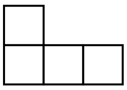
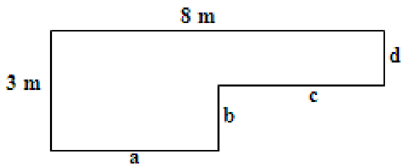
NOTES ON 2008 MEMORANDUM

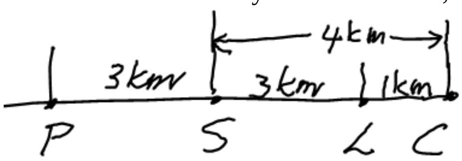
These notes are necessarily brief and often formal and symbolic.
Many questions could be answered using primitive methods, e.g. "If today is Wednesday, what day of the week will it be 100 days from now?" can be done by counting. That would be laborious, time-consuming and error-prone. The essence of a mathematical approach is to work more smartly by using appropriate representations to model the situation and to exploit the inherent structures and patterns in the situation.

GRADE 4(1)

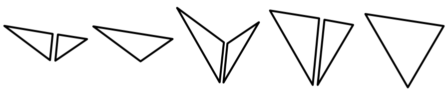
1. 1 pizza for 3 children
   12 pizzas × 3 children/pizza = 36 children
4. These are multiples of 6. Only 4182 = 6 ×697 is a multiple of 6
5. Look at the ones digit! The only possibility is 33×58 = 1914. Check it.
6. 09:47 to 10:18 = 31 minutes
   12:30 plus 31 min is 13:01
7. The pattern is 1 + **1** + 1 + **2** + 1 + **3** + 1 + **4** **+ 1 + (5 + 1 + 6 + 1 + 5)** + **2** + 1 + 8 + 1
8. 438 – 257 = 181 km
9. 438 + 169 = 607 km
10. Thabo takes 4 out of 12; 4/12 = 1/3. He has to pay 1/3 of R30 = R10
11.  The tower is on your left if you look at the object from the back
12. 6,8 ÷ 2 → 3,4 ÷ 2 → 1,7 ÷ 2 = 0,85
13. 24 – 24 = 0; 71 × 3 = 213 marbles
14. 37 will be opposite 38, therefore Con lives opposite Luke
15. R35,60 ÷40 = 89c so 89c × 15 = R13,35
16. 7; 9; 12; 16; 21; ~~28~~ **27. Pattern:** 7+**2**=9;9+**3**=12;12+**4**=16;16+**5**=21;21+**6**=**27**
17. 8 + 3 + a + b + c + d
    But a + c = 8 and b + d = 3
    So 8 + 3 + 8 + 3 = 22



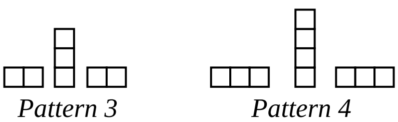
18. There are 36 tiles. 20 tiles are not covered (count halves). $\frac{20}{36} = \frac{5}{9}$
19. Draw it! Fill in the information as you read. Re-read, bit by bit!



20. Use trial and error to find that only 26 × 27 = 702. So 26 + 27 = 53
21.



22. Do not count or calculate – look for *structure*, e.g.
    For Pattern 3: 3 + 2×2
    For Pattern 4: 4 + 2×3
    For Pattern 100: 100 + 2×99

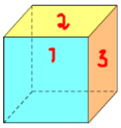


23. Division by 2, 3 and 4 leaves a remainder of 1. The smallest such number is 2×3×4 + 1 = 25.
    In general 1 more than a multiple of 24, i.e. 25, 49, 73, 97, 121, 145, … all leave a remainder of 1 when divided by 2, 3 or 4.
    Only 25, 145, … leaves a remainder of 0 when divided by 5, and of these only 25 is here given as a possibility.
24. 0×20 + 3×10 + 1×5
    1×20 + 1×10 + 1×5
25. The structure is 1 + 2 + 3 + 4 + 5 + 6 + ... + 48 + 49 + 50 = (1+50) + (2+49) + (3+48) + ... = 51 × 25

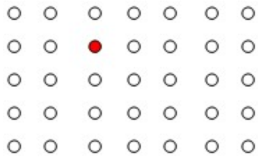
GRADE 4(F)

1. $\frac{1}{8} < \frac{1}{4} < \frac{1}{2}$
2. Check each answer, e.g. $9 = 3 \times 3$; $11 = 2 \times 3 + 5$; $13 = 2 \times 5 + 3$, etc. You know, or you learn from these calculations, that the sum of three odd numbers is odd, so 12, which is even, is not possible!
3. Straighten the string. Two loops of 1 cm make it 5 cm + 1 cm + 1 cm = 7 cm
4. 3 hours before 16:45 is 13:45, so 2 hours and 55 minutes (5 min less) is at 13:50
5. The watch gains 2 minutes every day (24 hours) for 7 days = 2 min/day× 7 days = 14 minutes
6. $10 \times 10 \times 10 = 1000$ or from 001 to 999 gives 999 combinations, plus 000 gives 1000

