## NOTES ON 2003 MEMORANDUM

These notes are necessarily brief and often formal and symbolic.
Many questions could be answered using primitive methods, e.g. "If today is Wednesday, what day of the week will it be 100 days from now?" can be done by counting. That would be laborious, time-consuming and error-prone. The essence of a mathematical approach is to work more smartly by using appropriate representations to model the situation and to exploit the inherent structures and patterns in the situation.

## GRADE 4(1)

3. $1 \mathrm{~km}=1000 \times 1 \mathrm{~m}=1000 \times 1000 \mathrm{~mm}$ (a million!)
4. Every letter becomes the next letter in the alphabet. $\mathrm{A} \rightarrow \mathrm{B}, \mathrm{B} \rightarrow \mathrm{C}$ etc.
5. January, February, March give $31+28+31=90$ days. Another 10 days brings us to 10 April
6. $102 \div 7=14$ rem 4 , so to make rem 7 , add 3 or $15 \times 7=105$, so 3 more marbles are needed
7. Test each of the given answers ...

Or, to do equal sharing, first put aside the 15 marbles that Zuki has more than Winkle. So, together they have $95-15=80$, so they each have $80 \div 2=40$. Now give back Zuke her extra 15 marbles Or, if Zinkle has $z$ marbles, $z+(z+15)=95$, so $2 \times z+15=95$, so $z=(95-15) / 2=40$
10. $50 \mathrm{c}+20 \mathrm{c}+10 \mathrm{c}+2 \mathrm{c}=82 \mathrm{c}$
$20 c+10 c+2 c=32 c$
$50 \mathrm{c}+10 \mathrm{c}+2 \mathrm{c}=62 \mathrm{c}$
$50 \mathrm{c}+2 \mathrm{c}=52 \mathrm{c}$
11.
12. $\mathrm{R} 2+\mathrm{R} 2=\mathrm{R} 4$ at the market

$6 \times 75 \mathrm{c}=\mathrm{R} 4,50$ at the greengrocer
13. $(5 \times 63) \div 9=5 \times(63 \div 9)=5 \times 7=35$. He can fill 7 bags from one box, so $5 \times 7$ from 5 boxes
14. $15 \mathrm{~min}+20 \mathrm{~min}+35 \mathrm{~min}=1$ hour and 10 minutes. 1 hour and 10 minutes before $08: 00$ is $06: 50$
15. Try each number, e.g. $10 \times 3 \rightarrow 30+3=33$. Or work backwards: $36-3 \rightarrow 33 \div 3 \rightarrow 11$
16.

17.

18. There are 18 rows and 10 columns, so $10 \times 18=180$ tiles
19. 3 凸 $=\nabla$, so 6 亿 $=2 \nabla$, so $X=2 \nabla=4 \Delta$
20. Represent it on paper, e.g. as horizontal measurement. Fill in information bit by bit:

Siva is shorter than Temba:
Aby is taller than Temba:
Siva is taller than Oscar:
Siva Temba

Siva is taller than Oscar.
Oscar Siva Temba Eby
Siva is shorter than Ram, Eby is taller than Ram: Oscar Siva Ram/Temba Eby
21. Make a systematic representation of all the possibilities, e.g.:
white bread with: egg
brown bread with:
polony
egg
colony
rye bread with egg polony
cheese
jam fishpaste peanut butter
cheese cheese
jam
fishpaste peanut butter
jam fishpaste peanut butter

So, number of sandwiches $=3 \times 6$
22. Draw it physically:

23. The smallest number is $1+1=2$ and the biggest $6+6=12$. From 2 to 12 is $12-2+1=11$ numbers
24. List all the possibilities systematically Notice any patterns?
Out of the 36 possibilities, a sum of 7 occurs six times, a sum of 6 occurs five times, etc.
You can expect a sum of 7 to occur 6 out of 36 times
You can expect a sum of 2 to occur 1 out of 36 times

| $1+1$ | $2+1$ | $3+1$ | $4+1$ | $5+1$ | $\mathbf{6 + 1}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $1+2$ | $2+2$ | $3+2$ | $4+2$ | $\mathbf{5}+\mathbf{2}$ | $6+2$ |
| $1+3$ | $2+3$ | $3+3$ | $\mathbf{4 + 3}$ | $5+3$ | $6+3$ |
| $1+4$ | $2+4$ | $\mathbf{3 + 4}$ | $4+4$ | $5+4$ | $6+4$ |
| $1+5$ | $\mathbf{2 + 5}$ | $3+5$ | $4+5$ | $5+5$ | $6+5$ |
| $\mathbf{1}+\mathbf{6}$ | $2+6$ | $3+6$ | $4+6$ | $5+6$ | $6+6$ |

