the Grades 3-5 where the children had been divided into teams and were tackling a variety of problems that were elaborate, but fun word problems, and/or involved number patterns. The teams scored points for the correct completion of problems, and children were asked to come and present their solutions in front to everyone else. The Group Leader (who was a Math Dept member) or children from the other groups would then point out errors in the presentations, if any. Though several students gave incorrect arguments and solutions, some really elegant and innovative ones were also presented. All the children were involved and interested in tackling the problems and were also critical of others’ solutions, and eager to point out errors, though applauded good solutions.

Some of the problems (puzzles) presented to the children in this group seemed to have been deliberately ‘ill-defined’ or ‘ambiguous’ such as the following one:

“Nicolas and his son and Peter with his son went fishing. Nicolas caught as many fish as his son, and Peter caught three times the number of fish that his son did. If the total number of fish caught were 30, how many fish did Nicolas catch?”

The ‘trick’ that the Group Leader apparently wanted from the students was to assume that Peter was the father of Nicolas, so that if the number of fish caught by Nicolas is represented by x , then Peter and his son would respectively have caught $3x$ and x respectively. This gives us the equation and a solution as follows: $x + x + 3x = 30 \Rightarrow x = 6$. In fact, one of the students very



shortly after the problem had been shown, immediately said that he ‘knew’ the problem and suggested that Peter was the father of Nicolas. In the subsequent discussion the Group Leader provided the solution above, and also pointed out that if Nicolas was the father of Peter, that would produce the following equation $x + x + x/3 = 30$, which does not have integer solutions. Then they moved on to the next problem.

This bothered me as in my opinion they should also have discussed the situation if neither of the two previous scenarios was the case. For example, if Peter’s son caught y fish, this would have given the equation $x + x + 3y + y = 30$, and explored the integer solutions for that as well. It’s all very well to encourage lateral, creative thinking, but given that no explicit relations between Nicolas and Peter were stated, one couldn’t just conveniently assume they existed so as to eliminate one variable.

Lastly, because two teams were tied, the children from those two teams were given the following ‘tiebreaker’: Continue the number pattern: 1; 11; 21; 1112; 3112; 211213; ...; ...; which I immediately recognized as one that, I think, the mathematician John Conway a few years proposed as a tricky joke. Again I thought this was an unfair question as it doesn’t involve any mathematics, but is simple a clever word play expressed in numbers. For example, the second one refers to the first and is ‘one one’, therefore written as 11. The third one similarly refers to the previous one and is ‘two ones’, hence written as 21. The next one is therefore ‘one one’ and ‘one two’ written as 1112, etc. Besides not involving any mathematics, if one has not seen this before, it’s not likely that one would decipher this pattern. Indeed the children couldn’t find the pattern, and the Group Leader eventually explained the solution and declared a tie. Though some puzzles like these are fun and do encourage some lateral thinking, and were clearly enjoyed by the children, a few more substantive problems could’ve been used that didn’t require some ingenious ‘trick’, but could be solved by simply applying creative, mathematical thinking.

All tutors, including the Group Leaders are paid a small stipend from funding they have for the Project. For the children, however, it is entirely free. The Director of the Programme who is from the Mathematics Dept. at UCLA is credited the equivalent workload of a yearlong module. One of the problems in South Africa currently is that similar enrichment programmes here often have to be done over and above other duties, and not many Mathematics Departments at University are as supportive. More over, funding in South Africa is usually not provided unless some strong research component of some sort is attached to the project.

Regarding selection of students for the programme, they do not have a formal admission test, but they do write to schools to inform them and to invite their top talented students for consideration to the programme. Quite often parents also hear of the programme, and then approach them directly. In admitting students to their programme they look at school marks, performance in mathematics competitions or other evidence of mathematical talent. If they feel that students are not keeping up sufficiently with their peers, they ‘gently’ inform the parents to withdraw them, but usually children who ‘fall behind’ become disinterested and drop out on their own.

Most of the notes of the LA Math Circle are available online at: <http://www.math.ucla.edu/~radko/circles/archive.shtml>
This is a valuable resource for not only our own teachers at school here in South Africa who want to start their own Mathematics Clubs or Circles, but also to the Siyanqoba Olympiad Training Programme of the South African Mathematics Foundation (SAMF).

International Seminar at the Park City Mathematics Institute (PCMI) in the USA

Report by Vasuthavan Govender

Introduction

I was one of two AMESA members were selected by the National Council of AMESA to attend an International Seminar in Mathematics Education (in June/July 2013), in Park City, Utah in the USA. Due to its limited size and highly structured agenda, participation in the International Seminar is by invitation. The International Seminar on Mathematics Education at PCMI began in 2001. The intent of the seminar is to foster conversation among six to eight nations from around the world about the teaching of mathematics. Participation of a mathematics educator and a high school teacher by each country is by invitation only. I attended in my capacity as a University Mathematics Education lecturer. Jobe Sambo from Mpumalanga was the other member. He attended as a mathematics teacher.

This seminar also serves as a resource for PCMI Summer Session participants to interact with outstanding professionals from around the world who study the teaching and learning of mathematics.



