MATHEMATICAL WORD PROBLEMS WITHOUT REAL CONTEXTS AND MEANING

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In this paper, I discuss my observations and concerns on how grade 9 learners solve word problems without real context and meaning. Issues of the languages used in the mathematics and when making sense of the word problems is briefly explored in relation to problem-solving. The results presented here are part of a wider study that followed a mixed-methods design with the qualitative results informing the quantitative data. For the purposes of this study, data were gathered via administering a test (in both home language [isiXhosa] and language of learning and teaching [English]) and conducting focus groups with few selected learners from a grade 9 mathematics classroom. The main finding of this study is that learners demonstrated tendencies to exclude real-world knowledge and realistic considerations from their solution processes to the task that was given to them in one language than the other.

INTRODUCTION

Problem-solving and integrated assessment are seen as the cornerstones of school mathematics and the Principles and Standards for School Mathematics (National Council of Teachers of Mathematics [NCTM], 2000) called for mathematics instruction and assessment to focus more on conceptual understanding than on procedural knowledge or rule-driven computation (Hamilton 2004; Kilpatrick, Swafford & Findell 2001). Major arguments for including word problems in the school mathematics curriculum have always been the potential role to promote realistic mathematical modelling and problem-solving and for the development in learners of the skills in being aware of when and how to apply classroom mathematical knowledge and everyday-life knowledge when solving problems.

Word problem-solving in school contexts serves as a game under tacitly agreed rules of interpretation (Greer 1997). According to Gatto (1992), these agreed rules are internalised in the learners' minds through the socio-mathematical norm, or hidden curriculum of traditional schooling that could influence many aspects of the intellectual activities in schools. Inoue (2009) suggested that instead of dismissing learners' computational answers, teachers should examine different sets of assumptions for solving word problems. Such a strategy may provide rich opportunities for learners to learn how to use their mathematical knowledge beyond school-based problem solving. Inoue points out that this could help the learners conceptualise word problem solving in terms of meaningful assumptions and conditions for modelling reality, rather than the assumptions imposed by textbooks, teachers, or authority figures.

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Some learners perform poorly in mathematics and have concurrent reading difficulties, whereas others perform poorly in mathematics yet relatively well in reading (Gross-Tsur, Manor & Shalev 1996). Learners may struggle concurrently with reading and mathematics due to weak phonological processing skills (Robinson, Menchetti & Torgensen 2002), whereas mathematics difficulties that occur without concurrent reading difficulties may be due to poor number sense (Robinson et al., 2002). Word problems may be challenging due to the variety of skills needed to solve these problems (Parmar, Cawley & Frazita 1996). That is, to solve a word problem, students must use text to identify missing information, construct a number sentence and set up a calculation problem for finding the missing information (Fuchs, Seethaler, Powell, Fuchs, Hamlett & Fletcher 2008).

SOLVING WORD PROBLEMS

Word problems have been defined differently in different studies. For the purpose of this study, the definition provided by Verschaffel, Greer, and De Corte (2000) as well as Sepeng & Webb (2012) is used. These researchers define word problems as "textual descriptions of situations assumed to be comprehensive to the reader, within which mathematical questions can be contextualised". They also highlight that word problems "provide, in convenient form, a possible link between the abstractions of pure mathematics and its applications to the real-world phenomena" (op.cit.: ix). According to Palm (2009), mathematical word problems include pure mathematical tasks "dressed up" in a real-world situation that require that the learners "undress" these tasks and solve them.

According to Lave (1992), word problems-solving describes stylised representations of hypothetical experiences separated from the students' experiences. In word problem-solving, students' minds could be torn between two types of knowledge systems that the word problem activates – one developed in the traditional mathematics classroom and the other developed through real-world experiences (Inoue 2005).

Socio-cultural and linguistic factors

One of the methodological issues, which socio-cultural approaches have yet to satisfactorily address, arises from the increasingly multicultural nature of mathematics classrooms in South Africa and perhaps elsewhere in the world. Students' interpretations of mathematics classroom interaction relate in part to their different social, cultural and linguistic backgrounds. Analysis of classroom interaction needs to find some way of taking account of this diversity, or it risks imposing a single cultural perspective, that of the researcher. Discursive psychology has the potential to address some of the above-mentioned issues.

