

From the President's Desk...



About 70% of our members now have e-mail. It therefore makes sense to increasingly move towards electronic communication with our members. For example, just like most of us receive our bank statements by e-mail and not as paper copies in the mail, so AMESA could be sending membership renewal notices and other communications by e-mail instead of paper copies through the mail.

Such a move would save us a substantial amount of money and save substantial manual labour. And we will save a few trees ...

Any move to electronic communication can only be successful if we have correct details of our members. However, our database is riddled with errors. When we attempt to send e-mails to any subset in the database, a substantial number gets returned. It is therefore vitally important that our members make sure that their details in our database are correct.

A major source of error in our database lies in the impossible task to decipher illegible handwriting in skew faxes. Please note that we provide Word Forms on our website for membership and congress registration, allowing members to type their detail, save the file, and attach it to an e-mail. Then our administrators can copy and paste instead of re-typing details, reducing the probability of errors. Please use these electronic forms and e-mail? Outdated fax machines should be thrown on the ash heap of history!

Starting with Congress 2014, we will be acknowledging the details of congress registration through e-mail and bulk SMSes and so try to improve communication with our members.

Alwyn Olivier

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Editor's Comment...



After a successful 2013, AMESA is once again ready to take up the challenge on behalf of its members and others in 2014. Our regions are committed to arranging high quality workshops, conferences and other mathematics happenings for its members. AMESA is the voice of Mathematics Education in South Africa and we do all in our power to uphold this important position. This issue reflects on our submissions to the DBE on ANA and the 2013 Grade 12 examinations. Should you like to comment or make suggestions, please contact me at news@amesa.org.za. Enjoy reading!

VG Govender

The South African Mathematics Foundation (SAMF) and its programmes for the First Quarter of 2014

SAMF is an initiative of AMESA (the Association for Mathematics Education of South Africa) and SAMS (the South African Mathematical Society) and its activities include teacher and learner programmes, publication of teacher and learner support materials, popularisation of mathematics and promoting public interest in mathematics research in South Africa. The SAMF ensures that the combined skills, know-how and professional capabilities of the members of these two societies are utilised in a professional manner. In this way the SAMF is a central office for Mathematics in the country and represents the entire mathematics community in South Africa.

With the recent spotlight on the 2013 matric results it has been reported that the Grade 12 final exam paper in mathematics is allegedly easier today than it was 20 years ago (Rapport, 5 January 2014, p. 4).

This, says Prof Johann Engelbrecht, Executive Director of the South African Mathematics Foundation (SAMF), is because there is no longer any difference between higher and standard grade. “The mathematics to which learners are exposed today is at a standard midway that of higher- and standard grade. This results in more learners taking the subject which in turn results in an increase in distinctions.”

He continued by saying that it is important for learners to study advanced mathematics as an additional subject. “It is the top learners who perform well at university and exposure to advanced mathematics will increase not only a learner’s chance of a distinction but also develop critical thinking and problem solving skills, which are two important skills needed to succeed at university.”

The SAMF manages a number of learner as well as teacher development programmes to better equip both learners and educators for the demands of advanced mathematics.

The annual South African Mathematical Olympiad (SAMO) is one such advanced mathematics programme. It takes place over three rounds and consists of separate papers for Grades 8 and 9 (junior division), and one paper for Grades 10 to 12 (senior division). The first round of the Olympiad is written at the individual schools on 13 March and the teachers mark the papers, which consist of 20 questions with multiple-choice answers each.

The Harmony Teacher Development Programme is a short course for teachers to develop their skills in problem solving Olympiad type material. It is available as a live taught (GET and FET phases) or DVD course (Level 1 for GET and FET phases and Level 2 for GET phase) and certificates are issued which may be used for continuous professional development. The entry fee is R57 per teacher. For more information on how to register, contact Patrick Rasehwete at the SAMF offices on 012 392-9348 or send an e-mail to rasehwetep@samf.ac.za.

Another teacher development project is the Primary Teacher Problem-Solving Course. The project aims to contribute towards improving the production of quality results in mathematics for Grades 4-7 learners, encouraging them to take mathematics as a subject in secondary school.

This 12-hour live taught problem solving course runs over two Saturdays when a group of 20 Grade 4 to 7 teachers in the same area register for participation. The programme is also offered as a distance learning course. The programme kicked off with a facilitator workshop, proudly offered in association with AMESA and Sasol, on 24 and 25 January. The course has been piloted in areas where Sasol has operations on 15 and 22 February. For more information on the SA Mathematics Primary School Project, please contact Thabo Ramaboea at the SAMF offices on 012 392 9342 or send an e-mail to info@samf.ac.za.

The cooperation of AMESA members to expand both learner and teacher projects are important and the SAMF requests AMESA regions to encourage schools to participate.

Find the SAMF online on:
Facebook: <http://www.facebook.com/MathematicsFoundation>
and Twitter: <http://twitter.com/MathsFoundation>



AMESA REGIONAL ACTIVITIES

As in previous years, AMESA regions are likely to be very busy in 2014. The organisation of the National Congress in Kimberly, Northern Cape is in full swing. In other regions, the focus will be on the regional conferences and teacher workshops.

The table below shows the dates and venues for the regional conferences for 2014.

Region (Province)	Date	Venue	Comments
Limpopo	22 March 2014	Tom Naude High School	Elections only for Secretary
North West	2-3 May 2014	Mabopane	No elections
Gauteng	10 May 2014	WITS Faculty of Education; Parktown	Elections for all positions
Mpumalanga	10-11 May 2014	Sybrand van Niekerk High School	No elections
Kwa-Zulu Natal	24 May 2014	Edgewood	No elections
Free State	23 August 2014	Lejweleputswa District, Welkom	No elections
Western Cape	6 September 2014	CLTI	No elections
Eastern Cape	13 September 2014	Mthatha	No elections

The formation of the Tshwane Branch in Gauteng

(information supplied by Thania Meiring)

Introduction and Background

The idea for a mini-conference in Tshwane took hold when Caroline Long attended the Gauteng Regional Conference in May 2013. She was so impressed by the enthusiasm and interest of the teachers and the quality of the presentations that she approached Khangelani Mdakane (Chairperson of AMESA Gauteng) with the idea. His vision had long been for a Tshwane branch. The idea was further discussed at the AMESA Congress in June, 2013, where there was great support from Rencia Lourens and other AMESA Gauteng members.

The first meeting of interested Tshwane people was on the 23rd July 2013 where the planning for the mini-conference was started. Professor Johann Engelbrecht assisted with getting support from the University of Pretoria for the branch. Edwin Smith, the Director of the Mamelodi Campus, gave the team his full support and the use of Mamelodi Campus for a mini-conference.

The first mini-conference

The first mini conference took place on 24th August 2013. The theme of this conference was "Mathematics teachers make a difference!" At this mini conference there were 6 strands and four presentations per strand. Casio did one of the presentations. Although no formal feedback was received, the general consensus was that the teachers enjoyed the conference and more importantly went back to their classrooms equipped with greater mathematics knowledge and the confidence from being part of a strong and vibrant AMESA community. The branch was launched here with an interim committee. On the 1st October 2013, the committee was formalised with office bearers.



Composition of the branch executive

A very inclusive and representative branch executive was elected. This is shown in the table below:

Position	Name
Chairperson	Caroline Long
Deputy Chairperson	William Chauke
Secretary	Thania Meiring
Treasurer	Ishaak Cassim
Foundation phase rep	Heidi Julies
Intermediate phase rep	Fatima Cassim
Senior phase rep	Caroline Long / Ronel Paulsen
FET rep	William Chauke
Colleges	Batseba Mofolo-Mbokane
Tertiary/universities	Johann Engelbrecht
Media liaison: Mamelodi Campus	Regina White
Media liaison: Groenkloof campus	Batseba Mofolo-Mbokane

The branch membership is said to be quite high although membership audits still have to be done. We look forward to some great activities from the Tshwane branch of AMESA.

Some branch activities

In 2013, the branch was involved in the review of the four Grade 12 exam papers. The planned activities for 2014 include a dedicated Geometry workshop for all phases on 22 February 2014.

Our remarks

We are pleased that the Tshwane branch has made great strides in its short existence. We are also impressed with the partnerships formed with the University of Pretoria.

Tshwane Branch mini-conference in pictures



The Plenary





A workshop



Another presentation





An interview with Mzwakhe Eric Sokutu, newly elected Chairperson of AMESA (Eastern Cape) and Eastern Cape Representative on the National Council

Mzwakhe is a mathematics teacher at Phakama Primary School in Zwide, Port Elizabeth. He has been a teacher since 1997 and is currently HOD for Intermediate phase and Grade 7.

Tell us something about Mzwakhe Eric Sokutu

I grew up on a farm in Alexandria along the coastal route to East London, Eastern Cape. I was in a farm school attending multi-grade classes where I passed standard 4 (Grade 6). It was in 1983. I wanted to complete my Grade 12 and become a teacher. This was realised when I was accepted at the Algoa College of Education in 1994.

Q. When did you join AMESA?

I joined AMESA in 2004. 2014 marks my tenth anniversary as a member of AMESA, an important milestone in my life.

Q. How many AMESA Congresses have you been to?

I have been to 7 congresses. My first one was at Polokwane in 2006. At the time Mamokgeti Setati was stepping down as President and I shed a tear when I listened to her speech and words of good bye. I loved her so much it was funny. I did not want her to step down. Ray Duba was just fine though as her successor. I only missed the one in Cape Town in 2013.

Q. What have been some of the highlights of being an AMESA member and how has AMESA helped you in your professional development?

I learnt a lot as an AMESA having a sense of belonging to a professional association which makes available information that enhances ones teaching skills in the classroom. I have also gained a lot of experience in organising some activities especially after getting exposure in being an LOC member for the PE based EC Congress held in 2008. I am a presenter without fear through experiences I got from being a member of the association. I also met a lot of good people and made friends.

Q. What would you say to mathematics teachers who would like to join AMESA but are hesitant to do so?

To teachers who want to join but are hesitant I say, they are losing out on a "good thing". Sometime it is not enough to just get told about something, just attend one congress and see if you will ever relax at home during the congress days, I bet one will not. I was even frustrated when I had to miss the congress in Cape Town due to being involved in CAPS training for Senior Phase teachers in the PE District.

AMESA 2014 Congress in Kimberly, Northern Cape

This year marks our coming of age as AMESA celebrates its 21st birthday. At the time of going to press, arrangements for our 20th Congress in the Northern Cape are in full swing. AMESA members have already received the first announcement of Congress 2014 and the second announcement will be available at the beginning of April.

Theme:	Demystifying Mathematics
Date:	7 - 11 July 2014
Venue:	Diamantvelt High School and Kimberly Boy's High School, Kimberly, Northern Cape

Note: The date for the submission of presentation proposals has been extended from 20 February to 18 March 2014



NOTICE OF AGM AND AGENDA

Notice is hereby given that the **20th Annual General Meeting** of the Association for Mathematics Education of South Africa, will be held on **Tuesday 8th July 2014** in conjunction with the Association's Annual National Congress at the **Diamantveldt High School, Kimberley.**

AGENDA

1. Welcome
2. Apologies
3. Finalising of the Agenda
4. Minutes of the Previous Meeting
5. National Council Report on the Activities of the Association
6. Financial Report and Budget
7. Presidential Address
8. Motions:
 - 8.1 That the AMESA membership fees be increased as follows:

Individual membership:	From R120 to R130
Associate (student) membership:	From R30 to R40
Institutional membership:	From R340 to R360
Life membership:	From R3 000 to R3 250
Other African countries, individual	From ZAR150 to ZAR170
Non-African countries, individual:	From USD65 to USD70
9. Nominations
 - 9.1 Are invited for the following **National Office Bearers** positions:
 - President
 - Vice President
 - Secretary
10. Any Other Business
11. Closure



APPENDIX A: (AMESA Constitution clause 18 - 33)

ANNUAL GENERAL MEETING

18. The supreme governing body of the Association shall be the Annual General Meeting, consisting of the following members:
 - 18.1 National Office Bearers.
 - 18.2 One representative from each branch, appointed by the branch
 - 18.3 $n \div 100$ (rounded to the next whole number) representatives from each region, appointed by the region, where n is the number of members in good standing in the region.
19. The Annual General Meeting shall meet at least once every year and shall be convened by the Council, who shall determine the place and date(s) of the Annual General Meeting. The Annual General Meeting shall normally meet during the national congress of the Association.
20. Notice of a meeting of the Annual General Meeting shall be given in writing and shall be posted to all members at least one month before the date of the meeting. The notice shall include the agenda of the meeting.
21. The Annual General Meeting shall adopt policies in furtherance of the aims and objects of the Association. The Annual General Meeting will specifically have the task of dealing with:
 - 21.1 the confirmation of the agenda;
 - 21.2 the credentials of delegates;
 - 21.3 amendments to the Constitution;
 - 21.4 reports from the National Council;
 - 21.5 reports from the National Office Bearers;
 - 21.6 financial reports, including the auditor's report;
 - 21.7 motions;
 - 21.8 election of the National Office Bearers;
 - 21.9 matters referred to it by the National Council;
 - 21.10 directives to be implemented by the National Council;
 - 21.11 any other matter entrusted to it under this Constitution.
22. The proceedings of the Annual General Meeting shall be open to all members of the Association. All members of the Association shall have the right to speak at the Annual General Meeting.
23. A majority of representatives shall constitute a quorum at the Annual General Meeting. In the event that there is no quorum within two hours of the time fixed for the meeting of the Conference, the meeting shall stand adjourned to the next day at the same time and place. At such adjourned meeting the representatives shall form a quorum. A notice of the adjourned meeting shall be telefaxed, telexed or conveyed by telephone to the Regional Secretaries of each region.

Procedures for nominations

24. Nominations for Office Bearers may be submitted by any two members, a branch or a region of the Association. Such nominations must reach the National Secretary not less than three calendar months before the next Annual General Meeting.
25. All nominations shall be made in writing and must contain the names of the proposers and the consent of the person



nominated. Nominations by a branch or region must be signed by at least four members of the branch committee or Regional committee.

26. The National Council shall make nominations in respect of every vacancy to which no other nomination has been received and may make nominations in addition to those received.
27. The names of all persons nominated for election at the Annual General Meeting shall be included in the agenda of the Annual General Meeting.

Procedures for motions and decision-making

28. Any two members, a branch or a region of the Association may submit a motion to the Annual General Meeting. Any motion on which the Annual General Meeting must vote, must reach the National Secretary not less than three calendar months before the date of the Annual General Meeting and must be included in the agenda for the Annual General Meeting.
29. Any motion that does not comply with paragraph 28 may be put to the Annual General Meeting, but may only be put to the vote if an order motion to this effect is carried unanimously by the Annual General Meeting.
30. All members of the Annual General Meeting shall have the right to vote at meetings of the Annual General Meeting and each member shall have one vote. Invited persons shall not have the right to vote.
31. Motions submitted to the Annual General Meeting, other than motions for election or for amending the Constitution, shall be determined by open vote, unless the chairperson or any other member demands a ballot.
32. Any motion submitted to the Annual General Meeting, with the exception of a motion for amending the Constitution, may be carried by a simple majority of votes. In the event of a tie the chairperson shall have a second and casting vote.
33. Motions for election in which the number of nominations is equal to the number of vacancies, shall be declared carried unopposed. Contested elections shall be determined by ballot.