25. "See" some structure in the sketches for $\mathrm{P}_{3}$ and $\mathrm{P}_{4}$, then extend backwards and forwards, e.g.:


## GRADE 4 (F)

2. $147 \mathrm{~mm}-103 \mathrm{~mm}=44 \mathrm{~mm}$
3. $9: 15+1$ hour $=10: 15 ; 10: 15+50 \mathrm{~min}=11: 05$
4. R9,89 $\times 4=\mathrm{R} 39,56 ; \mathrm{R} 42,37-\mathrm{R} 39,56=\mathrm{R} 2,81$
5. 

|  | 96000 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 160 |  | 60 |  |  |
|  |  | 20 |  | 30 |  |
| 2 | 4 |  | 5 | 5 | 6 |

7. $\mathrm{R} 4 \div 5=\mathrm{R} 0,80$ for 1 apple. $\mathrm{R} 9,60 \div \mathrm{R} 0,80=12$ apples
8. 

|  | Jim | Bongani |
| :---: | :---: | :---: |
| Round 1 | 125 | 100 |
| Round 2 | 125 | 125 |
| Round 3 | 150 | 100 |
| Round 4 | 50 | 150 |
| Total | 450 | 475 |

$475-450=25$. Bongani won by 25 points
9. $\frac{2}{10}=0,2$
10. $\frac{1}{3}=\frac{2}{6} ; \frac{2}{6}+\frac{1}{6}=\frac{3}{6}=\frac{1}{2}$
11.

