NOTES ON 2003 MEMORANDUM

These notes are necessarily brief and often formal and symbolic.

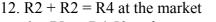
Many questions could be answered using primitive methods, e.g. "If today is Wednesday, what day of the week will it be 100 days from now?" can be done by counting. That would be laborious, time-consuming and error-prone. The essence of a mathematical approach is to work more smartly by using appropriate representations to model the situation and to exploit the inherent structures and patterns in the situation.

GRADE 4(1)

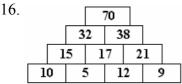
- 3. $1 \text{ km} = 1000 \times 1 \text{ m} = 1000 \times 1000 \text{ mm} \text{ (a million!)}$
- 5. Every letter becomes the next letter in the alphabet. $A \rightarrow B, B \rightarrow C$ etc.
- 7. January, February, March give 31 + 28 + 31 = 90 days. Another 10 days brings us to 10 April
- 8. $102 \div 7 = 14$ rem 4, so to make rem 7, add 3 or $15 \times 7 = 105$, so 3 more marbles are needed
- 9. Test each of the given answers ... Or, to do *equal* sharing, first put aside the 15 marbles that Zuki has more than Zinkle. So, together they have 95 - 15 = 80, so they each have $80 \div 2 = 40$. Now give back Zuke her extra 15 marbles Or, if Zinkle has z marbles, z + (z+15) = 95, so $2 \times z + 15 = 95$, so z = (95 - 15)/2 = 40

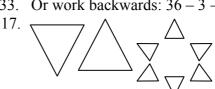
11.

10. 50c + 20c + 10c + 2c = 82c 20c + 10c + 2c = 32c 50c + 10c + 2c = 62c50c + 2c = 52c



- $6 \times 75c = R4,50$ at the greengrocer
- 13. $(5 \times 63) \div 9 = 5 \times (63 \div 9) = 5 \times 7 = 35$. He can fill 7 bags from one box, so 5×7 from 5 boxes
- 14. 15 min + 20 min + 35 min = 1 hour and 10 minutes. 1 hour and 10 minutes before 08:00 is 06:50
- 15. Try each number, e.g. $10 \times 3 \rightarrow 30 + 3 = 33$. Or work backwards: $36 3 \rightarrow 33 \div 3 \rightarrow 11$





- 18. There are 18 rows and 10 columns, so $10 \times 18 = 180$ tiles
- 19. 3 \heartsuit = \heartsuit , so 6 \heartsuit = 2 \heartsuit , so X = 2 \heartsuit = 4 \triangle

 20. Represent it on paper, e.g. as horizontal measurement. Fill in information bit by bit:

 Siva is shorter than Temba:
 Siva Temba

 Eby is taller than Temba:
 Siva Temba Eby

 Siva is taller than Oscar:
 Oscar Siva Temba Eby

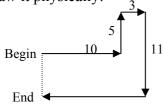
 Siva is shorter than Ram, Eby is taller than Ram:
 Oscar Siva Ram/Temba Eby

21. Make a systematic representation of all the possibilities. e.g.:

21. Wiake a Systema	ne representatio	in or an the possi	onnues, e.g		
white bread with:	egg	brown bread with:	egg	rye bread with	egg
	polony		polony		polony
	cheese		cheese		cheese
	jam		jam		jam
	fishpaste		fishpaste		fishpaste
	peanut butter		peanut butter		peanut butter
C 1 C	1 . 1	(



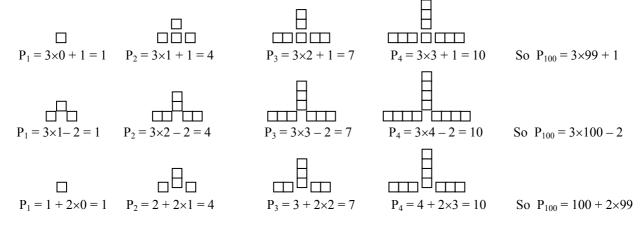
22. Draw it physically:



