Double Time

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A clock face is drawn on a sheet of transparent plastic, as shown in Figure 1, so that it can be viewed from either side. If you read the time from one side, can you work out what time would be shown on the other side?

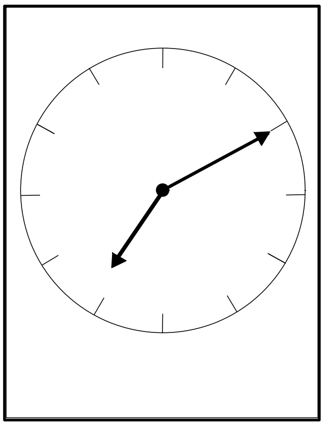

FIGURE 1: A clock face showing 7:10.

Pupils can explore this question by drawing clock faces on paper, and then turning the paper over and holding it up to the light to see what time is shown on the other side. Begin the exploration by considering only on-the-hour times such as 1:00, 2:00 and so on. Pupils could tabulate their observations, and then use the table to try to establish a general rule.

Time on side A	Time on side B
1:00	11:00
2:00	10:00
3:00	9:00
4:00	8:00
5:00	7:00
6:00	6:00
7:00	5:00
etc.	etc.

FIGURE 2: Tabulating the time shown on opposite sides for on-the-hour times.

Pupils should quickly discover that at 6:00 and 12:00 the time shown on both sides is the same. The table also highlights the pattern between the times shown on either side, namely that the two numbers add up to 12. We can thus arrive at a general statement that if the time shown on one side is $h{:}\,00$, then the time shown on the other side will be $(12-h){:}\,00$. Note that this general formula applies for h values from 1 to 12, and that 00:00 is equivalent to 12:00.

Pupils could then conjecture that a similar property holds for the minutes, i.e. that if the time shows m minutes on the one side then it will show $60 - m$ minutes on the other. So, for a time $h{:}m$ on one side we could conjecture that the time shown on the other side would be $(12 - h){:}(60 - m)$. Pupils could then test this conjecture to see if it works. Taking the time 7:10 as an example, the conjecture predicts that the time shown on the other side would be 5:50. However, when we try this out we discover that the time shown on the other side is in fact 4:50. Pupils should be encouraged to try to make sense of why the conjecture doesn't hold. Note that since the hour hand is between 7 and 8 on one side, it will be between 4 and 5 on the other side, so this is where the conjecture falls down. By adjusting the proposed formula for the hour calculation we arrive at the general result that if the time shown on one side of the clock is $h{:}m$, then the time shown on the other side is $(11 - h){:}(60 - m)$.

Pupils can then test this general formula with a number of different times to see if it correctly determines the time shown on the other side. As before, note that a time of $00{:}m$ is equivalent to $12{:}m$. So, for example, if the time shown on one side of the clock is 11:20, then the time shown on the other side is 12:40 – calculated as 00:40 by the general formula, but written as 12:40. However, if the time on one side is 12:40, then the formula calculates the time on the other side to be $-1{:}20$. So, we need one final adjustment, namely that if $h = -1$ as calculated by the formula then we simply add one multiple of 12 to get the correct value of $h = 11$.

We can summarise our results as follows. At 6:00 and 12:00 the time shown on both sides is the same. For other on-the hour times, if the time shown on one side is $h{:}00$, then the time shown on the other side will be $(12 - h){:}00$, and for times of the form $h{:}m$, then the time shown on the other side of the clock is $(11 - h){:}(60 - m)$. All of this with the understanding that an h value of 0 is written as 12, and that an h value of -1 is written as 11.

This is a fascinating hands-on activity to carry out with pupils, and it incorporates important aspects of mathematics such as investigating, data collection and organisation, spatial reasoning, pattern recognition, generalisation, testing and modifying conjectures, and the consideration of special cases.

FIGURE 3: An example of a see-through clock.