AMESA Curriculum Matters

The members of the curriculum committee are VG Govender and Rajen Govender. Both members of the committee are also members of the ACM (Advisory Committee for Mathematics) a subcommittee of the SAMF (South African Mathematics Foundation). The ACM commissioned three reports in 2013, all of which were accepted by the ACM and also published on the SAMF website.

ACM meeting with United States delegation

The People-to-People International (PTPI) was founded in 1956 by then United States President Dwight D. Eisenhower. PTPI promotes international understanding and friendship through educational, cultural and humanitarian activities. The ACM met with a People-to-People delegation from the USA in October 2013 in Cape Town.

The following persons were part of the USA delegation:

- Johnny Lott (Retired: University of Montana)
- Rebecca Adams (Retired Elementary teacher: Chesapeake Public Schools - Virginia)
- Carol Lerch (Worcester State University: Massachusetts)



Barbara Pond (Colorado Christian University)
Barbara Moses (Bowling Green State University)
Abdulim Shabazz (Grambling State University, Louisiana)

The 4 ACM members plus 6 others from Western Cape (mostly AMESA members) formed part of the South African delegation.

VG (Nico) Govender chaired the meeting alongside Johnny Lott. The meeting covered a variety of issues. Some of the issues discussed were:

- Common core standards in USA for mathematics 48 states; National Curriculum versus changes in the South African system (NATED 550; Curriculum 2005; NCS; CAPS).
- Changes in assessment; teacher training; materials development; stability in curriculum - changes linked to appointment of new ministers.
- Attracting new people to mathematics teaching - challenge in both countries.
- Preparation of learners for tertiary studies; system of colleges of education 2 years - primary school teacher - teacher education now a university competence in South Africa (as in the USA).
- In-service teachers - retrained for Mathematical Literacy ACE; Mathematical Literacy (quantitative literacy) also offered in some US states/school districts as an alternative to mathematics.
- Changes in minimum teacher education qualifications - from 2015 in South Africa; keeping teachers in the classroom rather than moving into admin positions; in both countries good mathematics teachers are “lost” to the profession due to their move to more admin positions (as a result of promotion).
- Performance of learners ; challenges of improving learner performance in both countries.
- Use of technology - USA graphic calculators; CAS, etc. changed over the years; South Africa also similar - use of cell phones now (Mxit competition).
- Geometry for high schools - Stand alone in the USA (after algebra 1); in South Africa Geometry is part of mathematics (as one of the sections such as Algebra, Trigonometry, etc.).
- In USA - Transformation Geometry is now part of common core; in South Africa - it has been moved to the Senior Phase; Euclidean Geometry compulsory in FET.
- Assessment - comparisons; in South Africa the focus is on Grade 12 exams; in the USA there are tests (set by states) at various grades; teacher is responsible for assessments; in South Africa there are the ANAs.

Our comments

The discussions were very fruitful and served to enlighten all those who were present. Both common issues and differences were discussed at length. The ACM was given a certificate of appreciation by the People-to-people delegation. This was handed over to SAMF at its board meeting in November 2013.

In addition to the work of the ACM, the AMESA curriculum committee has been very busy over the past year. Its main activities for 2013 were the review of the Grade 12 Mathematics and Mathematical Literacy papers and the review of the Grade 9 ANA for mathematics. The committee also made submissions to the Department of Basic Education about its intended “Technical Mathematics” for technical schools.





The Association for Mathematics Education
of South Africa

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26 November 2013

Mr Matanzima Mveli
Acting Director General
Curriculum Implementation & Support
Department of Basic Education
222 Struben Street
PRETORIA

AMESA report on the 2013 Mathematics & Mathematical Literacy Examination Papers

Dear Mr Mveli

On behalf of the National Council of the Association for Mathematics Education of South Africa (AMESA), I would like to commend the Department of Basic Education for the wonderful support afforded the Grade 12 learners and their teachers in the 2013 academic year.

As we have done for the past several years, I would like to make a formal submission of the AMESA Report on the 2013 Mathematics and Mathematical Literacy Examination Papers 1 and 2 that were written by the Grade 12 learners. The purpose of the report is to provide constructive feedback to the Department in the spirit of promoting mathematics education and enhancing the quality of the teaching and learning of Mathematics in South Africa.

It is our hope that the report, especially the question by question analyses, will be useful to the examiners, markers and moderators in our attempt to promote the high standard of mathematics education in our country.

Sincerely



Alwyn Olivier
AMESA President



AMESA report on the 2013 Mathematics & Mathematical Literacy Examination Papers

INTRODUCTION

The Grade 12 papers for Mathematics and Mathematical Literacy were written on Friday 1 November 2013 (Paper 1) and Tuesday 5 November 2013 (Paper 2).

AMESA regions (provinces) throughout South Africa soon after embarked on a workshop activity to review these papers according to specific criteria and guidelines. The provinces submitted their reports to the AMESA National Curriculum Committee. This report was then compiled by the Curriculum Committee and represents a summary of the findings and trends of the AMESA provincial reports.

The report covers specific comments on each paper focusing on the following aspects:

A. Overall Review

1. Technical aspects (typing, diagrams, etc.)
2. Language used and compliance with the cognitive levels of thinking
3. Curriculum coverage
4. Comparison with 2011 papers
5. Overall observations

B. Question by Question Analysis

Provincial leaders were “trained” in the analysis of questions using the analysis tool. Although we do not claim any validity of the analysis, we are nevertheless confident that it represents a fairly balanced and accurate perspective from a cross-section of teachers throughout the country.

MATHEMATICS PAPER 1

A. OVERALL REVIEW

1. Technical aspects (typing, diagrams, etc.)

As has been the trend for the past few years the paper was clear with no typing errors. All the diagrams, with the exception of the one for question 5, were well constructed, neat and clear. The arrows in question 5 seem to indicate intersection with axes which is fundamentally wrong for this type of graph.

Otherwise, the Department of Basic Education is to be complimented for its high technical standard.

2. Language used

The language used in the paper was clear and unambiguous and would be within the reach of most Grade 12 mathematics learners. Short sentences are used and are to the point. There is no complex English terminology in the paper. Words like “solve”, “simplify”, “calculate”, “write down”, “determine” and “sketch” have been used effectively throughout the examination paper.

All content was covered in the paper. However, a question on the inflection point was missing. But this is unlikely to have any impact on learner performance in the paper.



3. Syllabus coverage

Code	Content/Topic	Suggested	November 2013
1	Patterns & Sequences (LO1)	30	36
2	Annuities & Finance (LO1)	15	12
3	Functions & Graphs (LO2)	35	32
4	Algebraic manipulation; equations (LO2)	20	22
5	Calculus (LO2)	35	34
6	Linear Programming (LO2)	15	14
	Total	150	150

4.1 Standard of paper

The paper could be classified as a very reasonable paper. If learners were taught well, then a level 4 (minimum 50%) would be within reach. The table below shows that more questions were pitched at a routine procedures level (level 2) than a complex procedures level (level 3). In terms of balance, the combined level 1 and level 2 cognitive levels of the paper, is approximately 6% higher than the suggested 55%.

4.2 Compliance with levels of thinking

Code	Levels of thinking	Suggested	November 2012
1	Knowledge	25%	19%
2	Routine procedures	30%	42%
3	Complex procedures	30%	27%
4	Solving problems	15%	12%

The table above shows that the paper was well balanced and within the acceptable range for each level of thinking as prescribed by the Subject Assessment Guidelines.

4.3 Comparison with 2012 paper

Another column has been added to the above table to include our 2012 analyses of Paper 1.

Levels of thinking	Suggested	November 2013	November 2012	Difference
1 - Knowledge	25%	19%	22,7%	-3,7%
2 - Routine procedures	30%	42%	36%	6%
3 - Complex procedures	30%	27%	26%	1%
4 - Solving problems	15%	12%	15,3%	-3.3%

The paper, in terms of standard, was very similar to the 2012 paper, with marginal differences in the cognitive levels. These differences are not likely to impact significantly on learner performance.

6. Overall verdict

It was a fair, balanced paper. There was ample opportunity for learners to score well in this paper. However, learner performance would depend on a number of factors which are beyond the scope of this report.



B. QUESTION BY QUESTION ANALYSIS

Quest.	Content	Levels				Marks	Topic	Comment
		1	2	3	4			

Question 1: Algebraic manipulation; equations

1.1.1	Quadratic equation	2	1			3	4	Solving by factorisation
1.1.2(a)	Quadratic formula	2	2			4		Use quadratic formula
1.1.2(b)	Quadratic formula and common factor		2			2		Extension of 1.1.2a
1.1.3	Quadratic inequality	2	2			4		The impact of -3 on the inequality
1.2	Simultaneous equations	2	4			6		Express y in terms of x ; then routine
1.3	Simplification involving exponents			2	1	3		Express in terms of base 3; factorise numerator, then divide ("cancel") common factor
Total		8	11	2	1	22		

Question 2: Patterns and Sequences

2.1	Geometric sequence	1	2			3	1	Application; two values or x
2.2.1	Tenth term of a geometric sequence	1	2			3		Get a and r ; then substitute in formula
2.2.2	Sum to 9 terms of geometric sequence		1	1		2		Substitute in formula
2.3.1	Specific term of a combination sequence			1		1		The odd terms are all zero
2.3.2	Sum of first 500 terms of combination sequence		3	1		4		Consider only the non-zero terms (only take 250 terms; the other 250 terms are zero)
2.4.1	First term of a series given in sigma notation	1	1			2		Substitute in sigma notation formula
2.4.2	Existence of the sum to infinity			2	1	3		We note that $-1 < r < 1$
Total		3	9	5	1	18		



Quest.	Content	Levels				Marks	Topic	Comment
		1	2	3	4			

Question 3: Patterns & Sequences

3.1.1	nth term of arithmetic sequence	1	1			2	1	Substitute in formula
3.1.2	Specific terms of sequence	2				2		Add common difference
3.1.3	Finding the remainders when dividing by 3		2			2		Division
3.1.4	Sum of terms which are divisible by 3		3	2		5		Selecting certain terms of sequence
3.2.1	Number of dots in fifth figure	2				2		Following a pattern
3.2.2	Number of dots in fiftieth figure			3	2	5		Substitute in formula (which must be derived)
Total		5	6	5	2	18		

Question 4: Functions & Graphs

4.1	y-intercept of quadratic function	1				1	3	Simple; let $x = 0$
4.2	x-intercepts of quadratic function	1	2			3		Let $y = 0$, solve by factorisation
4.3	Turning point		3			3		Could use the formula
4.4	Sketching quadratic function (parabola)	1	2			3		Simple sketch
Total		3	7			10		

Question 5: Functions & Graphs; Logs

5.1.1	Determining the base in an exponential function			2		2	3	Substitute in graph equation; graphs not clear - show intersection with axes
5.1.2	Inverse of exponential function			2		2		The inverse of an exp function is a log function
5.1.3	Log inequality			1	1	2		Consider the intervals; cannot find log of negative number
5.1.4	Domain of a transformed log function				1	1		Consider values of x for which $x - 3 > 0$



Quest.	Content	Levels				Marks	Topic	Comment
		1	2	3	4			
5.2.1	Sketching the inverse of $y = 1$			2		2		The inverse is $x = 1$
5.2.2	Motivating whether the inverse is a function		2			2		Not a function
Total			2	7	2	11		

Question 6: Functions & Graphs

6.1.1	Determining values of constants in hyperbola			2		2	3	Manipulation of given equation then substitution or reading off from graph
6.1.2	Rewriting equation in another form	1	1			2		Linked to 6.1.1
6.1.3	Coordinates of the intersection of the asymptotes	2				2		Read off from graph or equation in 6.1.2
6.1.4	Reflection in the line $y = x + 2$			2		2		y - value of image: substitute $x = 2$ in $y = x + 2$; x - value of image: substitute $y = 0$ in $x = y - 2$
6.2	Calculation of constants in an exponential function				3	3		Interpretation of function; substitution
Total		3	1	4	3	11		

Question 7: Annuities & Finance

7.1.1	Effective annual interest rate		2			2	2	Use formula or do a simple calculation
7.1.2	Calculation of number of years k		3	2		5		Changing subject of formula
7.2.1	Calculation of amount to spend each month	1				1		Simple calculation
7.2.2	Present value		2	2		4		Substitute in present value formula
Total		1	7	4		12		



Quest.	Content	Levels				Marks	Topic	Comment
		1	2	3	4			

Question 8: Calculus

8.1.1	Derivative from first principles	5				5	5	Simple substitution
8.1.2	Average gradient		3	2		5		Interpretation; substitution
8.2	Using the rules of differentiation		2	1		3		Divide by first
8.3	Gradient of tangent		2	2		4		Substitute in derivative
Total		5	7	5		17		

Question 9: Calculus

9.1	Calculation of constants in cubic function			2	4	6	5	Obtained from two equations; $f(x)$ and $f'(x)$
9.2	Coordinates of other turning point		3			3		Use values for a and b from 9.1; simple substitution in $f'(x)$
9.3	Turning point of transformed function				2	2		A vertical stretch and a shift 4 units downward
Total			3	2	4	11		

Question 10: Calculus

10.1	Rate of change; maximum rate		2	1		3	5	Solve $r'=0$
10.2	Rate of change - stop flowing		2		1	3		Solve $r = 0$
Total			4	1	1	6		

Question 11: Linear Programming

11.1	Constraints			4		4	6	From descriptions
11.2	Sketching of constraints (inequalities)		5			5		Drawing of inequalities from 11.1
11.3.1	Profit equation	1				1		Simple equation
11.3.2	Coordinates which will give maximum profit		1	1		2		Use search line or substitution
11.4	Objective function				2	2		Calculation of if P is maximised when
Total		1	6	5	2	14		



Summary of marks and levels per question					
Question	Levels				Marks
	1	2	3	4	
1	8	11	2	1	22
2	3	9	5	1	18
3	5	6	5	2	18
4	3	7			10
5		2	7	2	11
6	3	1	4	3	11
7	1	7	4		12
8	5	7	5		17
9		3	2	4	11
10		4	1	1	6
11	1	6	5	2	14
Total	29	63	40	18	150
Percentage	19%	42%	27%	12%	100%

MATHEMATICS PAPER 2

A. OVERALL REVIEW

1. Technical aspects (typing, diagrams, etc.)

As far as the technical criteria of compliance are concerned, the typing was clear and error free, while the diagrams were clear and understandable. However, in some provinces the angle of rotation was not shown in the diagram for Question 9. Also, the square base for question did not show at least one right angle.

Despite these issues, the Department of Basic Education is to be complimented for its high technical standards for Mathematics paper 2.

2. Language used

As for paper 1, the language usage was clear and precise. However, the Transformation Geometry seemed unusually 'wordy' in comparison to the previous years. In addition, Question 8.4 seemed overly confusing as to what was required and how the answer should be presented. This question should have been better phrased.

3. Syllabus coverage

Code	Content/Topic	Suggested	November 2013
1	Coordinate Geometry	40	39
2	Transformation Geometry	25	26
3	Trigonometry	60	60
4	Data Handling	25	25
	Total	150	150



4.1 Standard of paper

Our analyses (in the table below) show that the combined level 1 and level 2 questions (knowledge and routine procedures) was 54% which was 1% lower than the suggested 55% for this two levels. This means that the level 3 and level 4 questions (complex procedures and solving problems) were 1% higher at 46%. A further interrogation of these levels shows the paper learning towards the problem solving level of thinking.

