THE USE OF AN ADVANCE ORGANISER TO IMPROVE MATHEMATICAL LEARNING – MOTIVATION FOR ALL TEACHERS

Sibis Mouton

Civil Engineering, Cape Peninsula University of Technology

Following the constructivist perspective on mathematical learning, this paper wants to show the didactical advantage of using a correct advance organiser to present certain topics in mathematics. By using a correct frame to hang the learner's knowledge on, the learner constructs his/her own knowledge from the activity that is given. Knowledge is then being constructed as a result of the learner's activity.

INTRODUCTION

When modelling is used as a teaching strategy in the classroom it entails that mathematics is developed directly from the context of a practical situation. The purpose of an "Advance Organiser" would be to provide the learner with a conceptual structure into which he or she will then integrate the new material (Bell 1978: 135). David Ausubel developed the theory of advance organiser as part of his Meaningful Verbal Learning. Ausubel defined advance organisers as

> appropriately relevant and inclusive introductory materials … introduced in *advance* of learning … and presented at a higher level of abstraction, generality, and inclusiveness that the information presented after it. The organiser serves to provide ideational scaffolding for the stable incorporation and retention of the more detailed and differentiated materials that follow. Thus, advance organisers are not the same as summaries or overviews, which comprise text at the same level of abstraction as the material to be learned, but rather are designed to bridge the gap between what the learner already knows and what he needs to know before he can successfully learn the task at hand (Ausubel, 1968: 148).

Here two topics in Mathematics will be used to illustrate how the advance organiser can be used to make mathematics more understandable.

THEORETICAL BACKGROUND OF CONSTRUCTIVIST PERSPECTIVES ON LEARNING

Cobb & Bauersfeld (1995: 2) wrote:

> The findings of numerous studies indicate that…many students develop conceptions that deviate significantly from those that the teacher intends.

This implies that the student constructs his/her own knowledge from the situations he/she experiences. Knowledge can be defined here as the mind-set used by a person to handle a situation or to cope with an experience. The knowledge which an individual possesses invariably leads to specific behaviour. This constructivist learning theory is based on the premise that knowledge is the result of the learner's activity rather than the passive reception of information or instruction. This model of

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the nineties is still valid in teaching today and can be a great help in improving learning in the mathematics classroom.

The radical wing of the constructivist front has adopted the revolutionary attitude of Jean Piaget. Von Glasersfeld stated that knowledge is an "adaptive function" (Von Glasersfeld 1991: xiv); our cognitive processes are intended to help us cope with the world of our experience.

This view of knowledge calls for teaching methods that generate activity and cause learners to reflect. Reflection, which should lead to understanding of contents, is necessary for conceptual advancement. Piaget maintained for five decades that all "operative" knowledge (that is, the know-how concerning mental operations) was the result of reflection. Locke, an empiricist, defined reflection as "the mind's ability to ponder its own operations" (Von Glasersfeld 1991: xviii). For the constructivists, reflective ability was a major source of knowledge at all levels of mathematics.

Von Glasersfeld defined learning as self-organization. However, he also acknowledged that this constructive activity occurs during interaction with other members of the community. It follows that verbalization, in the form of group discussions between the teacher and students and amongst students , is an important aspect of this classroom teaching paradigm.

Cobb and Bauersfeld called this environment the "inquiry mathematics classroom" (1995: 3). In this article I would like to call my approach in teaching these specific mathematical topics just that. This environment is then far removed from the normal declarative lecturing of mathematics where imitation and reproduction play a major role. I would like to stress that these inquiry based approaches tend to foster independent and critical thought. Modelling as a teaching strategy also typifies this kind of teaching approach.

MODELLING AS A TEACHING STRATEGY

Modelling as a teaching strategy is derived from mathematical modelling, a process which can be illustrated as in Figure 1 (adapted from Treilibs, Burkhardt & Low (1980: 2)). Modelling as a teaching strategy entails that mathematics is being developed directly from the context of practical situations in the classroom. This teaching strategy could then also call for a freer classroom atmosphere where there is input from both the teacher and the students.

