Book Review

Mathematical Thinking in the Lower Secondary Classroom. Edited by Christine Hopkins, Ingrid Mostert & Julia Anghileri. Cambridge University Press, 2016.

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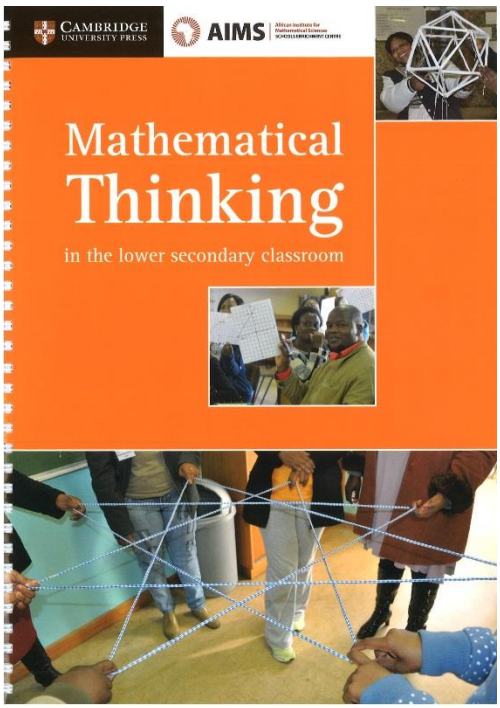
Reviewed by Duncan Samson

Mathematical Thinking in the Lower Secondary Classroom is the joint output of a team of lecturers and teacher trainers from around the world who teach courses in South Africa under the auspices of AIMSSEC, the African Institute for Mathematical Sciences Schools Enrichment Centre. The ideas presented in the book were originally developed for AIMSSEC's flagship Mathematical Thinking course – a 10 day residential course for educators who want to think more deeply about the way learners learn.



The activities described in the book draw on ideas of best practice from around the globe – Europe, USA, India, Australia, New Zealand and Africa – the result being a collection of rich activities that exemplify ways of teaching and learning that are universally relevant. The style of teaching embodied by the book is one of active participation, encouraging learning through problem solving and guided reinvention.

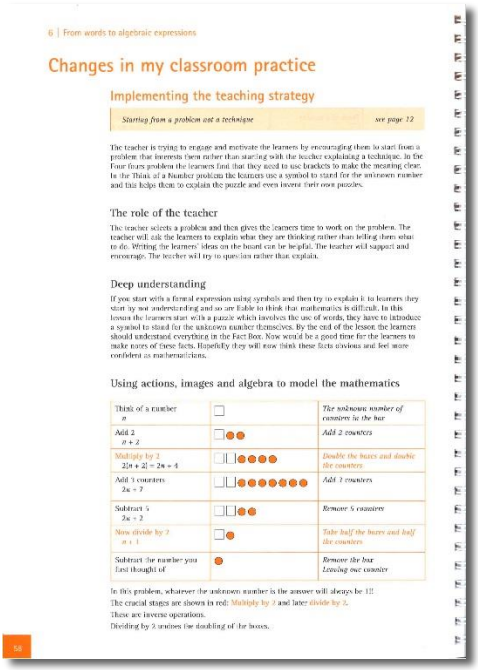
The purpose of *Mathematical Thinking in the Lower Secondary Classroom* is to support teachers in developing a deep understanding of the mathematics they teach, and in developing more effective ways of teaching. The book is structured to encourage professional development through collaboration, exploration, discussion and reflection. The book provides detailed guidance that should enable teachers to run their own teacher development workshops without the need of an expert leader. The underlying philosophy of the book is that teachers who are themselves learners are better equipped to help their students to be successful.

Each of the 20 chapters in the book presents an idea that can be used as the basis of a teacher-driven workshop. Each chapter is split into three parts – (i) *Workshop Activities for Teachers*, (ii) *Classroom Activities for Learners* and (iii) *Changes in my classroom practice*. The first part provides activities that the workshop participants should engage with, explore and discuss. The second part provides classroom activities that teachers can utilise in the planning and preparation of their lessons. The third part focuses on particular teaching strategies that complement the various activities – these include focusing on practical, hands-on and visual learning styles, the benefit of starting from a problem rather than a technique, as well as practical suggestions regarding questioning, classroom discussion, and getting feedback from learners.

Upon completion of the workshop each teacher then carries out the planned classroom activities with their own learners. A useful follow-up is then to begin the next professional development session with a discussion and critical reflection on the combined experiences of the group. This could perhaps be used to refine and extend the classroom activities. Importantly, having a formal follow-up develops the important habit of becoming a reflective as well as reflexive educator.

The authors of *Mathematical Thinking in the Lower Secondary Classroom* have extensive experience of teaching in developing countries around the globe. As such, one of the hallmarks of the book is the promotion of ideas that make use of readily available and cost-free resources.

While the primary idea of the book is to promote and support collaborative teacher-run workshops through the development of communities of practice, it is still an extremely useful book to delve into as an individual. Whether a fledgling teacher new to the profession or an established practitioner with years of experience, every mathematics educator is sure to find something of value in this useful book.



