THE USE OF THE IDEA OF *COHERENCE* IN DESCRIPTIONS AND ANALYSES OF SCHOOL MATHEMATICS CURRICULA, TEXTBOOKS AND PEDAGOGY

Zain Davis

School of Education, University of Cape Town

INTRODUCTION

At present the idea of *coherence* is enjoying a fair amount of attention in local discussions of school mathematics curricula, texts and pedagogy. Given that the results of students on mathematics tests are often cited in the introductory remarks in many papers, it seems that the appeal to coherence is intended to assist us to diagnose the state of mathematics education in schools, as well as to assist us to find a way to repair the apparently dismal state of school mathematics as indexed by the performances of South African school students on high stakes examinations. The take up of the idea of coherence as a diagnostic and explanatory tool appears to have been popularised by the large volume of work, mostly in the USA, that has emerged around TIMSS as well as around the work of the *Consortium on Chicago School Research*. So, while this essay is a reflection on local uses of *coherence*, a brief review of the relevant work of TIMSS is necessary.

TIMSS AND COHERENCE

What *coherence* might be is often dealt with by researchers by appealing to the widely-cited gloss of the term due to Schmidt and his collaborators: "We define content standards and curricula to be coherent if they are articulated over time as a sequence of topics and performances that are logical and reflect, where appropriate, the sequential or hierarchical nature of the disciplinary content from which the subject matter derives" (Schmidt, Houang & Cogan 2002: 19). This take on coherence emerges in a host of related papers (Beane 1995; Cogan, Schmidt & Wiley 2001; Ferrini-Mundy, Burrill & Schmidt 2007; Schmidt 2004; Schmidt 2008; Schmidt & Houang 2012; Schmidt, Wang & McKnight 2005; Schmidt & Prawat 2006; Schmidt, McKnight, Cogan, Jakwerth & Houang 1998; Valverde & Schmidt 2000). The TIMSS research pursed by Schmidt and his collaborators develops a measure of *incoherence*, by comparing the teaching of curriculum content with a stipulated curriculum which is taken as a model of coherence, using five indicators: (1) teaching a topic before the first grade for which it is indicated; (2) "sins of commission": teaching a topic in a grade for which it is not indicated; (3) "sins of omission": failing to teach a topic in a grade for which it is indicated; (4) teaching a topic later than the final grade for which is indicated; (5) breaks in the teaching of a topic across grades where such breaks are not indicated in the model (Schmidt & Houang 2012). The idea is that an overall measure of the degree of (mis-)alignment

Davis, Z. (2013). The use of the idea of *coherence* in descriptions and analyses of school mathematics curricula, textbooks and pedagogy. In Z. Davis & S. Jaffer (Eds.), *Proceedings of the 19th Annual Congress of the Association for Mathematics Education of South Africa*, Vol. 1. (pp. 35 – 46). Cape Town: AMESA.

with a stipulated curriculum (which is taken to be the model of coherence) can be derived by using the five indicators.

Related work on the TIMSS video study (Hiebert *et al*. 2003) addresses the issue of coherence in pedagogic contexts by examining the relations between subsequent problems in the sequence of problems that might be employed in a lesson, as well as considering the topic-relatedness of those problems. The nature of connections between successive problems were examined, and all but the first problem of a lesson were coded as being *one* of : (1) a repetition; (2) mathematically related; (3) thematically related; and (4) unrelated. They also calculated the proportion of problems related to a topic as part of their measure of the "clarity and coherence" of lessons.

Hiebert *et al*. attempt to develop and use a measure of problem complexity as part of their investigation of coherence, but their account of such complexity remains underdeveloped in its reliance on the number of decisions needed to be made and the presence or absence of sub-problems. What a "decision" might be in their scheme is rather unclear. Are decisions only explicitly made choices? Or are they also implicit made choices that have a regulative effect on the choices made at the local level of the particular problem? The same goes for their use of the idea of a "step" when they attempt to measure *procedural complexity* of problems by counting the "operational steps" per problem. What is a "step"? Is it a line of written text? Or might it be any computation, including those not written down but which are computationally necessary? Further, are computations always transparent, so that one can decide with a reasonable degree of certainty what the decisions and steps are? Problems are taken to be: (1) of *high complexity* if the student needs to make more than 4 decisions and solve 2 sub-problems; (2) of *moderate complexity* if more than 4 decisions are required along with the solution of 1 sub-problem; and (3) of *low complexity* if 4 or fewer decisions were required and no sub-problems.

