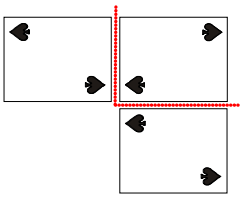
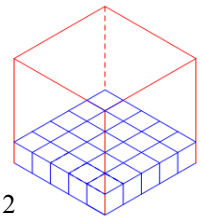
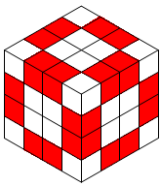
NOTES ON 2006 MEMORANDUM FIRST AND FINAL ROUNDS

These notes are necessarily brief and often formal and symbolic.
Many questions could be answered using primitive methods, e.g. "If today is Wednesday, what day of the week will it be 100 days from now?" can be done by counting. That would be laborious, time-consuming and error-prone. The essence of a mathematical approach is to work more smartly by using appropriate representations to model the situation and to exploit the inherent structures and patterns in the situation.

GRADE 4(1)

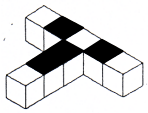
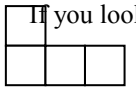


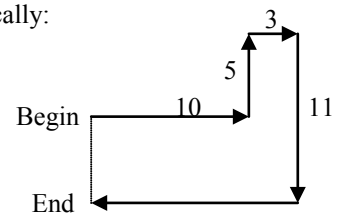




1. 5 + 3 = 8, while the others all have a sum of 7
2. Half of 8 × 8
6. There is a pattern of +14, +14, +14 in the numbers
7. $\frac{3}{4} \div \frac{1}{8} = \frac{6}{8} \div \frac{1}{8}$. How many $\frac{1}{8}$ are there in $\frac{6}{8}$?
8. B is a mirror-image in a horizontal or vertical line of symmetry, as shown
11. 35 000 mℓ ÷ 35 mℓ = 100
12. The figure can be divided into 32 equal triangles of which 16 are shaded.
13. 1, 4, 9, ... = 1×1, 2×2, 3×3, … So 8×8 = 64
14. 5 small cubes to a side. So 5×5 in bottom layer, with 5 layers, so 5×5×5
15. 274 – 246 + 1 = 29
16. 8 cubes on each of the 6 sides. But then they are all counted twice! So 6×8 ÷ 2
17. Bottom level: 3 × 3 = 9 blocks, Second level has 1 less: 8 blocks, Top level has 5 blocks
18. $x - 4 + 5 - 6 = 3$, so $x - 5 = 3$ so $x = 8$
19. $2 \times T + 2 = 38$, so $T = (38 - 2) \div 2 = 18$
20. 120 km in 60 min, so 20 km in 10 min, so 200 km in 100 min, so the time is 11:40
21. 7, 17, 27, 37, … 77 (two!), 87, 97 is 11, plus 70, 71, 72, … ~~77~~, 78, 79 is another 9, so 20
22. M + M + 30 = 114, so 2 × M = 84, so Monde weighs 42 kg
23. 50 × 2 – 1 = 99
24. Sum of rows = 1, 4, 9, … = 1×1, 2×2, 3×3, … So 50×50 = 2500
25. 3 + 8 + 3 + 8 = 22

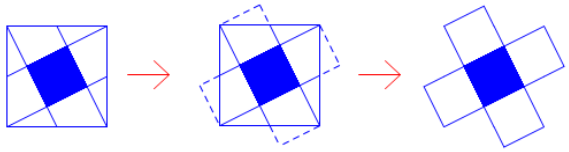
GRADE 4(F)

1. 
2. 2 hours and 55 minutes before 16:45 is 13:50
3. The watch gains 2 minutes every day (24 hours) for 7 days = 2 min/day× 7 days = 14 minutes
7. If Zuki has ♥ marbles, Zinkle has ♥ – 15. Together they have 2×♥ – 15 = 95 marbles. So ♥ = 55
8. 257 + Δ = 438, so Δ = 438 – 257 = 181 km
9. 438 + 169 = 607 km
10. 4 is 1/3 of 12. Thabo needs to pay 1/3 of R30 = R10
11.  If you look from *behind* the tower is on your *left*
12. 6,8 ÷ 2 = 3,4; 3,4 ÷ 2 = 1,7; 1,7 ÷ 2 = 0,85
13. Bingo : 71; Thandi: 71 – 24 = 47; Voyo: 71 + 24 = 95
    71 + 47 + 95 = 213 *or* 71 × 3 = 213 (24 – 24 = 0)
14. 37 will be opposite 38, therefore Con lives opposite Luke
15. R35,60 ÷40 = 89c so 89c × 15 = R13,35
16. 4 × 3 → 12 + 8 → 20 ÷ 2 → 10 – 6 = 4
17. 5 = 20 × ¼; 20 ÷ 3 = 6 2/3 (She cannot make 2/3 of a skirt → 6 skirts)
18. 15 × 2 = 30; 15 × 1/5 = 15/5 = 3; so 30 + 3 = 33 viennas
19. 2 people sit at the end → 56 ÷ 2 (2 people per table) → 28 tables
20. Draw it physically:
    11 – 5 = 6 km