9. Start "painting" (numbering) the sides …



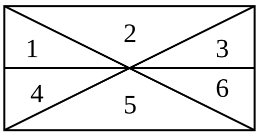
10. With 6 loose cubes, there would be 36 faces. Subtract the 10 non-visible faces ...
11. If Zuki has ♥ marbles, Zinkle has ♥ – 15. Together they have 2×♥ – 15 = 95 marbles. So ♥ = 55
12. The numbers must be different, so 99 + 98 + 97 = (100 – 1) + (100 – 2) + (100 – 3) = 300 – 6
13. $\frac{3}{4} + \frac{3}{4} \rightarrow 1\frac{1}{2} + \frac{3}{4} \rightarrow 2\frac{1}{4} + \frac{3}{4} \rightarrow 3 + \frac{3}{4} \rightarrow 3\frac{3}{4} + \frac{3}{4} \rightarrow 4\frac{1}{2}$
 ① ② ③ ④ ⑤ ⑥
14. Make a sketch of the situation!
 "2nd from front, 4th from back" means there are 5 rows. "3rd from left, 5th from right" means there are 7 learners per row. So 7 learners/row × 5 rows = 35 learners



15. If the 21th is a Monday, then also the 14th, 7th and 0th are Mondays.
 The 0th is the last day of the previous month, so the next day is the 1st of this month, so it is a Tuesday
16.

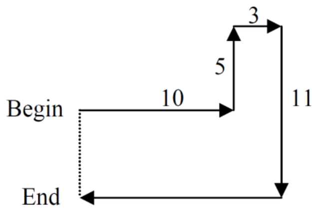
			32+38			
		32		**38**		
	15		**17**		**21**	
10		**5**		**12**		**9**

17. $4 \times 3 \rightarrow 12 + 8 \rightarrow 20 \div 2 \rightarrow 10 - 6 = 4$
18.



Invent some notation and count systematically, e.g.:
Areas 1, 2, 3, 4, 5 and 6 each form a triangle (6)
Two areas 1-4 and 3-6 each form a triangle (2)
Three areas 4-1-2, 2-3-6, 3-6-5 and 5-4-1each form a triangle (4)

19. If #T, then # people = 2 × T + 2 = 58, so T = (58 – 2) ÷ 2 = 28
20. Draw it physically! See diagram. It always helps to write!



21. $\frac{1}{5} = \frac{8}{40}$ and $\frac{1}{4} = \frac{10}{40}$, so $\frac{8}{40} < \mathbf{\frac{9}{40}} < \frac{10}{40}$
22. The first (left) digit changes the least – it just changes from 0 to 1, from 1 to 2 and from 2 to 0. This happens when the clock changes from 09:59:59 to 10:00:00; from 19:59:59 to 20:00:00 and from 23:59:59 to 00:00:00.
23. Let the children be A, B, C, D and E. List all the possibilities systematically, note patterns and structure:

A vs B	**B** vs C	**C** vs D	**D** vs E
A vs C	B vs D	C vs E	
A vs D	B vs E		
A vs E			

vs	A	B	C	D	E
A		X	X	X	X
B			X	X	X
C				X	X
D					X
E					

24.

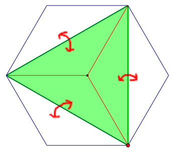
A vs B	**B** vs C	…	**H** vs I	**I** vs J
A vs C	B vs D	…	H vs J	
A vs D	B vs E			
A vs E	B vs F			
A vs F	B vs G			
A vs G	B vs H			
A vs H	B vs I			
A vs I	B vs J			
A vs J				

 The *structure* is: 9 + 8 + ... + 2 + 1 = **45**

25. The number sequence is 1; 1+3; 1+3+5; 1+3+5+7; … = 1; 4; 9, 16, ... = 1×1; 2×2; 3×3; 4×4, … So $P_{50} = 50 \times 50$

GRADE 5(1)

1. 11,23 s < 11,32 s < 11,4 s. So Peter Davids is third
2. Lucy's mother is 24 years older than Lucy
   In 8 years Lucy's mother will be 48
   Lucy is 24 now.
3. 07:20 + 45 min = 08:05
4. 500 ÷ 12 = 41 rem 8 (41.666… on the calculator), so 42 cartons are needed
5. Recognize and continue with pattern:
   1st layer = 1 = 1×1
   2nd layer = 4 = 2×2
   3rd layer = 9 = 3×3
   4tht layer = 16 = 4×4
   etc…
   10th layer = 10×10 = 100 grapefruit
6. 12 rounds × 3 minutes/round + 11 breaks × 1 minute/break = 36 minutes + 11 minutes
8. $\frac{5}{6} = \frac{?}{150}$
9. $\frac{1}{8}$ + 25 litres = $\frac{5}{8}$ of tank
   So 25 litres = $\frac{4}{8} = \frac{1}{2}$ of tank
   So the full tank holds 2 × 25 litres = 50 litres
10. See sketch



