

SA Mathematics Challenge

Wiskunde Uitdaging

MEMORANDUM

FIRST ROUND 2014
EERSTE RONDE 2014

QUESTION	4(1)	5(1)	6(1)	7(1)	VRAAG
1	B	B	D	C	1
2	B	C	B	E	2
3	A	D	B	B	3
4	C	E	B	D	4
5	C	D	A	A	5
6	B	E	E	E	6
7	E	E	C	E	7
8	B	D	D	A	8
9	D	A	E	B	9
10	E	C	D	E	10
11	B	D	E	B	11
12	E	B	E	A	12
13	D	C	C	C	13
14	C	C	D	C	14
15	B	B	D	B	15
16	A	D	A	A	16
17	C	A	A	C	17
18	B	E	D	D	18
19	B	D	A	D	19
20	E	B	E	C	20
QUESTION	4(1)	5(1)	6(1)	7(1)	VRAAG

Note:

We suggest that teachers make a transparency of a blank answer sheet and mark the answers for your grade on it. You can then use the transparency as an overlay to mark pupils' scripts quickly and painlessly.

But why not let the pupils mark and discuss it in class?

Nota:

Ons stel voor dat onderwysers 'n transparant maak van 'n skoon antwoordblad en die antwoorde vir u graad daarop merk. U kan dan die transparant as 'n masker gebruik om antwoordstelle vinnig en pynloos na te sien.

Eintlik stel ons voor dat leerlinge die vraestelle in die klas merk en bespreek!

NOTES ON 2014 MEMORANDUM FIRST ROUND

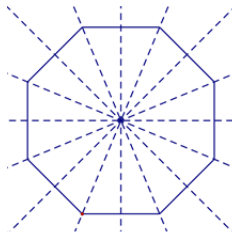
These notes are necessarily brief and often formal and symbolic.

Many questions could be answered using primitive methods, e.g. "If today is Wednesday, what day of the week will it be 100 days from now?" can be done by counting. That would be laborious, time-consuming and error-prone. The essence of a mathematical approach is to work more smartly by using appropriate representations to model the situation and to exploit the inherent structures and patterns in the situation.

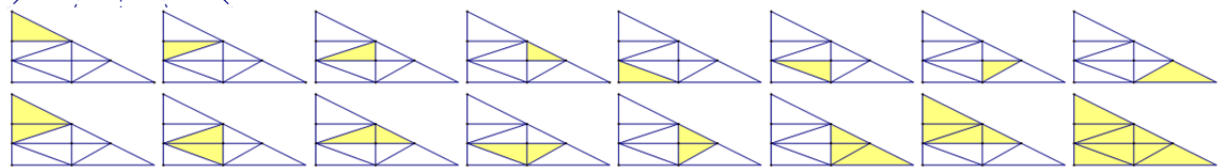
GRADE 4(1)

1. B. $5\,698 - 300 = 5\,398$
2. B. $13 - 3 + 6 - 5 + 2 - 3 = 13 + 3 - 3 - 3 = 10$
3. A. $2 \times 500 \text{ mL} = 1000 \text{ mL} = 1 \text{ litre}$. So from 20 litres you can fill $20 \times 2 = 40$ bottles.
4. C. 1 kg is sub-divided into 5 equal parts, so each sub-unit is 0,2 kg: $50 + 4 \times 0,2 = 50 + 0,8 = 50,8$
5. C. $73 - 68 = 5$
6. B. $230 \rightarrow -10 \rightarrow 220 \rightarrow -15 \rightarrow 205 \rightarrow -20 \rightarrow 185 \rightarrow -25 \rightarrow 160$
7. E. One ball costs $R60 \div 5 = R12$, so 3 balls cost $3 \times R12 = R36$
8. B. He has $R900 - R623 - R275 = R2$ left. So needs $R312 - R2 = R310$
9. D. Nine racks hold $9 \times 85 = 765$ oranges, so there are $785 - 765 = 20$ oranges left
10. E. In 60 minutes it travels 120 km. So in 30 minutes it travels half of 120 km, which is 60 km.

11. B.



12. E.



13. D. $\frac{16}{20} < \frac{18}{20} < \frac{19}{20}$

14. C. List them systematically:

247 427 724
274 472 742

15. B. Four layers of 5 boxes

16. A. Look at the *structure* in the pictures! Count the number of *triangles*.

The fourth shape is formed by $1+2+3+4 = 10$ triangles, so the number of straws = $10 \times 3 = 30$

17. C. You are equally likely to draw any one of the $7+1 = 8$ marbles, so $\frac{1}{8}$



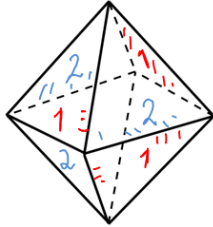
18. B. $\frac{2}{5}$ of R50 is R20, so he had R30 left. $\frac{1}{6}$ of R30 is R5, so he has $R30 - R5 - R25$ left.