23. The smallest number is 1 + 1 = 2 and the biggest 6 + 6 = 12. From 2 to 12 is 12 - 2 + 1 = 11 numbers 24. List all the possibilities systematically 1 + 12 + 13 + 14 + 15 + 16 + 1Notice any patterns? 1 + 22 + 23 + 24 + 25 + 26 + 2Out of the 36 possibilities, a sum of 7 occurs 1 + 32 + 33 + 3**4**+**3** 5 + 36 + 3six times, a sum of 6 occurs five times, etc. 1 + 42 + 43 + 44 + 45 + 46 + 4You can expect a sum of 7 to occur 6 out of 36 times 1 + 52 + 53 + 54 + 55 + 56 + 52 + 63+6 5 + 66 + 64 + 6

You can expect a sum of 2 to occur 1 out of 36 times 1 + 6

25. "See" some structure in the sketches for P₃ and P₄, then extend backwards and forwards, e.g.:



<u>GRADE 4 (F)</u>

- 2. 147 mm 103 mm = 44 mm
- 3. 9:15 + 1 hour = 10:15; 10:15 + 50 min = 11:05
- 5. $R9,89 \times 4 = R39,56; R42,37 R39,56 = R2,81$
- 6.

8.

96000 160 600 8 20 30

4 5

7. $R4 \div 5 = R0,80$ for 1 apple. $R9,60 \div R0,80 = 12$ apples

	Jim	Bongani
Round 1	125	100
Round 2	125	125
Round 3	150	100
Round 4	50	150
Total	450	475

б

475 - 450 = 25. Bongani won by 25 points

9. $\frac{2}{10} = 0,2$

2

10.
$$\frac{1}{3} = \frac{2}{6}$$
; $\frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

11.

13. School A: 8:00 8:30 9:00 9:30 10:00 10:30 11:00 11:30 School B: 8:00 8:35 9:10 9:45 10:20 10:55 11:30

14. $100 \times 5 = 500$

- $10 \times 5 = 50$
- $10 \times 5 =$ 50
- $4 \times 5 =$ 20; 500 + 50 + 50 + 20 = 620
- Or: $5 \times (100 + 10 + 10 + 4) = 5 \times 124 = 620$

- 15. List the possibilities systematically. Notice the number patterns and let the patterns help you: $0 \times 20c + 0 \times 10c + 10 \times 5c$ $1 \times 20c + 0 \times 10c + 6 \times 5c$ $2 \times 20c + 0 \times 10c + 2 \times 5c$ $0 \times 20c + 1 \times 10c + 8 \times 5c$ $1 \times 20c + 1 \times 10c + 4 \times 5c$ $2 \times 20c + 1 \times 10c + 0 \times 5c$ $0 \times 20c + 2 \times 10c + 6 \times 5c$ $1 \times 20c + 2 \times 10c + 2 \times 5c$ $0 \times 20c + 3 \times 10c + 4 \times 5c$ $1 \times 20c + 3 \times 10c + 0 \times 5c$ $0 \times 20c + 4 \times 10c + 2 \times 5c$ $0 \times 20c + 5 \times 10c + 0 \times 5c$ 16. $1 + 11 \rightarrow$ difference too much $2 + 10 \rightarrow$ difference too much $3 + 9 \rightarrow$ difference too much $4 + 8 \rightarrow \text{difference } 4$ 48 and not 84 because it must be less than 50 20. R15 + R6,75 = R21,75. R21,75 ÷ 3 × 4 = R29 21. 1100g - 680 g = 420 g. $420 g \div 12 = 35 g$. $35 g \times 30 = 1050g$. 1100 g - 1050 g = 50 g22. $100 - (4 \times 5) = 80$ 23. 1, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 31, 41, 51, 61, 71, 81, 91, 100 \rightarrow 21 ones 24. Pattern: 5 + 4n; $6129 - 5 \rightarrow 6124 \div 4 = 1531$ 25. Picture 1 = 1Picture 2 = 1 + 2Picture 3 = 1 + 2 + 3
 - Picture $100 = 1 + 2 + 3 + 4 + ... + 98 + 99 + 100 = (1 + 100) \div 2$

GRADE 5(1)

- 1. Three heaps of five blocks each = 15 blocks
- 3. $R100 (6 \times R4, 15 + 2 \times R12, 49)$
- 4. You should recognise most *without* actual division!

