USING THE 'KNOWLEDGE QUARTET' TO ANALYZE PRIMARY MATHEMATICS TEACHING IN SOUTH AFRICA: THE CASE OF SIBONGILE

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This paper presents results of the analysis of video data of a mathematics lesson taught by a Grade 5 teacher. The teacher is a participant in the Wits Maths Connect-Primary (WMC-P) professional development course. The Knowledge Quartet framework was used as a tool in providing a language of description to discuss on ways in which the teacher's content knowledge and pedagogical content knowledge come into play in the classroom. The analysis showed that the teacher used a number line as promoted by the WMC-P course and she believed that it is a useful tool in helping her learners to move to more efficient strategies for addition. There was evidence of strong connections between the actions on object, word, symbol and diagram representations within her explanation. However, her selection of number range within her examples were too low for Grade 5, which might have led to most learners being able to give answers to the questions mentally, which resulted in a situation where the need for the use of number line was not really motivated. We conclude this paper by identifying certain issues for later discussion with the teacher in order to expand our understanding of some of her pedagogical decisions.

INTRODUCTION

Widespread agreement exists that teaching and learning of primary mathematics in South Africa need improvement (Ensor et al. 2009; Fleisch 2008; Schollar 2008). This need is evident from the poor performance of primary school learners in the country as pointed out in the Annual National Assessment (ANA) report which indicated that the mean annual results of grades 3 and 6 stood at 41% and 27% respectively (Department for Basic Education 2012). Testing within the WMC-P project in 2011 with Grade 2 learners indicated that 75% of learner samples drawn from across the attainment range across the ten project schools were using 'count all'-based strategies for addition sums in the 1-20 number range (Venkat 2011).

The study reported in this paper is located within the Wits Maths Connect-Primary (WMC-P) project, a 5-year longitudinal research and development project targeting primary mathematics teachers from ten schools in Johannesburg, South Africa. The project is aimed at improving the quality of teaching and learning of primary school mathematics. To achieve this aim, the WMC-P project, among other activities, is piloting a 20-day professional development course focusing on developing and deepening primary school teachers' content knowledge from a pedagogical perspective. Research evidence indicates that teachers' mathematical knowledge is strongly related to the quality of their mathematics instruction (Hill et al. 2008) and learners' achievement gains (Hill, Rowan & Ball 2005; Jacobs, Megan, Carpenter,

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Linda & Dan 2007; Rowan, Chiang, & Miller 1997). These studies considered teachers' mathematical knowledge as both knowledge needed to do mathematics and the knowledge associated with teaching mathematics.

There is substantial evidence that primary mathematics is an under-researched area in South Africa (Venkat, Adler, Rollnick, Setati & Vhurumuku 2009). However, broader findings of gaps in teachers' Content Knowledge (CK) (Taylor 2008) and Pedagogical Content Knowledge (PCK) (Carnoy, Chisholm & Chilisa 2012) linked with small scale research has pointed to some problems in pedagogic practice. Amongst the problems that have been highlighted are low levels of teachers' content knowledge, and lack of consistency and coherence in the teachers' coordination of tasks and mediation means, which resulted in mathematical objects not emerging for many learners (Askew, Venkat & Mathews 2012). Hence, there is a need to explore primary school teachers' knowledge in teaching, as they participate in a professional development model that emphasises coherent connections within and across tasks and progression in teaching mathematical ideas, so as to improve teaching quality.

This research is part of a more extensive study on exploring ways in which teachers' content knowledge and pedagogical content knowledge come into play in the classroom using knowledge quartet framework. We begin the paper by describing and conceptualizing the framework, devised within the trajectory of work in primary mathematics teacher development by Tim Rowland and his team (Rowland, Huckstep & Thwaites 2003, 2005; Rowland, Turner, Thwaites & Huckstep 2009). We then use the framework to help us reflect on the teacher's mathematical knowledge using the four knowledge dimensions namely: foundation, transformation, connection and contingency knowledge.

CONCEPTUALIZING THE KNOWLEDGE QUARTET

The knowledge quartet framework was developed by the Subject Knowledge in Mathematics (SKIMA) research group at the University of Cambridge. They committed several years to the development of this framework, which has subsequently been revised many times (see Rowland et al. 2003, 2005; Rowland et al. 2009). A significant aspect of their framework is that it is not only aimed at defining what knowledge is needed for mathematics teaching and how such knowledge may be identified, but also provides a way of understanding how such knowledge is developed in teachers. Through categorizing situations from classrooms where mathematical knowledge surfaces in teaching from the detailed analysis of 24 lessons they observed (2 lessons each taught by 12 trainee teachers), this framework emerged. The framework consists of four categories of knowledge, namely, *foundation*, *transformation*, *connection* and *contingency*.

