THINKING ABOUT MATHEMATICS FOR TEACHING: CONFRONTING BLIND SPOTS

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This short paper describes some of my learning of mathematics and of teaching mathematics, through my participation in a professional development project. I focus on how I confronted my blind spots while working on solving equations with algebraic fractions; and on how I worked on my blind spots together with colleagues. I provide samples of learners' work to elaborate each of these aspects of professional development that stood out for me.

INTRODUCTION

The importance of creating a supportive working environment where teachers can learn from each other as professionals through confronting and interrogating one's blind spots cannot be stressed enough. In this short paper I present aspects which stood out for me and impacted on my practice as a mathematics teacher through my participation in the me programme offered by the Wits Maths Connect Secondary (WMCS) project. This happened in two ways: (a) by confronting my blind spots while working on restrictions on algebraic fractions that resulted in my understanding of the notion of a variable in the denominator; and, (b) by working on my blind spots together with colleagues. I begin the paper with a discussion of the WMCS project. I then provide samples of learners' work that elaborate on each of these two aspects of professional development that stood out for me.

THE MATHEMATICS DEVELOPMENT PROJECT

WMCS is the Wits Maths Connect Secondary project located at the Wits School of Education in the University of the Witwatersrand (Wits), Johannesburg. It is a research and development project, with the primary role of improving the quality and quantity of learner mathematical performance by working with teachers and strengthening teachers' content knowledge for teaching mathematics. The WMCS offers two different 16-day courses, with the 16 days spread over an academic year, to two groups of teachers. The one course caters for teachers who teach Grades 8, 9 and 10 and is called Transition Maths 1 (TM1); the other course is for Grade 11 and Grade 12 teachers, and is called Transition Maths 2 (TM2). The focus on transition points, from Grade 9 to Grade 10; and from Grade 12 into tertiary study, is both a focus and outcome of the on-going work of the WMCS. The wider goal is to strengthen the mathematics pipeline – enable more learners to succeed - within the school and between the school and tertiary studies.

When I participated in the TM1 course I was, and still am, teaching Grade 9 mathematics in my school. The mathematics presented in TM1 focuses on algebra

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and functions, together with some work on Euclidean geometry and trigonometry. The content we studied extended beyond Grade 10 mathematics into Grade 11 and 12, with a lot of emphasis on algebra and functions. The 16 day course was divided into eight 2-day sessions all of which were held at Wits. Most of the time in these sessions was spent on mathematics (for teaching). We focused on thinking about and doing mathematics, and the mathematics we did was related to the whole secondary curriculum, sometimes even with examples beyond what is covered in the curriculum. In addition, there was at least one part of the two day session that focused on aspects of teaching, particularly on choosing and using examples and working on explanations.

As I reflect on what I learnt in the WMCS project, I appreciate two things in particular. First was the mathematics I was learning for the first time; second was how important it is to be able to consistently work together as a team with other colleagues in your school, if you are to grow as a teacher. Both these aspects of the PD are what stand out for me and have impacted my teaching practice.

PRACTICAL EXAMPLES OF WHAT I LEARNT FROM WMCS PD

Confronting and working on blind spots with colleagues

I was fortunate that two of my colleagues from my school participated in the WMCS TM1 course with me. This was of great benefit to us in that we learnt to work together as professionals. We did not start out working as a team of teachers. I realised that we needed to support each other and help each other with the work from the course. This included doing the mathematics we had to work on between sessions at Wits, as well as studying aspects of our classroom practice. We also realised that we needed to create an environment where we could trust each other; where we could share ideas and confront our blind spots. By blind spots I mean mathematical concepts that teachers may frequently work with or teach without realizing that we do not emphasise them; and that failure to emphasize them will then result in learners not realising their importance. Two activities from the project that sensitized me to one of my blind spots: (1) the pre-test we wrote before we started with the project, and (2) the homework that we had to do in between contact sessions.

When I received my pre-test results I realised that there were certain issues I had never paid attention to in my own doing of mathematics. One of those issues that I elaborate on in this paper is the issue of writing restrictions on values of the unknown when solving equations with algebraic fractions. I used to solve an equation, and not consider the values of the unknown that I arrived at in my answer, and whether the values were possible solutions. I didn't realise that it was necessary to write the restrictions for the solutions. I was penalised for this omission in the test, and because I lost marks, it highlighted for me that writing restrictions was an essential part of the solution of the equation. This had an impact on me. I realised that as a teacher I did not pay attention to restrictions on possible solutions to equations with algebraic fractions, and this was a problem for my learners! I also realised that this was a

serious blind spot in my own mathematical thinking. It is not easy to face one's blind spots: no one wants to come across as lacking in any way. On the other hand, I found it very important to do so because it eventually leads to growth and realisation that we need to interact with each other as colleagues in order to sharpen our mathematics understanding, and improve our teaching.

