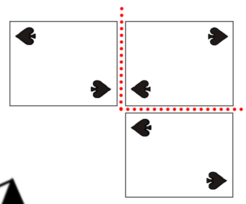
NOTES ON 2009 MEMORANDUM

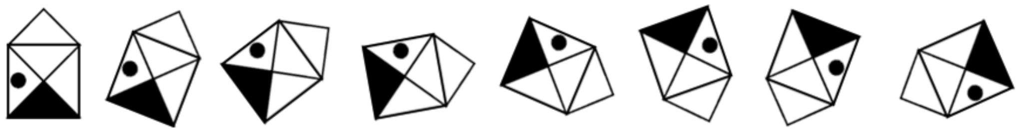
These notes are necessarily brief and often formal and symbolic.
Many questions could be answered using primitive methods, e.g. "If today is Wednesday, what day of the week will it be 100 days from now?" can be done by counting. That would be laborious, time-consuming and error-prone. The essence of a mathematical approach is to work more smartly by using appropriate representations to model the situation and to exploit the inherent structures and patterns in the situation.

GRADE 4(1)

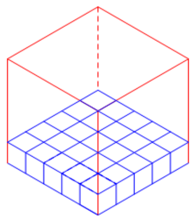
1. 5 + 3 = 8, while the others all have a sum of 7
2. Half of 8 × 8
4. Digital, 24 hour clock … it is in the evening, so 22:10
5. R35 ÷ 4 = (R32 ÷ 4) + (R3 ÷ 4) = R8 + R0,75 = R8,75
6. There is a pattern of +14, +14, +14 in the numbers
7. $\frac{3}{4} \div \frac{1}{8} = \frac{6}{8} \div \frac{1}{8}$. How many $\frac{1}{8}$ are there in $\frac{6}{8}$?
8. B is a mirror-image in a horizontal or vertical line of symmetry, as shown



9.

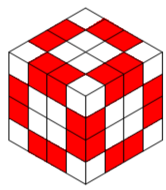


10. 20min + 15min = 35min. 08: 30 – 00:30 = 08:00 – 00:05 = 07:55
11. 35 000 mℓ ÷ 35 mℓ = 100
13. Investigate the *structure* by finding a pattern in special cases*:*



Stack number	1	2	3	4		*n*
Number of blocks	1	4	9	16		
	1×1	2×2	3×3	4×4		*n*×*n*

14. 5 small cubes to a side. So 5×5 in bottom layer, with 5 layers, so 5×5×5
15. 274 – 246 + 1 = 29
    Check – if you read page 1 and 2, you have read 2 pages, not 2–1 = 1 page



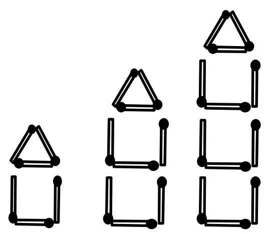
16. 8 cubes on each of the 6 sides. But then they are all counted twice! So 4×8 ÷ 2
17. Bottom level: 3 × 3 = 9 blocks, second level has 1 less = 8 blocks, top level has 5 blocks
18. $x - 4 + 5 - 6 = 3$, so $x - 5 = 3$ so $x = 8$
19. 2 × T + 2 = 38, so T = (38 – 2) ÷ 2 = 18
20. 120 km in 60 min, so 20 km in 10 min, so 200 km in 100 min, so the time is 11:40
21. 7, 17, 27, 37, … 77 (two!), 87, 97 is 11, plus 70, 71, 72, … ~~77~~, 78, 79 is another 9, so 20
22. M + (M + 30) = 114, so 2 × M = 84, so Monde weighs 42 kg
23. Investigate the *structure* by finding a pattern in special cases*:*

Row number	1	2	3	4		*n*
Number of triangles	1	3	5	7		2×*n* – 1

    So in Row 50 there are 2×50 – 1triangles
24. Look at the *structure* in the picture!
    First row: 1 = 1×1
    First 2 rows: 1 + 3 = 4 = 2×2
    First 3 rows: 1 + 3 + 5 = 9 = 3×3
    ….
    So, in 50 rows: 1 + 3 + 5 + 7 + ... to 50 numbers = 50×50 triangles
25. Investigate the *structure* by finding a pattern in special cases*:*

Number of storeys	2	3	4	5		*n*
Number of matches	6	9	12	15		3 × (*n* +1)

