Percentages – Nothing to be Scared of

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Introduction
For a period of three months (January to March 2005) we visited primary schools in Circuit 6 of the Winelands West Coast EMDC in the Western Cape. These visits formed part of an extended project aimed at improving mathematics teaching in the senior phase (which includes grades 7, 8 and 9).

It was observed that both learners and teachers struggled with calculations involving percentages. Bertus van Etten developed the four problems discussed below. By means of these problems four different scenarios that involve variations of working with percentages can be accessed. At different workshops teachers were requested to solve these. The strategies they used and the problems they encountered were discussed and analysed afterwards. The readers of this paper are requested to likewise try to solve the problems first, before reading through the solutions.

Four exercises on percentages
All of these problems deal with percentages in terms of money. Percentages, as we all know, appear in, and are used in, various contexts and professions: economists, health workers, business people, statisticians, etc. all make use of percentages at some stage. In his plenary address at the recent AMESA Congress 2006 in Limpopo Province, the MEC for Education referred to this concept as important for the medical profession.

The four exercises are the following:

<u>Exercise 1</u>
Eddie has bought a computer priced at 600 rand. He received 120 rand discount.
What percentage discount did Eddie get?

<u>Exercise 2</u>
Petra receives R60,00 pocket money every month. Of this she saves 15% every month.
How much money does she save every month?

<u>Exercise 3</u>
A shop owner gives 30% discount on CDs. So the client only has to pay 70% of the original price. After discount the client pays R 42,00. What is the price for a CD without discount ?

<u>Exercise 4</u>
A vendor sells apples at R56,00 per box. He makes 40% profit in the process. How much did he pay for the box of apples when he bought them?

It is remarkable that while most teachers managed to do the first two exercises without any real difficulty, the majority ran into problems with respect to the last two. With respect to exercises 1 and 2, they used the (only) method they seem to be familiar with: one value is divided into another and the result is multiplied by a third value. In each case two values are taken from the context, with the third value being 100. With respect to the first exercise, $\frac{120}{600} \times 100 = 20$, and for the second exercise, $\frac{15}{100} \times 60 = 9$. Exactly why they had approached the two problems in this way many of them could not explain. Some were also not sure as to whether the outcome or solution should be in terms of rand or percentage.

It is clear that this approach is based on the application of 'meaningless' knowledge, that is the application of an algorithm without having real insight into what it entails mathematically. It is essential that learners develop skills that can be applied easily but based on understanding and insight. For this reason teachers should refrain from merely providing learners with recipes, but instead should facilitate the development of mathematical concepts.

Next, we want to briefly discuss different ways of enhancing the understanding of percentages.

The common sense method

Only exposing learners to the use of a particular recipe, or algorithm, might interfere with a simple common sense approach based on logical reasoning. Without using a formula or other prescribed method one could reason as follows:

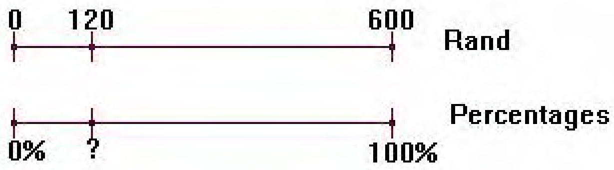
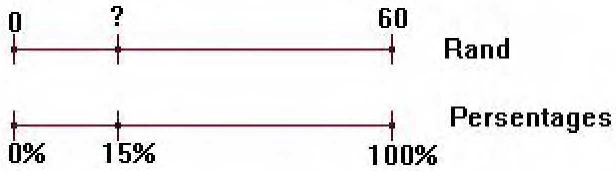
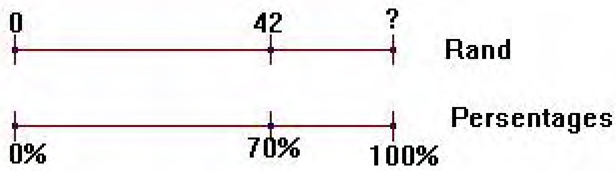
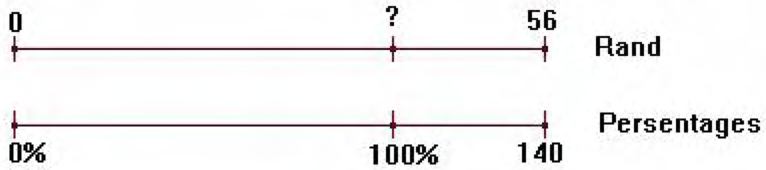
Exercise 1	600	Rand is	100 %	Divided by 10
	60	Rand is	10 %	Multiplied by 2
	120	Rand is	20 %	
Exercise 2	60	Rand is	100 %	Divided by 10
	6	Rand is	10 %	Divided by 2
	3	Rand is	5 %	adding 6 + 3 and 10 + 5
	9	Rand is	15 %	
Exercise 3	42	Rand is	70 %	Divided by 7
	6	Rand is	10 %	Multiplied by 10
	60	Rand is	100 %	
Exercise 4	56	Rand is	140 %	Divided by 7
	8	Rand is	20 %	Multiplied by 5
	40	Rand is	100 %	