Ellerton & Clements (1991) agree that while the process children use to solve word problems are clearly a psycholinguistic concern, much research in this area has been conducted by the persons primarily interested in the cognitive processes of problem-solving, and they have not focussed on the language of the problem or of the problem

solver. The overlap between ‘psycholinguistic’ and ‘problem-solving’ was the subject of comment by Rosenthal & Resnick (1974), who describe word problems in arithmetic as tasks which require the integration of linguistic and arithmetic processing skills. They argue that in word problems, a situation is described in which there is some modification, or combination of quantities.

Riley, Greeno & Heller (1983) demonstrated three information-processing models that simulate children’s different levels of performance on change, combine and compare problems, and applied these models to a sample in North America. This and other similar research (e.g., Adetula 1989; De Corte & Verschaffel 1989) has received acclaim with the result that in the US large teacher professional development programs are based on it. However, Clements and Del Campo (1987) present Australian data which could rarely, if ever, be caused by the strategies hypothesised by Riley et al. (1983), and a similar deduction was obtained by Lean, Clements & Del Campo (1990). Lean et al. showed that the differences in performance were clearly associated with sociolinguistic factors, arising from the questions being in English, which was, for almost all of the Papua New Guinea sub-sample, a second, third or even fourth language. Research (Clements & Del Campo 1987; Harris 1997) has shown that further investigations into cultural factors, including studies of the language and thinking patterns used by parents and teachers when they interact with young children, are needed.

Hater, Kane & Byrne (1974) have pointed out that, although teachers have often assumed that incorrect solutions to word problems have arisen from lack of understanding of mathematical concepts or a deficiency in computing skills, in fact, the errors have been caused by an inadequate understanding of the language of mathematics. Briars & Larkin (1984) presented a model of problem-solving ability that simulates solution performance characteristics. Although somewhat tempered with “set language” and memory resource constraints, the primary mechanisms contributing to solution performance in this model are deficiencies in conceptual knowledge. Unlike Riley et al. (1983), however, this conceptual knowledge includes such things as the ability to understand subset equivalences and the ability to understand that things can be undone in time.

A major source of difficulty with mathematical word problems is the fact that the language of mathematics and the language of common English usage often differ in important ways (Ellerton & Clements 1991). According to Kane (1968) there are four key difference between the two languages: (i) there are fewer redundancies in the language of mathematics; (ii) names given to mathematical objects usually have only a single denotation in mathematical language; (iii) adjectives are usually unimportant in mathematical language; and (iv) the grammar and syntax of mathematical language are far less flexible than is the case for general English. However, despite such differences being well-known, many children still encounter challenges in reading and writing mathematics because not much has been done by teachers to counter this (Durkin 1978).

Teachers' and learners' conceptions of real-wor(l)d problems

This topic of real-world knowledge and realistic considerations in students' solutions of arithmetic word problems has attracted the attention of many researchers in mathematics education. Several studies (Silver, Shapiro & Deutsch 1993; Verschaffel, De Corte & Lasure 1994) have addressed this issue by looking at students' approaches to, and solutions of *non-standard* or *problematic* arithmetic word problems wherein the appropriate solution or mathematical model is neither obvious nor indisputable, at least if one seriously takes into account the realities of the context evoked by the problem statement.

An increasing number of researchers have consistently suggested that current school instruction given for arithmetic word problems is likely to develop in students' tendency to exclude real-world knowledge and realistic considerations from their solution processes (Cooper & Harries 2005).

For many children in elementary school, emphasis has been put on syntax and arithmetic rules rather than treating the problem statement as a description of some real-world situation to be modelled mathematically (Xin 2009). For example, studies (Liu & Chen 2003) conducted on 148 Chinese students from 4th and 6th grade, reported that only one fourth (26%) of the students' solutions of problems were from a realistic point of view (attending to realistic considerations). Almost half (48%) of the responses revealed a strong tendency to exclude real-world knowledge, and in the rest of the cases, no answer was given.

Cooper (1998) offered a different explanation for the reason behind the *unrealistic* solutions, arguing that it stems from the socio-cultural norm of schooling that emphasises de-contexualised, calculation exercises. It is further reported that students tend to give more realistic answers if a real wor(l)d problem is presented as a social studies problem, rather than a mathematics problem (Säljö & Wyndhamn 1993).

