SA Mathematics Challenge
SA Wiskunde Uitdaging

MEMORANDUM 2012

QUESTION	4(1)	4(F)	5(1)	5(F)	6(1)	6(F)	7(1)	7(F)	VRAAG
1	B	C	D	C	E	D	C	B	1
2	B	C	B	A	E	B	B	C	2
3	C	B	C	B	C	C	C	D	3
4	C	D	D	B	D	E	B	A	4
5	D	C	E	A	C	E	A	C	5
6	E	E	A	B	E	A	C	B	6
7	B	D	B	C	A	A	D	E	7
8	C	D	C	A	D	C	A	D	8
9	C	B	B	C	B	E	B	A	9
10	B	C	D	A	E	B	C	A	10
11	D	B	D	D	D	A	E	A	11
12	D	B	B	A	B	C	E	D	12
13	A	A	C	E	E	E	B	D	13
14	E	A	A	E	C	D	B	A	14
15	B	B	D	D	E	A	B	D	15
16	A	C	B	**B**	A	C	D	B	16
17	B	B	B	B	B	D	B	C	17
18	C	E	D	A	D	C	B	A	18
19	D	D	B	C	A	B	D	C	19
20	A	E	C	D	E	C	E	B	20
21	B	D	D	B	B	B	A	D	21
22	B	D	B	E	B	A	D	E	22
23	D	B	E	A	C	B	A	B	23
24	B	A	B	A	D	D	B	A	24
25	A	A	D	A	A	B	C	A	25
QUESTION	4(1)	4(F)	5(1)	5(F)	6(1)	6(F)	7(1)	7(F)	VRAAG

Jump to worked answers:

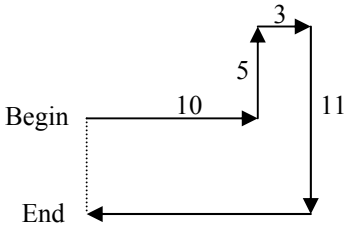
Grade 4(1) Grade 4(F) Grade 5(1) Grade 5(F) Grade 6(1) Grade 6(F) Grade 7(1) Grade 7(F)

NOTES ON 2012 MEMORANDUM

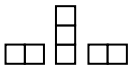
These notes are necessarily brief and often formal and symbolic.
Many questions could be answered using primitive methods, e.g. "If today is Wednesday, what day of the week will it be 100 days from now?" can be done by counting. That would be laborious, time-consuming and error-prone. The essence of a mathematical approach is to work more smartly by using appropriate representations to model the situation and to exploit the inherent structures and patterns in the situation.

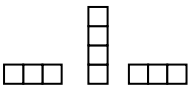
GRADE 4(1)

3. Straighten the string. Two loops of 1 cm make it 5 cm + 1 cm + 1 cm = 7 cm
4. 3 hours before 16:45 is 13:45, so 2 hours and 55 minutes (5 min less) is 13:50
5. The watch gains 2 minutes every day (24 hours) for 7 days = 2 min/day× 7 days = 14 minutes
6. Rotate (in your mind!) the pieces to fit ...
9. With 6 loose cubes, there would be 36 faces. Subtract the 10 non-visible faces ...
10. Place value!
11. If the number is 2 parts, then half the number is 1 part. These 3 equal parts total 96, so each part is 96÷3 = 32. So the number (2 parts) is 2×32 = 64. Test: 64 + 32 = 96
12. Do not rush into calculation – analyse the structure: 826×243 – 824×243 = (826 – 824)×243 = 2×243
13. The numbers must be different, so 99 + 98 + 97 = (100 – 1) + (100 – 2) + (100 – 3) = 300 – 6
14. 18 × 10
15. If the 21th is a Monday, then also the 14th, 7th and 0th are Mondays. The 0th is the last day of the previous month, so the next day is the 1st of this month, so it is a Tuesday
17. $\frac{3}{4} + \frac{3}{4} \rightarrow 1\frac{1}{2} + \frac{3}{4} \rightarrow 2\frac{1}{4} + \frac{3}{4} \rightarrow 3 + \frac{3}{4} \rightarrow 3\frac{3}{4} + \frac{3}{4} \rightarrow 4\frac{1}{2}$
    ① ② ③ ④ ⑤ ⑥
18. Compare question 11: If the tie costs one unit, then the shirt costs three units. The four equal units together cost R276. So each unit is R276÷4 = R69. The shirt costs three units, so 3×R69 = R207
19. 
20. If the number of tables is T, then 2 × T + 2 = 58, so T = (58 – 2) ÷ 2 = 28
21. Draw it physically! See diagram. It always helps to write!