A number of innovative questions are included in the paper and these may have “distracted” learners. However, the brighter learners would have been able to easily work through these distractions. Thus, we commend the Department of Basic Education for the high standards set.

4.2 Compliance with levels of thinking

Code	Levels of thinking	Suggested	November 2013
1	Knowledge	25%	23%
2	Routine procedures	30%	31%
3	Complex procedures	30%	27%
4	Solving problems	15%	19%

4.3 Comparison with 2012 paper

To compare the paper to the 2012 paper, a column was added to the above table to include our analyses of the 2012.

Code	Levels of thinking	Suggested	November 2013	November 2012	Difference
1	Knowledge	25%	23%	22%	1%
2	Routine procedures	30%	31%	33.3%	-2,3%
3	Complex procedures	30%	27%	30%	-3%
4	Solving problems	15%	19%	14.7%	5,7%

5. Overall verdict

It would appear from learners’ views after the paper that they found the questions quite challenging, possibly because of the higher level 4 proportion of the paper. At the same time if learners were taught properly and all the work was covered in class, then there were sufficient questions in the paper for learners to pass. In this regard, we would say that the paper was “not unfair”. The inclusion of some innovative and possibly challenging questions ensured that the paper was not predictable. This is likely to favour those who worked consistently well over the last few years.

Our overall verdict for this paper would be “a challenging but acceptable paper”.



Quest.	Content	Levels				Marks	Topic	Comment
		1	2	3	4			

Question 1: Data Handling

1.1	Interquartile range	2				2	4	IQR = $Q_3 - Q_1$
1.2	Interpretation of box and whisker-percentage	2				2		Simple percentage
1.3	Interpretation of box and whisker - quartiles			2		2		Calculation & interpretation
Total		4		2		6		

Question 2: Data Handling

2.1	Scatterplot	1	2			3	4	Simple plotting of points
2.2	Line/curve of best fit	1				1		Simple drawing
2.3	Data trends		1			1		Description
2.4	Probability		2			2		Estimating from table
Total		2	5			7		

Question 3: Data Handling

3.1	Estimation from Ogive	1	1			2	4	Read off from curve
3.2	Estimation from Ogive	1	1			2		Read off from curve
3.3	Modal class from Ogive	1				1		Check difference
Total		3	2			5		

Question 4: Data Handling

4.1	Interpretation of table and Bell curve			2		2	4	Noting mean and spread
4.2	Interpretation of Bell curve		1			2		Difference
4.3	Adjustment of marks				2			What should be added/subtracted
4.4	Effect on mean and standard deviation				2			Effect on new mean and standard deviation
Total			1	2	4	7		



Quest.	Content	Levels				Marks	Topic	Comment
		1	2	3	4			

Question 5: Coordinate Geometry

5.1.1	Gradient from angle	2				2	1	tan
5.1.2	Equation of line	1	1			2		Using x-intercept and gradient
5.1.3	Distance	1	1	1		3		Find S first then use distance formula
5.1.4	Coordinates of point T		2			2		From midpoints
5.2	x-intercept of line TR, written in coordinate form			2		2		Use ratio 2:3
5.3	Area of triangle	1	2	1		4		Use area rule
Total		5	6	4		15		

Question 6: Coordinate Geometry

6.1	Centre of circle	1	2	1		4	1	Write equation in terms of centre and radius
6.2	Equation of MR	1	2			3		Simple straight line
6.3	Relationship between p and q		2	2		4		Using equations
6.4	Calculate values of p and q			2	3	5		Using simultaneous equations
6.5	Equation of circle		2			2		Equation of circle centre O and radius ON, centre is (0;0)
6.6	Area of circle centre M	2				2		Use radius MR
6.7	Calculation of ratio				4	4		Work out NP and MP first
Total		4	8	5	7	24		

Question 7: Transformation Geometry

7.1	Rotation through 90° about the origin; then reflection	2				2	2	Rotation and reflection
7.2	Reflection and then rotation	2				2		Reflection then rotation
7.3	Checking whether order affects final position of transformation				2	2		Does the order matter?
Total		4			2	6		



Quest.	Content	Levels				Marks	Topic	Comment
		1	2	3	4			

Question 8: Transformation Geometry

8.1	Description of transformation	1	1			2	1	Move 4 units up then 4 units to the left; use of the word rigid is unique
8.2	Image of reflection	1	1			2		Reflection about $y = x$
8.3.1	Scale factor of enlargement		1			1		Reduce to one dimension;
8.3.2	Calculation of length of A'C'		1			1		Size of new size
8.4	Rigid transformation; equation in terms of s and t				4	4		New relationship
Total		2	4		4	10		

Question 9: Transformation Geometry

9.1	Rotation - calculation of angle			5		5	1	Using formula; x and y not indicated on axes
9.2	Speed of wheel in revolutions per minute				5	5		Calculation of arc distance and then speed as the time is given; very unique/unusual question
Total				5	5	10		

Question 10: Transformation Geometry

10.1	Trig ratio from diagram	2	1			3	3	Calculate r first
10.2	Trig ratio from diagram; reduction formula		2			2		Simplification then use values from diagram
10.3	Trig ratio from diagram; compound angle			3		3		Expand then substitute
Total		2	3	3		8		



Quest.	Content	Levels				Marks	Topic	Comment
		1	2	3	4			

Question 11: Trigonometry

11.1	Proving an identity	2	2	2		6	3	Work with LHS first; routine reductions with complex procedures
11.2	General solution of trig equation	2	3	2		7		Factorise with common factor then equate both factors to 0
11.3.1	Simplification of trig expression			1	2	3		Use identities; double angle not clearly evident
11.3.2	Maximum value of trig expression				1	1		Follow up of 11.3.1
11.4.1a	Trig ratio in terms of p and q		2	1		3		Expansion and manipulation
11.4.1b	Trig ratio in terms of p and q			2	2	4		Unique problem; manipulation
11.4.2	Simplification of an algebraic fraction (in terms of trig ratio)	2	2	2		6		Simplify expression; then substitute; then simplify again
Total		6	9	10	5	30		

Question 12: Trigonometry

12.1	Trig graphs	2	4			6	3	Simple graphs; routine
12.2	Period of g	1				1		The period is
12.3	Description of transformation			2		2		Shifting to the left then a reflection in the x-axis
12.3	Trig inequality; graph interpretation			2	2	4		Noting that $f'(x) > 0$ when $f(x)$ is increasing
Total		3	4	4	2	13		

Question 13: Trigonometry

13.1	Calculation of apex angle		2	1		3	3	Using cosine rule
13.2	Angle between face and base		2	4		6		Many steps; first get EF and GF
Total			4	5		9		



Summary of marks and levels per question					
Question	Levels				Marks
	1	2	3	4	
1	4		2		6
2	2	5			7
3	3	2			5
4		1	2	4	7
5	5	6	4		15
6	4	8	5	7	24
7	4			2	6
8	2	4		4	10
9			5	5	10
10	2	3	3		8
11	6	9	10	5	30
12	3	4	4	2	13
13		4	5		9
TOTAL	35	46	40	29	150
Percentage	23%	31%	27%	19%	100%

MATHEMATICAL LITERACY PAPER 1

A. OVERALL REVIEW

1. Technical aspects (typing, diagrams, etc.)

While all the technical aspects were in keeping with the usual high standards of the Department of Basic Education, the following issues were raised by AMESA regions:

- There was a printing error in question 4.2, where the keys shown were not correct since they all started at one.
- The scale marking for 125 km is not indicated in question 4.2. Since learners had to measure in question 4.2.6, they may not have been able to accurately obtain the scale for the map.
- In question 5.2, the key included the microscope. However, there was no question about the microscope. This may have been an unnecessary distraction to learners.

2. Language used

The use of language in the paper appeared to be within the reach of most Grade 12 Mathematical Literacy learners. However, the AMESA regions raised the following issues with respect to language:

- In question 2.2.3, the word *modus* was used. Most learners are not exposed to this word. Here *mode* would have been the better word.
- In question 4.1 the wording used (very safe, fairly safe, a bit unsafe and very unsafe), appeared to be confusing to learners.
- In question 5.2.2, the term “visually impaired” may have confused learners. “Visually impaired” could mean “blind” or “partially sighted”



3. Syllabus coverage

Code	Learning Outcomes	Suggested	November 2013
L01	Numbers and operations in context	37	40
L02	Functional relationships	38	37
L03	Space, Shape and measurement	38	36
L04	Data Handling	37	37
	Total	150	150

4.1 Standard of paper

The question paper was of a good and acceptable standard for Mathematical Literacy paper 1. The questions were set in such a way that it is easy to distinguish between the sub-sections. The questions ranged from very easy to slightly difficult. This is in keeping with the departmental requirement that only “knowledge” and “routine procedures” questions form part of Mathematical Literacy P1.

4.2 Compliance with levels of thinking

Code	Levels of thinking	Suggested	November 2013
1	Knowledge	60%	63%
2	Routine procedures	40%	37%
3	Non-routine procedures	0%	-
4	Reasoning and analysis	0%	-

4.3 Comparison with 2012 paper

Another column was added to the table above to include our 2012 analyses of the November 2012 paper.

Code	Levels of thinking	Suggested	November 2013	November 2012	Difference
1	Knowledge	60%	63%	56%	7%
2	Routine procedures	40%	37%	44%	-7%
3	Non-routine procedures	0%	-	-	-3%
4	Reasoning and analysis	0%	-	-	-

5. Overall verdict

Before deciding on the overall verdict, we would like to make note of the following issues which were raised by regions: The understanding is that in Mathematical Literacy examination papers, learners should be given formulas. All they need to do is to substitute into the given formulas. This was not the case in questions 1.3.2 and 6.3.1. Further, in question 2.3.4 learners are asked to derive a formula; teachers feel that this should not be expected of Mathematical Literacy learners in Paper 1. There was a possibility that question 2.3.2 may have confused learners. The fact that SEVEN is written in capital letters tended to mislead learners since they might divide by seven instead of eight, thus leaving out the owner of the car. Learners may have found the formula given in question 3.1.1 to be “a bit difficult” since it is not one of the usual “Mathematical Literacy” formulas.

These issues are valid one and should be considered for future references. However, these issues are likely to have minimal impact on learner performance.

With this in mind, we would like to proclaim that the paper was fair and within reach of most, if not all, Mathematical Literacy learners.



B. QUESTION BY QUESTION ANALYSIS

Quest.	Content	Levels				Marks	Topic	Comment
		1	2	3	4			

Question 1

1.1.1	Operations involving square root, percentages and decimals	2				2	LO1	Simple calculation using the calculator
1.1.2	Subtraction involving large numbers	1				1		Simple calculation
1.1.3	Calculations involving conversion of units		2			2		Ratio and proportion
1.1.4	Calculation of time given the distance and speed	2	3			2		Simple substitution
1.1.5a	Packing apples - calculation of time					3		Simple time calculation
1.1.5b	Average rate of working	2				2		Number of apples packed per minute
1.1.6	Probability		2			2		Simple probability
1.1.7	Ratio		2			2		Dividing the animals in given ratio; # sheep = $\frac{35}{36} \times 288$
1.2.1	Cost per CD	2				2		Division
1.2.2	Minimum number of CDs	3				3		Division
1.2.3	Writeable area of CD	3				3		Substitution in formula
1.3.1	Calculation of number of days	2				2		Division
1.3.2	Percentage discount	3				3		Percentage discount = $\frac{35}{304,99} \times 100\%$
1.3.3	Price excluding VAT		2			2		Price excluding VAT = $\frac{100}{114} \times R12,49$
1.3.4	Cost of goods	2				2	Multiplication and addition	
Total		22	11			33		



Quest.	Content	Levels				Marks	Topic	Comment
		1	2	3	4			

Question 2

2.1.1	Maximum length		2			2	LO1/ 3/4	Determining various distances
2.1.2	Total area of semi-circular sections		4			4		Simple calculation
2.1.3	Perimeter of garden	2				2		Simple substitution
2.1.4	Number of thyme plants	2				2		Substitution in formula
2.2.1	Ages in ascending order	2				2		From lowest to highest
2.2.2	Range		2			3		Highest - lowest
2.2.3	Mode	2				3		The one the appears the most
2.2.4	Mean	1	2			3		Add all up and divide by 15
2.2.5	Ages greater than upper quartile		2			2		Those who are older than 25
2.2.6	Depreciated value of toys	3				3		Substitution in given formula
2.3.1	Type of proportion	1				1		Inverse proportion
2.3.2	Monthly cost given the number of colleagues	2				2		Read off from graph
2.3.3	Sharing - given the cost	2				2		Read off from graph
2.3.4	Determining a formula		2			2		Division
Total		17	14			31		

Question 3

3.1.1	Area to be repainted	3				3	LO1/3	Substitution in formula; answer in correct units
3.1.2	Height of water	2	1			2		Substitution in formula; answer in correct units
3.2	Converting temperature from Celsius to Fahrenheit	2				3		Substitution; units in °F
3.3.1	Determining number of children	2				2		Subtraction
3.3.2	Total income		4			4		Substitution in given formula
3.4	Number of branded bags given the profit	3				3		Profit per bag was R22,00; divide R594 by 22



Quest.	Content	Levels				Marks	Topic	Comment
		1	2	3	4			

Question 3 (Continued)

3.5	Discounted price for pool pump	1	2			3		Discounted price = $\frac{88}{100} \times R4999$
3.6	Exchange rate	2				2		Division
Total		15	7			22		

Question 4

4.1.1	Calculate missing value from graph	1				1	LO1 /2/3	Subtraction
4.1.2	Percentage	2				2		Reading off from table
4.1.3	Comparison between graphs	2				2		Reading off from table; comparison
4.1.4	Interpretation from graph	2				2		Addition
4.1.5	Difference	2				2		Reading off appropriate values from table and subtracting
4.1.6	Ratio	2				2		Division; rounding off to nearest whole number
4.2.1	Identifying from map	2				2		Simple identification
4.2.2	Identifying from map	1				1		Simple identification
4.2.3	Identifying from map - percentage		2			2		Simple identification
4.2.4	Identifying from map - percentage	2				2		Simple identification
4.2.5	Identifying from map - direction	2				2		Simple identification
4.2.6	Scale		3			3		Scale calculation
Total		18	5			23		

Question 5

5.1.1	Calculating using table values	2				2	LO2/3	Doubling after every two hours: $K = 3200$
5.1.2	Time taken to multiply	2				2		Read off from table
5.1.3	Drawing of curve		5			5		Simple drawing
5.1.4	Average growth rate	2	1			3		Substitution
5.2.1	Location on plan/map		1			1		Identifying from map
5.2.2	Direction	3				3		Giving accurate directions



Quest.	Content	Levels				Marks	Topic	Comment
		1	2	3	4			

Question 5 (Continued)

5.2.3	Calculation of width	3				3		Substitution
5.2.4	Actual length from scale of plan	2				2		Scale calculations
Total		14	7			21		

Question 6

6.1.1	Interpretation of pie graph	2				2	LO2/4	Simple calculation
6.1.2	Interpretation of pie graph		3			3		Addition
6.2.1	Interpretation of table	1				1		Read off from table
6.2.2	Interpretation of table	2				2		Addition
6.2.3	Interpretation of table		2			2		Identification from table
6.3.1	Calculation of missing values from table	4				4		A - Division; B - Multiplication
6.3.2	Drawing of line graph		4			4		Plotting of given points
6.3.3	Break-even point from graph		2			2		Read off from graphs
Total		9	11			20		

Summary of marks and levels

Question	Levels				Marks
	1	2	3	4	
1	22	11			33
2	17	14			31
3	15	7			22
4	18	5			23
5	14	7			21
6	9	11			20
Total	95	55	-	-	150
Percentage	63%	37%	0%	0%	100%



MATHEMATICAL LITERACY PAPER 2

A. OVERALL REVIEW

1. Technical aspects (typing, diagrams, etc.)

Overall the technical aspects of the paper were in keeping with the high standards that we have come to expect from the Department of Basic Education over the years.