Countries and presentations of each delegate:

Country	Mathematics Teacher	Mathematics Education Lecturer
Honduras	Status of geometric transformations in the K-12 curriculum	The role of technology in developing understanding of geometric transformations
Finland	Status of geometric transformations in the K-12 curriculum	The role of technology in developing understanding of geometric transformations
Slovenia	Status of geometric transformations in the K-12 curriculum	Do you agree or disagree with the statement that the concept of transformation can be a unifying concept in geometry? Explain
China	Status of geometric transformations in the K-12 curriculum	Do you agree or disagree with the statement that the concept of transformation can be a unifying concept in geometry? Explain
South Africa	Status of geometric transformations in the K-12 curriculum	Training of prospective teachers on geometric transformations
USA	Status of geometric transformations in the K-12 curriculum	Training of prospective teachers on geometric transformations

This part of the seminar was very informative and enriching for all delegates. We learnt about mathematics teaching and learning in other countries, in general, and about transformation geometry, in particular.

Working on seminar brief

We were then divided into groups where we had to work on articles for international publication. I was part of a group which comprised the following persons: Johnny Lott (USA); Leif Osterberg (Finland); Shiqi Li (China).

We worked on the following article: “The Use of technology in the Teaching and Learning of Geometry through Transformations”. By the end of the seminar, we had completed the article and it has now gone out for review. It will be published in the next AMESA News.

Other information

The International Seminar was part of a wider Mathematics Institute Summer Programme. While we were busy with the international seminar, mathematics teachers (both primary and secondary) were attending mathematics courses/workshops in other venues. There were also those who were busy with their doctoral or post-doctoral research which they were sharing with others.

We attended a two hour session with the teachers. They were busy with problem solving activities. My group consisted mostly of young teachers. It was a very interesting interaction. I found them more interested in “trying to get an answer” rather than initially engaging with the problems and trying to make sense of the contexts in which the problems were set.

It was not all work. We had the opportunity to tour Salt Lake City (40 km away) and participated in the Park City Parade on American Independence Day (4 July)

I was very impressed with the objectives of this Summer programme and the way it was organised. I believe that we can learn from such a programme and it is possible for us to organise something similar in South Africa.



International delegates at the Park City Math Institute





Working on our article



The Mormon Tabernacle Choir Auditorium in Salt Lake City



The Old Mutual Foundation Advertorials

AMESA and the Old Mutual Foundation co-operate to publish a series of advertorials published in the *Mail and Guardian* supplement, *The Teacher*. The articles were written by Jacques du Plessis of the University of the Witwatersrand on behalf of AMESA.

The preamble to each article is:

Education is key to the future of South Africa, which is why the Old Mutual Foundation supports education initiatives that build excellence in mathematics and science at Secondary schooling level. A key priority is to increase the national mathematics and science pass rates to address critical skills shortage in the economy.

Our key education projects include AMESA (the Association for Mathematics Education of South Africa). The Old Mutual Foundation assists with the printing and distribution of AMESA publications which includes Pythagoras, Learning and Teaching Mathematics (LTM), AMESA News and AMESA Annual Congress materials.

We publish the next two articles in the series.

THE OLD MUTUAL FOUNDATION PRESENTS:

Approaching trigonometry from different angles Procedural and conceptual understanding

Jacques du Plessis | University of the Witwatersrand, Johannesburg



Trigonometry needs to be understood as more than just a set of procedures. Understanding *procedurally* means broadly to understand how and when to action a sequence of steps when solving problems (Rittle-Johnson, Siegler & Alibali, 2001). Procedural understanding denotes dynamic and successful utilization of particular rules, algorithms or procedures within relevant representation forms (Haapasalo, 2003). Kilpatrick, Swafford and Findell (2001) define procedural fluency as the skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. So procedural knowledge has to do with procedures and algorithms; the steps that are executed when solving a problem. Procedural knowledge is often described as fragile knowledge, as it is almost always connected to a particular context.

Understanding *conceptually* means to understand explicitly or implicitly the principles that govern a domain and the interrelations between units of knowledge in a domain (Rittle-Johnson et al., 2001). Conceptual understanding refers to the integrated and functional grasp of mathematical ideas, which enables learners to learn new ideas by connecting those ideas to what they already know (Kilpatrick et al., 2001). Conceptual understanding supports retention, and prevents common errors. It is knowledge that is flexible and can be applied across domains. Understanding conceptually implies that one understands relationships between mathematical ideas and entities.

It must be emphasised that procedural and conceptual understanding develop simultaneously in support of one another; they are two sides of the same coin. They do not have a dichotomous relationship, but exist in support of one another. One cannot understand conceptually, without being fluent in basic procedures, and procedures are always linked to some underlying conceptual understanding. Trigonometry is unique in this sense, since it is the domain in mathematics where this relationship can be used and developed simultaneously.

When teaching trigonometry we have to make conceptual relations clear to learners by attending explicitly to the connections between ideas in trigonometry and broadly across domains in mathematics. We must allow learners to wrestle with ideas and concepts, avoiding at all times the temptation to reduce conceptual problems to procedural problems. Each skill must be revisited



several times in different contexts, and in trigonometry it is almost always possible to approach a problem from several different perspectives. This can be used to the advantage of trigonometric learning in general, as we will show in the next illustrations.

Using – not just proving identities

As a starting point, let's look at two different approaches to the problem of proving the identity

$$\cos(45^\circ + x)\cos(45^\circ - x) = \frac{1}{2}\cos 2x.$$

Let's first answer the question using basic, intuitive responses to the given situation. Procedural thinking will probably lead us to start with the side that has the most information, which appears to be the left hand side of the identity. Then the next response will be to expand the compound angles and use special angles. I set out the approach below with some explanations about the trigonometric tools.