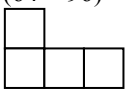
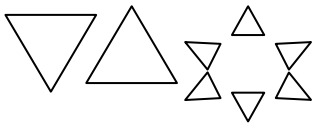
22. $\frac{1}{5} = \frac{8}{40}$ and $\frac{1}{4} = \frac{10}{40}$, so $\frac{8}{40} < \frac{9}{40} < \frac{10}{40}$, which means $\frac{1}{5} < \frac{9}{40} < \frac{1}{4}$
23. Do not count or calculate – look for structure, e.g.
    For Pattern 3: 3 + 2×2
    For Pattern 4: 4 + 2×3
    For Pattern 100: 100 + 2×99

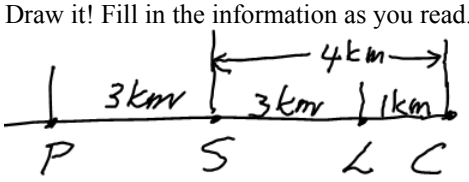

Pattern 3


Pattern 4

24. List all the possibilities and be systematic:
    1 + 1 = 2 2 + 2 = 4 3 + 3 = 6 4 + 4 = 8 5 + 5 = 10 6 + 6 = 12
    1 + 2 = 3 2 + 3 = 5 3 + 4 = 7 4 + 5 = 9 5 + 6 = 11
    Any other combination will be a repetition – therefore 11 possible answers
25. Look for structure, a clever way of counting, e.g. every point is connected to every other point except itself, so at each of the 18 points on the circle there are 17 lines, in total 18×17. But each line is counted twice, so 18×17 ÷ 2

GRADE 4(F)

1. Each letter moves one on: B→C, A→B, C→D, O→P, N→O. So EGGS → FHHT.
2. 3 hours before 16:45 is 13:45, so 2 hours and 55 minutes (5 min less) is 13:50
3. 8×9
4. (64 + 96) ÷ 2 = 80
6.  The tower is on your left if you look at the object from the back
7. 
8. 6, 11, 16, 21, 26, … are 1 more than 5, 10, 15, 20, 25, … Si the 100th number is 5×100 + 1 = 501
9. Meriams numbers are 1 more than a multiplr of 5.All her numbers end in 1 or 6. So she will count only 4526
10. The ones-digits must check, e.g. _8 × _9 must end in _2.
11. Between 09:47 and 10:18, 31 minutes pass. 31 minutes from 12:30 is 13:01
12. 257 + Δ = 438, so Δ = 438 – 257 = 181 km
13. 438 + 169 = 607 km
14. Draw it! Fill in the information as you read. Re-read, bit by bit!



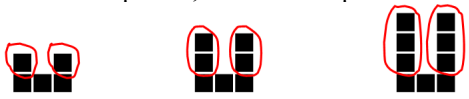
15. Bingo: 71; Thandi: 71 – 24 = 47; Voyo: 71 + 24 = 95
    71 + 47 + 95 = 213 *or* 71 × 3 = 213 (24 – 24 = 0)
16. If Zuki has ♥ marbles, Zinkle has ♥ – 15. Together they have ♥♥ – 15 = 95 marbles. So ♥♥ = 110 and ♥ = 55
17. R35,60 ÷40 = 89c so 89c × 15 = R13,35
18. Thabo takes 4 out of 12; 4/12 = 1/3. He has to pay 1/3 of R30 = R10
19. Let the children be A, B, C, D and E. List all the possibilities, be systematic, note patterns and structure:

    A vs B, A vs C, A vs D, A vs E
    B vs C, B vs D, B vs E
    C vs D, C vs E
    D vs E

vs	A	B	C	D	E
A		X	X	X	X
B			X	X	X
C				X	X
D					X
E					

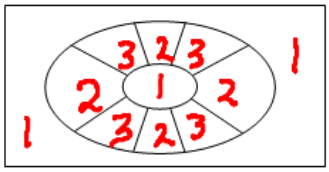
20. Try the numbers one by one, e.g. 20 × 3 → 60 + 8 → 68 ÷ 2 → 34 − 6 → 28 ≠ 20
21. At trees A, B and C the monkey arrived twice and left twice. At tree E he left twice and arrived once – so he started at tree E. At tree D he arrived twice and left only once – so the monkey is hiding in tree D!
22. Look at the *structure*!
    R_1: 1 = 1×1
    R_2: 1 + 3 = 2×2
    R_3: 1 + 3 + 5 = 3×3
    R_4: 1 + 3 + 5 + 7 = 4×4
    ….
    29 is the 15th odd number, so R_{15} = 15×15 = 225
23. Continue pattern of subtracting 4 cm/hour. Or the formula is Height = 32 – 4 × time
24. List all the possibilities and be systematic:

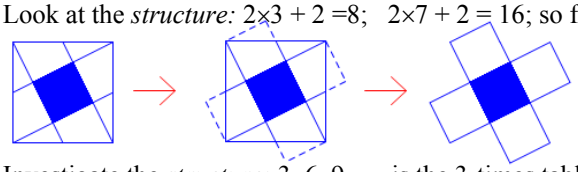
1 × 1 = 1	2 × 2 = **4**	3 × 3 = 9	4 × 4 = 16	5 × 5 = 25	6 × 6 = 36
1 × 2 = 2	2 × 3 = **6**	3 × 4 = **12**	4 × 5 = 20	5 × 6 = 30	
1 × 3 = 3	2 × 4 = 8	3 × 5 = 15	4 × 6 = 24		
1 × 4 = **4**	2 × 5 = 10	3 × 6 = 18			
1 × 5 = 5	2 × 6 = **12**				
1 × 6 = **6**					

    Because the order does not matter, any other combination (e.g. 2 × 1) will be a repetition. Also in the list above, 4, 6 and 10 are repeated, therefore 18 possible answers
25. 

    U_1 = 5 U_2 = 7 U_3 = 9
    $U_n = 2\times n + 3$, so $U_{20} = 2\times 20 + 3$

GRADE 5(1)

2. The numbers inside the square *and* the circle are 2 and 3. 2 is not inside the triangle
4. 147 mm – 103 mm = 44 mm
5. 100 ÷ 24 = 4 rem 4, i.e. 4 full days bringing us again to 10:00, plus 4 more hours, i.e. 11, 12, 13, 14:00
6. n^{th} row has $2\times n - 1$ dots, so 7^{th} row has $2\times 7 - 1$ dots
7. n^{th} row has $2\times n - 1$ dots, so 70^{th} row has $2\times 70 - 1$ dots
8. C – a rotation to the right through 90°
9. *Height* = 12 cm + 1,5 cm/day × *days*. So *Height after 30 days* = 12 + 1,5×30 = 57 cm
10. (150 cm – 12 cm) ÷ 1,5 cm/day = 92 days
11. One more than a multiple of 6, so it is odd, so it cannot be A or B. Test the others: 4 182÷6 = 697
12. 

13. If the cold drink costs Rx, then the ice cream costs R(x+3) and the burger R(x+7). So $3\times x + 10 = 19$, so $x = 3$
15. In the bottom layer there are $8 \times 4 = 32$ blocks, so in two layers there are 64 blocks
16. All the blocks of the bottom layer (32) and all the blocks round the side of the top layer (20)
17. Look at the *structure:* $2\times 3 + 2 = 8$; $2\times 7 + 2 = 16$; so for a rectangle with length 20: $2\times 20 + 2 = 42$
18. 
19. Investigate the *structure:* 3, 6, 9, … is the 3-times table:

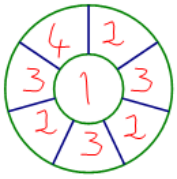
Pattern n	1	2	3	4		*n*
# coins	3	6	9	12		3×*n*

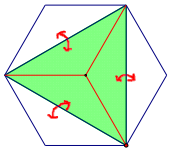
20. 4 reds = 10 greens = 3 purples. So 12 (3×4) reds = 9 (3×3) purples
21. The number must start and end with 1 so list them systematically:
    101 111 121 131 141 151 161 171 181 191
22. The structure is $1 + 2 + 3 + 4 + 5 + 6 + \ldots + 48 + 49 + 50 = (1+50) + (2+49) + (3+48) + \ldots = 51 \times 25$
23. If Penny has p coins and Alex has a coins: $p - 4 = a + 4$. But $p = 2\times a$, so $2\times a - 4 = a + 4$, so $a = 8$ and $p = 16$
24. Be systematic, e.g.

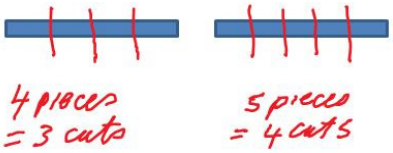
32	23	43	13
34	24	42	12
31	21	41	14

25. If a small pizza costs s rands and a large pizza costs L rands: $2s + 1L = 5s$, so $1L = 3s$, so the cost is $L = 3\times$R11,50 = R34,50

GRADE 5(F)

1. 11,23 s < 11,32 s < 11,4 s. So Peter Davids is third
2. Lucy's mother is 24 years older than Lucy
   In 8 years Lucy's mother will be 48
   Lucy is 24 now.
3. 07:20 + 45 min = 08:05
4. 500 ÷ 12 = 41 rem 8 (41.666… on the calculator), so 42 cartons are needed
5. Recognize and continue with pattern:
   1st layer = 1 = 1×1
   2nd layer = 4 = 2×2
   3rd layer = 9 = 3×3
   4tht layer = 16 = 4×4
   etc…
   10th layer = 10×10 = 100 grapefruit
6. 12 rounds × 3 minutes/round + 11 breaks × 1 minute/break = 36 minutes + 11 minutes
7. 