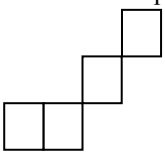
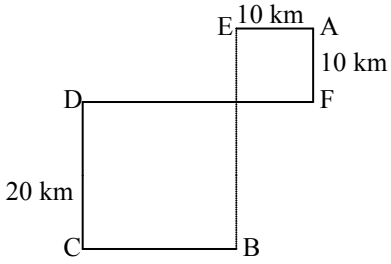
21. Arrange them: O S (R) T (R) E (Ram can be between Siva and Temba or between Temba and Eby)
    Oscar is the shortest
22. 1/4 = 10/40; 1/5 = 8/40 → 9/40
24. 2 ways: 2×10c + 5c; 1×10c + 1×20c + 5c
25. 2, 3, 4, 5, 6,7, 8, 9, 10, 11, 12 → 11 answers

GRADE 5(1)

1. There are 5 tiles in every metre because 1000 cm ÷ 200 cm = 5. So 15 × 10 = 150 tiles
2. The numbers inside the square *and* the circle are 2 and 3. 2 is not inside the triangle
3. Try and test each possible answer!
4. C – a rotation to the right through 90°
5. 4 reds – 10 greens – 3 purples. So 12 (3×4) reds – 9 (3×3) purples
6. n^{th} row has $2\times n - 1$ dots, so 7^{th} row has 13 dots
7. n^{th} row has $2\times n - 1$ dots, so 70^{th} row has $2\times 70 - 1 = 139$ dots
8. 100 ÷ 24 = 4 rem 4, i.e. 4 full days bringing us to 10:00, plus 4 more hours, i.e. 11, 12, 13, 14:00
   Or 10 + 100 = 110, 110 ÷ 24 = 6 rem 14
9. *Height* = 12 cm + 1,5 cm/day × *days*. So *Height after 30 days* = 12 + 1,5×30 = 57 cm
10. (150 cm – 12 cm) ÷ 1,5 cm/day = 92 days
11. One more than a multiple of 6, so it is odd, so it cannot be A or B. Test the others: 4 182÷6 = 697
12. If a sack weighs S kg, then $3S = S + 30$, so $2S = 30$, so $S = 15$. So $3S = 45$ kg
13. 3, 6, 9, … is the 3-times table. So 50 × 3 = 150
14. Mathematics is $\frac{1}{4}$ of his time, and this is 2 hours. So $\frac{4}{4}$ of his time is 4×2 hours = 8 hours
15. In the bottom layer there are 8 × 4 = 32 blocks, so in two layers there are 64 blocks
16. All the blocks of the bottom layer (32) and all the blocks round the side of the top layer (20)
17. 3×2 + 2 =8; 7×2 + 2 = 16; so for rectangle with length 20: 20×2 + 2 = 42
18. 
19. 75c more per week, so 12 × 75c =R9
20. Half the water weighs 21 kg – 12 kg = 9 kg, so all the water weighs 18 kg. So the bucket weighs 3 kg
21. The number must start and end with 1 so list them systematically:
    101 111 121 131 141 151 161 171 181 191
22. Share 30 litres in ratio 5 to 1, i.e. 25 to 5
23. If Penny has p coins and Alex has a coins:
    $p = 2\times a$, $p - 4 = a + 4$, so $2\times a - 4 = a + 4$, so $a = 8$, so $p = 16$, so $p + a = 24$
24. 4000
    3100, 3010, 3001
    2200, 2020, 2002
    2110, 2101, 2011
    ~~2020, 2002, 2011~~
    1300, 1030, 1003
    1210, 1201
    1120, 1102
    1111
    ~~1030, 1003~~
    1021, 1012
25. If a small pizza costs s rands and a large pizza costs L rands: $2s + 1L = 5s$, so $1L = 3s$, so the cost is $L = 3\times$R11,50 = R34,50