According to Inoue (2009), the *unrealistic* solutions may not simply stem from mindless or procedural problem-solving, but could originate in students' diverse efforts to make sense of the problem situation and the nature of the problem-solving activity in socio-cultural contexts. In actual fact, Verschaffel et al. (2000) have suggested that many students whose problem-solving did not seem to reflect familiar aspects of reality are known to defend their answers when their attention is drawn to the issue. Inoue (2005) argues that looking into students' justifications of their seemingly unrealistic answers can inform us of the various ways in which students interpret and make sense of the problem situation as well as the nature of problem-solving activity.

Language and achievement in mathematics

The mathematics achievement gap between English Second Language (ESL) learners and English First Language (EFL) speakers has been well documented (Secada 1992; Strutchens, Lubienski, McGraw & Westbrook 2004). Internationally and in South

Africa, there is no long history of research into the specific mathematics schooling experiences of English second language learners. However, in the past few decades a growing number of scholars in the (mathematics) education community have suggested expanding the sphere of mathematics education research into the socio-cultural arena in order to understand the schooling and mathematics outcomes of these learners more fully (e.g., see Atweh, Forgasz & Nebres 2001; Burton 2003). Such research originates outside the realm of 'traditional' mathematics education research and theory and supports Weissglass' (2002) assertion that the historical contexts and the socio-cultural structures in which mathematics and mathematics teaching and learning are embedded have a significant effect on learners' mathematics learning and performance, especially on those learners who have been historically marginalised (Sepeng 2011).

In South Africa, as in many previously colonised countries in Africa and Asia, there is an added level of complexity in terms of learner achievement in mathematics (Alidou & Brock-Utne 2005; Sepeng & Webb 2012). This added level of complexity hinges on the fact that mathematics is both taught and learned in a second language (English) in a majority of schools in both rural and urban areas (Taylor & Vinjevold 1999). For this reason issues of second language learning of mathematics are an integral part of this study and are discussed below.

REAL-LIFE MATHEMATICAL WORD PROBLEM WITHOUT REAL MEANING AND CONTEXTS

The situation in most parts of South Africa is similar to that in which many Western learners have been challenged by the famous "*shepherd's age*" problem (e.g., '*There are 125 sheep and 5 dogs in a flock. How old is the shepherd?'*(Nesher 1980). Most of these learners would answer "130", as they normally do in the classroom. Similar studies on realistic problem-solving have been conducted in China (see Liu & Chen 2003; Xin, Lin, Zhang & Yan 2007; Xin & Zhang 2009; Xu 2007), where learners are often confronted with word problems such as: '*There were 5 birds on a tree. If one bird was shot down by a hunter, how many birds are left?'.* A more realistic answer to the problem would be 'N*one, because all of the other birds would be frightened away by the sound of the shot'* (Xin 2009), however, most of these learners would give an answer as "4".

METHOD

Both quantitative and qualitative methods were used in this study. The study can be viewed as a mixed-methods design with quantitative data informing the qualitative results (Babbie & Mouton 2008). Quantitative data were gathered from the pre-testing of sense-making problem-solving abilities of learners. Six township secondary schools were chosen as a convenience sample of a cluster of similar schools in a metropolitan city. The sample consisted of ninth grade learners (n=107) from four schools.

Research Design

The study investigated what the situation was in terms of problem-solving abilities of Grade 9 English second language learners with respect to use of personal out-of-classroom mathematics knowledge in solving real-life mathematical word problems without real meaning. The task presented in both isiXhosa (learners' home language) and English (language of instruction) was used to establish how the learners engaged in problem-solving, and what difficulties they may have mathematically in making sense of this problem statement. Then I wanted to see if language used in the task had any effect on learners' problem-solving abilities and sense-making. In doing so, 54 of the 107 learners wrote the test in English and the rest in isiXhosa (EI group), and then vice versa (IE group). After marking the learners responses, 7 learners were purposefully selected to participate in a focus group with the aim of understanding why the solved the task the way they did.