The teaching style of modelling sets as two very important goals the following two aspects (which is similar to those stressed in the inquiry classroom approach):

1) The construction of increasingly powerful conceptual structures.
2) The development of intellectual autonomy.

Appropriate experiences are set up by the teacher, or text in the form of a practical experiment or a real life problem.

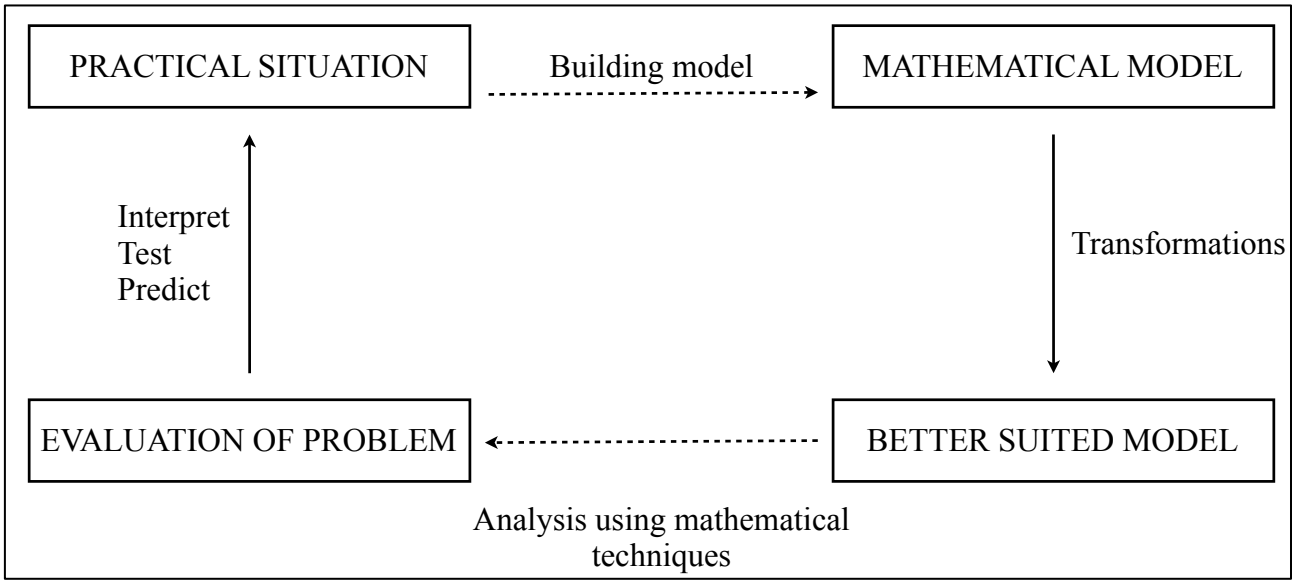

Figure 1: The modelling process

Mathematical knowledge is then developed by the learner either by actively taking part in the experiment and noticing a pattern that develops or by a problem related to real life where reflection on methods of solution leads to the construction of mathematical knowledge.

Starfield, Smith & Bleloch (1990: 28) stated:

> Struggle is a precursor of learning and being temporary perplexed is a natural phase of problem solving.

This is a way of saying that it is impossible to hand over the meaning of a theoretical concept to the student as a concrete object. Concepts cannot be taught directly, students should rather have to construct the meaning themselves.

Modelling as a teaching strategy is analogous to inquiry problem- centred approaches investigated by Cobb et al (1991) and by Olivier, Murray & Human (1992). Olivier, Murray and Human supplemented their theoretical framework by socio-constructivism, where sense-making is seen as an intrinsically social as well as an individual act. All of this is in line with the theoretical aspects of Cobb's emerging paradigm of mathematics teaching called the inquiry classroom. I see this style of the inquiry classroom as very close to the teaching strategy of modelling. Both these styles are inductive and thus it makes the mathematics more alive for the learner (see the quote below).

Lakatos (1976: 142) said:

> Deductivist style hides the struggle, hides the adventure.