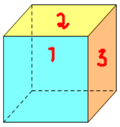
    So, for 10 storeys Thandi needs 3×11 matches



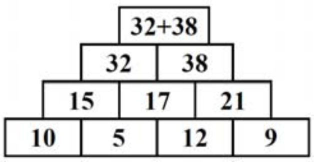
n=1 *n*=2 *n*=3

GRADE 4(F)

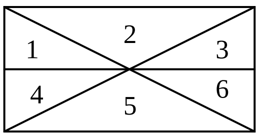
1. $\frac{1}{8} < \frac{1}{4} < \frac{1}{2}$
2. Check each answer, e.g. $9 = 3 \times 3$; $11 = 2 \times 3 + 5$; $13 = 2 \times 5 + 3$, etc. You know, or you learn from these calculations, that the sum of three odd numbers is odd, so 12 is not possible!
3. Straighten the string. Two loops of 1 cm make it 5 cm + 1 cm + 1 cm = 7 cm
4. 3 hours before 16:45 is 13:45, so 2 hours and 55 minutes (5 min less) is at 13:50
5. The watch gains 2 minutes every day (24 hours) for 7 days = 2 min/day× 7 days = 14 minutes
6. $10 \times 10 \times 10 = 1000$ or from 001 to 999 gives 999 combinations, plus 000 gives 1000
9. Start "painting" (numbering) the sides …



10. With 6 loose cubes, there would be 36 faces. Subtract the 10 non-visible faces ...
11. If Zuki has ♥ marbles, Zinkle has ♥ – 15. Together they have 2×♥ – 15 = 95 marbles. So ♥ = 55
12. The numbers must be different, so $99 + 98 + 97 = (100 - 1) + (100 - 2) + (100 - 3) = 300 - 6$
13. $\frac{3}{4} + \frac{3}{4} \rightarrow 1\frac{1}{2} + \frac{3}{4} \rightarrow 2\frac{1}{4} + \frac{3}{4} \rightarrow 3 + \frac{3}{4} \rightarrow 3\frac{3}{4} + \frac{3}{4} \rightarrow 4\frac{1}{2}$

    ① ② ③ ④ ⑤ ⑥
14. A rings on the hour and half-hour. B rings at 08:00, 08:35, 09:10, 09:45, 10:20, 10:55 and **11:30**
15. If the 21th is a Monday, then also the 14th, 7th and 0th are Mondays.
    The 0th is the last day of the previous month, so the next day is the 1st of this month, so it is a Tuesday
16. 

17. 6 and 4 is even, 6 is bigger than 4, 6 + 4 is a multiple of 5
18. For 6 milktarts she needs 8 cups of milk, so for 8 ($6 + 2 = 6 + \frac{1}{3}$ of 6) milktarts she needs $8 + \frac{1}{3}$ of $8 = 8 + \frac{1}{3}$ of $(6 + 2) =$

    $8 + \frac{1}{3}$ of $6 + \frac{1}{3}$ of $2 = 8 + 2 + \frac{2}{3} = 10\frac{2}{3}$ cups of milk.
19. With 15 eggs (9 + 6) you can make $6 + \frac{2}{3}$ of 6 milktarts = 6 + 4 = 10 milktarts
20. 

    Invent some notation and count systematically, e.g.:
    Areas 1, 2, 3, 4, 5 and 6 each form a triangle (6)
    Two areas 1-4 and 3-6 each form a triangle (2)
    Three areas 4-1-2, 2-3-6, 3-6-5 and 5-4-1each form a triangle (4)
21. $\frac{1}{5} = \frac{8}{40}$ and $\frac{1}{4} = \frac{10}{40}$, so $\frac{8}{40} < \frac{9}{40} < \frac{10}{40}$
22. Through systematic elimination, e.g.
    A in the top row must be 1, 8 or 6. But A in the right column cannot be 8 or 1, so A is 6
    B in the bottom row must be 9, 4 or 2. But A in the right column cannot be 9 or 4, so B is 2
    C in the top row must be 1 or 8. But C in the left column cannot be 1, so C is 8. So E is 1
    D in the bottom row must be 9 or 4. But D in the left column cannot be 4, so D is 9. So F is 4
    We only have 3, 5 and 7 left. But G cannot be 3 or 7, so G = 5. H cannot be 3, so H = 7 and X = 3

