CURRICULUM DEVELOPMENT MINUS TEACHER DEVELOPMENT ≠ MATHEMATICS EDUCATION

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INTRODUCTION

The theme of the 2013 AMESA curriculum is 'Mathematics versus the curriculum' with a tagline that asks 'What's the score?' The spectre that is raised in this title is of school mathematics, as recontextualised from the mathematics disciplinary field into curricula, working in ways that sit at odds with perspectives drawn from the originating field of mathematics. In this paper, I want to add a further layer to this picture of potential for distortion – and this further layer relates to the ways in which the curriculum comes to be enacted in classrooms. My empirical focus, as part of the work I am involved in at Wits, is on primary mathematics classrooms and particularly, in the teaching of early number. In research and development oriented terms, my interest is in exploring the nature of Foundation Phase teachers' mathematical knowledge, with attention that goes beyond what teachers know to the ways in which they appear to hold this knowledge.

Survey research looking at comparisons of quality across education systems is emphatic that this additional layer matters. An oft-cited section from the 2007 McKinsey report (Barber & Mourshed 2007) notes that high performing schools systems:

> 'get the right people to become teachers (the quality of an education system cannot exceed the quality of its teachers)
> 'develop these people into effective instructors (the only way to improve outcomes is to improve instruction) (op.cit.:13)

Research in mathematics education is equally emphatic about the limitations of reforms driven by changes at the level of curriculum, with little attention to teacher development:

> Although the typical methods of improving U.S. instructional quality have been to develop curriculum, and—especially in the last decade—to articulate standards for what students should learn, little improvement is possible without direct attention to the practice of teaching. Strong standards and quality curriculum are important. But no curriculum teaches itself, and standards do not operate independently of professionals' use of them. To implement standards and curriculum effectively, school systems depend upon the work of skilled teachers who understand the subject matter. How well teachers know mathematics is central to their capacity to use instructional materials wisely, to assess students' progress, and to make sound judgments about presentation, emphasis, and sequencing. (Ball, Hill & Bass 2005: 14)

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Extending the complexity further still, Schmidt & Burroughs (2013) point to the need for alignment between curricula attuned to the nature of mathematics in its intended, enacted and assessed forms.

My focus in this paper is to consider the current South African education policy context in relation to the emphasis on teacher development seen in research findings. Centrally, my interest is related to Ball et al.'s comment about the need to emphasize 'direct attention' to teacher development. I begin the next section by briefly overviewing the current South African education policy context, with particular attention to curriculum and teacher development. Given the widespread calls for focus on primary mathematics teacher content knowledge, I note within this discussion key aspects of mathematics teacher development policy at national and provincial levels, and discuss this in relation to theorizations of mathematics knowledge for teaching. In order to consider teaching in the context of curriculum reform, I draw from teacher reflection data collected in the context of a small scale teacher development intervention trialled within our broader Wits Maths Connect – Primary project. This data allows for the beginnings of an exploration about where our primary mathematics teacher development work might need to focus in order to capitalize on the potential of curriculum reform.

SOUTH AFRICAN EDUCATION POLICY CONTEXT – A BRIEF OVERVIEW

Poor mathematics performance on external assessments continues at all levels of the system in South Africa (DBE 2012a; 2012b). Recent responses to this problem have revolved around another wave of curriculum reform – the introduction of the Curriculum and Assessment Policy Statement (CAPS) (e.g., DBE 2011), which, in primary schools, was preceded by the introduction of the Foundations For Learning curriculum (DoE, 2008). Both curricula present increased specification of content, sequencing and pacing, with introduction of national systemic assessment aligned with curriculum and monitoring of this performance making in hard to resist adoption. In Gauteng, schools identified by districts as 'underperforming schools' have an additional layer of provision in the specification of lesson plans within the Gauteng Primary Literacy and Mathematics Strategy (GPLMS).

Prior research in mathematics education in South Africa has been largely critical of previous rounds of curriculum revision, noting problems at the levels of curriculum coverage and pacing (Reeves & Muller 2005), and under-specification of content, sequencing and progression within Curriculum 2005 (Chisholm et al. 2000), viewed as continuing into the Revised National Curriculum Statement (Hoadley, Murray, Drew & Setati 2010). This under-specification was viewed as particularly problematic in a context where significant gaps in teachers' content knowledge and pedagogic content knowledge had been identified (Taylor & Vinjevold 1999). Gaps in mathematics content knowledge continue to be noted as prevalent on the ground in research studies (Taylor 2011) and in the public arena (Jansen 2013), leading to calls

for increasing focus on content knowledge in pre-service teacher education and teacher testing.