19. B. Brian is half as old as Aunt Anna, so Brian is 21 years old. Cathy is 5 years younger than Brian, so Cathy is 16 years old.

20. E. Each of the digits can be 1, 3, 5, 7 or 9, giving $5 \times 5 \times 5 = 125$ possible combinations.

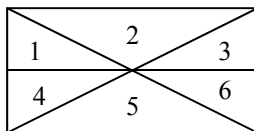
GRADE 5(1)

1. B. No need to calculate! $(999 + 1001) + (998 + 1002) = 2 \times 2000 = 4000$
2. C. Three hours after 10:30 is 13:30
3. D. $\frac{30 \times 30 \times 30}{299 + 1} = \frac{30 \times 30 \times 30}{10 \times 30} = 30 \times 3 = 90$
4. E. You can do it mentally, or test on your calculator or use the rule that *the digit sum must be a multiple of 3*
5. D. Lihle is 5 years older than Musa. So Musa is 5 years younger than Lihle, so if Lihle = 35, then Musa is 5 years younger (30)
6. E.



7. E. Bring everyday knowledge that ducks have 2 legs and sheep 4 legs
Check each of them, e.g. for (A): $60 \times 2 + 10 \times 4 = 160$ does not give 140 legs. But (E) does: $30 \times 2 + 20 \times 4 = 160$
8. D. Buses leave at 06:06 and 06:30. So when Anna arrives at 06:40 the bus left 10 minutes ago, so the next bus leave in 14 minutes.
9. A. $99 + 98 + 97 = 294$
10. C. With 6 loose cubes, there would be 36 faces. Subtract the 10 non-visible faces ...
11. D. The coin can land in 2 ways and the die in 6 ways, altogether the two together can land in 12 different ways (H, 1), (H, 2) ..., (H, 6); (T, 1), (T, 2) ..., (T, 6). There is only one (T, 6), so the probability is $1/12$.
12. B. January has 31 days of which 16 are odd days (1, 3, 5, ..., 29, 31)

13. C.



Invent some notation and count systematically, e.g.:

Areas 1, 2, 3, 4, 5 and 6 each form a triangle (6)

Two areas 1-4 and 3-6 each form a triangle (2)

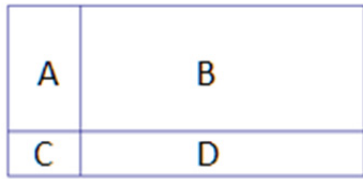
Three areas 4-1-2, 2-3-6, 3-6-5 and 5-4-1 each form a triangle (4)

14. C. $10 + 10 + 8 + 8 = 36$ or 4×10 minus the 4 corner poles that were counted twice
15. B. Make a systematic list:

24	42	72
27	47	74
16. D. Be systematic. Note structure and number patterns!
 $3 \times 12 + 0 \times 6 = 36$
 $2 \times 12 + 2 \times 6 = 36$
 $1 \times 12 + 4 \times 6 = 36$
 $0 \times 12 + 6 \times 6 = 36$
17. A. The next palindrome is 42424. So he must drive another $42424 \text{ km} - 42324 \text{ km} = 100 \text{ km}$
18. E. Work systematically!
101, 111, 121, 131, 141, 151, 161, 171, 181, 191 – this is 10
202, 212, 222, 232, 242, 252, 262, 272, 282, 292 – this is 10
....
909, 999, 929, 939, 949, 959, 969, 979, 989, 999 – this is 10
So $9 \times 10 = 90$
19. D. Look at the *structure* in the pictures!
 $P_1 = 4 \times 1 + 1 = 5$
 $P_2 = 4 \times 2 + 1 = 9$
 $P_3 = 4 \times 3 + 1 = 13$
 $P_{20} = 4 \times 20 + 1 = 81$
20. B. Make a list, varying the persons systematically. If the persons are A, B, C and D:
ABCD ACBD ADBC
ABDC ACDB ADCB
Similarly if the first person is B, C, or D.
So $4 \times 6 = 24$

GRADE 6(1)

- D. More than halfway between 80 and 90
- B.



