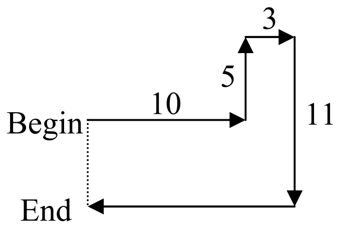
NOTES ON 2005 MEMORANDUM

These notes are necessarily brief and often formal and symbolic.
Many questions could be answered using primitive methods, e.g. "If today is Wednesday, what day of the week will it be 100 days from now?" can be done by counting. That would be laborious, time-consuming and error-prone. The essence of a mathematical approach is to work more smartly by using appropriate representations to model the situation and to exploit the inherent structures and patterns in the situation.

GRADE 4(1)

1. (64 + 96) ÷ 2
3. 5 = ∇ – 8, so ∇ = 13
4. 1,70 m – 1,05 m = 0,65 m = 65 cm
5. If you look from *behind* the tower is on your *left*
6. 08:00 – 70 min → 06:50
7. The figure can be divided into 32 equal triangles of which 16 are shaded.
8. If Mary has ∇ stamps, Jason has 2 × ∇ stamps. Together they gave 3 × ∇ = 96 stamps. So ∇ = 96 ÷ 3
9. 6 small ones and 2 big ones for a total of 8
10. 18 × 10 = 180
12. 257 + Δ = 438, so Δ = 438 – 257 = 181 km
13. 438 + 169 = 607 km
14. 3 + 8 + 3 + 8 = 22
15. Jan, Feb, March: 31 + 28 + 31 = 90 days, so the 100th day is on 10 April
16. Check answers systematically, e.g. 80 = 50 + 20 + 10; 32 = 20 + 10 + 2; 62 = 50 + 10 + 2; etc
17. Bottom level: 3 × 3 = 9 blocks, Second level has 1 less: 8 blocks, Top level has 5 blocks
18. A rings on the hour and half-hour. B rings at 08:00, 08:35, 09:10, 09:45, 10:20, 10:55 and 11:30
19. Draw it physically, as shown to the right!



20. Be systematic, e.g.

32	**2**3	**4**3	**1**3
34	24	42	12
31	21	41	14

21.

70			
32	38		
15	17	21	
10	5	12	9

22. The numbers are one more than a multiple of 4. Only 6129 is one more than a multiple of 4
23. Let the children be A, B, C, D and E. List all the possibilities, be systematic, note patterns and structure:

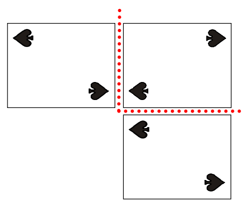
A vs B	**B** vs C	**C** vs D	**D** vs E
A vs C	B vs D	C vs E	
A vs D	B vs E		
A vs E			

vs	A	B	C	D	E
A		X	X	X	X
B			X	X	X
C				X	X
D					X
E					

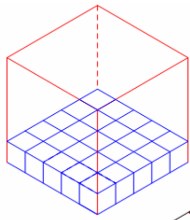
24. 3; 6; 9; 12; … = 1×3; 2×3; 3×3; 4×3; … So 53×3 = 159
25. 1; 4; 7; 10; … = 1 + 0×3; 1 + 1×3; 1 + 2×3; 1 + 3×3; … So 1 + 52×3 = 157

GRADE 4(F)

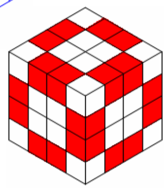
1. 5 + 3 = 8, while the others all have a sum of 7
2. Half of 8 × 8
6. There is a pattern of +14, +14, +14 in the numbers
7. $\frac{3}{4} \div \frac{1}{8} = \frac{6}{8} \div \frac{1}{8}$. How many $\frac{1}{8}$ are there in $\frac{6}{8}$?
8. B is a mirror-image in a horizontal or vertical line of symmetry, as shown



11. 35 000 mℓ ÷ 35 mℓ = 100

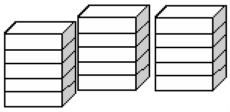


12. 6 × 180 mm = 1,080 m
13. 1, 4, 9, ... = 1×1, 2×2, 3×3, … So 8×8 = 64
14. 5 small cubes to a side. So 5×5 in bottom layer, with 5 layers, so 5×5×5
15. 274 – 246 + 1 = 29