C	E	A
G	X	H
D	F	B

23. Let the children be A, B, C, D and E. List all the possibilities systematically, note patterns and structure:

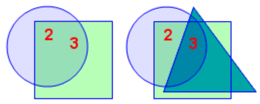
A vs B	**B** vs C	**C** vs D	**D** vs E
A vs C	B vs D	C vs E	
A vs D	B vs E		
A vs E			

vs	A	B	C	D	E
A		X	X	X	X
B			X	X	X
C				X	X
D					X
E					

A vs B	**B** vs C	…	**H** vs I	**I** vs J
A vs C	B vs D	…	H vs J	
A vs D	B vs E			
A vs E	B vs F			
A vs F	B vs G			
A vs G	B vs H			
A vs H	B vs I			
A vs I	B vs J			
A vs J				

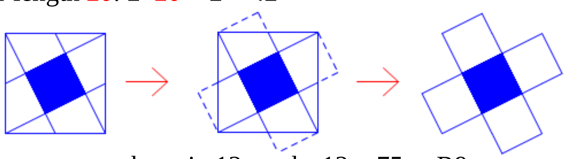
24. (see above) The structure is: 9 + 8 + ... + 2 + 1 = **45**
25. The structure is 1 + 2 + 3 + 4 + 5 + 6 + ... + 48 + 49 + 50 = (1+50) + (2+49) + (3+48) + ... = 51 × 25

GRADE 5(1)



1. There are 5 tiles in every metre because 1000 cm ÷ 200 cm = 5. So 15 × 10 = 150 tiles
2. The numbers inside the square *and* the circle are 2 and 3.
   2 is not inside the triangle
3. Check systematically: 360÷1 = 360, 360÷2 = 180, …, 360÷6 = 60…. But 360÷7 =51.428 …
4. C – a rotation to the right through 90°
5. 4 reds – 10 greens – 3 purples. So 12 (3×4) reds – 9 (3×3) purples
6. n^{th} row has $2\times n - 1$ dots, so 7^{th} row has $2\times 7 - 1 = 13$ dots
7. n^{th} row has $2\times n - 1$ dots, so 70^{th} row has $2\times 70 - 1 = 139$ dots
8. 100 ÷ 24 = 4 rem 4, i.e. 4 full days bringing us to 10:00, plus 4 more hours, i.e. 11, 12, 13, 14:00
   Or 10 + 100 = 110, 110 ÷ 24 = 6 rem 14
9. *Height* = 12 cm + 1,5 cm/day × *days*. So *Height after 30 days* = 12 + 1,5×30 = 57 cm
10. (150 cm – 12 cm) ÷ 1,5 cm/day = 92 days
11. One more than a multiple of 6, so it is odd, so it cannot be A or B. Test the others: 4 182÷6 = 697
12. If a sack weighs S kg, then $3S = S + 30$, so $2S = 30$, so $S = 15$. So $3S = 45$ kg
13. Investigate the *structure* by finding a pattern in special cases*:*

Pattern number	1	2	3	4		n
Number of coins	3	6	9	12		$3\times n$

14. Mathematics is $\frac{1}{4}$ of his time, and this is 2 hours. So $\frac{4}{4}$ of his time is 4×2 hours = 8 hours
15. In the bottom layer there are 8 × 4 = 32 blocks, so in two layers there are 64 blocks
16. All the blocks of the bottom layer (32) and all the blocks round the side of the top layer (20)
17. Look at the *structure:*
    Length 3: 2×3 + 2 =8
    Length 7: 2×7 + 2 = 16
    So for length 20: 2×20 + 2 = 42
18. 
19. 75c more per week, so in 12 weeks 12 × 75c =R9
20. You can it with 1, 2 or 3 shots …
    18 = 7 + 11
    20 = 9 + 11
    23 = 7 + 7 + 9
    27 = 9 + 9 + 9
21. The number must start and end with 1 so list them systematically:
    101 111 121 131 141 151 161 171 181 191
22. Share 30 litres in ratio 5 to 1, i.e. 25 to 5
23. If Penny has p coins and Alex has a coins:
    $p = 2\times a$, $p - 4 = a + 4$, so $2\times a - 4 = a + 4$, so $a = 8$, so $p = 16$, so $p + a = 24$

