Mathematical Literacy for Higher Education*

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Mathematical (or quantitative) literacy plays an important role in the university curriculum for many disciplines, and the object of this article is to enrich teachers’ understanding of what may be expected in terms of learners’ mathematical literacy in higher education.

Although I am writing mainly about mathematical literacy\(^1\), what I say is also relevant to Mathematics teachers, because learners in a Mathematics class should ideally be developing their mathematical literacy at the same time. However, it is easy for Mathematics teachers to lose sight of the applications of the mathematics concepts in real contexts, especially as the examinations do not emphasize these.

The nature of mathematical literacy

The school subject Mathematical Literacy deals with what is known elsewhere as quantitative literacy (especially in America) and as numeracy (in England and Australia). Any one of these terms refers to the same thing: the practice of using mathematics and statistics in real contexts.

The defining feature of mathematical literacy is that it has to do with thinking mathematically while simultaneously thinking about the context in which the mathematics is being used. I will consider a very simple example to illustrate the constant interplay between content and context: reading the newspaper a while ago I saw that construction workers working on the stadium for 2010 who had been on strike settled for a 12% pay rise. To begin to understand what this means, I need to know that the average daily wage of a construction worker is R140 (context) and I need to be able to accurately calculate what 12% of R140 is - and to realize that this calculation would be useful (mathematical content).

To understand more about the meaning of this quantity in the social context of the workers, I need to understand something about prices, the inflation rate, the average number of dependants a worker has and so forth (the context). Now I might feel that to derive greater insight I need to recalculate the daily wage figures as monthly or annual income so that I can make a comparison with salaries of workers in other sectors more easily, in which case the context will be dictating what mathematics to do. So the process of understanding and interpreting a very simple quantitative statement can lead to fairly complex trains of thought which involve competencies from the discipline of mathematics applied in an integrated way within a person’s thinking about a particular real context.

I think this quote from an American book about the urgent need to pay attention to the quantitative literacy of students in higher education makes the point about the interplay between mathematical content and the context very nicely:

“A quantitatively literate person must be able to think mathematically in context. This requires a dual duty, marrying the mathematical meaning of symbols and operations to their contextual meaning, and thinking simultaneously about both.” (Steen, 2004:25)

How is Mathematical Literacy different from Mathematics?

Firstly, I have already stressed the crucial role of contexts in mathematical literacy. Unlike mathematics, which can be abstract, mathematical literacy always operates within a real context. This quote from Deborah Hughes-Hallett (an eminent university mathematics educator) summarizes:

“…mathematics focuses on climbing the ladder of abstraction, while quantitative literacy clings to context. Mathematics asks students to rise above context, while quantitative

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\(^1\) I will use upper case only when I refer to the name of school subjects. Thus mathematical literacy is a kind of literacy, while Mathematical Literacy is the school subject.
literacy asks students to stay in context. Mathematics is about general principles that can be applied in a range of contexts; quantitative literacy is about seeing every context through a quantitative lens.” (Hughes-Hallett, 2001:94)

When Mathematics does pose contextualized problems they are often fairly artificial problems, posed principally to provide exercise in applying learned techniques (“story sums”). The real problems that Mathematical Literacy engages with are often not well defined, they may involve collecting or analyzing data and may not necessarily have only one answer.

A second difference between Mathematics and Mathematical Literacy is that data and statistics play an enormous role in Mathematical Literacy. Data handling is arguably the most important component in the Mathematical Literacy curriculum, while in the Mathematics curriculum most of it has been moved to the ‘optional’ topics.

Another aspect that makes Mathematical Literacy different from Mathematics is the greater stress on critical engagement with quantitative information. Compare these extracts from the definitions of Mathematical Literacy and of Mathematics in the curriculum statements:

**Mathematical Literacy** provides learners with an awareness and understanding of the role that mathematics plays in the modern world. …. It enables learners to develop the ability and confidence to think numerically and spatially in order to interpret and critically analyse everyday situations and to solve problems. (Department of Education, 2003a:9)

**Mathematics** enables creative and logical reasoning about problems in the physical and social world and in the context of mathematics itself. …. Mathematics is based on observing patterns; with rigorous logical thinking, this leads to theories of abstract relations. Mathematical problem solving enables us to understand the world and make use of that understanding in our daily lives. (Department of Education, 2003b:9)

In the Mathematical Literacy definition, words like “awareness and understanding” and “interpret and critically analyse” contrast with phrases like “rigorous logical thinking” and “understand the world” in the Mathematics definition.

