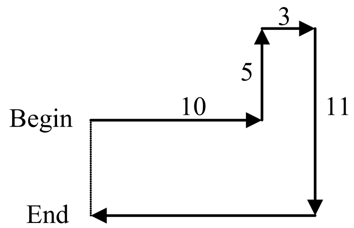
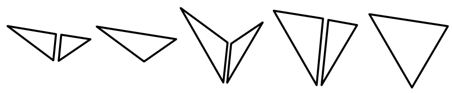
NOTES ON 2007 MEMORANDUM

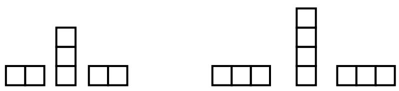
These notes are necessarily brief and often formal and symbolic.
Many questions could be answered using primitive methods, e.g. "If today is Wednesday, what day of the week will it be 100 days from now?" can be done by counting. That would be laborious, time-consuming and error-prone. The essence of a mathematical approach is to work more smartly by using appropriate representations to model the situation and to exploit the inherent structures and patterns in the situation.

GRADE 4(1)

3. Straighten the string. Two loops of 1 cm make it 5 cm + 1 cm + 1 cm = 7 cm
4. 3 hours before 16:45 is 13:45, so 2 hours and 55 minutes (5 min less) is 13:50
5. The watch gains 2 minutes every day (24 hours) for 7 days = 2 min/day× 7 days = 14 minutes
6. Rotate (in your mind!) the pieces to fit ...
9. With 6 loose cubes, there would be 36 faces. Subtract the 10 non-visible faces ...
10. Place value!
11. If Zuki has ♥ marbles, Zinkle has ♥ – 15. Together they have 2×♥ – 15 = 95 marbles. So ♥ = 55
12. Do not rush into calculation – analyse the structure: 826×243 – 824×243 = (826 – 824)×243 = 2×243
13. The numbers must be different, so 99 + 98 + 97 = (100 – 1) + (100 – 2) + (100 – 3) = 300 – 6
14. 18 × 10
15. If the 21^{th} is a Monday, then also the 14^{th}, 7^{th} and 0^{th} are Mondays. The 0^{th} is the last day of the previous month, so the next day is the 1^{st} of this month, so it is a Tuesday
17. $\frac{3}{4} + \frac{3}{4} \rightarrow 1\frac{1}{2} + \frac{3}{4} \rightarrow 2\frac{1}{4} + \frac{3}{4} \rightarrow 3 + \frac{3}{4} \rightarrow 3\frac{3}{4} + \frac{3}{4} \rightarrow 4\frac{1}{2}$
    ① ② ③ ④ ⑤ ⑥
18. Try the numbers one by one, e.g. 20 $\boxed{\times 3} \rightarrow 60$ $\boxed{+ 8} \rightarrow 68$ $\boxed{\div 2} \rightarrow 34$ $\boxed{- 6} \rightarrow 28 \neq 20$
19. If the number of tables is T, then 2 × T + 2 = 58, so T = (58 – 2) ÷ 2 = 28
20. Draw it physically! See diagram. It always helps to write!



21. $\frac{1}{5} = \frac{8}{40}$ and $\frac{1}{4} = \frac{10}{40}$, so $\frac{8}{40} < \frac{9}{40} < \frac{10}{40}$, which means $\frac{1}{5} < \frac{9}{40} < \frac{1}{4}$
22. 
23. Do not count or calculate – look for structure, e.g.
    For Pattern 3: 3 + 2×2
    For Pattern 4: 4 + 2×3
    For Pattern 100: 100 + 2×99



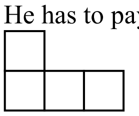
Pattern 3 *Pattern 4*

24. List all the possibilities and be systematic:

1 + 1 = 2	2 + 2 = 4	3 + 3 = 6	4 + 4 = 8	5 + 5 = 10	6 + 6 = 12
1 + 2 = 3	2 + 3 = 5	3 + 4 = 7	4 + 5 = 9	5 + 6 = 11	

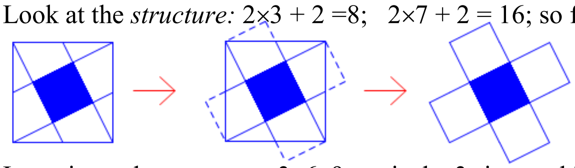
    Any other combination will be a repetition – therefore 11 possible answers
25. Look for structure, a clever way of counting, e.g. every point is connected to every other point except itself, so at each of the 18 points on the circle there are 17 lines, in total 18×17. But each line is counted twice, so 18×17 ÷ 2