The only concern raised in this regard was Annexure B (to be used for question 3.4) which appeared to be “not very clear”. However, this was not likely to impact negatively on learners’ performance.

2. Language used

Although the language usage in the paper appeared to be fair, learners had to do much reading in this paper. This has always been the case for Mathematical Literacy Paper 2 and teachers and learners have come to accept it. However, it is very likely that some questions would have posed a problem to English second language learners, especially the way some questions were formulated (questions 1.3; 1.4; 2.2; 3.1.1; 3.1.2; 4.3.1; 5.2).

Despite these concerns, the language used in the paper has been much simpler than in previous years.

3. Syllabus coverage

Code	Learning Outcomes	Suggested	November 2013
LO1	Numbers and operations in context	37	30
LO2	Functional relationships	38	44
LO3	Space, Shape and measurement	38	46
LO4	Data Handling	37	30
	Total	150	150

4.1 Standard of paper

The paper was of a good standard as expected for Mathematical Literacy Paper 2. A scrutiny of the cognitive levels table below shows a very close alignment with the suggested levels of thinking for the paper. Learners had to do a lot of reading as some of the questions were very wordy.

Some questions (1.3; 1.4; 2.2.2; 2.4; 3.1.1; 3.2.1; 4.1.2; 4.1.3; 4.2.2; 4.2.3; 5.2.6;) carry 5 marks or more. These questions usually involve multi-step calculations and learners should be given credit for constructive attempts. In this regard we ask that these and other similar questions be marked carefully, applying CA (consistent accuracy) where appropriate. In this way learners would not be unfairly penalized.

4.2 Compliance with levels of thinking

Code	Levels of thinking	Suggested	November 2013
1	Knowledge	0%	-
2	Routine procedures	20%	24%
3	Multi-step procedures	40%	39%
4	Reasoning and reflecting	40%	37%

We note that according to our analyses, the paper was set in line with prescribed guidelines for Mathematical Literacy Paper 2. The deviations are within the specified range.



4.3. Comparison with 2012 paper

To compare the paper to the 2012 paper, a column has been added to the above table:

Code	Levels of thinking	Suggested	November 2013	November 2012	Difference
1	Knowledge	0%	-	-	-
2	Routine procedures	20%	24%	20%	4%
3	Non-routine procedures	40%	39%	42%	-3%
4	Reasoning and analysis	40%	37%	38%	-1%

We note that the 2013 paper, in terms of our analyses, was marginally easier than the 2012. However, this is not likely to have an impact on the results.

5. Overall verdict

We note that learners may not be familiar with shapes such as the Sphere and Pentagon. Also in Question 3 large numbers are used and learners may be prone to making errors when doing their calculations.

Despite these and other issues such as language, we would like to compliment the Department of Basic Education on a well-balanced and challenging Mathematical Literacy Paper 2.

B. QUESTION BY QUESTION ANALYSIS

Quest.	Content	Levels				Marks	Topic	Comment
		1	2	3	4			

Question 1

1.1	Ratio and proportion		3			3	LO 3	Multiplication and division
1.2.1	Surface area		3			3		Substitution
1.2.2	Volume		2			2		Substitution
1.3	Verification of calculation			7		7		Multi- step calculations
1.4	Calculations - s options				9	9		Multi- step calculations - choosing option 1 or 2
Total		0	8	7	9	24		

Question 2

2.1.1	Formula to calculate claim		2			2	LO 1/2	Choosing appropriate data and inserting in formula
2.1.2	Verifying claim calculation			3		3		Division
2.2.1	Calculation of monthly cost			3		3		Addition
2.2.2	Calculation of difference in claims			9		9		Multi- step calculations
2.3	Calculation of fixed monthly deposit		4			4		Substitution in formula
2.4	Interpretation of tax tables			5		5		Calculation of tax payable
Total		0	6	20	0	26		



Quest.	Content	Levels				Marks	Topic	Comment
		1	2	3	4			

Question 3

3.1.1	Explanation of data in table form				5	5	LO 2/3/4	Making sense of numeric data
3.1.2	Calculations from table		4			4		Multi-step calculation
3.1.3	Probability		3			3		Division
3.2.1	Line graphs			6		6		Simple plotting of data
3.2.2	Description of trends from graphs				4	4		Making sense of drawn line graphs
3.3.1	Explanation of totals 100%				2	2		Rounding off
3.3.2	Median		2			2		The province in the middle
3.3.3	Identifying province - less than the lower quartile				2	2		Provinces below 22,55%
3.3.4a	Suitability of pie chart				2	2		Providing suitable reasons
3.3.4a	Suitability of line graph				2	2		Providing suitable reasons
3.4.1	Boundary changes				2	2		Reading from map
3.4.2	Scale measure			4		4		Scale calculations
Total		0	9	10	19	38		

Question 4

4.1.1	Perimeter of pentagon		2			2	LO 2/3	Multiply 270 mm by 5
4.1.2	Surface area			5		5		Addition of area then subtracting letter and newspaper opening
4.1.3	Area of circle			5		5		Conversion; multiple step calculations
4.2.1	Formula - delivery cost		3			3		Take into account the mass of the parcel
4.2.2	Missing values from table		6			6		Observing patterns in the table
4.2.3	Line graph			6		6		Simple plotting of points
4.3.1	Direction				3	3		Following directions carefully
4.3.2	Verification of claim using calculations				4	4		Multi-step calculations
Total		0	11	16	7	34		



Quest.	Content	Levels				Marks	Topic	Comment
		1	2	3	4			

Question 5

5.1.1	Possible explanation for most passing in December				2	2	LO 2/4	Learners are on holiday in December?
5.1.2	Range		2			2		Highest - Lowest
5.1.3	Justification why a claim is incorrect				3	3		Months not in order
5.2.1	Interpretation of horizontal section of graph				2	2		The cost is the same
5.2.2a	Explanation of a point				2	2		P is not part of the graph
5.2.2b	Cost of driving lesson - option B				3	3		Explanation of the graph for option B
5.2.3	Interpretation of point of intersection				2	2		The cost is the same
5.2.4a	Discussion of options				2	2		Reading off from graph
5.2.4b	Discussion of options				2	2		Reading off from graph
5.2.5	Identification of option; explanation				3	3		Reading off from graph for 6 hours
5.2.6	Difference in cost for option A and option B			5		5	Calculations for 30 hours of lessons	
Total			2	5	21	28		

Summary of marks and levels

Question	Levels				Marks
	1	2	3	4	
1		8	7	9	24
2		6	20	-	26
3		9	10	19	38
4		11	16	7	34
5		2	5	21	37
Total		36	58	56	150
Percentage		24%	39%	37%	100%





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26 November 2013

Mr Matanzima Mweli
Acting Director General: Curriculum Implementation & Support
Department of Basic Education
222 Struben Street
PRETORIA

AMESA REVIEW OF THE GRADE 9 ANA MATHEMATICS PAPER 2013

Dear Mr Mweli

Last year the Department (Mr Moloi) requested AMESA to give the Department “an independent opinion on the standard of the Grade 9 ANA Mathematics paper”, which we did. This year we have initiated the process ourselves. We find it a very fruitful professional development exercise for our participating members, and we believe that you will find our resulting review useful and constructive feedback into your evaluation of the annual assessment.

I therefore hereby submit our review to the Department in the spirit of promoting mathematics education and enhancing the quality of the teaching and learning of Mathematics in South Africa.

Sincerely



Alwyn Olivier
AMESA President



INTRODUCTION

All our AMESA regions (provinces) participated in a workshop activity to review the 2013 ANA Grade 9 Mathematics paper according to specific criteria and guidelines developed by our National Curriculum Committee. The provinces then submitted their reports to the AMESA National Curriculum Committee, and the Curriculum Committee then compiled this report as a summary of the findings and trends of the AMESA provincial reports.

The report covers specific comments on the paper, focusing on the following aspects:

A. Overall Review

1. Technical aspects (typing, diagrams, etc.)
2. Language used
3. Content area coverage
 - 4.1 Standard of paper
 - 4.2 Comment on the time allocated
 - 4.3 What did learners say
 - 4.4. Compliance with ANA Framework
 - 4.4.1 Difficulty level
 - 4.4.2 Cognitive level
 - 4.4.3 Format of questions

B. Question by Question Analysis

1. Question by question analysis
2. Weighting of content area (learning outcomes) per question
3. Difficulty levels per question
4. Cognitive levels per question

C. Conclusion

Although we do not claim any validity of the analysis, we are nevertheless confident that it represents a fairly balanced and accurate perspective from a cross-section of teachers throughout the country.

PART A. OVERALL REVIEW

1. Technical aspects (typing, diagrams, etc.)

All technical aspects of the paper were in keeping with the high standards we expect from the Department of Basic Education. The typing of the paper was of an exceptionally high standard. All diagrams and graphs were large and clear, leaving no room for misinterpretation by learners. Learners were given ample space in which to write their responses/solutions on the question paper.

2. Language used

The language used in the paper would be within reach of most Grade 9 learners. There were no unfamiliar words or terms in the paper. The instructions or stimulus in each question was in straightforward and simple English and did not involve any perceived ambiguity. Teachers were also happy with the Afrikaans translation of the paper, stating that this was a great improvement when compared to the previous year.



3. Content area coverage (in 2013 on NCS) from Learning Outcomes – See part B(1)

Code	Learning Outcomes/Content area	Suggested %	Actual
L01	Numbers, Operations and Relationships	15%	13%
L02	Patterns, Functions and Algebra	35%	35%
L03	Space and Shape	30%	19%
L04	Measurement	10%	14%
L05	Data Handling	10%	19%
	Total	100%	100%

4.1 Standard of paper

The paper was of a substantially good standard which tested the knowledge and skills which a Grade 9 mathematics learner should have. The DBE is to be commended for setting a well-balanced paper. All the questions were in accordance with the ANA framework for Grade 9. Learners who were taught well and worked hard should have no problem with passing the paper.

4.2 Comment on the time allocated: Did learners finish on time?

The time allocated (150 minutes) compared to the marks for the paper (140) appeared to be reasonable. This was confirmed by teachers who claimed that a majority of their learners finished the paper in the stipulated time.

4.3 What did learners say about the paper?

In general, learners claimed that the paper was “easier than expected”. However, a number of them were concerned about their ability to work through the Geometry questions in question 8.

4.4 Compliance with ANA Framework for Grade 9 Mathematics

4.4.1 Difficulty level – see part B(3)

Difficulty levels *	Easy E	Moderate M	Difficult D
Suggested %	25	60	15
Actual %	27	58	15

4.4.2 Cognitive level – see part B(4)

Cognitive levels **	Knowledge of basic concepts K	Application of concepts A	Non-routine problem solving N
Suggested %	25	60	15
Actual %	26	59	15



4.4.3 Format of questions

Format of question	Multiple choice M	Written response showing calculation C	Graph sketching G
Suggested %	10	80	10
Actual %	7	84	9

1. QUESTION BY QUESTION ANALYSIS

SECTION	QUESTION	TOPIC/CONTENT	LO	W/ SCHEDULE (TERM)	DIFFICULTY LEVEL (E, M, or D)	COGNITIVE LEVEL (1, 2, 3 or 4)	COMMENT	MARK	TOTAL MARK
QUEST 1									
	1.1	Number system	1	1	E	1	Types of numbers	1	
	1.2	Patterns	2	1	E	2	Observe denominators	1	
	1.3	Functions	2	1	E	2	Make $y = 0$	1	
	1.4	Algebra	2	1	E	1	Make sense of an expression	1	
	1.5	Exponents	2	1	E	2	Simplification	1	
	1.6	Scientific notation	1	1	E	1	Moving the decimal	1	
	1.7	Exponents	1	1	M	2	Division	1	
	1.8	Geometry	3	2	M	2	Choosing pairs of triangles with right description	1	
	1.9	Geometry	3	3	M	2	Calculation in terms of x	1	
	1.10	Probability	3	4	E	1	Probability of an event	1	10

QUEST 2									
	2.1	Algebraic fractions	2	1	M	2	Simplification	3	
	2.2	Simplifying brackets	2	2	M	2	Simplification	4	
	2.3	Square and cube roots	1	2	E	1	Simplification	5	
	2.4	Fractions	2	2	M	3	Simplification	4	16



SECTION	QUESTION	TOPIC/CONTENT	LO	W/ SCHEDULE (TERM)	DIFFICULTY LEVEL (E, M, or D)	COGNITIVE LEVEL (1, 2, 3 or 4)	COMMENT	MARK	TOTAL MARK
QUEST 3									
	3.1	Factorizing	2	1	E	1	Common factor	2	
	3.2	Factorizing	2	1	M	2	Difference of two squares	2	4

QUEST 4									
	4.1	Simple linear equation	2	1	E	1		2	
	4.2	Linear equation with brackets	2	2	M	2	Simplify both sides	4	
	4.3	Fractional linear equations	2	2	E	1	Multiply by LCM	4	
	4.4	Simple exponential equation	1	2	M	1	Find cube root on both sides	2	12

QUEST 5									
	5.1	Number patterns	2	1	E	1	Next two terms of sequence	2	
	5.2	Number patterns	2	1	M	1	General term (in terms of n)	2	
	5.3	Number patterns	2	1	M	1	Substitute in general term	2	6

QUEST 6									
	6.1	Distance, speed, time	1	2	M	1	Calculation of time	2	
	6.2	Simple interest	1	3	M	2	Simple interest after 3 years	5	
	6.3	Compound interest	1	3	M	2	Amount owing	4	11

QUEST 7									
	7.1.1	Coordinates on graph	2	2	E	1	Writing coordinates in a table	3	
	7.1.2	Finding the equation of graph	2	2	M	2	Using the table or any other method	2	
	7.2.1	Drawing graph	2	2	M	3	Use of ruler and pencil/pen	5	
	7.2.2	Reading off from graph	2	2	E	1	Knowing that "cut one another" means intersect	2	12



SECTION	QUESTION	TOPIC/CONTENT	LO	W/ SCHEDULE (TERM)	DIFFICULTY LEVEL (E, M, or D)	COGNITIVE LEVEL (1, 2, 3 or 4)	COMMENT	MARK	TOTAL MARK
QUEST 8									
	8.1.1	Basic geometry	3	2	M	2	Base angles of isosceles triangle	1	
	8.1.2	Basic geometry	3	2	M	2	Exterior angle of triangle	1	
	8.1.3	Basic geometry	3	2	M	2	Base angle of isosceles triangle	3	
	8.2.1	Congruency	3	3	M	2	Application of congruency	1	
	8.2.2	Congruency	3	3	E	1	Identifying a triangle	1	
	8.3	Congruency	3	3	D	3	Giving reasons for congruency	4	
	8.4.1	Similarity	3	3	M	2	Scaffolding question on similar triangles	4	
	8.4.2	Similarity	3	3	M	2	Application of similarity	3	18