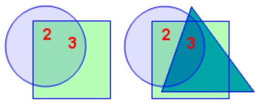


16. 8 cubes on each of the 6 sides. But then they are all counted twice! So 4×8 ÷ 2
17. 2 × (8 + 5) + 4 = 30
18. x – 4 + 5 – 6 = 3, so x – 5 = 3 so x = 8
19. 2 × T + 2 = 38, so T = (38 – 2) ÷ 2 = 18
20. 120 km in 60 min, so 20 km in 10 min, so 200 km in 100 min, so the time is 11:40
21. 7, 17, 27, 37, … 77 (two!), 87, 97 is 11, plus 70, 71, 72, … ~~77~~, 78, 79 is another 9, so 20
22. M + M + 30 = 114, so 2 × M = 84, so Monde weighs 42 kg
23. 50 × 2 – 1 = 99
24. Sum of rows = 1, 4, 9, … = 1×1, 2×2, 3×3, … So 50×50 = 2500
25. Diameter = 44 cm, so 2200 cm ÷ 44 cm/turn = 50 turns

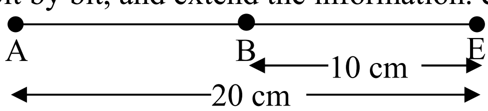
GRADE 5(1)



1. 3 columns, each with 5 blocks: 3 × 5 = 15 blocks



2. The numbers inside the square *and* the circle are 2 and 3.
   2 is not inside the triangle
3. These are multiples of 7. Test by division, use your calculator!
   4236 ÷ 7 = 605,1… so not a multiple. 4224 ÷ 7 = 603,4… 4235 ÷ 7 = 605
4. Try and test each possible answer!
5. In the bottom layer there are 8 × 4 = 32 blocks, so in two layers there 64 blocks
6. All the blocks of the bottom layer (32) and all the blocks round the side of the top layer (20)
7. If *C* children like chocolate, then 4 + 2×*C* = 40, so *C* = (40 – 4) ÷ 2 = 18
8. 274 – 246 + 1 = 29. Think of easier cases: if you start on page 1 and read to page 3, How many pages?
9. 6,8 ÷ 2 = 3,4; 3,4 ÷ 2 = 1,7; 1,7 ÷ 2 = 0,85
10. The number must start and end with 1 so list them systematically:
    101 111 121 131 141 151 161 171 181 191
11. 100 ÷ 12 = 8 rem 4, i.e. 8 years bringing us to Sept., plus 4 more months, i.e. Oct, Nov, Dec, Jan
    Or if September = 9, then 100 months further is 9 + 100 = 109. But
    January = 1, 13, 25, … *these leave a remainder of 1 if divided by 12*
    February = 2, 14, 26, … *these leave a remainder of 2 if divided by 12*
    March = 3, 15, 27, … *these leave a remainder of 3 if divided by 12,* etc.
    So 109 ÷ 12 = 9 rem 1, so January
12. 100 ÷ 7 = 14 rem 2, i.e. 14 full weeks bringing us to Wednesday, plus 2 more days, i.e. Friday
    Or if Monday = 1, Wednesday = 3, so 3 + 100 = 103, 103 ÷ 7 = 14 rem 5, and 5 is Friday
13. 100 ÷ 24 = 4 rem 4, i.e. 4 full days bringing us to 10:00, plus 4 more hours, i.e. 11, 12, 13, 14:00
    Or 10 + 100 = 110, 110 ÷ 24 = 6 rem 14
14. 12 chocolates weigh 1,1 kg – 680 g = 420 g, so 1 chocolate weighs 420 g ÷ 12 = 35 g
    so 30 chocolates weigh 30 × 35 g = 1050 g, so box weighs 1100 g – 1050 g = 50 g
15. 3 for 5 = ? for 90. You can build it up, e.g. 30 for 50 and 24 for 40, so 54 for 90. Or 3 × 18 for 5 × 18 (90)
16. Put the information in a sketch, fill in the details bit by bit, and extend the information. e.g.:
    The distance from A to E is 20 cm
    The distance from B to E is 10 cm
    You can deduce that AB = 10 cm!