$360 \div 1 = 360$	$360 \div 4 = 90$	$360 \div 7 = 51,42857$
$360 \div 2 = 180$	$360 \div 5 = 72$	$360 \div 8 = 45$
$360 \div 3 = 120$	$360 \div 6 = 60$	$360 \div 9 = 40$

- 5. 2 layers of 8 by 4 or 8 columns of 2 by 4 or 4 rows of 8 by 2
- 6. All the blocks of the bottom layer (32) and all the blocks round the side of the top layer (20)
- 7. 41000 g 725 g = 40275 g = 40,275 kg
- 8. Half of the previous number = $1,7 \div 2 = 0,85$
- 9. List all numbers containing a 9 systematically:
 9, 19, 29, 39, 49, 59, 69, 79, 89: 9 numbers
 90, 91, 92, 93, 94, 95, 96, 97, 98, 99: 10 numbers
 So the number of numbers without a 9 is 100 (9 + 10) = 81
- 10. List them systematically: 101, 111, 121, 131, 141, 151, 161, 171, 181, 191
- 11. 100 ÷ 12 = 8 rem 4, i.e. 8 years bringing us to Sept., plus 4 more months, i.e. Oct., Nov., Dec., Jan. Or if September = 9, then 100 months further is 9 + 100 = 109. But January = 1, 13, 25, ... these leave a remainder of 1 if divided by 12 February = 2, 14, 26, ... these leave a remainder of 2 if divided by 12 March = 3, 15, 27, ... these leave a remainder of 3 if divided by 12, etc. So 109 ÷ 12 = 9 rem 1, so January
- 12. $100 \div 7 = 14$ rem 2, i.e. 14 full weeks bringing us to Wednesday, plus 2 more days, i.e. Friday Or if Monday = 1, Wednesday = 3, so 3 + 100 = 103, $103 \div 7 = 14$ rem 5, and 5 is Friday
- 13. 100 ÷ 24 = 4 rem 4, i.e. 4 full days bringing us to 10:00, plus 4 more hours, i.e. 11, 12, 13, 14:00 Or 10 + 100 = 110, 110 ÷ 24 = 6 rem 14
- 14. September has 30 days so there are 20 days left in September. October has 31 days. November has 30 days. 20 + 31 + 30 = 81 days. 19 days to fill up 100 days, therefore 19 December

- 15. 3 for 5 = ? for 90. You can build it up, e.g. 30 for 50 and 24 for 40, so 54 for 90. Or 3×18 for 5×18 (90)
- 16. Put the information in a sketch, fill in the details bit by bit, and extend the information. e.g.: The distance from B to E is 10 cm You can deduce that AB = 10 cm!
- 18. 99 m = $\frac{9}{10}$, so 11 m = $\frac{1}{10}$. Therefore $\frac{10}{10} = 10 \times 11 = 110$ m
- 19. Make a sketch of the situation:
 "2nd from front, 4th from back" means there are 5 rows. "3rd from left, 5th from right" means there are 7 learners per row. So 7 learners/row × 5 rows = 35 learners
- 20. Imagine yourself looking at the card from behind. Or tear the corner from a piece of paper, turn it around!
- 21. Z8 6 X 5 8+6+X=18, so X=45+X+Z=18, so 5+4+Z=18, so Z=9
- 22. In second balance: 1B = 1A + 2C, so in first balance 5A = 3B = 3A + 6C, so 2A = 6C, so 1A = 3C
- 23. List systematically: 85 10, 86 11, 87 12, 88 13, ..., 99 24. From 85 to 99 is 15 possibilities
- 24. 2 Small + 1 Large = 5 Small, so 1 Large = 3 Small, costing $3 \times R11$, 50
- 25. Let the cost of a chocolate, a gum and a juice be c, g and j respectively Then 2g+2j = 10, so 1g+1j = 5. But 1c+1g+1j = 9, so 1c+(1g+1j) = 9, so 1c+5 = 9, so 1c = 4