11. If Joe's starting number is S, then he did S × 10 = 500. So S = 50. So correct answer is 50 ÷ 10 = 5
12. List them systematically: 300; 210; 201; 120; 111; 102
13. The numbers are all 1 less than a multiple of 6. On the calculator, if you divide by 6, they all end in .83333…
    365 + 1 → 366 ÷ 6 = 61 (365 ÷ 6 = 60.8333 …., but none of the others ends in 0.8333…)
14. Mary ate 24 ÷ 6 = 4 pieces, Veronica ate 24 ÷ 4 = 6 pieces, Ron ate 24 ÷ 3 = 8 pieces
    So John ate 24 – 8 – 6 – 4 = 6 pieces
    Or, the others ate $\frac{1}{6} + \frac{1}{4} + \frac{1}{3} = \frac{3}{4}$, so John ate $\frac{1}{4}$ of 24 pieces = 6 pieces
15. Two people at the end + 2 people/table: 20 tables × 2 people/table + 2 people = 42 people
16. If the number of tables is x, then 2×x + 2 = 58. So x = (58 – 2) ÷ 2 = 28
17. x boys × R4/boy + y girls × R5/girl = R12 + R20. So 3 boys and 4 girls
18. To tape the top of the box she will need 420 mm + 230 mm + 420 mm + 230 mm = 1,300m
    To tape the bottom of the box she will need the same as for the top - 1,300m
    To tape the 4 sides she will need 270 mm × 4 = 1,080m
    All together it will be 1,300 m + 1,300 m + 1,080 m = 3,680 m
19. Around the box from the one way will be: 420 mm + 270 mm + 420 mm + 270 mm = 1,380 m
    Around the box from the other way will be: 230 mm + 270 mm + 230 mm + 270 mm = 1 m
    Plus the bow: 1,380 m + 1 m + 40 cm = 1,380 m + 1,000 m + 0,400 m = 2,780 m
20. 100 – (4 × 10) = 60
21. The smallest number cannot be 1 or 2, otherwise the product of the other two must be 90 or 45, which is too big.
    One of the numbers cannot be 4, otherwise the product of the other two must be 90/4, which is not a whole number.
    If the smallest number is 3, the product of the other two must be 30, so 5× 6 = 30. So the sum is 3 + 5 + 6 = 14
22. List them systematically:

3241	3421	3124	2341	2431	2134	4321	4231	4132	1324	1234	1432
3214	3412	3142	2314	2413	2143	4312	4213	4123	1342	1243	1423

23. Look at the *structure* in the pictures!
    T_1: 1 = 1×1
    T_2: 1 + 3 = 4 = 2×2
    T_3: 1 + 3 + 5 = 9 = 3×3
    T_4: 1 + 3 + 5 + 7 = 16 = 4×4
    ….
    T_{50}: 1 + 3 + 5 + 7 + ... to 50 numbers = 50×50 = 2500
24. Look at the *structure* in the pictures!

Height (h)	1	2	3	4	…	b
# blocks in bottom row (b)	1	3	5	7		2 × h – 1

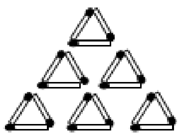
25. 1369 = 37×37

GRADE 5(F)

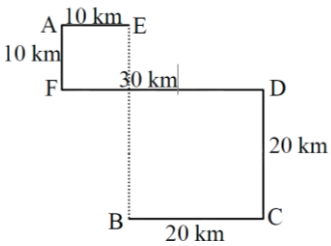
1. In the bottom layer there are $8 \times 4 = 32$ blocks, so in two layers there are 64 blocks
2. All the blocks of the bottom layer (32) and all the blocks round the side of the top layer (20)
3. The trip is 31 minutes, therefore 12:30 + 31 minutes → 13:01
5. $\frac{120}{360} = \frac{22}{?}$ So ? = 66 mm (The photo is enlarged 3 times)
6. If the loser had Δ votes, the winner had Δ + 1002 votes. Together $2\times\Delta + 1002 = 39218$
7. These are multiples of 6. Only $4182 = 6 \times 697$ is a multiple of 6
8. R5 less for you and R5 more for her is R10
9. $274 – 246 + 1 = 29$ (Check: if you read page 1 and 2, you have read 2 pages, not 2-1 = 1)
11. 41000 g – 725 g = 40275 g = 40,275 kg
12. Peter worked 3 out of the 6 hours, so he should get $\frac{3}{6} = \frac{1}{2}$ of R48 = R24
13. 99 m = $\frac{9}{10}$, so 11 m = $\frac{1}{10}$. Therefore $\frac{10}{10} = 10 \times \frac{1}{10} = 10 \times 11 = 110$ m
14. Consider the possible choices from the top row:
    If I choose 1, then the options are 1, 5, 9 or 1, 6, 8 giving products 45 or 48 respectively.
    If I choose 2, the options are 2, 4, 9 or 2, 6, 7 with products 72 or 84 respectively.
    If I choose 3, the options are 3, 4, 8 or 3, 5, 7 with products 96 or 105.
    So 105 is the maximum possible product.