From the table it is easy to see why exercises 3 and 4 do not fit into the recipe previously mentioned. In these two exercises 100% is not known and should therefore be calculated. Basically all that we need, apart from reasoning skills, to get to the solutions of all these exercises is knowledge about multiplication (tables), division, multiples, factors and equivalence. It is obvious that not all problems would work out so neatly, but this method could be effective in developing reasoning strategies.

Visualization

The following approach would be especially beneficial for learners who struggle with more theoretical approaches to problem-solving. By means of visualization these learners can be made to 'see' what actually happens in a much more practical way.

The approach to the solution is the same in all cases, namely: "… rand is … %". This brings us to a different approach, namely that of visualization by means of a double number line. This method is especially beneficial to learners who have problems conceptualising the mathematics involved. At the same time we extend the traditional use of the number line to represent values in context:

Exercise 1	
Exercise 2	
Exercise 3	
Exercise 4	

Now the similarities, as well as the differences, with respect to the four exercises are clearly visible. Apart from the zeros on the number lines, we have three values to work with in each case, two coming from the context and the third being 100%. The unknown value, represented by the question mark, needs to be determined every time.

Methods used

On each number line three 'columns' are visible, namely the nil-nil column, the column containing the question mark, and thirdly the column with the two values taken from the context. The number lines also clearly indicate whether the question mark represents a rand value or a percentage.

The calculations can now be done using a variety of different methods:

<u>Method 1</u>

Start with the sentence using the column that contains two values.
By means of multiplication and division determine the value represented by the question mark.
The value of the question mark (number) is then read off.

<u>Method 2</u>

Learners who are familiar with cross multiplication can now find the appropriate 'formulae' to solve the problem, although understanding might be lacking:

$\frac{120 \times 100}{600} = 20$, for the first exercise,

$\frac{15 \times 60}{100} = 9$, for the second exercise

$\frac{42 \times 100}{70} = 60$, for the third exercise

$\frac{56 \times 100}{140} = 40$, for the fourth exercise.

Method 3

The number lines can also be translated to ratio tables:

The first column contains the two values that correspond with the two values (one below the other) on the number lines.

The last column contains the question mark (value) and the value associated with it.

By means of multiplication and division the values in the empty columns are determined. The number below or above the question mark indicates what needs to be done to get to the solution.

The value (number) of the question mark can be read off.

price in rand	600				120
price in percentage	100				?

price in rand	60				?
price in percentage	100				15

price in rand	42				?
price in percentage	70				100

price in rand	56				?
price in percentage	140				100

Consequences for teaching

Before any technique is learned about how to calculate percentages, it is crucial for learners to know exactly what 'percentage' means. For example, prior to doing any exercises they should know that the expressions $\frac{120}{600} = 0{,}20 = \frac{20}{100} = 20\%$ represent four different ways of writing down the same value.

It can also be discussed and expressed as equivalent ratios, as follows: 120:600 = 20:100. By means of a ratio table as an alternative way of representing a fraction, ratios can be determined diagrammatically without much mathematical hocus-pocus. This is also a neat way of integration within the learning area by working with percentages, fractions, decimal fractions and ratios all at the same time.

÷ 10 × 2

price in rand	600	60	120		120
price in percentage	100	10	20		?

÷ 10 × 2

The information that appears in the first column (namely "price in rand" and "price in percentage"), of the ratio table ensures that the context remains visible to the learners. In this way the mathematics is made less abstract, with the emphasis being placed on using mathematics to solve a problem encountered in everyday life.

In the teaching-learning process it is not advisable to put too much emphasis on algorithms (multiplication and division) alone. Rather, the most important activity in the teaching-learning process is to translate the context into meaningful mathematics – in this case the use of the double number line or the ratio table. This can be used as a point of departure in deciding in what way the calculation should be done and makes more sense to the learners. It can also be accurately calculated by means of a calculator as long as the key sequences are explained