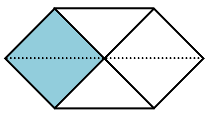
NOTES ON 2011 MEMORANDUM

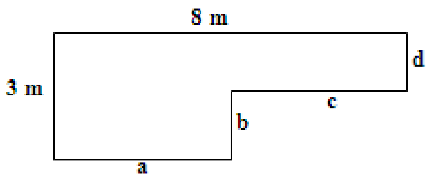
These notes are necessarily brief and often formal and symbolic.
Many questions could be answered using primitive methods, e.g. "If today is Wednesday, what day of the week will it be 100 days from now?" can be done by counting. That would be laborious, time-consuming and error-prone. The essence of a mathematical approach is to work more smartly by using appropriate representations to model the situation and to exploit the inherent structures and patterns in the situation.

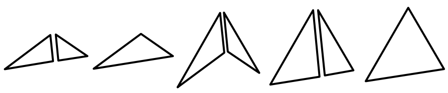
GRADE 4(1)

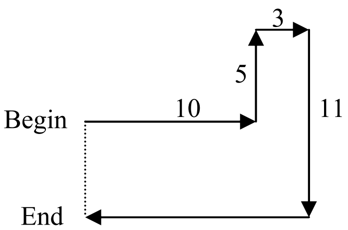
1. In D, 5 + 3 = 8, while the others all have a sum of 7
2. Half of 8 × 8
3. $\frac{32}{64} = \frac{1}{2}$
4. There is a pattern of +14, +14, +14 in the numbers
5. Place value!
6. Straighten the string. Two loops of 1 cm make it 5 cm + 1 cm + 1 cm = 7 cm
7. $\frac{1}{8} < \frac{1}{4} < \frac{1}{2}$
8. Rotate (in your mind!) the pieces to fit ... E!
9. 35 ÷ 4 = R8,75
10. Make *equal parts*: $\frac{2}{6} = \frac{1}{3}$



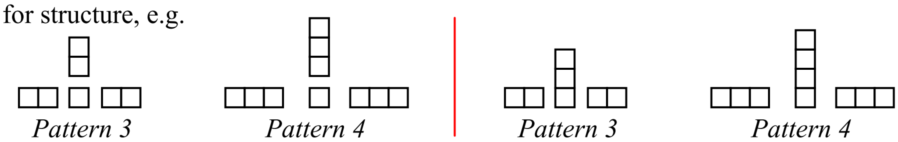
12. $\frac{1}{2} = 0{,}5$ and $\frac{3}{4} = 0{,}75$, so $0{,}2 < 0{,}4 < \frac{1}{2} < 0{,}6 < \frac{3}{4} < 0{,}8$
13. 8 + 3 + a + b + c + d
    But a + c = 8 and b + d = 3
    So 8 + 3 + 8 + 3 = 22



14. 08:00 – (15 + 20 + 35) min = 08c00 – 1 hour 10 minutes = 07:00 – 10 min = 06:50
17. With 6 loose cubes, there would be 6×6 = 36 faces. Subtract the 10 non-visible faces ...
18. 18 × 10
19. 
20. Do not rush into calculation – analyse the structure: 3826×243 – 3824×243 = (3826 – 3824)×243 = 2×243
21. The numbers must be different, so 99 + 98 + 97 = (100 – 1) + (100 – 2) + (100 – 3) = 300 – 6
22. Draw it physically! See diagram. It always helps to write!

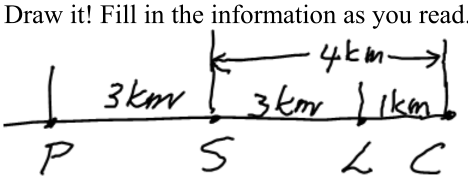


23. $x - 4 + 5 - 6 = 3$, so $x - 5 = 3$ so $x = 8$
24. If #T, then # people = 2 × T + 2 = 58, so T = (58 – 2) ÷ 2 = 28
25. Do not count or calculate – look for structure, e.g.
    For Pattern 3: 3×2 + 1
    For Pattern 4: 3×3 + 1
    For Pattern 100: 3×99+1



GRADE 4(F)