$\begin{aligned} &\cos(45^\circ + x)\cos(45^\circ - x) \\ &= (\cos 45^\circ \cos x - \sin 45^\circ \sin x)(\cos 45^\circ \cos x + \sin 45^\circ \sin x) \\ &= \left(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right)\left(\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right) \\ &= \frac{1}{\sqrt{2}}(\cos x - \sin x)\frac{1}{\sqrt{2}}(\cos x + \sin x) \\ &= \frac{1}{2}(\cos x - \sin x)(\cos x + \sin x) \\ &= \frac{1}{2}(\cos^2 x - \sin^2 x) \\ &= \frac{1}{2}\cos 2x \end{aligned}$	A direct application of the identity $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ Substituting special angle values. Or one can first distribute the brackets before replacing the special angle ratios Removing common factors from each bracket – this is algebraic and here is where learners could possibly start making mistakes, so algebraic rules must be foregrounded. A further algebraic step, using the difference of squares. Using the double angle identity to get to the right hand side.
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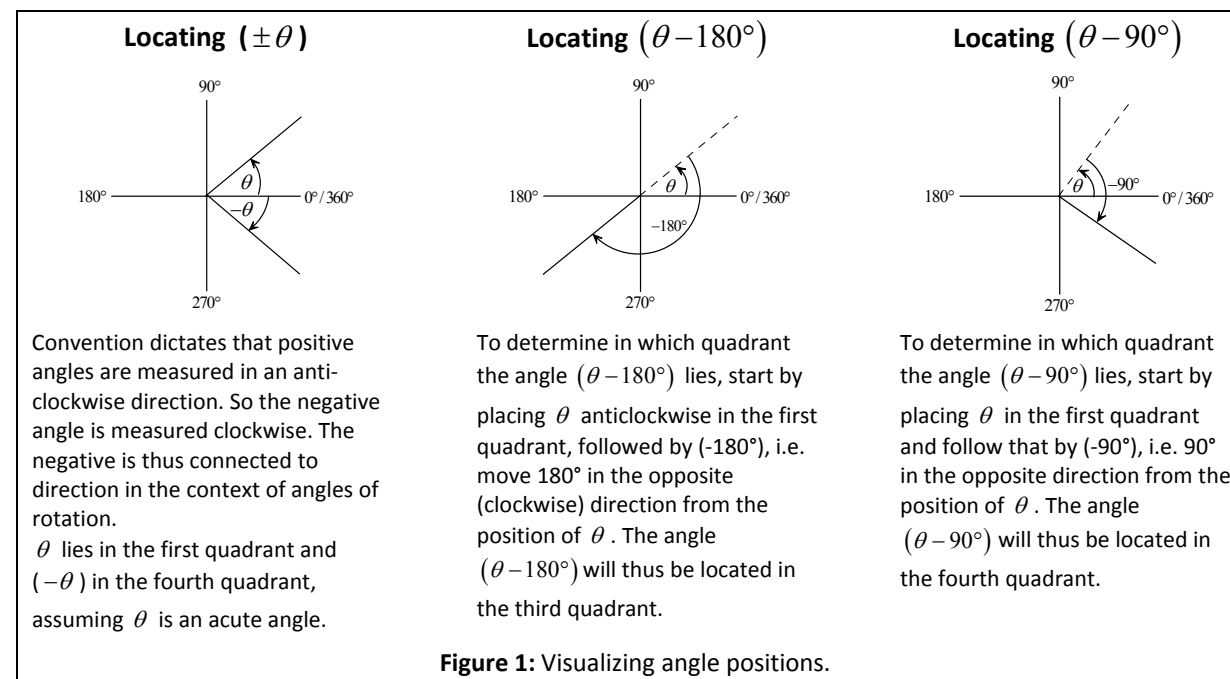
Let's now look at the same problem from a different perspective, namely to recognise and use the fact that the angles $(45^\circ + x)$ and $(45^\circ - x)$ differ only by the detail that one is addition and the other subtraction. In this approach we use the identity $\sin 2\theta = 2\sin \theta \cos \theta$. In order to transform the given expression to this form, we change the cosine to a sine, i.e. use complementary ratios – have you noticed that $(45^\circ + x)$ and $(45^\circ - x)$ are complementary angles?

$\begin{aligned} &\cos(45^\circ + x)\cos(45^\circ - x) \\ &= \sin[90^\circ - (45^\circ + x)]\cos(45^\circ - x) \\ &= \sin(45^\circ - x)\cos(45^\circ - x) \\ &= \frac{1}{2}[2\sin(45^\circ - x)\cos(45^\circ - x)] \\ &= \frac{1}{2}\sin 2(45^\circ - x) \\ &= \frac{1}{2}\sin(90^\circ - 2x) \\ &= \frac{1}{2}\cos 2x \end{aligned}$	Point out that it will be necessary to work in the first quadrant so that a negative can be introduced by subtraction using the identity $\cos(A + B) = \sin[90^\circ \pm (A + B)]$ This step uses both trigonometry and algebraic manipulation techniques. Now the angles have been manipulated by applying complementary functions and angles and it is ready to be manipulated to resemble the double angle identity for sine. Using algebraic manipulation to introduce the 2 and to undo this by multiplying with its reciprocal, brings us to the expanded version of the identity for $\sin 2\theta$. Applying the identity $2\sin \theta \cos \theta = \sin 2\theta$ and distributing $2(45^\circ - x)$. Finally the complementary angle is ready. Using the complementary ratio and angle to get to the right-hand side.
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This approach to working with identities leaves learners with a sense of choice and a need to work economically in their approach to trigonometry.