In the project one of the textbooks we used was Classroom Mathematics, Grade 11 (Laridon et al. 2006). After my pre-test results, and because of the work we had to do in the project, I studied the section on solving equations in this book (op.cit.: 69). This it opened my eyes. I see that this text book that we use in our school does place a lot of emphasis on restrictions when solving equations with these algebraic fractions.

My blind spots were also exposed when I looked at a video of my own teaching. This is a good way to confront blind spots – by seeing yourself teaching. This is important to do, not only on your own, but also with your colleagues. As I noted earlier, I was fortunate in my school that there were three of us on the TM1 course, and we all watched our own and each other's videos. It was very helpful and enlightening to have this opportunity to look at the videos of each other teaching in class. This enabled us to constructively review how we teach, and to discuss how we could improve on our teaching of particular topics. We learned when giving feedback to each other, that we needed to do this in as tactful a manner as possible. This was important for developing trust amongst each other as colleagues. It is not easy to sit and watch your own teaching in the company of others but it is important to allow colleagues to critically analyse your work – this is a productive way to grow as a professional. Participating in the project opened up opportunities for us as colleagues to engage with the mathematical content we were teaching, and interrogate our blind spots, both in mathematics, and in our teaching of mathematics.

Confronting blind spots when working on restrictions

In my previous teaching, I did not think about the examples of equations that I gave to learners. I accepted answers even when there needed to be a restriction – showing that as an educator I never regarded them as very important, that they formed part of the solution. It is not easy to be confronted with something you have ignored to emphasise with your learners. Nevertheless it challenged me to look at my practice and challenge myself even further.

As I have already mentioned, the algebra sessions in the professional development programme stood out for me, specifically the notion of restrictions when solving complex algebraic fractions. This work had to do with the meaning of algebraic expressions. Becoming more aware of x as a variable is a critical transition point from GET into FET. In my own learning of mathematics as a student in high school, and when I was a pre-service student doing my secondary teachers' diploma, there was never an emphasis for me on restrictions when dealing, for example, with algebraic fractions in equations. The WMCS-PD session that focused on algebraic

equations presented new area of knowledge for me; the manner in which it was explained and emphasised was such that it caught my attention and stood out for me. I was very happy with this session because it meant that I now began to reason about the equation and make sense of the solutions the equation may or may not have. I feel as a mathematics teacher that as part of the transition from Grade 9 to 10, awareness of this notion of restrictions is so important when teaching learners to solve complex algebraic fractions, and this knowledge has made me think about how and when this should be raised with learners.

I resolved that I will teach my learners to write restrictions at the beginning of the solution before they even begin to solve the equation, and that I will teach this as early as Grade 9. I also decided that I will penalise learners for not writing restrictions in their answers. The losing of marks will make the learners realise the importance of writing the restrictions with every solution as they are part of the solution.

It had never occurred to me that when working on the fractions it is extremely critical to emphasise these restrictions and take away marks from learners for not writing them. To explore this, as I prepared for this paper, I decided to ask a group of Grade 11 and 12 learners who agreed to do this with me, to solve three equations with algebraic fractions. As I analysed the answers from these learners I saw that the learners did not write the restrictions as part of the solution. Learners who participated in this activity are taught by other educators but their answers revealed to me that it is not only me who puts less emphasis on the restrictions – this was also what other educators did. The three problems in Figure 1 were given to the learners. In each case learners were supposed to solve for x.

a) $x+1=\dfrac{15}{x-1}$

b) $\dfrac{x}{x-2}=\dfrac{1}{x-3}-\dfrac{2}{2-x}$

c) $\dfrac{x-2}{x-1}-\dfrac{5}{x+2}=\dfrac{7}{x-1}$

Figure 1: Three problems that were given to learners to solve.

The answers I received from the learners showed that teachers need to put more emphasis on the restrictions because the learners never wrote any restrictions when they wrote answers to these questions. Figure 2 shows an example of learners' solution that was presented without restrictions.