1. There are 10 divisions from 1,50 to 1,70, so each decision is 0,20÷10 = 0,02 m.
2. 3 hours before 16:45 is 13:45, so 2 hours and 55 minutes (5 min less) is 13:50
4. (64 + 96) ÷ 2 = 80
6. 1,7 m – 1,05 m = 0,65 m = 65 cm
7. 12×3 = 36
8. 6×100 = 600
9. The number has to be divisible by 6: 7356 ÷ 6 = 1226
10. The ones-digits must check, e.g. _8 × _9 must end in _2.
11. Between 09:47 and 10:18, 31 minutes pass. 31 minutes from 12:30 is 13:01
12. 257 + Δ = 438, so Δ = 438 – 257 = 181 km
13. 438 + 169 = 607 km
14. Draw it! Fill in the information as you read. Re-read, bit by bit!



15. Bingo: 71; Thandi: 71 – 24 = 47; Voyo: 71 + 24 = 95
    71 + 47 + 95 = 213 *or* 71 × 3 = 213 (24 – 24 = 0)

16. If Zuki has ♥ marbles, Zinkle has ♥ – 15. Together they have ♥♥ – 15 = 95 marbles. So ♥♥ = 110 and ♥ = 55
17.

32+38			
32	**38**		
15	**17**	**21**	
10	**5**	**12**	**9**

18. $\frac{3}{4} + \frac{3}{4} \rightarrow 1\frac{1}{2} + \frac{3}{4} \rightarrow 2\frac{1}{4} + \frac{3}{4} \rightarrow 3 + \frac{3}{4} \rightarrow 3\frac{3}{4} + \frac{3}{4} \rightarrow 4\frac{1}{2}$
    1 2 3 4 5 6
19. Let the children be A, B, C, D and E. List all the possibilities, be systematic, note patterns and structure:

A vs B	**B** vs C	**C** vs D	**D** vs E
A vs C	B vs D	C vs E	
A vs D	B vs E		
A vs E			

vs	A	B	C	D	E
A		X	X	X	X
B			X	X	X
C				X	X
D					X
E					

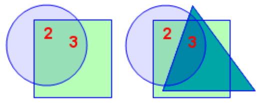
20. Name the girls a, b and c, and make a systematic list:
    abc bac cab
    acb bca cba
21. Try the numbers one by one, e.g. 20 $\boxed{\times 3} \rightarrow 60$ $\boxed{+ 8} \rightarrow 68$ $\boxed{\div 2} \rightarrow 34$ $\boxed{- 6} \rightarrow 28 \neq 20$
22. If the 21$^{\text{th}}$ is a Monday, then also the 14$^{\text{th}}$, 7$^{\text{th}}$ and 0$^{\text{th}}$ are Mondays.
    The 0$^{\text{th}}$ is the last day of the previous month, so the next day is the 1$^{\text{st}}$ of this month, so it is a Tuesday
23. Continue pattern of subtracting 4 cm/hour. Or the formula is Height = 32 – 4 × time
24. List all the possibilities and be systematic:

1 + 1 = 2	2 + 2 = 4	3 + 3 = 6	4 + 4 = 8	5 + 5 = 10	6 + 6 = 12
1 + 2 = 3	2 + 3 = 5	3 + 4 = 7	4 + 5 = 9	5 + 6 = 11	

    Any other combination (e.g. 2 + 1) will be a repetition – therefore 11 possible answers
25. $P_n = 4 \times n + 1$, so $P_{20} = 4 \times 20 + 1$

GRADE 5(1)