Design Type

Learners were given a real-life mathematical word problem without real meaning, similar to the problems discussed above as indicated in Table 1.

Table 1: Real-life Mathematical Word Problem Without Real Meaning (PWRM)

You have 10 red pencils in your left pocket and 10 blue pencils in your right pocket. How old are you?	Uneepensile ezilishumi ezibomvu kwipokotho yakho yasekhohlo, uphinde ubeneepensile eziluhlaza ezilishumi kwipokotho yakho yasekunene. Mingaphi iminyaka yakho?

RESULTS

The results revealed that learners had a string tendency to relegate real-world knowledge and realistic considerations from their solution processes to this task. The results of the English-isiXhosa (EI) and isiXhosa-English (IE) groups are discussed below. All the learners' responses were classified into four categories based on their written answers and their responses to the clinical interview questions. They are discussed in the next sub-sections.

Quantitative Data

Table 2 shows statistical results of learners' responses to a problematic word problem without real meaning for the 3 items. At a general level, both English and isiXhosa translation produced high percentages, respectively, 86% and 75% situational inaccurate responses. On average, the pre-test results illustrate that 81% of the responses showed a strong tendency to exclude real world knowledge and lack of common-sense understanding. Similar findings have been replicated for a wide variety of problems, across different age levels and socio-cultural settings (see Verschaffel et al. 2000). In fact and as can be seen in this study, learners solved the

'age' problem by a mere use of numbers given in a problem statement. As a result, only 2% and 5% of the responses considered realistic factors of the problem statement in both English and isiXhosa translations respectively.

Table 2: Percentages (and absolute numbers) of learners' responses

Item	Learner's Age	20YRS	Other Answer
English	2%	86%	12%
	(2)	(92)	(13)
isiXhosa	5%	75%	20%
	(5)	(80)	(22)
Total	3%	81%	16%
	(7)	(172)	(35)

Qualitative Data

The EI and IE groups

The data show that learners who wrote the English pre-test first had a stronger tendency to relegate and exclude realistic considerations and real world knowledge in their solution processes. The extracts below show learners' responses to and justifications of their "unrealistic" solutions after being probed.

Extract 1

R(esearcher): How did you approach this question?

L(earner) 3: My age is 20 years old, I added up 10 red pencils and 10 blue pencils and I got the answer 20.

R: OK, what about you L5? How did you approach the question?

L5: I added up 10 red pencils and 10 blue pencils then I got the answer 20.

R: Any other different approach?

Ls: None.

R: Your solution may not work in real life because of real factors. Why did you answer that way?

L6: The question said you have 10 pencils on this side and another 10 pencils on the other side, so I thought because of the question did not ask anything on personal details and then I thought when you add up the pencils from both sides, it bring up the total of your age or something.

L3: It's because the question didn't ask how old you are in real life.

The text in Extract 1 shows that these learners struggled and were unsuccessful to recognise everyday knowledge and their understanding of everyday practices described in the word problem. Cooper (1998) offers a different explanation for the reason behind the "unrealistic" solutions, suggesting that it originates from the socio-cultural norm of schooling that emphasises de-contextualised, calculation exercises. In actual fact, learner 3's argument that "*The question didn't ask how old you are in*

real life" confirms Cooper's view, that the learners' "unrealistic" responses reflect the learners' relationship to school mathematics and their willingness to employ the approaches stressed in school.

Reality in problem-solving

In solving this problem, the grade 9 learners readily responded "*I am 20 years old*", as if their own age could be determined by the reasoning that "*I added up 10 red pencils and 10 blue pencils*". Schoenfeld (1991) characterised this type of problem-solving as "*suspension of sense-making*", referring to the disconnect between learners' understanding of reality and problem-solving. As such, both EI and IE learners' problem-solving was lessened and relegated to a procedural, mechanical task with little or no sense-making beyond the number procedures used in this problem.