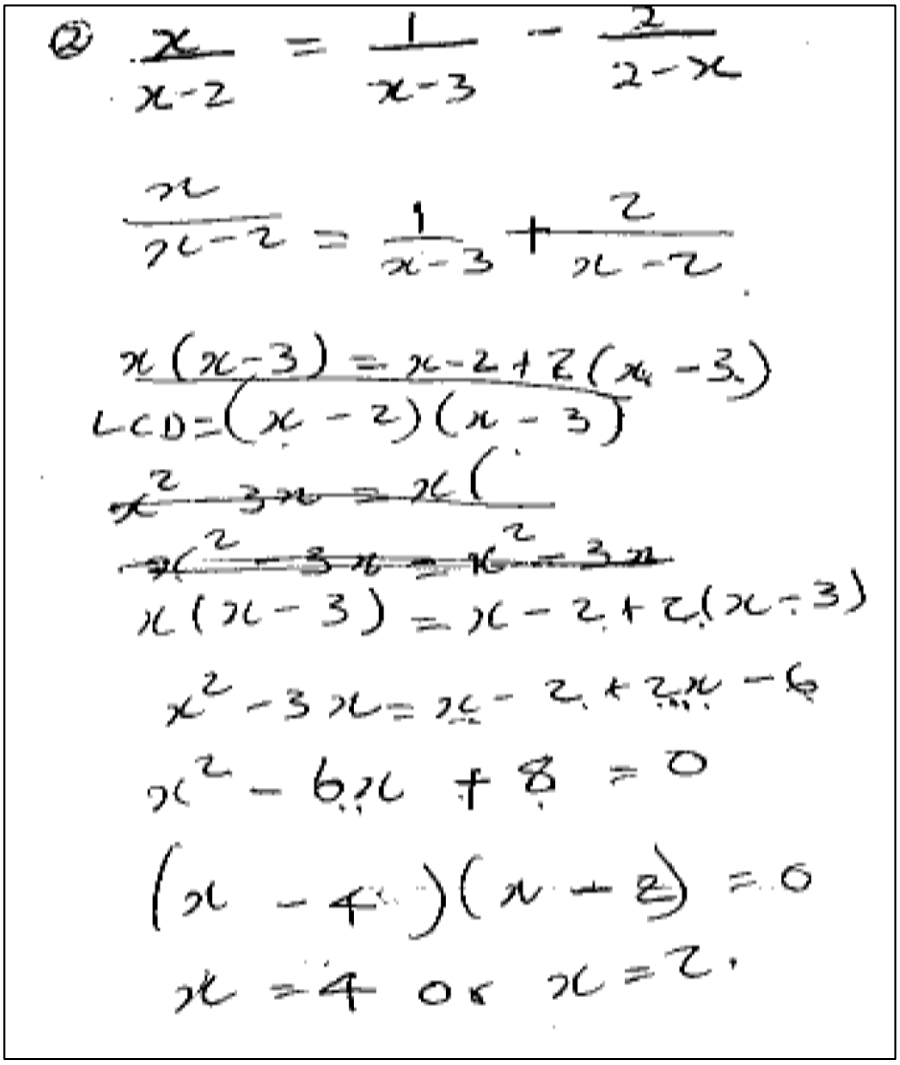

Figure 2: An example of a learner's solutions before the discussion.

I arranged a discussion with the group of learners who solved the three equations in Figure 1. In the session I made an effort to discuss issues such as division by zero and asking questions such as: *What is the answer when you have 0 divided by 6*? (which is zero) *and then when 6 is divided by zero*, (which is undefined). Then I went back to the equations with algebraic fractions and I asked learners what would be the effect of having zero as the denominator in these fractions. In this case learners began to realise that $x - 2$ in the second problem cannot be zero, which means that x cannot be 2. We then looked together at the other two equations. Shown in Figure 3 is the solution to the same problem after we had discussed the notion of division by zero.

This poses another issue for me. In Grade 8 and 9 we can use examples with learners that deal with the concept of real and non-real numbers. We can give examples where the denominator is a variable, and have them think about what it means. This will start to give them experience with restrictions on values of variables.

The learners I worked with did not find it difficult to understand the importance of having non-zero denominators when dealing with equations with algebraic fractions.

This brief experience with learners suggests to me that if we as educators emphasise the importance of writing restrictions as part of the solution to equations, we can make a difference to learners' performance.