1	2	**3**
4	**5**	6
7	8	9

15. Look at the *structure:*
    Length 3: $2\times3 + 2 = 8$
    Length 7: $2\times7 + 2 = 16$
    So for length 20: $2\times20 + 2 = 42$
16. If the length is x, then $2\times x + 2 = 220$. So $x = (220 – 2) \div 2 = 109$
17. From 1-9 you use 9 digits. From 10 to 99 you use $90 \times 2 = 180$ digits. For 100 you use 3 digits. 9 + 180 + 3 = 192
18. For 19 wheels we can have: 1 tricycle and 8 bicycles = total of 9 (too much)
    3 tricycles and 10 bicycles = total of 13 (too much)
    5 tricycles and 2 bicycles = total of 7 (just right)
19. Look at the *structure* in the pictures!
    T_1: $1 = 1\times1$
    T_2: $1 + 3 = 4 = 2\times2$
    T_3: $1 + 3 + 5 = 9 = 3\times3$
    ….
    T_{10}: 1 + 3 + 5 + 7 + ... to 10 numbers = 10×10 triangles
20. Look at the *structure* in the pictures!
    Count the number of *triangles:*
    # triangles in $T_1 = 1$
    # triangles in $T_2 = 1 + 2$
    # triangles in $T_3 = 1 + 2 + 3$
    # *triangles* in $T_{10} = 1 + 2 + 3 + 4 + ... + 9 + 10 = (1 + 10)\times10/2 = 55$
    So # *matches* = 55×3



21. They drink $\frac{1}{2} + (\frac{1}{3} \text{ of } \frac{1}{2}) = \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$ of the milk, so $\frac{1}{3}$ of the milk is left
22. Draw it! Fill in the information as you read. Re-read, bit by bit!

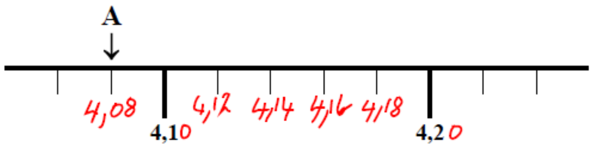


23. Be systematic, e.g.

32	23	43	13
34	24	42	12
31	21	41	14

24: Be systematic, e.g.
143, 142; 134, 132; 124, 123
413, 412; 431, 432; 421, 423
314, 312; 341, 342; 321, 324
214, 213; 241, 243; 231, 234
25. 99 – 36 = 63; 98 – 35 = 63; 97 – 34 = 63; 96 – 34 = 63; .... ; 73 – 10 = 63, so ...

GRADE 6(1)

1. Try trial and improvement, e.g. 50 + 52 + 54 ≠ 174; …. But 56 + 58 + 60 = 174
   Or test each of the given numbers …
   Or, if the smallest is x, then $x + (x + 2) + (x + 4) = 3 \times x + 6 = 174$, so $x = 56$
2. These are multiples of 8. 728 ÷ 8 = 91. All the others leave a remainder when divided by 8
3. These are 4 less than multiples of 8 (If you divide by 8 on a calculator, they give an answer of *.5). 724 ÷ 8 = 90 remainder 4. All the others leave other remainders
4. 

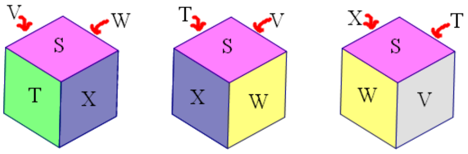
5. List them: 1, 2, 4, 5, 10, 20, 25, 50, 100
6. 5×2 = 10, so the last digit is 0
7. 25 + 20 + 30 + 15 + 35 = 125
8. Full lorry = 4653 kg; empty lorry = 2583 kg; 4653 – 2583 = 2070; 2070 kg ÷ 90 kg/bag = 23 bags
9. 1 × 2 – 1 = 1
   4 × 2 – 1 = 7
   7 × 2 – 1 = 13
   2 × 2 – 1 = **3**
11. Make a systematic list, e.g. 3579; 3597 | 3759; 3795 | 3957; 3975 | 9375; 9357 | 9537 …
    Or: He has 4 choices for the first number, then 3 choices for the second, 2 for the third and 1 for the fourth.
    So 4 × 3 × 2 × 1
12. ½ + 1/8 + 1/8 = ¾. So R15 is a ¼ of his money. So 4/4 of his money is 4 × R15 = R60
13. Trial and improvement: 30×31 = 930 is too small …. 35 × 36 = 1260 is too small …. 36 × 37 = 1332
14. 18 = 12 + 6 = 12 + ½ of 12. So $\frac{2}{3}$ + ½ of $\frac{2}{3} = \frac{2}{3} + \frac{1}{3} = 1$
15. 4 books = 2 books + 6 kg, so 2 books = 6 kg, so 1 book = 3 kg
16. May's R12 represents a 1/4 . Therefore Mark's 3/8 = ¼ + 1/8 is R12 + R6 = R18
17. Each number is the sum of the two numbers above it, e.g. 6 = 1 + 5, 15 = 5 + 10
18. Diana is 3 years older than Joe
    Joe is two years older than Cindy
    Diana is five years older than Cindy, so she was 8 + 5 = 13 years old
19. The smallest is 10 × 10 = 100. The largest, by guess-and-improvement = 31 × 31 = 961. So the total is 31−10+1=22
20. Mentally draw a straight line through the angle of each square the arrow is pointing at.
21. If her average score is 4,8, her total score is 5 × 4,8 = 24
    The first four judges gave her 4,5 + 4,6 + 4,7 + 5 = 18,8
    So the fifth judge gave her 24 – 18,8 = 5,2
22. Multiply all together: $(a \times b) \times (b \times c) \times (c \times d) \times (d \times a) = 20 \times 14 \times 35 \times 50 = 2 \times 2 \times 10 \times 10 \times 5 \times 5 \times 7 \times 7$
    So $a^2 \times b^2 \times c^2 \times d^2 = (a \times b \times c \times d)^2 = (20 \times 35)^2$
    So $a \times b \times c \times d = 700$
23. Find structure!

