TOWARDS EXPLICIT USE AND INTEGRATION OF AN OBJECT DEFINITION OF THE STATISTICAL MEAN WITH AVERAGE

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Extended abstract

The definition of the statistical mean is based on its add-and-divide algorithm, without logical recourse to the ontology of the mean. I argue that the mean is in need of a statistical object definition to complement the well-known process definition. To this end mean and average must be differentiated in colloquial discourse, so that the statistical mean can be defined as the origin or zero for the measurement of variation of data in a set.

MEAN AND AVERAGE

In her early work, Sfard (1991) argued that a mathematical concept has dual faces – a concept is a process as well as an object. In her later work, Sfard (2008) defines concepts as words (or other symbols) and their uses within different discourses. From a discursive perspective statistics is delineated as a discourse different from mathematics by its words and symbols, and its narratives and routines. Yet, when statistical objects like the mean are defined, they are defined as mathematical processes, where-after attention is drawn to some mathematical properties of the calculated numbers and some statistical uses. For example, *Collins Dictionary of Statistics* (Porkess, 2004) defines *mean* and *average* as follows:

> **Mean**. 1. A measure of an average value. There are several types of mean, used in appropriate circumstances, but unless stated otherwise the term "mean" is usually taken to be the arithmetic mean" (Porkess, 2004, p. 50) (The dictionary then proceeds to give the mean algorithm in algebraic notation).
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> **Average**. 1. In technical use, average usually has the same meaning as mean or arithmetic mean.
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> 3. In everyday use the word average is often used loosely to mean typical or representative, as in a statement like "William is average at football". … According to context, it may be any (or none) of mean, mode, median and midrange. (Porkess, 2004, p. 14)

As a statistical definition Porkess' definition of the mean as a "measure of an average value" fails to explain why the calculation of the arithmetic mean is a measure of an average value. Colloquial reference to objects like "an average car" bears witness to average as a subjective, context based judgment rather than an objective statistical measurement. The conflation of average and mean is problematic for teaching since it leaves the ontology of the mean as well as the average unexplained. In addition, the lack of a formal object definition of the mean leaves the process definition open to varied interpretations.

Lampen, E. (2013). Towards explicit use and integration of an object definition of the statistical mean with average. (Extended abstract) In Z. Davis & S. Jaffer (Eds.), *Proceedings of the 19th Annual Congress of the Association for Mathematics Education of South Africa*, Vol. 1. (pp. 97 – 99). Cape Town: AMESA.

PROBLEMATISING THE SHIFT FROM MATHEMATICAL TO STATISTICAL DISCOURSE ON THE MEAN

The first historical moment where the relationship of the mean to *differences* between measurements, rather than to the measurements themselves was foregrounded may provide opportunity for statistically meaningful disobjectification or re-enactment of the syntax of the mean algorithm. Bakker (2003, p. 9) describes the use of the term "averij" in ancient maritime law (around 700BC), where the purpose of calculating the "averij" was to *share* losses at sea *fairly* between customers.

The use of mean in discourse on variation, hence statistical discourse, developed late in history. The term *l'homme moyen*, the average man, was invented by the Belgian statistician Quetelet (1796-1874). Quetelet's use of the mean as a representative value rather than a 'real' value was an important step in the development of variation discourse on the mean. Alternative semantics for the mean that focuses on its statistical use and properties can be found, such as *mean as a measure of central tendency* or mean as a *balance point of a distribution.* These phrases are mostly encountered in statistics education texts and endeavour to provide cues for conceptual understanding of the statistical object, rather than the colloquial object *average*. It is rare to obtain such descriptions of the statistical mean from participants in statistics learning and teaching contexts (Watson & Moritz, 2000). The divide between colloquial and literate statistical discourse on the mean remain un-bridged.

DEVIATION DISCOURSE ABOUT AVERAGE AS A BRIDGE TO THE STATISTICAL MEAN

Colloquial understanding of the mean as the centre of an interval of average values indicates that deviation of actual values from the mean is intuitively understood as an important attribute of the mean. (Lampen, 2012) An educational goal is to develop intuitions of average as it refers to objects as well as data sets, to yield the necessary deviation discourse in which the statistical mean may develop object-meaning in addition to process-meaning. I propose the following didactical definition for the mean that will make its statistical use explicit and from which the calculation algorithm can be deduced:

The mean is a numerical value that indicates a hypothetical standard of zero variation among varying measures over all the cases in a data set. As such the mean is the origin for measuring variation in a data set.

Just as any measurement tool has a zero value from which deviations are quantified, so the mean is the zero point for measuring variation in a dataset.

CONCLUSION

I have argued the need for an origin or zero of such measurement of variation that can be utilized to derive the mean algorithm for statistical, rather than mathematical use. The implication of this approach to define the statistical mean as an object in addition

to a process is that educators have to develop discourse about variation and its measurement, which is the essence of statistical discourse.

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