ENHANCING LOW PERFORMING LEARNERS' ACHIEVEMENT IN FINDING THE GENERAL TERM OF A QUADRATIC SEQUENCE

[1]Eric Machisi, [2]Ugorji I. Ogbonnaya and [1]David Mogari

[1]University of South Africa, [2]Tshwane University of Technology

This study explores the use of alternative methods of solution to enhance low performing learners' achievement in finding the general term of quadratic sequence. A convenience sample of twenty-five low-performing grade twelve learners from a secondary school in Capricorn District in Limpopo Province took part in the study which adopted repeated measures research design. The learners were exposed to four methods of finding the general term of quadratics sequence. Data was collected using an achievement test and the data was analysed using one-way repeated measures analysis of variance. Findings indicate significant differences in the learners' average scores due to the methods used. The study therefore concludes that the achievement of low performing learners in finding the general term of quadratic sequence can be enhanced by offering them opportunities to explore a wide range of alternative methods of solving mathematics problems.

INTRODUCTION

A quadratic sequence is a number pattern in which the second difference is a constant. For example, the sequence: 3; 8; 15; 24; . . . is a quadratic sequence, its second difference is 2 (a constant).

Sequence	3		8		15		24
1st difference		5		7		9	
2nd difference			2		2		

Finding the general term of a quadratic sequence is a common question in the South African Matric mathematics paper one examination. However, our experiences show that many learners find it difficult to determine the general term of a quadratic sequence. The learners' difficulties could be attributed to the method the educators use to teach the topic. This study therefore explores the use of alternative methods of solution to enhance low performing learners' achievement on the general term of quadratic sequence.

THEORETICAL FOUNDATION

The study is founded on cognitive constructivists' theory of learning which sees learning as an active process in which learners construct meaning from their experiences in connection with their prior understandings (Driver, Asoko, Leach, Mortimer & Scott, 1994). To the cognitive constructivist, learning requires a change

Machisi, E., Ogbonnaya, U.I. & Mogari, D. (2013). Enhancing low performing learners' achievement in finding the general term of a quadratic sequence. In Z. Davis & S. Jaffer (Eds.), *Proceedings of the 19th Annual Congress of the Association for Mathematics Education of South Africa*, Vol. 1. (pp. 104 – 114). Cape Town: AMESA.

in the learner which can be achieved through the activities the learner engages in, including the consequences of those activities, and through reflection (Board, 2013). From this perspective, we believe that all learners (including low-performing learners) are capable of learning mathematics. This is possible provided the learners are offered opportunities to explore different methods of solving mathematics problems because as learners solve mathematical problems using different methods they will likely construct their own understanding which they can easily employ to solve such problems in future.

RESEARCH DESIGN

In this study, the repeated measures research design (Shuttleworth 2009) was employed. This research design (also known as within-subjects design) uses the same participants for each treatment condition and involves each participant being tested under all levels of the independent variable (Shuttleworth 2009). The researchers adopted the research design because it requires fewer participants and resources. The design allows statistical inference to be made with fewer participants. It also enables researchers to monitor the effect of each treatment upon participants easily. According to Minke (1997), the primary strengths of the repeated measures design are that it makes an experiment more efficient, maintains low variability and keeps the validity of the results higher while allowing for a smaller than usual sample to be used.

Participants

A convenient sample of 25 low performing learners (learners that persistently scored below pass mark in mathematics examinations over three years prior to this study) from a secondary school in the Capricorn District in Limpopo Province was used in this study. A convenient sample was chosen because of the willingness of the school to be used for the study. According to Tabachnick & Fidell (2006), the minimum sample size for detecting treatment effect(s) in a repeated measures design is 10 + the number of dependent variables. The recommended minimum sample size was therefore satisfied.

Data Collection Instruments

An achievement test was used to collect data to measure learners' understanding of the general term of a quadratic sequence. The test items were generated based on the topic, concept and depth of knowledge specified in the National Curriculum Statement, Mathematics Grades 10-12 (Department of Education [DoE], 2008). The test questions were essay questions in order to allow the learners showcase their understanding of the various methods of solution of quadratic sequence. The appropriateness of the test items was evaluated by six mathematics educators who had at least five years of mathematics teaching experience. After the evaluation process, the test was pilot tested on a sample of 10 learners in order to detect and correct any errors and ambiguities in the instruments before the actual fieldwork. The

final instrument was a 10 item test to assess individual learners' ability to use each of the four methods of determining the general term of a quadratic sequence.