Whole numbers	1–9	10–19	20–29	30–39	40–49	50–54
# of digits	9	20	20	20	20	10

    So, up to 54, 9+20+20+20+20+10 = 99 digits are used. So the 100th digit would be **5**: … 4849505152535455
24. A bit of logic! "Only one statement is true" is the same as "four of the five statements are false"!
25. Look at the *structure* in the pictures!
    V_1: 3 = 2×**1** + 1
    V_2: 5 = 2×**2** + 1
    V_3: 7 = 2×**3** + 1
    ….
    V_{50}: = 2×**50** + 1 tiles

GRADE 6(F)

1. Form a mental picture!



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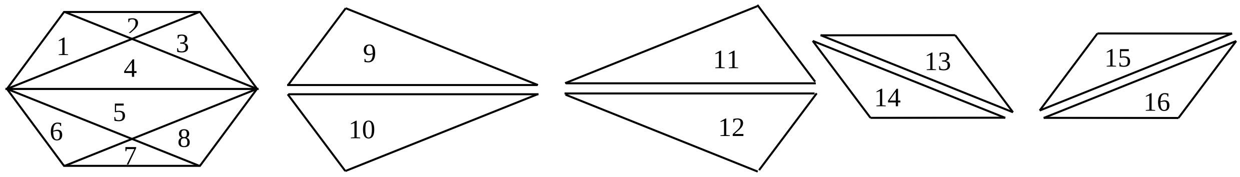
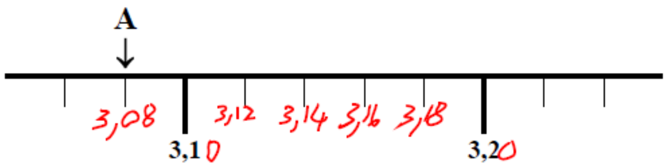


3. (C) $3\times3 - 3 + 3 = 9 - 3 + 3 = 9$

4.



5. List them systematically: 1, 2, 4, 5, 10, 20, 25, 50, 100
6. $4002 \div 4 = 1000$ rem 2
7. Count equal parts!
8. $\frac{5}{6} = \frac{40}{48}$ and $\frac{7}{8} = \frac{42}{48}$, so $\frac{40}{48} < \frac{41}{48} < \frac{42}{48}$
9. *Look* at the structure: For *n* dice, the number of visible faces is $n\times3 + 2$. If 50 faces are visible, $n\times3 + 2 = 50$, so n = 16
10. From half to full in 1 minute. So, after 59 minutes it was half-full
11. Write all the fractions as 1000ths: $\frac{399}{1000}$; $\frac{398}{1000}$; $\frac{410}{1000}$; $\frac{420}{1000}$; $\frac{300}{1000}$

    $\frac{2}{5} = \frac{400}{1000}$, so $\frac{399}{1000}$ is closest to $\frac{2}{5}$
12. ½ + 1/8 + 1/8 = 6/8 = ¾

    R15 = ¼ . So pocket money = 4 × R15 = R60
13. If the book costs R*x*, the CD costs R*x* + 60. Together they cost $x + x + 60 = 230$

    So $x = (230 - 60) \div 2 =$ R85
14. There is a general structure here: The denominators is twice the numerator + 1, i.e. $\frac{\lozenge}{2\times\lozenge+1}$

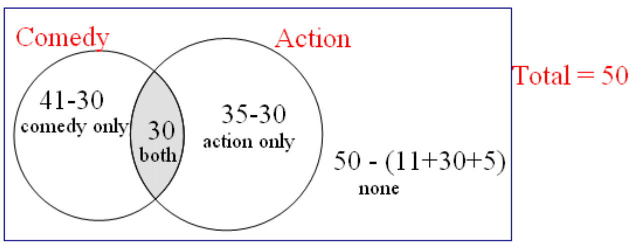
    We can therefore investigate a general pattern $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \frac{6}{13}, \frac{7}{15}, \ldots$

    Check with your calculator: $\frac{1}{3} = 0,333...$, $\frac{2}{5} = 0,4$, … So $\frac{1}{3} < \frac{2}{5} < \frac{3}{7} < \frac{4}{9} < \frac{5}{11} < \frac{6}{13} < \frac{7}{15} < \ldots$

