SYMBOL SENSE THROUGH A SPREADSHEET ALGEBRA VIEW OF $f(m,x)=mx$ AND $f(m,c,x)=mx+c$ IN PRE-SERVICE MATHEMATICS TEACHER EDUCATION

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This paper reports on the linear function $y=mx+c$ from the perspective of what is called spreadsheet algebra in the case of pre-service mathematics teacher education. Fostering a symbol sense through a spreadsheet algebra view (in mathematics teacher education) of this function can come about when using two spreadsheet algebra programmes that have been designed to explore what happens when the symbols m and c are varied. In addition to being able to draw the linear functions quickly, there is the possibility of using the programmes to draw attention to epistemic differences and relationships between symbols for the gradients and y-intercepts. Implications for the practices of mathematics teacher education and school mathematics teaching are considered, especially in light of calls for a focus on procedures.

INTRODUCTION

Working with linear functions represented through spreadsheets in (pre-service) mathematics teacher education is important because it affords the user/pre-service teacher opportunities to explore epistemic differences between the symbols *m* and *c* as in $y = mx + c$. Spreadsheet use has the potential to offer perspectives related to understanding these symbols that usually are not possible in the dominant pencil-paper environment common to school mathematics teaching. An examination of two versions of the linear function represented through spreadsheets is presented in this paper. When using the first one, it is possible to vary the gradient (*m*) represented in the form $\frac{a}{b}$ only, while the *y*-intercept equals zero. In the second one, it is possible to vary both the gradient and the *y*-intercept. The reason for the two versions relates to the idea of directing pre-service (or practising) teachers' attention to conceptual or epistemic differences specific to m and between m and c .The current call for 'the primacy of teaching procedures in school mathematics' (Julie 2012) makes it necessary to examine what a spreadsheet algebra view of linear functions has to offer.

TOWARDS A THEORETICAL FRAMEWORK

First, details on the context, viz., the practice of pre-service mathematics teacher education (at a university) followed by definitions of and connections between spreadsheet algebra and symbol sense are presented. Next there are descriptions of the two spreadsheet algebra programmes (SAPs) on linear functions. The paper

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concludes with a discussion that focuses on a pre-service teacher's pencil-paper recording of a particular behaviour of *m* and a conclusion based on implications for school mathematics teaching and mathematics teacher education in light of the current calls for the primacy of teaching procedures in school mathematics.

The context: pre-service mathematics teacher education

The present paper has its origins in pre-service mathematics teacher education in which we at the university explore, amongst other activities, school mathematics such as linear functions through the use of spreadsheet algebra programmes (SAPs) as 'resources' (Adler 2000). A spreadsheet can be used to point out links between arithmetic and algebra (Haspekian 2005). In other words, a spreadsheet can be used to 're-source' (as a verb) the algebra through arithmetic. One particular goal is to have pre-service teachers use the SAPs to build an initial repertoire (Feiman-Nemser 2001) with respect to (linear) functions and related symbols and their multiple (numerical, graphical, tabular and symbolic) representations. In addition, the SAPs in use can be regarded as didactic materials (Gellert 2004), meaning that they can serve as tools to enable pre-service teachers to acquire meanings for the various symbols. The pre-service teachers are also encouraged to move between spreadsheets and pencil-paper as mediums to write up their insights. Long-term thinking is that, beyond initial teacher education, the pre-service teachers extend their initial repertoire with respect to symbols in the case of linear and other functions.