Within this somewhat fraught terrain, mathematics teacher development work, focused particularly at the level of classroom skills, continues to be described as insufficient to deal with the scale of the problem (NEEDU 2013). This is noted in spite of acknowledgement of significant spending on upskilling programmes such as the Advanced Certificate in Education courses and others, and the increasing proportions of 'qualified' teachers in the system. Many of the current training offerings are short term in nature and 'off site' – district organized afternoon workshops or Saturday subject content knowledge sessions. Predictably, given the newness of the CAPS curriculum, much recent teacher development work has focused on CAPS-awareness related training. A combination of off-site and in-class support has been on offer to GPLMS schools in Gauteng with 'just-in-time' training aimed at supporting the teaching of content areas scheduled for the following term, and in-classroom mentor support working alongside teachers providing demonstration and assisted lessons and support aligned with GPLMS lesson plans.

The NEEDU report notes specifically the different models of teacher development underlying the GPLMS initiative in Gauteng and the LitNum Intervention in Foundation Phase in the Western Cape, both attempting to work at scale across primary schools. The former works to build content knowledge and classroom competence via classroom based support centred around the use of scripted lesson plans, whilst the latter offers a combination of 5 days training focused on mathematical content knowledge followed by in-school support. Importantly, both of these initiatives have an extended time frame, and have been subject to on-going revisions based on feedback from earlier implementation cycles on the ground.

THINKING ABOUT PRIMARY TEACHERS' MATHEMATICAL CONTENT KNOWLEDGE

These initiatives, and smaller-scale initiatives on the ground like our project, are grappling in a range of ways with issues around supporting the development of mathematical content knowledge, related pedagogical content knowledge and classroom competence. An issue that frames questions of mathematics teacher content knowledge development from the outset relates to questions around what content knowledge primary mathematics teachers require, to what depth and breadth, and to what level. These are questions that have been considered extensively in the mathematics teacher education literature (Ball et al. 2005; Baumert et al. 2010), so there is neither space nor necessity to rehearse them here. For the purposes of the argument I wish to make in this paper, I pick up on one strand of this research which emphasizes the need to pay attention to the ways in which teachers hold mathematics, rather than what mathematical content knowledge they hold. I use Watson & Barton's (2011) idea of teachers needing to know mathematics in ways that emphasize 'mathematical modes of enquiry' – which they delineate as follows:

> Knowing mathematics means being able to use mathematical concepts mathematically: the two cannot be separated. (op.cit.: 66)
>
> It is not just a question of what teachers know, but how they know it, how they are aware of it, how they use it and how they exemplify it. (op.cit.: 67)
>
> To become a fluent mathematician, a student must learn to act in certain ways in mathematical and other situations, to develop mathematical habits of mind, to enact mathematical modes of enquiry, and to think in terms of these underpinning key understandings.' (op.cit.: 70)

The idea of paying attention to the ways in which knowledge is held has substantial antecedents in the literature, and in particular, in writing that has dealt with the notion of syntactic knowledge of a discipline as necessary alongside substantive knowledge (Grossman, Wilson & Shulman 1989; Schwab 1978). In this body of work, substantive knowledge is described as encompassing facts, concepts, structures and explanatory principles within a discipline. In contrast, syntactic knowledge relates to the nature of inquiry and the ways in which new knowledge comes to be established. Watson & Barton's interest in mathematical modes of inquiry can therefore be viewed as an emphasis on syntactic knowledge as underpinning and interweaving substantive knowledge.

My interest in the notion of mathematical modes of enquiry is driven by evidence of Foundation Phase mathematics teaching within our project schools, and more broadly. Across the last two years of our project, a recurrent feature of much of the teaching we have observed relates to episodes in which correct answers are produced within teacher explanations, but produced in ways that show a lack of attunement to the ways in which knowledge is established in mathematics. Two episodes, re-created from fieldnotes and feedback from project team observers, illustrate this view.

Episode 1

'Halving' is the topic being dealt with in a Grade 2 class. The task in focus is working out 'Half of 26'. Each pair in the class is asked to make 26 balls from clay – which they do. The teacher's explanation proceeds as follows: 'I want us to count to 13, and move those balls aside. How many balls are on the other side? 13 as well. So 13 is half of 26.