Reliability and validity of the instrument

The reliability of the achievement test was established by calculating Kuder-Richardson (KR 20) reliability estimate, using data from a pilot study involving 10 grade twelve learners. The Kuder-Richardson method was chosen because it is less time consuming (Christmann & Badget 2008), it requires only the administration of a single test and overcomes any bias that might arise when a test is split into any number of ways as in the split-half method (Lenke, Wellens & Oswald 1977). From the Kuder-Richardson 20 calculations, a reliability value of 0.87 was obtained. According to Gay, Mill and Airasian (2011) a test is acceptable for use if its reliability coefficient exceeds 0.60. Hence, the KR20 value indicates that the test was reliable.

To check content validity of the test, it was validated by experts in the field of mathematics education. This panel comprised one subject advisor for Mathematics, one Head of Department (Mathematics) and four mathematics educators who were teaching grade twelve at the time the data was collected. The panelists were asked to independently judge if the test items reflected the content domain of the study. According to Gronlund (1998), this is just a matter of determining whether the tasks represent the larger domain of tasks it is supposed to represent. The content validity ratio of each item was calculated using the following formula:

$$CVR_i = \frac{\left[n_e - \left(\frac{N}{2}\right)\right]}{\left(\frac{N}{2}\right)}$$

CVR_i is the content validity ratio for the ith term; n_e is the number of judges rating the item as 'essential' to the domain and N is the total number of judges (Lawshe 1975). The mean of the test items' CVR_i was computed in order to find the content validity index (CVI) of the test. A CVI value of $+1.00$ was obtained, which indicates that there was complete agreement among the judges that the test items reflected the content domain of the study (Wynd, Schmidt & Schaefer 2003).

DATA COLLECTION PROCEDURES

After the learners were exposed to different methods of determining the general term of the quadratic sequence in the normal class teaching over one week period, the test was administered to assess individual learners' ability to use each of the four methods. Learners wrote the test four times, using a different method of solution each time, in their own order of preference. By the end of the session, each learner had used all the four methods, resulting in four different scores being recorded for each participant. The duration of the test was 1 hour and it was marked out of 50 marks.

The four methods the learners were exposed to are:

Method 1 (Treatment 1)

(Retrieved June 27, 2011from http://www.skoool.co.za/studynotes/maths/id635.htm)
This method uses the following formula:

$$T_n = a + (n-1)d_1 + \frac{1}{2}(n-1)(n-2)d_2$$

Where a =1st term of the sequence, d_1 =1st term of the 1st differences and d_2=2nd difference (which is constant). Applying this to the sequence 3; 8; 15; 25; … gives the following result: a = 3; d_1 = 5; d_2 = 2.

$$\therefore T_n = 3 + (n-1)5 + \frac{1}{2}(n-1)(n-2)2 = 3 + 5n - 5 + n^2 - 3n + 2 = 2n + n^2.$$

Method 2 (Treatment 2)

(Retrieved June 27, 2011from http://www.mathshelper.co.uk)

If the second difference is constant, then we know that we are dealing with a quadratic sequence which takes the form: $T_n = an^2 + bn + c$. The procedure for determining the general term of the sequence is as follows:

Step 1: Halve the second difference to get the value of a. Using the example above, $a = \frac{2}{2} = 1$. Therefore the first term of the required general formula would be $1.n^2$

Step 2: Now substitute $n = 1,2,3,4$ into $1.n^2$ to get the terms that must be subtracted from the original terms in order to get the residue. In this case, we obtain 1; 4; 9; 16.

Step 3: We write out the original sequence and the terms obtained in Step 2 and then subtract to obtain the following results:

Sequence:	3	8	15	24
n^2:	1	4	9	16
residue:	2	4	6	8

Step 4: Clearly, we see that the residues form a linear sequence. The formula of this linear sequence is: $T_n = a + (n-1)d = 2 + (n-1)2 = 2 + 2n - 2 = 2n$.

The residue is always either a constant (c) or a linear sequence $(bn + c)$. If the residue is a constant number, then the answer will be of the form $an^2 + c$.