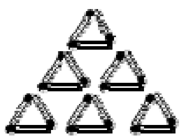
24. List them systematically:
    4000
    3100, 3010, 3001
    2200, 2020, 2002
    2110, 2101, 2011
    ~~2020, 2002. 2011~~
    1300, 1030, 1003
    1210, 1201
    1120, 1102
    1111
    ~~1030, 1003~~
    1021, 1012
25. If a small pizza costs *s* rands and a large pizza costs *L* rands: $2s + 1L -= 5s$, so $1L = 3s$, so the cost is $L = 3 \times$R11,50 = R34,50

GRADE 5(F)

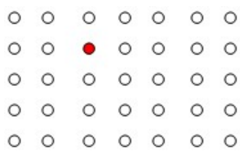
1. First pick up 7 then 1 then 6 etc
3. Build a mental picture! B, E & F
4. The trip is 31 minutes, so 12:30 + 31 minutes = 13:01
5. Use trial and error, i.e. try each of the given answers one by one
6. If the loser had Δ votes, the winner had $\Delta + 1002$ votes. Together $2\times\Delta + 1002 = 39218$
7. $(2 - 1) + (3 - 2) + (4 - 3) + \ldots + (100 - 99) + (101 - 100) = 1 + 1 + 1 + 1 + \ldots$ 100 times = 100
8. R5 less for you and R5 more for her is R10
9. $274 - 246 + 1 = 29$ (Check: if you read page 1 and 2, you have read 2 pages, not 2-1 = 1)
10. $3 + 3 + 1 + 3 + 3 + 1 + 3 + 1 + 3 + 3 = 24$cm
11. 41000 g – 725 g = 40275 g = 40,275 kg
12. Divide 420 into 7 equal parts:$420 \div 7 = 60$. 3 of these parts are dresses, i.e. $3 \times 60 = 180$
13. 99 m = $\frac{9}{10}$, so 11 m = $\frac{1}{10}$. Therefore $\frac{10}{10} = 10 \times \frac{1}{10}$ = $10 \times 11 = 110$ m
14. Consider the possible choices from the top row:
    If I choose 1, then the options are 1, 5, 9 or 1, 6, 8 giving products 45 or 48 respectively.
    If I choose 2, the options are 2, 4, 9 or 2, 6, 7 with products 72 or 84 respectively.
    If I choose 3, the options are 3, 4, 8 or 3, 5, 7 with products 96 or 105.
    So 105 is the maximum possible product.

1	2	**3**
4	**5**	6
7	8	9

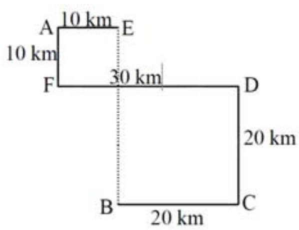
15. $T_n = 3 \times n + 1$
16. $X_n = 4 \times n + 1$
17. The first three may be blue, red and brown. Then the next one must match one of these colours
18. You can maybe take out, e.g. 10 red, then 10 brown, then 1 blue, then the next one is also blue
19. Look at the *structure* in the pictures!
    T_1: $1 = 1\times1$
    T_2: $1 + 3 = 4 = 2\times2$
    T_3: $1 + 3 + 5 = 9 = 3\times3$
    ….
    T_{10}: $1 + 3 + 5 + 7 + \ldots$ to 10 numbers = 10×10 triangles
20. Look at the *structure* in the pictures! Count the number of *triangles:*
    # triangles in $T_1 = 1$
    # triangles in $T_2 = 1 + 2$
    # triangles in $T_3 = 1 + 2 + 3$
    # *triangles* in $T_{10} = 1 + 2 + 3 + 4 + \ldots + 9 + 10 = (1 + 10)\times10/2 = 55$
    So # *matches* = 55×3



21. Make a sketch of the situation!
    "2nd from front, 4th from back" means there are 5 rows. "3rd from left, 5th from right" means there are 7 learners per row. So 7 learners/row × 5 rows = 35 learners



22. Draw it! Fill in the information as you read. Re-read, bit by bit!

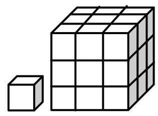


23. Be systematic, e.g.	32	23	43	13
	34	24	42	12
	31	21	41	14

24. Be systematic, e.g.
143, 142; 134, 132; 124, 123
413, 412; 431, 432; 421, 423
314, 312; 341, 342; 321, 324
214, 213; 241, 243; 231, 234
25. Debbie is first, Peter is second, Tom is third and Robert is fourth.

