THINKING ABOUT TEACHING MATHEMATICS: BEING WELL-PREPARED

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Through my participation in professional development I came to appreciate a well prepared lesson. I describe my experience of participating in the preparation and observation of a lesson, and how, despite being an experienced teacher, I began to work on more careful selection of examples and how these are explained.

INTRODUCTION

From participating in, and observing how the presenter went about presenting our sessions, and observing and reflecting on how he taught learners from my school in a 'lab' lesson, I realized that a lot of preparation went into each session and into the lab lesson. Therefore, the idea of a *well-prepared* lesson is one of the things that stood out strongly for me in the Wits Maths Connect Secondary Professional Development project (WMCS-PD). In this short paper I present practical examples of how participation in the WMCS-PD project impacted my teaching and understanding of what it means to be *well-prepared* for a lesson.

THE MATHEMATICS DEVELOPMENT PROJECT

The Wits Maths Connect Secondary (WMCS) project is located at the Wits School of Education, University of the Witwatersrand (Wits), Johannesburg. It is a research and development project, with the primary role of improving the quality and quantity of learner mathematical performance by working with teachers and strengthening teachers' content knowledge for teaching mathematics. The WMCS offers two different 16-day courses to two groups of teachers, with the 16 days spread over an academic year. The one course caters for teachers who teach Grades 8, 9 and 10 and is called Transition Maths 1 (TM1); the other course is for Grade 11 and Grade 12 teachers, and is called Transition Maths 2 (TM2). The focus on transition points, from Grade 9 to Grade 10; and from Grade 12 into tertiary study, is both a focus and outcome of the on-going work of the WMCS. The wider goal is to strengthen the mathematics pipeline, enabling more learners to succeed within the school and between the school and tertiary studies.

I participated in the TM1 course, and have responsibility for Grade 9 mathematics in my school. The mathematics presented in TM1 focused on algebra and functions, together with some work on Euclidean geometry and trigonometry. The contents we studied extended beyond Grade 10 mathematics into Grade 11 and 12 mathematics, particularly in algebra and functions. In each of the eight two day sessions we attended at the University, most time was spent on mathematics. We focused on thinking about and doing mathematics, and the mathematics we did was related to the

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whole secondary curriculum, sometimes even with examples beyond what is in the curriculum. In addition, we also spent time focused on aspects of teaching, particularly on examples and explanations. We discussed the choice of examples in textbooks and lessons, and the sequencing of those examples. Throughout the whole course we were also pushed into thinking about providing explanations to learners as to why certain concepts and their representations behaved the way they did, e.g. why the gradient of a vertical line is undefined. Many of us in the course thought that telling the learners *that* the gradient is undefined, was a full explanation. We learned to think first for ourselves, and then how we could explain more fully to learners, why this was the case, using the graphical representation of the vertical line, the definition of gradient, and so on. Through all this work in the course days, one of the things that I noticed was how well each session was prepared.

Two things are interesting for me about lesson preparation. First, I am an experienced teacher, and believed that I prepared appropriately for my teaching. So, it is interesting to me that I have been working now more on my preparation, as I could see some of what I ignored. Secondly, as I reflected on my learning in this project, I know I learned more mathematics. I also know we didn't focus directly on lesson preparation. Yet, this is what has mostly impacted my practice.

NOTICING A WELL PREPARED LESSON: LEARNER ACTIVITY, EXAMPLES AND RESOURCES

As I mentioned, functions were focused on in the course, and two of the two-day sessions were on functions. In the first one, on the first day we worked on our own understanding of functions as learners of mathematics; on the second day, we observed our lecturer teach a group of 60 Grade 10 learners (from mine and other neighbouring schools). This was called a lab lesson, as the group of teachers in the professional development course all observed the lesson. In addition, the lecturer went through the lesson he had planned with us before we moved to the observation session. We had an observation sheet for our comments, and this asked us to take note of the examples and explanations in the lesson, as well as what the learners were doing. And through these activities, I noticed things about the sessions in which I was a learner of mathematics, as well as in the lab lesson.

I observed that the examples and resources the presenter used in our session, and in the lab lesson had been prepared well in advance. All resources used, whether they were the teaching aids or the introductory example, or a set of exercises, these had been thought about and were readily available. For example, I noticed that our lecturer began our session with an example that we had to work on and it got us all involved. He gave us a rational function which is not in the secondary curriculum – and many of us had not worked with a function like this. Yet, we were all able to get involved, exploring what happened as we changed values of x, and then explored properties of functions, including asymptotes, for example.

