What's the Whole?

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On the facing page is the work of a fourth grade student from a class that I taught in a university town in the Midwest in the United States (US). At the time I was busy completing my doctoral studies and took advantage of an opportunity to teach at a local primary or elementary school. In the US the school year begins during late August and ends in June of the following year. The entry of the 8th December date on the student's work shows that the school year had been going for more than three months. The class had not yet done operations using fractions.

The class had to do the following task:

> Using a 6 × 4 grid as a whole, explain how you will shade different fractions such as $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{12}, \frac{1}{24}, \frac{1}{3}$ and $\frac{1}{48}$

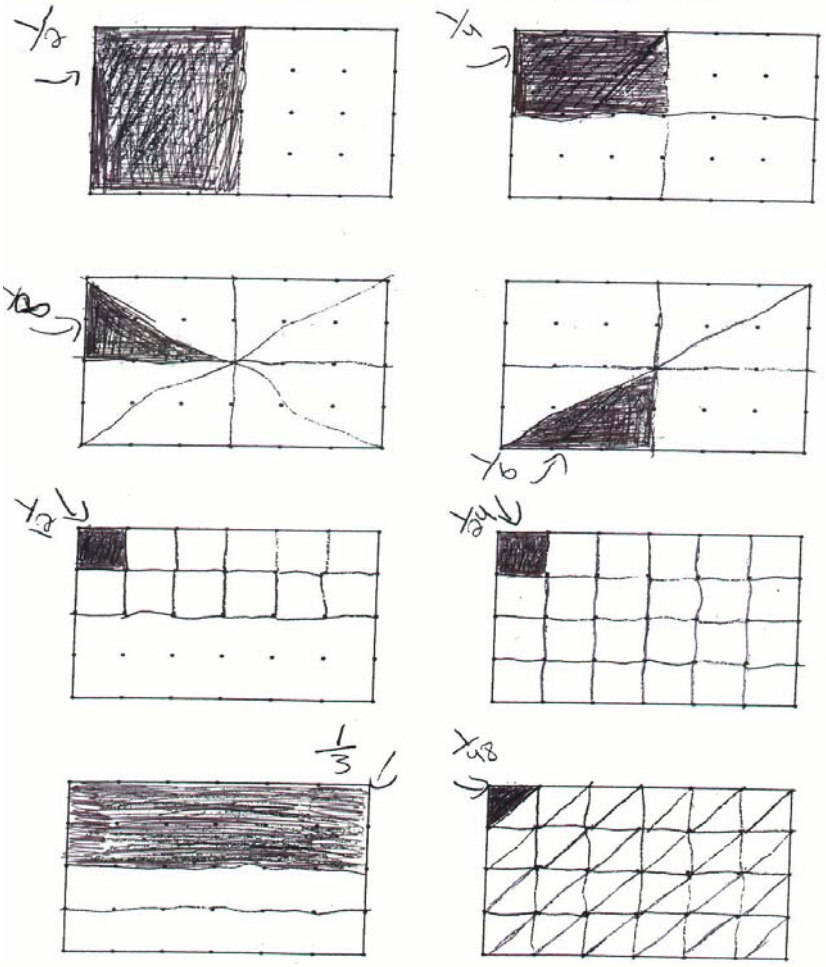
I experimented with making mathematical reasoning central to my teaching. To do this I made use of area models as representations because they are applicable across different grade levels. For example, they can be used to illustrate mathematical ideas in the different learning outcomes in the Revised National Curriculum Statement (RNCS). Teaching and learning fractions via area models, for example, connects the learning outcomes on number and shape and space. Area models are also applicable in the case of geometric probability (data handling)

The student's explanations reveal her insights about fractions. For example, she wrote "you cut the rectangle into four equal pieces and then shade 1," in order to show $\frac{1}{4}$. She did not always consider the 6 × 4 rectangle or grid as the whole.

It is interesting to note that the student at times shows 'flawed reasoning' as opposed to 'poor reasoning' (Russell, 1999, 9). For example, see the case where she shaded $\frac{1}{6}$ and $\frac{1}{3}$.

A noteworthy observation is that she has "tacit knowledge" (Polanyi, 1967) that $\frac{1}{24} = \frac{1}{48} + \frac{1}{48}$. For example, her reasoning: 'You cut the whole rectangle in 24 and then you split the 24 in half and you shade 1 because 24 + 24 = 48' This reveals that she knows more than she can say. She does not know the "rule" $\frac{1}{24} = \frac{1}{48} + \frac{1}{48}$, but is aware of it in a tacit sense.

Fractions on Grids (6-by-4 squares)



12-8-00 Math Test

Using the (6x4) grid's as a whole, Explain how you will shade: (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{6}$ (e) $\frac{1}{12}$ (f) $\frac{1}{24}$ (g) $\frac{1}{3}$

!! Explanations !!

(a) $\frac{1}{2}$: You cut the rectangle in half and shade 1.

(b) $\frac{1}{4}$: You cut the rectangle in 4 equal pieces and shade 1.

(c) $\frac{1}{8}$ You cut the rectangle in 8 equal pieces and shade 1.

(d) $\frac{1}{6}$: You cut the rectangle in 6 equal pieces and shade 1.

(e) $\frac{1}{12}$: You cut the rectangle in 12 equal pieces and shade 1.

(f) $\frac{1}{24}$ You cut the rectangle in 24 equal pieces which is the whole rectangle and shade 1.

(g) $\frac{1}{3}$: You cut the rectangle in 3 equal pieces and shade 1.

(h) $\frac{1}{48}$: You cut the whole rectangle in 24 and then you split the 24 in half and you shade 1, becau 24 + 24 = 48.

Such student work could potentially be used for professional development with teachers. Stimulating questions to ask teachers would be:

Briefly describe what you observe in this artefact (student's work)?
What does this student know? What is your evidence?
What does this student not know? What is your evidence?
How would you improve or change the student's explanations?
Describe how you think the student was taught fractions? What is your evidence?
What have you learned from examining the student's work?

References:

Polanyi, M., 1967. *The tacit dimension*. New York: Doubleday.

Russell S.J., 1999. Mathematical reasoning in the elementary grades. In *Developing Mathematical Reasoning in Grades K – 12*, Yearbook of the National Council of Teachers of Mathematics, edited by Lee V. Stiff. pp. 1 – 12. Reston, Va.: National Council of Teachers of Mathematics.

Schultz J., 1991. Area models spanning the mathematics of grades 3 – 9. *Arithmetic Teacher*, **39**, pp. 392-398.