Spreadsheet algebra, linear functions and symbol sense

Spreadsheet algebra is defined in the paper as school algebra objects such as (linear) functions which could be expressions or equations (a comparison of functions) that are represented in multiple ways, e.g. tables, graphs and formulas, through the use of 'electronic' spreadsheets (Pea 1985). This definition is adapted from Leung (2006), who defines spreadsheet algebra as the algebra learned through spreadsheets. In particular, spreadsheet algebra is instantiated in the form of SAPs (think applets) showing (linear) functions or 'number recipes' (Schwartz & Yerushalmy 1992; Yerushalmy & Schwartz 1993). The number recipes, i.e., functions, can be represented as discrete or real number inputs that are paired with corresponding outputs, depending on the particular recipe. To paraphrase Leung, the use of spreadsheet algebra (in the form of a SAP) demands a new vision of school algebra where there is a shift in emphasis from symbolic manipulation, as in pencil-paper school algebra, towards the symbol sense (Arcavi 1994; 2005) needed for understanding of objects such as (linear) functions.

A definition of symbol sense through a spreadsheet algebra view of linear functions will be far from closed and fixed (Arcavi 1994; 2005). In the present paper, symbol sense refers to the literal symbols *m* and *c* as found in high school algebra. Symbol sense is dependent on the context, whether it is pre-service mathematics teacher

education or school mathematics teaching. Through spreadsheets use, for example, the idea is to have the pre-service teachers 'read through' (Arcavi, 2005) the forms of the gradient m represented as $\frac{a}{b}$ and to hopefully 'see' equivalent forms or to distinguish between m and c in the case of, say $y = 3x + 3$. In school mathematics, teaching such distinctions will be useful, especially where learners are required to follow procedures to plot the linear function as in a pencil-paper environment. In the case where learners have to do symbolic manipulation on linear expressions only, distinctions between m and c might not be needed because they are not necessarily coupled with graphical meanings. Not surprisingly, a symbol sense will be dictated in high schools by the content of high stakes examinations, i.e. legitimate school mathematics (Julie 2012). In a spreadsheet algebra view, the symbols m and c can play different roles, namely as variables or parameters, and there is the possibility of the development of an 'intuitive feel' (Arcavi 2005) for those differences.

Spreadsheet algebra view of the linear function $G(a,b,x) = \left(\frac{a}{b}\right)x$ *or* $f(m,x) = mx$

In the linear function shown in Figure 1, there are three linked representations—a graph, a formula and a table. For the sake of convenience, the graph has its x- and y-axes calibrated in units ranging from -5 to 5 for each axis. Each of the four quadrants has been split symmetrically into halves, resulting in eight regions that can be identified through corresponding reflections, i.e. transformations of the numerical values of the gradients of the form $\frac{a}{b}$.

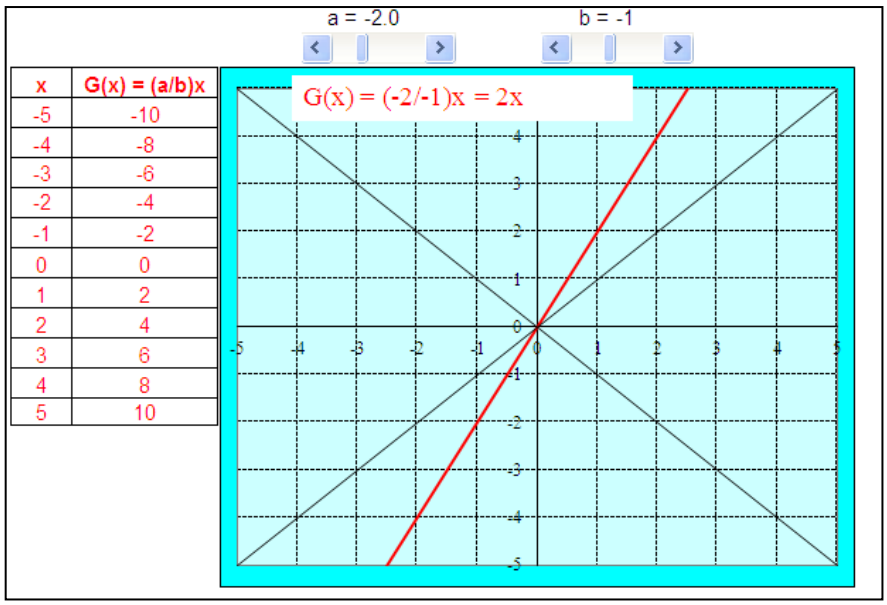

Figure 1: A spreadsheet algebra view of the linear function $G(a,b,x) = \left(\frac{a}{b}\right)x$, or $f(m,x) = mx$.