13. School A: 8:00 8:30 9:00 9:30 10:00 10:30 11:00 $11: 30$
$\begin{array}{llllllll}\text { School B: } & 8: 00 & 8: 35 & 9: 10 & 9: 45 & 10: 20 & 10: 55 & 11: 30\end{array}$
14. $100 \times 5=500$
$10 \times 5=50$
$10 \times 5=50$
$4 \times 5=20 ; 500+50+50+20=620$
Or: $5 \times(100+10+10+4)=5 \times 124=620$
15. List the possibilities systematically. Notice the number patterns and let the patterns help you:

```
\(0 \times 20 c+0 \times 10 c+10 \times 5 c \quad 1 \times 20 c+0 \times 10 c+6 \times 5 c\)
\(2 \times 20 c+0 \times 10 c+2 \times 5 c\)
\(0 \times 20 c+1 \times 10 c+8 \times 5 c \quad 1 \times 20 c+1 \times 10 c+4 \times 5 c\)
\(2 \times 20 \mathrm{c}+1 \times 10 \mathrm{c}+0 \times 5 \mathrm{c}\)
\(0 \times 20 \mathrm{c}+2 \times 10 \mathrm{c}+6 \times 5 \mathrm{c}\)
\(1 \times 20 c+1 \times 10 c+4 \times 5 c\)
\(0 \times 20 \mathrm{c}+3 \times 10 \mathrm{c}+4 \times 5 \mathrm{c}\)
\(1 \times 20 c+3 \times 10 c+0 \times 5 c\)
\(0 \times 20 c+4 \times 10 c+2 \times 5 c\)
\(0 \times 20 \mathrm{c}+5 \times 10 \mathrm{c}+0 \times 5 \mathrm{c}\)
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16. $1+11 \rightarrow$ difference too much
$2+10 \rightarrow$ difference too much
$3+9 \rightarrow$ difference too much
$4+8 \rightarrow$ difference 4
48 and not 84 because it must be less than 50
17. $\mathrm{R} 15+\mathrm{R} 6,75=\mathrm{R} 21,75 . \mathrm{R} 21,75 \div 3 \times 4=\mathrm{R} 29$
18. $1100 \mathrm{~g}-680 \mathrm{~g}=420 \mathrm{~g} .420 \mathrm{~g} \div 12=35 \mathrm{~g} .35 \mathrm{~g} \times 30=1050 \mathrm{~g} .1100 \mathrm{~g}-1050 \mathrm{~g}=50 \mathrm{~g}$
19. $100-(4 \times 5)=80$
20. $1,10,11,12,13,14,15,16,17,18,19,21,31,41,51,61,71,81,91,100 \rightarrow 21$ ones
21. Pattern: $5+4 n ; 6129-5 \rightarrow 6124 \div 4=1531$
22. Picture $1=1$

Picture $2=1+2$
Picture $3=1+2+3$
Picture $100=1+2+3+4+\ldots+98+99+100=(1+100) \div 2$

## GRADE 5(1)

1. Three heaps of five blocks each $=15$ blocks
2. R100 - $(6 \times \mathrm{R} 4,15+2 \times \mathrm{R} 12,49)$
3. You should recognise most without actual division!

| $360 \div 1=360$ | $360 \div 4=90$ | $360 \div 7=51,42857$ |
| :--- | :--- | :--- |
| $360 \div 2=180$ | $360 \div 5=72$ | $360 \div 8=45$ |
| $360 \div 3=120$ | $360 \div 6=60$ | $360 \div 9=40$ |

5. 2 layers of 8 by 4 or 8 columns of 2 by 4 or 4 rows of 8 by 2
6. All the blocks of the bottom layer (32) and all the blocks round the side of the top layer (20)
7. $41000 \mathrm{~g}-725 \mathrm{~g}=40275 \mathrm{~g}=40,275 \mathrm{~kg}$
8. Half of the previous number $=1,7 \div 2=0,85$
9. List all numbers containing a 9 systematically:
$9,19,29,39,49,59,69,79,89$ : 9 numbers
$90,91,92,93,94,95,96,97,98,99: 10$ numbers
So the number of numbers without a 9 is $100-(9+10)=81$
10. List them systematically: $101,111,121,131,141,151,161,171,181,191$
11. $100 \div 12=8$ rem 4, i.e. 8 years bringing us to Sept., plus 4 more months, i.e. Oct., Nov., Dec., Jan. Or if September $=9$, then 100 months further is $9+100=109$. But
January $=1,13,25, \ldots$ these leave a remainder of 1 if divided by 12
February $=2,14,26, \ldots$ these leave a remainder of 2 if divided by 12
March $=3,15,27, \ldots \quad$ these leave a remainder of 3 if divided by 12, etc.
So $109 \div 12=9$ rem 1, so January
12. $100 \div 7=14$ rem 2, i.e. 14 full weeks bringing us to Wednesday, plus 2 more days, i.e. Friday Or if Monday = 1 , Wednesday $=3$, so $3+100=103,103 \div 7=14$ rem 5, and 5 is Friday
13. $100 \div 24=4$ rem 4 , i.e. 4 full days bringing us to $10: 00$, plus 4 more hours, i.e. $11,12,13,14: 00$ Or $10+100=110,110 \div 24=6$ rem 14