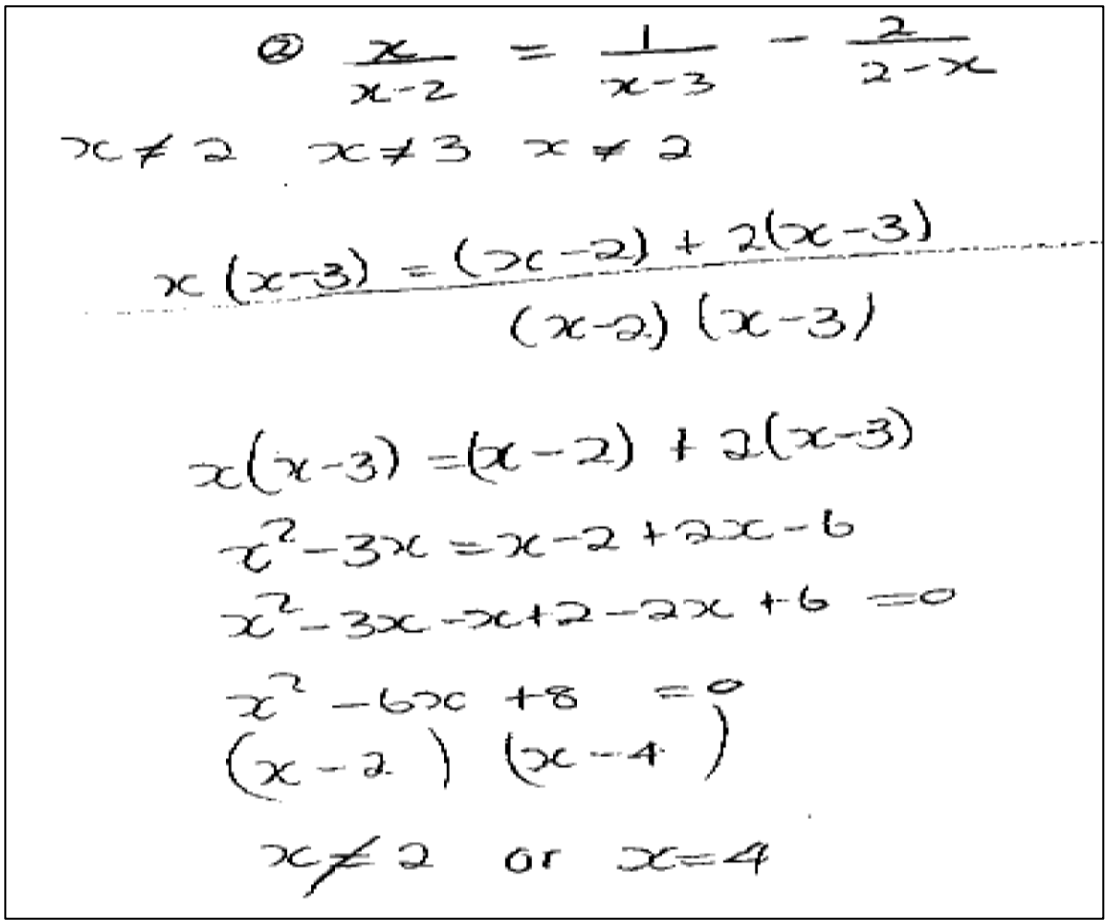

Figure 3: A learner's solution after the discussion.

I also took a decision to discuss this small exploration with my colleagues. I raised the importance of emphasising restrictions on the solution to equations. I advised on ensuring that marks are deducted for not writing the restrictions when the learners answer questions of this nature. The project thus further afforded me and my colleagues the opportunity to engage with each other on aspects of our teaching, and the opportunity to learn to be comfortable to talk to and learn from each other.

CONCLUSION

I learnt a lot from participating in the WMCS-PD programme, and for the scope of this paper I have chosen to focus on one aspect that is discussed above. The discussion above highlights the importance of creating a supportive working environment where teachers can learn from each other as professionals through confronting and interrogating each other's blind spots. In this paper, the importance of writing restrictions when dealing with algebraic equations with fractions was also highlighted. This has been a great opportunity for us and I know my colleagues share these sentiments. In sharing this experience at AMESA, we hope to be ambassadors for the growth of our profession and our practices as mathematics teachers.

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REFERENCE

Laridon, P., Brown, M., Jawurek, A., Mbatha, P., Scheiber, J., Sigabi, M. & van Noort, D. (2006). *Classroom Mathematics Grade 11*. Sandton: Heinemann.