STUDENTS' INTERPRETATIONS IN LEARNING DERIVATIVES IN A UNIVERSITY MATHEMATICS CLASSROOM

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This is part of a study which investigated students' interpretations in learning derivatives owing to errors they displayed in an extended curriculum programme. This article focuses on errors that are displayed by students in derivatives of trigonometric functions. Thirty students registered for mathematics in chemical engineering were selected to participate in this project. The researcher used a qualitative case study approach and collected data through students' audio and video recordings. The study used Dubinsky's 1991 Actions; Processes; Objects and schema (APOS) theory to analyse the data collected. Students demonstrated different types of errors, namely, conceptual, interpretation and arbitrary. The use of APOS theory in instructional practices showed that students gain understanding of derivatives of trigonometric functions.

INTRODUCTION AND BACKGROUND

The poor performance in mathematics persists despite several attempts done to tackle factors considered as the 'key' in perpetuating weaknesses among students in South Africa. Feza-Piyose (2012: 62) asserts that "one of the contributing factors to the poor performance of students in mathematics is quality of instruction received by the majority of South African students". This proposes that there should be special attention given to the design of lesson plans to increase opportunities of students to gain understanding in mathematics instructions. Reynolds & Wheatley (1996: 564) claim that "in order for lecturers to provide learning opportunities for students, lecturers should reflect on what it means for the student to learn mathematics".

It is difficult to conclude that students understand lessons presented by teachers in their classrooms. Siyepu (2013: 13) asserts that "the meaning of the word 'understanding' is controversial in teaching and learning of mathematics". Researchers such as Hiebert & Carpenter (1992) and Stylianides & Stylianides (2007) emphasise that students understand mathematics when they link their existing mathematical concepts and procedures to make connections to solve new problems that require solutions based from previously learned strategies.

This study focuses on errors exhibited by students in audio and video recordings when they differentiate trigonometric functions. Poor performance of students in mathematics is also ascribed to misconceptions and errors that they bring to higher education from elementary levels (Brodie 2006; 2010; Kiat 2005; Luneta & Makonye 2010).

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A misconception is a conceptual structure, constructed by the learner, which makes sense in relation to her/his current knowledge, which is not aligned with conventional mathematical knowledge (Nesher 1987; Smith, DiSessa, & Roschelle 1993). In this study the researcher uses the word 'errors' to refer to systematic, persistent and pervasive patterns of mistakes performed by students across a range of contexts (Nesher 1987).

The essence of this study is to investigate students' interpretations in their learning of derivatives owing to errors and misconceptions they display in their solutions.

LITERATURE REVIEW

Students' poor performance in mathematics raises uneasiness among mathematics teachers; educators; subject advisors; curriculum developers; department of education officials and other interested parties. In South Africa there is no single cause of students' poor understanding of mathematics (Reddy 2004). Reddy (2004) highlights several reasons such as resources and infrastructure of schools, low teacher qualifications, and poor learning cultures in schools. Siyepu (2012) asserts that one of the factors that contributes in students' poor performance in mathematics originates from errors they make in their attempts to solve mathematical problems.

Students' difficulties in calculus

The study of calculus, with its fundamental concepts of limit, derivative and integral, requires an ability to understand algebraic variables as generalised numbers and as functionally related varying quantities (Gray, Loud & Sokolowski 2009). Gray et al. (2009: 71) suggest that "calculus instruction should continue to emphasise the differing uses of variables in various contexts and strive to develop students' conceptions of variables as changing and co-varying quantities". Luneta & Makonye (2010: 167) state that "students' performance in calculus is undermined by weak basic algebraic skills of factorisation, handling operations in directed numbers, solving equations, and poor understanding of indices". They further argue that algebraic incompetence has a direct impact on learning calculus. They suggest that lecturers should be aware of the educational backgrounds of the first-year university students in order to design learning activities that may close any gaps that exist between matriculation and the first-year university levels.

Calculus has been a problematic and a difficult subject for many students in many parts of the world (Barnes 1995). Students often misunderstand the notion of a function and the concept limit which is the cornerstone of several related concepts such as continuity, differentiability, integration and convergence of sequences and series (Tarmizi 2010).

Students' understanding of first principles of differentiation

The understanding of the first principles of differentiation is the building block to the understanding of various rules of differentiation. Ryan (1992) indicates that most students have a limited concept image for the gradient. He recommends that the

development of global ideas associated with the gradient of a straight line should be a focus of learning before the idea of gradient of a curve is introduced in beginner calculus. He further elaborates that some students are still indicating fundamental problems with slope as a rate of change and are keying into the x-axis rather than the nature of the slope to state whether it is positive, zero or negative.