GRADE 5 (F)

- 1. 2,06 m = 206 cm; 2,06 m 1 cm = 2,05 m
- 3. 4 around the top, 4 around the bottom and 4 around the middle.
- 4. 1 on top, 4 around the middle and 1 at the bottom.
- 5. 4 at the bottom and 4 at the top.
- 6. $10 \times 95c = R9,50$
 - $10 \times R1,20 = R12,00$

R9,50 + R12,00 = R21,50 ; R40,00 - R21,50 = R18,50

7. Half of target C is shaded and that is more than on any other target

9. $\frac{1}{6} = 4$ slices, $\frac{1}{4}$	$= 6$ slices $\frac{1}{3} = 8$ sli	ces; $24 - 18 = 6$	
10. $150 \div 6 \times 5 = 12$	25		
$13. \bigcirc 0 2$ O P L	3 • •	$\stackrel{6}{\longrightarrow}$ N	
14. $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$			
15. Light: 6 12 18	24		
Bell: 8 16 24			
16. $E(1) = 2 = 1 \times 2$	O(1) = 1 =	$1 \times 2 - 1$	
$E(2) = 4 = 2 \times 2$	O(2) = 3 =	$= 2 \times 2 - 1$	
$E(3) = 6 = 3 \times 2$	O(3) = 5 =	$= 3 \times 2 - 1$	
$E(4) = 8 = 4 \times 2$	O(4) = 3 =		
:	$\mathbf{\hat{s}}$	100 2 1 100	
$E(100) = 100 \times 2 =$		$100 \times 2 - 1 = 199$	
18. List the possibilit			
	ible more than 9	1 + 16	not possible more than 9
	ible more than 9	2 + 15	not possible more than 9
1	ible more than 9	3 + 14	not possible more than 9
-	ible more than 9	4 + 13	not possible more than 9
-	ible more than 9	5 + 12	not possible more than 9
6+9 Possible		6+11	not possible more than 9
7+8 not possi	ible no card is a 7	7 + 10	not possible no card is a 7
		8 + 9	Possible

- 19. A = B + 2C, so A + C = B + 3CBut in middle A + C = B + D, so D = 3C, so B = 4CBut A = B + 2C, so A = 4C + 2C = 6C
- 20. The difference in age 40 16 = 24 remains the same So Mom – Me = 24 and Mom = $2 \times Me$ So $2 \times Me - Me = 24$, so Me = 24
- 21. 988, 898, 889, 997, 979, 799
- $22.99 36 = 63; 98 35 = 63; 97 34 = 63; 96 34 = 63; \dots; 73 10 = 63, \text{ so } \dots$
- 23. Make a table of information:

	10	11	Total
Boys	18	13	31
Girls	14	5	19
Total	32	18	50

24. $10 \times 10 = 100$

25. $T_1 = 3 = 1 \times 3 = 1 \times 3$

 $T_2 = 9 = 3 \times 3 = (1+2) \times 3$ $T_2 = 18 = 6 \times 3 = (1+2) \times 3$

$$T_3 = 18 = 6 \times 3 = (1 + 2 + 3) \times 3$$

 $T_{10} = (1 + 2 + 3 + ... + 9 + 10) \times 3 = 55 \times 3$

GRADE 6(1)