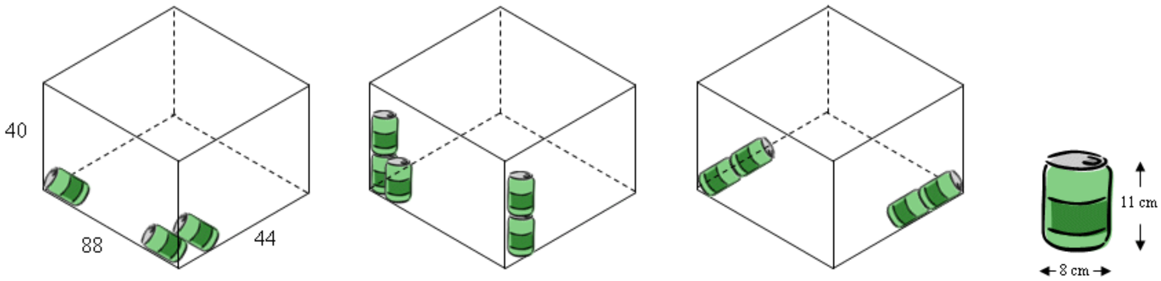
    Conclusion: the larger the denominator, the larger this kind of fraction, so $\frac{11}{23}$ is the largest
15. Trial and improvement: $30\times31 = 930$ is too small …. $35 \times 36 = 1260$ is too small …. $36 \times 37 = 1332$
16. Structure! $50\times51 = 2550$

P_1	P_2	P_3	P_4	…	P_{50}
1×2	2×3	3×4	4×5	…	?

17. Make a representation of the situation (draw it):



18. Consider different possible packings:



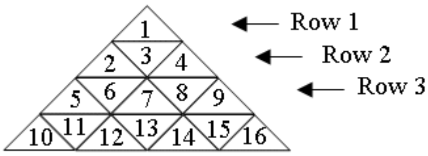
The number of cans in each of these packings is:
8×5×5 = 200 11×5×3 = 165 11×4×5 = 220

19. $\frac{4}{6} = \frac{6}{?}$ So ? = 9 cm

20. Work systematically!
101, 111, 121, 131, 141, 151, 161, 171, 181, 191 – this is 10
202, 212, 222, 232, 242, 252, 262, 272, 282, 292 – this is 10
….
909, 999, 929, 939, 949, 959, 969, 979, 989, 999 – this is 10
So 9×10 = 90

21. Investigate the *structure* by finding a pattern in special cases*:*

Row number	1	2	3	4		n
Number of numbers	1	3	5	7		$2\times n - 1$



22. In question 23 you see that the last number in Row 49 is 49×49
So the first number in Row 50 is 49×49 + 1 = 2402

23. Investigate the *structure* by finding a pattern in special cases*:*

Row number	1	2	3	4		n
Last number	1	4	9	16		$n\times n$

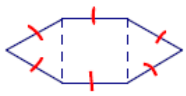
24. You can try to find a formula for the pattern 1, 3, 7, 13, ...
But note that the middle number is the *average* of the first and last numbers. So (2402+2500)/2 = 2451

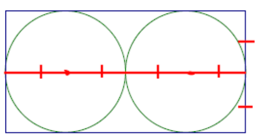
25. Investigate the *structure* by finding a pattern in special cases*:*

Row number	1	2	3	4		n
Sum	1	9	35	91		
	1x1	3x3	5x7	7x13		$(2\times n\text{-}1) \times (n^2\text{-}n\text{+}1)$

But note that the sum is the middle number × number od numbers – remember, the middle number is the average!
So 2451×99 = 242649
Or you can use the Gauss method: (First + Last) × Number of numbers ÷ 2, so (2402+2500) × 99 ÷ 2 = 242649

GRADE 7(1)

1. Use trial and improvement to find that only 26 × 27 = 702. So 26 + 27 = 53
2. 3×3–3+3 = 9
3. 

   6 × 2 = 12
4. 

   Length × Breadth = 8 × 4 = 32
5. $1+\frac{1}{1+\frac{2}{3}} = 1+\frac{1}{\frac{5}{3}} = 1+\frac{3}{5}$

6. There is a general structure here: The denominators is twice the numerator + 1, i.e. $\frac{\lozenge}{2\times\lozenge+1}$

We can therefore investigate a general pattern $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \frac{6}{13}, \frac{7}{15}, \ldots$

Check with your calculator: $\frac{1}{3}$ = 0, 333..., $\frac{2}{5}$ = 0,4, … So $\frac{1}{3} < \frac{2}{5} < \frac{3}{7} < \frac{4}{9} < \frac{5}{11} < \frac{6}{13} < \ldots$

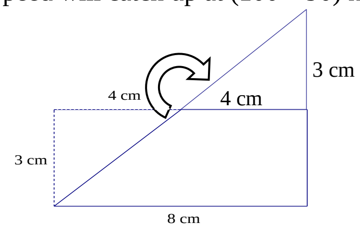
Conclusion: the larger the denominator, the larger this kind of fraction, so $\frac{11}{23}$ is the largest

7. We know: $\frac{\text{Sum of numbers}}{5} = 30$, so Sum of numbers = 30 × 5 = 150

If she erases the number x, then $\frac{150 - x}{4} = 28$. So x = 150 –112 = 38

8. Speed will catch up at (100 – 90) km/h = 10 km/h, so he will catch up 30 km in 3 hours

9.