GRADE 6(1)

1. Make equal parts. Each small square is half of the next bigger square
So half of half of the big square is a quarter of the big square
2. There are 8 columns, each with 2 + 4 + 6 cubes. So 8 × 12 = 96 cubes
3. In middle row the missing number is 18 – (11 + 6) = 1, so in right column A = 18 – (1 + 10) = 7
5. $\frac{1}{7} = \frac{5}{35}$ and $\frac{1}{5} = \frac{7}{35}$ so $\frac{5}{35} < \frac{6}{35} < \frac{7}{35}$. Or $(\frac{1}{5} + \frac{1}{7}) \div 2 = (\frac{7}{35} + \frac{5}{35}) \div 2 = \frac{6}{35}$
7. Use trial and error, i.e. try each of the given answers one by one
8. Continue the patterns: 17, 22, 27, 32, 37, 42, 47, 52, … and 17, 24, 31, 38, 45, 52, …
9. For n dice, the number of visible faces is n×3 + 2. So for 75 dice, 75×3 + 2 = 27
10. Imagine or draw the cube! If the side is 3 times as long, the big cube contains 27 of the small cubes.
So its mass is 27 times as large!



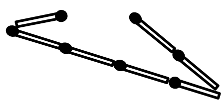
11. 0×20 + 3×10 + 1×5
1×20 + 1×10 + 1×5
12. 102 ÷ 7 = 14 remainder 4, so adding 3, we have 105 ÷ 7 = 15
14.

B	C	M
A	2	N
1	D	

In the middle row, N cannot be 2, so N is 1 or 3
Suppose N = 3. Then A = 1 which is impossible (already a 1 in left column).
So N = 1, A = 3. In left column B = 2. Then C = 1 (D ≠ 1), so M = 3, so M+N = 4
15. 3 × (1 + 2 + 3) = 18
16. Vary the possibilities systematically. First note that she could not draw 1, 3 or 5 games, otherwise her total would be a fraction. If she drew 6 games her total was $6\times\frac{1}{2} = 3$. If she drew 4 and won 2 her total was $2\times1 + 4\times\frac{1}{2} = 4$. If she drew 2 and won 4 her total was $4\times1 + 2\times\frac{1}{2} = 5$
17. Vary the numbers systematically and note the behaviour of the product of the numbers:

1 + 17 = 18 and 1 × 17 = 17	6 + 12 = 18 and 6 × 12 = 72
2 + 16 = 18 and 2 × 16 = 32	7 + 11 = 18 and 7 × 11 = 77
3 + 15 = 18 and 3 × 15 = 45	8 + 10 = 18 and 8 × 10 = 80
4 + 14 = 18 and 4 × 14 = 56	**9 + 9 = 18 and 9 × 9 = 81**
5 + 13 = 18 and 5 × 13 = 65	10 + 8 = 18 and 10 × 8 = 80

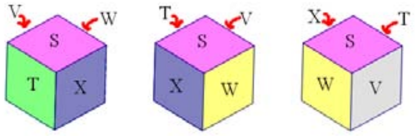
18. ? = 000ΔΔΔ = 0ΔΔΔ + $\frac{1}{2}$(0000ΔΔ) = 6□ + 4□ from first two balances



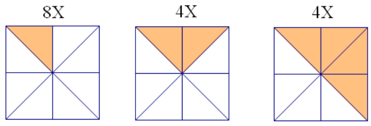
19. 3, 3, 1 and 3, 2, 2 (the sum of any two sides must be greater than the third side – why?)
20. If the numbers are x and y: 6×x + y = 17. So 17 – y must be a multiple of 6, i.e. 12, so y = 5
21. In each case the remainder is 2 less than the divisor. So if we add 2 to the number, it is divisible by 3, 4, 5 and 9.
3 × 3 × 4 × 5 = 180 is the smallest number divisible by 3, 4, 5, and 9. So my number is 178
22. If the empty glass has a mass of g gram and milk m gram, then $g + m = 370$ and $g + \frac{1}{2}m = 290$.
So $\frac{1}{2}m = 370 - 290 = 80$ grams, so m = 160 grams and g = 370 – 160 = 210 grams
23. Each number is the sum of the two numbers above it, e.g. 6 = 1 + 5, 15 = 5 + 10
24. If a bubble gum cost B cents and a chocolate costs C cents:
$B + C = 90$ and $10B + 5C = 470$, so $5B + 5(B + C) = 470$, so $5B + 5\times90 = 470$, so $B = 4$, so C = R0,86
25. 1, 1+3, 1+3+5, … = 1, 4, 9, … = 1×1, 2×2, 3×3, … So *Pattern 20* has 20×20 cubes