And

> All propositions are true and all inferences valid. Mathematics is presented as an ever- increasing set of eternal, immutable truths. Counterexamples, refutations, criticism cannot possibly enter. An authoritarian air is secured for the subject by beginning with disguised monster-barring and

> proof-generated definitions and with the fully-fledged theorem, and by suppressing the primitive conjecture, the refutations, and the criticism of the proof.

USE OF AN ADVANCE ORGANISER

I would like to start this practical teaching methodology with a very apt quote from Bertrand Russell (1927: 89):

> When it comes to algebra and we have to operate with *x* and *y* there is a natural desire to know what *x* and *y* really are. That, at least, was me feeling: I always thought the teacher knew what they were but wouldn't tell me.

The use of the advance organiser is then to help the student in forming the correct constructs which will act as a long term advantage to understand further mathematical concepts.

I want to illustrate the use of the advance organiser in two examples that I currently use in the mathematics classroom at the Cape Peninsula University of Technology in the Civil Engineering Department.

Example 1

The use of a choc chip cookie as an advance organiser for explaining the determination of volume by Integration in our Mathematics 2 classes. As an introduction to this lesson, the students are first asked to work out the volume of the choc chip cookie that they have all received on their desks. This they do by measuring the diameter and the height of the cookie and using the formula for a cylinder where Volume = $\pi r^2 h$. The associated worksheet in shown in Appendix 1.

Ideally the students should work in pairs so as to accord with the principles of the socio-constructivist model of Oliver, Murray and Human. It is worthwhile noticing that the lecturer is leading the students from the known to the unknown. The unknown then becomes familiar and then the student should be able to use the volume of the choc chip cookie which they did beforehand to determine the definite integral as was set out in the worksheet above. In other words, the calculation of the volume of the concrete choc chip cookie acts as an advance organiser for the volume of the cylinders that the students have to calculate in the integration process. Also notice here that the worksheet starts with the revolution around the y- axis which is easier for students to visualize. Lastly, the lecturer is simply the facilitator and the students are supposed to figure out the method for themselves. It is clear that the ‘choc chip cookie’ acts as the scaffolding for the student to do the eventual integration. Thus the use of this advance organiser and the way the worksheet is set up, will help the student tremendously to master the eventual abstract mathematical techniques.

Example 2

The use of an outdoor Trigonometry Project to act as an advance organiser for concepts in the introduction of trigonometry. This project can be used in grade 10 or

11. At CPUT, this project is done with the Extended Curriculum students (not with the mainstream).

This is a didactically sound project to establish the real use and meaning of trigonometry. There are 5 tasks that need to be executed outside in a real life situation. The project is shown in Appendix 2.

USING THE FUNCTION CONCEPT

As a lecturer it is essential to me that we introduce trigonometric functions as a mathematical function that gives us the relationship between an input (the angle) and the output (the ratio). I use as analogy for a function the picture of a Bow and Arrow where the arrow is our input (x) and the target is our output (y). The input is then the domain and the target (the y-values) is the range.

As we know, a mathematical function describes the relationship between two variables in a situation. In 1908 Klein (in Offner (1978: 214)) said:

> We, who are called reformers, would put the function concept at the very centre of instruction, because of all the concepts of the mathematics of the past two centuries, this one plays the leading role whenever mathematical thought is used.

I, as a tertiary lecturer and ex-school teacher, feel that the explicit functional relationship between two variables is best explained to high school pupils and students by dealing with phenomena from their own world, be it an everyday occurrence or one with a scientific connotation. Then at least they know what is x and y and are not confused like Mr Bertrand Russell in the quote given above when he was at school! It is only in studying these kind of relationships between two variables in the classroom, that the *dynamic quality* of variables is adequately illustrated.

To enable learners to use algebraic language with understanding, it is my opinion that a mathematical modelling approach offers dynamic possibilities in order to overcome the dichotomy between language and reality. Harper (1979:294) in his research study on children's interpretation of numerical variables, complained that a talk- and- chalk approach represents mathematics in a static way. The functional relationship of cause and effect between variables is best illustrated with concrete examples and lots of table work where x is the input and y the output.