Step 5: Finally, you add the results of step 1 and step 4 to get the general term of the quadratic sequence, $T_n = n^2 + 2n$, which is the same as what we obtained previously.

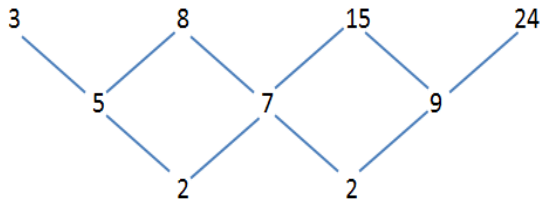
Method 3 (Treatment 3)

(Retrieved June 27, 2011 from http://www.mathforum.org/library/drmath/view/56440.html)

Suppose we have an arbitrary quadratic sequence, $T_n = an^2 + bn + c$, then the differences will look like this,

$n=1$		$n=2$		$n=3$
$a+b+c$		$4a+2b+c$		$9a+3b+c$
	$3a+b$		$5a+b$	
		$2a$		

Now considering the sequence 3; 8; 15; 24; … we obtain the following results:



$$a+b+c=3$$
$$3a+b=5$$
$$2a=2$$

Working from the bottom to the top, we have, $a=1$, $b=2$, $c=0$. Therefore the required formula (general term) is $T_n = n^2 + 2n$ as before.

Method 4 (Treatment 4)

(Retrieved June 27, 2011from http://www.cbv.ns.ca/mathhelp/quadratic1.htm)

The general form of a quadratic sequence is $T_n = an^2 + bn + c$. Then we have to figure out the value of a, b, and c and put them into this equation. The value of a is obtained by using the formula: $d_2 = 2a$ where d_2 is the 2nd difference. Applying this to our example gives: $2a = 2 \Rightarrow a = 1 \therefore T_n = n^2 + bn + c$ *

Since T_1 is 3 and T_2 is 8, we can make two ordered pairs of the form $(n;T_n)$. That is, (1;3), (2;8), … .

Substituting these points into equation * gives the following two results:

$$3 = (1)^2 + b + c_2 \Rightarrow 2 = b + c \qquad \ldots\ldots \quad (1)$$

$$8 = (2)^2 + b(2) + c_4 \Rightarrow 4 = 2b + c \qquad \ldots\ldots \quad (2)$$

Solving equation (1) and equation (2) simultaneously gives: $b = 2$ and $c = 0$. Therefore the formula for the general term of the sequence is $T_n = n^2 + 2n$ as required.

It is important to note that Method 4 is the method that is common in most grade twelve school mathematics textbooks.

FINDINGS

Table 1 shows the achievement test scores of the learners using each of the four methods (i.e. M_1; M_2; M_3 & M_4).

Table 1: Learners' percentage scores per method.

	Scores			
Learners	M_1	M_2	M_3	M_4
L_1	94	98	96	78
L_2	90	90	92	94
L_3	98	96	100	78
L_4	50	70	88	20
L_5	88	92	100	86
L_6	100	34	86	100
L_7	78	96	100	64
L_8	82	68	90	90
L_9	58	54	68	48
L_{10}	100	72	92	52
L_{11}	54	80	68	40
L_{12}	60	14	32	12
L_{13}	94	68	82	44
L_{14}	60	76	98	86
L_{15}	90	100	92	44
L_{16}	72	92	88	56
L_{17}	52	40	80	2
L_{18}	94	100	100	80
L_{19}	92	100	88	84
L_{20}	80	74	90	68
L_{21}	76	64	70	54
L_{22}	92	90	100	88
L_{23}	70	60	76	6
L_{24}	76	50	84	6
L_{25}	78	64	100	34

A one-way repeated measures analysis of variance (RM ANOVA) in SPSS was performed on the data to evaluate the following hypotheses:

$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$

H_A : At least on treatment or observation mean (μ) is different from the others.

Results of repeated measures ANOVA

Descriptive statistics

Table 2 shows the initial output from the repeated measures ANOVA analysis.

Table 2: Descriptive statistics.

	Mean	Std. Deviation	N
M 1	79.1200	15.93821	25
M 2	73.6800	22.90327	25
M 3	86.4000	15.19868	25
M 4	56.5600	30.37828	25

Table 2: Descriptive statistics.

The Descriptive Statistics contains the number of cases available for analysis per each level of the independent variable.