2. The numbers inside the square *and* the circle are 2 and 3.
   2 is not inside the triangle



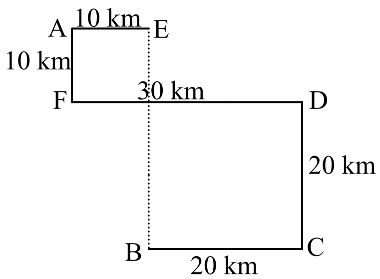
4. 147 mm – 103 mm = 44 mm
5. 100 ÷ 24 = 4 rem 4, i.e. 4 full days when it is 10:00 again, plus 4 more hours, i.e. 11, 12, 13, 14:00
6. $\nabla - 8 = 5$, so $\nabla = 13$, then we have 8 – 3 = 13 – 8, so 5 = 5
8. 20×20 = 40. So there are 20 tiles along each side, and along the diagonal
9. n^{th} row has $2 \times n - 1$ dots, so 7^{th} row has 2×7 – 1 dots
10. n^{th} row has $2 \times n - 1$ dots, so 70^{th} row has 2×70 – 1 dots
11. 6, 12, 18, 24, … are multiples of 6, so 7, 13, 19, 25, …are 1 more than a multiple of 6.
    The 100^{th} number in 6, 12, 18, 24, … is 100×6 = 500, so the 100^{th} number in 7, 13, 19, 25, … is 100×6 + 1
12. $\frac{5}{6} = \frac{?}{150}$
13. $0, \frac{2}{7}, \frac{4}{7}, \frac{6}{7}, \frac{8}{7}, \frac{10}{7}, \frac{12}{7}, \frac{14}{7}$
14. In the bottom layer there are 8 × 4 = 32 blocks, so in two layers there are 64 blocks
15. All the blocks of the bottom layer (32) and all the blocks round the side of the top layer (20)
16. List them systematically: 7, 17, 27, 37, … 77 (two!), 87, 97 is 11, plus 70, 71, 72, … ~~77~~, 78, 79 is another 9, so 20
18. *Height* = 12 cm + 1,5 cm/day × *days*. So *Height* after 30 days = 12 + 1,5×30 = 12 + 45
19. (150 cm – 12 cm) ÷ 1,5 cm/day = 92 days
20. The 3 seconds duration for ringing 4 o'clock correspond to the *intervals* between bell tolls, not the number of tolls:

Ring number	1	2	3	**4**	5	6	7	**8**
Seconds	0	1	2	**3**	4	5	6	**7**

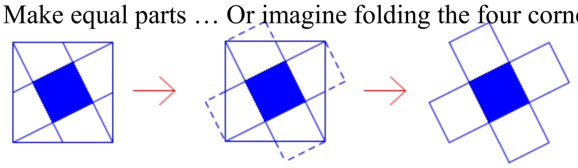
21. If a sack weighs *S* kg, then $3 \times S = S + 30$, so $2 \times S = 30$, so $S = 15$. So $3 \times S = 45$ kg
22. In 10 years time, each one will be 10 years older. So together they will be 5×10 = 50 years older, so 121 + 50
23. Be systematic, for example: 32 23 43 13
    Note: the order matters! 34 24 42 12
    31 21 41 14
24. Xholili will walk 4/7 and Thandi 3/7 of the 21 km. So Xholili will walk 12 km. To walk 12 km (4 + 4 +4), Xholili will take 1+1+1 = 3 hours
25. If a small pizza costs *s* rands and a large pizza costs *L* rands: $2s + 1L = 5s$, so $1L = 3s$, so the cost is $L = 3 \times$R11,50

GRADE 5(F)

1. C – rotate C to the left through 90°
2. 4, 8, 12, 16, … are multiples of 4, so 5, 9, 13, 17, …are 1 more than a multiple of 4.
   The 100^{th} number in 4, 8, 12, 16, … is 100×4 = 400, so the 100^{th} number in 5, 9, 13, 17, … is 100×4 + 1
3. Only 4065 is 1 more than a multiple of 4 (if you divide by 4 on a calculator, the decimal part is .25). All the other are 3 more than a multiple of 4 (if you divide by 4 on a calculator, the decimal part is .75).
4. Draw it! Fill in the information as you read. Re-read, bit by bit!



5. 9 small cubes fit onto the bottom, then there are 3 such layers, so 9 × 3 = 27
6. 8 – 7,93 = 0,07 < 8,08 – 8 = 0,08
7. (234469 + 234562) ÷ 2 = 234515,5
8. –– ×5 + 2 ––›, so 20×5 + 2 = 102
9. –– -2 ÷ 5 ––›, so (152 – 2) ÷ 5 = 150 ÷ 5 = 30
10. Look at the *structure:* 2×3 + 2 =8; 2×7 + 2 = 16; so for a rectangle with length 20: 2×20 + 2 = 42
11. Try and test each possible answer!
12. 4 reds = 10 greens = 3 purples. So 12 (3×4) reds = 9 (3×3) purples
13. Make equal parts … Or imagine folding the four corners to the inside …
14.