A common error is to assume that students who have done Mathematics will necessarily be mathematically literate. When I was giving workshops for first year Science students, people would ask “But why? Don’t they all do Maths 1?” They do, but some of them still do not know how to calculate a percentage increase or how to interpret a stacked bar chart (because this is not done in the school or university Mathematics curriculum). However, even if they did have these skills they would not have learned anything in Maths 1 about data analysis and interpretation. In addition, the Mathematics curriculum includes very little practice within life-related (or other) contexts and does not focus on the critical engagement mentioned above.

**The importance of mathematical literacy for higher education: Why do we teach it?**

The University of Cape Town, in a concept paper *Enhancing the Quality and Profile of UCT Graduates* (2009) says, “We will seek opportunities within existing curricula to strengthen the basic ‘literacies’ – academic, quantitative and information – both as a means of improving learning capacity and as a contribution to informed citizenship.”

This last quote points to two reasons for teaching mathematical literacy:

- **Improving learning capacity.** In other words helping learners and students to cope with the quantitative demands of their programmes of study.
- **Informed citizenship.** We want learners and students to develop the skills to make informed decisions in society. This in particular also links to the idea of developing a critical capacity.

Another three purposes we have for teaching quantitative literacy are:

- To prepare students for the quantitative demands of their professional life and work.
- To enable students to manage their everyday lives. For example, they need to be prepared to deal with personal finances and make decisions about health management.
To use quantitative data and information to sensitize our students to the extensive social problems in our country.

The only one of these purposes that is not identified in the section under the heading “Purpose” in the FET curriculum statements for Mathematical Literacy is that of preparing students for higher education (unlike in the equivalent section in the Mathematics curriculum statement). What I want to argue here is that school Mathematical Literacy, properly taught according to the curriculum, goes a long way towards meeting this purpose as well. For this reason we in the Numeracy Centre were excited when Mathematical Literacy was introduced as a subject, because we felt it was a very positive step towards promoting our goals of having quantitatively literate students and graduates.

Examples of quantitative literacy from the first year university curriculum

I will use three examples from the first year curriculum in various disciplines to illustrate some points about the importance of Mathematical Literacy in preparing students for higher education.

Example 1 is from the Speech and Hearing Science first year curriculum. Students on this course have not necessarily done Mathematics at school and do not do a mathematics course at university:

**EXAMPLE 1:**

This graphical representation is introduced to first year students of audiology and speech therapy and underlies the understanding of some of the most fundamental concepts in the study of the science of human hearing. It is a two-dimensional representation of a three-dimensional surface (using non-linear scales) and students in first year are expected to use it to understand and explain the ways in which the human ear reacts differently under different circumstances. This assumes that students can interpret the meaning of a graph that is changing in slope and that they have a flexible understanding of the idea of rate of change. This is a competency that is required in many different disciplines and it is very important that Mathematical Literacy teachers do not focus only on linear graphs. They need to help their learners develop a good intuitive understanding of the idea of rate of change and how it relates to slope on a curve.

Example 2 is from the first year Law curriculum:

**EXAMPLE 2:**

Extract from judgment in: **Government of South Africa v Grootboom 2001 (1) SA 46(CC)**

“The housing shortage in the Cape Metro is acute. About 206 000 housing units are required . . . the number of shacks in this area increased by 111% during the period 1993 to 1996 and by 21% from then until 1998.

. . . The scope of the problem is perhaps most sharply illustrated by this: about 22 000 houses are built in the Western Cape each year while demand grows at a rate of 20 000 family units per year. The backlog is therefore likely to be reduced . . . only by 2 000 houses a year.”
Students are introduced to the Grootboom case in their first year. The significance of this case is that it was the first in which a decision was made by the Constitutional Court on the so-called 'socio-economic rights', which are rights such as the right to housing and education that are relatively unique to our Constitution. Even this very brief extract from the judgment contains a number of tricky quantitative concepts (such as 111% increase, compounding effect of consecutive % increases, growth rates). Percentage increase is a concept that many students really struggle with (as well as the idea of using a growth factor). It is really important that they encounter this idea in contexts like population growth and not only as interest on investments and loans.

Example 3 is a typical reading task from first-year Medical students’ prescribed readings:

**EXAMPLE 3:**
Concerning victim gender and age, Table 1 shows that for both homicide and suicide, males predominate over females in almost all age ranges. In line with universal trends homicidal violence shows a sharp increase at age 15 and remains high into the mid-40s, whereafter there is a dramatic decline. In the over 65 year age group more females are victims of homicide but this probably reflects the greater proportion of elderly women in the country.