GRADE 5(F)

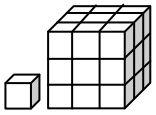
1. In the bottom layer there are 8 × 4 = 32 blocks, so in two layers there 64 blocks
2. In the bottom layer there are 8 × 4 = 32 blocks, so in two layers there 64 blocks
3. Between 09:47 and 10:18, 31 minutes pass. 31 minutes from 12:30 is 13:01
4. 500 ÷ 12 = 41 rem 8. The 8 bottles need to be put into another crate. 41 + 1 = 41 crates
5. 360 ÷ 120 = 3; 270 ÷ 90 = 3; 22 × 3 = 66 mm
6. If the loser had Δ votes, the winner had Δ + 1002 votes. Together 2×Δ + 1002 = 39218
7. The number has to be divisible by 6: 7356 ÷ 6 = 1226

8. R5 less for you and R5 more for her is R10
9. Chapter six ended on page 245. 274 – 245 = 29 pages
10. 
11. 41 kg – 725 g = 40,275 kg
12. 3 h (Peter) + 2 h (Paul) + 1 h (David) = 6 hours
    R48 ÷ 6 = 8. R8 × 3 = R24
13. 600 ÷ 10 = 60. This is the correction to get the original number
    60 ÷ 10 = 6. This is the correct computation
14. 0 × 5 + 2 = 2
    1 × 5 + 2 = 7
    2 × 5 + 2 = 12 etc
15. 37 – 2 = 35. 35 ÷ 5 = 7
16. 3 × 2 + 2 = 8
    7 × 2 + 2 = 10
    10 × 2 + 2 = 22
17. 600 ÷ 6 × 5 = 500
    120 ÷ 6 × 5 = 100
    300 ÷ 6 × 5 = 250
18. 9 × 1 = 9; 90 × 2 = 180; 1 × 3 = 3
    9 + 180 + 3 = 192
19. Make a sketch of the situation:
    "2nd from front, 4th from back" means there are 5 rows. "3rd from left, 5th from right" means there are 7 learners per row. So 7 learners/row × 5 rows = 35 learners
20. One dark square : 8 light squares → 1/9
21. 6 × 8 → 48 + 8 → 56
22. 

23. 12, 13, 14; 21, 23, 24; 31, 32, 34; 41, 42, 43. 12 numbers
24. -10 years for each sister: 100 – 30 = 70
25. 1st row + last row = 51; 2nd row + 2nd to last row = 51
    50 ÷ 2 = 25 → 51 × 25 = 1275

GRADE 6(1)

1. Make equal parts. Each small square is half of the next bigger square
   So half of half of the big square is a quarter of the big square
2. There are 8 columns, each with 2 + 4 + 6 cubes. So 8 × 12 = 96 cubes
3. In middle row the missing number is 18 – (11 + 6) = 1, so in right column A = 18 – (1 + 10) = 7
5. $\frac{1}{7} = \frac{5}{35}$ and $\frac{1}{5} = \frac{7}{35}$ so $\frac{6}{35}$ is exactly in between them. Or $(\frac{1}{5} + \frac{1}{7}) \div 2 = (\frac{7}{35} + \frac{5}{35}) \div 2 = \frac{6}{35}$
7. Use trial and error, i.e. try each of the given answers one by one
8. Continue the patterns: 17, 22, 27, 32, 37, 42, 47, 52, … and 17, 24, 31, 38, 45, 52, …
9. For *n* dice, the number of visible faces is $n \times 3 + 2$. So for 75 dice, $75 \times 3 + 2 = 27$
10. Imagine or draw the cube! If the side is 3 times as long, the big cube contains 27 of the small cubes. So its mass is 27 times as large!

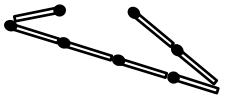


11. 0×20 + 3×10 + 1×5
    1×20 + 1×10 + 1×5
12. 102 ÷ 7 = 14 remainder 4, so adding 3, we have 105 ÷ 7 = 15
14.