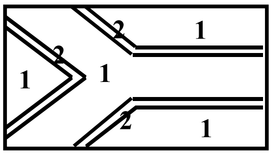
15. If the drink costs Rx, then the ice cream costs R(x+3) and the burger R(x+7). Altogether So $3 \times x + 10 = 19$, so $x = 3$
Or check the answers, e.g. can the ice cream cost R12? Then the drink costs R9, and the burger costs R16, but 12+9+16 is not 19, so ...
16.

1	***4(a)***	***2(b)***	**3**
3	**2**	**4**	**1**
4	**1**	**3**	**2**
2	**3**	**1**	**4**

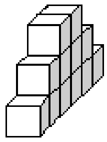
17.
18. List them systematically: 997; 988; 979   898; 889   799
19. The number must start and end with 1 so list them systematically:
101   111   121   131   141   151   161   171   181   191
20. 99 m = $\frac{9}{10}$ of roll, so 11 m = $\frac{1}{10}$ of roll. Therefore $\frac{10}{10}$ of role = 10 × 11 m = 110 m
21.
22.
23. The structure is 1 + 2 + 3 + 4 + 5 + 6 + ... + 48 + 49 + 50 = (1+50) + (2+49) + (3+48) + ... = 51 × 25
24. If Penny has p coins and Alex has a coins: $p - 4 = a + 4$. But $p = 2 \times a$, so $2 \times a - 4 = a + 4$, so $a = 8$ and $p = 16$
25: Be systematic, e.g.
143, 142;   134, 132;   124, 123
413, 412;   431, 432;   421, 423
314, 312;   341, 342;   321, 324
214, 213;   241, 243;   231, 234

GRADE 6(1)

1. Make equal parts. Each small square is half of the next bigger square.
   So half of half of the big square is a quarter of the big square
2. $15 + 3 \div 2 = 16{,}5$ mm or $(15+18) \div 2$
3. Start numbering (colouring) the regions, e.g. as shown …



4. $\frac{1}{7} = \frac{5}{35}$ and $\frac{1}{5} = \frac{7}{35}$ so $\frac{6}{35}$ is exactly in between them. Or $(\frac{1}{5} + \frac{1}{7}) \div 2 = (\frac{7}{35} + \frac{5}{35}) \div 2 = \frac{6}{35}$
7. The pattern is 1 + **1** + 1 + **2** + 1 + **3** + 1 + **4 + 1 + (5 + 1 + 6 + 1 + 5)** + **2** + 1 + 8 + 1
8. Continue the patterns: 17, 22, 27, 32, 37, 42, 47, 52, … and 17, 24, 31, 38, 45, 52, …
   Or, the lowest common multiple of 5 and 7 is 35, so 17 + 35 will be common and every 35 after that
9. There are 8 columns, each with 2 + 4 + 6 cubes.
   So $8 \times 12 = 96$ cubes



10. Vary the numbers systematically and note the behaviour of the product of the numbers:

$1 + 17 = 18$ and $1 \times 17 = 17$	$6 + 12 = 18$ and $6 \times 12 = 72$
$2 + 16 = 18$ and $2 \times 16 = 32$	$7 + 11 = 18$ and $7 \times 11 = 77$
$3 + 15 = 18$ and $3 \times 15 = 45$	$8 + 10 = 18$ and $8 \times 10 = 80$
$4 + 14 = 18$ and $4 \times 14 = 56$	$9 + 9 = 18$ and $9 \times 9 = 81$
$5 + 13 = 18$ and $5 \times 13 = 65$	$10 + 8 = 18$ and $10 \times 8 = 80$

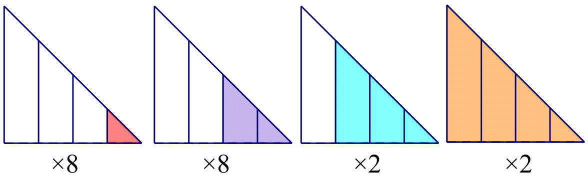
11. Every date is one weekday later in the next year, because $365 \div 7 = 52$ rem 1. Then we must account for leap years (*):

