OPERATIONS WITH INTEGERS

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This paper looks at replacing the trusted pedagogy about how to operate with integers found in textbooks approved by the Education department with a pedagogy that foregrounds the properties of number as resources to perform the different operations on integers. This pedagogy has been successfully employed in Grade 7 and Grade 8. The learners' ability to do different computations was measured by worksheets and test without the use of a calculator. This approach did not reveal any disadvantages to learner and the analysis of the test revealed and average mark of above 70%.

INTRODUCTION

Many GET mathematics teachers derive their pedagogy from mathematics textbooks approved by the Education Department. These texts generally use the number line, various manipulatives (e.g. red and black counters) and other everyday tasks (e.g. ascending and descending a ladder), to 'explain' how integers are added, subtracted, multiplied and divided. This pedagogy, whilst having some flaws, is usually applied with such trust. This talk is about performing the operations with integers, in a way that foregrounds the properties of number as resources to perform the different operations.

Having encountered the ideas of using the properties of number as resources for performing the operations on integers, in a GET Mathematics ACE lecture, we successfully employed this pedagogy in Grade 7 (at St. Augustine's Primary School) and Grade 8 (Fairmount Secondary School) mathematics classes. The success was measured by learners' ability to do different computations, using the properties of number as resources. More importantly, the lecture revealed a way in which learners did not have to use procedures or operations instructions like "$-\times-=+$ " and "when adding a negative number and a positive number, subtract the smaller number from the bigger number, and take the sign of the bigger number".

In the talk, we will present the resources, and then show how these were used to perform the different operations, focusing on integer subtraction (due to time constraints).

CONTENT

The following will be displayed, as examples of conversations that usually accompany the dedicated trust in the above textbook pedagogy approved by the Education Department:

- both numbers are positive, so the sum is positive or both numbers are negative, so the sum is negative
- numbers have opposite signs, so the sum is the difference between the numbers, with a sign similar to the number with the 'greater size'

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- both numbers are negative, so the difference is the same as adding its additive inverse (opposite)
- both numbers are positive, so the product or quotient is positive
- both numbers are negative, so the product or quotient is negative

The above exist only because of a trust we have in what we have been taught. We are interested in the 'why' the operations results in an answer that is correct. Somehow the pedagogy is trusted to justify the correct answer. This talk will show how the resources of the number system will justify the correct answer.

We will then indicate that the application of the above pedagogy involves a shift to a different set of numbers to perform the integer operations, and the learners' activity is consequently based on applying intricate procedures rather than on sound mathematical principles and ideas. To elaborate, the following examples of integer subtraction (see Figure 1) will be discussed, with the discussion drawing on the following properties of number:

- The sum of an integer and its additive inverse is 0: *inverse*
- The difference between the same two integers is 0: *every element of* $\mathbb{Z}$ *has its own additive inverse*
- If $a,b,c \in Z$, then $(a+b)+c = a+(b+c)$: *associativity*
- If $a,b \in Z$, then $a+b = b+a$: *commutativity*

Example 1: Subtracting a positive number from a positive number: $8 - 3 = 5$ No problem!	Example 2: Subtracting a positive number from a negative number: $-5 - 3 = ?$ $= -8 + 3 - 3$ $= -8 + 3 - 3$ $= -8 + 0$ $= -8$

Example 3: Subtracting a negative number from a positive number:	Example 4: Subtracting a negative number from a negative number:
$8 - (-3)$ $= 11 - 3 - (-3)$ $= 11 + (-3) - (-3)$ $= 11 + 0$ $= 11$	$-7 - (-5)$ $= (-5) + (-2) - (-5)$ $= (-5) - (-5) + (-2)$ $= 0 + (-2)$ $= -2$

Figure 1: Examples of integer subtraction

The following sample worksheet (adapted for this presentation) was designed to prompt learners to use the properties of numbers as resources in integer subtraction, rather than performing an operation based on an observation. The intended outcome is to instill and develop a way of thinking that does not rely on the above observations. These sample worksheets focuses the participants attention on integers as numbers in their own right and not shifting the mathematics to constitute a repertoire of procedures where little attention is paid to different sets of numbers and their properties. Intuitive notions of numbers are circulating among teachers and learners so it is not unusual for both parties to rely on those notions in order to perform operations with integers. The integers are then transformed to fit the intuitive notions of whole numbers. Learners and educators most probably perform operations on integers like they would with whole numbers. For them the integer is used as an object consisting of two symbols, a digit and an operator and not as one object. In Questions 5 and 6 (refer to Figure 2), it is anticipated that the learners will freely and successfully change the forms of the expressions to facilitate the use of the acquired resources.

SUBTRACTION OF INTEGERS

1.

$$(+4)-(+8)$$
$$=[(-\quad)+(\quad)]-(+8)$$
$$=(-\quad)+[(\quad)-(+8)]$$
$$=(-\quad)-(0)$$
$$=\ldots\ldots\ldots$$

2.

$$(-4)-(+12)$$
$$=[(-\quad)+(+\quad)]-(+12)$$
$$=(-\quad)+[(\quad)-(+12)]$$
$$=(-\quad)-(0)$$
$$=\ldots\ldots\ldots$$

3.

$$(+4)-(-8)$$
$$=[(+12)+(-\quad)]-(-8)$$
$$=(+12)+[(-\quad)-(-8)]$$
$$=(+12)+(0)$$

4.

$$(-9)-(-4)$$
$$=[(\quad)+(\quad)]-(-4)$$
$$=(\quad)+[\qquad\qquad]$$
$$=(\quad)+(\quad)$$
$$=\ldots\ldots\ldots\ldots$$

5. Show that: $(-7)-(-15)=8$.

6. Calculate the value of each of the following expressions.

(a) $(-12)-7$ (b) $(9)-(-13)$ (c) $(5)-(21)$
(d) $(-8)-(-5)$ (e) $(-3)-(-10)-(6)$ (f) $(15)-(7)-(-6)$

Figure 2: Sample Worksheet

CONCLUSION

The purpose of this talk, while not discounting the value of resources like the number line and counters to introduce learners to the integer operations, indicated that the operations could be performed by foregrounding the properties of number as primary mathematical resources instead of expecting learners to conduct their mathematical activity based on a large number of observations.

At St. Augustine's Primary school, a Grade 7 of 2012 class struggled to complete integer computations by using the observations presented by the teacher. However, after re-teaching integer operations, using the properties of number as resources, the learners exhibited greater competence and more successfully completed assigned tasks. Calculators were not used.

At Fairmount Secondary School, the series of integer operation lessons were taught during the first term of 2013. The 'integer operation worksheet' activities were followed by a mixed operations worksheet and a test. The activities served to consolidate computation skills and to assess the success of this approach. The worksheet and test were done without the use of a calculator and the learners were encouraged to use fewer intermediate steps. This approach has not revealed any disadvantages and the analysis of the test revealed an average mark above 70%. Other Grade 8 teachers at the school also used this approach and reported a marked improvement in learner competence.

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