17. The computer rule is *Output number* = *Input number* × 5 + 2. So if *Input* is 20, *Output* = 20 × 5 + 2
18. 99 m = $\frac{9}{10}$ of roll, so 11 m = $\frac{1}{10}$ of roll. Therefore $\frac{10}{10}$ of roll = 10×$\frac{1}{10}$ of roll =10×11 m = 110 m

19. Imagine yourself looking at the card from behind. Or tear the corner from a piece of paper, turn it around!
20. If Joe's starting number is S, then he did $S \times 10 = 9000$. So $S = 900$. So correct answer is $900 \div 10 = 90$
21. 1Δ = 6 and 1Δ + 1☺ = 10. So 6 + 1☺ = 10, so 1☺ = 4 and 2☺ = 8
22. 3×2 + 2 =8; 7×2 + 2 = 16; so for rectangle with length 20: 20×2 + 2 = 42
23. List them systematically: **9**97; 988; 979 **8**98; 889 **7**99
24. Make an appropriate representation, e.g. take the special case of 4 teams A, B, C and D and make a systematic list or drawing as shown here.
4 teams play $2 \times (\mathbf{3} + 2 + 1)$ games or $\mathbf{4} \times 4 - 4$ or $\mathbf{4} \times 3$ games
So **22** teams play $2 \times (\mathbf{21} + 20 + \ldots + 1)$ or $\mathbf{22} \times 22 - 22$ games or $\mathbf{22} \times 21$ games

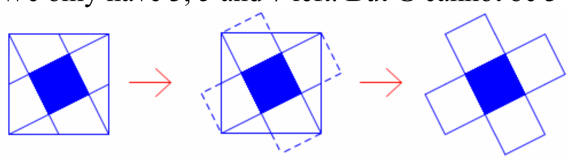
vs	**A**	**B**	**C**	**D**
A		X	X	X
B	X		X	X
C	X	X		X
D	X	X	X	

25. $1 + 2 + 3 + 4 + 5 + 6 + \ldots + 47 + 48 + 49 + 50$
$= 51 + 51 + 51 + \ldots$ 25 times
$= 25 \times 51$
$= 1275$

GRADE 5(F)

1. There are 5 tiles in every metre because 1000 cm ÷ 200 cm = 5. So 15 × 10 = 150 tiles
2. Do not count the poles on the corners twice! 4 × 10 – 4 = 36
4. C – a rotation to the right through 90°
5. 4 reds – 10 greens – 3 purples. So 12 (3×4) reds – 9 (3×3) purples
6. n^{th} row has $2 \times n - 1$ dots, so 7^{th} row has 13 dots
7. n^{th} row has $2 \times n - 1$ dots, so 70^{th} row has $2 \times 70 - 1 = 139$ dots
8. Total in rows is 1, 4, 9, 16, … = 1×1, 2×2, 3×3, 4×4, … So in 70 rows there are 70×70 dots
9. *Height* = 12 cm + 1,5 cm/day × *days*. So *Height after 30 days* = 12 + 1,5×30 = 57 cm
10. (150 cm – 12 cm) ÷ 1,5 cm/day = 92 days
11. One more than a multiple of 6, so it is odd, so it cannot be A or B. Test the others: 4 182÷6 = 697
12. If a sack weighs S kg, then $3S = S + 30$, so $2S = 30$, so $S = 15$. So $3S = 45$ kg
13. 3, 6, 9, … is the 3-times table. So 50 × 3 = 150
14. Mathematics is $\frac{1}{4}$ of his time, and this is 2 hours. So $\frac{4}{4}$ of his time is 4×2 hours = 8 hours
15. 8 milktarts = 6 + 2 milktarts = $6 + \frac{1}{3}$ of 6. So she need $8 + \frac{1}{3}$ of 8 cups = $10\frac{2}{3}$ cups of milk
16. 15 eggs = 9 + 6 eggs = $9 + \frac{2}{3}$ of 9 eggs. So she can bake $6 + \frac{2}{3}$ of 6 tarts = 6 + 4 milktarts
17. Through systematic elimination, e.g.
A in the top row must be 1, 8 or 6. But A in the right column cannot be 8 or 1, so A is 6
B in the bottom row must be 9, 4 or 2. But A in the right column cannot be 9 or 4, so B is 2
C in the top row must be 1 or 8. But C in the left column cannot be 1, so C is 8. So E is 1
D in the bottom row must be 9 or 4. But D in the left column cannot be 4, so D is 9. So F is 4
We only have 3, 5 and 7 left. But G cannot be 3 or 7, so G = 5. H cannot be 3, so H = 7 and X = 3

C	E	A
G	X	H
D	F	B

18. 