10. The area is of course the same as the area of the original rectangle: 3 cm × 8 cm = 24 cm^2
11. 21 horizontals + 21 verticals = 42 lines
12. 64 = 8 × 8 = P_8, so 9 + 9 = 18 lines
13. 64 lines means it is P_{31}, which has 31 × 31 = 961 squares
14. Make a list, varying the numbers systematically. If the digits are a, b, c and d:

abcd, abdc, acbd, acdb, adbc, adcb and similarly if the first digit is b, c, and d. So 6 × 4 = 24. Or 4 × 3 × 2 × 1 = 24

15. 1/3 + (1/4 of 2/3) = 4/12 + 2/12 = 6/12 ; 24 = 6/12 ; Penny originally had 48 marbles; 1/3 of 48 = 16
16.

	c	d
a	**12**	**20**
b	**21**	D

Using a representation like this, Area D = b × d

We know a × c = 12, b × c = 21, a × d = 20

Multiply them all together: $a^2 \times c^2 \times b{\times}d$= 12 × 20 × 21

But a×c = 12, so $a^2{\times}c^2$ = 144, so b×d = 12×20× 21 ÷ 144 = 35 = Area D

17. Let the width be w metres, then the length is 4×w metres, so the Perimeter = 10 ×w = 100, so w = 10

So the area is length × width = 40 metres × 10 metres= 400 m^2

18. The 6$^{\text{th}}$ column is given by 6 × row n.

So the last number in row 80 is 6 × 80 = 480. Then row 81 is 481, 482, 483, …

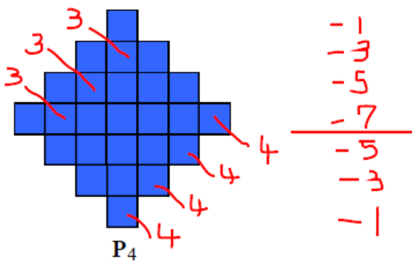
19. 4321 ÷ 6 = 720 remainder 1; so 721$^{\text{st}}$ row
20. From 1, 3, 5, 7, to 99 are 50 numbers, so 1+3+5+ … +97+99 = 50 × 50 = 2500
21. The first digit can be 1, 3, 5, 7, 9. The second digit can be 1, 3, 5, 7, 9, which gives 5 × 5 = 25 possible combinations
22. If the cost of a drink is D rands and the cost of an ice-cream is C rands:

2D + 1C = 15 (1)
1D + 2C = 12
Add: 3D +3C = 27
÷3: 1D +1C = 9 (2)

23. In 22 above: (1) – (2): 1D = R15 – R9 = R6
24. Let the weights be M, D and C kilograms. Then M+D = 90, M+C = 70, D+C = 40 kg.

Add all together: 2×M + 2×D + 2×C = 200
So M + D + C = 100

25. Find structure!



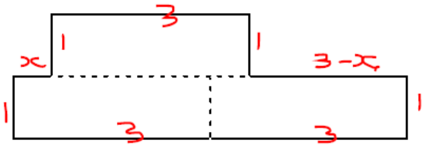
P_4

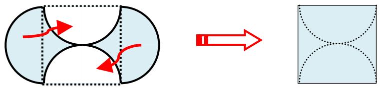
One structure for P_4 is 4×4 + 3×3. Generalise, then P_{50} = 50×50 + 49×49

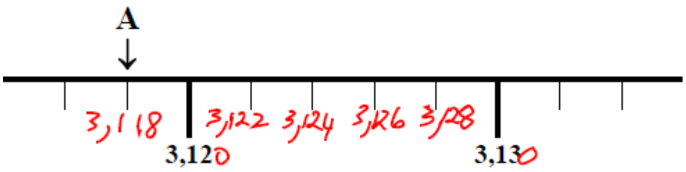
Another structure for P_4 is (1+3+5+7) + (1+3+5) = 4×4 + 3×3 …..

GRADE 7(F)

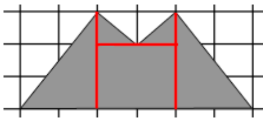
1. Redraw it and fill in the measurements!



2.  So 6 cm × 6 cm = 36 cm^2

3. Average = Total mass ÷ number of children = (3×75 + 6×66) kg ÷ 9 children = 69 kg/child
4. The "vertical" formula is $2×a + 2$. Find a so that $2×a + 2 = 64$
   Or the "horizontal" formula is $4 + 2×(a – 1)$, so find a so that $4 + 2×(a – 1) = 64$
5. If the price without VAT is Rx, then 1,14 × x = 36,15. So x = 31,71
6. 