<u>**GRADE 6(F)**</u>

1. Form a mental picture!

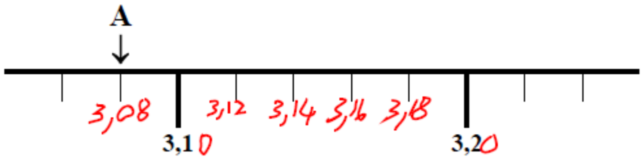


2. List the triangles systematically - *notation* and a *system* will help!



3. Try trial and error, e.g. 8 + 9 + 10 + …
   Or test each of the given numbers …
   Or, if the smallest is x, then $x + (x + 1) + (x + 2) + \ldots + (x + 6) = 7 \times x + 21 = 63$, so $x = 6$

4.



5. List them systematically: 1, 2, 4, 5, 10, 20, 25, 50, 100
6. 4002 ÷ 4 = 1000 rem 2
7. Count equal parts!
8. $\frac{5}{6} = \frac{40}{48}$ and $\frac{7}{8} = \frac{42}{48}$, so $\frac{40}{48} < \mathbf{\frac{41}{48}} < \frac{42}{48}$
9. Vary the numbers systematically and note the behaviour of the product of the numbers:

1 + 29 = 30 and 1 × 29 = 29	9 + 21 = 30 and 9 × 21 = 309
2 + 28 = 30 and 2 × 28 = 46	10 + 20 = 30 and 10 × 20 = 200
3 + 27 = 30 and 3 × 27 = 81	11 + 19 = 30 and 11 × 19 = 209
4 + 26 = 30 and 4 × 26 = 56	12 + 18 = 30 and 12 × 18 = 216
5 + 25 = 30 and 5 × 25 = 104	13 + 17 = 30 and 13 × 17 = 221
6 + 24 = 30 and 6 × 24 = 144	14 + 16 = 30 and 14 × 16 = 224
7 + 23 = 30 and 7 × 23 = 161	**15 + 15 = 30 and 15 × 15 = 225**
8 + 22 = 30 and 8 × 22 = 176	

10. From half to full in 1 minute. So, after 59 minutes it was half-full
11. Write all the fractions as 1000ths: $\frac{399}{1000}$; $\frac{398}{1000}$; $\frac{410}{1000}$; $\frac{420}{1000}$; $\frac{300}{1000}$. $\frac{2}{5} = \frac{400}{1000}$, so $\frac{399}{1000}$ is closest to $\frac{2}{5}$
12. $\frac{13}{20}$ is more than $\frac{12}{20}$ $(\frac{3}{5})$ and less than $\frac{16}{20}$ $(\frac{4}{5})$, so he is on side DE
13. He still has $\frac{7}{20}$ of the distance to go, so $\frac{7}{20}$ of 25 cm = (25 cm ÷ 20) × 7 = 8,75 cm
14. There is a general structure here: The denominators is twice the numerator + 1, i.e. $\frac{\Diamond}{2\times\Diamond+1}$

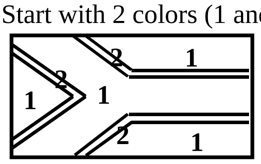
    We can therefore investigate a general pattern $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \frac{6}{13}, \frac{7}{15}, \ldots$

    Check with your calculator: $\frac{1}{3} = 0,333...$, $\frac{2}{5} = 0,4$, … So $\frac{1}{3} < \frac{2}{5} < \frac{3}{7} < \frac{4}{9} < \frac{5}{11} < \frac{6}{13} < \frac{7}{15} < \ldots$

    Conclusion: the larger the denominator, the larger this kind of fraction, so $\frac{11}{23}$ is the largest
15. Trial and improvement: 30×31 = 930 is too small …. 35 × 36 = 1260 is too small …. 36 × 37 = 1332
16. Structure! 50×51 = 2550

P_1	P_2	P_3	P_4	…	P_{50}
1×2	2×3	3×4	4×5	…	?