- 2. 25 + 20 + 30 + 15 + 35 = 125
- 4. $0,13 0,01 \rightarrow 0,12 0,01 \rightarrow 0,11 0,01 \rightarrow 0,10 0,01 \rightarrow 0,09$
- 5. We know that $\frac{1}{5} > \frac{1}{6} > \frac{1}{7}$ and $\frac{7}{30} > \frac{7}{36}$. So compare $\frac{1}{5}$ and $\frac{7}{30}$: $\frac{1}{5} = \frac{6}{30} < \frac{7}{30}$
- 6. $(15 \times 12 20) \times R2,40/8 15 \times R2,40 = R12$

or $(15 \times 12 - 20) \times 30c - 15 \times 12 \times 20c = 15 \times 12 \times 10c - 20 \times 30c = R12$

7. The watch gains 5 min in 6 hours, so 1 min in 6/5 h = 1 h 12 min, so it gains 2 min in 2 h 24 min 8. You can test them one by one, but it takes time! Clearly $11 \div 3$ = leaves a remainder of 2.

 $111 \div 3 = 37$, so $111 111 \div 3 = 37 037$ and 1111 = 1110 + 1 leaves a remainder of 1. Maybe you know, or discover, that if the digit sum is divisible by 3, then the number is divisible by 3

- 11. B, A, D, in that order, shows the line around the cube. C views B and D together
- 12. Imagine or draw the cube! If the side is 3 times as long, the big cube contains 27 of the small cubes! So its mass is 27 times as large!
- 13. The 6 small triangles (6), the one big one (7) and these:
- small cubes! 89 10 11 12 14 15 16
- 14. $10 \times 10 \times 10 \times 10 = 10000$ or from 001 to 999 give 999 combinations, plus 000 give 10000 15. List all the possibilities systematically. Seeing the patterns also helps!

16. In the first balance: 3 circles = 6 squares, so 1 circle = 2 squares.In the second balance: 3 triangles = 2 circles, so 9 triangles = 6 circles = 12 squares

- 17. You should know from experience that (A) and (B) are true. For the others, test several different numbers trying to find one that is not true, and use your calculator for quick calculation, e.g.:
 (C): 112, 212, 5712 ... are all divisible by 4; (D): 1232, 2232, 3232, 29232 ... are all divisible by 8. In (C) and (D) we can't test *all* cases, so we can't be 100% sure, but it can be proved in other ways.
 (E): 163 is not divisible by 9, so the statement is false! *Some* numbers are divisible, but not *all*!
- 18. Corrie walks 60 m for every 45 of Denise, i.e. 4 m for every 3 of Denise. So Corrie walks 4 out

of every 7 metres of the distance, i.e. $\frac{4}{7}$ of 3570 m

Or: In 1 minute they come 60 + 45 = 105 m closer to one another. So they will cover 3570 m in 3570 m \div 105 m/min = 34 minutes. In 34 minutes Corrie walks 34 min× 60 m/min = 2040 m 20. Make a sketch, e.g.:

41 - 30 = 11 children like only Comedy

35 - 30 = 5 children like only Action

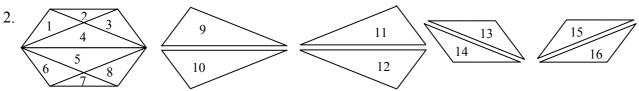
50 - (11 + 30 + 5) = 4 children don't like either

 $\mathbf{C}_{11} \mathbf{A}$ Total = 50

- 21. List the possibilities systematically: 300 | 210, 201 | 120, 102 | 111
- 22. If the middle number is m, then (m-1) + m + (m+1) = 54, so $3 \times m = 54$, so m = 18So the consecutive numbers are 17, 18 and 19. So $17 \times 18 \times 19 = 5814$
- 23. Put the information in a sketch like this. Use the information bit by bit, and extend the info, e.g.: Elize finished 2 m ahead of Daphne, but 3 m behind Fay Elize beat Candy by 6 metres, etc. 4 m 2 m 3 m Candy
- 24. $P_n = 4 \times n + 1$, so $P_{20} = 4 \times 20 + 1$
- 25. If Zander caught *x* fish, then Vincent caught x + 8 fish, all together x + (x + 8) + 20 + 24 + 18 = 100. Or, Vincent and Zander together caught 100 - 62 = 38 fish. But Vincent caught 8 more than Zander

GRADE 6 (F)

11.