8. $\frac{5}{6} = \frac{?}{150}$
9. $\frac{1}{8}$ + 25 litres = $\frac{5}{8}$ of tank
   So 25 litres = $\frac{4}{8} = \frac{1}{2}$ of tank
   So the full tank holds 2 × 25 litres = 50 litres
10. See sketch


11. If Joe's starting number is *S*, then he did *S* × 10 = 500. So *S* = 50. So correct answer is 50 ÷ 10 = 5
12. List them systematically: 300; 210; 201; 120; 111; 102
13. The numbers are all 1 less than a multiple of 6. On the calculator, if you divide by 6, they all end in .83333…
    365 + 1 → 366 ÷ 6 = 61 (365 ÷ 6 = 60.8333 …., but none of the others ends in 0.8333…)
14. Mary ate 24 ÷ 6 = 4 pieces, Veronica ate 24 ÷ 4 = 6 pieces, Ron ate 24 ÷ 3 = 8 pieces
    So John ate 24 – 8 – 6 – 4 = 6 pieces
    Or, the others ate $\frac{1}{6} + \frac{1}{4} + \frac{1}{3} = \frac{3}{4}$, so John ate $\frac{1}{4}$ of 24 pieces = 6 pieces
15. From 1-9 you use 9 digits. From 10 to 99 you use 90 × 2 = 180 digits. For 100 you use 3 digits. 9 + 180 + 3 = 192
16. For 19 wheels we can have: 1 tricycle and 8 bicycles = total of 9 (too much)
    3 tricycles and 10 bicycles = total of 13 (too much)
    5 tricycles and 2 bicycles = total of 7 (just right)
17. *x* boys × R4/boy + *y* girls × R5/girl = R12 + R20. So 3 boys and 4 girls
18. Make sense of the situation, e.g. draw it!


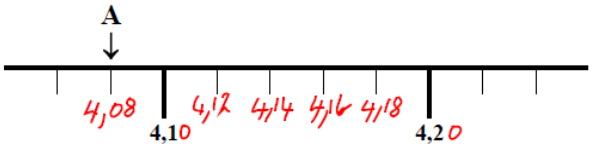
    It takes 12 min ÷ 3 cuts = 4 minutes per cut
    So 4 cuts will take 4x4 minutes!
19. $\frac{835+835+835+835+835}{5} = \frac{835\times 5}{5} = 835\times\frac{5}{5} = 835\times 1 = 835$
    or $\frac{835\times 5}{5} = 835\times 5\div 5 = 835\times(5\div 5) = 835\times 1 = 835$
20. 100 – (4 × 10) = 60
21. The smallest number cannot be 1 or 2, otherwise the product of the other two must be 90 or 45, which is too big.
    One of the numbers cannot be 4, otherwise the product of the other two must be 90/4, which is not a whole number.
    If the smallest number is 3, the product of the other two must be 30, so 5× 6 = 30. So the sum is 3 + 5 + 6 = 14

22. Study the structure:
    $4 \boxtimes 3 = 16 = 4\times3 + 4 = 4\times4$
    $6 \boxtimes 3 = 24 = 6\times3 + 6 = 6\times4$
    $7 \boxtimes 5 = 42 = 7\times5 + 7 = 7\times6$
    $8 \boxtimes 7 = 64 = 8\times7 + 8 = 8\times8$
    So $6 \boxtimes 8 = 6\times9 = 54$
23. Look at the *structure* in the pictures!
    T_1: $1 = 1\times1$
    T_2: $1 + 3 = 4 = 2\times2$
    T_3: $1 + 3 + 5 = 9 = 3\times3$
    T_4: $1 + 3 + 5 + 7 = 16 = 4\times4$
    ….
    T_{50}: $1 + 3 + 5 + 7 + ...$ to 50 numbers $= 50\times50 = 2500$
24. Look at the *structure* in the pictures!