17. Make a systematic list, e.g. 3579; 3597 | 3759; 3795 | 3957; 3975 | 9375; 9357 | 9537 …
    Or: He has 4 choices for the first number, then 3 choices for the second, 2 for the third and 1 for the fourth. So 4 × 3 × 2 × 1
18. Start with 2 colors (1 and 2) and draw it:

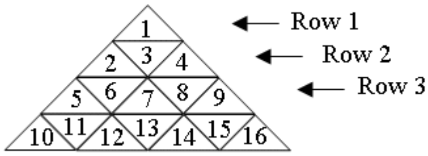


19. $\frac{4}{6} = \frac{6}{?}$ So ? = 9 cm

20. Work systematically!
101, 111, 121, 131, 141, 151, 161, 171, 181, 191 – this is 10
202, 212, 222, 232, 242, 252, 262, 272, 282, 292 – this is 10
….
909, 999, 929, 939, 949, 959, 969, 979, 989, 999 – this is 10
So 9×10 = 90

21. Investigate the *structure* by finding a pattern in special cases*:*

Row number	1	2	3	4		n
Number of numbers	1	3	5	7		$2\times n - 1$



22. In question 23 you see that the last number in Row 49 is 49×49
So the first number in Row 50 is 49×49 + 1 = 2402

23. Investigate the *structure* by finding a pattern in special cases*:*

Row number	1	2	3	4		n
Last number	1	4	9	16		$n\times n$

24. You can try to find a formula for the pattern 1, 3, 7, 13, ...
But note that the middle number is the *average* of the first and last numbers. So (2402+2500)/2 = 2451

25. Investigate the *structure* by finding a pattern in special cases*:*

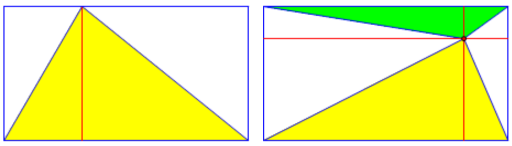
Row number	1	2	3	4		n
Sum	1	9	35	91		
	1x1	3x3	5x7	7x13		$(2\times n-1) \times (n^2-n+1)$

But note that the sum is the middle number × number od numbers – remember, the middle number is the average!
So 2451×99 = 242649
Or you can use the Gauss method: (First + Last) × Number of numbers ÷ 2, so (2402+2500) × 99 ÷ 2 = 242649

<u>GRADE 7(1)</u>

1. (5,6 + 5,65) ÷ 2 = 11,25 ÷ 2 = 5,625
2. 3×3 – 3 + 3 = 9 – 3 + 3 = 6 + 3 = 9
3. nth number = $2\times n - 1$, so 83rd number = 2×83 – 1 = 165

4. & 5.



6. $1+\dfrac{1}{1+\frac{2}{3}} = 1+\dfrac{1}{\frac{5}{3}} = 1+\frac{3}{5}$

7. We know: $\dfrac{\text{Sum of numbers}}{11} = 8$, so Sum of numbers = 11 × 8 = 88
If the new number is x, then $\dfrac{88+x}{12} = 11$. So x = 12 × 11 – 88 = 44

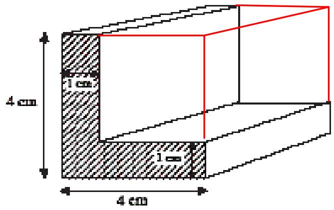
8. Add all together: 2A + 2B + 2C = 42, 2×(A + B + C) = 42, so A + B + C = 21
9. B + A + C = 21 and A + C = 16, so B + 16 = 21
10. The smallest is 10 × 10 = 100. The largest, by guess-and-improvement is 31 × 31 = 961. Count them!
11. # *Triangles* = 2×*squares* + 2, or 2×(*squares* + 1). So *Triangles (6)* = 2×6 + 2 = 14
12. *Triangles (60)* = 2×60 + 2 = 122
13. $2\times x + 2 = 60$, so $x = 29$
14. Make a list, varying the numbers systematically. If the digits are a, b, c and d:
abcd, abdc, acbd, acdb, adbc, adcb and similarly if the first digit is b, c, and d. So 6 × 4 = 24. Or 4 × 3 × 2 × 1 = 24
15. 2 × (7 + 8 + 9) = 2 × 24

