Long Division: A Conceptual Approach using Bowls and Coins

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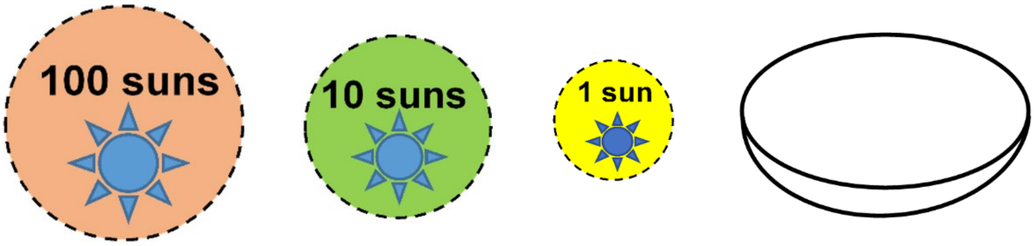
INTRODUCTION

Mastering mathematical skills and developing a sound understanding of fundamental mathematical properties and relationships between concepts is one of the most important didactic goals in teaching primary school mathematics. When teaching procedural techniques (algorithms) it is important that we make the links to previous knowledge clear in order to ensure a sound conceptual understanding of the algorithm. In this article we present a hands-on approach to introducing the long division algorithm which clearly demonstrates the mathematical foundations of the technique. This article offers clear and didactic explanations for each step of the long division algorithm by using physical coins and bowls. The coins are representative of the dividend, while the bowls represent the divisor. Illustrating the long division technique through this hands-on approach helps to develop the underlying concepts associated with the long division algorithm so that the decimal structure of the numbers and the arithmetic operations in each stage of the process become clearer.

MATERIALS

For the purposes of this article we will demonstrate the idea using three different coins known as "Suns". The coins, "100 suns", "10 suns" and "1 sun" represent the decimal structure of the dividend. The value of each coin is denoted by its inscription, size and colour. In addition, a number of bowls are used to represent the divisor.

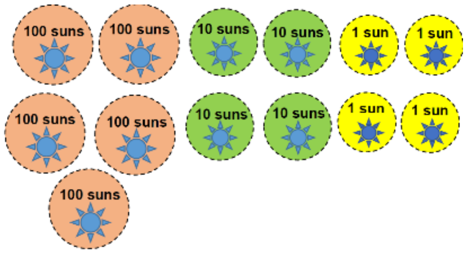


Although the coins, or "Suns", cannot be broken up, they can nonetheless be exchanged for smaller value coins. So, a "10 suns" coin can be replaced with ten "1 sun" coins, and a "100 suns" coin can be replaced with ten "10 suns" coins. For a given calculation, each pupil (or pair of pupils) should be given coins to represent the dividend and bowls to represent the divisor. Pupils will then physically distribute the dividend value (coins) over the divisor (bowls). This hands-on process then serves to enable the transition to the arithmetic long division algorithm. Two examples follow to illustrate the use of the coins and bowls.

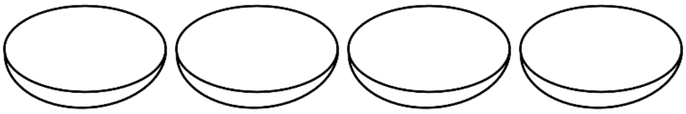
EXAMPLE 1: $544 \div 4$

The "Sun" coins represent the decimal structure of the dividend 544. Thus, we use five "100 suns" coins, four "10 suns" coins and four "1 sun" coins:

$$544 = (5 \times 100) + (4 \times 10) + (4 \times 1)$$

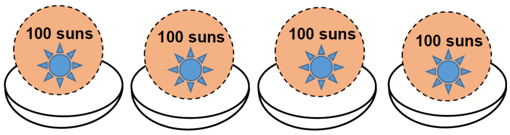


The divisor of 4 is represented by four bowls:



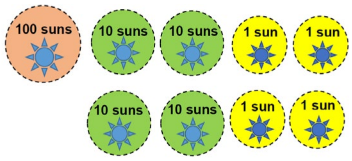
To divide 544 by 4 using coins and bowls, our aim is distribute the 544 Suns evenly between the four bowls. We begin with the highest value (hundreds) and continue through to the lowest value without skipping any digits. If a larger value cannot be divided evenly, then it can be replaced (exchanged) with an equivalent value made up of smaller coins. It is important to maintain the value of the place in the quotient by paying attention to every digit of the dividend in each stage of the process. The following flowchart illustrates the process:

A1:
In the **hundreds stage** we have five 100-Sun coins. We can distribute four of them evenly amongst the four bowls, one coin per bowl.