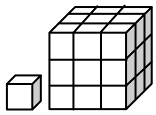
19. 75c more per week, so 12 × 75c =R9
20. Half the water weighs 21 kg – 12 kg = 9 kg, so all the water weighs 18 kg. So the bucket weighs 3 kg
21. The 7 cookies that was left was one less than the children, so there were 8 children who each got 7, i.e. 8 × 7 = 56 cookies, minus the one that was short, so 55 cookies
22. Share 30 litres in ratio 5 to 1, i.e. 25 to 5
23. If Penny has p coins and Alex has a coins:
$p = 2 \times a$, $p - 4 = a + 4$, so $2 \times a - 4 = a + 4$, so $a = 8$, so $p = 16$, so $p + a = 24$

24. 4000
3100, 3010, 3001
2200, 2020, 2002
2110, 2101, 2011
~~2020, 2002. 2011~~
1300, 1030, 1003
1210, 1201
1120, 1102
1111
~~1030, 1003~~
1021, 1012

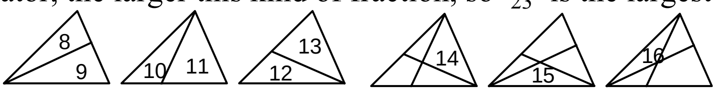
25. If a small pizza costs *s* rands and a large pizza costs *L* rands:
$2s + 1L$ -= $5s$, so $1L = 3s$, so the cost is $L = 3 \times$ R11,50 = R34,50

GRADE 6(1)

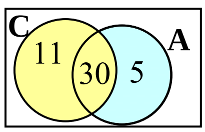
1. 8 – 7,93 = 0,07 is smaller than 8,08 – 8 = 0,08
3. 302400 **÷ 6** = 50400, 50400 **÷ 7** = 7200, 7200 **÷ 8** = 900, 900 **÷ 9** = 100, 100 **÷ 10** = 10
4. $\frac{13}{20}$ is more than $\frac{12}{20}$ ($\frac{3}{5}$) and less than $\frac{16}{20}$ ($\frac{4}{5}$), so he is on side DE
5. He still has $\frac{7}{20}$ of the distance to go, so $\frac{7}{20}$ of 25 cm = (25 cm ÷ 20) × 7 = 8,75 cm
6. From the ground, over the length, to the ground again is 6 m + 8 m + 6 m = 20 m, and from the ground, over the width, to the ground again is 6 m + 10 m + 6 m = 22 m
7. Use trial and error, i.e. try each of the given answers one by one
8. Use trial-and-improvement. Or 54 ÷ 3 =17. 17 + 18 + 19 = 54, so 17 × 18 × 19 = 5814
9. $\frac{8}{11} - \frac{5}{8} = \frac{9}{88}$ of tank = 135 ℓ, so $\frac{1}{88}$ of tank = 135 ℓ ÷ 9 = 15 ℓ, so $\frac{88}{88}$ of tank = 15 ℓ × 88 = 1320 ℓ



10. 4 books = 2 books + 6 kg, so 2 books = 6 kg, so 1 book = 3 kg
11. Imagine or draw the cube! If the side is 3 times as long, the big cube contains 27 of the small cubes. So its mass is 27 times as large!
12. There is a general structure here: The denominators are twice the numerator + 1, i.e. $\frac{n}{2 \times n+1}$
We can therefore investigate a general pattern $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \frac{6}{13}, \frac{7}{15}, \ldots$
Check with your calculator: $\frac{1}{3}$ = 0, 333..., $\frac{2}{5}$ = 0,4, … So $\frac{1}{3} < \frac{2}{5} < \frac{3}{7} < \frac{4}{9} < \frac{5}{11} < \frac{6}{13} < \frac{7}{15} < \ldots$
Conclusion: the larger the denominator, the larger this kind of fraction, so $\frac{11}{23}$ is the largest
13. The 6 small triangles (6),
the one big one (7) and these:



14. Make a sketch, e.g. the one shown here
41 – 30 = 11 children like only **C**omedy
35 – 30 = 5 children like only **A**ction
50 – (11 + 30 + 5) = 4 children don't like either

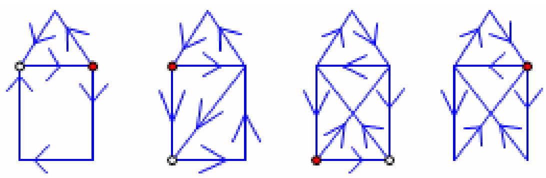


Total = 50

15. James is now 2 × 5 = 10 years old, so 15 years from now he will be 10 + 15 = 25 years old
16. Make a systematic list, e.g. 3579; 3597 | 3759; 3795 | 3957; 3975 | 9375; 9357 | 9537 …
17.