Ultimately, though, it is *content continuity* that is to be used to gain an impression of the degree of coherence of mathematics lessons. Content continuity, accompanied by increasing procedural complexity and non-repetitive mathematical relatedness across a segment of teaching, seems to be the acme of mathematical coherence in lessons for Hiebert *et al*. (2003).

OPPORTUNITY-TO-LEARN AND COHERENCE

In the work of Reeves and her collaborators we find the idea of coherence aligned with the notion of opportunity-to-learn (also referred to as 'OTL'; see Smith, Smith & Bryk 1998) in the form of "curricular coherence", one of the central dimensions of OTL (see also, Gillies & Quijada 2008; McDonnell 1995; Reeves 2005; Smith 1998; Stevens 1996; Stevens no date; Wang 1998).

> More recently, the OTL construct has been expanded to include measures of 'curricular coherence', that is, the degree to which domain-specific or disciplinary content is systematically presented to learners in terms of the conceptual coherence of its organization, and 'curricular

> pacing', the structuring and organization of curriculum across adjacent grades. The idea is that curricular pacing and coherence helps prevent a cumulative deficit in breadth and depth of domain specific knowledge and conceptual advancement of specialized skills and concepts across grades improving the likelihood of learners having the prerequisite content knowledge for the next year […] (Reeves & Muller 2005: 107-108)

Reeves and her collaborators detect a general lack of coherence at the level of curriculum implementation in South African mathematics teaching (Reeves & Muller 2005; Reeves & McAuliffe 2012). The message is that the curriculum is coherent, but that pedagogy is very often problematic in South African schooling contexts because, they aver, mathematics teaching tends to lack coherence and consequently robs students of OTL. Reeves and her collaborators also refer to coherence as "the degree to which domain-specific or disciplinary content is systematically presented to learners in terms of the conceptual coherence of its organization" (Reeves & Muller 2005: 107) but they are silent on what such "conceptual coherence" might be. Reeves & McAuliffe (2012: 20) arrive at a measure of coherence by asking an expert—"a highly qualified and experienced mathematics teacher, who had served on the committee that developed the revised *National Curriculum Statement* for mathematics"—to assess whether or not a representation of a sequence of pedagogic activity indicates coherence. At the end of the day, for them, incoherence at the level of curriculum implementation is indicated by the presence of two pedagogic failings, one of which is the disruption of the official curriculum sequencing of content, and the other, a dramatic decrease in the pace of content delivery, both of which result in a loss of OTL for students.

Schmidt and Reeves, along with their respective collaborators, argue that the organisation of the implemented curriculum in local pedagogic contexts can produce pedagogic effects that result in weak curricular coherence, with the consequent negative effects on school students' opportunities to learn mathematics, when the sequencing and pacing of content delivery fails to align with curriculum statements. Outside of the work of Hiebert *et al*. (2003), neither of the two groups really say much about coherence in terms of the detail of the mathematics content that is taught. There are, however, local attempts at engaging with aspects of pedagogy beyond sequence, pace and alignment. One example of which is Venkat & Naidoo (2012), who use elements of Systemic Functional Linguistics to measure the coherence of the language use of teachers in mathematics lessons.

SYSTEMIC FUNCTIONAL LINGUISTICS AND COHERENCE

Venkat and Naidoo attempt to measure the linguistic coherence of pedagogic communication of mathematics teaching by studying the linguistic referencing that emerges in pedagogic communication in terms of what Halliday & Hasan (1985) refer to as *co-reference*, *co-classification* and *co-extension*. We can describe the three terms in a simplified manner as follows, taking pairs of linguistic references as our object: co-reference refers to different utterances referring to the same thing; co-classification refers to different utterances referring to non-identical things, but both

of which belong to the same category; and co-extension refers to different utterances referring to non-identical things that belong to non-identical categories, but with the non-identical classes both belonging to a more general category. For Halliday and Hasan, the coherence of a text is realised through the generation of *cohesive chains*. Such chains are sequences of language that hang together by means of relations of identity and similarity, which Halliday and Hasan describe as *identity chains* and *similarity chains*. The linguistic chains are composed of *cohesive ties* established through the use of co-reference, co-classification and co-extension, with co-reference realising identity chains, and co-classification and co-extension realising similarity chains.