QUEST 9									
	9.1	Reflection	3	3	M	2	Reflecting a triangle	2	
	9.2	Image of a point	3	3	E	1	From the reflection	1	
	9.3	Rotation through 180° about the origin	3	3	M	2	Drawing of triangle	2	
	9.4	Length of segment AA'	3	3	M	1	Properties of reflection	1	6



SECTION	QUESTION	TOPIC/CONTENT	LO	W/ SCHEDULE (TERM)	DIFFICULTY LEVEL (E, M, or D)	COGNITIVE LEVEL (1, 2, 3 or 4)	COMMENT	MARK	TOTAL MARK
QUEST 10									
	10.1.1	Area (of circle)	4	1	D	4	Using area of two circles	2	
	10.1.2	Substitution in formula	4	1	D	3	Area of a ring	2	
	10.2.1	Calculation in triangular prism	4	1	E	2	Use of theorem of Pythagoras	2	
	10.2.2	Use of theorem of Pythagoras to calculate PT	4	1	M	2	Acknowledge that PQR is a triangle with height PT	4	
	10.2.3	Area of triangle	4	1	M	2	Use of area formula	3	
	10.2.4	Volume of prism	4	1	M	2	Use of volume formula	2	
	10.2.5	Surface area of prism	4	1	D	3	Use of surface area formula	5	20

QUEST 11									
	11.1	Completing frequency table	5	3	D	4	Use histogram	4	
	11.2	Reading off from histogram or table	5	3	E	1	Interpretation of histogram or table	1	
	11.3	Calculating mean from table	5	3	M	2	Summing of all products and dividing by the number of learners	3	
	11.4	Reading off from table or histogram and calculating %	5	3	M	2	Interpretation of table or histogram	2	10

QUEST 12									
	12.1	Stem and leaf display	5	3	M	2	Writing the leaves in descending order	5	
	12.2.1	Range	5	3	M	2	Highest - lowest	1	
	12.2.2	Mode	5	3	E	1	The number which occurs the most	1	
	12.2.3	Median	5	3	M	2	The number in the middle	1	
	12.2.4	Calculation from data or stem-and-leaf display	5	3	E	1	Interpretation of stem and leaf display	1	9



SECTION	QUESTION	TOPIC/CONTENT	LO	W/ SCHEDULE (TERM)	DIFFICULTY LEVEL (E, M, or D)	COGNITIVE LEVEL (1, 2, 3 or 4)	COMMENT	MARK	TOTAL MARK
QUEST 13									
	13.1	Probability	5	4	E	1	Simple calculation	1	
	13.2	Probability	5	4	E	1	Simple calculation	1	
	13.3	Probability	5	4	E	1	Simple calculation	1	3
QUEST 14									
	12.1	Problem solving	5	4	D	4	Using Venn diagrams or linear equation	5	3
TOTAL									140

2. WEIGHTING OF CONTENT AREA (LEARNING OUTCOMES) PER QUESTION

Key:

- 1 - Numbers, operations and relationships
- 2 - Patterns functions and algebra
- 3 - Space and shape
- 4 - Measurement
- 5 - Data handling

QUESTION/CONTENT	1	2	3	4	5	TOTAL
1	3	4	2		1	10
2	5	11				16
3		4				4
4		12				12
5		6				6
6	11					11
7		12				12
8			18			18
9			6			6
10				20		20
11					10	10
12					9	9
13					3	3
14					3	3
Total	19	49	26	20	26	140
%	13%	35%	19%	14%	19%	100%
Requirement	15%	35%	30%	10%	10%	100%



3. DIFFICULTY LEVELS PER QUESTION

Key:

E - Easy; M - Moderate; D - Difficult

QUESTION/DIFFICULTY LEVEL	E	M	D	TOTAL
1	5	5		10
2	3	13		16
3	2	2		4
4	4	8		12
5	2	4		6
6		11		11
7	5	7		12
8		14	4	18
9	2	4		6
10		16	4	20
11		6	4	10
12	4	5		9
13	3			3
14	3 (Non-routine)			3
Total	33	95	12	140
%	23%	68%	09%	100%
Requirement	25%	60%	15%	100%

4. COGNITIVE LEVELS PER QUESTION

Key:

K - Knowledge; A - Application; N - Non-routine problem solving

QUESTION/LEVEL	K	A	N	TOTAL
1	03	04	03	10
2	03	13	0	16
3	04	0	0	04
4	0	12	0	12
5	06	0	0	06
6	0	11	0	11
7	05	07	0	12
8	0	13	05	18
9	02	04	0	06
10	0	16	04	20
11	01	09	0	10
12	04	05	0	09
13	03	0	0	03
14	0	0	03	03
Total	31	94	15	140
%	22%	67%	11%	100%
Requirement	25%	65%	15%	100%



PART C: CONCLUSION

1. Highlights

- It was a well- balanced paper, catering for a wide range of ability levels.
- The paper was generally in keeping with the Grade 9 ANA framework in terms of cognitive and difficulty levels; the questions were well constructed and thought provoking.
- Exemplar papers arrived earlier than usual to schools and so teachers were able to conduct revision sessions with learners; thus learners were better prepared when compared to 2012.
- The time allocation appeared to be “spot-on” with most learners finishing within the stipulated 150 minutes.

2. Challenges

- It remains a challenge to complete a whole years’ work to answer the paper in the third term. The timing for the writing of ANA put teachers under severe pressure, firstly to complete the work and secondly to mark the work within a specified period. They also had to do a question-by-question analysis for the DBE. Regions felt that it would be better if the paper would be written in the fourth term. A similar view was given in 2012. The DBE assured teachers that the 4th term part of the paper had come from Grade 7 & 8. While this is true, many schools tend not to do justice to the 4th term part of the syllabus in Grades 7 & 8 and depend on the Grade 9 teacher to “put things right”. Thus learners tend to be disadvantaged as they do not have the necessary background to handle these 4th term part of the ANA paper and consequently perform poorly in these sections.
- Although there were some murmurs of the dissatisfaction that the ANA did not count, advocacy by the DBE and schools appeared to have had the desired results of the ANA being taken more seriously by teachers and learners in 2012.
- Parents do not understand why there was a difference in marks obtained by their children in the ANA and the final exam paper. They need to be informed about the similarities and differences between the ANA mark and the final mark. In this regard, the ANA is an “exam” only; the final examination consisted of an exam mark plus a school based assessment (SBA) mark. The current ratio of 75:25 in favour of SBA tends to inflate the final mark.
- Teachers felt that the mark allocation for question 8 should ensure that learners get credit for both statements and reasons as learners spend quite some time when writing reasons for their geometry statements.
- Teachers are under enormous pressure to mark and analyse the papers within a short period; and submit their analyses to the districts.

3. Comparison with 2012

- Although the paper was similar in standard to the 2012 paper, it would appear that learners responded better this year when compared to 2012. This was probably due to better support and better preparation.
- Learners tended to perform better in certain sections such as number patterns and graphs, when compared to 2012. However, challenges still remain in some of the algebra topics and geometry.
- Afrikaans medium schools were very complimentary of the Afrikaans translation of the paper which was far better when compared to 2012.

4. Overall verdict

It was a very well-balanced, fair paper with very substantial coverage of the Grade 9 curriculum.

5. Concluding remarks

- The marking memorandum for Grade 9 ANA should be more detailed and alternative responses should be included. This will help teachers in their development and expose their learners to different methods of working through various questions.
- Learners should be given practice on working with papers of two hour (+) duration prior to the ANA as it will prepare them to sit for two hours; also this practice of sitting for more than 2 hours for an exam paper will hold them in good stead for the FET where papers are usually 2 or more hours.
- The DBE should ensure that there is substantial teacher development for senior phase mathematics teachers. It would appear that some teachers are not very confident about teaching some of the algebra topics and geometry. Support for teachers in the senior phase is likely to result in better learner performance and possibly increase the pool of learners for mathematics in the FET.



The Old Mutual Foundation Advertorials

AMESA and the Old Mutual Foundation co-operate to publish a series of advertorials published in the *Mail and Guardian* supplement, *The Teacher*. The articles were written by Jacques du Plessis of the University of the Witwatersrand on behalf of AMESA.

The preamble to each article is:

Education is key to the future of South Africa, which is why the Old Mutual Foundation supports education initiatives that build excellence in mathematics and science at Secondary schooling level. A key priority is to increase the national mathematics and science pass rates to address critical skills shortage in the economy.

*Our key education projects include AMESA (the Association for Mathematics Education of South Africa). The Old Mutual Foundation assists with the printing and distribution of AMESA publications which includes *Pythagoras*, *Learning and Teaching Mathematics (LTM)*, *AMESA News* and *AMESA Annual Congress materials*.*

We publish the last of the articles in the 2013 series.

The Old Mutual Foundation presents

Timelines and annuity formulae as pedagogical content tools in financial mathematics

Jacques du Plessis | University of the Witwatersrand, Johannesburg



Education is key to the future of South Africa, which is why the Old Mutual Foundation supports education initiatives that build excellence in mathematics and science at Secondary schooling level. A key priority is to increase the national mathematics and science pass rates to address critical skills shortage in the economy. Our key education projects include AMESA (the Association for Mathematics Education of South Africa). The Old Mutual Foundation assists with the printing and distribution of AMESA publications which include *Pythagoras*, *Learning and Teaching Mathematics*, *AMESA News* and *Annual Congress materials*.

Financial mathematics is an elegant domain of mathematics. To fully understand how things fit together in the mathematics of finance, one needs to understand visually in both a relational and instrumental way. Financial mathematics provides a perfect example of how these two concepts introduced by Richard Skemp (1976) coexists in support of one another. In this article I will focus on using timelines and the annuity formulae as *pedagogical content tools* (Rasmussen & Marrongelle, 2006). A pedagogical content tool is a graph, an equation, a diagram or even a mathematical statement that the teacher uses to advance the understanding of some piece of mathematics. The focus in this article is the use of the timeline as a pedagogical content tool, so I assume that the reader is familiar with the financial mathematics formulae as well as the concepts of future (accumulated) value (F_V) and present (principal) value (P_V).

Timelines – a tool for understanding structure

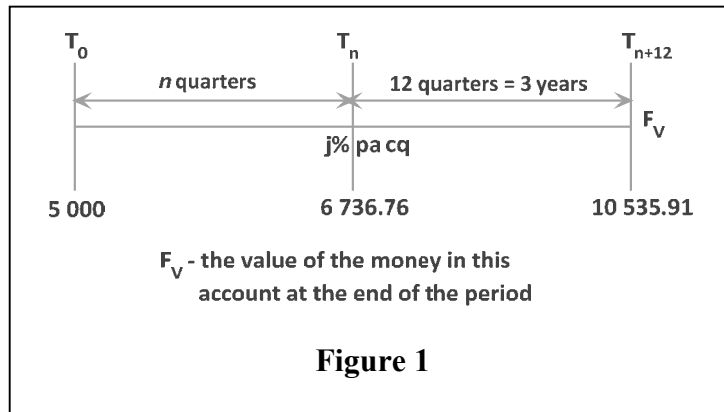
Timelines in financial mathematics create the necessary structure that enhances both relational and instrumental understanding. The golden rule in problems where timelines are used is that all calculations need to converge to the same point on the timeline. The following example shows how useful the timeline is in building the problem structure and explicating what otherwise would have been hidden to the problem solver.



Example 1: R5 000 was invested into a fund that pays j % per annum compounded quarterly. After n quarters the fund stood at R6 736,76 and after a further 3 years, it stood at R10 535,91. Determine the values of j and n .

Solution:

The timeline (Figure 1) represents the situation. The timeline clearly shows that the problem has two distinct components – a growth of n quarters for R5 000 to become R6 736,76. The second half of this investment runs for three years where the accumulated amount of R6 736,76 grows to R10 535,91 at the same rate.



The timeline becomes the pedagogical content tool that creates the necessary structure that help us decide on a strategy. The interest rate in our calculations will be the effective quarterly rate equivalent to a rate of j % per annum compounded quarterly which is a nominal quarterly rate. The timeline shows that the final three years of this investment is the period where the rate j % p.a. compounded quarterly, is the only unknown. Over the first n quarters both j and n are unknown, as is the case over the whole period of $n + 12$ quarters. We therefore will start by focusing on the last three years of this investment and we have a choice of working either in years or in quarters. Whichever period we choose to work with, the rate will be an effective rate per period. It is more time efficient here to work with the effective rate per quarter.

For the final three years:

$$F_V = P_V (1+i)^n \text{ where } i = \frac{j}{400}$$

$$\therefore 10535.91 = 6736.76(1+i)^{12}$$

$$\therefore (1+i)^{12} = 1.563943201$$

$$\therefore i = 0.037970669$$

Thus $j = 400 \times 0.037970669 = 15,19\%$ per annum compounded quarterly. This is the nominal quarterly rate. We need to work with an effective quarterly rate in order to calculate the value of n which is located in the first part of the timeline. We simply need to divide the rate by four to obtain the effective quarterly rate – that is the rate per quarter compounded quarterly. In our case we will simply work with the value of i as we calculated it in the above, as it is an effective quarterly rate.



To find n :

$$F_V = P_V (1+i)^n$$

$$\therefore 6736.76 = 5000(1.03797066897)^n$$

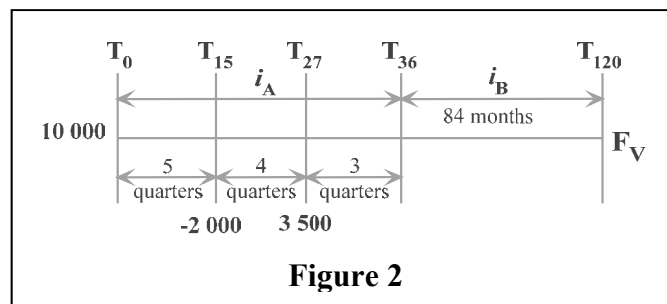
$$\therefore n = \frac{\log 1.347352}{\log 1.03797066897} = 8 \text{ quarters}$$

The timeline organises the information in such a way that the mathematical processes used to solve the problem, become visible. The next example highlights the choices that can be made when working with a timeline.

Example 2: R10 000 is originally invested for a period of ten years at an effective rate of 15% per annum. During the period of the investment, R2 000 was withdrawn at the end of the 15th month, R3 500 was invested at the end of the 27th month, and three years after the initial deposit, the interest rate was adjusted to 13% per annum compounded monthly. Determine the future value of this investment after ten years.

Solution:

This example shows how the timeline transforms seamlessly to the structure of the problem that needs to be solved. Here the timeline as a pedagogical content tool effectively illustrates how we can move to any place in time as long as we work with the effective rates per period, and all calculations converge to the same point in time (on the timeline) – see Figure 2.