14. September has 30 days - so there are 20 days left in September. October has 31 days. November has 30 days. $20+31+30=81$ days. 19 days to fill up 100 days, therefore 19 December
15. 3 for $5=$ ? for 90 . You can build it up, e.g. 30 for 50 and 24 for 40 , so 54 for 90 . Or $3 \times 18$ for $5 \times 18$ ( 90 )
16. Put the information in a sketch, fill in the details bit by bit, and extend the information. e.g.: The distance from A to E is 20 cm The distance from $B$ to $E$ is 10 cm You can deduce that $\mathrm{AB}=10 \mathrm{~cm}$ !

17. $99 \mathrm{~m}=\frac{9}{10}$, so $11 \mathrm{~m}=\frac{1}{10}$. Therefore $\frac{10}{10}=10 \times 11=110 \mathrm{~m}$
18. Make a sketch of the situation:
" 2 nd from front, 4 th from back" means there are 5 rows. " 3 rd from left, 5 trom right" means there are 7 learners per row. So 7 learners/row $\times 5$ rows $=35$ learners
19. Imagine yourself looking at the card from behind. Or tear the corner from a piece of paper, turn it around!
20. 

|  |  | $Z$ |
| :--- | :--- | :--- |
| 8 | 6 | $X$ |
|  |  | 5 | $8+6+X=18$, so $X=4$

$5+X+Z=18$, so $5+4+Z=18$, so $Z=9$
22. In second balance: $1 B=1 A+2 C$, so in first balance $5 A=3 B=3 A+6 C$, so $2 A=6 C$, so $1 A=3 C$
23. List systematically: $85-10,86-11,87-12,88-13, \ldots, 99-24$. From 85 to 99 is 15 possibilities
24. 2 Small +1 Large $=5$ Small, so 1 Large $=3$ Small, costing $3 \times$ R11, 50
25. Let the cost of a chocolate, a gum and a juice be $c, g$ and $j$ respectively

Then $2 g+2 j=10$, so $1 g+1 j=5$. But $1 c+1 g+1 j=9$, so $1 c+(1 g+1 j)=9$, so $1 c+5=9$, so $1 c=4$

## GRADE 5 (F)

1. $2,06 \mathrm{~m}=206 \mathrm{~cm} ; 2,06 \mathrm{~m}-1 \mathrm{~cm}=2,05 \mathrm{~m}$
2. 4 around the top, 4 around the bottom and 4 around the middle.
3. 1 on top, 4 around the middle and 1 at the bottom.
4. 4 at the bottom and 4 at the top.
5. $10 \times 95 \mathrm{c}=\mathrm{R} 9,50$
$10 \times \mathrm{R} 1,20=\mathrm{R} 12,00$
$\mathrm{R} 9,50+\mathrm{R} 12,00=\mathrm{R} 21,50 ; \mathrm{R} 40,00-\mathrm{R} 21,50=\mathrm{R} 18,50$
6. Half of target C is shaded and that is more than on any other target
7. $\frac{1}{6}=4$ slices, $\frac{1}{4}=6$ slices $\frac{1}{3}=8$ slices; $24-18=6$
8. $150 \div 6 \times 5=125$

9. $\frac{1}{2}$ of $\frac{1}{3}=\frac{1}{6}$
10. Light: $\begin{array}{llll}12 & 18 & 24\end{array}$

Bell: 81624
16. $\mathrm{E}(1)=2=1 \times 2$
$\mathrm{O}(1)=1=1 \times 2-1$
$\mathrm{E}(2)=4=2 \times 2$
$\mathrm{O}(2)=3=2 \times 2-1$
$\mathrm{E}(3)=6=3 \times 2$ $O(3)=5=3 \times 2-1$
$\mathrm{E}(4)=8=4 \times 2$
$\mathrm{O}(4)=3=4 \times 2-1$
:
:
$\mathrm{E}(100)=100 \times 2=200$
$\mathrm{O}(100)=100 \times 2-1=199$
18. List the possibilities

| $1+14$ | not possible more than 9 | $1+16$ | not possible more than 9 |
| :--- | :--- | :--- | :--- |
| $2+13$ | not possible more than 9 | $2+15$ | not possible more than 9 |
| $3+12$ | not possible more than 9 | $3+14$ | not possible more than 9 |
| $4+11$ | not possible more than 9 | $4+13$ | not possible more than 9 |
| $5+10$ | not possible more than 9 | $5+12$ | not possible more than 9 |
| $6+9$ | Possible | $6+11$ | not possible more than 9 |
| $7+8$ | not possible no card is a 7 | $7+10$ | not possible no card is a 7 |
|  |  | $8+9$ | Possible |

19. $A=B+2 C$, so $A+C=B+3 C$

But in middle $A+C=B+D$, so $D=3 C$, so $B=4 C$
But $A=B+2 C$, so $A=4 C+2 C=6 C$
20. The difference in age $40-16=24$ remains the same

So Mom $-\mathrm{Me}=24$ and $\mathrm{Mom}=2 \times \mathrm{Me}$
So $2 \times \mathrm{Me}-\mathrm{Me}=24$, so $\mathrm{Me}=24$
21. 988, 898, 889, 997, 979, 799
