

Effective Interest Rates: Making Sense or Cents?⁵

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An effective interest rate enables us to compare different interest rates over different periods, and with different compounding frequencies. Consequently, an understanding of effective rates is important for personal finance. In the SA school curriculum, effective interest rates are introduced in Grade 11 in both Mathematics and Mathematical Literacy. However, text books generally do not extend their discussion to engage with the relationship between effective rates and actual banking products. In this article I explore the notion of effective interest by focusing on its definition, its connection to percentage increase, and the derivation of the effective interest formula. I then discuss the interest rates of notice deposits as advertised by two big South African banks and show how effective rates are used in each case. Through this I show that the effective rate formula used at school is not the formula generally used by banks. Rather, banks quote an ‘*average* effective annual rate’ as this enables them to quote seemingly higher effective rates.

DEFINING THE EFFECTIVE INTEREST RATE

An effective annual interest rate refers to the annual rate of interest when compounding occurs more frequently than once a year. It therefore takes into consideration the effect of compounding. Definitions of effective interest typically refer also to nominal rate. For example, the following definition is provided in the National Curriculum Statement:

Effective interest – the annual rate which is equivalent to a **nominal rate** when compounding is effected more often than once a year (e.g. 12% p.a. compounded monthly is equivalent to 12,68% p.a.; the nominal interest rate (i) is 0,12 and the effective interest rate is 0,1268).

(Department of Education (DoE), 2003, p. 87, emphasis in original)

In order to provide a formula for calculating the effective annual interest rate it is first necessary to define a number of terms. If i is the nominal interest rate per annum, then $i_m = \frac{i}{m}$ represents the effective rate per period, where m is the number of compounding periods in a year. Thus i_4 is an effective quarterly interest rate, i_{12} is an effective monthly interest rate, and i_1 is an annual interest rate with annual compounding (in this case the nominal and effective rates are the same). Based on these definitions of the various terms, we have the following formula for the effective annual interest rate:

$$i_e = \left(1 + \frac{i}{m}\right)^m - 1$$

MAKING SENSE OF EFFECTIVE INTEREST RATE THROUGH NUMERICAL EXAMPLES

It is helpful to use an example to illustrate the notion of effective annual interest rate. Consider a scenario where I invest R800 for a year at 6% p.a. compounded monthly. Each month I receive 0.5% interest on the latest balance. So, each month I get more interest than the previous month because the interest of 0.5% is calculated on a slightly larger amount each time. By contrast, if we consider a simple interest scenario then I still get 0.5% each month but this is always calculated on the original balance of R800.

⁵ An earlier version of this article was presented at the 2014 AMESA Annual National Congress (Pournara, 2014).

In Table 1 I compare these two scenarios. I include month zero to indicate the starting amount for the 12-month period. In the simple interest section I get R4 interest each month and this accumulates to R48 over the year. In the compound interest section, the amount of interest earned increases each month, and the total interest for the year is R49.34.

Simple interest			Compound interest		
End of month	Balance	Interest	End of month	Balance	Interest
0	800.00		0	800.00	
1	804.00	4.00	1	804.00	4.00
2	808.00	4.00	2	808.02	4.02
3	812.00	4.00	3	812.06	4.04
4	816.00	4.00	4	816.12	4.06
5	820.00	4.00	5	820.20	4.08
6	824.00	4.00	6	824.30	4.10
7	828.00	4.00	7	828.42	4.12
8	832.00	4.00	8	832.57	4.14
9	836.00	4.00	9	836.73	4.16
10	840.00	4.00	10	840.91	4.18
11	844.00	4.00	11	845.12	4.20
12	848.00	4.00	12	849.34	4.23

TABLE 1: Comparison of simple and compound interest for 12 months

I now show the connection between this accumulated amount, percentage increase and the effective annual rate. If we calculate the percentage increase on the principal amount in each case, we get:

$$\text{SI increase} = \frac{848-800}{800} = 6.0\% \quad \text{CI increase} = \frac{849.34-800}{800} = 6.1675\%$$

It is important to note that the amount of R849.34 has been rounded to 2 decimal places. If we work to higher levels of accuracy the percentage increase will be 6.167781% (rounded to 6 decimal places).