In their analysis of the mathematics teaching in a Grade 2 lesson, Venkat and Naidoo discover instances of the "ambiguous use of co-reference", of "disconnections in co-classification" and of "incoherent co-extensions", each indicating a weakening of linguistic coherence in pedagogic communication because the formation of cohesive ties are disrupted. If their analysis is valid, it might well be the case that the students who participated in the lesson experienced difficulties in grasping and learning some of the mathematics content because of the general incoherence of the pedagogic communication to which they were subjected.

While the analytic method offered in Venkat & Naidoo (2012) engages with the mathematics of classrooms in terms of the referential aspects of pedagogic communication in an attempt to measure coherence, it doesn't really engage directly with the mathematical processing of teachers and their students. In Venkat & Adler (2013), however, we do find an attempt to deal with the problem of coherence by engaging with the mathematics work of teachers and students more directly.

PROCEDURAL TRANSFORMATIONS AND COHERENCE

Venkat & Adler (2013) construct a method for measuring the coherence of what they refer to as *mathematical discourses in instruction* (essentially, pedagogic communication) by examining transformations of the representations associated with typical mathematics problems used in the teaching of curriculum content. The core elements of the method consist in: (1) identifying a stated mathematics problem (*stated problem*); (2) identifying the initial expression/representation that is used to start work on the problem (*initial representation*); (3) describing the method used to work through the problem, starting with the initial representation (*transformation activity*); (4) answering four yes/no questions: (4.1) does the initial representation connect with the stated problem appropriately? (4.2) does the transformation activity connect with the initial representation in mathematical terms at interim stages? (4.3) does the transformation activity connect with the initial representation in mathematical terms at the final stage? (4.4) does the transformation activity and the associated explanation "establish connections between its steps with the initial representation?" The number of 'yes'/'no' answers to the four questions listed under

(4) indicates the degree of coherence/incoherence of the particular "mathematical discourse in instruction".

Venkat and Adler's method centres on deciding at the various points in the solution of a problem whether a mathematically defensible link, or "connection", is maintained with previous transformations of the successive representations that emerge as a problem is worked through. The methodological problem that Venkat and Adler might consider exploring in greater depth is whether or not their four questions (4.1 to 4.4) can really be answered in the manner that they do. The central issue of interest here can be expressed in the form of a question: "What constitutes a *connection* between initial, interim and terminal representations?" The problem is that the idea of connection, which is made to do so much analytic work in the method, remains underspecified.

In mounting their arguments for measuring the degree of mathematical coherence in pedagogy and problem solving, both Venkat & Naidoo (2012) and Venkat & Adler (2013) imply an acceptance of a view of school mathematics that reads the content that is realised in schooling as that described in official curriculum statements. The same can be said of the implicit views of school mathematics in the discussions of coherence in the work of Schmidt and Reeves and their collaborators. One of the effects of such—perfectly reasonable—views of school mathematics is that there are computational features of the mathematical activity of teachers and their students that are central to the realisation of mathematical coherence but which escape consideration in the analyses discussed thus far.

TOPIC, CONTENT AND COHERENCE

In Jaffer (2012) we find an approach to the question of coherence that does not treat the mathematics constituted in pedagogic contexts as always-already given by official curriculum statements. Drawing on Eco's (1984) discussion of textual coherence, Jaffer argues for using the terms of *topic* and *content* in a way that takes topic names to be labels used to indicate that some or other selection of mathematics contents should be focused on, and contents are what come to constituted in response to those labels. For Jaffer, how a topic name comes to be associated with content depends in part on what Eco refers to as *isotopies*. An isotopy is a textual marker the reader uses as a clue to the topic, enabling them to work towards constituting a coherent text. Jaffer argues that, just as Eco's readers search texts for isotopies, so mathematics students read the computational activity that they are routinely required to engage in for isotopic markers to get clues about the mathematics topics they engage with. As in the case of Eco's readers, the isotopic markers recognised by mathematics students enable them to work towards constituting coherence. What this means is (1) that the content constituted in the name of an announced topic in part comes to be through the interpretative work of the participants as they work on generating isotopic coherence, and (2) the texts produced can possess internal coherence which might or might not be aligned with some or other external model of coherence.