22. $99-36=63 ; 98-35=63 ; 97-34=63 ; 96-34=63$; ; $73-10=63$, so ...
23. Make a table of information:

|  | 10 | 11 | Total |
| :--- | :---: | :---: | :---: |
| Boys | 18 | 13 | 31 |
| Girls | 14 | 5 | 19 |
| Total | 32 | 18 | 50 |

24. $10 \times 10=100$
25. $\mathrm{T}_{1}=3=1 \times 3=1 \times 3$
$\mathrm{T}_{2}=9=3 \times 3=(1+2) \times 3$
$\mathrm{T}_{3}=18=6 \times 3=(1+2+3) \times 3$
:
$\mathrm{T}_{10}=(1+2+3+\ldots+9+10) \times 3=55 \times 3$

## GRADE 6(1)

2. $25+20+30+15+35=125$
3. $0,13-0,01 \rightarrow 0,12-0,01 \rightarrow 0,11-0,01 \rightarrow 0,10-0,01 \rightarrow 0,09$
4. We know that $\frac{1}{5}>\frac{1}{6}>\frac{1}{7}$ and $\frac{7}{30}>\frac{7}{36}$. So compare $\frac{1}{5}$ and $\frac{7}{30}: \frac{1}{5}=\frac{6}{30}<\frac{7}{30}$
5. $(15 \times 12-20) \times \mathrm{R} 2,40 / 8-15 \times \mathrm{R} 2,40=\mathrm{R} 12$ or $(15 \times 12-20) \times 30 \mathrm{c}-15 \times 12 \times 20 \mathrm{c}=15 \times 12 \times 10 \mathrm{c}-20 \times 30 \mathrm{c}=\mathrm{R} 12$
6. The watch gains 5 min in 6 hours, so 1 min in $6 / 5 \mathrm{~h}=1 \mathrm{~h} 12 \mathrm{~min}$, so it gains 2 min in 2 h 24 min
7. You can test them one by one, but it takes time! Clearly $11 \div 3=$ leaves a remainder of 2 . $111 \div 3=37$, so $111111 \div 3=37037$ and $1111=1110+1$ leaves a remainder of 1 .
Maybe you know, or discover, that if the digit sum is divisible by 3 , then the number is divisible by 3
8. $\mathrm{B}, \mathrm{A}, \mathrm{D}$, in that order, shows the line around the cube. C views B and D together
9. Imagine or draw the cube! If the side is 3 times as long, the big cube contains 27 of the small cubes! So its mass is 27 times as large!

10. The 6 small triangles (6), the one big one (7) and these:

11. $10 \times 10 \times 10 \times 10=10000$ or from 001 to 999 give 999 combinations, plus 000 give 10000
12. List all the possibilities systematically. Seeing the patterns also helps!
$10 \times 1 \mathrm{c}+0 \times 2 \mathrm{c}$
$1 \times 5 \mathrm{c}+2 \times 2 \mathrm{c}+1 \times 1 \mathrm{c}$
$2 \times 5 \mathrm{c}$
$1 \times 10 \mathrm{c}$
$8 \times 1 \mathrm{c}+1 \times 2 \mathrm{c}$
$1 \times 5 \mathrm{c}+1 \times 2 \mathrm{c}+3 \times 1 \mathrm{c}$
$6 \times 1 \mathrm{c}+2 \times 2 \mathrm{c}$
$1 \times 5 \mathrm{c}+0 \times 2 \mathrm{c}+5 \times 1 \mathrm{c}$
$4 \times 1 \mathrm{c}+3 \times 2 \mathrm{c}$
$2 \times 1 \mathrm{c}+4 \times 2 \mathrm{c}$
$0 \times 1 \mathrm{c}+5 \times 2 \mathrm{c}$
13. In the first balance: 3 circles $=6$ squares, so 1 circle $=2$ squares.

In the second balance: 3 triangles $=2$ circles, so 9 triangles $=6$ circles $=12$ squares
17. You should know from experience that (A) and (B) are true. For the others, test several different numbers trying to find one that is not true, and use your calculator for quick calculation, e.g.:
(C): $112,212,5712 \ldots$ are all divisible by 4 ; (D): 1232, $2232,3232,29232 \ldots$ are all divisible by 8 . In (C) and (D) we can't test all cases, so we can't be $100 \%$ sure, but it can be proved in other ways. (E): 163 is not divisible by 9 , so the statement is false! Some numbers are divisible, but not all!
18. Corrie walks 60 m for every 45 of Denise, i.e. 4 m for every 3 of Denise. So Corrie walks 4 out of every 7 metres of the distance, i.e. $\frac{4}{7}$ of 3570 m
Or: In 1 minute they come $60+45=105 \mathrm{~m}$ closer to one another. So they will cover 3570 m in $3570 \mathrm{~m} \div 105 \mathrm{~m} / \mathrm{min}=34$ minutes. In 34 minutes Corrie walks $34 \mathrm{~min} \times 60 \mathrm{~m} / \mathrm{min}=2040 \mathrm{~m}$
20. Make a sketch, e.g.:
$41-30=11$ children like only Comedy
$35-30=5$ children like only Action
$50-(11+30+5)=4$ children don't like either

21. List the possibilities systematically: $300|210,201| 120,102 \mid 111$
22. If the middle number is $m$, then $(m-1)+m+(m+1)=54$, so $3 \times m=54$, so $m=18$ So the consecutive numbers are 17,18 and 19 . So $17 \times 18 \times 19=5814$
23. Put the information in a sketch like this. Use the information bit by bit, and extend the info, e.g.: Elize finished 2 m ahead of Daphne, but 3 m behind Fay Elize beat Candy by 6 metres, etc.

24. $\mathrm{P}_{n}=4 \times n+1$, so $\mathrm{P}_{20}=4 \times 20+1$
25. If Zander caught $x$ fish, then Vincent caught $x+8$ fish, all together $x+(x+8)+20+24+18=100$.

Or, Vincent and Zander together caught $100-62=38$ fish. But Vincent caught 8 more than Zander

## GRADE 6 (F)

2. 



3. $1,2,4,5,10,20,25,50,100$
8. $\frac{1}{8}=\frac{10}{80}$ and $\frac{1}{10}=\frac{8}{80}$, therefore $\frac{9}{80}$ will be in between
9. $100 \div 20=5 ; 3000 \div 20=150$
10. 1 will be opposite 16,2 opposite 17,3 opposite 18,4 opposite 19,5 opposite 20,6 opposite 21 , 7 opposite 22
11.

| 2 | 3 |
| :---: | :---: |
| 3 | 5 |
| 4 | $?$ |
| 5 | 12 |
| 6 | 17 |$+4$

12. Dick is first, Peter is second, Tom is third and Harry is fourth.
13. Find the three numbers by trial and improvement. Use your calculator, e.g. $10 \times 11 \times 12=1320$ is too small, etc. But $14 \times 15 \times 16=3360$, so $14+15+16=45$
14. $2 p+3 ; 2(30)+3=63$
$15.370 \mathrm{~g}-290 \mathrm{~g}=80 \mathrm{~g}$; half of the milk weighs 80 g , so all the milk will weigh 160 g . $370 \mathrm{~g}-160 \mathrm{~g}=210 \mathrm{~g}$
15. $35-6=29$ ( 29 children either has a cat, a dog or both); $29-24=5$ ( 5 children has only dogs); $18-5=13$
$17.60 \div 5(1$ part +4 parts $)=12 ; 12 \times 6(1$ part +5 parts $)=72$ glasses
16. $(2000-1999)+(1998-1997)+\ldots+(2-1)=1+1+1+1+\ldots+1(1000$ times $)$
17. $(1+100)+(2+99)+(3+98)+(4+97)+\ldots+(50+51)=101 \times 50=5050$
18. Use the repeated structure of the system, e.g. imagine writing down all the numbers from 0 to 99 . Then the sum of the units digits is $10 \times(0+1+2+\ldots+8+9)$
The sum of the tens-digits is $10 \times 1+10 \times 2+\ldots+10 \times 9=10 \times(1+2+\ldots+8+9)$
So all together $2 \times 10 \times(1+2+\ldots+8+9)=2 \times 10 \times 45=900$. Adding the 1 of 100 gives 901 .
19. If the cheaper article costs $\mathrm{R} a$, the second article costs . Together they cost $a+a+6=11$

So $2 \times a+6=11$, so $2 \times a=5$. so $a=2,50$
22. Make a table of the differences:

|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{1}$ | 0 | 1 | 2 | 3 | 4 | 5 |
| $\mathbf{2}$ | 1 | 0 | 1 | 2 | 3 | 4 |
| $\mathbf{3}$ | 2 | 1 | 0 | 1 | 2 | 3 |
| $\mathbf{4}$ | 3 | 2 | 1 | 0 | 1 | 2 |
| $\mathbf{5}$ | 4 | 3 | 2 | 1 | 0 | 1 |
| $\mathbf{6}$ | 5 | 4 | 3 | 2 | 1 | 0 |

23. $1+3+5+7+\ldots+25+27+29=15 \times 15=225$
24. "All but six where white" means 6 were not white, so $b+y=6$.

Similarly $w+y=6$
and $w+b=6$
Add them together: $2 w+2 b+2 y=18$, so $w+b+y=9$
25. $w+b+y=9$

So $w+(b+y)=9$
so $w+6=9$, so $w=3$

## GRADE 7(1)

2. $114 \%=\mathrm{R} 36,15$ so $1 \%$ is $\mathrm{R} 36,15 \div 114$ and $100 \%=\mathrm{R} 36,15 \div 114 \times 100=\mathrm{R} 31,71$
3. $(3 \times 75)+(6 \times 66)=621 \mathrm{~kg}$ all together. So the average is $621 \mathrm{~kg} \div 9$ children $=69 \mathrm{~kg} /$ child
4. The "vertical" formula is $2 \times a+2$. Find $a$ so that $2 \times a+2=64$

Or the "horizontal" formula is $4+2 \times(a-1)$, so find $a$ so that $4+2 \times(a-1)=64$