The input in the case of Trigonometry is an angle and the output is the ratio. The inverse trigonometric function is then easily explained by now wanting to make the output the input and the input the output. One defines the new domain for the angle to ensure that the inverse trigonometric function is indeed a function. So, looking for the solution tan x = 0,34 we have to use the inverse function to find the reference angle $\tan^{-1}(0{,}34) = 18{,}78^{o}$ and then derive all the solutions according to the domain given.

The *outdoor project* above clearly illustrate to learners that when one has the input one can calculate the ratio (or part thereof). It also illustrate practically that when one has the output (that is the ratio) one can obtain the input. This project as such is then an advance organiser; it provides the scaffolding to hang and develop further

knowledge about trigonometric functions. It has been used successfully at CPUT over the last 8 years.

In the case of the logarithmic function the input is a number and the output is a power (normally the power of 10). The inverse logarithmic function can then be defined as the opposite action; now one puts the power in and the number comes out.

In the case of problem solving the student can now easily realize that if for instance x is required then log $10x = 0.321$ is the wrong function; one should have the 10x as the output so the inverse function should be used namely $10^{.321} = 10x$. Or if one has to solve for y and one is given $x = \ln\sqrt{1-e^y}$, then one changes the function every time to get the correct output:

$$\text{since } x = \ln\sqrt{1-e^y},$$
$$2x = \ln(1-e^y),$$
$$\text{which means that } e^{2x} = 1-e^y$$
$$\text{and } e^y = 1-e^{2x},$$
$$\text{so that } y = \ln(1-e^{2x}).$$

It is interesting to note that in a recent study done by Coupland, Gardner and Carmody (2008) at the University of Technology in Sydney, students were asked what mathematics they learnt in engineering was the most useful to them. The two mathematical topics they rated most highly were trigonometric functions and linear algebra. I believe that civil engineering students should have a good understanding of trigonometry and its wonderful applications in everyday life.

CONCLUSION

The two examples of the use of an advance organiser I discussed have not been part of a formal research study. However, my intuition tells me that the use of advance organisers in the mathematical classroom has sound didactical merit. Students who have been exposed to good advance organisers tend to have a better grasp of the mathematical concepts and content that follows such exposure. It might be productive to experiment with this didactical method in the classroom. Such experimentation could provide a bridge for the student to understand mathematical concepts.

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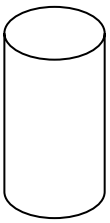
APPENDIX 1

VOLUME CALCULATION – MATHEMATICS 2

EXERCISE FOR LAYING A FOUNDATION

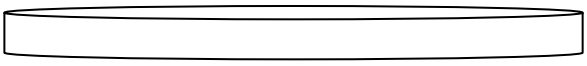
Find the volume of:

1



H (height) = 10 cm
r (radius) = 1 cm

2 H = 2 cm, r = 4 cm



3

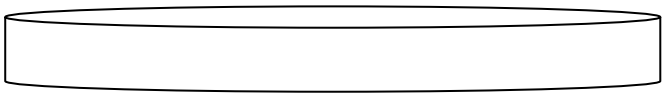
H = 12 cm, r = 5 cm

4.1 Sketch a cylinder in the same position as no 3, but make it with a small height and a BIG radius.

4.2 If H = 2 cm and r = 20 cm, what is the volume?

5 Diameter = 20 cm

H = 3 cm. What is the volume?