Episode 2

A Grade 1 class are working on number bonds of 5. Each learner pair has a set of 5 bottle tops in front of them. The teacher asks for two numbers that add up to 5. Learners produce a 4/1 split. The teacher asks learners to separate the two groups. She then says: 'Put them all together and count how many you have altogether.

In both of these instances, teachers are working with content that is grade appropriate in relation to the curriculum and its sequencing. No overtly wrong answers are produced in either case, but mathematical purpose, direction, connection and the

ways in which given information can be used to derive unknowns are disrupted. Writing at both primary and secondary levels (Bansilal 2012; Venkat & Adler 2012) provides evidence suggesting that the phenomena is a broad one. In previous writing emanating from the project, we have discussed elements of this prevalence, noting that our data suggests that it is not content knowledge in any simplistic sense that appears to be at the root of the problem in Foundation Phase – given that Foundation Phase teachers appear able to produce the correct answers to the problems they are working with (Askew, Venkat & Mathews 2012). One way of conceptualizing this problem is in relation to temporal scope, with production of the right answer dealing with the temporal present in ways that allow the teacher to move on to the next problem, whilst not dealing with the connections that locate mathematical concepts within networks of ideas that build and broaden over time. In a forthcoming paper, I describe this phenomena in the following terms:

> teaching often either accepts, or produces, the answer to the immediate problem without attention to the broader understandings and longer term efficiencies needed for autonomous student work with similar and related problems. This production allows lessons to progress without any need for learning to progress within them. (Venkat, forthcoming)

Across this evidence, the 'problem' seems to relate more to ways in which mathematics knowledge is known rather than the knowledge components per se. This point leads back to this conference's concern with curriculum formulations in relation to mathematics. Watson & Barton's concern relates to the ways in which teachers' work aligns with disciplinary ideas about mathematical working. The policy context outline above notes that the CAPS curriculum has moved in the direction of increasing specification of content and sequence, with this increasing specification viewed largely as a necessary corrective in an arena of content knowledge gaps. My question in this paper is with the nature of the gaps in Foundation Phase teachers' content knowledge and whether increasing specification of (primarily substantive) content can fill gaps that appear at the syntactic level.

DATA SOURCES

In order to consider this issue, I share data drawn from our experiences of collaborating with two colleagues, Anna-Lena Ekdahl and Ulla Runesson from Jönköping University, Sweden running a teacher development project based on a 'Learning Study' (Lo & Pong, 2005). Learning studies are a teacher development model based on the Japanese lesson study model, which involves teachers working with an experienced teacher and researcher to understand learner understandings of a specific 'object of learning', and then planning a series of lessons together that build these understandings.

Variation theory forms the underpinning theory of learning in Learning Studies. A central tenet of variation theory is that learning is made possible through discernment of variation of the critical features that constitute an 'object of learning'. Further, objects of learning have both specific and general features. Specific features can be related to the ability to select actions that allow for completion of the problem at

hand, with general features relating to the longer term understandings and competences that working with particular tasks is intending to develop. In line with its antecedents in phenomenography (Marton & Booth 1997), what is critical to perceive about objects of learning is viewed 'relationally' – i.e. as emergent in encounters between learners and what it is they are trying to learn. Thus, the critical features in focus often shift across the course of a Learning Study.

Our Learning Study was located in a suburban Johannesburg primary school serving a township, inner city and local area learner body. The school is one of the primary schools in our broader Wits Maths Connect-Primary project, a 5 year research and development project working with ten primary schools in one district. Data collected during 2012 from the Grade 3 cohort in a Maths Club at this school suggested that whilst there had been some shift from 'count all' to 'count on' based strategies, grouped counting strategies were only used in the context of availability of specific resources (money, bead strings), with reversions to unit counting if these resources were unavailable (Takane 2013). Further, Foundation Phase teachers in this school had told us during a 2012 focus group interview that whilst learners were able to add or subtract two given numbers, they were usually unable to solve problems involving either missing addends/subtrahends or missing start numbers:

e.g. $7 + __ = 12$ or $__ - 6 = 15$

When we asked the teachers how they dealt with learner inability to solve these problems, one noted that she would then tell her learners that they had to take away 7 from 12 or add 6 to 15 in the above instances. She noted though, that many learners were not able to follow these rules, and either wrote incorrect answers, or left out these kinds of problems. These findings, coupled with the broader South African literature detailing the on-going use of concrete counting based strategies with addition/ subtraction (Ensor et al. 2009; Schollar 2008) led to our focus on solving part-part-whole relations in additive situations and the partitioning competence that underlies these solutions for a small scale intervention project.