Height (h)	1	2	3	4	…	b
# blocks in bottom row (b)	1	3	5	7		$2 \times h - 1$

25. $1369 = 37\times37$

GRADE 6(1)

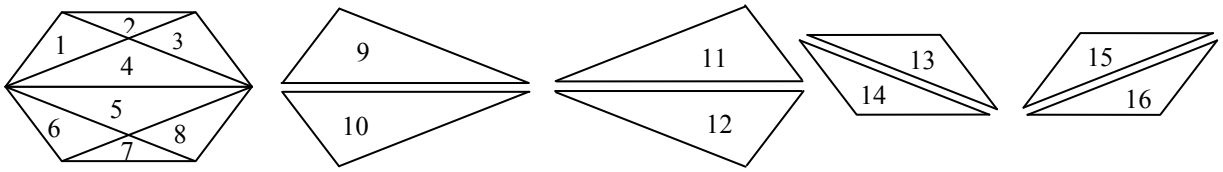
1. Try trial and improvement, e.g. 50 + 52 + 54 ≠ 174; …. But 56 + 58 + 60 = 174
   Or test each of the given numbers …
   Or, if the smallest is x, then $x + (x + 2) + (x + 4) = 3 \times x + 6 = 174$, so $x = 56$
2. These are multiples of 8. 728 ÷ 8 = 91. All the others leave a remainder when divided by 8
3. These are 4 less than multiples of 8 (If you divide by 8 on a calculator, they give an answer of *.5). 724 ÷ 8 = 90 remainder 4. All the others leave other remainders
4. 

5. List them: 1, 2, 4, 5, 10, 20, 25, 50, 100
6. 5×2 = 10, so the last digit is 0
7. 25 + 20 + 30 + 15 + 35 = 125
8. Full lorry = 4653 kg; empty lorry = 2583 kg; 4653 – 2583 = 2070; 2070 kg ÷ 90 kg/bag = 23 bags
9. 1 × 2 – 1 = 1
   4 × 2 – 1 = 7
   7 × 2 – 1 = 13
   2 × 2 – 1 = **3**
11. Make a systematic list, e.g. 3579; 3597 | 3759; 3795 | 3957; 3975 | 9375; 9357 | 9537 …
    Or: He has 4 choices for the first number, then 3 choices for the second, 2 for the third and 1 for the fourth.
    So 4 × 3 × 2 × 1
12. ½ + 1/8 + 1/8 = ¾. So R15 is a ¼ of his money. So 4/4 of his money is 4 × R15 = R60
13. Trial and improvement: 30×31 = 930 is too small …. 35 × 36 = 1260 is too small …. 36 × 37 = 1332
14. You can continue the counting: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, … and 5, 18, 31, 44, 57, 70,
15. In the hundreds position Xoli will write 3 a 100 times (from 300 to 399).
    In the tens position Xoli will write 3 10 times (from 30 to 39)
    In the one position Xoli will write 3 10 times (from 03, 13, 23, …. to 93)
16. May's R12 represents a 1/4 . Therefore Mark's 3/8 = ¼ + 1/8 is R12 + R6 = R18
17. Each number is the sum of the two numbers above it, e.g. 6 = 1 + 5, 15 = 5 + 10
18. Diana is 3 years older than Joe
    Joe is two years older than Cindy
    Diana is five years older than Cindy, so she was 8 + 5 = 13 years old
19. The largest, by guess-and-improvement is 31 × 31 = 961. 32 × 32 > 1000
20. If her average score is 4,8, her total score is 5 × 4,8 = 24
    The first four judges gave her 4,5 + 4,6 + 4,7 + 5 = 18,8
    So the fifth judge gave her 24 – 18,8 = 5,2
21. Mentally draw a straight line through the angle of each square the arrow is pointing at.
22. Multiply all together: $(a \times b) \times (b \times c) \times (c \times d) \times (d \times a) = 20 \times 14 \times 35 \times 50 = 2{\times}2{\times}10{\times}10{\times}5{\times}5{\times}7{\times}7$
    So $a^2 \times b^2 \times c^2 \times d^2 = (a \times b \times c \times d)^2 = (20{\times}35)^2$
    So $a \times b \times c \times d = 700$
23. Find structure!

Whole numbers	1–9	10–19	20–29	30–39	40–49	50–54
# of digits	9	20	20	20	20	10

    So, up to 54, 9+20+20+20+20+10 = 99 digits are used. So the 100th digit would be **5**: … 484950515253545**5**
24. A bit of logic! "Only one statement is true" is the same as "four of the five statements are false"!
25. Look at the *structure* in the pictures!
    V_1: 3 = 2×**1** + 1
    V_2: 5 = 2×**2** + 1
    V_3: 7 = 2×**3** + 1
    ….
    V_{50}: = 2×**50** + 1 tiles

GRADE 6(F)

2. (5,6 + 5,65) ÷ 2 = 5,625
3. 3×3 – 3 + 3 = 9
4. 