The effective quarterly rate i_A is obtained through the conversion formula:

$$\left(1 + \frac{i_A}{4}\right)^4 = (1+i)$$

$$\therefore \frac{i_A}{4} = \sqrt[4]{1.15} - 1$$

$$\therefore i_A = 0.03555807634$$

The effective monthly rate i_B is given by the calculation $i_B = \frac{13}{1200} = 0.0108\dot{3}$. We can have the following equivalent scenarios from the timeline, which all lead to the same answer of $F_V = \text{R}40\,893,47$.

$$\text{Moving to } T_{36}: 10000(1+i_A)^{12} - 2000(1+i_A)^7 + 3500(1+i_A)^3 = F_V (1+i_B)^{-84}$$

$$\text{Moving to } T_{120}: \{10000(1+i_A)^{12} - 2000(1+i_A)^7 + 3500(1+i_A)^3\}(1+i_B)^{84} = F_V$$



$$\text{Moving to } T_0: 10000 - 2000(1+i_A)^{-5} + 3500(1+i_A)^{-9} = F_V (1+i_B)^{-84} (1+i_A)^{-12}$$

$$\text{Moving to } T_{15}: 10000(1+i_A)^5 - 2000 + 3500(1+i_A)^{-4} = F_V (1+i_B)^{-84} (1+i_A)^{-7}$$

$$\text{Moving to } T_{27}: 10000(1+i_A)^9 - 2000(1+i_A)^4 + 3500 = F_V (1+i_B)^{-84} (1+i_A)^{-3}$$

Annuities and timelines

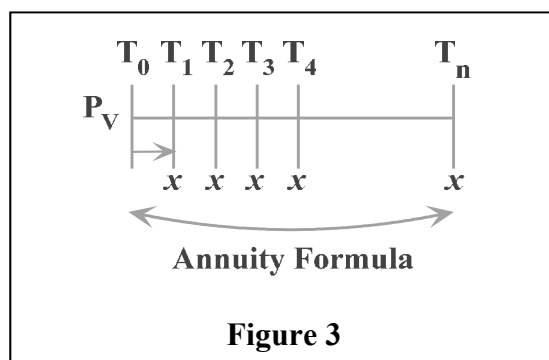
The standard annuity formulae that learners have to know are

$$P_V = x \left[\frac{1 - (1+i)^{-n}}{i} \right] \text{ when working with the present value of an annuity, and}$$

$$F_V = x \left[\frac{(1+i)^n - 1}{i} \right] \text{ when working with the future value of an annuity.}$$

One very important aspect is to note is that n in these formulae do not denote the period of the loan, but rather the number of payments made in the annuity structure.

These formulae are easy to remember and they provide useful *shortcuts* in the calculation phase. What needs to be understood is how the timelines relate to the formulae for the annuities.



When working with the present value of an annuity (e.g. loans) one usually starts paying back at the end of the first period. So it is expected that there will be an *open period* before the payments, which amortises the loan, starts. The timeline is represented in Figure 3. At the beginning of the timeline, there is one period where *nothing happens*. This is the standard timeline for a scenario

where the formula $P_V = x \left[\frac{1 - (1+i)^{-n}}{i} \right]$ applies. Any variation away from this happens *outside the*

annuity, and is treated as normal compound interest using one of formulae $P_V = F_V (1+i)^{-n}$ or

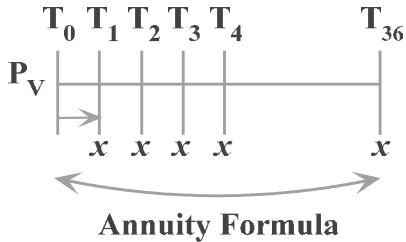
or $F_V = P_V (1+i)^n$. What follows are a few scenarios and how the present value annuity formula is adjusted.



Problem scenario 1:

A loan is amortised by making monthly payments for 3 years into an account, starting at the end of the first month.

Timeline:



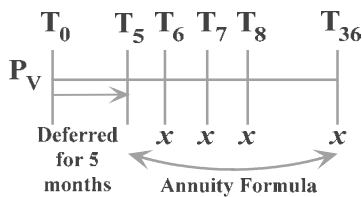
Formula:

This is the standard formula with 36 payments in the annuity: $P_V = x \left[\frac{1 - (1+i)^{-36}}{i} \right]$

Problem scenario 2:

A loan is amortised over a three year period by making monthly payments starting six months after the loan is granted.

Timeline:



Formula:

There are five months at the beginning where no payments are made. Payments start at the beginning of the sixth month. Thus a deferment of 5 months leaving the one space open for the annuity formula to apply. So 31 payments happen in the annuity:

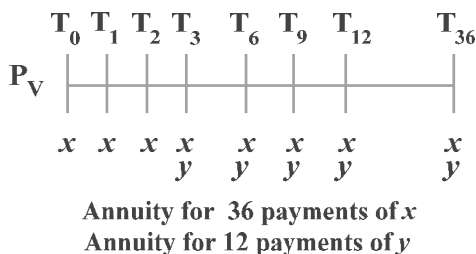
$$P_V (1+i)^5 = x \left[\frac{1 - (1+i)^{-31}}{i} \right]$$



Problem scenario 3:

A loan is amortised by making 37 monthly payments of Rx starting immediately. Five additional payments of Ry are made at the end of each quarter for the full period of the loan.

Timeline:



Formula:

There are two separate annuities running simultaneously. Only 36 payments are in the annuity for x (the first payment is immediate and lies outside the annuity structure) and there are 12 payments in the annuity for y . i_A is the effective monthly rate and i_B is the effective quarterly rate.

$$P_V = x + x \left[\frac{1 - (1 + i_A)^{-36}}{i_A} \right] + y \left[\frac{1 - (1 + i_B)^{-12}}{i_B} \right]$$

For the future value formula, there is no space open. So it works on the notion that the annuity matures the moment the last payment is made.

To end

Using timelines as a pedagogical content tool provides the teacher with a didactical instrument that makes the structural aspects of financial mathematics visible in the class. It builds on both relational and instrumental aspects of the mathematics experience by using the structural aspects of financial mathematics.

To share your experience or to comment on this article, please feel free to email me at jacques.duplessis@wits.ac.za.

Also join the discussion on the AMESA Facebook page at <https://www.facebook.com/amesa93>.

References

Rasmussen, C., & Marrongelle, K. (2006). Pedagogical content tools: Integrating student reasoning and mathematics in instruction. *Journal for Research in Mathematics Education*, 37(5), 388–420.

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The Use of Dynamic Geometry Software in the Teaching and Learning of Geometry through Transformations

By Johnny Lott (USA), Nico Govender (South Africa), Yexin Tao (China); Leif Osterberg (Finland)

This paper was drawn up during the Park City Mathematics Institute (PCMI) International Seminar in July 2013 and is reproduced here with kind permission of the PCMI.

Introduction

Dynamic geometry technology should be used to maximise student learning in geometry. Such technology may help students in their learning processes by enabling them to visualize geometry concepts being studied. With dynamic geometry packages, students explore properties of congruence and similarity, focusing on the role of transformations. The use of technology depends on the teacher, and the teacher's role in making curriculum decisions is very important. Recognising that today's students are using technology outside of the classroom and the study of mathematics should keep pace, teachers need to take advantage of the technology for learning. Thus, one goal for using dynamic geometry is to enable students to examine mathematical ideas on their own, allowing them not to be just consumers of technology but producers of knowledge through technology. By allowing freedom and free access to technology at any time, students can be creative, be involved, and can advance their learning as they study geometry. (It is noted that allowing students free use of technology may cause concern for some teachers because with this free access, some class control may be lost.)

Moving from Concept to Construction

Dynamic geometry can be used in different ways. One approach is to move from concept to construction with technology. The following example from geometry illustrates the approach.

Congruent Triangles by Side-Angle-Side (SAS)

Concept

Consider an investigation of a fundamental congruence test for triangles such as the Side-Angle-Side theorem: *If two sides and the included angle of one triangle are congruent to two corresponding sides and the included angle of another triangle, the triangles are congruent.* Students can be asked: How can you determine if two triangles are congruent? For example, what if two sides and the included angle of one triangle are congruent to two sides and the included angle of the second triangle?

Construction (Modelling)

In most situations, the students determine an answer by investigation. They may construct triangles with two sides of certain lengths and a certain included angle measure. They might draw the figure on patty paper and then overlay it with a partner's drawing to observe if the triangles match exactly. Sometimes the students cut out the triangles they draw and put them together to compare whether they match. They may complete the process without formally thinking about transformations, but students should discover that the triangles appear congruent and conjecture the Side-Angle-Side theorem.

Dynamic Geometry Software

Generally speaking, the SAS theorem for congruent triangles can be introduced as above. However, if dynamic geometry software were used in the class, what new or different learning of SAS could be accomplished?



Imagine a classroom scene where a teacher has students sketch triangles using dynamic geometry with two sides of given lengths and an included angle between them with a given measure. If student sketches are displayed for all to see, most should recognize that the triangles are the “same size and shape”, or appear to be congruent even though they may have different orientations and be at different locations on individual screens. By having students copy any two of the triangles on their own screens, they could be challenged to try and make one fit atop the other using transformation properties of the geometry software. For example, the student may use a translation to drag one triangle toward the other until two vertices of congruent appearing angles match. The student might then rotate the dragged triangle around the matched vertex until the angles match. The triangles may match exactly after this movement, or set of movements, but sometimes the “translated” triangle may also need to be reflected to get them to match exactly. In any event, after no more than three movements, the two triangles should be made to match and thus appear congruent.

In the second approach, the students actively move the triangles electronically using the mathematical notions of transformations given in the dynamic geometry. Visualising the triangles moving on the screen using transformations they have chosen, students not only see the geometric transformations intuitively, they know precisely which transformations have been used to make them match exactly. With this background, the teacher can explain the congruent triangles in a mathematical way- two triangles are congruent if one can be mapped onto the other by a sequence of isometries (rigid transformations). The two approaches aid each other. It is important to note that working with patty paper gives credence to the movements that take place with the dynamic geometry package, but the use of the dynamic geometry mathematizes the informal movement with patty paper.

The second approach using dynamic geometry allows students to engage in more formal reasoning. Consider the above example constructed with technology where two triangles in Figure 1 below have $AB = A'B'$, $AC = A'C'$, and $\angle BAC$ is equal to $\angle B'A'C'$. This figure is used to examine SAS with dynamic geometry software:

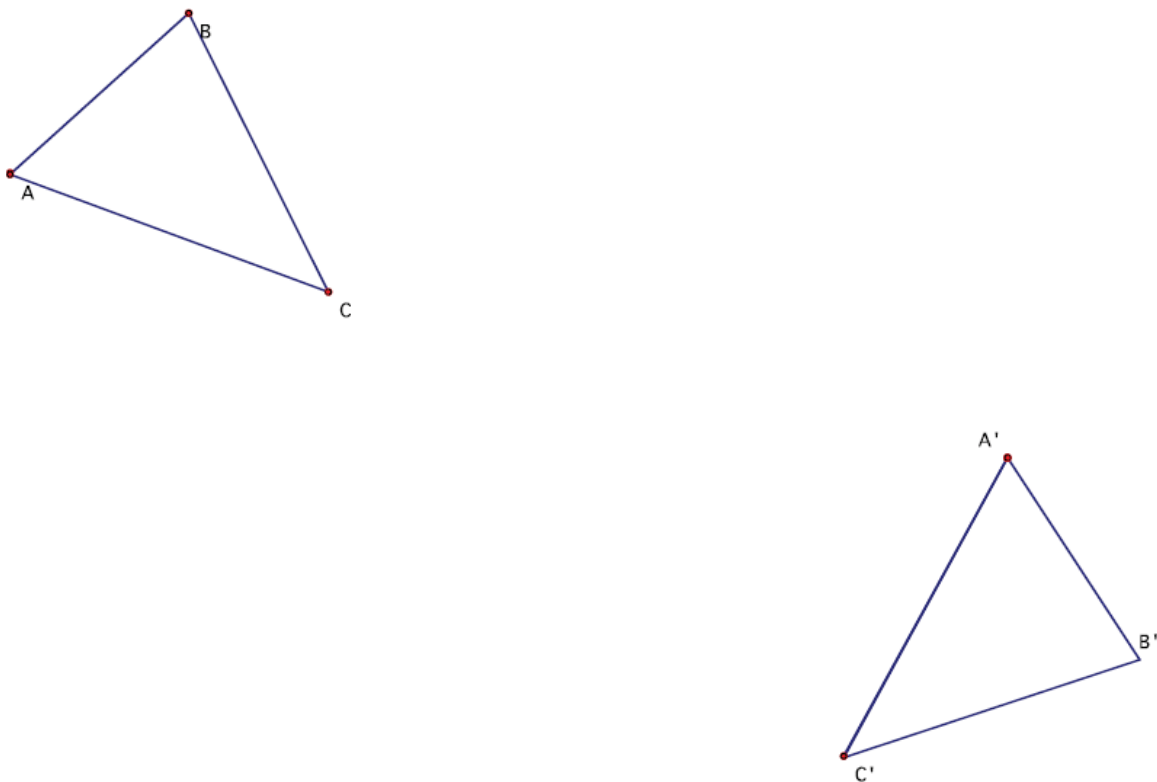


Figure 1: Two triangles with respective two sides and an included angle equal

To investigate the informal description above more systematically, students first find a line that reflects A to A' as in Figure 2. An intuitive way to reflect a point is to reflect the point over the perpendicular bisector, n , of the segment connecting the two points, in this case, the perpendicular bisector of AA' .



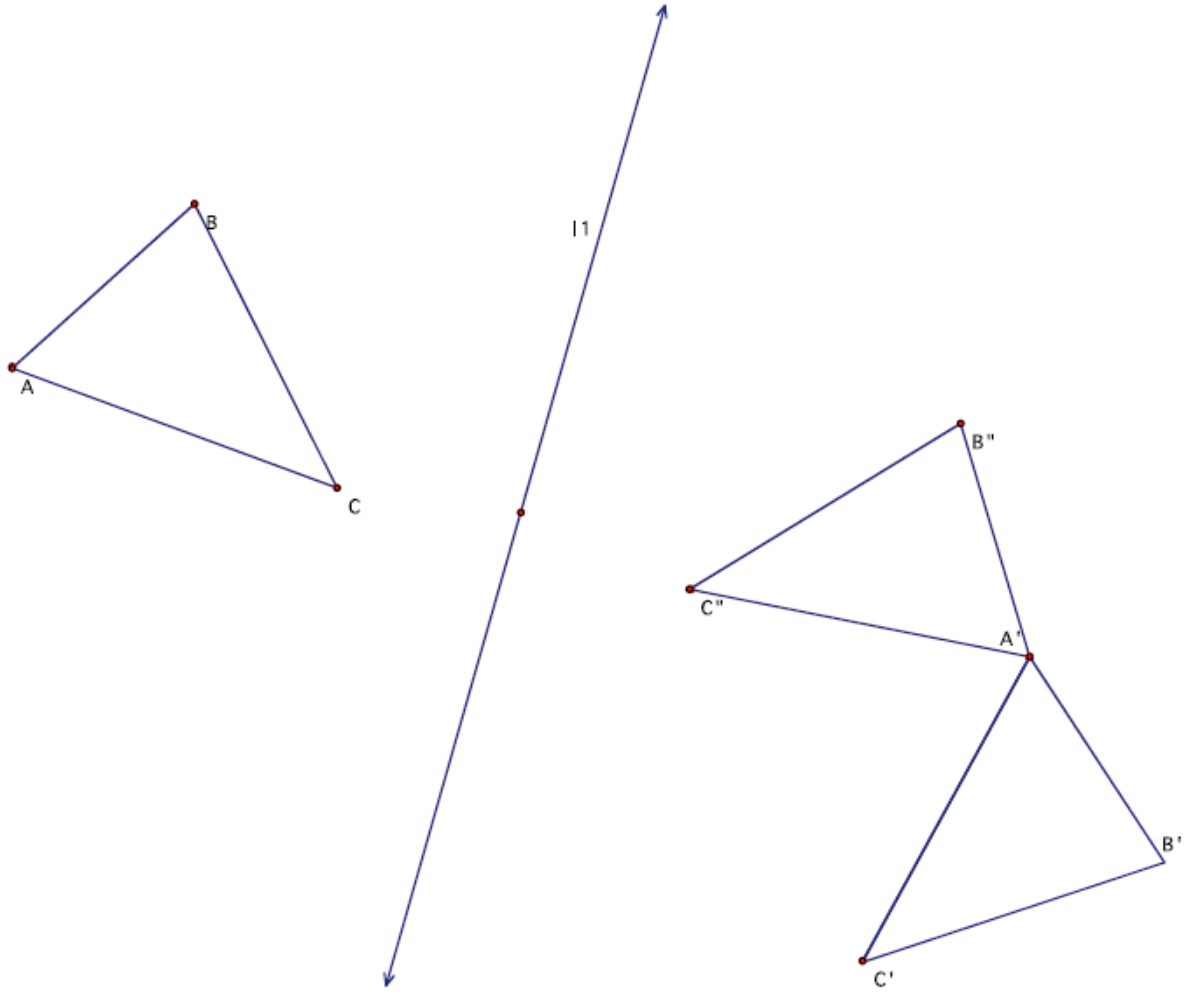


Figure 2: Triangle ABC reflected in line n , where C is reflected to C''

As pictured, line n is the perpendicular bisector of AA' . The image of triangle ABC under this reflection is triangle $A'B''C''$. There are many ways to continue; one is to find a line that will reflect $A'B''$ onto $A'B'$. Because the segments share an endpoint, any reflecting line that maps one segment to the other must contain the common point A' . Thus, to map a line onto an intersecting line, an angle bisector containing the intersection point can be used. In Figure 3, students might use the angle bisector, j , of $B''A'B'$. Now because $A'B''$ and $A'B'$ are equal, and the angle bisector reflects $A'B''$ onto $A'B'$, point B'' is mapped onto point B' . This is shown in Figure 3 yielding the image of triangle $A'B''C''$ as triangle $A'B'C'$.