Jaffer's take on coherence is clearly different from the other positions discussed in that it focuses in the first instance on reading the coherence produced by the interpretative work of pedagogic agents rather than immediately measuring the texts that they produce for coherence by way of comparison with an external model of coherence. What this methodological move enables her to do is to gain some insight into *what* does come to be constituted as the content, *how* the content comes to be constituted, as well as being suggestive of *why* the content takes the form it does, all of which can productively inform our understanding of the educational and pedagogic problems that plague us as well as suggest ways to proceed. That said, we must note that without some model of coherence we cannot proceed very far beyond the initial level of description of the mathematical work of teachers and their students generated by Jaffer's approach.

COMPUTATIONAL STRUCTURES AND COHERENCE

In this section I introduce an additional dimension of coherence into the discussion—one that I believe remains largely ignored, or unnoticed, in the research literature. Whatever comes to be constituted as the mathematics content of a topic, whether in curricula, textbooks or pedagogic activity, is either a clear restatement of some or other mathematics content, or it is something that is intended to effect the same outcome as some or other mathematics content.

For example, the term *cancellation* is used routinely in the teaching of school mathematics, where most teachers intend the term to refer to the composition of inverses (often, additive or multiplicative). Many students do, however, read their teachers' uses of the term cancellation as referring to the deletion of elements of expressions that are spatially situated in particular ways, thus generating 'operations' that are different from the arithmetic compositions intended by their teachers. Much of the time such 'cancellations' produce outcomes that are consistent with the composition of inverses, but there are times when they produce outcomes that cannot be integrated with the arithmetic of the real numbers and, for an adept, the result is an immediately recognisable lack of mathematical coherence at the local computational level.

Consider the work of an otherwise very successful Grade 10 student, referred to by her teachers at the private school she attended as one of their top Grade 10 students, and who consistently scored above 80% on her mathematics tests and examinations. On her mid-year examination she encountered a problem requiring her to simplify the expression $2+\frac{3}{a}+\frac{4}{a-1}$. Her attempted solution to the problem, along with her teacher's marking of the solution, is shown in Figure 1. The student computed what she believed to be a 'lowest common denominator', $a-1$, from the terms in the expression and then attempted to multiply each of those terms by $a-1$, 'cancelling' where she thought it was appropriate to do so. From other work done by the student it appeared that she computed lowest common denominators by finding the smallest

value into which all of the denominators divide. Her computation of the lowest common denominator in this instance is curious because it, apparently, doesn't agree with her usual procedure. However, once we understand what she means by cancellation we can see that has applied her procedure for computing lowest common denominators faithfully.

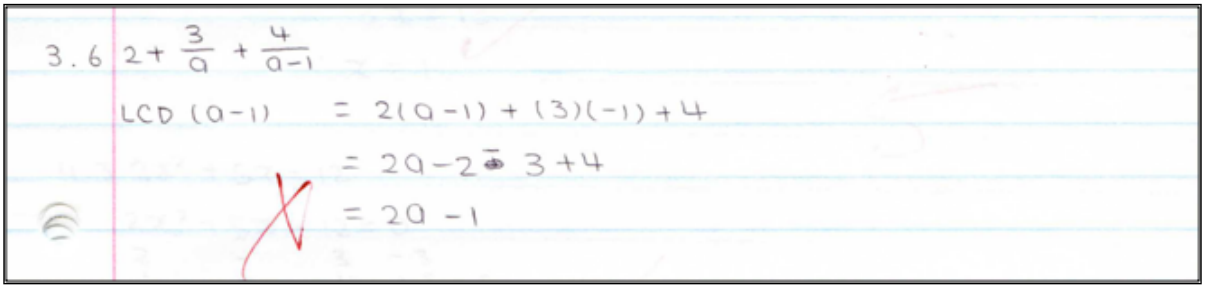

Figure 1: 'Multiplying' $2+\frac{3}{a}+\frac{4}{a-1}$ by the 'LCD' $(a-1)$.