⇩

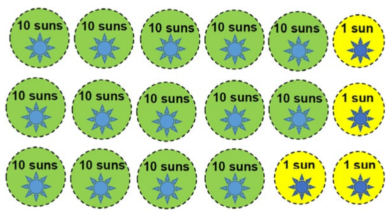
A2:
We now have one 100-Sun coin remaining, plus 44 more Suns: $(1 \times 100) + (4 \times 10) + (4 \times 1) = 144$.
(400 Suns have already been distributed)



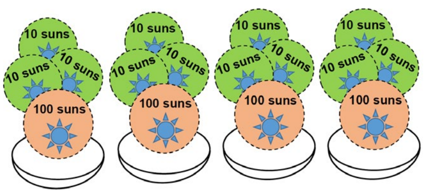
This single 100-Sun coin cannot be distributed evenly between the four bowls, therefore…

⇩

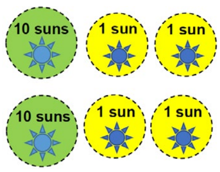
A3:
We replace the 100-Sun coin with ten 10-Sun coins. We now have $(14 \times 10) + (4 \times 1) = 144$ in the dividend.



A4:
In the **tens stage** we have fourteen 10-Sun coins. We can distribute twelve of them evenly amongst the four bowls, three coins per bowl. Each bowl now has 130 Suns.

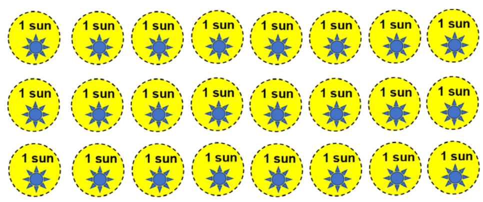


A5:
There are now two 10-Sun coins remaining, plus four 1-Sun coins: $(2 \times 10) + (4 \times 1) = 24$.
(520 Suns have already been distributed)



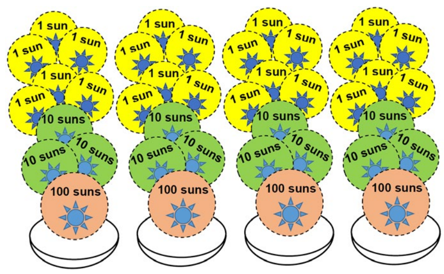
The two 10-Sun coins cannot be distributed evenly between the four bowls, therefore…

A6:
We replace the two 10-Sun coins with twenty 1-Sun coins. We now have 24 Suns (all 1-Sun coins) remaining.



A7:
In the **units stage** we have twenty-four 1-Sun coins. We can distribute them evenly between the four bowls, six coins per bowl.

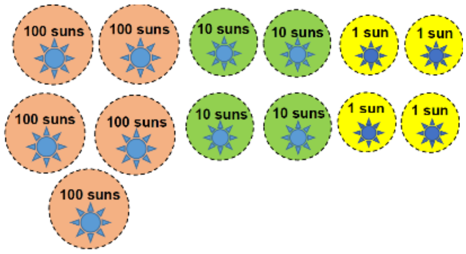
Each bowl now has 136 Suns. This is the **quotient**.