Sphericity

Sphericity is the condition where the variances of the differences between all combinations of the repeated measures levels are equal. Violation of this assumption causes the Repeated Measures ANOVA test to increase Type I error rate (Laerd 2012). The SPSS computed significance value for the ANOVA test would be too low and thus we risk rejecting the null hypothesis when actually we should not.

Table 3: Testing for sphericity.

Mauchly's test of sphericity

Measure: MEASURE_1

Within Subjects Effect	Mauchly's W	Approx. Chi-Square	df	Sig.	Epsilon Greenhouse-Geisser	Epsilon Huynh-Feldt	Epsilon Lower-bound
Method	.563	13.064	5	.023	.761	.845	.333

The Mauchly's test of sphericity tests the null hypothesis that the variances of the differences between levels of the repeated measures factor are equal. The above test indicated that the assumption of sphericity had been violated because the significance value ($\chi^2(5) = 13.1$, $p = .023$) is less than the criterion value of .05. To account for the violation of sphericity, the degrees of freedom of the F-test were corrected using Huynh-Feldt epsilon value ($\varepsilon = 0.845$). The Huynh-Feldt correction was used because ε was greater than .75 (see Field 2008).

ANOVA F-test

Table 4 shows the repeated measures ANOVA. The Huynh-Feldt corrected results indicate that there was a statistically significant main effect of the independent variable (method) on the dependent variable (learners mathematics scores) (F (2.54, 60.8) = 16.74, p = .000). Therefore, the null hypothesis that the averages for the four methods are the same is rejected and we conclude that at least one of the treatment means (μ) is different.

Table 4: ANOVA F-test

Tests of Within-Subjects Effects

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
Method	Sphericity Assumed	12105.400	3	4035.133	16.744	.000
	Greenhouse-Geisser	12105.400	2.282	5304.767	16.744	.000
	Huynh-Feldt	12105.400	2.535	4775.786	16.744	.000
	Lower-bound	12105.400	1.000	12105.400	16.744	.000
Error(Method)	Sphericity Assumed	17351.600	72	240.994		
	Greenhouse-Geisser	17351.600	54.768	316.822		
	Huynh-Feldt	17351.600	60.834	285.229		
	Lower-bound	17351.600	24.000	722.983		

Since a statistically significant result was found the Bonferroni Post Hoc test was conducted to compare each method with all other methods in order to determine where exactly the significant differences exist.

Bonferroni Pairwise Comparisons

From the significance values of each pairwise comparison, we obtain the following:

The difference between the mean scores of the learners on Method 1 and Method 2 had a probability (p = 1.000) far greater than alpha (.05). This implies that the difference is not significant. Therefore, the null hypothesis that the means scores were equal was accepted.

The difference between the mean scores of the learners on Method 1 and Method 3 had a probability (p = .155) greater than alpha (.05). This implies that the difference is also not significant. Hence, the null hypothesis that the two means scores were equal was also accepted.

The difference between the mean scores of the learners on Method 1 and Method 4 is statistically significant. The difference had a probability (p) = .001 which is less than alpha (.05). Therefore, the null hypothesis that the two means were equal was rejected.

The difference between the mean scores of the learners on Method 2 and Method 3 was statistically significant (p = .004). Therefore, the null hypothesis was rejected.

The difference between the mean scores of the learners on Method 2 and Method 4 had a probability (p) = .022. This being less than alpha (.05) implies that the difference between the mean scores of the learners on the two methods is statistically significant. Therefore, the null hypothesis that the two mean scores were equal was rejected.

The difference between the mean scores of the learners on Method 3 and Method 4 was statistically significant (p = .000). Hence, the null hypothesis was rejected.

Table 5: Bonferroni pairwise comparisons

Pairwise Comparisons Measure: MEASURE_1						
(I) Method	(J) Method	Mean Difference (I-J)	Std. Error	Sig.[b]	95% Confidence Interval for Difference Lower Bound	Upper Bound
1	2	5.440	4.281	1.000	-6.868	17.748
	3	-7.280	3.064	.155	-16.088	1.528
	4	22.560*	4.813	.001	8.722	36.398
2	1	-5.440	4.281	1.000	-17.748	6.868
	3	-12.720*	3.225	.004	-21.991	-3.449
	4	17.120*	5.315	.022	1.837	32.403
3	1	7.280	3.064	.155	-1.528	16.088
	2	12.720*	3.225	.004	3.449	21.991
	4	29.840*	5.113	.000	15.139	44.541
4	1	-22.560*	4.813	.001	-36.398	-8.722
	2	-17.120*	5.315	.022	-32.403	-1.837
	3	-29.840*	5.113	.000	-44.541	-15.139