When the student was shown her attempted solution and asked to explain how she went about solving it, an interesting and revealing exchange between the student and the researcher about her 'cancellation' of terms ensued.

Student: I thought that if you times by *a* minus one … it is gonna be … of each other … I thought it is gonna give minus three.

Interviewer: Now, explain what you did here … You divided? … You cancelled? … What did you do to the *a*?

Student: Um. It is the same as if you … it's minus … it's four. I thought it is the same as if you have fraction three over four and you times by four, it is going to be cancelled each other because …

She writes down the computation shown in Figure 2 as she speaks.

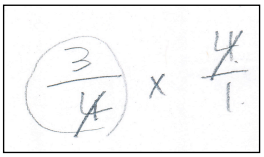

Figure 2: Cancelling the 4s in $\frac{3}{4}\times\frac{4}{1}$.

Interviewer: But you didn't cancel the minus one there …

Student: Ja, so that is … Oh, no … But … Yes … *a* minus one… Isn't it … there different thing, there difference between *a* and minus one?

Interviewer: Explain. I am not quite sure what you mean.

Student: Um … I thought the *a* and the minus one is separate things … Do you know what I mean?

Interviewer: Is it, *a* times minus one?

Student: No, it is not *a* times minus one because if it was it is going to be minus *a* … Right? They wrote *a* minus one. They mean … The teacher wants to me to …

> They just wanted … They just wanted *a* subtract … They say *a* minus one. So, I thought that they are separate things.

She writes down the computation shown in Figure 3 as she speaks.

Interviewer: Because of the way it is said?

Student: I said that it can be cancelled … each other.

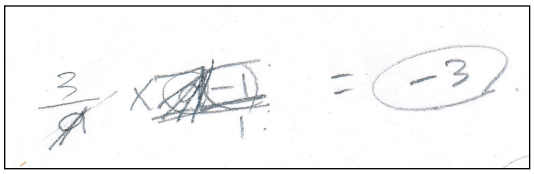

Figure 3: 'Cancelling' the *a*s in $\frac{3}{a} \times \frac{a-1}{1}$.

The type of cancellation performed by the student doesn't produce any conflicts with arithmetic in instances like her example of $\frac{3}{4} \times \frac{4}{1}$, but it clearly does violate our real number arithmetic in her example of $\frac{3}{a} \times \frac{a-1}{1} = -3$, where she 'cancels' the *a*s to produce $3 \times -1 = -3$. Her cancellation 'operation' is one that takes as its arguments (inputs) the spatial positions of what she determines to be identical objects—4 in the instance, *a* in the other—and its value (output) is a void. The 'operation' is sensitive to the vertical position of identical objects, constituting them as pairs of arguments—top and bottom—which are deleted from expressions to leave a void. We can thus describe the students cancellation 'operation' as one that maps pairs of identical objects that are spatially situated so that they are vertically distinguishable as (top, bottom) or (bottom, top), to a void. The scriptural indicators of the 'operation' are usually oblique slashes drawn through its arguments. We can also describe the 'operation' using a more concise symbolism by writing CAN:(a_{top}, a_{bottom}) → *void*, which is to be read as CAN maps the pair (a_{top}, a_{bottom}) to *void*. Since CAN is clearly commutative, we can also write CAN:(a_{bottom}, a_{top}) → *void*. In Figure 4 we see a good example of her use of cancellation to produce a void, which appears to be the outcome of the 'operation' consuming the two arguments.