GRADE 4(F)

1. 1 pizza for 3 children
    12 pizzas × 3 children/pizza = 36 children
4. These are multiples of 6. Only 4182 = 6 ×697 is a multiple of 6
5. 33×58 = 1914
6. 09:47 to 10:18 = 31 minutes
    12:30 = 31 min. = 13:01
7. Jason has 2/3 of the stamps and Mary has 1/3 of the stamps
    96 ÷ 3 = 32 stamps
8. 438 – 257 = 181 km
9. 438 + 169 = 607 km
10. Thabo takes 4 out of 12; 4/12 = 1/3
    He has to pay 1/3 of R30 → R10
11.  The tower is on your left if you look at the object from the back
12. 6,8 ÷ 2 → 3,4 ÷ 2 → 1,7 ÷ 2 = 0,85
13. 24 – 24 = 0; 71 × 3 = 213 marbles

14. R35,60c ÷ 40 = R0,89c
    R0,89c × 15 = R13,25c
17. 8 × 2 + 3 × 2 = 22m
22. 100 × 3 = 300
24. 20c + 10c + 5c
    3 × 10c + 5c
25. Number of blocks = 1 + 2+ 3+ 4+ …. + 48 + 49 + 50 = (1+50) + (2 + 49) + … = 25 × 51 = 1275

GRADE 5(1)

2. The numbers inside the square *and* the circle are 2 and 3. 2 is not inside the triangle
4. 147 mm – 103 mm = 44 mm
5. 100 ÷ 24 = 4 rem 4, i.e. 4 full days bringing us again to 10:00, plus 4 more hours, i.e. 11, 12, 13, 14:00
6. n^{th} row has $2 \times n - 1$ dots, so 7^{th} row has $2 \times 7 - 1$ dots
7. n^{th} row has $2 \times n - 1$ dots, so 70^{th} row has $2 \times 70 - 1$ dots
8. C – a rotation to the right through 90°
9. *Height* = 12 cm + 1,5 cm/day × *days*. So *Height after 30 days* = 12 + 1,5×30 = 57 cm
10. (150 cm – 12 cm) ÷ 1,5 cm/day = 92 days
11. One more than a multiple of 6, so it is odd, so it cannot be A or B. Test the others: 4 182÷6 = 697
12. If a sack weighs S kg, then $3 \times S = S + 30$, so $2 \times S = 30$, so $S = 15$. So $3 \times S = 45$ kg
13. If the cold drink costs Rx, then the ice cream costs R(x+3) and the burger R(x+7). So $3 \times x + 10 = 19$, so $x = 3$
15. In the bottom layer there are 8 × 4 = 32 blocks, so in two layers there are 64 blocks
16. All the blocks of the bottom layer (32) and all the blocks round the side of the top layer (20)
17. Look at the *structure:* 2×3 + 2 =8; 2×7 + 2 = 16; so for a rectangle with length 20: 2×20 + 2 = 42
18. 
19. Investigate the *structure:* 3, 6, 9, … is the 3-times table:

Pattern n	1	2	3	4		*n*
# coins	3	6	9	12		$3 \times n$

20. 4 reds = 10 greens = 3 purples. So 12 (3×4) reds = 9 (3×3) purples
21. The number must start and end with 1 so list them systematically:
    101 111 121 131 141 151 161 171 181 191
22. The structure is 1 + 2 + 3 + 4 + 5 + 6 + ... + 48 + 49 + 50 = (1+50) + (2+49) + (3+48) + ... = 51 × 25
23. If Penny has p coins and Alex has a coins: $p - 4 = a + 4$. But $p = 2 \times a$, so $2 \times a - 4 = a + 4$, so $a = 8$ and $p = 16$
24. Be systematic, e.g.