Year	2011	2012*	2013	2014	2015	2016*	2017	2018	2019	2020*	2021	2022
Day	Tue	Wed	Fri	Sa	Su	Mo	We	Thu	Fri	Sa	Mo	Tue

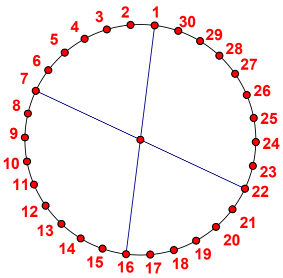
12. List all the numbers systematically: 10, 20-21, 30-32, 40-43, ... , 90-98. So there are 1+2+3+4+5+6+7+8+9 = 45
13. 16 out of 24 marbles are not blue, so the probability of choosing a not-blue marble is $\frac{16}{24} = \frac{2}{3}$.
14. $\frac{13}{20}$ is more than $\frac{12}{20}$ ($\frac{3}{5}$) and less than $\frac{16}{20}$ ($\frac{4}{5}$), so he is on side DE
15. He still has $\frac{7}{20}$ of the distance to go, so $\frac{7}{20}$ of 25 cm = (25 cm ÷ 20) × 7 = 8,75 cm
16. $64 = 1 \times 64 = 2 \times 32 = 4 \times 16 = 8 \times 8$
17. 18 = 12 + 6 = 12 + ½ of 12. So $\frac{2}{3}$ + ½ of $\frac{2}{3} = \frac{2}{3} + \frac{1}{3}$
18. 3 Boys + 4 Girls = (3B x 4rands) + (4G x 5rands) = 12rand s+ 20rands = R32
19. Out of every 8 vehicles sold, 3 are bakkies. So the number of bakkies sold = $\frac{3}{8}$ of 96 = (96 ÷ 8) × 3 = 36
20. If the first number is x, then $9 \times x + 36 = 135$, so $9 \times x = 99$, so $x = 11$
21.
22. 4 books = 2 books + 6 kg, so 2 books = 6 kg, so 1 book = 3 kg
23. ? = 000ΔΔΔ = 0ΔΔ + $\frac{1}{2}$(0000ΔΔ) = 6□ + 4□ from first two balances
24. If the sides are a and b, then $2a + 2b = 480$, so $a + b = 240$. But one side is double the other, so $a + 2a = 240$, so $3a = 240$
    Or try trial and improvement: $2 \times 100 + 2 \times 50 = 300 < 480$; $2 \times 120 + 2 \times 60 = 360 < 480$; …
25. Do not count or calculate, investigate the *structure:* 1, 4, 9, … = 1×1, 2×2, 3×3, … 20×20

GRADE 6(F)

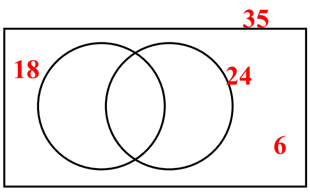
1. There are 4 different *sizes* of triangles as shown. Total = 8 + 8 + 2 + 2:



2. (5,6 + 5,65) ÷ 2 = 11,25 ÷ 2 = 5,625
3. $\frac{1}{10} = \frac{8}{80}$ and $\frac{1}{8} = \frac{10}{80}$, so $\frac{9}{80}$ will be exactly inbetween
4. 2×35 cm (top and bottom) + 4×10 cm (sides) + 47 cm (ribbon) = 70+40+47 =157 cm
7. The rule is "halve". $\frac{1}{2}$ of $(1 + \frac{1}{2}) = \frac{1}{2}$ of 1 + $\frac{1}{2}$ of $\frac{1}{2}$ $= \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$
8. $\frac{5}{100} = \frac{x}{3000}$
9. Draw it! 1 is opposite 16, 2 opposite 17, …. 7 opposite 22



