THEOREMS AND CONVERSES

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Vula Programme at Hilton College

TARGET AUDIENCE: FET Phase

DURATION: 2 hours

MAXIMUM NO. OF PARTICIPANTS: 30

MOTIVATION

Circle geometry is back. Because most schools did not offer Paper 3 in the previous curriculum, many educators have not taught circle geometry for at least six years. For various reasons, including time constraints, very little emphasis is placed on the formal proofs of the circle theorems. The proofs of the converses of the theorems are largely ignored. In this workshop we will discuss the converses of the standard circle geometry theorems with an emphasis on the reductio ad absurdum technique. In addition we will investigate the interesting converse of the 'angle at the centre' theorem – which does not exist. We will use *Geometers Sketchpad* to do this. The workshop will include techniques of presentation with *Geometers Sketchpad*.

CONTENT

The workshop will be divided into two parts: the first half hour being a demonstration and discussion. The next ninety minutes will be hands-on the computers while the participants create their own demonstrations.

Demonstration 1: "If a quadrilateral is cyclic, then its opposite angles are supplementary." The proof of this theorem uses the angle at centre theorem and isosceles triangles. It is straightforward as is its explanation using *Geometers Sketchpad*.

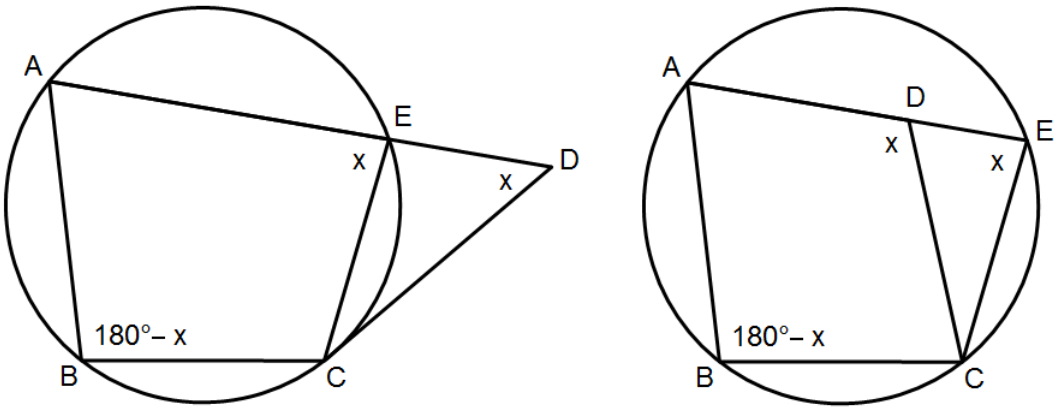
Demonstration 2: The converse of this theorem states that "If the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic." The proof of this is more interesting as it uses the method of contradiction.

Given a quadrilateral with opposite angles supplementary.

Call the quadrilateral ABCD and assume that it is not cyclic. Construct a circle through points A, B and C. Point D will lie either inside or outside the circle. The circle cuts AD or AD produced at a point E. $A\hat{D}C$ and $\hat{B}$ are given supplementary. $A\hat{E}C$ and $\hat{B}$ are supplementary by construction.

But this is impossible because an exterior angle of a triangle cannot be equal to an interior angle. Therefore the original assumption must be false and quadrilateral ABCD is cyclic.

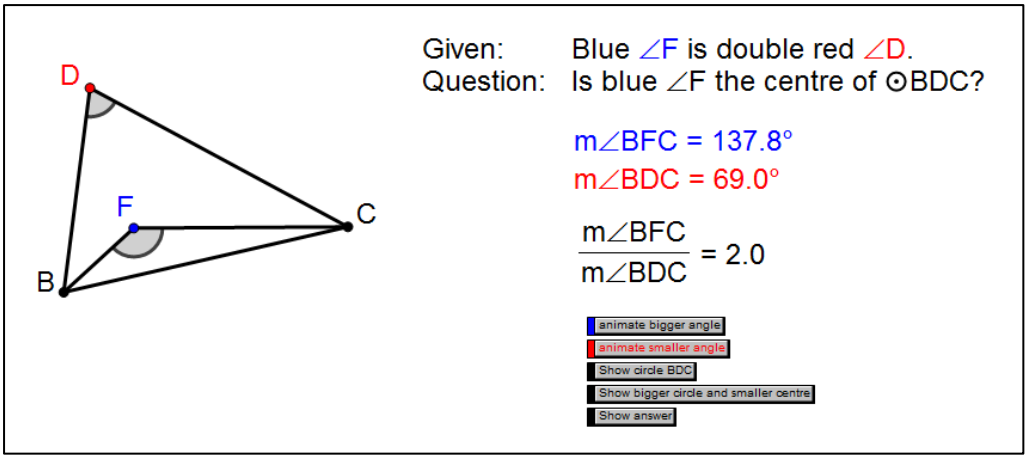
Southwood, S. (2013). Theorems and converses. In Z. Davis & S. Jaffer (Eds.), *Proceedings of the 19th Annual Congress of the Association for Mathematics Education of South Africa*, Vol. 2. (pp. 59 – 60). Cape Town: AMESA.



Demonstration 3: "If an arc (or chord) subtends an angle at the centre of the circle and an angle at the circumference of the same segment, the angle at the centre is double the angle at the circumference." The proof of this theorem is also straightforward as is its explanation using *Geometers Sketchpad*.

Demonstration 4: "If a side of a triangle subtends an angle inside the triangle which is double the opposite angle of the triangle then the angle inside the triangle is the centre of the circumscribed circle." We use a Sketchpad demonstration to prove that this is NOT true.

The screen copied below shows part of the method of explaining this.



Copies of detailed step-by-step instructions for each demonstration will be provided by the presenter.

ABSTRACT

The workshop is aimed at educators in the FET phase who are interested in circle geometry, the use of technology in its teaching and also in learning techniques in Geometers Sketchpad. The demonstration, discussion and activities will focus on the proofs of the circle theorems and their converses with a particular emphasis on proof by contradiction.