B	C	M
A	2	N
1	D	

In the middle row, N cannot be 2, so N is 1 or 3
Suppose N = 3. Then A = 1 which is impossible (already a 1 in left column).
So N = 1, A = 3. In left column B = 2. Then C = 1 (D ≠ 1), so M = 3, so M+N = 4

15. $3 \times (1 + 2 + 3) = 18$
16. Vary the possibilities systematically. First note that she could not draw 1, 3 or 5 games, otherwise her total would be a fraction. If she drew 6 games her total was $6 \times \frac{1}{2} = 3$. If she drew 4 and won 2 her total was $2 \times 1 + 4 \times \frac{1}{2} = 4$. If she drew 2 and won 4 her total was $4 \times 1 + 2 \times \frac{1}{2} = 5$
17. Vary the numbers systematically and note the behaviour of the product of the numbers:
    $1 + 17 = 18$ and $1 \times 17 = 17$ $\quad 6 + 12 = 18$ and $6 \times 12 = 72$
    $2 + 16 = 18$ and $2 \times 16 = 32$ $\quad 7 + 11 = 18$ and $7 \times 11 = 77$
    $3 + 15 = 18$ and $3 \times 15 = 45$ $\quad 8 + 10 = 18$ and $8 \times 10 = 80$
    $4 + 14 = 18$ and $4 \times 14 = 56$ $\quad 9 + 9 = 18$ and $9 \times 9 = 81$
    $5 + 13 = 18$ and $5 \times 13 = 65$ $\quad 10 + 8 = 18$ and $10 \times 8 = 80$
18. ? = 000ΔΔΔΔ = 0ΔΔΔ + $\frac{1}{2}$(0000ΔΔ) = 6□ + 4□ from first two balances



19. 331 and 322 (the sum of any two sides must be greater than the third side – why?)
20. If the numbers are x and y: $6 \times x + y = 17$. So $17 - y$ must be a multiple of 6, i.e. 12, so $y = 5$
21. In each case the remainder is 2 less than the divisor. So if we add 2 to the number, it is divisible by 3, 4, 5 and 9. $3 \times 3 \times 4 \times 5 = 180$ is the smallest number divisible by 3, 4, 5, and 9. So my number is 178
22. If the empty glass has a mass of g gram and the milk has a mass of m gram, then
    $g + m = 370$
    $g + \frac{1}{2} m = 290$
    So $\frac{1}{2} m = 370 - 290 = 80$ gram, so $m = 160$ gram and $g = 370 - 160 = 210$ gram
23. Each number is the sum of the two numbers above it, e.g. $6 = 1 + 5$, $15 = 5 + 10$
24. If a bubble gum cost B cents and a chocolate costs C cents:
    $B + C = 90$ and $10B + 5C = 470$, so $5B + 5(B + C) = 470$, so $5B + 5 \times 90 = 470$, so $B = 4$, so $C =$ R0,86
25. 1, 4, 9, … = 1×1, 2×2, 3×3, … 20×20

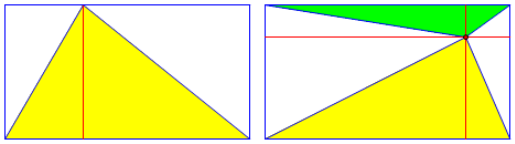
GRADE 6(F)

1. 1^{st} number + (1^{st} number + 2) + (1^{st} number + 4) = 174
   (174 – 6) ÷3 = 1^{st} number = 56
   Biggest number = 56 + 6 = 62
3. $3 \times 3 - 3 + 3 = 9$
5. Full lorry = 4653 kg; empty lorry = 2583 kg; 4653 – 2583 = 2070
   2070 ÷ 90 = 23
6. 4002 ÷ 4 = 1000 rem 2
8. 5/6 = 40/48; 7/8 = 42/48
   40/48; 41/48; 42/48
9. $2 \times 4 + 0 \times 3 = 8$
   $2 \times 4 + 1 \times 3 = 11$
   $2 \times 4 + 2 \times 3 = 14$
   $50 - (2 \times 3) \rightarrow 44 \div 4 = 11$
   11 + 2 = 13 dice
11. 399/1000; 398/500; 410/1000; 420/1000; 300/1000
    2/5 = 400/1000 → 399/1000 is closest to 2/5
12. ½ + 1/8 + 1/8 = 6/8 = ¾
    R15 = ¼ → Pocket money = 4 × R15 = R60
13. 230 – 60 = 170; 170 ÷ 2 = R85
15. $36 \times 37 = 1332$
16. Jane eats 2×12 sweets in 5 minutes; she eats 2×24 sweets in 10 minutes. Jane eats 48 sweets in 10 minutes
17. 50 – 30 = 20; 41 – 30 = 11; 35 – 30= 5
    Of the 20 learners who do not like both, 11 like comedy and 5 like action films
    20 – 11 – 5 = 4
18. Length = 88 cm → 11 tins ; Width = 44 cm → 4 ; Height = 40 cm → 5 tins
    11 × 4 × 5 = 220
19. 7 × 2 = 14 → 5 extra wheels → 5 tricycles → 2 bicycles
20. 4 ÷ 2 × 3 = 6; 6 ÷ 2 × 3 = 9 cm
22. Row 2 = 2 + 1
    Row 3 = 3 + 2
    Row 4 = 4 + 3
    Row 50 = 50 + 49 = 99