- 3. 1, 2, 4, 5, 10, 20, 25, 50, 100
- 8. $\frac{1}{8} = \frac{10}{80}$ and $\frac{1}{10} = \frac{8}{80}$, therefore $\frac{9}{80}$ will be in between
- 9. $100 \div 20 = 5; 3000 \div 20 = 150$
- 10. 1 will be opposite 16, 2 opposite 17, 3 opposite 18, 4 opposite 19, 5 opposite 20, 6 opposite 21, 7 opposite 22

2	3	
3	5	+2
4	?	+3
5	12	+4
6	17	+5

- 12. Dick is first, Peter is second, Tom is third and Harry is fourth.
- 13. Find the three numbers by trial and improvement. Use your calculator, e.g. $10 \times 11 \times 12 = 1320$ is too small, etc. But $14 \times 15 \times 16 = 3360$, so 14 + 15 + 16 = 45
- 14. 2p + 3; 2(30) + 3 = 63
- 15. 370 g 290 g = 80 g; half of the milk weighs 80 g, so all the milk will weigh 160 g.
- 370 g 160 g = 210 g
- 16. 35-6=29 (29 children either has a cat, a dog or both); 29-24=5 (5 children has only dogs); 18-5=13
- 17. $60 \div 5 (1 \text{ part} + 4 \text{ parts}) = 12; 12 \times 6 (1 \text{ part} + 5 \text{ parts}) = 72 \text{ glasses}$

- 18.(2000 1999) + (1998 1997) + ... + (2 1) = 1 + 1 + 1 + 1 + ... + 1 (1000 times)
- $19. (1 + 100) + (2 + 99) + (3 + 98) + (4 + 97) + \dots + (50 + 51) = 101 \times 50 = 5050$
- 20. Use the repeated structure of the system, e.g. imagine writing down all the numbers from 0 to 99. Then the sum of the units digits is $10 \times (0 + 1 + 2 + ... + 8 + 9)$ The sum of the tens-digits is $10 \times 1 + 10 \times 2 + ... + 10 \times 9 = 10 \times (1 + 2 + ... + 8 + 9)$ So all together $2 \times 10 \times (1 + 2 + ... + 8 + 9) = 2 \times 10 \times 45 = 900$. Adding the 1 of 100 gives 901.
- 21. If the cheaper article costs R*a*, the second article costs . Together they cost a + a + 6 = 11So $2 \times a + 6 = 11$, so $2 \times a = 5$. so a = 2,50
- 22. Make a table of the differences:

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

- 23. $1 + 3 + 5 + 7 + \ldots + 25 + 27 + 29 = 15 \times 15 = 225$
- 24. "All but six where white" means 6 were not white, so b + y = 6.

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Similarly w + y = 6
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and w + b = 6
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- Add them together: 2w + 2b + 2y = 18, so w + b + y = 9
- 25. w + b + y = 9So w + (b + y) = 9so w + 6 = 9, so w = 3

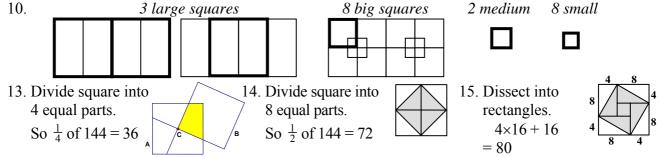
GRADE 7(1)

- 2. 114% = R36,15 so 1% is R36,15 ÷ 114 and 100% = R36,15 ÷ 114 × 100 = R31,71
- 3. $(3 \times 75) + (6 \times 66) = 621$ kg all together. So the average is 621 kg ÷ 9 children= 69 kg/child
- 4. The "vertical" formula is $2 \times a + 2$. Find *a* so that $2 \times a + 2 = 64$ Or the "horizontal" formula is $4 + 2 \times (a - 1)$, so find *a* so that $4 + 2 \times (a - 1) = 64$
- 5. We do not know whether he has had his birthday in 1524. So he can be 64 or 63
- 6. $p + q + p + q = p + p + q + q = 2 \times p + 2 \times q = (p + q) \times 2 \neq p \times q + p \times q$
- 7. Let the width be *w* metres, then the length is $4 \times w$ metres, so the area = $4 \times w \times w = 100$, so w = 5Then the perimeter is $2 \times (4 \times w + w) = 10 \times w = 10 \times 5 = 50$ metres