The first category, *foundation*, consists of the theoretical background related to mathematics knowledge, beliefs and understanding that teachers possess or acquire during training, in preparation for their roles in the classroom. It is about knowledge possessed regardless of whether it is being put to purposive use. The key components

of this theoretical background are: knowledge and understanding of mathematics *per se* (i.e. content knowledge) and knowledge of mathematics specific pedagogy (i.e. pedagogical content knowledge), together with beliefs concerning the nature of mathematical knowledge, the purposes of mathematics education, and the situations that provide conducive environments for mathematical learning (Rowland et al., 2003). The remaining three categories, unlike foundation knowledge, refer to "ways and contexts in which knowledge is brought to bear on the preparation and conduct of teaching" (Turner & Rowland 2011: 200).

The second category, *transformation*, lies at the heart of the knowledge quartet. It is concerned with mathematical knowledge-in-action as demonstrated both in planning to teach mathematics and in the act of teaching itself. In their discussion of this category, Turner and Rowland (2011) cited Shulman's notion that this category refers to the "capacity of a teacher to transform the content knowledge he or she possesses into form[s] that are pedagogically powerful" (Shulman 1987: 15). The teacher's choice and use of examples, representations, use of instructional materials and demonstrations in teaching mathematics and explanations of mathematical ideas are the critical components of this category.

Connection, the third category, is concerned with the coherence of the planning or teaching mathematics displayed across an episode, lesson or series of lessons. Coherence refers to the sequencing of materials for instruction, and an awareness of the relative cognitive demands of different mathematics topics and tasks (Rowland et al. 2003). The key features of this category include: decisions about sequencing, making connections between procedures and concepts. Of particular importance in the conceptualization of the knowledge quartet in this study, is the need noted in the literature for building coherent connections between actions on mathematical objects, word, diagram and symbolic representations within the teachers' explanation and also connections between episodes (Askew, Brown, Rhodes, William, & Johnson, 1997; Haylock & Cockburn 2008). This follows the identification of disconnection and lack of teaching for progression as prevalent in the South African primary school teaching landscape (Askew et al. 2012; Ensor et al. 2009; Venkat, Abdulhamid & Morrison 2012; Venkat & Naidoo 2012)

Contingency, the last category, focuses on the teacher's responses to classroom events that were not anticipated in the planning of how practice would unfold in the mathematics classroom (Turner & Rowland 2011). The features of this category are evident when the teacher deviates from the agenda set out in the prepared mathematics lesson plan, and responds to learners' mathematical ideas and provides insights during instruction. Rowland et al. (2003) argue that teachers with limited mathematical knowledge find it more difficult to cope within this category. Contingency refers to knowledge that is demanded during classroom interaction. This is prompted when the teacher encounters surprise from a learner's misconception or demonstration of a concept in ways that are unfamiliar to the teacher. In the

following paragraph, we highlighted the usefulness of knowledge quartet in relation to South African primary mathematics teaching.

The empirical problems identified in South African primary school mathematics teaching suggest that the knowledge quartet framework may be useful. The teachers' mathematical knowledge and understanding in relation to connections and progression fit into the foundation category as part of the teacher's theoretical background knowledge. When put into planning and in the act of teaching, teacher's knowledge of "analogies, illustrations, examples, explanations, and demonstrations *that are presented in a way that the content knowledge is transformed to be* comprehensible to others" (as described by Shulman (1986)) fit into the transformation category of the knowledge quartet. The coherence of the connections between mathematical ideas and progression as conceived in terms of use of representations and strategies in the enactment of a lesson and across sequences of lessons fit into the connection category. The ways in which a teacher mediates empirical problems as they arise in classrooms fit into the contingency category. This is the teacher's knowledge of interaction in the use of pupils' descriptions of their methods/strategies and their reasoning. According to the literature, these help to establish and emphasise connections of mathematical ideas and establish misconceptions, in ways that are not always possible for the teacher to anticipate in his/her lesson planning.