Students' understanding of the concept *derivative*

Pillay (2008) and Zandieh (1997a; 1997b; 2000) suggest that the concept of a derivative can be represented in many ways, for instance graphically as the slope of a tangent line to the curve at a point, verbally as the instantaneous rate of change, physically as speed or velocity and symbolically as the limit of the difference quotient.

The symbolic representation of the derivative is an expression for the average gradient, which is written as $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$. The derivative as the rate of change uses the Leibniz's notation. In Leibniz's notation for differentiation the derivative of the function $f(x)$ is written $\frac{d(f(x))}{dx}$. If we have a variable representing a function, for example if we set $y = f(x)$ then we can write the derivative as $\frac{dy}{dx}$.

Calculus requires a high level of conceptual understanding, yet many students struggle to make sense of differentiation (Parameswaran 2007). Some students show an inadequate understanding of the concepts of function and variable (Barnes, 1995). She also claims that there is a lack of awareness that a derivative is a rate of change. Uygur & Ozdas (2005) assert that the derivative is a difficult concept for many students. They further explain that it is worse when the function considered is a composite function. Tall (1993: 19) indicates that "the Leibniz notation $\frac{dy}{dx}$ proves to be almost indispensable in the calculus". Yet it causes serious conceptual problems with students whether it represents a fraction or a single individual symbol (Tall, 1993). He further explains that one difficulty with the notion of the chain rule is the dilemma of whether the du can be cancelled in the equation $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$.

Students' understanding of rules of differentiation

Studies of students' difficulties with calculus topics may offer insights into understanding misconceptions and errors which are committed by students when doing calculus (Clark, Gordero, Cottrill, Czarnocha, DeVries, John, Tolias & Vidakovic 1997). Mundy (1984: 171) asserts that there is "a tendency of calculus students to operate at a rote level of procedures and symbol manipulation, which is not supported by an understanding of the concepts involved". As a result, students fail to use calculus strategies when dealing with non-routine problems (Clark, et al. 1997).

Barnes (1995: 4) suggests that "students should not be taught rules for differentiation until they have developed a good understanding of what a derivative is, and a familiarity with the relationship between a function and its derivative". She further elaborates that students should explore techniques on how to find and investigate derivatives of a variety of different functions. She emphasises that this may help to avoid what Ryan (1992) has described as 'the rush to the rule', where the meaning is ignored or forgotten, and students operate on a purely mechanical level, pushing symbols around on paper.

The chain rule is a calculus concept that causes difficulties for many students (Wangberg, Engelke & Karakok 2010). The derivatives of trigonometric functions become complicated when they need the application of the chain rule. Literature also shows that function composition is particularly problematic for students (Engelke, Oehrtman & Carleson 2005). Some students are introduced to the chain rule as merely a rule that should be applied without much attempt to reveal the reasons for and justification of the procedure (Orton 1983). The chain rule concept, as an example of a schema, requires students' understanding of standard derivatives and the basic rules of differentiation (Clark, et al. 1997).

The literature related to studies in calculus provides evidence that students develop more procedural than conceptual understanding in differentiation with regard to the application of the chain rule (Wangberg et al. 2010). Wangberg, et al. argue that some students fail to recognise that differentiating functions such as $y = \cos \pi x$ require the use of the chain rule.

THEORETICAL FRAMEWORK

This study is based on APOS theory. The acronym APOS, denoting actions, processes, objects, and schemas refers to mental structures an individual may build in responses to mathematical problem situations (Weller, Arnon & Dubinsky 2011). Research based on this theory requires that for a given concept the likely mental structures need to be detected, and then suitable learning activities should be designed to support the construction of those mental structures (Maharaj 2013). The aim of applying APOS theory is to reveal the nature of students' understanding rather than to provide a statistical comparison of students' mathematical scores (Weller, Clark, Dubinsky, Loch, McDonald & Merkovsky 2003).

For this study the researcher adopted the investigation cycle of Asiala, Brown, De Vries, Dubinsky, Mathews & Thomas (1996) to develop theoretical analysis, design and implementation of instruction and to collect and analyse the data. The investigation cycle through the steps is as shown in Figure 1.

Stages of APOS theory

APOS theory postulates that a mathematical concept develops as one tries to transform existing physical or mental objects (Maharaj 2013). The descriptions of

action, process, object and schema given below are based on those given by Weller, et al. (2011: 131).