Visualizing angle positions to locate reduction rules

Learners often think that in identities such as $\sin(-\theta) = -\sin \theta$ and $\sin(\theta - 180^\circ) = -\sin(180^\circ - \theta)$ we are removing (taking out) the negative, i.e. they see it as an algebraic operation that they perform (inappropriately) and this leads to errors like $\cos(\theta - 180^\circ) = -\cos(180^\circ - \theta)$. Learners must come to realise that a negative in an angle indicates direction. It is useful to visualise the angle that is suggested by the way it is expressed. In trigonometry, when there is a negative in an angle, it indicates direction (change) as illustrated in Figure 1.



By focusing on visualizing angles of rotation when reducing angles from quadrants outside the first quadrant to its equivalent first quadrant angle, the process becomes immediate and is not tainted by procedures that could be applied incorrectly.

Traditionally learners would write $\sin(\theta - 180^\circ) = \sin[360^\circ + (\theta - 180^\circ)] = \sin(180^\circ + \theta) = -\sin \theta$.

This process of reduction will reduce to $\sin(\theta - 180^\circ) = -\sin \theta$ based on conceptually sound trigonometric principles. By visualising angles in the quadrant system, learners will soon see that in a particular quadrant angles could be described in several different ways.

Periodicity in trigonometric equations and angle reduction

The trigonometric functions are the only periodic functions that we work with in the school curriculum, and we simply do not explore their behaviour enough with learners. Periodic behaviour can be used to illustrate many of the aspects of school trigonometry that learners do not understand conceptually. Learners are introduced to the three basic trigonometric functions in Grade 10. In Grade 11, the basic trigonometry functions can be used to demonstrate how the reduction formulae relate angles to one another. It can also be used to conceptualise trigonometric equations and their multitude of possible solutions.

As clearly illustrated in Figure 2, the period of a function is determined by the amount of degrees it takes to complete one cycle. In the case of the sine function, the period is 360° . This means that every 360° the behaviour (increasing/decreasing) and the value of $\sin \theta$ is the same. As the diagram indicates, A and A_1 , B and B_1 and C and C_1 are all angles that are related.

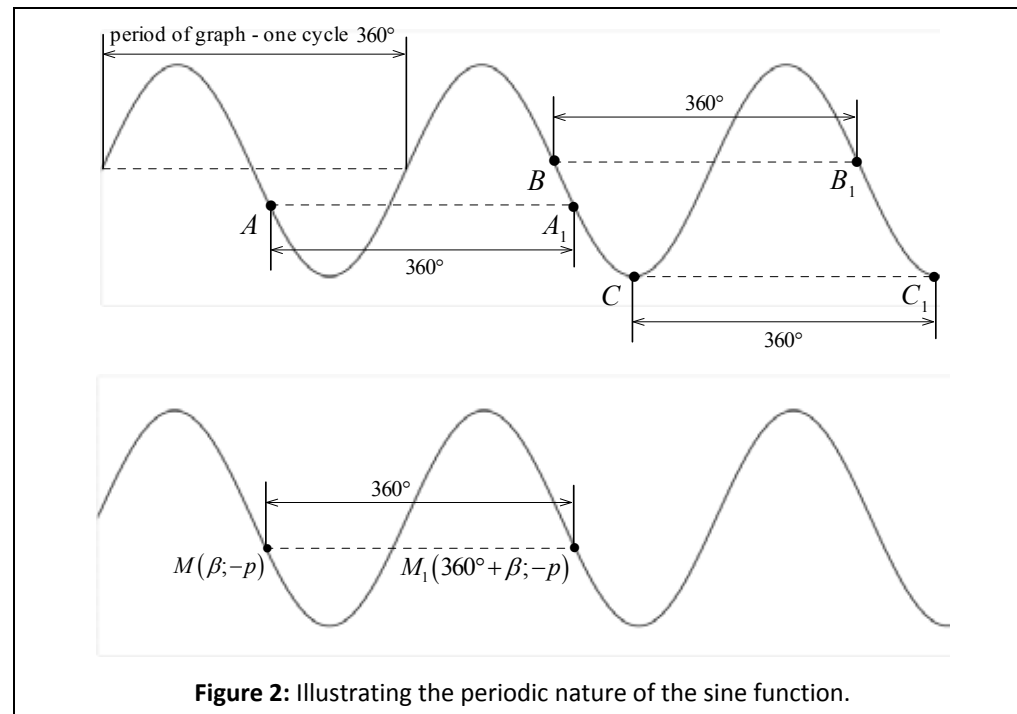


Figure 2: Illustrating the periodic nature of the sine function.

The angles in Figure 2 are furthermore primarily related to angles over the domain $[-90^\circ; 90^\circ]$ as Figure 3 indicates. The *smallest* positive angle and the *largest* negative angle for a particular value is taken as its primary positions for the particular ratio. Working with angles of rotation (using the unit circle) and the symmetry in a circle, one can demonstrate this even better.