32	23	43	13
34	24	42	12
31	21	41	14

25. If a small pizza costs s rands and a large pizza costs L rands: $2s + 1L = 5s$, so $1L = 3s$, so the cost is L = 3×R11,50 = R34,50

GRADE 5(F)

2. Lucy's mother is 24 years older than Lucy
   16 + 8 = 24
   In 8 years Lucy's mother will be 48
   Lucy is 24 now.
3. 07:20 + 45 min = 08:05
4. 500 ÷ 12 = 41 rem 8
   42 cartons
5. 360 ÷ 120 = 3
   270 ÷ 90 = 3
   22 × 3 = 66
6. Chapter 6 ends on page 245
   274 – 245 = 29 (she has to read page 246 as well)
8. 150 ÷ 6 = 25
   25 × 5 = 125
9. 1/8 + 25 litres = 5/8
   25 litres = 4/8 = ½
   25 × 2 = 50 litre tank

11. $500 \div 10 = 50$ (original number)
    $50 \div 10 = 5$ (correct answer)
13. $365 - 5 \rightarrow 360 \div 6 = 120$
14. Marty ate $24 \div 6 = 4$ slices
    Veronica ate $24 \div 4 = 6$ slices
    Ron ate $24 \div 3 = 8$ slices
    $24 - 8 - 6 - 4 = 6$ slices for Justin
15. Two people at the end + 2 people per table: $20 \times 2 + 2 = 42$ people
16. $58 - 2 = 56$
    $56 \div 2 = 28$ tables
17. 8 kids: $8 \times 6 = 48$
    $48 + 7 = 55$
18. $420 \times 4 = 1680$
    $230 \times 4 = 920$
    $270 \times 4 = 1080$
    $1680 + 920 + 1080 = 3{,}68$ m
19. $270 \times 4 = 1080$
    $420 \times 2 = 840$
    $230 \times 2 = 460$
    Ribbon and knot = 400 mm
    $1080 + 840 + 460 + 400 = 2{,}78$ m
23. Last row = 99 blocks, 2nd to last row = 97 blocks
    Last row + 1st row = 100
    2nd row + 2nd to last row = 100
    25 (pairs of rows) × 100 = 2500 blocks
24. $50 + 49 = 99$
25. Last row = $37 + 36 = 73$
    72×18 pairs $+ 73 = 1369$

GRADE 6(1)

1. Make equal parts. Each small square is half of the next bigger square.
   So half of half of the big square is a quarter of the big square
2. There are 8 columns, each with $2 + 4 + 6$ cubes. So $8 \times 12 = 96$ cubes
5. $\frac{1}{7} = \frac{5}{35}$ and $\frac{1}{5} = \frac{7}{35}$ so $\frac{6}{35}$ is exactly in between them. Or $(\frac{1}{5} + \frac{1}{7}) \div 2 = (\frac{7}{35} + \frac{5}{35}) \div 2 = \frac{6}{35}$
7. $\frac{5}{100} = \frac{x}{3000}$
8. Continue the patterns: 17, 22, 27, 32, 37, 42, 47, 52, … and 17, 24, 31, 38, 45, 52, …
   Or, the lowest common multiple of 5 and 7 is 35, so 17 + 35 will be common and every 35 after that
9. *Look* at the structure: For n dice, the number of visible faces is $n\times3 + 2$. So for 75 dice, $75\times3 + 2$
10. Every date is one weekday later in the next year, because $365 \div 7 = 52$ rem 1. Then we must account for leap years (*):

Year	1995	1996*	1997	1998	1999	2000*	2001	2002	2003
Day	Tue	Wed	Fri	Sa	Su	Mo	We	Thu	Fri

12. "Manually" try different orientations to find the maximum!
13. If the sides are a and b, then $2a + 2b = 480$, so $a + b = 240$. But one side is double the other, so $a + 2a = 240$, so $3a = 240$
14.

B	C	M
A	2	N
1	D	

In the middle row, N cannot be 2, so N is 1 or 3
Suppose N = 3. Then A = 1 which is impossible (already a 1 in left column).
So N = 1, A = 3. In left column B = 2. Then C = 1 (D ≠ 1), so M = 3, so M+N = 4

15. $3\times(1 + 2 + 3)$
16. Vary the possibilities systematically. First note that she could not draw 1, 3 or 5 games, otherwise her total would be a fraction. If she drew 6 games her total was $6\times\frac{1}{2} = 3$. If she drew 4 and won 2 her total was $2\times1 + 4\times\frac{1}{2} = 4$. If she drew 2 and won 4 her total was $4\times1 + 2\times\frac{1}{2} = 5$
17. Vary the numbers systematically and note the behaviour of the product of the numbers:

$1 + 17 = 18$ and $1 \times 17 = 17$	$6 + 12 = 18$ and $6 \times 12 = 72$
$2 + 16 = 18$ and $2 \times 16 = 32$	$7 + 11 = 18$ and $7 \times 11 = 77$
$3 + 15 = 18$ and $3 \times 15 = 45$	$8 + 10 = 18$ and $8 \times 10 = 80$
$4 + 14 = 18$ and $4 \times 14 = 56$	$9 + 9 = 18$ and $9 \times 9 = 81$
$5 + 13 = 18$ and $5 \times 13 = 65$	$10 + 8 = 18$ and $10 \times 8 = 80$

18. ? = 000ΔΔΔ = 0ΔΔΔ + $\frac{1}{2}$(0000ΔΔ) = 6□ + 4□ from first two balances

19. If Sunday's date is x, then $x + (x+1) + (x+2) + (x+3) + (x+4) + (x+5) + (x+6) = 126$, so $7\times x + 21 = 126$, so $x = 15$

20. The same *structure* as 19! $9\times x + 36 = 135$, so $9\times x = 99$, so $x = 11$

21. Represent and organise the info in a table:

Fill in what they are *not*:

	5	**6**	**7**	**8**
Ali				
Oli	**x**		**x**	
Uli	**x**	**x**		**x**
Eli		**x**		**x**

So Uli is 7, the others not:

	5	**6**	**7**	**8**
Ali			**x**	
Oli	x		x	
Uli	x	x	yes	x
Eli		x	**x**	x

So Eli is 5, Ali is not 5:

	5	**6**	**7**	**8**
Ali	**x**		x	
Oli	x		x	
Uli	x	x	yes	x
Eli	yes	x	x	x

22. See 21

23. Investigate the *structure:* The sums in the *Rows* are 1, 2, 4, 8, 16, ...Use this pattern!

Row n	0	1	2	3	4	5	6		*n*
Sum	2^0	2^1	2^2	2^3	2^4	2^5	2^6		2^n

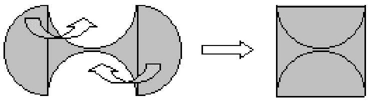
24. If a bubble gum cost *B* cents and a chocolate costs *C* cents:
$B + C = 90$ and $10B + 5C = 470$, so $5B + 5(B + C) = 470$, so $5B + 5\times 90 = 470$, so $B = 4$, so $C = 86$c

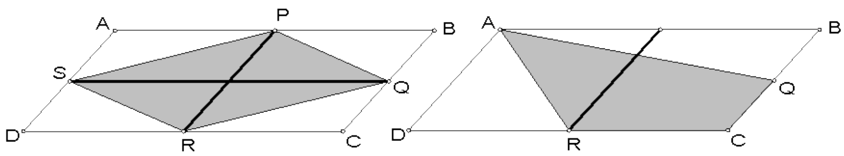
25. Do not count or calculate, investigate the *structure:* 1, 4, 9, … = 1×1, 2×2, 3×3, … 20×20

GRADE 6(F)

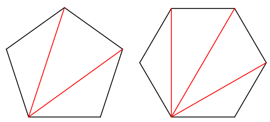
1. 56 + 58 + 60 = 174
2. 728 ÷ 8 = 91
3. 724 – 4 → 720 ÷ 8 = 90
4. 6 × 6 = 36
   6 + 6 = 12
7. 25 + 20 + 30 + 15 + 35 = 125
8. 4653 – 2583 → 2070 ÷ 90 = 23
9. 1 × 2 – 1 = 1
   4 × 2 – 1 = 7
   7 × 2 – 1 = 13
   2 × 2 – 1 = 3
12. ½ + 1/8 + +1/8 = ¾
    R15 = ¼
    R15 × 4 = R60
13. 36 × 37 = 1332
14. 2/3 ÷ 2 = 1/3
    12 ÷ 2 = 6
    6 × 3 = 18
    1/3 × 3 = 1 cup of milk
15. 2 books = 6 kg
    1 book = 3 kg
16. R12 = ¼
    R12 × 4 = R48
    ¾ ÷ 2 = 3/8
    3/8 of R48 = R18
17. 10 + 10 = 20
18. Diana is 3 years older than Joe
    Joe is two years older than Cindy
    Diana is five years older than Cindy
    8 + 5 = 13
19. A: R800 × 12 = 9600
    B: R110 × 80 = 8800
21. 5 × 4,8 = 24
    4,5 + 4,6 + 4,7 + 5 = 18,8
    24 – 18,8 = 5,2
24. 15 + 20 + 25 + 30 = 90
    15 bananas in the last hour
25. 50 × 4 + 1 = 201