There was a similar process with the learners in the lab lesson. In the lab lesson he used resources we have in the project and our classrooms, like laminated white boards, and had prepared his graphs in advance. I was surprised to see how learners participated. It was different from my class. In the presentation of a well prepared lesson the learners are continuously involved in activity through graded examples which are sequenced and organized according to the needs of the learners and the content. The needs of the learners here do not necessarily refer to adjusting to the level they are working at, but it means knowing where the learners are and where it is that you want them to be. Thus the use of examples and resources in a well prepared lesson helps you connect with your learners. I have used this experience, which I came to appreciate through all the sessions in the professional development programme, to be well prepared for my mathematics teaching.

Below I share an example of how the examples that were chosen in the lab lesson facilitated learner involvement and the formation of concepts

Example 1: Observing a lab class on functions

As I have mentioned, the lab lesson was organized as part of the professional development programme, and it was with learners from my school and others in the same township that were on the campus as part of the Learner Mentoring Project[15]. The lesson was an hour long the teachers in the professional development programme sat around the edge of the lecture room. The resources that were given to learners included a worksheet with the activities they had to engage in, so they could write all that they needed on that paper. Each learner also had an A4 sized laminated white board (this is just laminated white cardboard), with a pen to write on it. The white boards and pens are useful resources that a teacher can use when he/she wants to see all learners' work at once. These are especially useful for schools like mine where a Grade 9 or 10 class has up to 75 learners and there is no space left to move around checking learners' solutions. Learners are instructed to write answers or suggestions large enough for the teacher standing in front to see, and then hold them up for the teacher and other learners to see.

The first activity for learners was to consider the problem: "Two numbers multiplied together equals 12, what are the numbers". The learners had many suggestions. As the learners offered pairs of numbers, the lecturer filled them in on a table that he quickly drew on the board. Learners had a similar table in their worksheet. Through the lesson, the presenter used four different representations of the hyperbola to explain the concept. The verbal: *When two numbers are multiplied the product is 12. Which two numbers are those?* The algebraic: *Algebraically the verbal above can be written as:* $xy = 12$. The tabular: see Table 1. The graphical: *The graph was drawn by both the presenter and learners.*

[15] The Learner Mentoring Project is also part of Wits Maths Connect. Learners from our schools meet with students from the university who support them with their maths learning. Some Saturdays the learners come to Wits. And this week, the project organised for them to be part of a demonstration or lab lesson.

Table 1: Generating products of 12.

x									
y									
xy	12	12	12	12	12	12	12	12	12

In this first activity, as the learners offered pairs of numbers whose product was 12, the wrote these different responses into the table on the board. He also asked questions, for example: "Can one of the numbers be a quarter?" This pushed learners to use fractions and not only think about products with natural numbers. One learner even went as far as suggesting two fractions, $\frac{8}{3}$ and $\frac{9}{2}$, to give $\frac{8}{3}\times\frac{9}{2}$. Some learners used negative numbers (e.g., -4 and -3) without being told to do so. The presenter also asked learners to think about the number zero, with the questions: "Can x be zero?" and "Can y be zero?"

The learners were expected to draw the graph using their own values. And the questions now asked by the presenter were: "Is the graph curvy?" and "Is it a straight line?" In conclusion the presenter also drew the graph on the board, and later told the learners that the name of the graph is called "the hyperbola".

In my view the objectives were met by letting the learners see that when the relationship between x and y was a product, the graph was a curve. What struck me the most was how he taught the lesson without actually giving the learners all the information at the start. Instead, learners were able to participate in developing the graph and various representations of the hyperbola. The questions asked to push learners, and the examples given, indicated to me that this was a well thought out lesson. I observed the way learners were engaging with all the activities. Observing this lab lesson also indicated to me that to present a well prepared lesson involved knowing what questions to ask and when it is appropriate to ask these; what examples to give and at what time. This implies that in the preparation one has to think about the suitability of the exercises and examples and of possible questions to ask learners. For example, the lecturer was prepared and ready to suggest fraction values, and zero so as to push learners' thinking about number, and about multiplication and division by zero and so the emerging asymptotes.