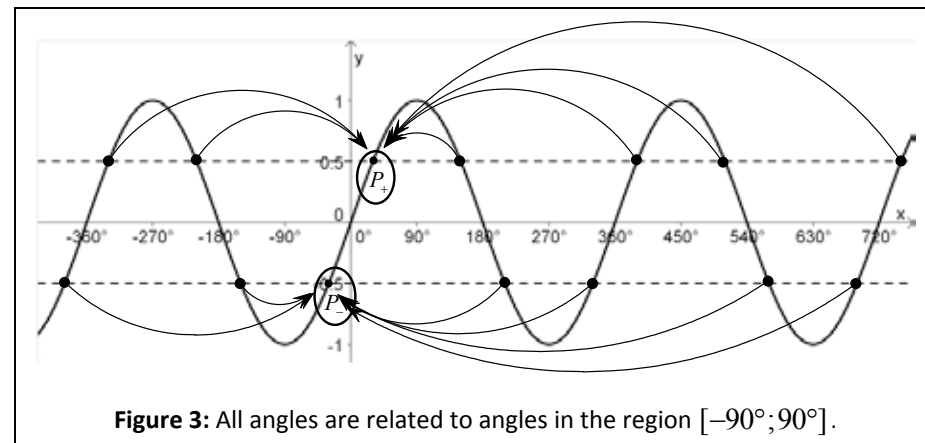


Figure 3: All angles are related to angles in the region $[-90^\circ; 90^\circ]$.

Both horizontal and vertical reduction rules can now be visualised using the trigonometric functions. The illustrations in Figure 4 clearly show how the angles are deconstructed in relation to either the 180° or 90° points on the graph of $y = \sin \theta$.

In the diagram to the left in Figure 4 we illustrate the reduction rule $\sin(180^\circ - \theta) = \sin \theta$. It is easy to see how the two angles θ and $(180^\circ - \theta)$ are related. In the middle diagram we illustrate the reduction rules for $\sin(90^\circ \pm \theta)$. In order to show that both of these rules result in $\cos \theta$, the graphs of $y = \cos \theta$ and $y = \sin \theta$ can be used simultaneously. In the diagram to the right we illustrate that $\sin 30^\circ = \cos 60^\circ = \frac{1}{2}$.

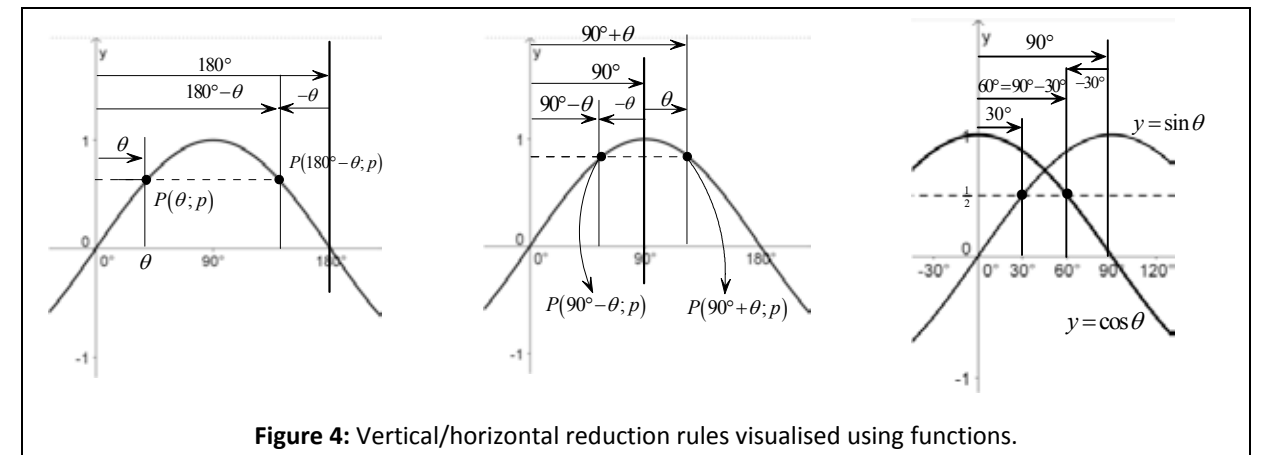


Figure 4: Vertical/horizontal reduction rules visualised using functions.

The periodicity of solutions to trigonometric equations are also illustrated by the functions concerned. For example, when we consider the values for which $\sin(x - 45^\circ) = \cos(2x + 30^\circ)$, through some working we get to the solutions $x = 35^\circ + n \cdot 120^\circ$ or $x = -165^\circ + n \cdot 120^\circ$; $n \in \mathbb{Z}$. Visualising the solution on the graphs of the two functions clearly shows how these solutions themselves follow periodic behaviour (Figure 5).

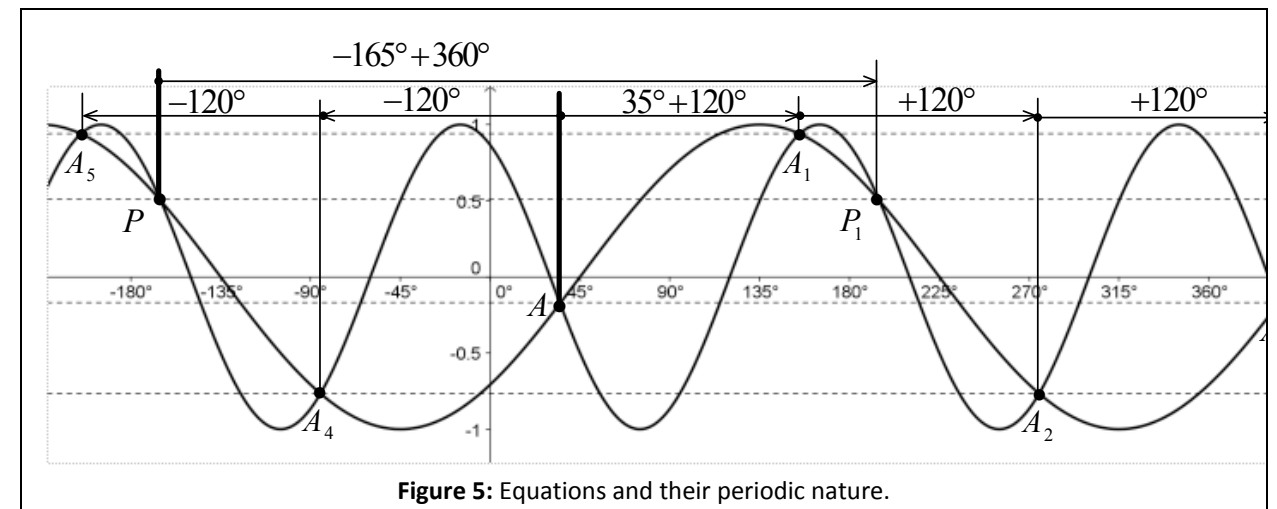


Figure 5: Equations and their periodic nature.