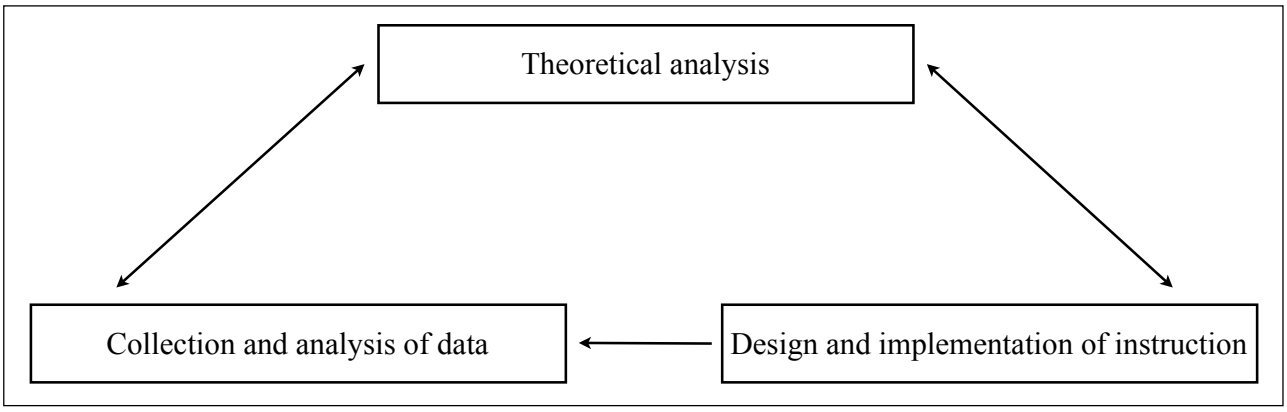

Figure 1: Investigation cycle used in APOS theory.

A transformation is first conceived as an *action*, in that it requires specific instructions and the need to perform each step of the transformation explicitly. As an individual repeats an action and reflects on it, it may be interiorised into a mental process. A *process* is a mental structure that performs the same operation as the action but wholly in the mind of the individual. If one becomes aware of a process as a totality, realises that transformations can act on that totality, and can actually construct such transformations, and then we say the individual has encapsulated the process into a cognitive *object*.

A mathematical topic often involves many actions, processes, and objects that need to be organised and linked into a coherent framework, called a *schema*. An APOS analysis of a mathematical concept consists of the actions; processes; objects; schemas and their relationships that might make up an individual's understanding of the concept.

RESEARCH METHODS

This is a single qualitative case study research. Merriam's (1988: 16) definition is: "Qualitative case study can be defined as an intensive, holistic description and analysis of a single entity, phenomenon or social unit. Case studies are particularistic, descriptive and heuristic, and rely heavily on inductive reasoning in handling multiple data sources." Particularistic means that a case study focuses on a particular situation, event, program, individual, institution or phenomenon. The case itself is important for what it reveals about the phenomenon and for what it might represent. This specificity of focus makes it an especially good design for practical problems (ibid.).

Research participants

The research participants of the study were the thirty students who were registered for mathematics in the department of chemical engineering in an extended curriculum programme in the academic year of 2009. They were borderline cases as they did not meet the minimum requirements of the main engineering stream.

Data collection

Data were collected by means of audio and video recordings. Audio and video recordings encompass observations of students solving differentiation problems in the form of classroom exercises. There were five lessons observed in the classroom but only lesson reported in this study.

Audio and video recordings

Audio and video recordings can provide researchers with a more complete sense of who the participants are, and acquaint researchers with the setting in which the people function and types of activities that participants do (Dufon 2002). Replaying the event also allows researchers more time to study and think about the data before drawing conclusions (Dufon 2002).

The students did calculations on a whiteboard, while the researcher and other students observed and made additions to correct errors and misconceptions. They also asked questions and assisted in explanation.

The audio and video recordings supported the data collection process through bringing a high level of detail regarding the researcher's and students' interactions (Pelling & Renard 1999). The researcher used a tutorial in the audio and video recordings to revise for the upcoming test towards the completion of differentiation. The use of audio and video recordings was to capture the lesson as it occurred and, it is then also possible to relive the lesson at the convenience of the researcher (Mbekwa 2003). The session was audio and video recorded in a lecture room.

The researcher and students in the sample group gathered in a lecture room to revise the previous test as given in Figure 2.

Find the derivatives of the following functions and leave your answer in the simplest form.

1.1 $y = \dfrac{\sec\theta}{1 - \cos ec\theta}$

1.2 $y = x^3 e^{2x+3}\sqrt{\cos x}$

1.3 $y = \sec^2 x e^{-\tan^2 x}$

Figure 2: Test revised by the students for audio and video recordings.

The revision of the previous test was to find students' interpretations and to give them a feedback to correct their misconceptions and errors as they worked with the researcher and other students to differentiate these problems during a class discussion. The students were free to ask questions when they did not understand the concepts and procedures that were applied to reach a correct solution.

Data analysis

In audio and video recordings, several strategies that were used by students to solve differentiation problems were identified. As this had been audio and video recorded the CD was replayed and transcribed with errors and misconceptions noted as the students showed their calculations in the video. This also depicted the researcher and other students intervening to correct and rectify errors and misconceptions made by other students through open discussion.