2	3	
3	5	+2
4	?	+3
5	12	+4
6	17	+5

18. Start at ● and end at ○:



19. If the empty glass has a mass of g gram and the milk has a mass of m gram, then

$g + m = 370$

$g + \frac{1}{2}m = 290$

So $\frac{1}{2}m = 370 - 290 = 80$ gram, so $m = 160$ gram and $g = 370 - 160 = 210$ gram

20. $\frac{2005+2004}{2005-2004} = 4009$

21. $(2000 - 1999) + (1998 - 1997) + \ldots + (4 - 3) + (2 - 1) = 1 + 1 + 1 + 1 + \ldots + 1$ (1000 times)

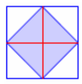
22. Let c be the cost of a coke and d the cost of a packet of chips. The cost of the first buy is $6c + 7d$ and of the second is $8c + 4d$. So $6c + 7d = 8c + 4d$. This means you bought 2 Cokes more, but 3 chips less, so 2 cokes cost just as much as 3 chips. So 8 cokes cost just as much as 12 packets of chips. So 8 cokes and 4 packets of chips cost just as much as 16 packets of chips

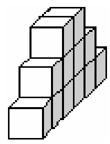
23. There are 5 possible first digits (1, 3, 5, 7, 9) and 5 possible second digits, so in total $5 \times 5 = 25$

24. $P_n = 4 \times n + 1$, so $P_{20} = 4 \times 20 + 1$

25. Let the length of a tile be x cm and the width y cm. The perimeter of the floor is $8x + 8y = 800$ cm. So the perimeter of one tile is $2x + 2y = 200$ cm

GRADE 6(F)





1. Make equal parts. Each small square is half of the next bigger square. So half of half of the big square is a quarter of the big square
2. There are 8 columns, each with 2 + 4 + 6 cubes. So $8 \times 12 = 96$ cubes
3. In middle row the missing number is $18 - (11+6) = 1$, so in right column $A = 18 - (1+10) = 7$
5. $\frac{1}{7} = \frac{5}{35}$ and $\frac{1}{5} = \frac{7}{35}$ so $\frac{6}{35}$ is exactly in between them. Or $(\frac{1}{5} + \frac{1}{7}) \div 2 = (\frac{7}{35} + \frac{5}{35}) \div 2 = \frac{6}{35}$
7. List them systematically and you will find $3 \times 2 \times 3 = 18$ different combos
8. Continue the patterns: 17, 22, 27, 32, 37, 42, 47, 52, … and 17, 24, 31, 38, 45, 52, …
9. For n dice, the number of visible faces is $n \times 3 + 2$. So for 75 dice, $75 \times 3 + 2 = 27$
10. $(359 - 2) \div 3 = 119$
11. $0 \times 20 + 3 \times 10 + 1 \times 5$

    $1 \times 20 + 1 \times 10 + 1 \times 5$
12. $102 \div 7 = 14$ rem 4, so adding 3, we have $105 \div 7 = 15$
14.

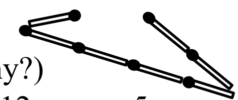
B	C	M
A	2	N
1	D	

In the middle row, N cannot be 2, so N is 1 or 3
Suppose N = 3. Then A = 1 which is impossible (already a 1 in left column).
So N = 1, A = 3. In left column B = 2. Then C = 1 (D ≠ 1), so M = 3, so M+N = 4

15. $3 \times (1 + 2 + 3) = 18$
16. Vary the possibilities systematically. First note that she could not draw 1, 3 or 5 games, otherwise her total would be a fraction. If she drew 6 games her total was $6 \times \frac{1}{2} = 3$. If she drew 4 and won 2 her total was $2 \times 1 + 4 \times \frac{1}{2} = 4$. If she drew 2 and won 4 her total was $4 \times 1 + 2 \times \frac{1}{2} = 5$
17. Vary the numbers systematically and note the behaviour of the product of the numbers:

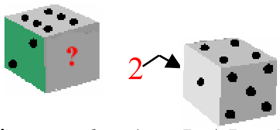
$1 + 17 = 18$ and $1 \times 17 = 17$	$6 + 12 = 18$ and $6 \times 12 = 72$
$2 + 16 = 18$ and $2 \times 16 = 32$	$7 + 11 = 18$ and $7 \times 11 = 77$
$3 + 15 = 18$ and $3 \times 15 = 45$	$8 + 10 = 18$ and $8 \times 10 = 80$
$4 + 14 = 18$ and $4 \times 14 = 56$	$9 + 9 = 18$ and $9 \times 9 = 81$
$5 + 13 = 18$ and $5 \times 13 = 65$	$10 + 8 = 18$ and $10 \times 8 = 80$

18. ? = 000ΔΔΔΔ = 0ΔΔΔ + $\frac{1}{2}$(0000ΔΔ) = 6□ + 4□ from first two balances



19. 331 and 322 (the sum of any two sides must be greater than the third side – why?)
20. If the numbers are x and y: $6 \times x + y = 17$. So $17 - y$ must be a multiple of 6, i.e. 12, so $y = 5$
21. In each case the remainder is 2 less than the divisor. So if we add 2 to the number, it is divisible by 3, 4, 5 and 9. $3 \times 3 \times 4 \times 5 = 180$ is the smallest number divisible by 3, 4, 5, and 9. So my number is 178

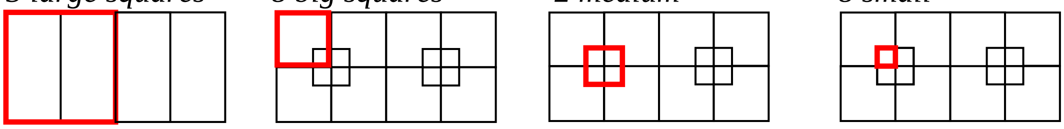
22. The number on the front die is 2 (5 + 2 =7).
The number on back die cannot be 6, 1, 2 or 5.
So it can be 3 or 4. So the sum is 2 + 3 or 2 + 4



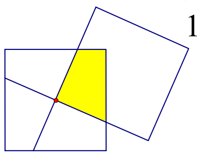
23. Each number is the sum of the two numbers above it, e.g. 6 = 1 + 5, 15 = 5 + 10
24. If a bubble gum cost B cents and a chocolate costs C cents:
$B + C = 90$ and $10B + 5C = 470$, so $5B + 5(B + C) = 470$, so $5B + 5\times90 = 470$, so $B = 4$, so $C =$ R0,86
25. 1, 4, 9, … = 1×1, 2×2, 3×3, … 20×20

GRADE 7(1)

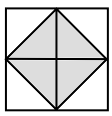
1. Looking right from the top: 5 faces. Right from the bottom: 5 faces. Right from the front: 4 faces. Right from the back: 4 faces. From the right: 4 faces. From the left: 4 faces. So 4 + 4 + 4 + 4 + 5 + 5
2. If the price without VAT is Rx, then $1{,}14 \times x = 36{,}15$. So $x = 31{,}71$
3. Average = Total mass ÷ number of children = (3×75 + 6×66) kg ÷ 9 = 69 kg
4. The "vertical" formula is $2\times a + 2$. Find a so that $2\times a + 2 = 64$
Or the "horizontal" formula is $4 + 2\times(a - 1)$, so find a so that $4 + 2\times(a - 1) = 64$
5. Final price = 120% of (110% of *Price*) = 1,2×1,1 of *Price* =1,32 of *Price* which is a 32% increase
6. $p + q + p + q \;=\; p + p + q + q \;=\; 2\times p \;+\; 2\times q \;=\; (p + q)\times 2 \;\neq\; p\times q \;+\; p\times q$
7. Let the width of the room be w meters. Then Area = $4\times w\times w = 100$, so $4\times w^2 = 100$, so $w^2 = 25$, so $w = 5$
Perimeter = $4w + w + 4w + w = 10w = 50$ m
8. $\dfrac{n}{6{,}34} = \dfrac{a}{1}$. Divide both sides by 100, then $\dfrac{n}{634} = \dfrac{a}{100}$
9. The number is a multiple of 7. So check which of 7, 14, 21, 28, 35, 42, 49, 56, … leave a remainder of 1 when divided by 3 and 5
Quicker: the first two conditions means that the number is one more than a multiple of 3×5 = 15
So the possible numbers are 16, 31, 46, 61, 76, 91. Of these, only 91 is also a multiple of 7
10. *3 large squares* *8 big squares* *2 medium* *8 small*