There are many ways to make school trigonometry a more conceptual experience for learners. What has been illustrated here was merely an attempt to open the discussion around trigonometry and the procedural approach with which it is taught.

If you have ideas that you would like to share and/or or if you would like to comment, feel free to email me at jacques.duplessis@wits.ac.za or join the conversation on the AMESA Facebook page at <https://www.facebook.com/amesa93>.

References

- Haapasalo, L. (2003). The conflict between conceptual and procedural knowledge: Should we need to understand in order to be able to do, or vice versa? In L. Haapasalo, & K. Sormunen (Eds.), *Towards Meaningful Mathematics and Science Education. Proceedings on the IXX Symposium of the Finnish Mathematics and Science Education Research Association*. University of Joensuu. Bulletins of the Faculty of Education, 86, 1-20.
- Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington DC: National Academy Press.
- Rittle-Johnson, B., Siegler, R.S., & Alibali, M.W. (2001). Developing conceptual knowledge and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology*, 93(2), 346-362.

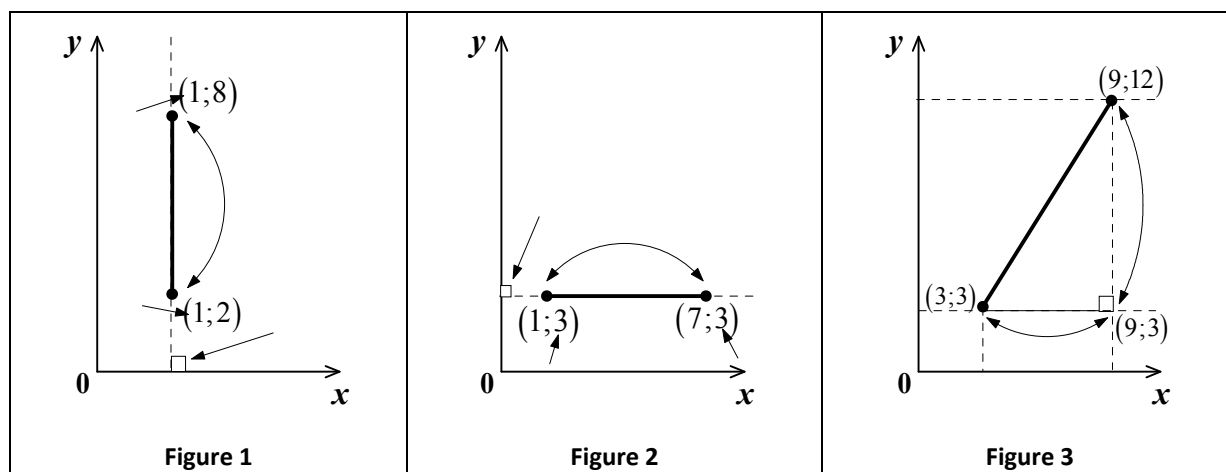
Richard Skemp (1977) distinguishes between *instrumental* and *relational* understanding as two very different ways of understanding mathematics. Very briefly: instrumental understanding is knowing *what* to do without understanding *why*, while relational understanding is knowing both what to do and why. Relational understanding forms a meaningful network of concepts and procedures, and there is little doubt that teachers should always aim to develop relational understanding in every context.

This article emphasises the importance of developing relational understanding when working with functions, with particular reference to diagrams and the important links diagrams create between concept and relation to other mathematical tools. We focus on the conceptual use of lengths and distances between functions in diagrams to demonstrate *relational understanding* of algebraic concepts in the domain of functions.

Horizontal and vertical lines

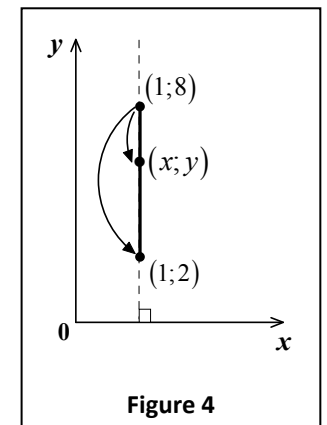
Working with functions relies on the understanding of the Cartesian axes as an orthogonal projection system is essential, as it establishes grounding relationships between the axes, and the reference positions of points on the plane in relation to the axes. Learning about points as vertical projections on the Cartesian plane can be a powerful tool in understanding the relationships between gradients, lengths of line segments (both specific and general) and also equations of line graphs. Describing the distance formula that is used to find the lengths of oblique lines on a plane given two distinct coordinates, is directly related to understanding both vertical and horizontal lengths on the Cartesian plane.