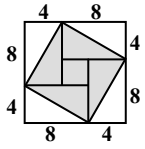
23. Row 50 + Row 1 = 99 + 1 = 100
    Row 49 + Row 2 = 98 + 2 = 100
    25 × 100 = 2500
24. 15 × 15 =225
25. 50 × 51 = 2550

GRADE 7(1)

2. $3\times3 - 3 + 3 = 9 - 3 + 3 = 6 + 3 = 9$
3. nth number = $2\times n - 1$, so 83rd number = $2 \times 83 - 1 = 165$
4. & 5.



6. $1+\dfrac{1}{1+\frac{2}{3}}=1+\dfrac{1}{\frac{5}{3}}=1+\frac{3}{5}$
7. We know: $\dfrac{\text{Sum of numbers}}{11}=8$, so Sum of numbers = 11 × 8 = 88
   If the new number is x, then $\dfrac{88+x}{12}=11$. So x = 12 × 11 – 88 = 44
8. Add all together: 2A + 2B + 2C = 42, so A + B + C = 21
9. B + A + C = 21 and A + C = 16, so B + 16 = 21
10. The smallest is 10 × 10 = 100. The largest, by guess-and-improvement = 31 × 31 = 961. Count them!
11. # *Triangles* = 2×*squares* + 2, or 2×(*squares* + 1). So *Triangles (6)* = 2×6 + 2 = 14
12. *Triangles (60)* = 2×60 + 2 = 122
13. $2\times x + 2 = 60$, so x = 29
14. Make a list, varying the numbers systematically. If the digits are a, b, c and d:
    abcd, abdc, acbd, acdb, adbc, adcb and similarly if the first digit is b, c, and d. So 6 × 4 = 24

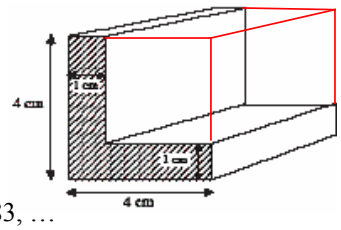


15. 2 × (7 + 8 + 9) = 2 × 24
16.

	c	d
a	12	20
b	21	D

Using a representation like this, Area D = b × d
We know a × c = 12, b × c = 21, a × d = 20
Multiply them all together: $a^2 \times c^2 \times b\times d = 12 \times 20 \times 21$
But $a\times c = 12$, so $a^2\times c^2 = 144$, so $b\times d = 12\times20\times 21 \div 144 = 35$

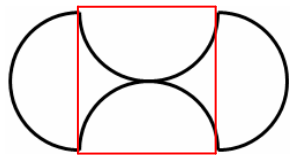
17. Volume = area of base × length = 7 cm^2 × 12 cm = 84 cm^3
    Or think of cutting out a rectangular prism:
    Volume = 4×4 × 12 – 3×3 × 12 = 7 × 12



18. The first digit can be 2, 4, 6, 8. The second digit can be
    0, 2, 4, 6, 8, which gives 4 × 5 = 20 possible combinations
19. The 6th column is given by 6 × row n.
    So the last number in row 80 is 6 × 80 = 480. Then row 81 is 481, 482, 483, …
20. Filling: In 1 minute $\frac{1}{12}$ of bath fills
    Emptying: In 1 minute $\frac{1}{18}$ of bath empties
    Together: In 1 minute $\frac{1}{12} - \frac{1}{18} = \frac{1}{36}$ of batch fills. So the whole bath ($\frac{36}{36}$) fills in 36 minutes
21. Fill in numbers in the calendar, and test each statement with the numbers.
22. We know $a + d = c + b$, so $a + b + c + d = a + d + c + b = 2\times(a + d) = 52$.
    So $a + d = 26$, so $a + (a + 8) = 26$, so $a = 9$
24. 3 lines from two corners divide the triangle in 4 × 4 sections
    10 lines from two corners will divide the triangle in 11 × 11 sections = 121
25. $\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\ldots\times\frac{2004}{2005}\times\frac{2005}{2006} = 1\times\frac{2}{2}\times\frac{3}{3}\times\ \ldots\times\frac{2005}{2005}\times\frac{1}{2006} = \frac{1}{2006}$