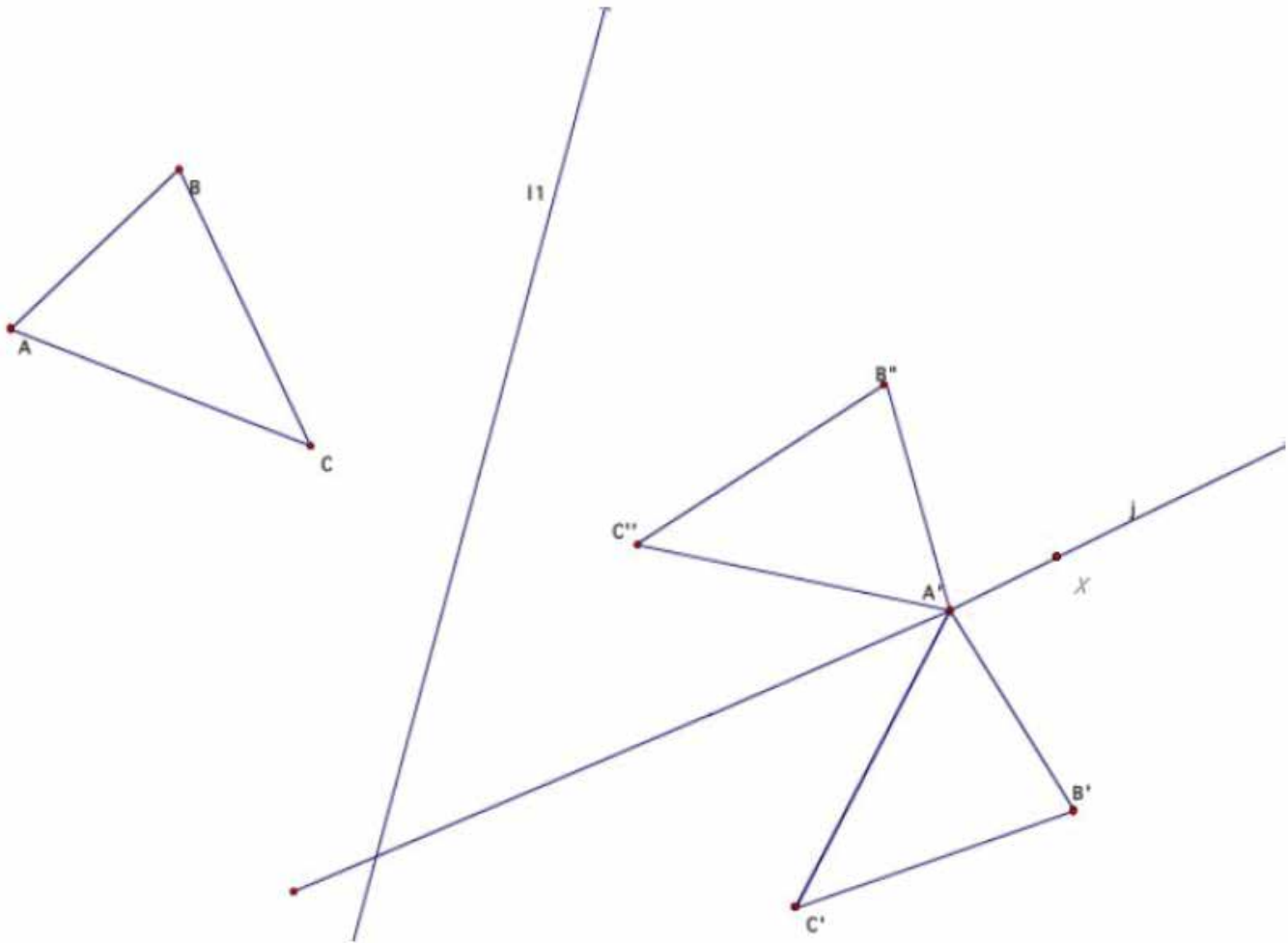


Figure 3: Triangle $A'BC$ reflected to triangle $A'B'C'$

In this situation, it appears that $r_{jn}(\text{triangle } ABC) = \text{triangle } A'B'C'$. The remaining question to be sure this is true is to consider what happens to point C . Is its image point C' ? If so, the triangles are congruent. To answer this question, students may consider what can possibly happen to point C . They know that AC is congruent to $A'C'$ because the latter is the image of the former in a reflection in line n . And they know that the image of $A'C''$ is mapped to a congruent segment in the reflection in line j . The only question is whether the image of C is C' . They know that any isometry preserves angle measures, that line j is the bisector of $\angle B''A'B'$ making $\angle B''A'X$ equal to $B'A'X$. Additionally students know that $B''A'C''$ maps to an equal angle in the reflection in line j . Having equal angles and with $A'C''$ equal to $A'C'$ gives point C' as the image of point C'' and thus the image of point C . Hence, the transformation to make triangle ABC congruent to triangle $A'B'C'$ used two reflections in intersecting lines, n and j .

Suppose lines n and j intersect in point O . What else can be learned about the original (triangle ABC) and the final image (triangle $A'B'C'$)? Consider the measures of angles AOA' , BOB' , and COC' as seen in Figure 4.



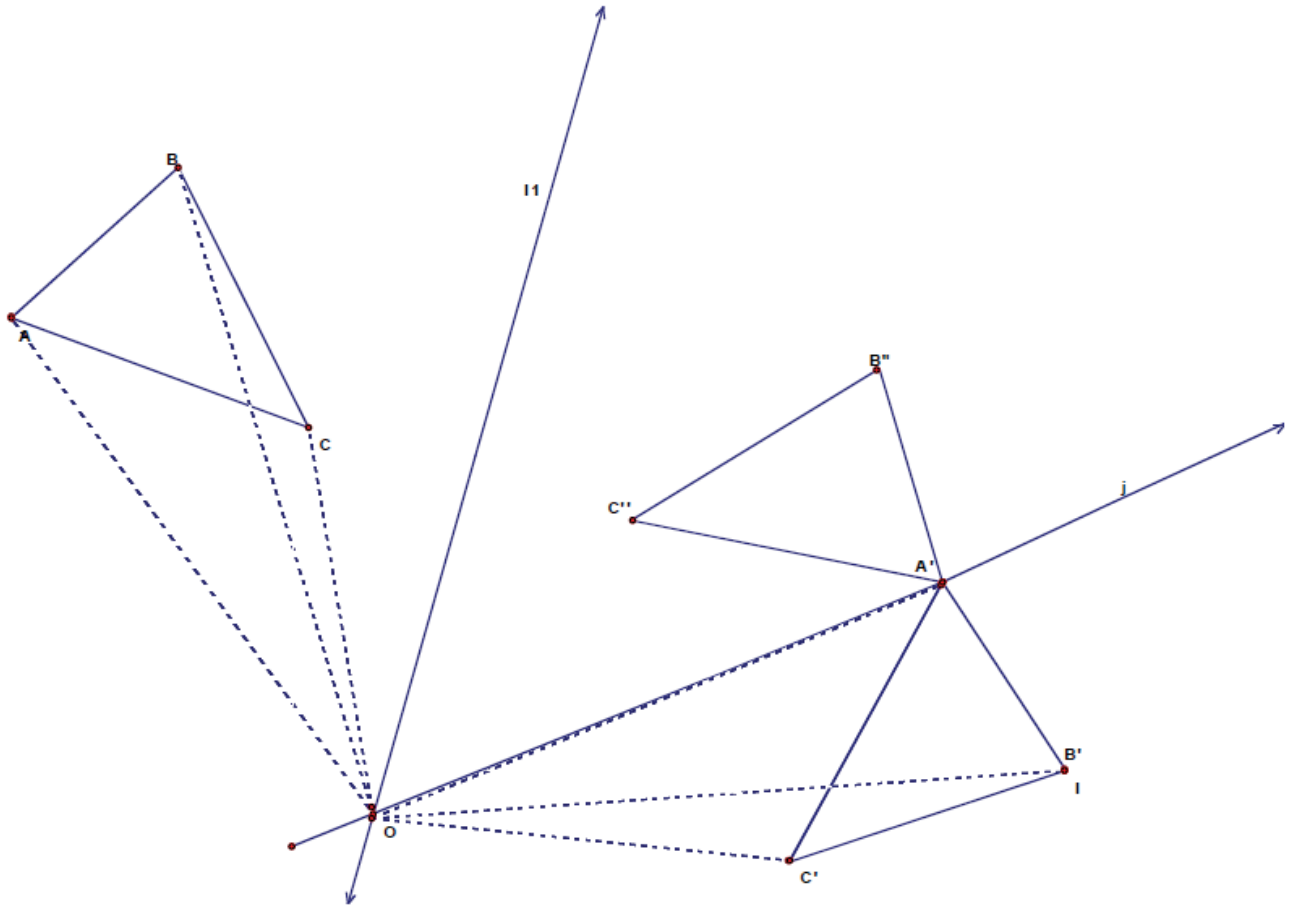


Figure 4: Measures of rotation angles are approximately -104°

Because $OA = OA'$ (O is the perpendicular bisector of AA' and any point on n is equidistant from A and A'), there is a circle with centre O and radius OA containing points A'' and A' . Additionally because $OB = OB'' = OB'$ using properties of isometries, there is a circle with centre O and radius OB containing points B , B'' , and B' , and similarly because $OC = OC'' = OC'$, there is a circle with centre O and radius OC containing points C , C'' , and C' . Using the dynamic geometry measuring tool, a rotation of -104° with centre O can be accomplished by the composition of reflections in intersecting lines n and j . Moreover, the measure of the angle between the intersecting lines is 52° . (To consider why the angle has measure 52° , think about the reflecting lines as angle bisectors)

In this way, technology can be used to build a rotation in a step-by-step development that is the composition in two intersecting lines, n and j . Further, the angle of rotation is twice the measure of the angle formed by the two reflecting lines. Though this is a specific example, it is possible to use the same reasoning to prove that the composition of two reflections in intersecting lines is a rotation with centre at the point of intersection of the lines and the angle of rotation twice the measure of the angle formed by the lines. [Actually there are two different angle measures formed by the lines, but consider the points and their images to determine the angle measure being used.]



Using Transformational Geometry to Obtain Properties of Figures

Transformational geometry may be used to make and prove conjectures about properties of geometric figures.

Example Show that the angle bisectors of a triangle are concurrent.

As a class, students are encouraged to investigate the statement on their own and make conjectures. It may be advantageous to have students investigate this using dynamic geometry with their choice of triangles. Once they have made the conjecture that all angle bisectors are concurrent, a teacher may use the following to reason and prove the conjecture. In Figure 5, let ABC be any triangle.

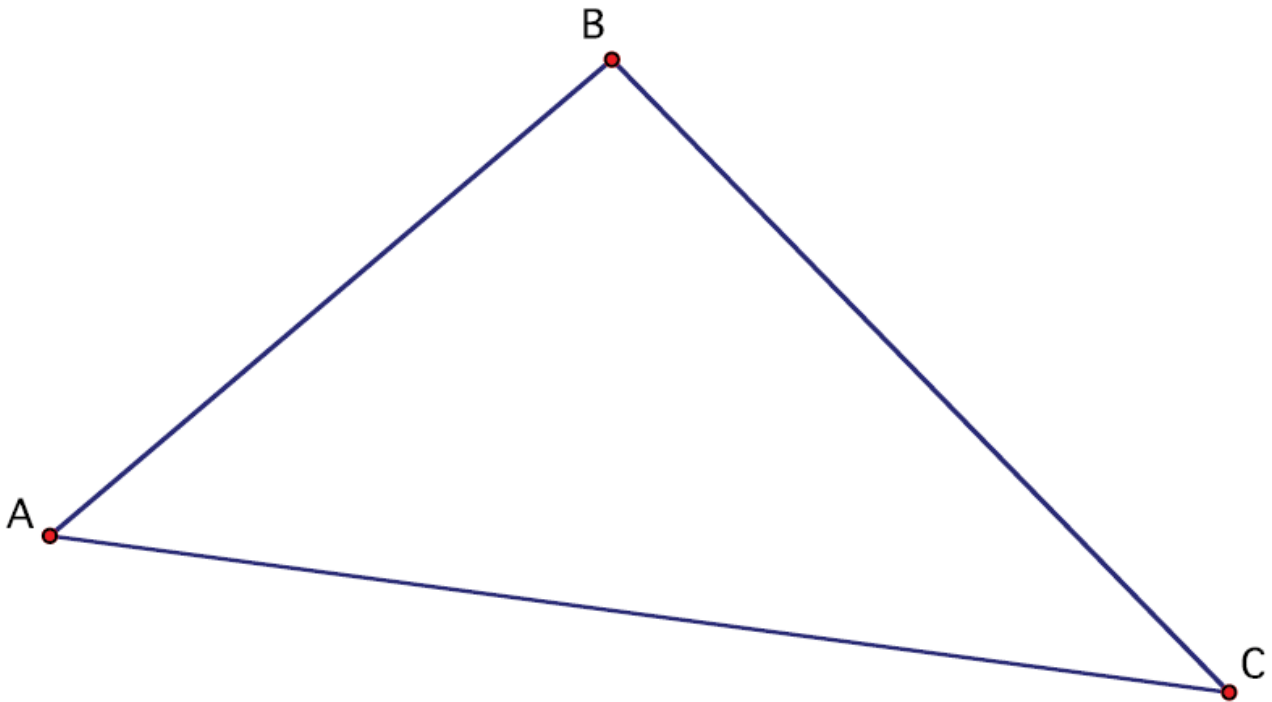


Figure 5: Scalene triangle ABC

Construct the bisector of $\angle CAB$. Reflect AC in this angle bisector. The image of AC must be AB to preserve angle measures as in Figure 6.