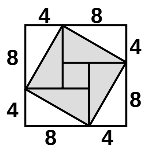
11. $x_1 + x_2 + \ldots + x_7 = 7 \times 49$
So $(x_1 + 1) + (x_2 + 2) + \ldots + (x_7 + 7) = (x_1 + x_2 + \ldots + x_7) + (1 + 2 + \ldots + 7) = 7\times49 + 4\times7$
The new average = (7×49 + 4×7) ÷ 7 = 49 + 4
12. The smallest is 10 × 10 = 100. The largest, by guess-and-improvement = 31 × 31 = 961. Count them!
13. Divide square into 4 equal parts.
So $\frac{1}{4}$ of 144 = 36



14. Divide square into 8 equal parts.
So $\frac{1}{2}$ of 144 = 72



15. Dissect into rectangles.
4×16 + 16 = 80



16. If 60% die, then after the first year there are 40% left
After two years 40% of 40% survive, that is 0,4 of 40% = 16% survive
After three years 40% of 16% survive, that is 0,4 of 16% = 6,4% survive
17. Make an appropriate representation, e.g. take the easier special case of 4 teams, A, B, C and D, as shown here.
4 teams play a total of **3** + 2 + 1 games –note the *structure!*
So **12** teams play a total of **11** + 10 + 9 + … + 3 + 2 + 1 games
Do you have a short method to find the answer?

vs	**A**	**B**	**C**	**D**
A		X	X	X
B			X	X
C				X
D				

18. For 1 step: 5 matches, for 2 steps: 8 + 3 matches, for 3 steps: 8 + 3 + 3 matches …
Pattern: 3×(no. of steps) + 2
19. If April had x eggs, Peter had $x + 2$, Melanie had $x + 7$, Jack had $x + 1$
Together: $x + (x + 2) + (x + 7) + (x + 1) = 38$, so $4\times x + 10 = 38$, so $x = 7$

20. If he bought x apples and y oranges, then the cost is $2 \times x + 1 \times y = 52$ and the total fruit is $x + y = 32$
$2 \times x + y = 52$ can be written as $x + x + y = 52$, so $x + (x + y) = 52$, so $x + 32 = 52$, so $x = 20$

21. $\frac{5^{14}}{5^{17}} = \frac{5 \times 5 \times 5 \times 5 \ldots 14 \text{ times}}{5 \times 5 \times 5 \times 5 \ldots 17 \text{ times}} = \frac{1}{5^3} = \frac{1}{125}$

22. List the units digits of the first few powers:

Exponent	1	2	3	4	5	6	7	8	9
Units digit	3	9	7	1	3	9	7	1	3

The last digits has a recurring pattern 3, 9, 7, 1

Reorganise the results like this and analyse the sequences:

Units digit	**Exponents giving the units digit**	
3	1, 5, 9, 13, …	These have a remainder of 1 when divided by 4
9	2, 6, 10, 14, …	These have a remainder of 2 when divided by 4
7	3, 7, 11, 15, …	These have a remainder of 3 when divided by 4
1	4, 8, 12, 16, …	These are multiples of 4

We simply have to decide in which sequence 2005 will be …

23. Look for *structure* in the denominator:

	F_1	**F_2**	**F_3**	**F_4**	**…**	**F_{10}**
Numbers	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{20}$		?
Structure	$\frac{1}{1 \times 2}$	$\frac{1}{2 \times 3}$	$\frac{1}{3 \times 4}$	$\frac{1}{4 \times 5}$		$\frac{1}{10 \times 11}$

24. Calculate intermediate answers and look for *structure* and *patterns*:

Sum of **1** fraction $= \frac{1}{2}$

Sum of **2** fractions $= \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$