GRADE 7(1)

3. $(4 \times 75) + (6 \times 65) = 690$ kg all together. So the average is 690 kg $\div$ 10 children= 69 kg/child
4. The "vertical" formula is $2\times a + 2$. Find a so that $2\times a + 2 = 64$. Or $(a + 1)\times 2 = 64$, so $a = 64\div 2 - 1$
   Or the "horizontal" formula is $4 + 2\times(a - 1)$, so find a so that $4 + 2\times(a - 1) = 64$
5.  So 6 cm × 6 cm = 36 cm^2
6. Volume = 15 cm × 8 cm × x cm = 120 cm^3, so $x = 1$. So area is (15 cm + 2 cm) × (8 cm + 2 cm) = 17 cm × 10 cm
7. $(2 - 1) + (3 - 2) + (4 - 3) + \ldots + (100 - 99) + (101 - 100) = 1 + 1 + 1 + 1 + \ldots$ 100 times = 100
8. If the *prices* are Rp and Rb, then $3p + 5b = 44$, so $3p + 3b + 2b = 44$, so $3(p + b) + 2b = 44$, so $3\times 10 + 2b = 44$
   Or: $p + b = 10$, so $3p + 3b = 30$. But $3p + 5b = 44$, so $2b = 14$
9. $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{16}$
10. That is what is meant with a *random* event – it is independent of what happened before, it has no memory!
11. List them all: 20, 22, 24, 26, 28 and similarly for the 40s. 60s and 80s, so $4 \times 5 = 20$
12. If the old price is x, then the cash price is $0{,}9 \times (1{,}1\times x) = 0{,}99 \times x < x$
13. The order in which we add numbers does not matter! So the final number is $1 + 2 + 3 + 4 + \ldots + 99 + 100$
    $1 + 2 + 3 + 4 + \ldots + 99 + 100 = (1+100) + (2+99) + (3+ 98) + \ldots = 101 \times 50$
14. Do not rush into calculation! *Look for structure!* $\frac{24\times18\times15+24\times18\times13+24\times18\times7}{24\times18}=\frac{24\times18\times(15+13+7)}{24\times18}=35$
15. If the dimensions of the room is a by b by c, then the area to paint is $A = 2ab + 2ac + 2bc$
    Double the dimensions are $2a$ by $2b$ by $2c$, so the area to paint is $D = 2(2a)(2b) + 2(2a)(2c) + 2(2b)(2c) = 4\times A$
16. If the length and width are l m and w m, then Area = $l\times w$ =100. But $l = 4\times w$, so $4\times w \times w = 100$, so $w = 5$ and $l = 20$
17. In middle row the missing number is $18 - (11+6) = 1$, so in right column $X = 18 - (1+10) = 7$
18. *Make* equal parts! $\frac{4}{8}=\frac{1}{2}$
19. $\Delta ADR = \Delta ABQ = \frac{1}{4}ABCD$, so *subtract* the two white triangles: $1-\frac{1}{2}=\frac{1}{2}$



20. If they mine 5%, then 95% is left. So:
    After 1 year, 95% is left
    After 2 years, 95% of 95% = $0{,}95 \times 0{,}95 = 0{,}95^2$ is left
    After 3 years, 95% of 95% of 95% = $0{,}95 \times 0{,}95 \times 0{,}95 = 0{,}95^3$ is left
    After 10 years, $0{,}95^{10}$ is left. Use a calculator: $0{,}95^{10} = 0{,}598$ = 59,8% is left
    After 13 years, $0{,}95^{13}$ is left. $0{,}95^{13} = 0{,}513$ = 51,3%, more than half, is left
    After 14 years, $0{,}95^{14}$ is left. $0{,}95^{14} = 0{,}487$ = 48,7%, less than half, is left
21. You can draw it, or investigate numerical patterns for a triangle, square, pentagon, hexagon, etc.
    Or you can reason it out: At each vertex of an n-gon there are n–3 diagonals because the point is connected to every other point, except to itself and to the two adjacent points (these are *sides* of the n-gon). So at n vertices there are $n\times(n-3)$ diagonals, counted twice. So the formula is $D(n) = n\times(n-3) \div 2$, so $D(8) = 8\times(8 - 3) \div 2$
22. Divide the polygon into triangles, because we know the angles of a triangle totals 180°:
    A square (4-gon) into 2 triangles, so each angle is $2\times180° \div 4$
    A pentagon (5-gon) into 3 triangles, so each angle is $3\times180° \div 5$
    A hexagon (6-gon) into 4 triangles, so each angle is $4\times180° \div 6$
    An n-gon into n–2 triangles, so each angle is $(n-2)\times180° \div n$