| Table 1: Homicide and suicide by gender and age, first quarter, 1999 |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| **Age ranges**  | **Males**       | **Females**     | **Males**       | **Females**     |
|                 | (% of total)    | (% of total)    | (% of total)    | (% of total)    |
| < 15            | 10 (76.9%)      | 3 (23.1%)       | 1 (50.0%)       | 1 (50.0%)       |
| 15 – 24         | 166 (91.2%)     | 16 (8.8%)       | 22 (91.7%)      | 2 (8.3%)        |
| 25 – 34         | 231 (91.3%)     | 22 (8.7%)       | 23 (85.2%)      | 4 (14.8%)       |
| 35 – 44         | 141 (82.9%)     | 29 (17.1%)      | 11 (68.8%)      | 5 (31.2%)       |
| 45 – 54         | 48 (84.2%)      | 9 (15.8%)       | 9 (81.8%)       | 2 (18.2%)       |
| 55 – 64         | 20 (87.0%)      | 3 (13.0%)       | 8 (100%)        | 0 (0%)          |
| 65+             | 3 (42.9%)       | 4 (57.1%)       | 5 (62.5%)       | 3 (37.5%)       |

This example includes some text and a table that the text refers to. It makes use of quantitative words and phrases that students are assumed to understand such as “predominate”, “sharp increase” and “dramatic decline”. The last sentence draws on an understanding of the relationship between numbers and proportions, something that we find students have enormous difficulty with.

This table also illustrates the need for a student to be familiar with percentages and to be able to interrogate the table critically and to ask questions like “What are the percentages percentages of?” A student needs to be able to formulate a statement like (referring to the percentage at the top left) “76.9% of the homicides of people under 15 years in age were males” (not 76.9% of the male homicides were under 15” or “76.9% of under 15 year old males were homicides” or some other variation).

The above three examples demonstrate clear links between the competencies learners should develop in Mathematical Literacy and the quantitative demands of many disciplines studied in higher education. But what about Mathematics? Does it prepare them to cope with the demands of examples like the ones shown above? I believe that if Mathematics is taught and assessed in accordance with the high ideals in the original curriculum statement, then a student graduating with Mathematics would be highly mathematically literate, but in my experience this is often not the case.

I have worked with students who, having done Mathematics in Grade 12, then pursue further studies which make quantitative demands that do not draw directly on their mathematics knowledge. For these students, Mathematical Literacy could have been a more suitable preparation. On the other hand, many students in disciplines which do require Mathematics as preparation still face quantitative requirements in their studies for which Mathematics has not prepared them, particularly in the area of data analysis.
Some suggestions for how to prepare students for the quantitative demands of higher education

In my opinion the most important thing a Mathematical Literacy or Mathematics teacher can do to prepare students for the quantitative demands of Higher Education is to strive to achieve the more idealistic goals of the curriculum. The curriculum statements for both Mathematics and Mathematical Literacy are full of really wonderful ideals and objectives. There are statements about learners developing communicative competence, learning through authentic activities, doing problem solving, developing sensitivity to societal concerns, and above all learning how to think critically. There are so many pressures in everyday teaching that discourage these things, such as the learners themselves and especially the nature of the final assessment, but my advice would be continuously to strive to achieve these more idealistic objectives wherever possible. This can be done by:

- Requiring learners to communicate clearly, using language as well as graphs, symbols and formulae. Especially in Mathematics, they need to learn to use symbols rigorously. For example, learners should never get away with putting an equal sign between two things that are not equal.
- Keeping the contexts real and authentic. This can be complicated and sometimes makes the underlying mathematics more difficult, but in real life problems do not come neatly packaged with only the necessary information (and all of it), with no assumptions to be made and all the necessary data provided in a neatly organised form. Learners need to learn how to organise their approach to problems that are not just artificial “word sums” with all the formulae given.
- Giving problems where there is not always only one right answer. Sometimes real problems have a range of possible solutions and one has to make a choice of solution for a particular purpose.
- Requiring learners always to examine their answers to see if they make sense in the context.
- Teaching data analysis seriously. Many of the quantitative demands of higher education involve data and statistics and in fact all citizens need to be able to critique misuses of data and statistics in society.
- Stressing proportional thinking. Many students really struggle with reasoning with proportions and the different representations there are for proportions. They need to understand that quantities are either absolute or relative and ratios, fractions, proportions, percentages and other expressions of rate are all just different ways of representing relative quantities.
- Fostering critical thinking. Learners and students need to develop the ability to resist persuasive arguments that are not sufficiently supported by fact. They need to learn to ask pertinent, insightful, probing questions that will help them get to the bottom of understanding issues. This is probably the hardest task for any teacher, but ultimately absolutely crucial for the development of informed and thinking citizens!

I have presented these suggestions because I believe that teachers may be interested in and motivated by my observations, but with no intention of implying that I know better than anyone else how to cope with the challenges that real teachers face in real classrooms!

References