Figure 1, Figure 2 and Figure 3 show three related cases of the length of a line segment. What is key here is to understand the importance of vertically- and horizontally orientated lines and the indicators that inform us of the orientation of the line. Figure 1 and Figure 2 show how lines are related with respect to the axes, by indicating that the line is orientated vertically or horizontally in relation to one of the axes. This is indicated by the right angle positioned where the extended segment meets the axis. Another, and alternative, indicator lies in the coordinates of the vertices of the line segments. Here either both the x values or both the y values are equal.



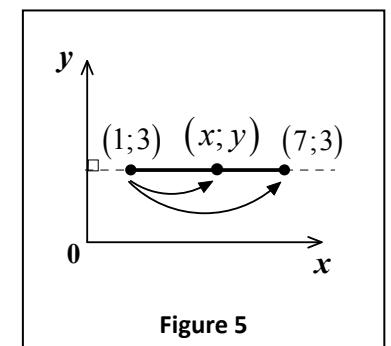
Only one of the two indicators needs to be present to suggest that the lines are either vertical or horizontal. The ideas related to different gradients emerge quite clearly from these two figures.

Figure 1 illustrates the distance between the two vertically related points as an ordinate change (a change in y values), but also needs emphasis of the fact that there is no change in the abscissas (x values) of the vertices of the segment. Here the gradient is $m = \frac{8-2}{1-1} = \frac{6}{0}$, which is undefined. The reason for this undefined gradient is now both visually and algebraically clear: for vertical lines the gradient is undefined because $\Delta x = 0$. The length of a vertical line segment is calculated by the vertical distance between the ordinates of the segment vertices: $\Delta y = 8 - 2 = 6$. Another important characteristic that follows is the fact that coordinate pairs on vertical lines share their x -values so that vertical lines will always be expressed as an equation of the form $x = c$, where c is a constant. This can easily be confirmed by using the principle of colinearity. Choosing a variable point $(x; y)$ that lies on the segment (Figure 4), we can use the gradient to find the equation of the line.



Now $m = \frac{8-y}{1-x} = \frac{8-2}{1-1} = \frac{6}{0}$. The gradient is thus undefined which is an indicator that $\Delta x = 0$. Using this we get that $x - 1 = 0$, so the equation of the line is $x = 1$.

Figure 2 shows a line segment that is oriented horizontally, and here the gradient is $m = \frac{3-3}{7-1} = \frac{0}{6} = 0$. The length of a horizontal line segment is calculated by the horizontal distance between the abscissas of the segment vertices. $\Delta x = 7 - 1 = 6$. This also becomes an indicator for horizontal lines: If the gradient $m = 0$ since $\Delta y = 0$, then the line is horizontal. Horizontal lines all share the same ordinates in each coordinate pair and will thus always be expressed as an equation of the form $y = c$, where c is a constant. This again can easily be confirmed by using the concept of colinearity applied to the vertices and some variable point $(x; y)$ on the line (see Figure 5). Here $\frac{3-y}{1-x} = \frac{3-3}{1-7}$, so $\frac{3-y}{1-x} = \frac{0}{-6} = 0$ and thus the gradient of the line is zero which is indicated by $\Delta y = 0$. Finally we get that $y - 3 = 0$, so the equation of the line is $y = 3$.



Oblique lines

Up to now we have developed ideas around horizontal and vertical lines and used these ideas to build relationships between the orientations, the coordinate pairs forming the lines, the gradients and components of the gradients of the lines, the equations of such lines and, most importantly, the underlying orthogonal projections that determine particular relationships. When considering oblique lines, the orientation of lines has now been established as key in finding the lengths of segments. It will thus be more natural to create vertical and horizontal distances that can be used to find the length of any oblique segment.

Figure 3 shows how these vertical and horizontal lines are drawn to create a right-angled triangle with a vertex $(9;3)$. Using Pythagoras one can conclude the length of the line segment easily without *imposing* the distance formula: $l = \sqrt{9^2 + 6^2} = 3\sqrt{13}$

The equation of this oblique line segment (Figure 6) is found in the same way as before. We have that $\frac{12-y}{9-x} = \frac{12-3}{9-3} = \frac{9}{3} = 3$ which simplifies

to $12 - y = 27 - 3x$, so the equation of the line is $y = 3x - 15$. The concept of colinearity of points on a line replaces the instrumental understanding of the so-called *two-point formula* with a relational understanding focusing on a third variable point that *generalises* the equation of lines through two points.

Objectifying length with functions

Moving to a more algebraic situation, let's look at the two functions $g(x) = \frac{1}{2}x - \frac{7}{2}$ and $f(x) = \frac{1}{2}x^2 - 3x - \frac{7}{2}$ which are shown in Figure 7. Let's ask the question: What is the length of the line segment AB?

To answer the question, we must realise that here we do not have numerical vertices, but we have to work with algebraic relationships between the abscissa and the ordinate of each of the points on the graph. The distance AB is a vertical distance, and is thus calculated by using the difference between the ordinates of point A and point B. A further relational understanding of coordinates is now necessary. Each coordinate pair that lies on a particular function is described by the pathway mapped out by the function. So all points on the function $g(x) = \frac{1}{2}x - \frac{7}{2}$ will have the relationship in the coordinate pair that is described by $(x; g(x)) = (x; \frac{1}{2}x - \frac{7}{2})$. The ordinate is dependent on the abscissa, which is the independent variable in the pair. So if $x = a$, then clearly $(a; g(a)) = (a; \frac{1}{2}a - \frac{7}{2})$. Point B on the other hand is located on the parabola. It is clearly indicated that the relationship between the two points is orthogonal, which indicates that the abscissa is shared, or otherwise that $\Delta x = 0$. So for $x = a$, point B will be described by the relationship $(a; f(a)) = (a; \frac{1}{2}a^2 - 3a - \frac{7}{2})$. So to now find the length of the line segment AB, we need the difference in the ordinates of points A and B. Thus $L_{AB}|_{x=a} = g(a) - f(a) = \frac{1}{2}a - \frac{7}{2} - (\frac{1}{2}a^2 - 3a - \frac{7}{2}) = -\frac{1}{2}a^2 + \frac{7}{2}a$ where $a \in [0; 7]$.