In this paper, we present an initial analysis of a single lesson taught by one of the teachers participating in the WMC-P professional development course focused on addition of whole numbers. Within the course sessions on addition and subtraction, tutors emphasized the use of closed and empty number lines (Beishuizen 1993) as useful models for addition and subtraction problems, and also discussed the use of part-part-whole models (Carpenter, Fennema, Franke & Levi 1999). The lesson was video recorded, and we created a verbatim transcript that captured all the teacher talk, teacher-learner interactions, and descriptions of the tasks and representations that were produced and used by the teacher during the course of the lesson. In the following section, we first present the summary of the lesson organised into episodes. Each episode consists of a specific focus and activity. We decide on the focus based on the activity and/or lesson format (teacher work, learner group or individual work) that took place within the lesson, not necessarily what the teacher says. Hence, the activity and/or lesson format drives the focus. As the focus of the lesson shifted, this marked the beginning of the new episode.

SIBONGILE'S LESSON

Sibongile's[1] focus was on teaching grade 5 lesson on addition. There were 40 learners in the class with average age of 11 years. The main focus of this lesson was

[1] All names used in this paper, except the authors are pseudonyms intended to preserve the anonymity of the participants

teaching addition using empty number line—a topic and representation that was emphasised in the course. Table 1 presents the summary of the lesson.

Table 1: Lesson summary

Episode	Content Outline
E1	*Focus*: Mental addition *Activity*: Teacher started the lesson with questions challenging her learners' mental addition abilities, by saying "who can give me the answer for eight plus nine, three plus four, ten plus seven, nine plus nine, seven plus seven, five plus two, nine plus eight, twelve plus nine, thirty-two plus eight, forty-two plus two, nineteen plus seven and twenty-six plus seven?" Most learners appeared able to answer these questions immediately and correctly. Some learners were seen counting-on using their fingers. Teacher demonstrated the use of tally counting with example of eight plus nine and discouraged using it to solve problems by saying "let us say you have twenty sums and you are required to do it in fifteen minutes. Can you be able to finish?" Learners responded in chorus, "No!"
E2	*Focus*: Use of closed/structured number line *Activity*: As an alternative to tally counting, the teacher introduced the use of a closed number line with unit intervals across the (1-10) range and started with an example of the sum 4+4. One learner said "start at four and make four jumps on the number line to get to eight." Teacher then pointed out that even the use of closed number line would not provide efficient way of working out the sum, especially when dealing with large numbers, e.g. 33+9.
E3	*Focus*: Use of empty number line *Activity*: Teacher explained the use of the empty number line as a better alternative; a quicker and more efficient way of adding numbers. She illustrated with the sum 5+2 using a count on strategy. The next example was 19+8. The teacher pointed out that it was quite different from the previous example, and then introduced the notion of friendly number. She defined a friendly number as "a number that is a multiple of ten", and then she asked "which number is a multiple of ten and nearer to nineteen?" Learners' responded saying '"twenty". She then explained adding one to nineteen on the empty number line by making a unit jump from 19 to land on a friendly number 20 and then, adding the remaining 7 as one jump to get to 27. Using the same procedure, teacher worked out another example with the sum 23+19.
E4	*Focus*: Learners work out sum on the board *Activity*: Teacher asked three learners to come and work out the sums 8+6, 19+8 and 19+21, respectively, on the board using the empty number line. Two learners did the sums correctly as expected by the teacher, but the other one made an error for the sum 19+21. He drew an empty number line and started with 21 and he made a jump to 30 as to land on a friendly number, but he wrote "+8" on top of the jump (instead of +9 because 21+9 gives 30). The learner's error appeared to have taken the teacher by surprise. She said to the class, "I don't think what you are doing is right." And asked, "Who can help him?"

RESULTS AND DISCUSSION

By selecting episodes from Sibongile's lesson, we attempt to unpack aspects of mathematical knowledge in teaching relevant to the four dimensions of the knowledge quartet.

Foundation

There are basically two types of number line; structured (or closed) and empty number line. These two types of number line are particularly relevant to Sibongile's lesson. Freudenthal (1973) suggested that a structured number line with marks for every number was a more natural model of children's informal counting strategies than arithmetic blocks. The structured number line was constituted in measurement situations and so was associated with rigid fixed distances. Gravemeijer (1994) argued that the use of the structured number line caused counting to be a passive reading of the answer on the number line, which did not raise the level of the strategies the learners used to solve a problem. This criticism of structured number line led to the origin of the empty number line in the Dutch Realistic Mathematics Education movement (RME). The empty number line allows learners to draw marks for themselves instead of the fixed marks in structured number lines. Following a comprehensive review of literature on the importance of empty number line, Anton, Beishuizen & Treffers (1998) articulate four reasons for using the empty number line in teaching:

- It is well-situated to link up with informal solution procedures because of the linear character of the number line.
- It provides opportunities to raise the level of the learners' activity.
- It stimulates a mental representation of numbers and operations (addition and subtraction in particular).
- It eventually helped learners to develop an internalised model, where learners keep track of what they are doing without dependence on visualization, leading to a reduction of the memory load while solving a problem.