Students demonstrated errors namely conceptual, interpretation and arbitrary errors. Conceptual errors were identified as what Kiat (2005) describes as those that show a failure to grasp the concepts in a problem and also failure to appreciate the relationships in a problem.

Interpretation errors were identified as what Olivier (1989) describes as interference, where students tend to wrongly interpret the concept due to over-generalisation of the already existing schema. Arbitrary errors were identified as what Orton (1983) describes as those in which the subject behaved arbitrarily and failed to take account of the constraints laid down in what was given.

RESULTS RELATED TO STUDENTS' WORK AS REFLECTED FROM THE AUDIO AND VIDEO RECORDINGS

In the audio and video recordings three students showed their solutions on a whiteboard, whilst representing their groups. The students demonstrated errors that were already described above.

The researcher intervened by explaining appropriate procedures, describing concepts that were interpreted incorrectly as the students expressed themselves explaining their understanding of the derivatives of trigonometric functions.

The first student argued that her understanding is that logarithmic differentiation is applied only when the base of the function is in the form of a variable, and the index is also a variable. In the case of $y = x^3 e^{2x+3}\sqrt{\cos x}$, all the terms are not in a transcendental form. This student's question indicated that she confused the application of the chain rule with the application of the logarithmic differentiation rule. This problem contains three functions whilst the students were familiar with differentiation of two functions which make it easy for them to apply the product rule.

The second student questioned why do we not apply the power rule to differentiate $y = \ln x^3$. His question might be asked for clarity purposes or else it might show that this student did not know the difference between the power rule and logarithmic differentiation. This question might symbolise poor conceptualisation.

The researcher explained that in the case of a natural logarithmic function we do not apply the power rule. In the explanation the lecturer made examples of x^3 and $\ln x^3$ showing techniques of differentiating these two different functions. He further

explained that the first function x^3 requires the application of the power rule with its derivative equal to $3x^2$ and the second function $\ln x^3$ requires application of logarithmic differentiation to obtain its derivative which is equal to $\frac{3}{x}$.

This intervention assisted the student working on audio and video recordings to rectify her mistake. The lecturer also intervened by correcting errors as he explained the appropriate procedure involved in cancelling trigonometric functions under addition.

The third student raised a question, which reflected a conceptual error. He wanted to know whether it is appropriate to substitute $\sec^2 x$ with $\tan x$.

This question showed that the student did not understand that although the derivative of $y = \tan x$ is $\frac{dy}{dx} = \sec^2 x$, the derivative of $\sec^2 x$ is not $\tan x$. The researcher explained the appropriate procedure of obtaining the derivative of $y = \sec^2 x$. Another student suggested further simplification of

$y' = [2\tan x - 2\tan^2 x \sec x]\sec^2 x e^{-\tan^2 x}$ to $y' = 2\tan x[1 - \sec^2 x]\sec^2 x e^{-\tan^2 x}$. The researcher explained that to remove a common factor would be an undesirable closure, as it is the opposite of simplification.

The fourth student requested the use of the product rule to differentiate this problem. The same student attempted the problem by using the product rule in audio and video recorded observations. The student showed that she had poor understanding of the standard derivatives.

DISCUSSION AND IMPLICATIONS FOR TEACHING

In the case of $y = x^3 e^{2x+3}\sqrt{\cos x}$, students who participated in this study showed confusion with regard to this problem. All the terms are not in a transcendental form but still the nature of the problem requires application of logarithmic differentiation. Students' responses indicate that they are at an action level of APOS theory with regard to this problem. Their interpretations show that they assume the nature of this problem requires application of the product rule of which it is not in this case. These students worked with the product rule for a long time when they encounter two functions joined by a multiplication sign. As a result it came first in their minds that even in a problem with three functions joined by a multiplication, the product rule applies. This suggests that lecturers should familiarise students with the application of a logarithmic differentiation in problems that are not in transcendental form as some cannot be differentiated with the application of the product rule. This can be done by providing tutorials or exercises that have a variety of problems requiring the application of logarithmic differentiation.

In case of $y = \sec^2 x \cdot e^{-\tan^2 x}$ two different approaches apply. Some few students applied the product, and obtained a correct solution and many students applied the logarithmic differentiation. Although both of these two methods work some students indicated that they thought only logarithmic differentiation that works.

All students who participated in this study demonstrated understanding with regard to the differentiation of $y = \dfrac{\sec\theta}{1 - \cos ec\theta}$. This shows that these students are at a schema level of the APOS theory with regard to the quotient rule of trigonometric functions.

This study showed that students' poor understanding of differentiation is based on poor conceptualisation. They demonstrate inadequate interpretation of concepts; as a result they fail to link mathematical symbols and formulae with appropriate procedures to be applied. This study reveals that classroom interactions that focus on making sense of mathematical symbols; rules and formulae might assist students to develop meaningful understanding of mathematics.

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