MATHEMATICAL MODELLING IN THE SCHOOL CURRICULUM

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Extended abstract

Modelling begins with a real problem. The first step in the process is simplifying the problem. I have indicated the steps by arrows (Figure 1). The next step is translating the simplified problem into mathematics (i.e., finding a mathematical model). The third step is to work with the mathematics. Then, once a solution has been found to the mathematical problem, there is the step of translating the solution back to the original situation, and then asking whether the solution is feasible. If there is no feasible solution, we would have had to go back perhaps even to the beginning of the problem, maybe modifying it so that a solution can be found.

Many examples of modelling are tentatively scheduled to be discussed in this presentation, each for a particular reason (see Table 1).

Table 1: Examples of modelling.

Example of modelling	To illustrate
Scheduling golfers into foursomes	The modelling process
Grading students	Teachers model all the time!
Value-added models	An economic model that some want to apply in the U.S. to evaluate teachers
How big is an object?	The many choices we have for a model
IQ for intelligence	A very poor model
Scale models of objects	Geometric models
Path of a projectile	Simplifications made to model algebraically
Cost of mailing a letter	An exact model
Temperature in an oven	An almost-exact, theory-based model
Population of a city over time	An impressionistic model
World record times in the mile	Between impressionistic and almost-exact
Linear regression in much research	An impressionistic model

Many "word problems" in books that seem to be applications (coin problems, age problems, etc.) are phony because they ask for information that needed to be known before the problem could be created; they are reverse-given find. Instead of teaching students how to apply mathematics, they teach students that mathematics has no real

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applications. This particular emerges when students study algebra, because – unlike arithmetic – students do not see algebra in the world around them.

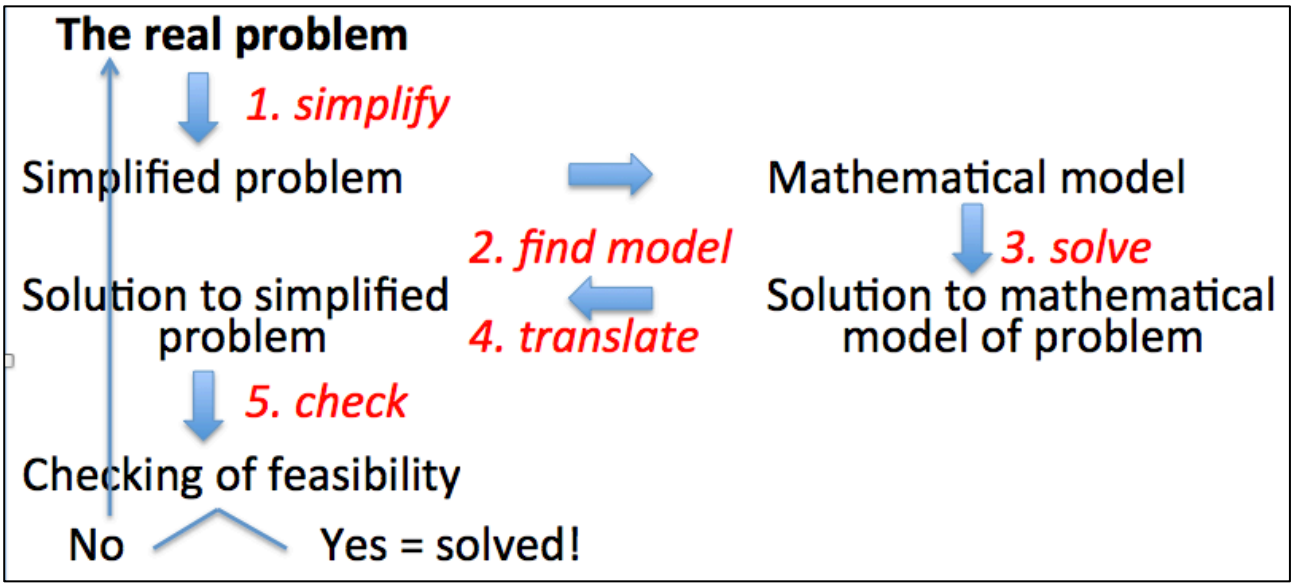

Figure 1: The modelling process.

Even though in the diagram, the mathematical modelling process goes from the real world situation to its mathematical counterpart, in order to teach mathematical modelling we must go in the reverse direction and study the basic uses of each mathematical idea that is being discussed, because most important mathematical ideas have more than one basic use meaning. This is what makes them important.

Table 2: Modelled situations and associated arithmetic.

The arithmetic	Situations the mathematics models
Numbers	Counts, measures, ratio comparisons, locations, identifications or codes, nominal uses
Addition	Putting-together, slide
Subtraction	Take-away, comparison
Multiplication	Acting across (area, array, etc.), rate factor, scale change
Division	Rate, ratio

It is important to keep track of the units in situations; "unit arithmetic" is not only important in science; it is found in everyday arithmetic. It is also important to use calculator and computer technology, for this technology has replaced complicated paper-and-pencil algorithms in the real world.

As a learning progression, it is fundamentally important that in the primary school these uses of numbers and operations go beyond counts to include non-integers. Then, in middle school, these uses can be employed to give meaning to algebraic expressions. From the meanings of algebraic expressions come the situations that functions model. The broad kinds of situations that the various types of functions

model should be as much a part of the curriculum as the mathematical properties of these functions, for it is almost certain we would not be studying these functions were it not for their applications.

Applications are to applied mathematics as theoretical properties are to pure mathematics. Mathematical modelling is the basic process by which we work with applications; the basic process by which we develop pure mathematics is that of proving. From our experience with the teaching and learning of proof, we can learn many things about the teaching and learning of modelling.

Table 3: Modelled situations and associated geometry.

The geometry	Situations the mathematics models
Points	Locations, ordered pairs or ordered n-tuples (data points), nodes of a network, dots
Lines	Shortest path between points, straightness, connections
Figures when geometric transformations are discussed	Drawings and graphs, real objects

The case for modelling as a standard part of the everyday experience in the mathematics classroom is grounded not only on the importance and motivational qualities of good applications, but also on the contributions that modelling and applications make to a deeper and richer understanding of mathematical concepts.