In Figure 1 the dark lines indicate the functions $y = x$ and $y = -x$. Because the *y*-intercept is zero, the user's (pre-service teacher's) attention can thus be focused on the gradient which takes the form of a rational number $\frac{a}{b}$. In Figure 1 we have $G(a,b,x) = \left(\frac{-2}{-1}\right)x$ with its linked table of values and graph. The choice of -2 and -1 was made to show a graphical representation of the gradient when both *a* and *b* have negative values. The numerical values of *a* and *b* can be varied by the technique or procedure of 'leaning' on the scrollbars indicated above the graph in Figure 1. Through this technique, the linear function can be conceptualised as a two-parameter 'family of (linear) functions' (Yerushalmy & Schwartz 1993: 61) that can be represented symbolically as $G(a,b,x) = \left(\frac{a}{b}\right)x$.

In summary, procedures such as systematically varying the numerical values of *a* and b and noting corresponding changes are about developing a symbol sense with respect to the gradient of the linear function.

Spreadsheet algebra view of $y = ax + b$ ***or*** $f(m,c,x) = mx + c$

As with the previous linear function we have the three linked representations, viz. a table, a graph and a formula (see Figure 2). The use of the scrollbars enables the user to vary the gradient (*a*) and the *y*-intercept (*b*), which is not possible in the previous linear function. By leaning on the scrollbars we can produce a two-parameter "family of linear functions". We can vary the gradient (*a*) and keep the y-intercept (*b*) constant and vice versa. We can write the two-parameter linear function as $f(m,c,x) = mx + c$**,** although we have *a* and *b* in $y = ax + b$ in the actual SAP. The three-column table (See Figure 2) is designed in a way to show the gradient (Δy) numerically (See third column) which is then also represented graphically in the form of the two triangles on the graph. Such a representation of the gradient is not present in the case in Figure 1. The instructions in the lower right-hand corner make the SAP usable as didactic material that, when used, is aimed at guiding the user/pre-service teacher to explore connections between tabular, numerical, graphical and algebraic representations of $y = ax + b$**.**

In summary, the forms of the linear functions in the above differ from and are similar to the dominant pencil-paper environment specific to school mathematics teaching. First, they are represented through spreadsheets and the variables, in addition to x and y, are in the form of parameters (*m* and *c*) that can be varied through the use of the scrollbars.