Two things about this length are important. First, the length expression takes on the nature of a quadratic function and as a result behaves like a quadratic function. Length has now become an *object* that is expressed by an algebraic expression, which introduces variability. Second, we can therefore use this variability to explore the increasing and decreasing nature of this length algebraically for

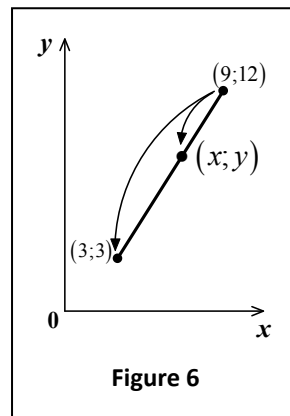


Figure 6

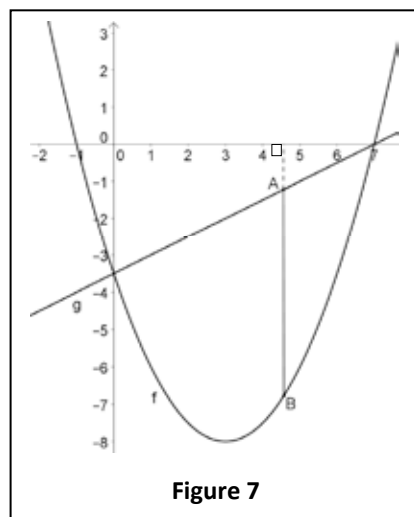


Figure 7

different values of a in the given interval. An important misinterpretation that the diagram brings about, is that learners mistakenly take the value of $x = 4\frac{1}{2}$ to calculate the length of AB. It must be emphasised that the position of the line is merely to indicate the orthogonal relationship of the line with the horizontal axis – in fact that the line is vertical and thus that $\Delta x = 0$.

Questions that follow can explore specific lengths (problems A and B) of relational lengths (Problem C) and Problem D will investigate the relational understanding within the components of the problem.

Problem A: What is the length of AB at the point where $x = 3$?

Solution: Since the expression for length is known, the abscissa is fixed as 3, and the length is calculated:

$$L_{AB}|_{x=3} = -\frac{1}{2}(3)^2 + \frac{7}{2}(3) = 6$$

Problem B: For which value(s) of x is the length of segment AB equal to 3 units?

Solution: Here the length is fixed as 3. The result is a quadratic *equation* where the abscissa is the *unknown*, giving two possibilities near the extreme points of the given interval:

$$\begin{aligned} L_{AB}|_{x=a} &= 3 \\ \therefore -\frac{1}{2}a^2 + \frac{7}{2}a &= 3 \\ \therefore a^2 - 7a + 6 &= 0 \\ \therefore (a-6)(a-1) &= 0 \\ \therefore a &= 6 \text{ or } a = 1 \end{aligned}$$

Problems A and B are merely presenting two possible cases based on either the length (ordinate value) given and the abscissa is unknown, or the abscissa is given and the ordinate length is unknown. A further extension that can build on the relational aspect of the length of AB is to search for values of x for which the length of the line is bigger or smaller than 3.

Problem C uses the relational aspect of the length of line segment AB.

Problem C: For which value of x in the interval $[0; 7]$ is the line segment AB of maximum length?

Solution:

$$L_{AB}|_{x=a} = -\frac{1}{2}a^2 + \frac{7}{2}a$$

The maximum will occur at the turning point, since the leading term is negative and the nature of the length expression is quadratic. This leaves the choice of either using the formula $x = -\frac{b}{2a}$ or

$$\text{completing the square to find the abscissa: } x = -\frac{b}{2a} = -\frac{\frac{7}{2}}{2(-\frac{1}{2})} = \frac{7}{2}.$$

In problem D we illustrate the relational aspect within the components of the problem.



Skemp, R.R. (1977). Relational understanding and instrumental understanding. *Mathematics Teaching*, 77, 20-6.

Learners working on the group question

OBITUARY

It is with very sad news that we report on the untimely passing of two of our members of the past few months.



Sushie Naidoo

Sushie was a very active member of AMESA in KwaZulu-Natal. She attended many congresses and conferences over the years.

She also was one of the facilitators in our ESP project and is a past editor of AMESA News.

Merriam Ramatsobane Ngoana

Merriam was a very active member of AMESA in Limpopo and was a treasurer in that region. She also participated in the ESP project. She wrote an article about her experiences in the ESP project for AMESA News.

Lucia J. van Rensburg

Lucia was the Free State coordinator for the STATS SA project maths4stats, and a long-time AMESA member and presenter at AMESA congresses. She tragically died in the week after attending and presenting a workshop at the AMESA 2013 national congress in Cape Town (picture below).



We offer our deepest condolences to the families of these and other members who passed away. We will miss their dedicated service to AMESA. It will be appreciated if members could send me information about other members of AMESA who have passed away recently. A brief description of their AMESA involvement and a recent photograph will be appreciated. Send the information to news@amesa.org.za.



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