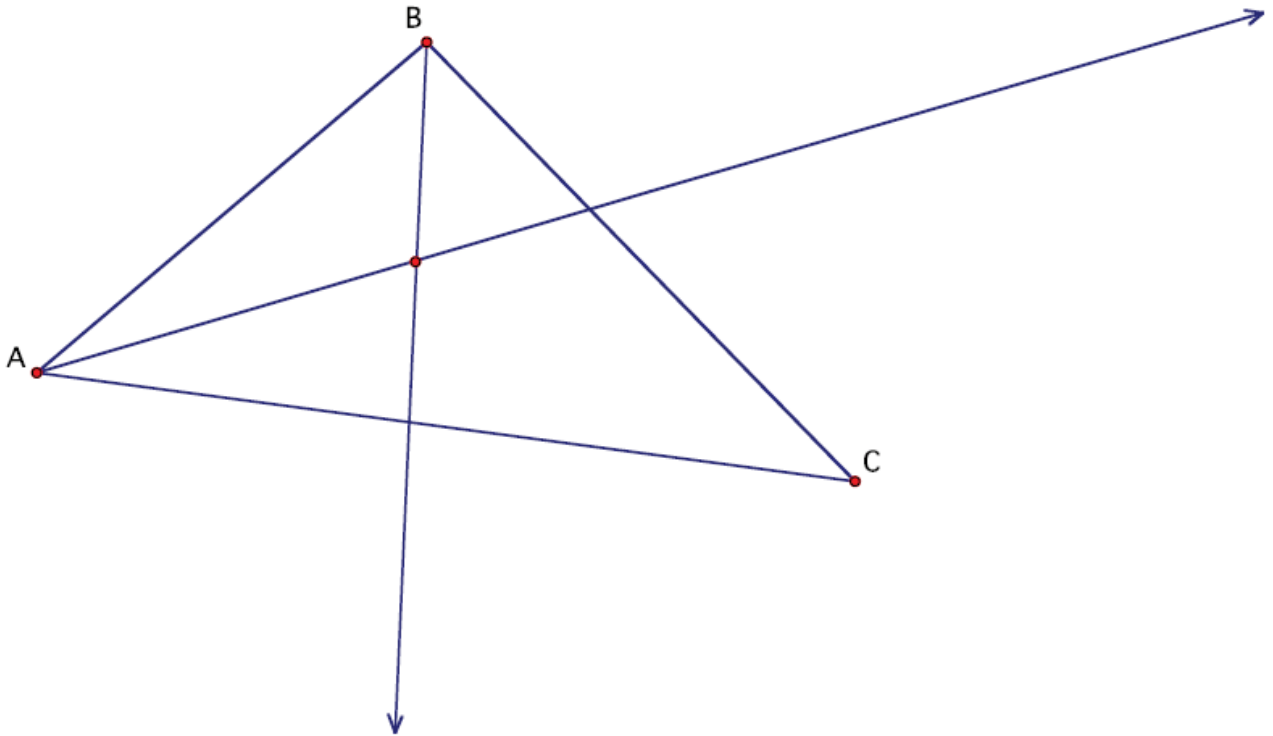


Figure 6: Triangle ABC with bisectors of angles A and B

Now the bisector of $\angle ABC$ is constructed. The image of AB in this bisector is CB (Why?). The final step in thinking about how this may be done is to find a line that contains the point of intersection of the first two angle bisectors and makes CB its own image. [The reason for doing this involves a basic theorem of transformational geometry; that is, a composition of reflections in three concurrent lines may be replaced by a single line concurrent with those three original lines. The teacher may wish to have students try this result with dynamic geometry before this problem is given.] A natural line that meets both conditions is the perpendicular from the point of intersection of the two angle bisectors to CB . This line is constructed as in Figure 7, and the image of CB in a reflection in the perpendicular is CB itself.



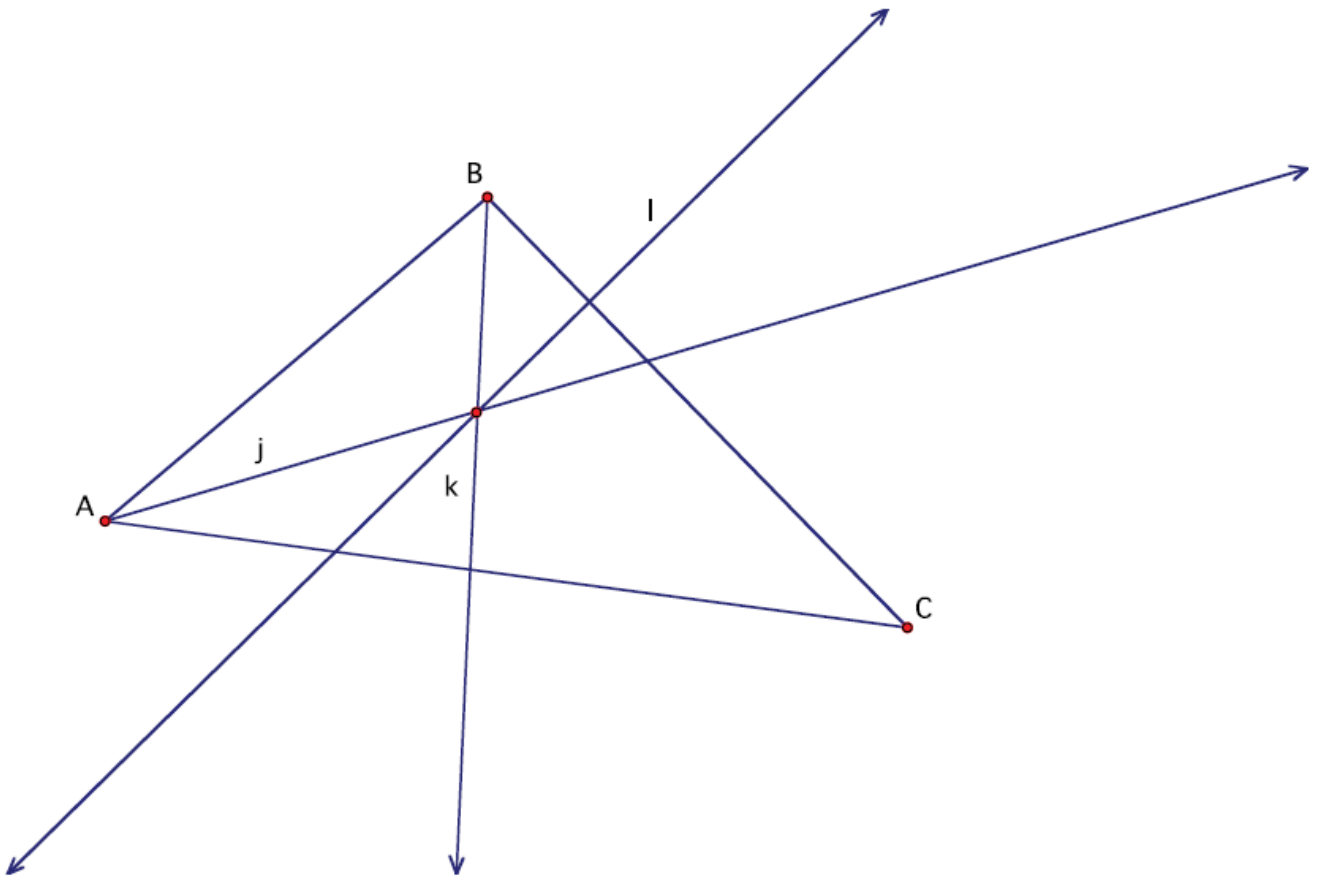


Figure 7: Triangle ABC with bisectors of angles A and B with a perpendicular to CB through the point of intersection of the angle bisectors

The point of intersection of the angle bisectors is its own image in each of the reflections. Moreover, $r_{lrkj}(AC) = CB$. But the image of AC in the bisector of $\angle ACB$ is also CB. . Thus the angle bisectors of the angles of triangle ABC are concurrent; that is, contain the same point.

Advantages of Using Technology

Using technology to work with geometric transformations provides many advantages to students. Students are able to generate fast, accurate and easy arguments leading to conjectures and suggested ways of reasoning, to problem solutions, and to explorations of new and unknown mathematics. They may create their own conjectures and make a construction with technology to write a proof or argument. Teachers on the other hand may need examples to encourage students to create conjectures.

As an example, consider the following:

Given $AB \parallel CD$ with $AE = 5$ cm; $BE = 3$ cm and $EC = 9$ cm, calculate the value of x (ED in the diagram) in Figure 8.



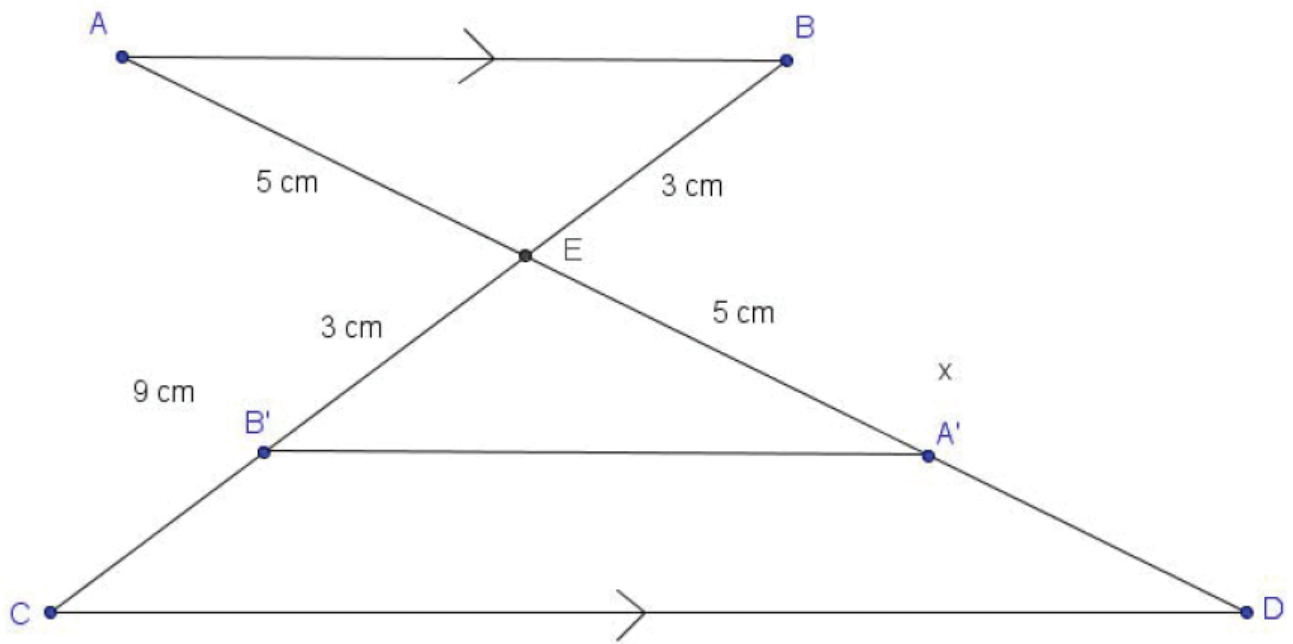


Figure 8: Geometric figure with $AB \parallel CD$

Students may solve this problem by first proving that $\triangle ABE$ is similar to $\triangle DCE$ using theorems related to parallel lines and then deducing the ratios:

$$\frac{5}{x} = \frac{5}{x}$$

$$x = 15\text{cm}$$

However, suppose students use geometric transformations and rotate $\triangle ABE$ 180° about E to produce $\triangle EA'B'$ which sits on $\triangle EDC$.

(This can be shown using properties of the rotation.) Now $\triangle EA'B'$ is a dilation of $\triangle EDC$ with centre E and scale factor, $\frac{3}{9}$ or $\frac{1}{3}$. (Again, students can show this is true using properties of a dilation.) Thus $x = 3(5\text{ cm}) = 15\text{ cm}$.

Another investigation that can be made easier is to consider properties of a quadrilateral by constructing it using transformations and a dynamic geometry and beginning with any isosceles triangle as in Figure 9. Choose the base, and either rotating the triangle 180° about a midpoint of the base or reflect the triangle in the base. The pre-image and the image from the reflection form a quadrilateral. What are the properties of this quadrilateral? What would happen for a scalene triangle, an equilateral triangle or a right triangle if similar transformations were used?



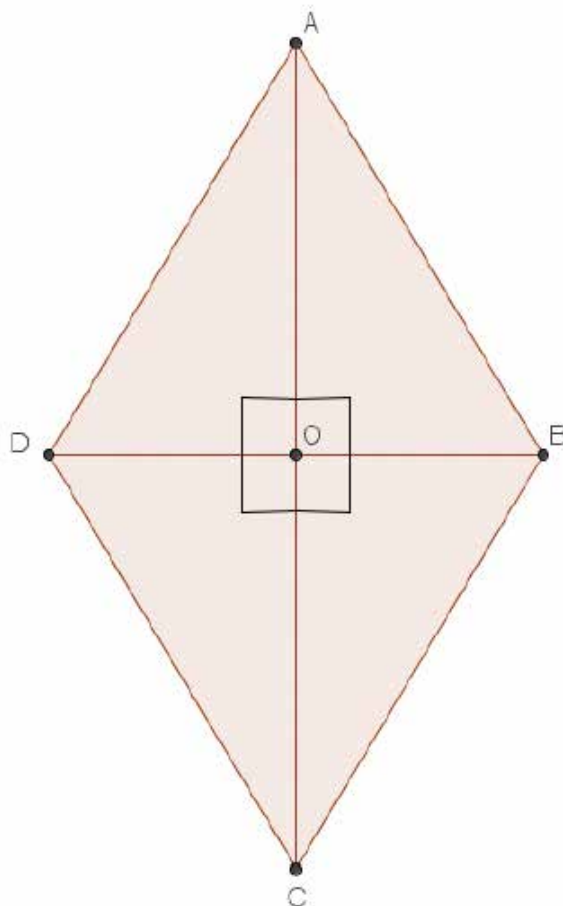


Figure 9: Isosceles triangle DAB is reflected in base DB

Disadvantages of Using Technology

The discussion above illustrated some advantages of using technology but did not mention disadvantages. For example, when do measurements using dynamic geometry software, typically approximations, possibly lead to incorrect results and is this important to consider in terms of student learning?

Another concern with a dynamic geometry package, students only see a small portion of the plane displayed but do not see what happens to the rest of the plane under the transformation being considered. As a result, they may forget that an entire plane is being transformed.

Conclusion

Dynamic geometry software can be a beneficial tool for using geometric transformations in the study of geometry. An important consideration is to use technology to develop background knowledge and ways of thinking as well as help students move further in their understanding of core geometry concepts.

To maximise the potential of technology, teachers need to understand the how, what and why of teaching with technology. They need experience in considering when technology can most effectively be used. Transformational geometry provides one area where the technology can be used for more learning as long as student have free access to dynamic geometric software as they explore problems and develop understanding.



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