THE LEARNING OF GEOMETRY AS MOVING FROM ONE THINKING LEVEL TO THE NEXT: REVISITING VAN HIELE

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Extended abstract

The purpose of this paper is to revisit ideas of van Hiele on the learning and teaching of geometry. I explore how this classical theorist viewed the development of geometrical thought in learners and provide some examples on how the theory can be applied in the teaching of Euclidean geometry in the Curriculum and Assessment Policy Statements (CAPS) for the Further Education and Training (FET) phase. Such a re-examination is made imperative by the re-introduction of geometry in the mainstream FET curriculum. That reintroduction compels teachers, teacher educators and curriculum material writers to refresh their understanding of the importance and implications of this leading geometry education theory. By consolidating the theoretical foundations of geometry education, the paper implicitly aims to counter moves by the establishment to deskill and reduce teachers to technicians for instrumental rather than professional considerations.

In this paper I revisit the ideas of van Hiele on the learning of (Euclidean) geometry as it 'stresses the importance of the teaching learning act' (Teppo 1991: 212). I also critique the theory briefly to alert the reader of some of the pitfalls in theory building. More importantly I attempt to illustrate how the theory can be applied to help learners progress through different levels of geometry understanding. Pegg & Davey (1991: 10) point out that the ideas of van Hiele were the catalyst for much of the renewed interest in the teaching of geometry during the 1980's, evolving largely as a reaction to the deficiencies perceived in the views of Piaget. The reintroduction of Euclidean geometry into the mainstream Further Education and Training (FET) mathematics curriculum in South Africa's Curriculum and Assessment Policy Statements (CAPS) puts educators and teacher educators in a challenge similar to that of the past. Many educators wonder, in the first instance, why geometry is being brought back to the curriculum yet they had difficulties with it in the past. They doubt whether those impediments of the past have been fixed and thus appeal for support (e.g., Ndlovu 2011). To address some of the concerns this paper will be guided by the following questions: What are the basic tenets of the theories of the van Hiele model of geometrical though development? What are the criticisms of these theories that should be taken into account? How can the learning phases of the theory be exemplified in the teaching of geometrical content in the FET phase?

The theory is in three parts that describe five sequential and discrete levels that students pass through as geometrical thought develops, discuss the nature or properties of insight into geometric concepts and present a guide to the phased

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development of geometric lessons. Van Hiele (1986: 39-47) distinguishes the five different thought levels as 1) visualisation, 2) analysis, 3) informal deduction, 4) formal deduction and 5) rigour, respectively. Although the descriptions are specific to geometry van Hiele (1986: 41) concedes that they are actually stages of cognitive development since "the levels are situated, not in the subject matter but in the thinking of man".

Usiskin (1982: 5) identifies five properties namely a) fixed sequence, b) adjacency, c) distinction, d) separation, and e) attainment through instruction. Level attainment through instruction proceeds through five phases: i) inquiry or information, ii) guided/directed orientation, iii) explicitation, iv) free orientation, and v) integration.

Some studies that have raised questions about some of the characteristics of the theory. Burger & Shaughnessy (1986: 45), for example, report that they failed to detect the discontinuity between levels and found instead that the levels were dynamic and of a continuous nature rather than static and discrete. Fuys, Geddes & Tischer (1988) also found students who progressed through the levels at different rates, or even oscillated between levels, for different geometric content. Gutierrez, Jaime & Fortuny (1991) found that students can develop more than one level at the same time. In other words, van Hiele's broad statements 'are not as black and white as they are often portrayed to be' (Pegg & Davey 1998: 114). Regarding levels even van Hiele (1986) himself has doubted the existence or testability of levels higher than the fourth and considered them as of no practical value. Yet other studies have suggested the addition of a pre-recognition level at the lower end (e.g., Clements & Battista 1992).

At the grade 10 level the theory can be applied to revise basic results established in earlier grades regarding lines, angles and triangles, especially the similarity and congruence of triangles required by the new CAPS for the grade (DBE 2010: 26). Those basic concepts or results can be used to help learners come to grips with the mid-point theorem and its proof, the definitions and theorems about quadrilaterals all of which can be dealt with at levels 2 and 3. At the grade 11 level learners can be scaffold according to the learning levels to understand riders and execute proofs about circle geometry such as: angles subtended by a chord of the circle on the same side of the chord are equal, the opposite angles of a cyclic quadrilateral are supplementary, the angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment, etc. (DBE 2010: 35). In grade 12 the theory can be applied to prove (accepting the results established in earlier grades): that a line drawn parallel to one side of a triangle divides the other two sides proportionally (and the Mid-point Theorem as a special case of this theorem), that equiangular triangles are similar, that triangles with sides in proportion are similar and prove the Pythagorean Theorem by similar triangles (DBE 2010: 48). Specific activities will be used to illustrate these possibilities.

In this paper I have discussed the ideas of van Hiele with regards to levels of geometrical learning. I have cited some common criticisms of the theory but admit that it has been influential in the design of geometrical curricula and will continue to do so for the foreseeable future. I have also highlighted examples of classroom activities that can be used in the FET curriculum.

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