The Learning Study afternoon planning meetings involved the three Grade 2 and three Grade 3 teachers in the school, with follow up videotaping of each of the three jointly planned lessons taught by the Grade 3 teachers. The whole process took 3 weeks and was conducted in February 2013. Pre- and post-tests focused on learner awareness of partitions and part-part-whole situations were conducted at the start and end of this process. Follow up learner interviews and group and individual teacher interviews completed the Learning Study. Video records of planning meetings and interviews, and then video records of all three Grade 3 teachers teaching the planned lessons formed the central data sources for the Learning Study.

Whilst the research focus within learning study research is more usually on the ways in which differences in what is made available to learn impacts on what is actually learned (Liljestrand & Runesson 2006), my interest here is primarily on analyzing the final focus group and individual teacher reflections on the process of involvement in

the Learning Study. The learner performance overview of pre- to post-test findings is included simply to show that it is possible, in relatively short periods of time, to shift learner understandings with changes in teaching emphases, with the need to remain realistic about what these shifts consist of. The analysis of the detail of the Learning Study data is on-going and in progress currently. Central to this paper are data on teacher reflections that point to limitations in their understandings of what acting mathematically consists of with important gaps that this intervention brought some awareness of.

Given my interest in the alignments between teaching of these ideas, and the ways in which they are described mathematically and pedagogically in the literature and inscribed in the curriculum, I briefly outline relevant literature and the Foundation Phase CAPS curriculum treatment of additive relations.

ADDITIVE RELATIONS – LITERATURE

Carpenter et al (1999) have identified the range of additive relation situations as: join/separate (change) situations, part-part-whole (combine) situations and compare situations. Within each of these problem types, the 'missing' quantity can be varied to represent total, initial value or change value in join/separate situations, and part or whole quantities in combination and comparison situations. Carpenter et al have noted that whilst 'start unknown' problems are much harder for learners given the absence of an initial reference, making 'direct modelling' of the situation through a concrete replay involving count actions redundant and requiring a more abstract sense of the structural relations between whole and parts.

An influential piece of writing in the phenomenographic literature focused on additive relations is Dagmar Neuman's (1987) work which aimed to understand why learners perform differently on early addition problems. Whilst the original work was in Swedish, the results have been reported in other studies (Marton & Booth 1997). Neuman's work argues for teaching that encourages structural, rather than operational conceptions of number relationships. This work pushes the need to discern the nature of the number relationships in additive relation problems.

Literature notes the need, for the sake of efficiency and for instances where operational options cannot be directly applied, for teaching to incorporate a focus on structural models that show part-part-whole relationships. Two kinds of diagrams of part-part-whole relationships are common in the literature and in textbooks – a rectangular model and a triad model. Both are shown in Figure 1, applied to the context of the following problem:

Thandeka has seven cakes. She has three more cakes than Kim. How many cakes does Kim have?

As the number range in additive problems increases, knowledge of partitions of numbers in the lower number ranges supports fluent problem solving in higher number ranges: e.g. to solve 18 + 7, selecting 2 + 5 as a useful partition of 7, and then

applying this partition to the original problem to give 18 + 2 + 5 allows for efficient problem-solving.