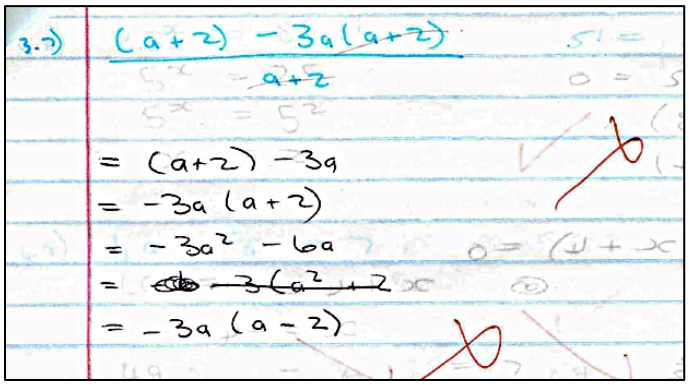

Figure 4: The use of the pair $(a-2, a-2)$ once causes each $a-2$ to vanish.

Elsewhere in the student's work, however, we do find instances where she uses the term *cancel* to refer to an operation that looks like CAN, but which produces 1 as a value, even when she's dealing with letters rather than numbers (see Figure 5).

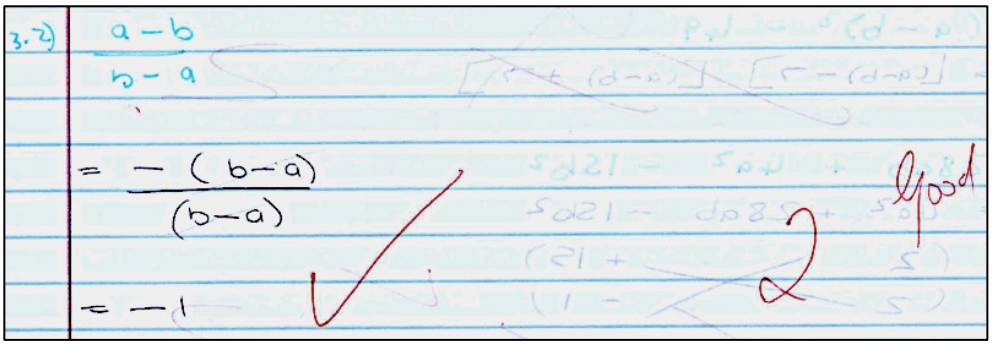

Figure 5: Cancelling $(b-a)$ in $\dfrac{-(b-a)}{(b-a)}$

Given that she uses both CAN:(a_{top}, a_{bottom}) → *void* and CAN:(a_{top}, a_{bottom}) → 1, we need to distinguish them. Let's rename the first CAN_v:(a_{top}, a_{bottom}) → *void* and the rename the second CAN_1:(a_{top}, a_{bottom}) → 1. When we examine the instances in which the two 'operations' occur in the student's work, we find that she uses CAN_1 whenever she has expressions or sub-expressions of the form $\dfrac{x}{x}$, and CAN_v whenever she believes that there are identical terms that meet her vertical position criterion, but which are not part of the same sub-expression, like $\dfrac{3}{a}\times\dfrac{a-1}{1}$ or $\dfrac{-12a-1}{20a}$ (see Figure 6). This difference appears to be the result of her criterion for recognising and constituting arguments for cancellation: whenever she encounters expressions from which she can constitute a pair of terms as (x_{top}, x_{bottom}), they can be cancelled.

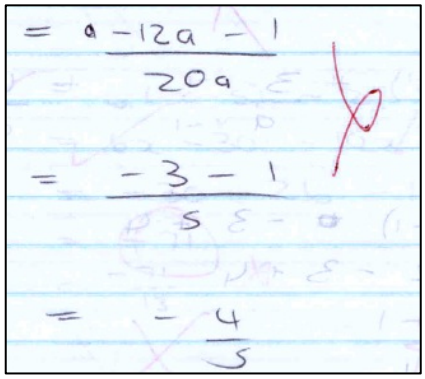

Figure 6: 'Cancelling' the *a*s and the 'common factor' 4 in $\dfrac{-12a-1}{20a}$.

In each of the instances discussed, as far as the student is concerned, she's merely cancelling, which is an operation she has seen her teachers do hundreds of times. The student has four different computational resources for dealing with the composition of multiplicative inverses in her computational repertoire:

(1) division, as in $x \div x = 1$, where $x \neq 0, x \in \mathbb{R}$;

(2) multiplication, as in $x \times \frac{1}{x} = 1$, where $x \neq 0, x \in \mathbb{R}$;

(3) CAN_1, as in $\frac{-(b-a)}{(b-a)} = -1$; and

(4) CAN_v, as in $\frac{(a+2)-3a(a+2)}{a+2} = (a+2)-3a$.