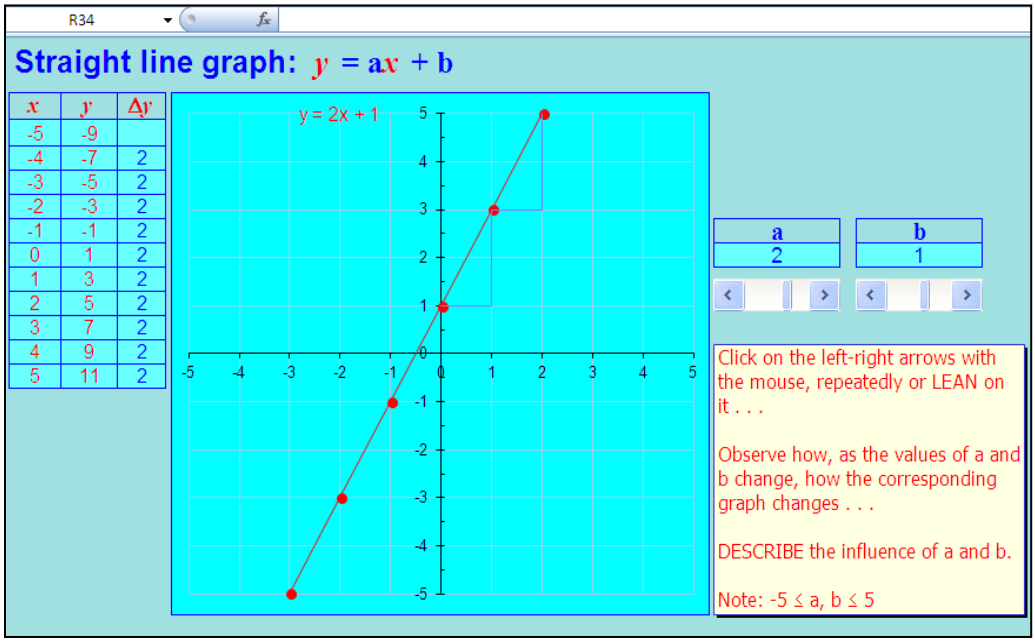

Figure 2: A spreadsheet algebra view of the linear function $y = ax + b$ or $f(m,c,x) = mx + c$.

The linear functions also differ because they do not look exactly like linear functions represented in legitimate school mathematics characteristic of pencil-paper tests and examinations. They are recognisable as 'straight line graphs' when it comes to school mathematics teaching. These differences and similarities point to issues connected to the practices in (pre-service) mathematics teacher education and school mathematics teaching to which we now turn.

FURTHER EXEMPLIFICATION AND CONCLUSION

Fostering symbol sense through a spreadsheet algebra view of the linear functions in (pre-service) mathematics teacher education implies that we consider the following two inter-related points. First, the SAPs containing the linear functions can serve as resources (Adler 2000) and are also didactic materials (Gellert 2004) peculiar to the practice of mathematics teacher education. Second, working with the SAPs requires 'mediated use' (Adler 2000) where the pre-service teachers are directed to find out what happens when doing procedures such as leaning on the scrollbars and taking into account the dominant pencil-paper environment of school mathematics teaching.

Through mediated use in mathematics teacher education, the two linear functions can serve as a resource for developing a symbol sense connected to the gradient and between the gradient and the y-intercept. After exploring $G(a,b,x) = \left(\frac{a}{b}\right)x$, a pre-service teacher summarised in pencil-paper form the behaviour of and relationships between linear functions in instances where the gradients are inverted. The reason for

showing a pencil-paper recording has to do with mediated use, i.e. efforts to have pre-service teachers work between a pencil-paper environment specific to school mathematics teaching and a spreadsheet environment as found in (pre-service) mathematics teacher education. In particular, it meant that the pre-service teacher followed the procedure of 'leaning on', i.e. changing the parameters *a* and *b* on the scrollbar in ways where she noted what happened when 2 became $\frac{1}{2}$ and $\frac{2}{4}$, $\frac{3}{5}$ became $\frac{5}{3}$ and $\frac{5}{6}$ became $\frac{6}{5}$ (see Figure 3).