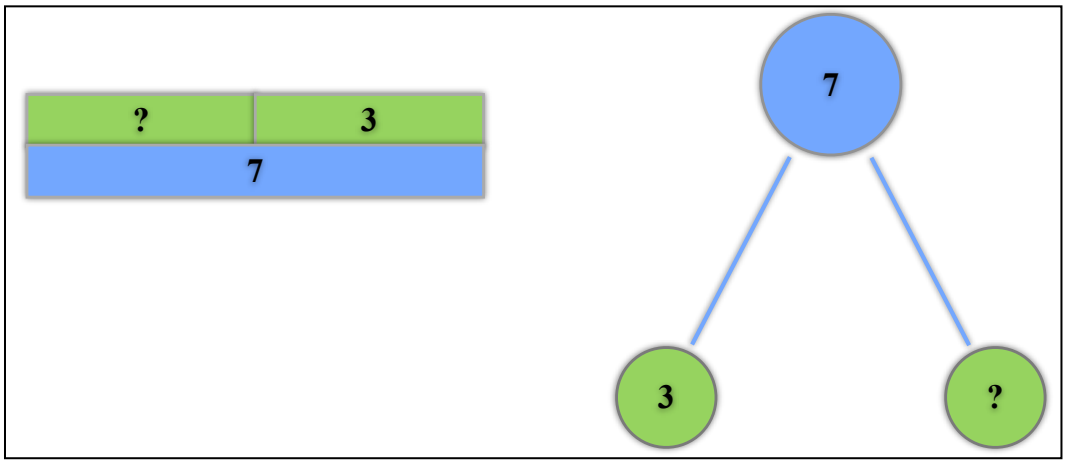

Figure 1: Rectangular and triad models to represent part-part-whole relations.

Similarly, to work out 25 = 18 + __ , it is useful recognize that a jump of 2 from 18 produces 20, with a further 5 producing 25. These analyses lead to broad agreement in the literature that teaching should work to support the development of all additive relation facts at the level of recall to 20.

ADDITIVE RELATIONS – CAPS FOUNDATION PHASE

Number-related concepts make up the majority of the Foundation Phase Mathematics curriculum, allocated a 65%, 60% and 58% content weighting respectively across Grades 1 to 3 (DBE 2011: 12). Addition and subtraction are dealt with across all three grades, in the midst of 'word problems and 'context-free' problems. The comparison and ordering of numbers that underlies competence with part-part-whole relations also figures across all three grades. The need to include change, combine and compare examples is noted explicitly across all three grades, as is the need to incorporate problems where the missing value varies across start, addend/subtrahend and total.

Partitioning skills are focused on 'breaking down' numbers into constituent parts. This idea is explicitly mentioned in the CAPS curriculum across all three grades as a 'technique when solving problems' (op.cit.: 23). Extension of number range and building towards more efficient abstract strategies are two key recurring features of progression, with the expectation by the end of Grade 2, of 'rapid recall' of all number pairs that add up to give 20 (op.cit.: 356).

Of interest within my focus on part-part-whole relations and partitioning numbers, is the limited explicit attention to how learners can be supported to identify and model different problem types or to identify wholes and parts in a range of additive relation situations (there is mention in Grade 1 on 'getting learners to think about numbers as composed of other numbers' (op.cit.: 110). This understanding is required in order to select the appropriate strategy. A further feature, given the problem-solving imperative that requires a learner to both identify parts and wholes and then select an

appropriate decomposition, is the lack of emphasis on working systematically to note all or at least many of the possible decompositions.

FINDINGS AND ANALYSIS

In Table 1 I provide an overview of shifts in performance between the pre- and post-tests that book-ended the Learning Study intervention. Matched pre- and post-test data were available for 123 learners across the three classes in Grade 3 in the school.

Table 1: Pre- and post-tests results of the Learning Study intervention.

Question 1	Number of learners in this category	
No of partitions of 9 into 2 boxes	Pre-test	Post-test
0	44	24
1	18	4
2	17	6
3	11	6
4 to 9	33	54
10	0	29
Question 2	**Number of learners in this category**	
No of partitions of 12	Pre-test	Post-test
0 to 4	45	22
5 to 9	59	43
10	19	58
Question 3	**Number of learners in this category**	
Mixed part-part-whole problems	Pre-test	Post-test
8 + 7 = _	118	109
10 - _ = 7	103	99
_ - 2 = 3	70	89
14 = _ + 6	64	72
_ + 9 = 13	67	77
_ - 4 = 6	34	66
6 + _ = 11	63	83
10 = _ - 6	16	13

On Q3, which consisted of a variety of missing part/missing whole problems, some learners left some problems unanswered, so the figures in the table show the numbers getting that problem correct.

Some features of the test are noteworthy. Firstly, the partitioning of number expected in this test (of 9 and 12) fall below what is expected in the CAPS curriculum for

Grade 3. Similarly, the number range of the part-part-whole problems in Q3 falls within the number and problem range expected in Grade 2.