5. 1, 2, 4, 5, 10, 20, 25, 50, 100
6. 4002 ÷ 4 = 1000 rem 2
8. $\frac{5}{6}=\frac{40}{48}; \frac{7}{8}=\frac{42}{48}$
   $\frac{40}{48}<\frac{41}{48}<\frac{42}{48}$
9. *Look* at the structure: For *n* dice, the number of visible faces is $n\times3 + 2 = 50$. So $n = 16$
10. From half to full in 1 minute ∴ 59 minutes
11. 399/1000; 398/500; 410/1000; 420/1000; 300/1000
    2/5 = 400/1000 → 399/1000 is closest to 2/5
12.

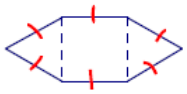
2	3	
3	5	+2
4	?	+3
5	12	+4
6	17	+5

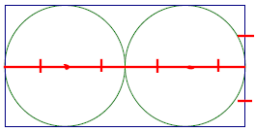
13. 230 – 60 = 170; 170 ÷ 2 = R85
15. Find the three numbers by trial and improvement. Use your calculator, e.g. 10×11×12 = 1320 is too small, etc. But 14×15×16 = 3360, so 14 + 15 + 16 = 45
16. Jane eats 2 × 12 sweets in 5 minutes; she eats 2 × 24 sweets in 10 minutes. Jane eats 48 sweets in 10 minutes
17. 50 – 30 = 20; 41 – 30 = 11; 35 – 30= 5
    Of the 20 learners who do not like both, 11 like comedy and 5 like action films
    20 – 11 – 5 = 4
18. (2000 – 1999) + (1998 – 1997) + … + (2 – 1) = 1 + 1 + 1 + 1 + … + 1 (1000 times)
19. 7 × 2 = 14 → 5 extra wheels → 5 tricycles → 2 bicycles
20. 4 ÷ 2 × 3 = 6; 6 ÷ 2 × 3 = 9 cm
21. 11, 22, 33, 44, 55, 66, 77, 88, 99, (9)
    101, 111, 121, 131, 141, 151, 161, 171, 181, 191, (10)
    202, 212, 222, 232, 242, 252, 262, 272, 282, 292, (10)
    303, etc. (10)
    404, etc. (10)
    So the total is 49
22. Make a table of the data, find a pattern and use the pattern to predict the answer:

Row number	Number of triangles
1	1 = 2 × 1 – 1
2	3 = 2 × 2 – 1
3	5 = 2 × 3 – 1
4	7 = 2 × 4 – 1
50	2 × 50 – 1 = 99

23. Look systematically at special cases and find a pattern:
    Total triangles in 1 row = 1
    Total triangles in 2 rows = 1 + 3 = 4 = 2^2
    Total triangles in 3 rows = 1 + 3 + 5 = 9 = 3^2
    Total triangles in 4 rows = 1 + 3 + 5 + 7 = 16 = 4^2
    So, total triangles in 50 rows = 50^2= 50 × 50 = 2 500
24. It really is the same question as 23 – the numbers and the number of triangles are the same!
    So 15^2 = 15 × 15 = 225
25. X*n* = 4 × *n* + 1, so X_{50} = 4 × 50 + 1 = 201

GRADE 7(1)

1. Use trial and improvement to find that only $26 \times 27 = 702$. So $26 + 27 = 53$
2. $\frac{1}{2}$ of $\frac{1}{5}$ is $\frac{1}{10}$, so the number is $\frac{1}{5}$. Then $\frac{5}{7}$ of $\frac{1}{5} = \frac{1}{7}$
3. 

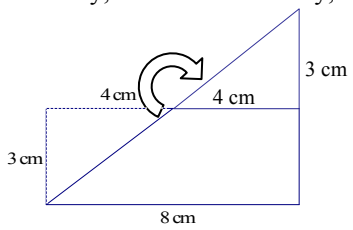
   $6 \times 2 = 12$
4. 

   Length × Breadth $= 8 \times 4 = 32$
5. $1+\dfrac{1}{1+\frac{3}{4}} = 1+\dfrac{1}{\frac{7}{4}} = 1+\frac{4}{7}$
6. There is a general structure here: The denominators is twice the numerator + 1, i.e. $\frac{\Diamond}{2\times\Diamond+1}$

   We can therefore investigate a general pattern $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \frac{6}{13}, \frac{7}{15}, \ldots$

   Check with your calculator: $\frac{1}{3} = 0,333..., \frac{2}{5} = 0,4, \ldots$ So $\frac{1}{3} < \frac{2}{5} < \frac{3}{7} < \frac{4}{9} < \frac{5}{11} < \frac{6}{13} < \ldots$

   Conclusion: the larger the denominator, the larger this kind of fraction, so $\frac{11}{23}$ is the largest
7. We know: $\frac{\text{Sum of numbers}}{5} = 30$, so Sum of numbers $= 30 \times 5 = 150$

   If she erases the number x, then $\frac{150-x}{4} = 28$. So $x = 150 - 112 = 38$
8. Because $365 \div 7 = 52$ rem 1, the day of the week is one day later every year. So in 2013 28 July is on a Sunday, in 2014 on Monday, in 2015 on Tuesday, in 2016 (a leap year!) on Thursday, in 2017 on Friday, and in 2018 on a Saturday
9. 