11. 0×20 + 3×10 + 1×5
    1×20 + 1×10 + 1×5
12. 101, 111, 121, 131, 141, 151, 161, 171, 181, 191, (10)
    202, 212, 222, 232, 242, 252, 262, 272, 282, 292, (10)
    303, etc. (10)
    404, etc. (10)
    So the total is 40
13. May's R12 represents a 1/4 . Therefore Mark's 3/8 = ¼ + 1/8 is R12 + R6 = R18
14. 35 – 6 = 29 (29 children either has a cat, a dog or both); 29 – 24 = 5 (5 children has only dogs); 18 – 5 = 13



15. 48 = 3×16, so 3×80 min = 240 min = 4 h for the whole job, so 4 hours – 80 min for the rest
16. 3, 6, 9, … is the 3-times table. So 50 × 3 = 150
17. Structure! 50×51 = 2550

P_1	P_2	P_3	P_4	…	P_{50}
1×2	2×3	3×4	4×5	…	?

18. Trial and improvement: 30×31 = 930 is too small …. 35 × 36 = 1260 is too small …. 36 × 37 = 1332
19. There are 5 possible first digits (1, 3, 5, 7, 9) and 5 possible second digits, so in total 5 × 5 = 25
20. *Look* at the structure: For n dice, the number of visible faces is n×3 + 2. So for 75 dice, 75×3 + 2
21.

B	C	M
A	2	N
1	D	

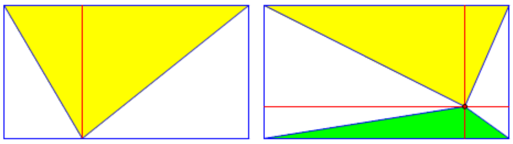
In the middle row, N cannot be 2, so N is 1 or 3
Suppose N = 3. Then A = 1 which is impossible (already a 1 in left column).
So N = 1, A = 3. In left column B = 2. Then C = 1 (D ≠ 1), so M = 3, so M+N = 4

22. 3×(1 + 2+ 3)
23. Investigate the *structure:* The sums in the *Rows* are 1, 2, 4, 8, 16, ...Use this pattern!

Row n	0	1	2	3	4	5	6		n
Sum	2^0	2^1	2^2	2^3	2^4	2^5	2^6		2^n

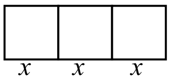
24. If a bubble gum cost B cents and a chocolate costs C cents:
    $B + C = 90$ and $10B + 5C = 470$, so $5B + 5(B + C) = 470$, so $5B + 5×90 = 470$, so $B = 4$, so $C = 86$c
25. Half the water weighs 21 kg – 12 kg = 9 kg, so all the water weighs 18 kg. So the bucket weighs 3 kg

GRADE 7(1)

1. & 2. 

3. $1+\frac{1}{1+\frac{2}{3}}=1+\frac{1}{\frac{5}{3}}=1+\frac{3}{5}$

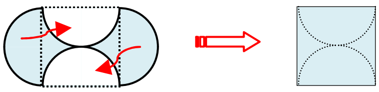
4. Let the side length of a square be x cm
Then $8\times x = 24$ and $x = 3$
So Area = 9 cm × 3 cm = 27 cm^2



6. Check them one by one … All square numbers must have an odd number of factors – why?
7. Total mass = (4 × 75) + (6 × 65) = 690 kg. So the average is 690 kg ÷ 10 children= 69 kg/child

8. $\frac{20+18+16+14+12+10+8+6+4+2}{10+9+8+7+6+5+4+3+2+1}=\frac{2\times(10+9+8+7+6+5+4+3+2+1)}{(10+9+8+7+6+5+4+3+2+1)}=2$

9. 1+3+6+10+15+21 = 56
10. $\frac{1}{3} < n < 4\frac{1}{3}$, so n = 1, 2, 3, 4
Or: Multiples of 3 between 2 and 13 are 3; 6; 9; 12
Or: Use trial and checking, e.g. if n = 1, then 2 < 3 <13 is true. If n = 5, then 2 < 15 <13 is false, …
11. They have won 14/20 games. To win 80% of 30 games, they must win 24/30 games. So they must win 10/10 of the remaining games.