Sum of **3** fractions $= \frac{2}{3} + \frac{1}{12} = \frac{3}{4}$

Sum of **4** fractions $= \frac{3}{4} + \frac{1}{20} = \frac{4}{5}$

Sum of **10** fractions $= \frac{10}{11}$

25. $(1 + 1) \times (1 + \frac{1}{2}) \times \ldots \times (1 + \frac{1}{100}) = \frac{2}{1} \times \frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \ldots. \times \frac{100}{99} \times \frac{101}{100}$

$= \frac{2 \times 3 \times 4 \times 5 \times \ldots \times 99 \times 100 \times 101}{2 \times 3 \times 4 \times 5 \times \ldots \times 99 \times 100}$

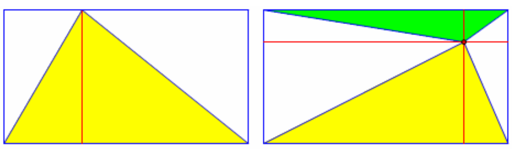
$= 101$

GRADE 7(F)

2. $3 \times 3 - 3 + 3 = 9 - 3 + 3 = 6 + 3 = 9$

3. nth number $= 2 \times n - 1$, so 83rd number $= 2 \times 83 - 1 = 165$

4. & 5.



6. $1 + \frac{1}{1 + \frac{2}{3}} = 1 + \frac{1}{\frac{5}{3}} = 1 + \frac{3}{5}$

7. We know: $\frac{\text{Sum of numbers}}{11} = 8$, so Sum of numbers = 11 × 8 = 88

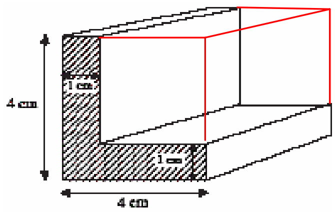
If the new number is x, then $\frac{88 + x}{12} = 11$. So x = 12 × 11 – 88 = 44

8. Add all together: 2A + 2B + 2C = 42, so A + B + C = 21
9. B + A + C = 21 and A + C = 16, so B + 16 = 21
10. 12:9. So 3 revolutions will be 36 clicks, which will revolve B 4 times
11. # *Triangles* = 2×*squares* + 2, or 2×(*squares* + 1). So *Triangles (6)* = 2×6 + 2 = 14
12. *Triangles (60)* = 2×60 + 2 = 122
13. 2×x + 2 = 60, so x = 29
14. Make a list, varying the numbers systematically. If the digits are a, b, c and d:
abcd, abdc, acbd, acdb, adbc, adcb and similarly if the first digit is b, c, and d. So 6 × 4 = 24
15. 2 × (7 + 8 + 9) = 2 × 24
16.

	c	d
a	12	20
b	21	D

Using a representation like this, Area D = b × d
We know a × c = 12, b × c = 21, a × d = 20
Multiply them all together: $a^2 \times c^2 \times b \times d$ = 12 × 20 × 21
But a×c = 12, so $a^2 \times c^2$ = 144, so b×d = 12×20× 21 ÷ 144 = 35

17. Volume = area of base × length = 7 cm^2 × 12 cm = 84 cm^3
Or think of cutting out a rectangular prism:
Volume = 4×4 × 12 – 3×3 × 12 = 7 × 12



18. The first digit can be 2, 4, 6, 8. The second digit can be 0, 2, 4, 6, 8, which gives 4 × 5 = 20 possible combinations
19. The 6^{th} column is given by 6 × row n.
So the last number in row 80 is 6 × 80 = 480. Then row 81 is 481, 482, 483, …
20. Filling: In 1 minute $\frac{1}{12}$ of bath fills

Emptying: In 1 minute $\frac{1}{18}$ of bath empties

Together: In 1 minute $\frac{1}{12} - \frac{1}{18} = \frac{1}{36}$ of batch fills. So the whole bath ($\frac{36}{36}$) fills in 36 minutes

21. Fill in numbers in the calendar, and test each statement with the numbers.
22. We know $a + d = c + b$, so $a + b + c + d = a + d + c + b = 2\times(a + d) = 52$.
So $a + d = 26$, so $a + (a + 8) = 26$, so $a = 9$
24. 3 lines from two corners divide the triangle in 4 × 4 sections
10 lines from two corners will divide the triangle in 11 × 11 sections = 121
25. $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \ldots \times \frac{2003}{2004} \times \frac{2004}{2005} = 1 \times \frac{2}{2} \times \frac{3}{3} \times \ldots \times \frac{2004}{2004} \times \frac{1}{2005} = \frac{1}{2005}$