These theoretical considerations are part of the foundation knowledge relevant to Sibongile's lesson. This is not to suggest that Sibongile would know or articulate them as presented. However, it is clear from her lesson as pointed out in episode 2, that she is aware of the two types of the number line, and that empty number line provides more efficient level of strategy of addition than the structured number line.

Sibongile:	Using such a number line that has got numbers on it like this [*showing a structured 1 – 10 number line on the board*]. Let us assume that you are given a sum like this [*33+9 written on the board*]. Will you be able to draw a number line that is too long up to there [*pointing to the end of the board*]? You can still do it, but you will waste lot of time. Do you want us to try it?
Learners:	[*in chorus*] No

Sibongile: I don't think so, either. If you draw a number line that is too long, you will be going to take time to put the numbers up to thirty-three and then jump forward nine times before you get the answer.

The fact that Sibongile states the use of empty number line as a tool for building more efficient strategies, suggest the nature of her belief in the usefulness of the model. The knowledge that underpins Sibongile's recognition that the empty number line model provides a more efficient way of addition is conceptualized within the domain of foundation knowledge. In her explanation of the solution of the sum 19+8 in episode 3 using empty number line, Sibongile introduced the notion of friendly number.

Sibongile: To make it easier for us, we look for a number that we are going to call a friendly number. Someone who is friendly is the one who you can easily work with… numbers that are multiple of ten are easier to work with, so they are called friendly numbers. For our sum [referring to 19+8 written on the board], If I look for nineteen and I am supposed to add eight, which number is a multiple of ten that is nearer to nineteen? Sipho.
Sipho: Twenty.
Sibongile: It is twenty, so we say that the friendly number is twenty, so we can add one to nineteen and get twenty [teacher made a unit jump from 19 to 20 on the board]. So what number remains to add? Thandi.
Thandi: Seven.
Sibongile: So, we can make a big jump of seven from twenty, which is easier to add on a multiple of ten [*teacher made a big jump from 20 and wrote 7 on top*]. So what is our answer?
Learners: [*in chorus*] Twenty-seven.

It is evident from the above, that Sibongile demonstrated her knowledge of the usefulness of bridging through a multiple of 10, which she called 'friendly number'. Adding on multiple of 10 or 100 is a core skill in fluency with addition and/or subtraction.

Transformation

As described in episodes 3, Sibongile demonstrated the idea of using number line representation in a well-organized manner. There is evidence of transformation of the idea of using number line with the introduction of the notion of 'friendly number'. It was explicit from her explanation on how and why the friendly number is used. Even though, in her example of the sum 19+8, there was no explicit statement of the fact that 8 is split into 1 and 7, but it was clear to the learners that 1 is taken from 8 to make 19 a friendly number and what remains to add is 7.

Sibongile: … so we can add one to nineteen and get twenty [*teacher made a unit jump from 19 to 20 on the board*]. So what number remains to add? Thandi.
Thandi: Seven.

Sibongile's commencement of her lesson with questions checking her learners' addition abilities merits some consideration and comments. Her first set of questions (referring to the sums 8+9, then 3+4, 10+7, 9+9, 7+7, 5+2, 9+8, 12+9, 32+8, 42+2, 19+7 and 26+7) seem to indicate a deliberate level of progression in the kinds of questions she asked. It is clear that she started sums with two single digit numbers except (10+7) and moved to double and single digits numbers. The selection of this sequence seems to be deliberate, as indicated by Sibongile.