12.  So 6 cm × 6 cm = 36 cm^2

13. Fill in numbers in the calendar, and test each statement with the numbers.
14. We know $a + d = c + b$, so $a + b + c + d = a + d + c + b = 2\times(a + d) = 52$.
So $a + d = 26$, so $a + (a + 8) = 26$, so $a = 9$, so $a + b = 9 + 10$
15. If the date on Wednesday is x, then $(x\text{-}3) + (x\text{-}2) + (x\text{-}1) + x + (x+1) + (x+2) + (x+3) = 112$
So $7x = 112$, so $x = 112/7 = 16$, so ...
16. Distance = speed × time, so the distance from A to C = 7/4 h × 4 km/h = 7 km
So the distance from C to B is 5 km, and they have 5/4 h to get there. 5/4 h × x km/h = 5 km, so x = 4 km/h
17. If the *prices* are Rp and Rb, then $3p + 5b = 44$, so $3p + 3b + 2b = 44$, so $3(p + b) + 2b = 44$, so $3\times10 + 2b = 44$
Or: $p + b = 10$, so $3p + 3b = 30$. But $3p + 5b = 44$, so $2b = 14$
18. $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{16}$
19. There are many patterns to see, e.g. in *Row n* the 2nd number is $2n$. So *Row 30* is 59 + 60 + 61 + 62
Or, there is a constant difference of 8 in the sums; the sum is 2 more than a multiple of 8. Sum in *Row 30* = 30×8+2
20. List them all: 20, 22, 24, 26, 28 and similarly for the 40s. 60s and 80s, so 4 × 5 = 20
21. 5, 5, 1   5, 4, 2   5, 3, 3   4, 4, 3   The sum of any two sides must be greater than the third side – why?
22. 6, 11, 16, … The formula for $P_n = 5n + 1$
23. Look for *structure* in the denominator:

	F_1	F_2	F_3	F_4	…	F_{10}
Numbers	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{20}$		?
Structure	$\frac{1}{1\times2}$	$\frac{1}{2\times3}$	$\frac{1}{3\times4}$	$\frac{1}{4\times5}$		$\frac{1}{10\times11}$

24. Calculate intermediate answers and look for *structure* and *patterns*:

Sum of **1** fraction $= \frac{1}{2}$

Sum of **2** fractions $= \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$

Sum of **3** fractions $= \frac{2}{3} + \frac{1}{12} = \frac{3}{4}$

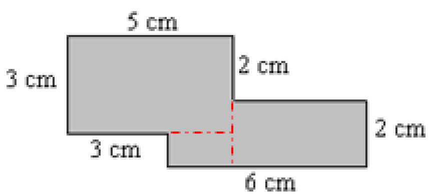
Sum of **4** fractions $= \frac{3}{4} + \frac{1}{20} = \frac{4}{5}$

Sum of **10** fractions $= \frac{10}{11}$

25. $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \ldots \times \frac{2009}{2010} \times \frac{2010}{2011} = \frac{2}{2} \times \frac{3}{3} \times \frac{4}{4} \times \frac{5}{5} \times \ldots \times \frac{2010}{2010} \times \frac{1}{2011} = \frac{1}{2011}$

GRADE 7(F)

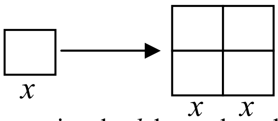
1. $5{\times}3 + 4{\times}2 + 2{\times}1 = 25\text{ cm}^2$ or $5{\times}3 + 6{\times}2 - 2{\times}1 = 25\text{ cm}^2$



2. 5 + 2 + 4+ 2 + 6 + 1 + 3 + 3 = 26 cm
3. (2 – 1) + (3 – 2) + (4 – 3) + … + (100 – 99) + (101 – 100) = 1 + 1 + 1 + 1 + ... 100 times = 100
4. 10 000 000 m = 10 000 km
5. n^{th} number $= 2{\times}n - 1$, so 83^{rd} number $= 2 \times 83 - 1 = 165$
6. In middle row the missing number is 18 – (11+6) = 1, so in right column $x = 18 - (1+10) = 7$
7. Do not rush into calculation! *Look for structure!* $\frac{24\times18\times15+24\times18\times13+24\times18\times7}{24\times18} = \frac{24\times18\times(15+13+7)}{24\times18} = 35$
8. b and c are both less than 1, so $b{\times}c$ is less than both b and c.
9. We know: $\frac{\text{Sum of numbers}}{11} = 8$, so Sum of numbers = 11 × 8 = 88