7. Use fact that the area of a triangle is half of area of a rectangle …



8. (2 + 4+ 6 + 8 + … + 98 + 100) – (1 + 3 + 5 + … + 97 + 99)
   = (2 – 1) + (4 – 3) + (6 – 5) + … + (98 – 97) + (100 – 99)
   = 1 + 1+ 1 + 1 + … 50 times = 50
9. If Sandy is x years old, Mandy is $x + 7$ years old. In 4 years time Sandy is $x + 4$ years old and Mandy is $x + 11$. Then $x + 4 = ½ × (x + 11)$. So x, i.e. Sandy is 3 and Mandy is 10. 3 + 10 = 13
10. If the length of each rectangle is x, and the width is y:
    $3 × y = 2 × x$ and $x + y = 15$
    So $2x + 2y = 30$, so $3y + 2y = 30$, so $5y = 30$, so $y = 6$, and $x = 9$.
    Area of 1 rectangle is 9×6 = 54 cm^2, so the area of 5 rectangles is 5×54 cm^2
11. If the dimensions of the room is a by b by c, then the area to paint is A = $2ab + 2ac + 2bc$
    Double the dimensions are $2a$ by $2b$ by $2c$, so the area to paint is D = $2(2a)(2b) + 2(2a)(2c) + 2(2b)(2c)$ = 4×A
12. Test all the cases systematically: 1 × 17 = 17; 2 × 16 = 32; 3 × 15 = 45; … 9 × 9 = 81, then the answer repeats, because the order does not matter (e.g. 2 × 16 = 16 × 2).
13. List them systematically: 1, 2, 4, 5, 8, 10, 16, 20, 25, 40, 50, 80, 100, 125, 200, 250, 400, 500, 1000, 2000
14. Volume = 15 cm × 8 cm × x cm = 120 cm^3, so x = 1. So area is (15 cm + 2 cm) × (8 cm + 2 cm) = 17 cm × 10 cm
15. In middle row the missing number is 18 – (11 + 6) = 1, so in right column z = 18 – (1 + 10) = 7
16. $p×1×\frac{1}{8} = 1$, so $p = 8$

    $q×1×4= 1$, so $q = \frac{1}{4}$

    $u×4×\frac{1}{8} = 1$, so $u = 2$

    $p×s× u = 1$, so $8× s×2= 1$, so $s = \frac{1}{16}$

    $r + s = \frac{1}{2} + \frac{1}{16} = \frac{9}{16}$

p	q	r
s	1	t
u	4	$\frac{1}{8}$

17. List them systematically: 799; 979; 997; 889; 898; 988
18. You can draw it, or investigate numerical patterns for a triangle, square, pentagon, hexagon, etc.
    Or you can reason it out: At each vertex of an n-gon there are n–3 diagonals because the point is connected to every other point, except to itself and to the two adjacent points (these are *sides* of the n-gon). So at n vertices there are $n × (n–3)$ diagonals, counted twice. So the formula is $D(n) = n × (n–3) ÷ 2$, so $D(8) = 8 × (8 – 3) ÷ 2$
19. D(80) = 80× (80 – 3) ÷ 2 = 3080
20. Look for structure, a clever way of counting, e.g. every point is connected to every other point except itself, so at each of the 18 points on the circle there are 17 lines, in total 18×17. But each line is counted twice, so 18×17 ÷ 2

21. Multiply them all together: $(a \times b) \times (b \times c) \times (c \times a) = 2 \times 24 \times 3 = 144$
Transform: $(a \times b \times c)^2 = 12^2$, so $a \times b \times c = 12$
$a \times b \times c = 12$, and $b \times c = 24$, so $a \times 24 = 12$, so $a = \frac{1}{2}$
$a \times b \times c = 12$, and $a \times c = 3$, so $3 \times b = 12$, so $b = 4$
$a \times b \times c = 12$, and $a \times b = 2$, so $2 \times c = 12$, so $c = 6$

22. Look for structure! $T_{50} = 1 + 2 + 3 + 4 + \ldots + 49 + 50 = (1+50) + (2+49) + \ldots + (25+26) = 25 \times 51 = 1275$

23. Look for structure and pattern!
$N_1 = 2 = 1 \times 1 + 1$
$N_2 = 5 = 2 \times 2 + 1$
$N_3 = 10 = 3 \times 3 + 1$
$N_4 = 17 = 4 \times 4 + 1$
Test the numbers! $30 \times 30 + 1 = 901$ is the only one which fits the pattern.

24. If they mine 5%, then 95% = 0,95 is left. So:
After 1 year, 95% is left
After 2 years, 95% of 95% = $0{,}95 \times 0{,}95$ is left
After 3 years, 95% of 95% of 95% = $0{,}95 \times 0{,}95 \times 0{,}95 = 0{,}95^3$ is left
After 10 years, $0{,}95^{10}$ is left. Use a calculator: $0{,}95^{10} = 0{,}598 = 59{,}8\%$ is left
After 13 years, $0{,}95^{13}$ is left. $0{,}95^{13} = 0{,}513 = 51{,}3\%$, more than half, is left
After 14 years, $0{,}95^{14}$ is left. $0{,}95^{14} = 0{,}487 = 48{,}7\%$, less than half, is left

25. $(1 + 1) \times (1 + \frac{1}{2}) \times \ldots \times (1 + \frac{1}{100}) = \frac{2}{1} \times \frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \ldots \times \frac{100}{99} \times \frac{101}{100}$

$$= \frac{2 \times 3 \times 4 \times 5 \times \ldots \times 99 \times 100 \times 101}{2 \times 3 \times 4 \times 5 \times \ldots \times 99 \times 100}$$

$$= 101$$