8.
$$\frac{n}{6,34} = \frac{100 \times n}{634} = \frac{a}{1}$$
, so $\frac{n}{634} = \frac{a}{100}$

9. The number is a multiple of 7. So check which of 7, 14, 21, 28, 35, 42, 49, 56, ... leave a remainder of 1 when divided by 3 or 5

Quicker: the first two conditions means that the number is one more than a multiple of $3 \times 5 = 15$ So the possible numbers are 16, 31, 46, 61, 76, 91. Of these, only 91 is also a multiple of 7



16. Represent and organise the info in a table:

	5 6 7 8						
Ali							
Oli	Х		Х				
Uli	Х	x		X			
Eli		X		X			

So Uli is 7, the others not:						
	5 6 7 8					
Ali			X			
Oli	Х		Х			
Uli	Х	Х	yes	х		
Eli		Х	X	Х		

0	T 11	•	_		•		_
V'A		10	4		10	not	4.
N ()	E .III	18)	AIL	18	1101)
00		10	~,	1 111	10	not	<i>·</i> · ·

	5	6	7	8
Ali	х		х	
Oli	Х		Х	
Uli	Х	Х	yes	х
Eli	yes	Х	х	х

17. See previous answer

- 18. $L_n = 3 \times n + 2$, so $L_{15} = 3 \times 15 + 2 = 47$
- 19. If April had *x* eggs, Peter had x + 2, Melanie had x + 7, Jack had x + 1Together: x + (x + 2) + (x + 7) + (x + 1) = 38, so $4 \times x + 10 = 38$, so x = 7
- 20. If he bought x apples and y oranges, then the cost is $2 \times x + 1 \times y = 52$ and the total fruit is x + y = 32 $2 \times x + y = 52$ can be written as x + x + y = 52, so x + (x + y) = 52, so x + 32 = 52, so x = 20

21.
$$\frac{5^{17}}{5^{17}} = \frac{1}{5^3} = \frac{1}{125}$$

22. List the units digits of the first few powers:

Exponent	1	2	3	4	5	6	7	8	9
Units digit	3	9	7	1	3	9	7	1	3

The last digits has a recurring pattern 3, 9, 7, 1

Reorganise the results like this and analyse the sequences:

Units digit	Exponents giving the units digit				
3	1, 5, 9, 13, These have a remainder of 1 when divided by 4				
9	2, 6, 10, 14, These have a remainder of 2 when divided by 4				
7	3, 7, 11, 15, These have a remainder of 3 when divided by 4				
1	4, 8, 12, 16, These are multiples of 4				

We simply have to decide in which sequence 2003 will be ...

23. Look for *structure* in the denominator:

	F ₁	\mathbf{F}_2	F ₃	\mathbf{F}_4	•••	F ₁₀
Numbers	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{20}$?
Structure	$\frac{1}{1 \times 2}$	$\frac{1}{2 \times 3}$	$\frac{1}{3 \times 4}$	$\frac{1}{4 \times 5}$		$\frac{1}{10 \times 11}$

24. Calculate intermediate answers and look for structure and patterns:

Sum of **1** fraction $= \frac{1}{2}$ Sum of **2** fractions $= \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$ Sum of **3** fractions $= \frac{2}{3} + \frac{1}{12} = \frac{3}{4}$ Sum of **4** fractions $= \frac{3}{4} + \frac{1}{20} = \frac{4}{5}$ So, Sum of **10** fractions $= \frac{10}{11}$ 25. $(1+1) \times (1+\frac{1}{2}) \times \dots = \frac{2}{1} \times \frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \dots \times \frac{100}{99} \times \frac{101}{100}$ $= \frac{2}{2} \times \frac{3}{3} \times \frac{4}{4} \times \frac{5}{5} \times \dots \times \frac{100}{100} \times \frac{101}{1}$ = 101

GRADE 7 (F)

2. $23 \times 25 = \frac{46}{4} \times \frac{a}{2} = 23 \times \frac{a}{4}$, so $25 = \frac{a}{4}$, so ...