6 Diameter = 30 cm

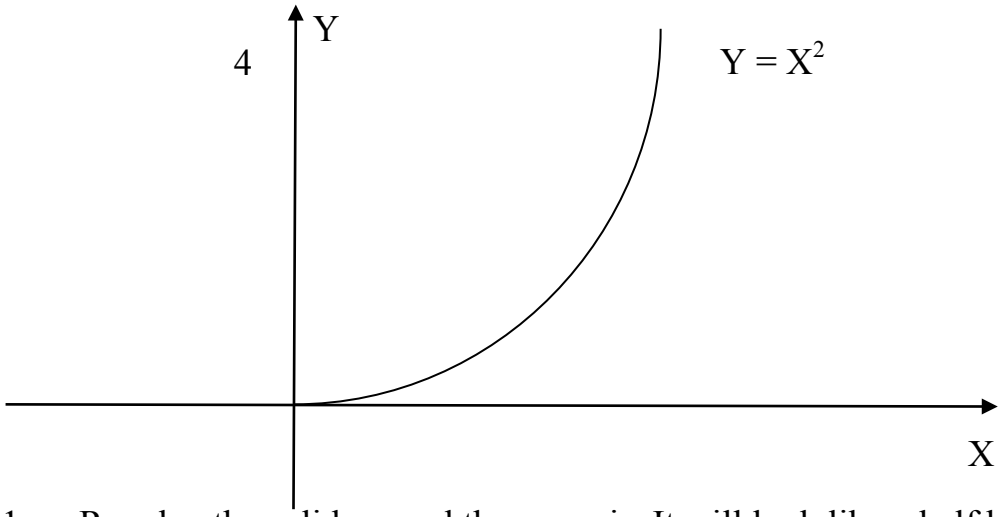
H = 2 cm. What is the volume?

NOTICE: Number 5 is a short cylinder that looks like a choc chip cookie.

NOTICE: Number 6 is also a short cylinder but it is turned on its side. It looks like a choc chip cookie on its side.

WE ARE NOW GOING TO WORK OUT THE VOLUME OF A SOLID REVOLVING AROUND THE Y – AXIS BY FILLING THE SPACE WITH CHOC CHIP COOKIES THAT LOOK LIKE THE ONE IN NUMBER 5

7



7.1 Revolve the solid around the y – axis. It will look like a half ball (show). We can now fill up the space with choc chip cookies, small ones at bottom and bigger ones at top. First we need a top boundary. Draw the line y = 4.

7.2 Now colour in the volume generated between y = 4 and $y = x^2$

7.3 Put in (draw them in) 3 cylinders which looks like 3 of your flat choc chip cookies.

7.4 Volume of one of these choc chip cookies: $d V = \pi r^2 . h =$ ..... (NB: We read the radius (the width) on the x axis. If $y = x^2$, then $x = \sqrt{y}$. The height of one cookie we read on the y- axis; it is small, so it is *dy*)

7.5 Our final integral (which gives us the sum of all the cookies that is the total volume) will be in terms of x or y?

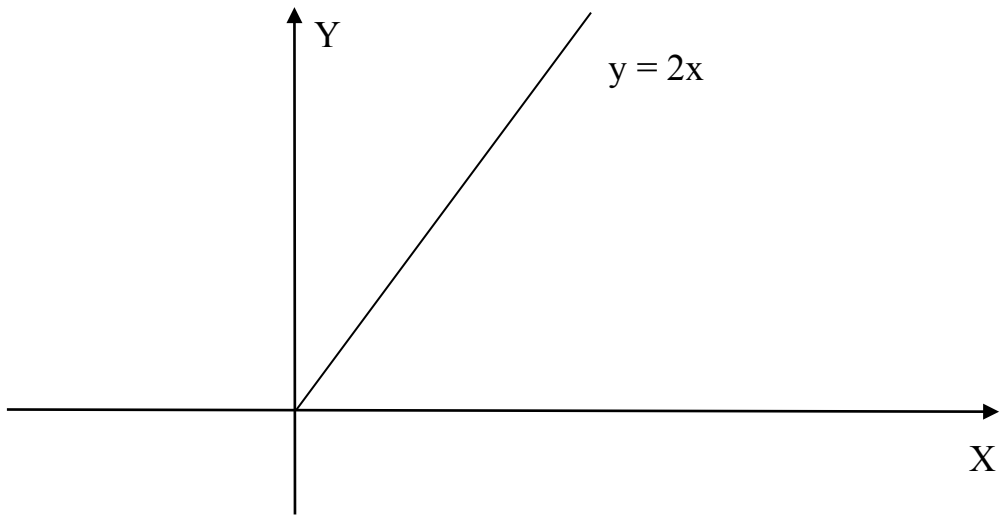
7.6 Give the boundaries of this integral: $\int \pi y \, dy$.