> Sibongile: So people know how to add numbers and can quickly add a single digit and a single digit number. Let us look at the number like twelve plus nine …

However, we suggest that the sequence of the question is not well-chosen to some extent for the following reason. As core skill for addition, learners often easily add numbers to make 10. The first and third questions (8+9 and 10+7) are similar expression for the number 17. It will be well-chosen if the sequence start with 10+7 then 8+9, because the latter requires more cognitive demand than the former. However, in the last two questions (19+7 and 26+7), the sequence appeared to be well-chosen. Another important structure that was visible in the sequence is the idea of doubling being a key strategy for mental addition. Her choice of practice examples and exercise also calls for some comments. The sequence of practice examples are 4+4, 5+2, 19+8, 23+19 and then she asked two learners to come to the board and do 8+6 and 19+8. There was some sense of progression in the sequence of the examples from doubling to counting on. This also appeared to be well-chosen, but the example selection does not reflect the range of concept structure for addition. For example, there were no examples that require learners to use commutative property of addition. Rowland (2008) emphasises the need for the range of examples selection to reflect the range of concept structures related to an idea. For instance, in the example of addition situation above, a variation of order of addend is not opened. By not opening this variation, a feature of the concept of addition, that the sum is independent of order of the addends is not discerned (Marton, Runesson & Tsui 2004).

The number range used in her selection of examples was much lower than the range specified in the Grade 5 curriculum. Learners seemed able to produce answers to initial mental questions quite quickly, some using counting on and many as recalled fact. It may not be useful to promote an alternative strategy using examples that learners know the answers to mentally.

Connection

In terms of connection, it seems that Sibongile offers two important connections in her lesson. The first connection relates to the sequence of the lesson between the different ways of adding numbers; starting with mental addition to the use of empty number line. But there are some contradictions in the link between the mental addition and the use of number line. Most learners seemed able to give answers to the first set of questions mentally; resulting in a situation where the need for the number line was not really motivated. Literature suggests the need for new representations or

tools to be motivated by problems for which existing tools are insufficient or inefficient (Anton et al. 1998). It can be noted here that example and representations are coherent; but the point is that they seem not to connect or progress well from where learners are.

The second connection is that between the actions on object described with jumps on the number line, the word representation, the symbolic form of the sum and the number line within her explanation. As described in the previous section in the solution for the sum 19+8, Sibongile established the connections between her talk about the use of friendly number linked to the action on the object described with the jumps and the diagram representation (i.e. the number line), in which the symbolic form follows. The second connection therefore appeared to be coherent.

Contingency

Sibongile does not seem ready to deviate from the agenda set out for her lesson. When learners give incorrect answers, she keeps on asking until she gets the correct answer. She does not probe or allow learners' to explain their answers and use that to establish learners' misconceptions. For example, in Episode 4, Themba's error in adding the sum 19+21 on the board appears to have taken Sibongile by surprise.

Themba:	I will look for a friendly number, which is thirty [*he made a jump from 21 to 30 on an empty number line and landed on a friendly number 30. He then wrote +8 on top*].
Sibongile:	That's mean you people are not looking at what Themba is doing? I don't think what you are doing is right. Who can help him? Thandi.
Thandi:	He is supposed to write plus nine.

This would seem to be a case where Themba might be encouraged to reconsider what he has written by asking him some well-chosen questions. One such question might ask Themba what is the answer to 21+8? But Sibongile said, "I don't think what you are doing is right." She then asks the class, "Who can help him?" Thandi gave the right answer, but there was no diagnosis of why Themba might have written "+8".

CONCLUDING REMARK

In this paper, we have used the knowledge quartet framework as a tool to provide initial analysis of Sibongile's lesson while reflecting on the foundation, transformation, connection and contingency dimensions of the quartet. The analysis showed that Sibongile enacted the idea of using an empty number line as promoted by the WMC-PD course and she believed that it is a useful tool in helping her learners to move to more efficient strategy for addition. There was evidence of strong connections between the actions on object described with jumps on the number line, the word representation, the symbolic form of the sum and the number line within her explanation. However, her selection of number range within her examples were too low for Grade 5, which might have led to most learners seemed able to give answers

to the questions mentally; hence resulting in a situation where the need for the use of number line was not really motivated.

We intend to expand further on this understanding by identifying, for later discussion with Sibongile, matters such as:

- Does Sibongile understand that for the kind of sums in the number range used, most learners could do them quicker mentally than with the use of the empty number line? What was her intention in the choice of these particular examples and exercise? Was she aware that the number range here was much lower than the range specified in the Grade 5 curriculum? If so, what was the rationale for this selection? Where might she have extended to after this work?
- Does Sibongile understand that her selection of examples does not recognize the need to establish the commutative property of addition?
- Does Sibongile notice the connections she made between the action on object, word, symbol, and diagram representations as important in facilitating learners understanding?
- Does Sibongile consider learners' incorrect answers as useful for teaching? Does she consider deviating from the agenda set for her lesson as important for facilitating learners' understanding?

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