5. We do not know whether he has had his birthday in 1524. So he can be 64 or 63
6. $p+q+p+q=p+p+q+q=2 \times p+2 \times q=(p+q) \times 2 \neq p \times q+p \times q$
7. Let the width be $w$ metres, then the length is $4 \times w$ metres, so the area $=4 \times w \times w=100$, so $w=5$ Then the perimeter is $2 \times(4 \times w+w)=10 \times w=10 \times 5=50$ metres
8. $\frac{n}{6,34}=\frac{100 \times n}{634}=\frac{a}{1}$, so $\frac{n}{634}=\frac{a}{100}$
9. The number is a multiple of 7 . So check which of $7,14,21,28,35,42,49,56, \ldots$ leave a remainder of 1 when divided by 3 or 5
Quicker: the first two conditions means that the number is one more than a multiple of $3 \times 5=15$
So the possible numbers are $16,31,46,61,76,91$. Of these, only 91 is also a multiple of 7
10.


13. Divide square into 4 equal parts.
So $\frac{1}{4}$ of $144=36$
 14. Divide square into 8 equal parts. So $\frac{1}{2}$ of $144=72$

15. Dissect into rectangles. $4 \times 16+16$ $=80$

16. Represent and organise the info in a table:

Fill in what they are not:

|  | 5 | 6 | 7 | 8 |
| :--- | :---: | :---: | :---: | :---: |
| Ali |  |  |  |  |
| Oli | x |  | x |  |
| Uli | x | x |  | x |
| Eli |  | x |  | x |

So Uli is 7, the others not:

|  | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |
| :---: | :---: | :---: | :---: | :---: |
| Ali |  |  | x |  |
| Oli | x |  | x |  |
| Uli | x | x | yes | x |
| Eli |  | x | x | x |

So Eli is 5, Ali is not 5:

|  | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |
| :---: | :---: | :---: | :---: | :---: |
| Ali | $\mathbf{x}$ |  | x |  |
| Oli | x |  | x |  |
| Uli | x | x | yes | x |
| Eli | yes | x | x | x |

17. See previous answer
18. $\mathrm{L}_{n}=3 \times n+2$, so $\mathrm{L}_{15}=3 \times 15+2=47$
19. If April had $x$ eggs, Peter had $x+2$, Melanie had $x+7$, Jack had $x+1$

Together: $x+(x+2)+(x+7)+(x+1)=38$, so $4 \times x+10=38$, so $x=7$
20. If he bought $x$ apples and $y$ oranges, then the cost is $2 \times x+1 \times y=52$ and the total fruit is $x+y=32$ $2 \times x+y=52$ can be written as $x+x+y=52$, so $x+(x+y)=52$, so $x+32=52$, so $x=20$
21. $\frac{5^{14}}{5^{17}}=\frac{1}{5^{3}}=\frac{1}{125}$
22. List the units digits of the first few powers:

| Exponent | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Units digit | 3 | 9 | 7 | 1 | 3 | 9 | 7 | 1 | 3 |

The last digits has a recurring pattern $3,9,7,1$
Reorganise the results like this and analyse the sequences:

| Units digit | Exponents giving the units digit |  |
| :---: | :--- | :--- |
| 3 | $1,5,9,13, \ldots$ | These have a remainder of 1 when divided by 4 |
| 9 | $2,6,10,14, \ldots$ | These have a remainder of 2 when divided by 4 |
| 7 | $3,7,11,15, \ldots$ | These have a remainder of 3 when divided by 4 |
| 1 | $4,8,12,16, \ldots$ | These are multiples of 4 |

We simply have to decide in which sequence 2003 will be ...
23. Look for structure in the denominator:

|  | $\mathbf{F}_{1}$ | $\mathbf{F}_{2}$ | $\mathbf{F}_{3}$ | $\mathbf{F}_{4}$ | $\ldots$ | $\mathbf{F}_{10}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Numbers | $\frac{1}{2}$ | $\frac{1}{6}$ | $\frac{1}{12}$ | $\frac{1}{20}$ |  | $?$ |
| Structure | $\frac{1}{1 \times 2}$ | $\frac{1}{2 \times 3}$ | $\frac{1}{3 \times 4}$ | $\frac{1}{4 \times 5}$ |  | $\frac{1}{10 \times 11}$ |

24. Calculate intermediate answers and look for structure and patterns:

Sum of $\mathbf{1}$ fraction $=\frac{1}{2}$
Sum of 2 fractions $=\frac{1}{2}+\frac{1}{6}=\frac{2}{3}$
Sum of 3 fractions $=\frac{2}{3}+\frac{1}{12}=\frac{3}{4}$
Sum of 4 fractions $=\frac{3}{4}+\frac{1}{20}=\frac{4}{5}$
So, Sum of 10 fractions $=\frac{10}{11}$