There are two implications flowing from the students computational repertoire that I wish to draw attention to here. The first is that it is clear that the 'operation' CAN_v:(a_{top}, a_{bottom}) → *void* introduces incoherence into the computational space in which the student works because the mapping does not do what division or multiplication do, and is therefore inconsistent with real number arithmetic. Secondly, CAN_v:(a_{top}, a_{bottom}) → *void*, and probably also CAN_1:(a_{top}, a_{bottom}) → 1, are not taking elements of the real numbers as arguments and values. The arguments are probably alphanumeric characters, so that CAN_1 and CAN_v do not have $\mathbb{R}$ as domain but rather some implicitly recognised collection consisting of symbols that are commonly used in school mathematics. What this means is that 'operations' like CAN_1 and CAN_v insert non-numerical objects and processes into the computational space, but those processes cannot be integrated with the fundamental computational resources of the field of real numbers because they do not take numbers as arguments and values.

We thus have computational resources for arithmetic consisting of numbers and arithmetic operations used in tandem with 'operations' that have arguments and values that are not elements of the reals. This state of affairs produces a fragmented computational space, consisting of different regions of objects and associated operations—real numbers with arithmetic operations on the one hand, and alphanumeric characters used with CAN_1 and CAN_v on the other—but with none of the operations able to range over *all* the objects in the computational space. Such a computational space seems to be tainted by a lack of mathematical coherence at a both more general and fundamental level than the local level of computational work unfolding in the process of solution a problem, even if many of its resources enable teachers and students to, so to speak, 'get the job done'.

CONCLUDING REMARKS

Each of the diverse investigations of coherence discussed in this presentation covers a dimension of coherence that is important for school mathematics, indicating that we should take a multifaceted approach to coherence if we believe that it is an issue of great significance to the teaching and learning of school mathematics. Alongside the question of whether or not the taught curriculum is aligned with the prescribed curriculum, we also have to consider the coherence of the curriculum itself, the coherence of the language used to elaborate mathematics in mathematics lessons and

in textbooks, the coherence of approaches to the solution of mathematics problems in textbooks and lessons, the coherence of lesson structure, the manner in which students attempt to construct mathematical coherence for themselves, as well as the general and local computational coherence of teacher and student mathematical activity.

REFERENCES

Beane, J. (Ed.). (1995). *Towards a coherent curriculum. The 1995 ASCD Yearbook*. Alexandria, VA: Association for Supervision and Curriculum Development.

Cogan, L.S., Schmidt, W. H. & Wiley, D. E. (2001). Who Takes What Math and in Which Track? Using TIMSS to Characterize U.S. Students' Eighth-Grade Mathematics Learning Opportunities, *Educational Evaluation and Policy Analysis*, 23(4), pp. 323–341.

Eco, U. (1984). *Semiotics and the Philosophy of Language*. London: Macmillan.

Ferrini-Mundy, J., Burrill, G. & Schmidt, W. H. (2007). Building teacher capacity for implementing curricular coherence: mathematics teacher professional development tasks, *Journal of Mathematics Teacher Education*, 10, pp. 311–324

Gillies, J. & Quijada, J. (2008). *Opportunity to learn: a high impact strategy for improving educational outcomes in developing countries.* Washington, DC: USAID, EQUIP2.

Halliday, M.A.K. & Hasan, R. (1985). *Language, context, and text: Aspects of language in a social semiotic perspective*. Deakin University Press.

Hiebert, J., Gallimore, R., Garnier, H., Givvin, K.B., Hollingsworth, H., Jacobs, J., Miu-Ying Chui, A., Wearne, D., Smith, M., Kersting, N., Manaster, A., Tseng, E., Etterbeek, W., Manaster, C. Gonzales, P. & Stigler, J. (2003). *Teaching mathematics in seven countries: Results from the TIMSS 1999 video study*. Washington, DC: U.S. Department of Education, National Center for Education Statistics.