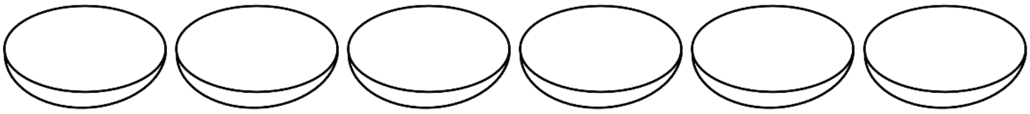
Example 2: $544 \div 6$

The "Sun" coins represent the decimal structure of the dividend 544. Thus, we use five "100 suns" coins, four "10 suns" coins and four "1 sun" coins as before:

$$544 = (5 \times 100) + (4 \times 10) + (4 \times 1)$$



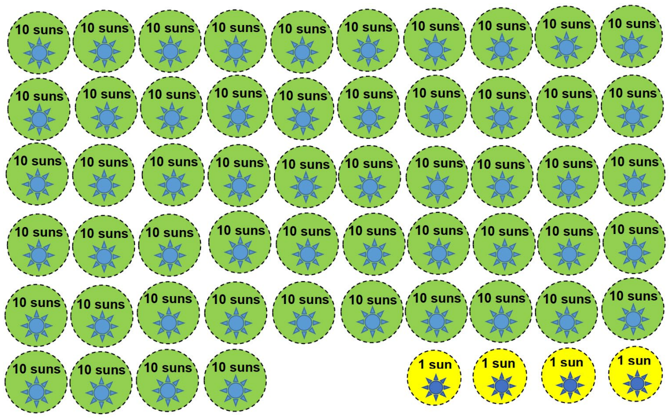
The divisor of 6 is represented by six bowls:



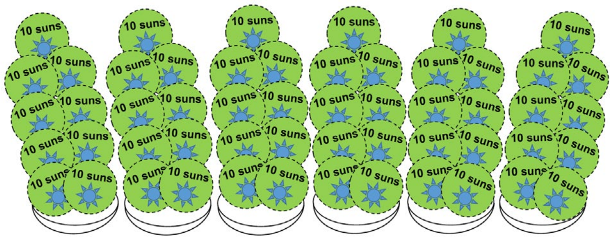
A1:
In the **hundreds stage** we have five 100-Sun coins. We wish to distribute them evenly between the six bowls.

These five 100-Sun coins cannot be distributed evenly between the six bowls, therefore…

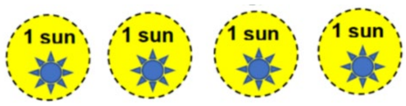
A2:
We replace the five 100-Sun coins with fifty 10-Sun coins: $500 = 50 \times 10$.



A3:
In the **tens stage** we have fifty-four 10-Sun coins. We can distribute all of them evenly between the six bowls, nine coins per bowl. Each bowl now has 90 Suns.



A4:
There are now four 1-Sun coins remaining.
(540 Suns have already been distributed)



A5:
In the **units stage**, the four remaining 1-Sun coins cannot be evenly distributed between the six bowls.

Each bowl has 90 Suns, and this represents the **quotient**.

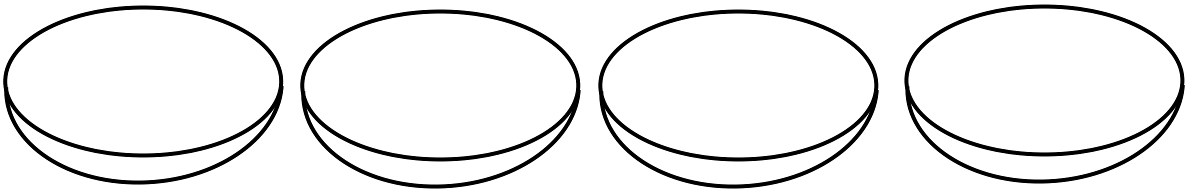
The 4 Suns that are left over represent the **remainder.**

CONCLUDING COMMENTS

The approach of using coins and bowls as a physical counterpart to the more formal long-division algorithmic procedure is a powerful way to introduce long division to primary school pupils. The bowls and coins help develop the idea of sharing equally, and then determining what is left of the dividend. Replacing coins (that cannot be distributed equally between the bowls) with an equivalent value of smaller coins deepens conceptual understanding that will strengthen the underlying process of the formal long-division algorithm. In the South African context, the bowls and coins could be used as a strategy to informally introduce long division in Grade 4 or 5, and then used to support the arithmetic notation when it is introduced later in Grade 6.

APPENDIX – MATERIALS

Bowls



Coins