Meaning-making of problem statement

All the written responses that reflected the common-sense understanding of everyday practice were categorised as sense-making of problem statement. In extract 1, learner 6 argued that "*So I thought because of the question did not ask anything on personal details and then I thought when you add up the pencils from both sides, it bring up the total of your age or something"*, symbolises Lave's (1992) description of word problem-solving as stylised representation of hypothetical experiences separated from learners' experiences. According to Inoue (2005), learners' minds could be torn between two types of knowledge systems that the word problem activates: one developed in the traditional mathematics classroom and the other developed in through real-world experiences. In this study, the learners who gave calculation answers appeared mindless and mechanical; however, pre-intervention observations into these classrooms revealed that what is really problematic seems to be the lack of the opportunity for the learners to freely bridge the calculation answers and their everyday life knowledge.

Personal interpretation of problem situation

Most of the learners' responses were largely influenced by personal interpretation of the problem statement based on quantitative information that led to word problem-solving resulting in calculation exercises, and not a solution that makes sense in terms of their everyday knowledge and experiences. Most of the learners in the EI and IE groups interpreted the problem situation the same way, with justifications pointing the same direction of reasoning. Extract 1 shows that when the learners answered: "*20 years old*", it was not because they did not know their actual age, or they did not understand the relevant mathematical concept (Frankenstein 2009). Rather, it was, as (Pulchaska & Semadini 1987) suggest, because learners give illogical answers to problems with irrelevant questions or irrelevant data is that those learners believe mathematics does not make any sense.

DISCUSSION

The overall results obtained word problems without real meaning task illustrate that learners' performance on word problems differs dramatically depending on how the problems are designed (Verschaffel, Greer & Van Dooren 2009). In addition, such word problems require more extensive consideration of how the situation (or context) should be modelled, and if the information provided is relevant and sufficient for solving the problem (Säljö, Riesbeck & Wyndham 2009). The quantitative results of this study demostrate that learners have a tendency to respond to the problems even if the information given is irrelevant to answering the question given (Sepeng 2011). In fact, it is interesting to see that intercultural comparison studies show similar findings (Säljö et.al. 2009; Verschaffel et al. 2000; Xin 2009).

A report by Julie & Mbekwa (2005) raises concerns with the way in which the notion of what constitutes a *relevant context* might not be in the same for curriculum developers, teachers, and learners. In other words, the overall results of the study suggest that some of the mathematics word problems used in school curricula are not relevant to and do not address the socio-cultural situations faced by and known to the learners from poor socio-economic backgrounds. Sethole (2004) suggests that foregrounding of context may lead to a loss of focus on the development of conceptual mathematics knowledge and render the mathematics invisible or inaccessible. Contrary to this, the results of this study suggest that with well-planned and effective teacher development interventions, the issue of context and meaning in mathematics teaching may play a pivotal role in the development of learners' problem-solving abilities. The pre-test results indicated that current school instruction given for mathematical word problems is likely to develop in students a tendency to exclude real-world knowledge and/or reality in their solution processes (Cooper & Harries 2005; Greer 1997; Verschaffel, De Corte, Lasure, Vaerenbergh, Bogaerts & Ratinckx 1999).

CONCLUSION

Computational errors made by the learners, in particular number skills, appear to stem from the inability to use language(s) (home and/or LoLT) effectively in order to solve problems in a realistic situation. This finding suggests that mathematics practitioners (i.e., teachers, teacher educators, textbook authors, etc.) should generate mathematics activities that take into consideration the reality of the situations that are related to learners' out-of-school mathematics. Learners should be afforded opportunities to use their everyday life experiences and knowledge when making sense of problems. In doing so, there should be a smoother movement between learners' informal language and formal written mathematical language.

As such, recognition of the role of common sense and out-of-school knowledge in learning mathematics word problems, coupled with considerations of the complementary roles of learners' home language and the language of learning and teaching in multilingual classrooms, appear to be potentially fruitful approaches to

teacher development in the multilingual contexts found in South African mathematics classrooms. In addition, the findings of this study should provide insights for individuals and groups who strive to empower mathematics teachers with innovative and effective pedagogies, particularly those who attempt to assist second language learners to use their everyday life knowledge, experiences, and common-sense understanding freely when solving mathematics word problems.

The data generated via word problems task in this study revealed that an inappropriate use of contexts, particularly when learners are invited to engage in the real world, but then penalised for doing so, results in classroom inequalities (Boaler 2009). As learners learn to answer nonsensical questions about 'number of blue and red pencils in your pocket', they come to believe that mathematics classrooms are strange places in which common sense cannot be used.

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