Name the small rectangles A, B, C and D. List all the possibilities systematically:
 One region: A, B, C, D
 Two regions: AB, CD, AC, BD
 Four regions: ABCD

- B. $? \rightarrow \div 9 \rightarrow 2100 \rightarrow \div 7 \rightarrow 300 \rightarrow \div 5 \rightarrow 60 \rightarrow \div 3 \rightarrow 20$
- B. Clockwise: $1 + 5 + 1 + 1 + 4 + 4 + 5 + 1 = 22$. Ant-clockwise: $5 + 5 + 6 + 6 = 22$
- A. $2 \times (6+8) + 8 \times (6+8) = 2 \times 14 + 8 \times 14 = 10 \times 14 = 140$
- E. If the numbers are equal, they each are $138 \div 3 = 46$. Then $45+46+47=138$
- C. $(\frac{1}{4} + \frac{1}{12}) \div 2 = \frac{1}{3} \div 2 = \frac{1}{6}$
- D. The distance from $\frac{1}{3}$ to each of the fractions are $\frac{1}{6}, \frac{1}{120}, \frac{1}{60}, \frac{1}{270}$ and $\frac{1}{39}$, of which $\frac{1}{270}$ is the smallest
- E. From the first two equations $A = 15$ and $B = 2$, or $A = 2$ and $B = 15$. But from the third equation B cannot be 15, so $B = 2$, and then $2 + C = 13$, so $C = 11$
- D. The first three may be blue, red and brown. Then the next one must match one of these colours
- E. You can maybe take out, e.g. 10 red, then 10 brown, then 1 blue, then the next one must also blue
- E. Replace the 5 numbers with 20, 20, 10, 10, 30. Then only $20+20+30 = 70$.
- C. $\frac{1}{4}$ of the balls sold are cricket balls, so the total number of balls sold are $4 \times 60 = 240$. If x rugby balls are sold, then $5x$ soccer balls were sold, Then $x + 5x + 60 = 240$, $x = 30$, and $5x = 150$.
- D. $\triangle MDN = \frac{1}{8}$ of square and $\triangle BCN = \frac{1}{4}$ of square. So $ABNM = 1 - \frac{1}{8} - \frac{1}{4} = \frac{5}{8}$ of square
- D. Investigate the *structure* by finding a pattern in special cases:

Number of cuts	0	1	2	3		n
Number of pieces	1	3	5	7		$2 \times n + 1$

Therefore, there will be $2 \times 9 + 1 = 19$ pieces of string

- A. $147 = 12 \times 12 + 3$, but $122 = 11 \times 11 + 1$, $102 = 10 \times 10 + 2$, $81 = 9 \times 9 + 0$, $401 = 20 \times 20 + 1$
- A. Think of green-amber-red as a repeating block of 3. Up to 99 this block is repeated 33 times, and then the next flash is green, so there are 34 greens.
- D. Let the children be A, B, C and D. List all the possibilities systematically. Note that a child cannot play against itself, and order does not matter, i.e. A vs B is the same as B vs A:

A vs **B** **B** vs **C** **C** vs **D** **D** vs **E**
 A vs C B vs D C vs E
 A vs D

- A. Be systematic:

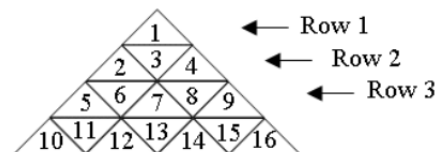
1	$6 = 4+2$	$11 = 8+2+1$
2	$7 = 4+2+1$	$12 = 8+4$
$3 = 2+1$	8	$13 = 8+4+1$
4	$9 = 8+1$	$14 = 8+4+2$
$5 = 4+1$	$10 = 8+2$	$15 = 8+4+2+1$

- E. $U_1 = 1 + 3 + 4$
 $U_2 = 2 + 4 + 5$
 $U_3 = 3 + 5 + 6$
 $U_{80} = 80 + 82 + 83 = 245$

GRADE 7(1)

- C. $2 - 2 \div 2 + 2 = 2 - 1 + 2 = 3$
- E. Sum of 4 consecutive numbers is 26, i.e. $5+6+7+8$. Therefore, Mon is the 5th
- B. Each hour is equivalent to 30° , thus the four hours between 08:00 and 12:00 is equivalent to $4 \times 30^\circ$
- D. Investigate the *structure* by finding a pattern in special cases:

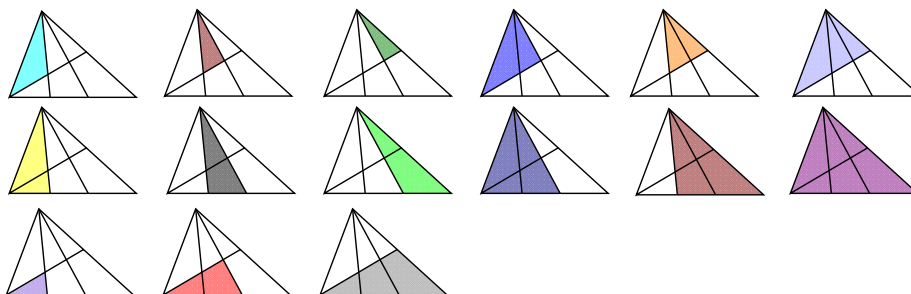
Row number	1	2	3	4		n
Number of numbers	1	3	5	7		$2 \times n - 1$



Therefore, there will be $2 \times 20 - 1 = 39$ numbers in Row 20

- A. The last number in each Row is the sequence 1, 4, 9, 16, n^2
The last number in Row 19 is 19×19 . So the first number in Row 20 is $19 \times 19 + 1 = 362$

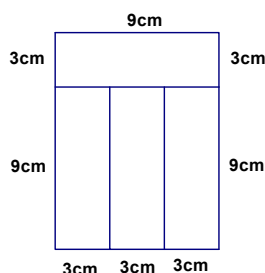
- E.



- E. $1 + \frac{1}{1+\frac{1}{3}} = 1 + \frac{1}{\frac{4}{3}} = 1 + \frac{3}{4} = 1\frac{3}{4}$

- A. $\frac{60 \text{ km}}{1 \text{ h}} = \frac{60\,000 \text{ m}}{3600 \text{ sec}} = \frac{600 \text{ m}}{36 \text{ sec}} = \frac{200 \text{ m}}{12 \text{ sec}}$

- B.



- E. X567Y is a multiple of 3, and $5+6+7 = 18$ is a multiple of 3, so X+Y must be a multiple of 3. So the largest value of Y is 9, e.g. $3+9$, $6+9$, $9+9$.
- B. Use trial and improvement:
 $20 \times 6 - 0 \times 2 = 120 \neq 88$ He did not have all correct
 $19 \times 6 - 1 \times 2 = 112 \neq 88$ He did not have 19 correct
 $17 \times 6 - 3 \times 2 = 96 \neq 88$ He did not have 17 correct
 $16 \times 6 - 4 \times 2 = 88$ He had 16 correct!

- A. $3x - 5 = 25$, so $x = 10$

- C. Structure!

P_1	P_2	P_3	P_4	...	P_{50}
1×2	2×3	3×4	4×5	...	?

- C. Difference in mass of water when half full and one third full:

$$\frac{1}{2} - \frac{1}{3} \equiv 12 - 10 \text{ kg}, \text{ so } \frac{1}{6} \equiv 2 \text{ kg}$$

Thus, when half full, the water will be 6 kg, which means that the bucket has a mass of 6 kg.

- B. The sum of the five numbers is $a + b + c + d + e = 60 \times 5 = 300$
The new sum is $80 + b + c + d + e = 65 \times 5 = 325$
So $80 - a = 25$, so $a = 55$

- A. $\frac{20!}{19!} = \frac{20 \times 19 \times 18 \times \dots \times 1}{19 \times 18 \times \dots \times 1} = 20$

- C. Note that the numbers in column G are multiples of 7, in column F are all one less than a multiple of 7, etc. 2014 is 2 less than a multiple of 7 ($2014 = 287 \times 7 + 2$), so 2014 will be in column E, which are all 2 less than a multiple of 7 (5, 12, 19...)

$$\begin{array}{rcl}
 18. \text{ D.} & 4 \text{ choc} + 2 \text{ cool} & = \text{R}35 \\
 & \underline{2 \text{ choc} + 4 \text{ cool} = \text{R}43} & \\
 & 6 \text{ choc} + 6 \text{ cool} & = \text{R}78 \\
 & 1 \text{ choc} + 1 \text{ cool} & = \text{R}13
 \end{array}$$

19. D. The total number of dots that are not visible = total dots – visible dots

The total of the numbers on one die = $1+2+3+4+5+6 = 21$, so the total on the three dice is 63.

Numbers 1, 1, 2, 3, 4, 5, 6 are visible, and these total 22.

So the total dots *not* visible = $63 - 22 = 41$

20. C. Suppose there are 100 people

	Sick	Well
Month 1	10	90
Month 2	$3+27=30$	$7+63=70$

So $\frac{30}{100}$ of the people is sick