23. The volume of the offices = 10 m × 8 m × 2,4 m = 192 m^3 and 180 m^3 < 192 m^3 < 280 m^3
24. The formula for $P_n = 5n + 1$
25. For every 10 °C there is a *constant difference* of 18 °F. So a 1 °C change equals a 1,8 °F change. So 32 °C = 86 + 2×1,8 °F
    Or the formula is F = 1,8×C + 32

GRADE 7(F)

1. Use trial and error to find that only $26 \times 27 = 702$. So $26 + 27 = 53$
2. $4 + 4 - 4 \div 4 = 4 + 4 - 1 = 8 - 1 \neq 1$
3. $6 \times 2 = 12$
4. Length × Breadth = $8 \times 4 = 32$
5. $1+\frac{1}{1+\frac{2}{3}}=1+\frac{1}{\frac{5}{3}}=1+\frac{3}{5}$

6. There is a general structure here: The denominators is twice the numerator + 1, i.e. $\frac{\lozenge}{2\times\lozenge+1}$

   We can therefore investigate a general pattern $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \frac{6}{13}, \frac{7}{15}, \ldots$

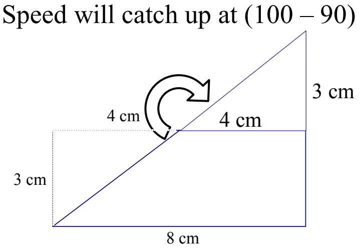
   Check with your calculator: $\frac{1}{3}$ = 0, 333..., $\frac{2}{5}$ = 0,4, … So $\frac{1}{3} < \frac{2}{5} < \frac{3}{7} < \frac{4}{9} < \frac{5}{11} < \frac{6}{13} < \ldots$

   Conclusion: the larger the denominator, the larger this kind of fraction, so $\frac{11}{23}$ is the largest

7. We know: $\frac{\text{Sum of numbers}}{11} = 8$, so Sum of numbers = 11 × 8 = 88

   If the new number is x, then $\frac{88+x}{12} = 11$. So x = 12 × 11 – 88 = 44

8. Speed will catch up at (100 – 90) km/h = 10 km/h, so he will catch up 30 km in 3 hours

9. 

10. Area = ½ × 8 × 6
11. 21 + 21 = 42
12. 8 × 8 = 64 ; 9 + 9 = 18
13. 64 ÷ 2 = 32 ; 31 × 31 = 961
15. 1/3 + (1/4 of 2/3) = 4/12 + 2/12 = 6/12 ; 24 = 6/12 ; Penny originally had 48 marbles; 1/3 of 48 = 16
16. 7 × 5 = 35
17. length = 4 × width ; perimeter = 4 × width + 4× width + 1 × width + 1 × width = 10 widths
    100 ÷ 10 = 10 ; 10 × 40 = 400
18. 6 × 81 – 3 = 483
19. 4321 ÷ 6 = 720 remainder 1; 721^{st} row
20. 50 × 50 = 2500
22. 2 drinks + ice cream = R15 ; 1 drink + 2 ice creams = R12;
    3 drinks + 3 ice creams = R15 + R12 = R27
    R27 ÷ 3 = R9
23. 2 drinks + 1 ice cream = R15 ; 1 drink + 1 ice cream = R9 ; 1 drink = R15 – R9 = R6
24. Test systematically:
    (1, 31) and (1, 31): 1 + 11 + 1×31 = 63
    (3, 15) and (15, 3): 3 + 15 + 3×15 = 63
    (7, 7): 7 + 7 + 7×7 = 63
25. Alphans use remainders when dividing by 6!
    So 2×5 + 4 = 14_{10} = Rem(14 ÷ 6) = 2 on Alpha