7.7 Now we can work out the total volume by integration of this definite integral. Remember that the integration sign ($\int$)simply means to add up the volume of all the cookies - $\sum$.

[Answer = 8π units^3]

WE ARE NOW GOING TO WORK OUT THE VOLUME OF A SOLID REVOLVING AROUND THE X – AXIS BY FILLING THE SPACE WITH CHOC CHIP COOKIES THAT LOOK LIKE THE ONE IN NUMBER 6, in other words the cookies will now be on their side (have a look again at number 6).

8



8.1 Revolve the solid around the x – axis. Will look like a cone on its side (show). We can now fill up the space with choc chip cookies, small ones on left and bigger ones on right. First we need a boundary on the right. Draw the line x = 3.

8.2 Now colour in the volume generated between x = 3 and y = 2x

8.3 Put in (draw them in) 3 cylinders which looks like 3 of your flat choc chip cookies on their side.

8.4 Volume of one of these choc chip cookies: d V = πr^2 . h = ... (NB: We read the radius (the width) on the y axis. We know y = 2x . The height of one cookie on its side we read on the x - axis; it is small, so it is *dx*)

8.5 Our final integral (which gives us the sum of all the cookies that is the total volume) will be in terms of x or y?

8.6 Give the boundaries of this integral: $\int \pi(2x)^2$ dx.

8.7 Now we can work out the total volume by integration of this definite integral. Remember that the integration sign ($\int$)simply means to add up the volume of all the cookies - $\sum$.

[Answer = 36 π units^3]

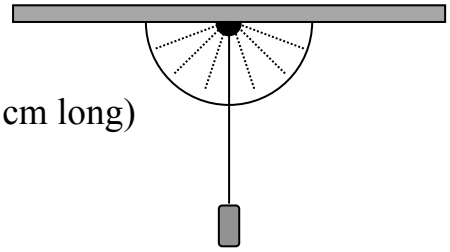
APPENDIX 2

PRACTICAL APPLICATION OF TRIGONOMETRY

This project is to be completed in pairs.

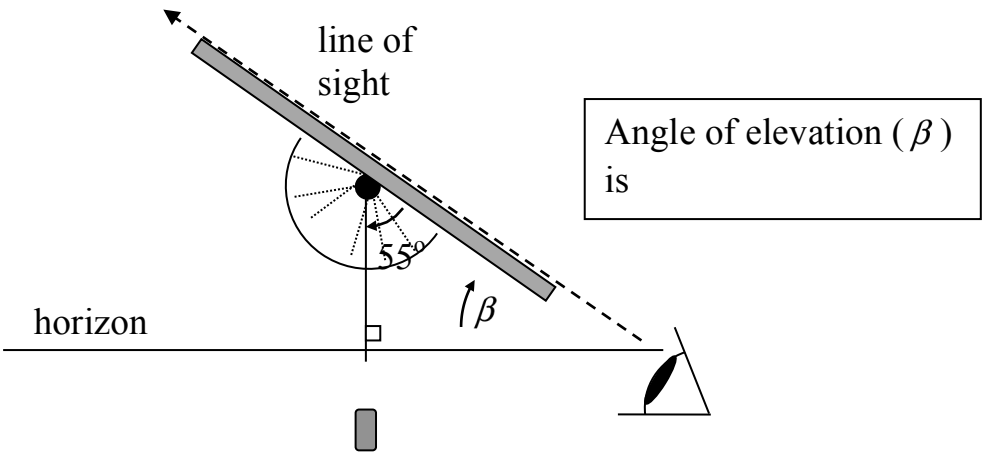
TASK 1: MAKE A CLINOMETER (SIGHTING DEVICE)
EQUIPMENT
You will need …

1. a ruler
2. a protractor
3. a piece of string (15 – 20 cm long)
4. a weight (e.g. an eraser)
5. a piece of Cellotape
6. a piece of Prestik



INSTRUCTIONS
1. Stick the protractor on to the ruler using Cellotape.
2. Tie the weight to one end of the string.
3. Attach the other end of the string to the centre point of the protractor using the Prestik.

