Using Different Representations to Compare and Order Fractions

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INTRODUCTION

With reference to the CAPS document for the Intermediate Phase, one of the first aspects to consider when teaching fractions is that learners need to be able to compare and order common fractions with different denominators (up to at least eighths in Grade 4, and up to at least twelfths in Grade 5). In this article I discuss a number of different representations which I have found useful for learners as they engage with these types of fraction tasks.

SEQUENCING OF FRACTION RELATIONSHIPS WHEN COMPARING

Before discussing the representations themselves, something to keep in mind is the value of comparing two fractions in a sequence of increasing complexity. This way learners can gradually build their conceptual understanding of fractions. A useful sequence is the following:

1. Both fractions are unit fractions, e.g. $\frac{1}{5}$ and $\frac{1}{8}$.
2. Both fractions have the same denominator, e.g. $\frac{2}{5}$ and $\frac{4}{5}$.
3. Both fractions have the same numerator, e.g. $\frac{4}{5}$ and $\frac{4}{6}$.
4. One fraction is smaller than a half while the other is greater than a half, e.g. $\frac{4}{9}$ and $\frac{7}{10}$.
5. Both fractions are greater (or smaller) than a half, e.g. $\frac{6}{9}$ and $\frac{7}{12}$.

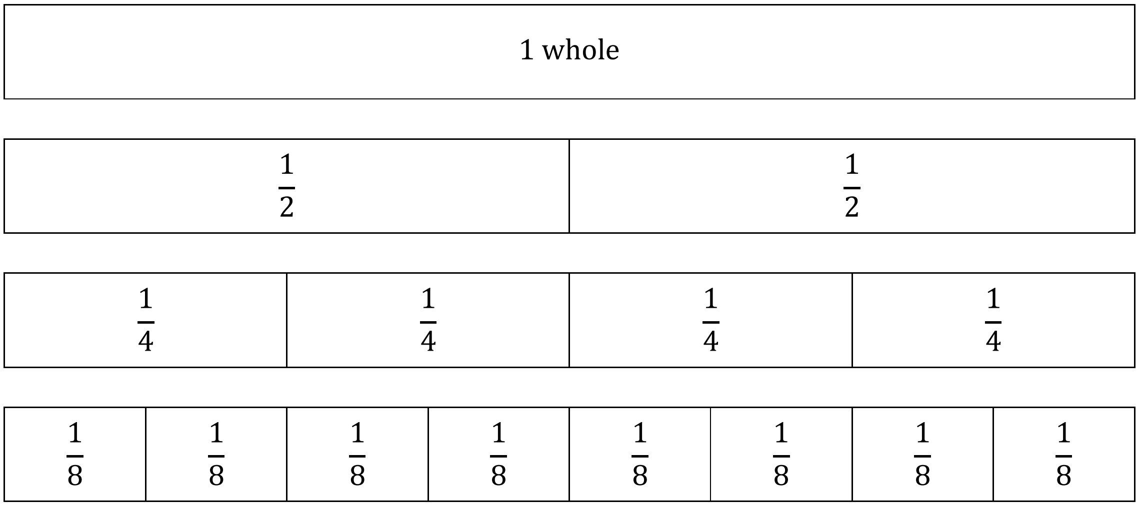
DIFFERENT REPRESENTATIONS

The value of engaging with fractions using different representations is that it allows learners to 'see' fractions in a variety of different ways, thereby strengthening their fundamental conceptualisation of what fractions are. The different representations that I will discuss in this article are:

- Fraction walls
- Bar models
- Number lines
- Equivalent fractions
- Lowest common multiple or lowest common denominator
- Using decimal fractions or percentages
- Fractions as a part of a collection of objects

FRACTION WALLS

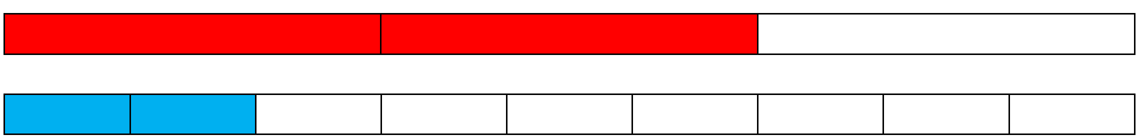
Fraction walls are a very useful way to engage with unit fractions. Most teachers are no doubt familiar with this representation, but it needs to be made more explicit and user-friendly by giving learners the opportunity to physically manipulate the fraction pieces themselves. An A4 piece of paper is ideal when dealing with fractions that are multiples of 2 for example.



It is important for learners to think about unit fractions as **sizes** relative to a whole. For example, if  is $\frac{1}{3}$ of a whole stick, how long will the whole stick be? When working with unit fractions (i.e. numerators of 1), attention should be drawn to the observation that the bigger the denominator the smaller the fraction.

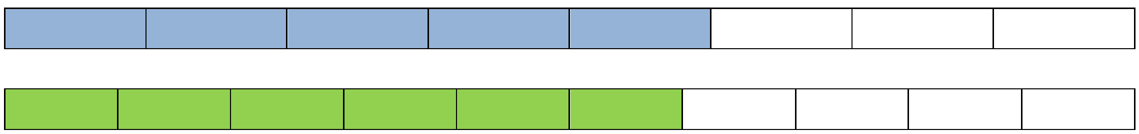
BAR MODELS

Bar models can be used to divide equally sized wholes into different fractions. Each whole is divided into the required number of equal parts, and the two fractions can then be compared visually. This is demonstrated below for $\frac{2}{3}$ and $\frac{2}{9}$, the models clearly showing which of the two fractions is the larger.



