USING WOODEN CUBES TO HELP LEARNERS INVESTIGATE NUMBER PATTERNS AND TO UNDERSTAND SUM OF TERMS OF AN ARITHMETIC SERIES

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The number patterns, sequences and series topic is one of the ten main content areas of the FET mathematics curriculum. According to the requirements of Curriculum and Assessment Policy Statement (CAPS) FET Band, Grade 12 learners are expected to identify and solve problems involving number patterns that lead to arithmetic and geometric sequences and series, including infinite geometric series. In this presentation we will demonstrate how Grade 12 learners used wooden cubes to investigate patterns. This also supported them to understand the sum of terms of an arithmetic series.

WHY WE CHOSE TO TALK ABOUT THIS?

One of the specific aims of the curriculum is to provide a meaningful opportunity to develop in learners the ability to be methodical, to generalize, make conjectures and try to justify and prove them. We chose this activity because we found that it not only helped our learners with finding the sum of terms of an arithmetic series, but it also helped them to understand why the method works. With this activity learners do not simply 'plug in' numbers in a formula without understanding, but are engaged in meaningful application of the sum concept. Through this activity, we are hoping to expose learners to methods of finding the sum of terms of an arithmetic series, other than the one prescribed in most mathematics textbooks. According to the diagnostic report of the 2011 National Senior Certificate Examination most learners struggled with questions involving both arithmetic and geometric series. Suggestion for improvement according to this report is that learners undertake investigations to enhance their understanding of number patterns, sequence and series. Research (Moyer, 2001; Chiphambo, 2011) shows that the use of physical manipulatives support knowledge construction when used appropriately. We hoped that the use of wooden cubes will support learner's understanding of patterns, sequences and series.

The worksheet we used and given below was designed by Dr Duncan Samson and was workshopped with participating teachers of the FRF Mathematics Education Chair based at Rhodes University.

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INVESTIGATION 1

Gauss's method is a fast and efficient way of determining the sum of an arithmetic series. Let's illustrate the method using the sum of the first 100 natural numbers, i.e. the series 1 + 2 + 3 + 4 + … + 98 + 99 + 100. Gauss's method involves writing down the sequence to be summed *twice*, once *forwards* and once *backwards*, carefully aligning the terms as follows:

$$\begin{array}{ccccccccccccc} 1 & + & 2 & + & 3 & + & \ldots & + & 98 & + & 99 & + & 100 \\ 100 & + & 99 & + & 98 & + & \ldots & + & 3 & + & 2 & + & 1 \end{array}$$

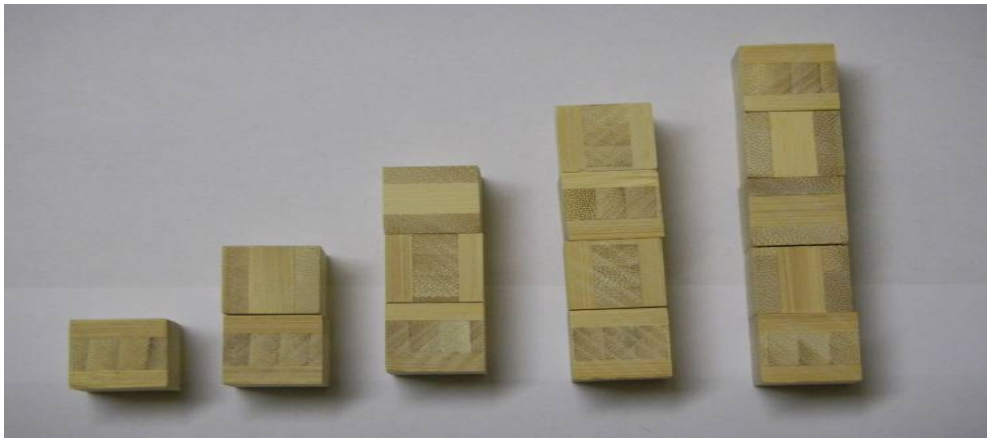
The two terms in each column are then added, giving a sum of 101 for each of the 100 columns. This gives a total tally of 10100. However, this is double the required answer, so the sum of the original sequence is 5050.

$$\begin{array}{ccccccccccccc} 1 & + & 2 & + & 3 & + & \ldots & + & 98 & + & 99 & + & 100 \\ 100 & + & 99 & + & 98 & + & \ldots & + & 3 & + & 2 & + & 1 \\ \hline 101 & + & 101 & + & 101 & + & \ldots & + & 101 & + & 101 & + & 101 \end{array}$$

$$\therefore \quad \text{Sum of first 100 natural numbers} = \frac{101 \times 100}{2}$$

$$= 5050$$

Use the wooden cubes to investigate Gauss's method in order to develop a visual appreciation for why the method works. Begin with the series 1+2+3+4+5 as shown below, and then investigate on your own.



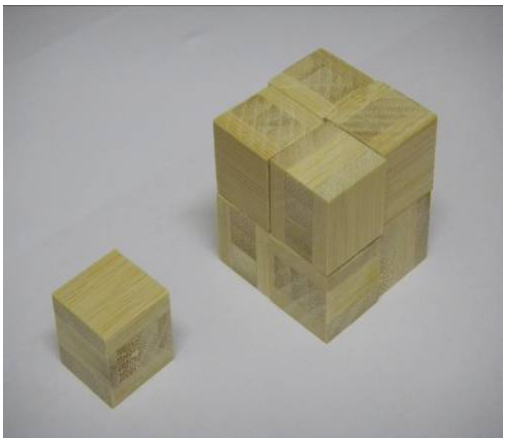
INVESTIGATION 2

(a) Complete the following table:

Column A	Column B
$1^3 + 2^3 =$	$1 + 2 =$
$1^3 + 2^3 + 3^3 =$	$1 + 2 + 3 =$
$1^3 + 2^3 + 3^3 + 4^3 =$	$1 + 2 + 3 + 4 =$

(b) Describe in words any observed patterns.

(c) Use the wooden cubes to investigate the patterns you have observed. An example is shown below to get you started.



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(d) Determine $1 + 2 + 3 + 4 + \ldots + 98 + 99 + 100$ using the S_n formula for an arithmetic series: $S_n = \frac{n}{2}[2a + (n-1)d]$.

Hence determine $\sqrt{1^3 + 2^3 + 3^3 + 4^3 + \cdots + 98^3 + 99^3 + 100^3}$.

(e) Given that $\sum_{i=1}^{n} i^3 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$, prove that $\left(\sum_{i=1}^{n} i\right)^2 = \sum_{i=1}^{n} i^3$.

CONCLUSION

At the time of submitting this abstract, lessons in class were still going on. We will provide a detailed account of how the lessons were conducted, and how the Grade 12 learners experienced this activity in due course.

REFERENCES

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