Now, since the effective rate is an annual interest rate that will make R800 grow to R849.34 in one year, we can use the compound interest formula ($A = P(1 + i)^n$) to determine the effective rate:

$$849.34 = 800(1 + i)^1$$

$$\therefore i = \frac{849.34}{800} - 1 = 1.06167781 \dots - 1 = 6.167781\%$$

This is the same answer obtained with the percentage increase calculation, which is not surprising since $i = \frac{849.34}{800} - 1$ is an equivalent form of that calculation. We thus see that the effective annual rate is the same as the percentage increase over a one-year period.

DERIVING A FORMULA FOR EFFECTIVE RATE

In many school texts the formula for effective rate is simply stated without derivation. However, the formula may be less intimidating if learners are aware of its derivation, which begins with equating two versions of the compound interest formula.

We begin with a general form of the compound interest formula with m compounding periods per year: $A_1 = P \left(1 + \frac{i}{m}\right)^m$. When we work with the effective rate, we compound once at the end of the year, so we have $A_2 = P \left(1 + i_e\right)^1$. Since the two calculations must produce the same output, we can equate the two equations: $P \left(1 + \frac{i}{m}\right)^m = P \left(1 + i_e\right)^1$. Since P is common we get: $1 + i_e = \left(1 + \frac{i}{m}\right)^m$ which leads to the formula $i_e = \left(1 + \frac{i}{m}\right)^m - 1$. From the formula we can see that the effective interest rate is independent of the principal amount but dependent on the number of compounding periods. Testing our earlier scenario with R800 invested at 6% p.a. compounded monthly, we get $i_e = \left(1 + \frac{0.06}{12}\right)^{12} - 1 = 6.16778118\dots\%$ as expected.

It is important to note that effective interest rates are defined for a one-year period. When we refer to effective rates for a period shorter or longer than one year, we are manipulating the annual effective rate in some way.

APPLICATION TO BANK INTEREST RATES

In this section I consider two adverts for fixed period investments from two South African banks. When banks advertise notice deposits they are required to indicate whether the advertised rate is a nominal or an effective rate. They also generally offer different rates depending on the period of the investment, and may offer different rates depending on when interest is paid out. For example, if interest is paid out at the end of the investment (i.e. on maturity), the rate indicated will be higher than if interest is paid out at regular intervals during the life of the investment.

Figure 1 provides nominal and effective rates for a fixed deposit account from First National Bank (First National Bank, 2015). According to the information provided, if an amount is invested for 12 months, the nominal rate is 6.20%, and the effective rate is 6.38%, based on the convention of monthly compounding.

Fixed Deposit		
Period	Interest (nominal)	Interest (effective)
1 month	4.00%	4.00%
2 months	4.80%	4.81%
3 months	5.10%	5.12%
4 months	5.10%	5.13%
5 months	5.10%	5.14%
6 months	5.75%	5.82%
7 months	5.75%	5.83%
8 months	5.75%	5.85%
9 months	5.75%	5.86%
10 months	5.75%	5.88%
11 months	5.75%	5.89%
12 months	6.20%	6.38%
13 months	6.20%	6.40%
18 months	6.25%	6.53%
24 months	6.35%	6.75%
36 months	6.70%	7.40%
48 months	7.00%	8.05%
60 months	7.20%	8.64%

FIGURE 1: FNB advertised rates for Fixed Deposit Account

We can confirm these figures by using the formula: $i_e = \left(1 + \frac{0.62}{12}\right)^{12} - 1 = 6.38\%$ (to 2 decimal places). However, if we try to confirm figures for periods shorter than or longer than one year, we do not get the effective rates advertised. For example, consider the case of 36 months: the bank quotes a nominal annual rate of 6.7% and an effective annual rate of 7.4%. We can check this by adapting the formula derived above. We know that 6.7% is the nominal rate that will be compounded 36 times over 3 years, so we have: $A_1 = P \left(1 + \frac{0.067}{12}\right)^{36}$. We need to determine an effective rate that will be compounded annually for 3 years to obtain the same final amount, so we have $A_2 = P(1 + i_e)^3$. Since $A_1 = A_2$ and the principal amounts are common, we have $(1 + i_e)^3 = \left(1 + \frac{0.067}{12}\right)^{36}$. Solving for i_e we get $i_e = 6.91\%$ (to 2 decimal places). This is the rate which is compounded annually for 3 years and which produces the same amount as 6.7%

compounded monthly for 36 months. But this is not the effective rate quoted by the bank (7.4%). There is, however, a second possibility for calculating an effective rate.