GRADE 7(F)

1. $5 \times 3 - 2 \times 1 = 13$
   $6 \times 2 - 2 \times 1 = 10$
   $13 + 10 = 23$
2. $3 + 3 + 5 + 2 + 4 + 2 + 6 + 1 = 26$
3. Red square = 3 x 3 = 9 (The two semi circles outside the 3x3 square cancel out the two inside the square



4. $3 \times 75 + 6 \times 66 = 225 + 396 = 621$
   $621 \div 9 = 69$ kg
5. $64 - 2 = 62$
   $62 \div 2 = 31$
6. $R36{,}15 \div 1114 \times 100 = R31{,}71$
8. 2×3 {2 big triangles make one rectangle}$+ 2 \times 2$ {square between two big triangles} $+ 1$ {two small triangles}
   $= 6 + 4 + 1 = 11$
9. Sum of even numbers $= 102 \times 25 = 2550$
   Sum of odd numbers $= 100 \times 25 = 2500$
   $2550 - 2500 = 50$
10. In four years time Sandy will be 7 and Mandy will be 14. At this moment Sandy is 3 years old and Mandy is 10 years old. $3 + 10 = 13$
13. If the dimensions of the room is a by b by c, then the area to paint is $A = 2ab + 2ac + 2bc$
    Double the dimensions are $2a$ by $2b$ by $2c$, so the area to paint is $D = 2(2a)(2b) + 2(2a)(2c) + 2(2b)(2c) = 4 \times A$
14. $9 \times 9 = 81$
15. 1, 2, 4, 5, 8, 10, 16, 20, 25, 40, 50, 80, 100, 125, 200, 250, 400, 500, 1000, 2000 (20)
17. 799; 979; 997; 889; 898; 988 (6)
19. $11 \times 8 = 88$; $11 \times 12 = 132$; $132 - 88 = 44$
20. $12 \div 4 = 3$; $3 \times 3 = 9$
21. Filling: In 1 minute $\frac{1}{12}$ of bath fills

    Emptying: In 1 minute $\frac{1}{18}$ of bath empties

    Together: In 1 minute $\frac{1}{12} - \frac{1}{18} = \frac{1}{36}$ of batch fills. So the whole bath $(\frac{36}{36})$ fills in 36 minutes
22. Row 1 = 1 dot; row 50 = 50 dots; row 1 + row 50 = 51 dots
    Row 2 = 2 dots; row 49 = 49 dots; row 2 + row 49 = 51 dots
    $51 \times 25 = 1275$ dots
23. Look for structure and pattern!
    $N_1 = 1$
    $N_2 = 5 = 2^2 + 1$
    $N_3 = 10 = 3^2 + 1$
    $N_4 = 17 = 4^2 + 1$
    Test the numbers! $30^2 + 1 = 901$ is the only one
24. If they mine 5%, then 95% = 0,95 is left. So:
    After 1 year, 95% is left
    After 2 years, 95% of 95% = $0{,}95 \times 0{,}95$ is left
    After 3 years, 95% of 95% of 95% = $0{,}95 \times 0{,}95 \times 0{,}95 = 0{,}95^3$ is left
    After 10 years, $0{,}95^{10}$ is left. Use a calculator: $0{,}95^{10} = 0{,}598 = 59{,}8\%$ is left
    After 13 years, $0{,}95^{13}$ is left. $0{,}95^{13} = 0{,}513 = 51{,}3\%$, more than half, is left
    After 14 years, $0{,}95^{14}$ is left. $0{,}95^{14} = 0{,}487 = 48{,}7\%$, less than half, is left
25. $(1 + 1) \times (1 + \frac{1}{2}) \times \ldots = \frac{2}{1} \times \frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \ldots \times \frac{100}{99} \times \frac{101}{100}$
    $= \frac{2}{2} \times \frac{3}{3} \times \frac{4}{4} \times \frac{5}{5} \times \ldots \times \frac{100}{100} \times \frac{101}{1}$
    $= 101$