25. $(1+1) \times\left(1+\frac{1}{2}\right) \times \ldots=\frac{2}{1} \times \frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \ldots \times \frac{100}{99} \times \frac{101}{100}$

$$
\begin{aligned}
& =\frac{2}{2} \times \frac{3}{3} \times \frac{4}{4} \times \frac{5}{5} \times \ldots \times \frac{100}{100} \times \frac{101}{1} \\
& =101
\end{aligned}
$$

## GRADE 7 (F)

2. $23 \times 25=\frac{46}{4} \times \frac{a}{2}=23 \times \frac{a}{4}$, so $25=\frac{a}{4}$, so $\ldots$
3. $6 \times 4=24 \mathrm{~cm}^{2} ; 24 \mathrm{~cm}^{2} \div 2=12 \mathrm{~cm}^{2}$
4. $6 \times 4=24 \mathrm{~cm}^{2}$
$24 \mathrm{~cm}^{2} \div 2=12 \mathrm{~cm}^{2}$
5. $6 \times 4=24 \mathrm{~cm}^{2}$
$24 \mathrm{~cm}^{2} \div 2=12 \mathrm{~cm}^{2}$

6. 24 is three-quarters of the marbles she had before giving anything to Sonny; $24 \div 3 \times 4=32$ 32 is two -thirds of the marbles she had before giving anything to Ronnie; $32 \div 2 \times 3=48$
7. Volume $=15 \times 8 \times x=120$, so $x=1 \mathrm{~cm}$. So area is $(15 \mathrm{~cm}+2 \mathrm{~cm}) \times(8 \mathrm{~cm}+2 \mathrm{~cm})=170 \mathrm{~cm}^{2}$
8. Divisible by 7 and 8 means it is divisible by $7 \times 8=56$.

First multiple of 56 bigger than 800 is $15 \times 56=840$, then 896 , then 952 . So 3 .
10.

| width | Length | Perimeter |
| :---: | :---: | :---: |
| 1 | 7 | 16 |
| 2 | 6 | 16 |
| 3 | 5 | 16 |
| 4 | 4 | 16 |

11. $\frac{1}{8}$

12. List the possibilities:

| $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ |
| :--- | :--- | :--- | :--- |
| 1 | 2 | 3 | 9 |
| 1 | 2 | 4 | 8 |
| 1 | 2 | 5 | 7 |
| 1 | 3 | 4 | 7 |
| 1 | 3 | 5 | 6 |
| 2 | 3 | 4 | 6 |

13. If $\mathrm{A}=4 \times 4$ and $\mathrm{B}=5 \times 5$, then $\mathrm{E}=1 \times 1$ and $\mathrm{C}=6 \times 6$ and $\mathrm{D}=7 \times 7$. So the area is $=13 \times 11$
14. 



I "invent" a notation as either a single letter, or naming the top-left bottom-right vertices. So the squares are:
C, E, F, G, H, I
AE, AJ, DE, DI, EI, IJ
15. R1,50 for first km; $90 \mathrm{c} \times 10 \mathrm{~km} ; \mathrm{R} 1,50+\mathrm{R} 9,00=\mathrm{R} 10,50$
16. $\mathrm{R} 20-\mathrm{R} 2-\mathrm{R} 1,50=\mathrm{R} 16,50 ; \mathrm{R} 16,50 \div \mathrm{R} 0,90=18 ; 18+1=19 \mathrm{~km}$
17. $25 \times 25=625 \mathrm{~m}^{2}$
18. $\left(1-\frac{1}{2}\right) \times\left(1-\frac{1}{3}\right) \times\left(1-\frac{1}{4}\right) \times \ldots \times\left(1-\frac{1}{2003}\right)$
$=\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \ldots \times \frac{2001}{2002} \times \frac{2002}{2003}=\frac{1}{2003}$
21. 3 lines from two corners divide the triangle in $4 \times 4$ sections

10 lines from two corners will divide the triangle in $11 \times 11$ sections $=121$
22. 18 balls left to go into 7 boxes. If you put two balls in each box there are still four balls left over.
23. The sides of a square are equal. So list the possibilities of the rectangle until 4 times the width is equal to the length: $4(3)=12 ; 12 \times 4=48 \mathrm{~cm}$
25. Record the information in a table (matrix):

| says | Ahmed | Bruce | Delia | Clare | Eldon |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Ahmed |  |  | yes |  |  |
| Bruce |  |  |  | yes |  |
| Delia |  | yes |  |  |  |
| Clare |  |  |  | no |  |
| Eldon |  |  |  |  | yes |

Decide if Clare is lying or not:
If Clare is telling the truth, it means she did not do it, so Bruce and everyone else is lying, so nobody did it, which is not true. So Clare cannot be telling the truth!
So Clare is lying, which means she did it, so Bruce is the one telling the truth (Clare did it) and everyone else is lying.