10. You can calculate the areas and get $\frac{16}{32}$
11. Let the width be w metres, then the length is $4 \times w$ metres, so the Perimeter $= 10 \times w = 100$, so $w = 10$
    So the area is length × width = 40 metres × 10 metres = 400 m^2
12. 21 horizontals + 21 verticals = 42 lines
13. $64 = 8 \times 8 = P_8$, so 9 + 9 = 18 lines
14. 64 lines means it is P_{31}, which has $31 \times 31 = 961$ squares
15. The perimeter of the square is 4×15 cm = 60 cm. So the perimeter of the pentagon is also 60 cm, and each side is 60 cm ÷ 5 = 12 cm
16. 1/3 + (1/4 of 2/3) = 4/12 + 2/12 = 6/12 ; 24 = 6/12 ; Penny originally had 48 marbles; 1/3 of 48 = 16
17.

	c	d
a	12	20
b	21	D

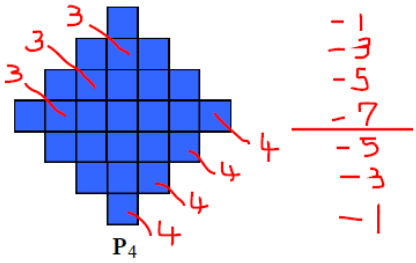
    Using a representation like this, Area D = b × d

    We know a × c = 12, b × c = 21, a × d = 20

    Multiply them all together: $a^2 \times c^2 \times b \times d = 12 \times 20 \times 21$

    But $a \times c = 12$, so $a^2 \times c^2 = 144$, so $b \times d = 12 \times 20 \times 21 \div 144 = 35$ = Area D
18. The first digit can be 1, 3, 5, 7, 9. The second digit can be 1, 3, 5, 7, 9, which gives $5 \times 5 = 25$ possible combinations
19. The 1st column is multiples of 6, given by $6 \times n$, where $n = 0, 1, 2, 3, \ldots$ like in the row numbers.
    So the first number in row 80 is $6 \times 80 = 480$. Then row 80 is 480, 481, 482, …
20. $4321 \div 6 = 720$ remainder 1; so the second number in row 720
21. From 1, 3, 5, 7, to 99 are 50 numbers, so $1+3+5+ \ldots +97+99 = 50 \times 50 = 2500$

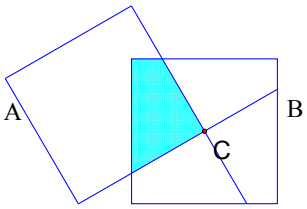
22. If the cost of a drink is D rands and the cost of an ice-cream is C rands:
    2D + 1C = 15 (1)
    1D + 2C = 12
    Add: 3D +3C = 27
    ÷3: 1D +1C = 9 (2)
23. In 22 above: (1) – (2): 1D = R15 – R9 = R6
24. Let the weights be M, D and C kilograms. Then M+D = 90, M+C = 70, D+C = 40 kg.
    Add all together: 2×M + 2×D + 2×C = 200
    So M + D + C = 100
25. Find structure!

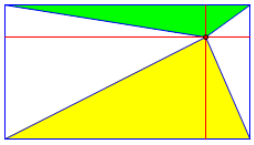


    One structure for P_4 is $4\times4 + 3\times3$. Generalise, then $P_{50} = 50\times50 + 49\times49$
    Another structure for P_4 is $(1+3+5+7) + (1+3+5) = 4\times4 + 3\times3$ …..