   If the new number is x, then $\frac{88+x}{12} = 11$. So $x = 12 \times 11 - 88 = 44$
10. $\frac{7}{8} - \frac{1}{2} = \frac{3}{8}$ = 420 litres, so $\frac{1}{8}$ = 420 litres ÷ 3 = 140 litres. So the full tank = $\frac{8}{8}$ = 140 litres × 8 = 1120 litres
11. # *Triangles* = 2×*squares* + 2, or 2×(*squares* + 1). So *Triangles (6)* = 2×6 + 2 = 14
12. *Triangles (60)* = 2×60 + 2 = 122
13. $2{\times}x + 2 = 60$, so $x = 29$
14. $\frac{1}{4} + (\frac{1}{4})^2 + (\frac{1}{4})^3 + (\frac{1}{4})^4 = \frac{4^3+4^2+4^1+4^0}{4^4} = \frac{85}{256}$
15. Volume = 15 cm × 8 cm × x cm = 120 cm^3, so $x = 1$. So area is (15 cm + 2 cm) × (8 cm + 2 cm) = 17 cm × 10 cm
16. Starting with the primes: 23, 29, 31, 37, 41, 43, 47,53, 59, 61, 67, 71, 73, 79, these are also prime 13, 73, 17, 37, 97, and of the five 31+1, 37+1, 71+1, 73+1 and 79+1, only 71+1 is a multiple of 3
17. Write as the product of factors, but do not repeat factors, e.g. do not write 6 = 2×3, because it is already there!
    $5\times7\times8\times9 = 5\times7\times2\times2\times2\times3\times3$
18. You must take all possible combinations of the numbers 19, 29, 59 and 79 (they are all prime). Do not calculate, simply systematically count all possible combinations. It is the same as taking all possible combinations of the letters A, B, C and D if the order does not matter:
    1 at a time: A, B, C, D – so 4
    2 at a time: AB, AC, AD, BC, BD, CD – so 6
    3 at a time: ABC, ABD, ACD, BCD – so 4

19. If the length of a square doubles (×2), then the area quadruples (x4), as illustrated in this simple example



If the dimensions of the room is a by b by c, then the area to paint is $A = ab + 2ac + 2bc$

Double the dimensions are $2a$ by $2b$ by $2c$, so the area to paint is $D = (2a)(2b) + 2(2a)(2c) + 2(2b)(2c) = 4 \times A$

20. Add all together: $2A + 2B + 2C = 42$, so $A + B + C = 21$
21. $B + A + C = 21$ and $A + C = 16$, so $B + 16 = 21$
22. You can draw it, or investigate numerical patterns for a triangle, square, pentagon, hexagon, etc.
    Or you can reason it out: At each vertex of an n-gon there are $n–3$ diagonals because the point is connected to every other point, except to itself and to the two adjacent points (these are *sides* of the n-gon). So at n vertices there are $n\times(n–3)$ diagonals, counted twice. So the formula is $D(n) = n\times(n–3) \div 2$, so $D(8) = 8\times(8 – 3) \div 2$
23. $D(8) = 80\times(80 – 3) \div 2 = 3080$
24. After 1 year, 95% is left
    After 2 years, 95% of 95% = $0{,}95 \times 0{,}95 = 0{,}95^2$ is left
    After 3 years, 95% of 95% of 95% = $0{,}95 \times 0{,}95 \times 0{,}95 = 0{,}95^3$ is left
    After 13 years, $0{,}95^{13}$ is left. $0{,}95^{13} = 0{,}513 = 51{,}3\%$, more than half, is left
    After 14 years, $0{,}95^{14}$ is left. $0{,}95^{14} = 0{,}487 = 48{,}7\%$, less than half, is left
25. The order in which we add numbers does not matter! So the final number is $1 + 2 + 3 + 4 + ... + 99 + 100$
    $1 + 2 + 3 + 4 + ... + 99 + 100 = (1+100) + (2+99) + (3+ 98) + ... = 101 \times 50$