Jaffer, S. (2012). The notion of 'topic' in school mathematics and the problem of isotopic coherence. In Nieuwoudt, S., Laubscher, D. & Dreyer, H. (Eds.), (2012). *Proceedings of the Eighteenth National Congress of the Association for Mathematics Education of South Africa*, *Proceedings of Long and Short Papers*, 24-28 June 2012, North-West University, Potchefstroom, pp.178-185.

McDonnell, L. (1995). Opportunity-to-learn as a research concept and policy instrument, *Educational Evaluation and Policy Analysis*, 17(3), pp. 305–322.

Reeves, C. (2005). *The effect of 'Opportunity-to-Learn' and classroom pedagogy on mathematics achievement in schools serving low socio-economic status communities in the Cape Peninsula*. Unpublished PhD thesis, School of Education, Faculty of Humanities, University of Cape Town.

Reeves, C. & McAuliffe, S. (2012). Is curricular incoherence slowing down the pace of school mathematics in South Africa? A methodology for assessing coherence in the implemented curriculum and some implications for teacher education, *Journal of Education,* 61(5), pp. 6–11.

Reeves, C. & Muller, J. (2005). Picking up the pace: variation in the structure and organization of learning school mathematics, *Journal of Education*, 37, pp. 103–130.

Schmidt, W. (2004). A Vision for Mathematics, *Educational Leadership,* 37(5), pp. 525–559.

Schmidt, W. (2008). What's Missing from Math Standards? Focus, Rigor, and Coherence, *American Educator,* (Spring), pp. 22–24.

Schmidt, W. & Houang R. (2012). Curriculum Coherence and the Common Core State Standards for Mathematics, *Educational Researcher,* 41, pp. 294–308.

Schmidt, W., Houang R. & Cogan, R. (2002). A coherent curriculum: the case of mathematics, *American Educator,* (Summer), 1–18.

Schmidt, W., Wang, H. & McKnight, C. (2005). Curricular coherence: an examination of US Mathematics

and Science contents standards from an international perspective, *Curriculum Studies,* 37(5), 525–559.

Schmidt, W.H. & Prawat, R.S. (2006). Curriculum coherence and national control of education: issue or non-issue? *Journal of Curriculum Studies*, 38(6), 641–658.

Schmidt, W.H., McKnight, C.C., Cogan, L.S., Jakwerth, P.M. & Houang, R.T. (1998). *Facing the Consequences: Using TIMSS for a Closer Look at United States Mathematics and Science Education*. Boston: Kluwer.

Smith, B. (1998). *It's about Time: Opportunities To Learn in Chicago's Elementary Schools. Improving Chicago's Schools*. ED 439 216. UD 033 439. http://www.consortium-chicago.org/publications/pdfs/

Smith, J.B., Smith B. & Bryk A. S. (1998). *Setting the pace: opportunities to learn in Chicago public elementary schools. Consortium on Chicago School Research Report*. http://www.consortium-chicago.org/publications/pdfs/p0d04.pdf

Stevens, F. (1996). Opportunity to learn science: connecting research knowledge to classroom practices, *Journal of Teacher Education*, 48(3), 222–232.

Stevens, F. (no date). Teacher evaluation of the sustainability of Opportunity-to-Learn assessment strategies. ERIC Clearinghouse on Urban Education. http://ericweb.tc.columbia.edu/teachers/teachereva/#burstein1989

Valverde, G.A. & Schmidt, W.H. (2000). Greater expectations: Learning from other nations in the quest for 'worldclass standards' in US school mathematics and science, *Journal of Curriculum Studies*, 32(5), 651–687.

Venkat, H. (2010). Exploring the nature and coherence of mathematical work in South African Mathematical Literacy classrooms, *Research in Mathematics Education*, 12(1), 53–68.

Venkat, H. & Adler, J.B. (2013). Coherence and connections in teachers' mathematical discourses in instruction, *Pythagoras*, 23(3), 25–32.

Venkat, H. & Naidoo, D. (2012). Analyzing coherence for conceptual learning in a Grade 2 numeracy lesson, *Education as Change*, 16(1), 21–33.

Wang, J. (1998). Opportunity to Learn: the impacts and policy implications educational evaluation and policy analysis, *American Educational Research Association*, 20(3), 137–156.