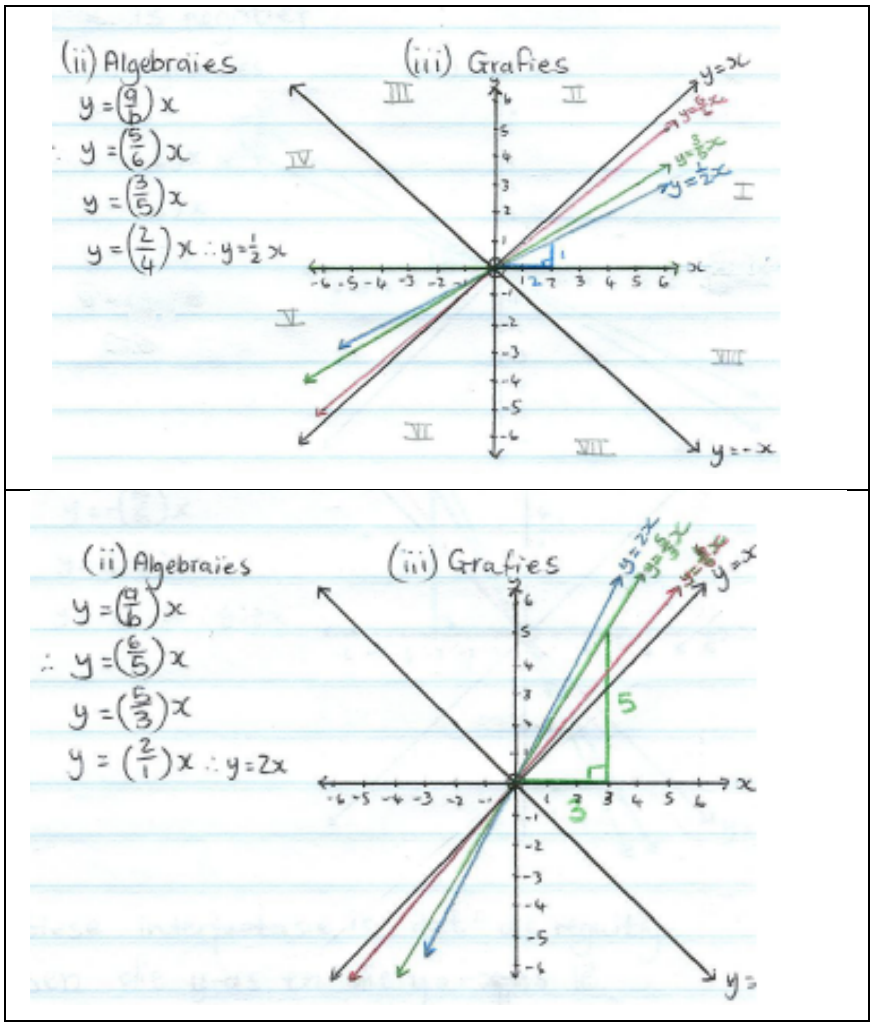

Figure 3: Evidence of pencil-paper symbol sense in instances where gradients are inverted in $G(a,b,x)=\left(\frac{a}{b}\right)x$.

She recorded in pencil-paper the reflections in the line $y=x$ for the three examples she chose to write up as numerical changes in the parameters *a* and *b* of the gradient $\frac{a}{b}$ and their associated graphical effects. No claim is made that what she recorded is

proof of her knowledge. She may have obtained what she drew from a source other than the SAP shown in the above. What can be said is that the pencil-paper drawings she made can be used to complement the mathematical potential of the SAP as didactic material and what is valued in mathematics teacher education. The drawings or recordings in Figure 3 can be used to 'read through' the symbols $\frac{a}{b}$ and, thus there is the possibility of understanding what procedures such as manipulating or changing them numerically is about graphically, viz., transformations or reflections in $y = x$.

Symbol sense with respect to linear functions espoused in the paper has to be viewed pragmatically when it comes to school mathematics teaching. In the case of the latter, symbol sense implies taking into consideration a framework related to the daily routines of high school classrooms, i.e. legitimate school mathematics. The symbol sense associated with the two SAPs is peculiar to the practice of mathematics teacher education but is aimed at illuminating school mathematics teaching. It can be argued that the teaching of procedures with respect to linear functions in school mathematics teaching has to accompany an emerging and often pragmatic symbol sense when it comes to symbolic manipulation peculiar to high stakes examinations and pencil-paper-dominated environments where learners' progress is what counts.

Further investigation is needed to see how, when and where the fostering of symbol sense present in the proceduralising embedded in the scrollbars designed to vary the symbols *m* and *c* (in the two SAPs) and accompanying conceptualisation are linked when it comes to school mathematics teaching.

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