Here we think about an *average* rate over the 3 years. In other words we determine the total percentage increase over the 3 years and then divide this by 3. This is the same as working with simple interest. So we have A_1 as defined above giving us the total amount accumulated over 3 years. We then use the simple interest formula over 3 years with an effective rate as follows: $A_2 = P(1 + 3 \times i_e)$. Once again, since $A_1 = A_2$ and the principal amounts are common, we have: $1 + 3 \times i_e = \left(1 + \frac{0.067}{12}\right)^{36}$. Solving for i_e we get $i_e = 7.4\%$ (to 2 decimal places). This is the rate quoted by the bank. So it appears that the bank is quoting *average* rates for the 36 month period. This rate is higher than the rate calculated previously because it works off the principal amount rather than from the closing balance at the end of each year. It is not surprising that a bank would choose to quote a higher rate to draw clients.

Now consider Figure 2 where Nedbank offers the following rates on an investment product (Nedbank, 2015), and indicates different rates depending on when interest is paid out.

Nedbank Green Savings Bonds				
18, 24, 36 or 60 month deposits, for clients less than 60 years of age.				
Balance	Period of Investment	Interest Monthly	Interest Half-yearly	Interest on Expiry
R1 000 or more	18 months	6.90%	7.00%	7.24%
	24 months	7.14%	7.25%	7.65%
	36 months	7.39%	7.50%	8.24%
	60 months	8.11%	8.25%	9.96%

FIGURE 2: Nedbank advertised rates for investment product

If interest is paid out monthly then we are working with a nominal annual rate since no interest accumulates in the account. If interest is paid on expiry (i.e. at maturity), we are working with an effective annual rate. In other words, the interest rate on expiry should generate the same amount of interest as we would get by receiving interest monthly and withdrawing it from the account.

Once again we need to consider both methods of calculating the effective rate. For illustrative purposes I focus on 24 months and 60 months.

$$\begin{array}{ll}
 \text{24 months} & \text{60 months} \\
 (1 + i_e)^2 = \left(1 + \frac{0.0714}{12}\right)^{24} & (1 + i_e)^5 = \left(1 + \frac{0.0811}{12}\right)^{60} \\
 i_e = 7.378353\% \text{ to 6 dp} & i_e = 8.418351\% \text{ to 6 dp}
 \end{array}$$

Neither of these figures agrees with the advertised rates for interest paid on expiry.

Below I provide the calculations for an *average* effective rate:

$$\begin{array}{ll}
 1 + 2 \times i_e = \left(1 + \frac{0.0714}{12}\right)^{2 \times 12} & 1 + 5 \times i_e = \left(1 + \frac{0.0811}{12}\right)^{5 \times 12} \\
 2i_e = 0.1530110762 & 5i_e = 0.4980074982 \\
 i_e = 7.650553\% \text{ to 6 dp} & i_e = 9.960150\% \text{ to 6 dp}
 \end{array}$$

If we truncate the answers (to 2 decimal places), we get the quoted rates for interest paid on expiry. Once again, it appears that the bank is quoting *average* annual rates.

In the case where interest is paid half-yearly, it appears that the quoted rates have also been calculated using an average rate. I illustrate this for the case of 18 months by making the relevant substitutions:

$$1 + \frac{1}{2}i_e = \left(1 + \frac{0.069}{12}\right)^{0.5 \times 12}$$

$$\frac{1}{2}i_e = \left(1 + \frac{0.0661}{12}\right)^6 - 1$$

$$\frac{1}{2}i_e = 0.03499975612 \dots$$

So $i_e = 7.00\%$, truncated to 2 decimal places as quoted in the advert. In the first line of the above calculations, the right-hand side represents the accumulated amount after six months of monthly compounding. The left-hand side must therefore yield the same amount over a six-month period, using simple interest. This can be achieved by halving the annual interest rate which means we obtain 3.5% interest over six months, and this equates to an annual rate of 7.0%. Using the same calculation with the relevant monthly rate, we can obtain all the quoted rates for interest paid half-yearly.

Based on these calculations, it seems reasonable to conclude that banks are advertising ‘average effective annual rates’, rather than the effective rate obtained from the formula used at school, when dealing with periods that are longer or shorter than one year. By comparing the answers obtained in the calculations we see that the average effective rates are higher than those that come from the school formula, but it is important to recognise that the higher rates quoted do not mean that we earn more interest. The average rate is based on a simple interest calculation which will yield the same amount of interest as the effective rate obtained from the formula we use at school. It is thus important to emphasise that an effective rate is defined for **one** year, and any “effective rate” quoted for a period longer or shorter than a year has been manipulated in some way. These manipulations may differ from bank to bank and from country to country.

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