In the professional development session immediately after the lab lesson, we discussed the notes we had taken with other teachers on the course. The first point we discussed was the starting example. Most of us said we would not start work with the hyperbola with a verbal statement. Instead, we would just put up an equation like $y=\frac{3}{x}$, fill in a table of values, plot the points and say this is a hyperbola, with general form $y=\frac{k}{x}$. We could see that the choice of example for the introduction provided the opportunity for learners to get engaged in the lesson and to see how the

pairs of values for x and y they had suggested, when plotted, formed the curve; that the curve was a 'picture' of the relationship between x and y, or input and output. The example and the values also unlocked the understanding of some of the properties of the hyperbola, e.g. why the graph does not touch the axes.

Below I give a description of how this impacted on my own practice as a mathematics teacher.

Example 2: working with algebraic expressions in my Grade 9 class

I presented a one hour lesson on polynomials to my Grade 9 learners, focusing on the difference between monomials, binomials and trinomials. I prepared for the lesson by choosing the introductory activity and examples, and the resources needed. I wanted learners to be involved, and so I made sets of cards for learners in the class. On each card there was one algebraic expression from the list in Table 2, or the words: 'monomial', 'binomial', 'trinomial', 'one', 'two', and 'three'. I guided the learners by telling them that 'mono', 'bi' and 'tri' mean 1, 2 and 3, respectively, and we talked about how to see different terms in the expressions. The task for learners was to decide which of the various expressions were monomials, binomials or trinomials. Learners had to work in pairs, and each pair had cards to sort and match.

This activity has some similarities with how polynomials are dealt with in Grade 9 Classroom Mathematics (Laridon et al 2006), but mine was not to complete a table, but rather to match cards, and so choose between different expressions that might not be immediately obvious to learners, whether they were monomials, binomials or trinomials. Like the textbook, I included a range of examples.

Some learners managed to do this correctly, for all the different expressions in the table. But there were interesting errors that I could see and that I could work with.

Table 2: Detail on matching cards.

Algebraic expression	Terminology	Meaning
$3xy,\ 2a,\ a^2,\ \sqrt{4x},\ \frac{x}{2}$	Monomial	1 term
$3p + q,\ 7m^2 - n^2$	Binomial	2 terms
$5xp - 3pq + 7pr,\ x - 2 + \frac{y}{5}$	Trinomial	3 terms

A few learners thought that $3p+q$ was a trinomial, because they saw the 3, p and q as separate terms. This led me to deal with factors as well as terms, and that in mathematics, $3p$ is one term, but $3 + p$ is a binomial, that terms are separated by the operations of addition or subtraction. In a similar way, some thought that a^2 was not a monomial because of the square, and so we looked at the meaning of a^2.

I prepared more cards for learners which had questions and answers to match. The emphasis was more on learners finding their own like and unlike terms, e.g.,

(a) $5p^2q^2 + 6p^2q^2 = 11p^2q^2$

(b) $3x^2y + 3xy^2 = 3x^2y + 3xy^2$

I asked the question: "Why do we add 5 and 6 to get 11 in (a) but we do not add 3 and 3 to get 6 in (b)?" This forced the learners to explain like terms and unlike terms. I gave them an activity to do in class involving addition and subtraction of like and unlike terms. In his paper of the design, purpose and use of examples, Rowland (2008) discusses how all mathematical objects have components or variables, and learners need opportunities to see how these can vary. I focused on this in the examples. These two examples, look similar. Seeing how they are different—how the powers vary—can help learners identify like terms.

In my presentation I will give more detail on the lesson, the cards and what the learners were able to do, and also what I learned from trying out a lesson like this myself, where I had an activity for learners to start with where I could get them involved, and see what they were thinking. I will also discuss how the lesson shifted from a focus on sorting polynomials, to working with like and unlike terms.

CONCLUSION

Through the WMCS professional development programme I learned that the selection and sequencing of examples reflect your very own awareness and knowledge of the concept you are teaching. I also learnt that the selection and sequencing of examples presented in a well prepared lesson is not a simple matter of picking any example that comes into your head – or that is in the textbook. You need to think about each step of the lesson and what examples will help the learners and help you see whether learners understand. I am a very experienced teacher, and it becomes easy to think you can just take any example from the textbook, or from what you have taught before. I now prepare lessons more carefully, and focus on what examples I will use, how I will use them, and why this will help learners learn.

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