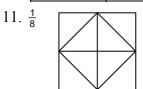
4.
$$6 \times 4 = 24 \text{ cm}^2$$
; $24 \text{ cm}^2 \div 2 = 12 \text{ cm}^2$

5.
$$6 \times 4 = 24 \text{ cm}^2$$

 $24 \text{ cm}^2 \div 2 = 12 \text{ cm}^2$
6. $6 \times 4 = 24 \text{ cm}^2$

- 24 cm² \div 2 = 12 cm² 7. 24 is three-quarters of the marbles she had before a
- 7. 24 is three-quarters of the marbles she had before giving anything to Sonny; $24 \div 3 \times 4 = 32$ 32 is two –thirds of the marbles she had before giving anything to Ronnie; $32 \div 2 \times 3 = 48$
- 8. Volume = $15 \times 8 \times x = 120$, so x = 1 cm. So area is $(15 \text{ cm} + 2 \text{ cm}) \times (8 \text{ cm} + 2 \text{ cm}) = 170 \text{ cm}^2$
- 9. Divisible by 7 and 8 means it is divisible by $7 \times 8 = 56$. First multiple of 56 bigger than 800 is $15 \times 56 = 840$, then 896, then 952. So 3.

10.	width	Length	Perimeter
	1	7	16
	2	6	16
	3	5	16
	4	4	16



12. List the possibilities:

1	2	3	4
1	2	3	9
1	2	4	8
1	2	5	7
1	3	4	7
1	3	5	6
2	3	4	6

13. If $A = 4 \times 4$ and $B = 5 \times 5$, then $\overline{E = 1} \times 1$ and $C = 6 \times 6$ and $D = 7 \times 7$. So the area is $= 13 \times 11$ 14.

А			В	
	D		С	
		Е	F	
	G	Н	Ι	
				J

I "invent" a notation as either a single letter, or naming the top-left bottom-right vertices. So the squares are:

C, E, F, G, H, I

AE, AJ, DE, DI, EI, IJ

- 15. R1,50 for first km; $90c \times 10$ km; R1,50 + R9,00 = R10,50
- 16. R20 R2 R1,50 = R16,50; $R16,50 \div R0,90 = 18$; 18 + 1 = 19 km
- $17.25 \times 25 = 625 \text{ m}^2$

18.
$$(1 - \frac{1}{2}) \times (1 - \frac{1}{3}) \times (1 - \frac{1}{4}) \times \dots \times (1 - \frac{1}{2003})$$

= $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \dots \times \frac{2001}{2002} \times \frac{2002}{2003} = \frac{1}{2003}$

21. 3 lines from two corners divide the triangle in 4×4 sections

- 10 lines from two corners will divide the triangle in 11×11 sections = 121
- 22. 18 balls left to go into 7 boxes. If you put two balls in each box there are still four balls left over.
- 23. The sides of a square are equal. So list the possibilities of the rectangle until 4 times the width is equal to the length: 4(3) = 12; $12 \times 4 = 48$ cm
- 25. Record the information in a table (matrix):

says	Ahmed	Bruce	Delia	Clare	Eldon
Ahmed			yes		
Bruce				yes	
Delia		yes			
Clare				no	
Eldon					yes

Decide if Clare is lying or not:

If Clare is telling the truth, it means she did not do it, so Bruce and everyone else is lying, so nobody did it, which is not true. So Clare cannot be telling the truth!

So Clare is lying, which means she did it, so Bruce is the one telling the truth (Clare did it) and everyone else is lying.