GRADE 7(F)

1. Square total area: (5 cm + 3 cm) × (5 m + 3 cm) = 8 cm × 8 cm = 64 cm²
   Rectangles total area: 4×5 cm × 3 cm = 60cm²
   Area of small square: 64 cm² - 60 cm² = 4 cm²
2. $\frac{2^{24}}{2} = 2^{23}$
3. Average = Total mass ÷ number of children = (3×75 + 6×66) kg ÷ 9 = 69 kg
4. The "vertical" formula is $2\times a + 2$. Find a so that $2\times a + 2 = 64$
   Or the "horizontal" formula is $4 + 2\times(a - 1)$, so find a so that $4 + 2\times(a - 1) = 64$
5. If x is the price without VAT, then $1{,}14 \times x = 9{,}46$. So x = R8,30, so VAT = R1,16
6. Let the overlapping region have an area of x cm².
   Area of larger square is 36 cm², So area A = $36 - x$
   Area of smaller square is 16 cm², So area B = $16 - x$
   area A − area B = $(36 - x) - (16 - x) = 36 - x - 16 + x = 36 - 16 = 20$
8. Divide square into 4 equal parts. So $\frac{1}{4}$ of 144 = 36



9. If her average for the first 4 tests was 67%, we can also express it as (67 + 67 + 67 + 67) ÷ 4.
   So 67 + 67 + 67 + 67 + 63 + 67 = 398. 398 ÷ 6 = 66,33
10. If the length of each rectangle is x, and the width is y:
    $3 \times y = 2 \times x$ and $x + y = 15$
    So $2x + 2y = 30$, so $3y + 2y = 30$, so $5y = 30$, so $y = 6$, and $x = 9$.
    Area of 1 rectangle is 9×6 = 54 cm², so the area of 5 rectangles is 5×54 cm²
11. Each of the 26 letters in the alphabet can be paired with itself (e.g. BB for Barry Brown) and paired with each of the other letters. Order matters - PG is different from GP! There are 26 × 26 combinations.
12. Test all the cases systematically: 1 × 17 = 17; 2 × 16 = 32; 3 × 15 = 45; … 9 × 9 = 81, then the answer repeats, because the order does not matter (e.g. 2 × 16 = 16 × 2).
13. $\frac{8}{11} - \frac{5}{8} = \frac{9}{88}$ of tank is 135 ℓ. So $\frac{1}{88}$ of tank = 135 ℓ ÷9 = 15 ℓ. So $\frac{88}{88}$ of the tank = $88 \times \frac{1}{88}$ of the tank = 88 × 15 ℓ
14. 
15. In middle row the missing number is 18 – (11 + 6) = 1, so in right column z = 18 – (1 + 10) = 7
16. $p\times 1\times\frac{1}{8} = 1$, so $p = 8$

    $q\times 1\times 4 = 1$, so $q = \frac{1}{4}$

    $u\times 4\times\frac{1}{8} = 1$, so $u = 2$

    $p\times s\times u = 1$, so $8\times s\times 2 = 1$, so $s = \frac{1}{16}$

    $r + s = \frac{1}{2} + \frac{1}{16} = \frac{9}{16}$

p	q	r
s	1	t
u	4	$\frac{1}{8}$

17. The smallest is 1x4 = 4 and the biggest is 24x4 = 96, so there are 24 multiples of 4
18. $$\frac{4+8+12+\ldots 92+96}{24} = \frac{24\times(4+96)/2}{24} = \frac{4+96}{2} = 50$$
19. List them systematically: 799; 979; 997; 889; 898; 988

20. Investigate the *structure* by finding a pattern in special cases:

# houses	1	2	3	4		n
# matches	5	9	13	17		$4\times n-1$

21. $4\times n - 1 = 225$, so $n = (225 - 1) \div 4 = 56$
22. Multiply them all together: $(a \times b) \times (b \times c) \times (c \times a) = 2 \times 24 \times 3 = 144$
    Transform: $(a\times b\times c)^2 = 12^2$, so $a\times b\times c = 12$
    $a\times b\times c = 12$, and $b \times c = 24$, so $a \times 24 = 12$, so $a = \frac{1}{2}$
    $a\times b\times c = 12$, and $a \times c = 3$, so $3 \times b = 12$, so $b = 4$
    $a\times b\times c = 12$, and $a \times b = 2$, so $2 \times c = 12$, so $c = 6$
23. Look for structure!
    $T_{50} = 1 + 2 + 3 + 4 + \ldots + 49 + 50 = (1+50) + (2+49) + \ldots + (25+26) = 25 \times 51 = 1275$
24. Look for structure and pattern!
    $N_1 = 2 = 1\times 1 + 1$
    $N_2 = 5 = 2\times 2 + 1$
    $N_3 = 10 = 3\times 3 + 1$
    $N_4 = 17 = 4\times 4 + 1$
    Test the numbers! $30\times 30 + 1 = 901$ is the only one which fits the pattern
25. $\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times \ldots \times\frac{2010}{2011}\times\frac{2011}{2012} = 1\times\frac{2}{2}\times\frac{3}{3}\times \ldots\times\frac{2011}{2011}\times\frac{1}{2012} = \frac{1}{2012}$