Learner performance shows large improvement across the period of the Learning Study on producing all or many partitions of 9 and 12 (up from 0 to 29 learners for 9, and 19 to 58 learners for 12). Within the intervention lessons, we saw in learners' worksheet responses on tasks looking at breaking numbers down into two parts, that there were shifts towards more systematic production of partitions using either commutativity or compensation strategies. However, using partitioning skills strategically across the range of part-part-whole problems given in Q3 shows more mixed results, with some small increases or decreases. Of interest is that relatively large positive shifts in performance are seen in missing start problems, and especially so in the missing start problems involving subtraction.

The increases in the number of learners able to produce many or all partitions of numbers through systematic working suggested that features such as systematicity and completeness had not featured as prominently in these teachers' handling of additive relations ideas previously. A post intervention focus group meeting with the Grade 2 and Grade 3 teachers indicated some corroboration for this view. Reflecting on the interventions, one Grade 2 teacher commented, with others agreeing with her:

> If you are doing the bonds of 10, you'll just go in the class and say: 'Now we are going to do the bonds of 10, and you are going to give me two numbers that is going to build 10. […]
>
> And then, to realize or identify – like the one that you say? Systematic. We didn't know that we supposed to do that systematically. We are just giving the numbers haphazardly.

Differences in relation to how they taught a topic that all of them had taught before were also acknowledged:

> 3C teacher: 'The way we were teaching didn't help much because children were making mistakes, a lot of them. But now, with this one, as we moved from one stage to another, making combinations, then the structure, then the part-part, I can see that children were getting it right. And they could solve the problems. [..] This method is better than what we have been doing. We didn't know what to do, but this one is much better.
>
> 3B teacher: 'Before it was nearly the same but I didn't stress the word of splitting. And then the part-part-whole, I didn't use it. But they did do it. Like 15, you can say 10 + 5. And again, I didn't mention the word – u swap, when I taught them before. We did it, but I didn't stress it.'

In addition to seeing the importance of working systematically, some comments also indicated a lack of awareness of 'efficiency' or 'brevity' as an important component of mathematical development:

> 3A teacher: I learned that they [learners] can get answers without counting. As long as they know that these two parts can interchange, they will easily get their answers.

Given the prevalence of concrete counting-based strategies in primary mathematics in South Africa, there are indications that a lack of attunement to efficiency within mathematical working is widespread.

Across all teachers, there were positive comments about the openings to discuss and plan lessons together, as well as time to watch excerpts of each others' teaching. This comment and elements of the previous answer suggests that some shifts may have been effected at the level of substantive knowledge. Importantly, in the recognition that we saw at the end of the Learning Study of the need for systematic working and completeness, I would also suggest that some learning has been effected at the level of syntactic knowledge – in relation to the 'norms' of what is entailed in working in mathematically attuned ways.

DISCUSSION

What do these findings suggest for the possibilities of seeing improvements in learning in the CAPS context – a context dominated by a highly specified and sequenced curricular form? Firstly, it would appear that increased specification of content cannot, in and of itself, address gaps at the level of syntactic knowledge – the level that seems to relate most centrally to mathematical modes of inquiry and the kinds of practices that point towards the practices of mathematics as a discipline. Watson & Barton (2011) note this, cautioning that:

> For us, lists of required mathematics content knowledge are hypothetical until mathematical modes of seeking, using, and exemplifying understanding are understood. (op.cit.: 66)

Secondly, it appears highly unlikely that once off, 'just in time' models of training can change teachers' modes of thinking, as their focus on supporting immediate delivery tends to push towards a 'how to complete tasks' orientation rather than an orientation towards mathematical inquiry. A long trajectory of research in teacher development suggests that successful teacher development extends over time. Finally, Watson & Barton's notion of mathematical modes of thinking rests on extended opportunities to do mathematics in mathematical ways – working within programmes to build both substantive and syntactic knowledge over time. Current models of teacher development in South Africa suggest that this framework is not currently in view.

Anna-Lena Ekdahl's closing comment to the teachers following thanking them for their participation in the Learning Study was as follows:

> It really matters how you teach. It makes a difference.

My sense is that part of what shifted in relation to these teachers' handling of additive relations was related not just to the concepts and skills needed to complete problems, but also to a shift towards seeing systematic working, completeness of a set of solutions, and attention to supporting efficiency as integral to this working. All of these features are important aspects of working mathematically whilst working with mathematical content. In a terrain where there is broader evidence from Foundation Phase mathematics teaching that suggests that this attunement towards working mathematically may be lacking, indications of this shift are potentially as important as growth in substantive knowledge of mathematics.

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