Another way to compare two fractions that have the same numerator, as above, is to make the link back to unit fractions. Since $\frac{2}{3} = \frac{1}{3} + \frac{1}{3} = 2 \times \frac{1}{3}$ and $\frac{2}{9} = \frac{1}{9} + \frac{1}{9} = 2 \times \frac{1}{9}$, then since $\frac{1}{3}$ is bigger than $\frac{1}{9}$ it should be clear to deduce that $\frac{2}{3}$ is bigger than $\frac{2}{9}$.

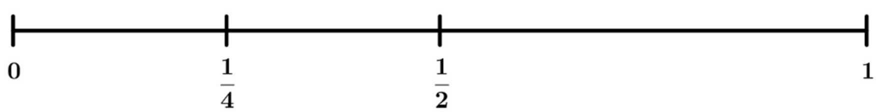
The most difficult fractions to compare are those that have different numerators as well as denominators. Bar models can still be used to compare such fractions. Let us consider the fractions $\frac{5}{8}$ and $\frac{6}{10}$. Learners might initially notice that both fractions are bigger than a half. While this is an important observation, it doesn't help determine which one is the bigger fraction. To do this we take two bars of equal length and subdivide one into 8 equal parts and the other into 10 equal parts. It is important that this is done accurately, so learners should use a ruler to do this. By shading 5 of the 8 equal parts in the first bar, and 6 of the 10 equal parts in the second bar, the model clearly shows that $\frac{5}{8}$ is bigger than $\frac{6}{10}$.



NUMBER LINES

There are many different fraction manipulatives that can be bought or built. Although undoubtedly useful, one drawback of such physical manipulatives is that they have limited flexibility, with only certain subdivisions of the whole being catered for. This is where a simple number line can be a very useful resource to employ since it can be subdivided into any number of equal parts. By using simple reference points on a number line learners can also estimate whether fractions are, for example, bigger than a quarter or bigger than a half.

Take an empty number line and mark off reference points that learners will be familiar with such as 0, $\frac{1}{4}$, $\frac{1}{2}$ and 1.



If learners wanted to compare two fractions with the same denominator, such as $\frac{3}{8}$ and $\frac{7}{8}$, they could use the number line as follows. Building on their knowledge of equivalent fractions and unit fractions, learners could reason that since $\frac{4}{8}$ is equivalent to $\frac{1}{2}$, and $\frac{8}{8}$ is equivalent to 1, it follows that $\frac{3}{8}$ is smaller than $\frac{7}{8}$.

If learners wanted to compare two fractions with different numerators *and* denominators, such as $\frac{3}{5}$ and $\frac{4}{10}$, they could use the number line as follows. Building on their knowledge of doubling and halving fractions, learners could reason that since $\frac{3}{5}$ doubled is more than 1, and since $\frac{4}{10}$ doubled is less than 1, it follows that $\frac{3}{5}$ is greater than $\frac{4}{10}$. One could of course simply convert $\frac{3}{5}$ into its equivalent fraction $\frac{6}{10}$, from which it is clear that $\frac{3}{5}$ is greater than $\frac{4}{10}$. Using the number line is simply a different way of comparing the fractions, and brings something slightly different to the discussion.

EQUIVALENT FRACTIONS

Converting fractions into convenient equivalent fractions is a very useful way of comparing two fractions. If one wanted to compare $\frac{4}{5}$ and $\frac{7}{10}$, one could convert $\frac{4}{5}$ into an equivalent fraction $\frac{8}{10}$, from which it is clear that $\frac{4}{5}$ is the bigger fraction.

A useful way of engaging with equivalent fractions is by using the fraction wall (or bar models) to look for fractions that are equivalent in size. Learners can also focus on patterns to help them find equivalent fractions, for example $\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16}$ where all numerators are multiples of 3 and all denominators are multiples of 4.

LOWEST COMMON MULTIPLE OR LOWEST COMMON DENOMINATOR

Once learners are able to see the patterns in the multiples of numbers being used, the vocabulary of LCM or LCD can be introduced. One can build on these concepts to compare fractions that have different numerators as well as different denominators. To compare $\frac{2}{3}$ and $\frac{3}{4}$ one could convert $\frac{2}{3}$ into $\frac{8}{12}$ and $\frac{3}{4}$ into $\frac{9}{12}$, from which it is clear that $\frac{3}{4}$ is bigger than $\frac{2}{3}$.

USING DECIMAL FRACTIONS OR PERCENTAGES

Learners sometimes feel more comfortable converting common fractions into either decimal fractions or percentages. For example, if we wanted to order the factions $\frac{3}{4}, \frac{5}{8}, \frac{1}{2}$ and $\frac{7}{10}$ we could use standard approaches to convert them into the decimal fractions 0,75 ; 0,625 ; 0,5 and 0,7. Arranged smallest to biggest we thus have $\frac{1}{2}; \frac{5}{8}; \frac{7}{10}; \frac{3}{4}$.

FRACTIONS AS A PART OF A COLLECTION OF OBJECTS

This strategy relies on learners being aware of common multiples, or at least having a strategy of trial-and-error of finding a suitably sized collection of objects. If one wanted to compare $\frac{2}{3}$ and $\frac{3}{5}$, for example, one could decide on a collection of 30 objects (e.g. sweets) and then determine $\frac{2}{3} \times 30 = 20$ and $\frac{3}{5} \times 30 = 18$ thus showing that $\frac{2}{3}$ is bigger than $\frac{3}{5}$.

CONCLUDING COMMENTS

In order for learners to make sense of fractions, they first need to understand that computation with fractions is different from computation with whole numbers. They then need to understand the relative size of fractions and understand the relationship between numerators and denominators (which must be seen together) before the introduction of traditional fraction algorithms. In this article I have described a number of different representations for engaging with fractions. The teaching and use of different representations within a context, e.g. the comparison and ordering of fractions, has the potential to highlight important conceptual connections.