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Changes in my classroom practice

Implementing the teaching strategy

Starting from a problem not a technique *see page 12*

The teacher is trying to engage and motivate the learners by encouraging them to start from a problem that interests them rather than starting with the teacher explaining a technique. In the Four fours problem the learners find that they need to use brackets to make the meaning clear. In the Think of a Number problem the learners use a symbol to stand for the unknown number and this helps them to explain the puzzle and even invent their own puzzles.

The role of the teacher

The teacher selects a problem and then gives the learners time to work on the problem. The teacher will ask the learners to explain what they are thinking rather than telling them what to do. Writing the learners' ideas on the board can be helpful. The teacher will support and encourage. The teacher will try to question rather than explain.

Deep understanding

If you start with a formal expression using symbols and then try to explain it to learners they start by not understanding and so are liable to think that mathematics is difficult. In this lesson the learners start with a puzzle which involves the use of words, they have to introduce a symbol to stand for the unknown number themselves. By the end of the lesson the learners should understand everything in the Fact Box. Now would be a good time for the learners to make notes of these facts. Hopefully they will now think these facts obvious and feel more confident as mathematicians.

Using actions, images and algebra to model the mathematics

Think of a number n	☐	*The unknown number of counters in the box*
Add 2 $n + 2$	☐●●	*Add 2 counters*
Multiply by 2 $2(n + 2) = 2n + 4$	☐☐●●●●	*Double the boxes and double the counters*
Add 3 counters $2n + 7$	☐☐●●●●●●●	*Add 3 counters*
Subtract 5 $2n + 2$	☐☐●●●	*Remove 5 counters*
Now divide by 2 $n + 1$	☐●	*Take half the boxes and half the counters*
Subtract the number you first thought of	●	*Remove the box* *Leaving one counter*

In this problem, whatever the unknown number is the answer will always be 1!!
The crucial stages are shown in red: Multiply by 2 and later divide by 2.
These are inverse operations.
Dividing by 2 undoes the doubling of the boxes.

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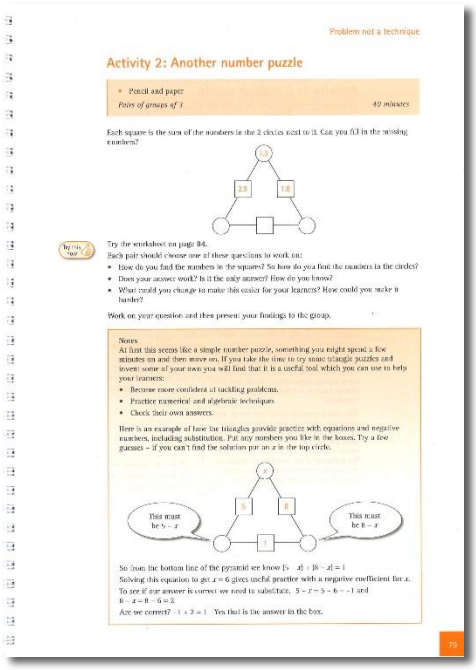

Problem not a technique

Activity 2: Another number puzzle

- Pencil and paper

Pairs of groups of 3 *40 minutes*

Each square is the sum of the numbers in the 2 circles next to it. Can you fill in the missing numbers?

Try the worksheet on page 84.

Each pair should choose one of these questions to work on:

- How do you find the numbers in the squares? So how do you find the numbers in the circles?
- Does your answer work? Is it the only answer? How do you know?
- What could you change to make this easier for your learners? How could you make it harder?

Work on your question and then present your findings to the group.

Notes

At first this seems like a simple number puzzle, something you might spend a few minutes on and then move on. If you take the time to try some triangle puzzles and invent some of your own you will find that it is a useful tool which you can use to help your learners:

- Become more confident at tackling problems.
- Practice numerical and algebraic techniques.
- Check their own answers.

Here is an example of how the triangles provide practice with equations and negative numbers, including substitution. Put any numbers you like in the boxes. Try a few guesses – if you can't find the solution put an x in the top circle.

This must be $5 - x$

This must be $8 - x$

So from the bottom line of the pyramid we know $|5 - x| + |8 - x| = 1$

Solving this equation to get $x = 6$ gives useful practice with a negative coefficient for x.

To see if our answer is correct we need to substitute. $5 - x = 5 - 6 = -1$ and $8 - x = 8 - 6 = 2$.

Are we correct? $-1 + 2 = 1$. Yes that is the answer in the box.

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While I was working through *Mathematical Thinking in the Lower Secondary Classroom* I was serendipitously preparing my own Grade 8 workbook for the upcoming term. I found some the ideas in the book pertaining to the introduction of algebra particularly useful, and I have adapted and developed some of these into what I hope will be a meaningful first glimpse for our Grade 8s into the value of algebra as a mathematical idea.

In addition to *Mathematical Thinking in the Lower Secondary Classroom*, aimed specifically at Senior Phase teachers, companion volumes for FET teachers (*Mathematical Thinking in the Upper Secondary Classroom*), for Intermediate Phase teachers (*Mathematical Thinking in the Upper Primary Classroom*) and Foundation Phase teachers (*Mathematical Thinking in the Lower Primary Classroom*) are in preparation.

For further information about AIMSSEC and its various activities visit http://www.aimssec.ac.za. Follow the Professional Development link for information about the various courses offered by AIMSSEC, or follow the Resources link for learning activities, interactive mathematical challenges, forums for teachers to exchange ideas, as well as selected further reading.