* The mean difference is significant at the .05 level.

b. Adjustment for multiple comparisons: Bonferroni

These results suggest that the low performing learners achieved better on the test using methods 1 and 3 than using methods 2 and 4. It is important to note here that

method 4 which had the lowest average percentage score for the learners is the most common method in current grade twelve mathematics textbooks.

RECOMMENDATIONS

Based on the findings of this study, we recommend that in order to enhance the learning and achievement of low performing learners in finding the general term of quadratic sequence in particular and in mathematics in general, educators should expose the learners to a wide range of methods of solving mathematics problems rather than confine them to only the methods in their textbooks. This implies that the educators need to have a wide repertoire of methods of solving problems beyond what is available in one textbooks to enable them create an environment that will enable learners explore alternative methods of solving mathematics problems. The experience of a wide range of methods of solving mathematical problems will offer learners opportunities to explore various techniques of dealing with mathematical problems and help them to build their own knowledge and meanings. In so doing, learners will find methods of solutions they understand better and that will enhance their mathematics achievements.

We also recommend that mathematics textbooks be reviewed regularly to incorporate wider range of methods of solving mathematics problems in order to cater for the needs of all students including low performing learners.

In conclusion, we argue that unless learners are exposed to a variety of methods of solving mathematics problems, they may not be able to discover the solution methods that will make learning mathematics easier for them and the high failure rate in mathematics will not likely be a thing of the past.

REFERENCES

Badget, J. L. & Christmann, E. P. (2008). *Interpreting assessment data: Statistical techniques you can use. National Science Teachers Association*. Retrieved June 6, 2011 from http://www.books.google.co.za.

Board, J. (2013). *Learning theory-Constructivist Approach*. Retrieved February 19, 2013 from http://education.stateuniversity.com/pages/2174/Learning-Theory-constructivist-approach.html.

Department of Education (2008). *National Curriculum Statement Grades 10-12 (General). Learning Programme Guidelines - Mathematics.* Pretoria: Department of Education.

Driver, R., Asoko, H., Leach, J., Mortimer, E. & Scott, P. (1994). Constructing scientific knowledge in the classroom. *Educational Researcher*, 23(7), 5-12.

Field, A. (2008). *A Bluffer's Guide to Sphericity: BPS-MSC Newsletter 6(1).* Sussex: University of Sussex.

Gay, L. R.; Mill, G. E. & Airasian, P. (2011). *Educational Research: Competencies for Analysis and Application*. Upper Saddle River, NJ: Pearson Education.

Gronlund, N. E. (1998). *Assessment of student achievement*. Massachusetts: Allyn & Bacon.

Laerd (2012). *Sphericity*. Lund Research. Retrieved August 20, 2012 from http://statistics.laerd.com/statistical-guides/sphericity-statistical-guides.php

Lawshe, C. H. (1975). A quantitative approach to content validity. *Personnel Psychology,* 28, 563-575.

Lenke, J. M., Wellens, B. & Oswald, J.H (1977). *Difference between Kuder-Richardson Formula 20 and Formula 21: Reliability Coefficients for Short Tests with different item variability.* New York: The Psychological Corporation.

Minke, A. (1997). Conducting Repeated Measures Analyses: Experimental Design Considerations. Retrieved August 17, 2012 from: http://ericae.net/ft/tamu /Rm.htm.

Shuttleworth, M. (2009). Repeated Measures Design. Retrieved August 7, 2011 from http://www.experiment-resources.com/repeated-measures-design.

Tabachnick, B. G. & Fidell, L.S. (2006). *Using multivariate statistics* (5th ed.). New Jersey: Prentice-Hall Inc.

Van de Walle, J. A. (2004). *Elementary and middle school mathematics: Teaching developmentally*. New York: Pearson.

Wynd, C. A., Schmidt, B. & Schaefer, M. A. (2003). Two Quantitative approaches for Estimating Content Validity. *Western Journal of Nursing Research*, 25(5), 508-518.