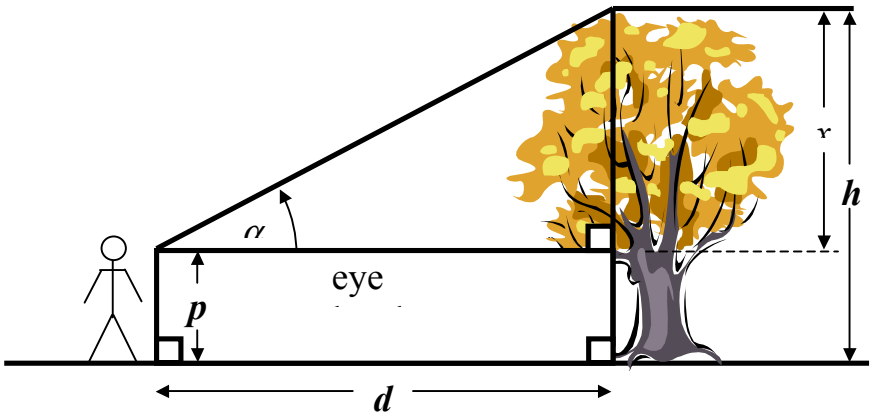
USING THE CLINOMETER TO MEASURE ANGLES OF ELEVATION
USE THIS METHOD: AMOUNT OF ROTATION IS THE DIFFFERENCE FROM WHERE YOU STARTED (90°) TO WHERE YOU END SAY 122° SO $122^O - 90^O$ IS 32° OR USING A TRIANGLE SKETCH, THE EXPLANATION IS:



TASK 2: DETERMINE THE HEIGHT OF THE TALLEST TREE ON THE FIELDS (4)

Materials required: Sighting device (clinometer), tape measure

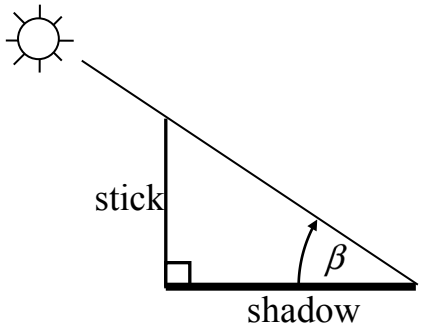
Method:
1. Measure the distance along the ground from you to the tree (*d* in metres).
2. Use the sighting device to determine Error! Objects cannot be created from editing field codes., the angle of elevation of the top of the tree.
3. Find *p*, your own eye level height.
4. Now calculate *h*, the height of the tree.



Note: Tasks 3 and 4 must be carried out in quick succession, on a sunny day. Why?

TASK 3: DETERMINE THE ANGLE OF ELEVATION OF THE SUN (4)

Materials required: Stick, tape measure



Method:
1. Measure *l*, the length of the stick.
2. Have your partner hold the stick vertically. Why is it important that the stick is perpendicular to the surface of the ground? How can you ensure that the stick *is* perpendicular to the ground?
3. Measure *s*, the length of the shadow of the stick.
4. Calculate the angle of elevation of the Sun.

TASK 4: DETERMINE THE HEIGHT OF A LAMP POST . (4)

Materials required: Tape measure

Method: Measure the length of the shadow of the lamp post and use your findings in TASK 3 to find the height of the lamp post.

NB TASK 5 MUST BE CARRIED OUT IN YOUR OWN TIME WITH YOUR PARTNER BY FINDING A RUGBY FIELD AND DOING THE SAME AS IN TASK 3 AND 4. PLEASE WRITE DOWN ALL IMPORTANT FACTS E.G WHAT WAS TIME OF DAY ETC AND SHOW ALL YOUR CALCULATIONS AND DESCRIBE YOUR METHOD.

TASK 5: DETERMINE THE DISTANCE FROM THE TOP OF THE CROSSBAR TO THE TOP OF THE RUGBY POSTS (8)

Materials required: Tape measure
Method: Devise your own method.