16.

	c	d
a	**12**	**20**
b	**21**	D

Using a representation like this, Area D = b × d
We know a × c = 12, b × c = 21, a × d = 20
Multiply them all together: $a^2 \times c^2 \times b \times d = 12 \times 20 \times 21$
But a×c = 12, so $a^2 \times c^2 = 144$, so b×d = 12×20× 21 ÷ 144 = 35 = Area D

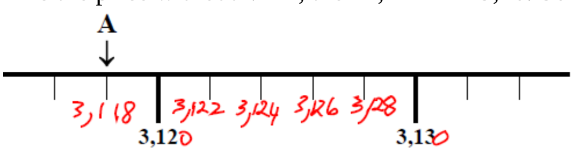
17. Volume = area of base × length = 7 cm^2 × 12 cm = 84 cm^3
Or, think of cutting out a rectangular prism:
Volume = 4×4 × 12 – 3×3 × 12 = 12 × (16 – 9) = 12 × 7



18. The first digit can be 2, 4, 6, 8. The second digit can be 0, 2, 4, 6, 8, which gives 4 × 5 = 20 possible combinations
19. The 6^{th} column is given by 6 × row n.
So the last number in row 80 is 6 × 80 = 480. Then row 81 is 481, 482, 483, …
20. T + F = 17 + R (Tom and Fred together had 17 more apples than Rhoda)
T = 7 and R = 5 (Tom had 7 apples. Rhoda had 5 apples)
7 + F = 17 + 5
7 + F = 22
21. Fill in numbers in the calendar, and test each statement with the numbers.
22. We know $a + d = c + b$, so $a + b + c + d = a + d + c + b = 2\times(a + d) = 52$.
So $a + d = 26$, so $a + (a + 8) = 26$, so $a = 9$
24. 3 lines from two corners divide the triangle in 4 × 4 sections
10 lines from two corners will divide the triangle in 11 × 11 sections = 121
25. $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \ldots \times \frac{2007}{2008} \times \frac{2008}{2009} = 1 \times \frac{2}{2} \times \frac{3}{3} \times \ldots \times \frac{2008}{2008} \times \frac{1}{2009} = \frac{1}{2009}$

GRADE 7(F)

1. Square total area: (5 cm + 3 cm) × (5 m + 3 cm) = 8 cm × 8 cm = 64 cm^2
Rectangles total area: 4×5 cm × 3 cm = 60cm^2
Area of small square: 64 cm^2 - 60 cm^2 = 4 cm^2
2. If we had 6 loose cubes, it would have 6×6 = 36 faces. In the given figure, 5×2 = 10 faces overlap, so there are 26 visible faces, each with an area of 1 cm^2
3. Average = Total mass ÷ number of children = (3×75 + 6×66) kg ÷ 9 = 69 kg
4. The "vertical" formula is $2\times a + 2$. Find a so that $2\times a + 2 = 64$
Or the "horizontal" formula is $4 + 2\times(a – 1)$, so find a so that $4 + 2\times(a – 1) = 64$
5. If x is the price without VAT, then 1,14 × x = 9,46. So x = R8,30, so VAT = R1,16
6.



8. (2 + 4+ 6 + 8 + … + 98 + 100) – (1 + 3 + 5 + … + 97 + 99)
= (2 – 1) + (4 – 3) + (6 – 5) + … + (98 – 97) + (100 – 99)
= 1 + 1+ 1 + 1 + … 50 times = 50
9. If her average for the first 4 tests was 67%, we can also express it as (67 + 67 + 67 + 67) ÷ 4.
So 67 + 67 + 67 + 67 + 63 + 67 = 398. 398 ÷ 6 = 66,33
10. If the length of each rectangle is x, and the width is y:
$3 \times y = 2 \times x$ and $x + y = 15$
So $2x + 2y = 30$, so $3y + 2y = 30$, so $5y = 30$, so $y = 6$, and $x = 9$.
Area of 1 rectangle is 9×6 = 54 cm^2, so the area of 5 rectangles is 5×54 cm^2
11. Each of the 26 letters in the alphabet can be paired with itself (e.g. BB for Barry Brown) and paired with each of the other letters. Order matters - PG is different from GP! There are 26 × 26 combinations.
12. Test all the cases systematically: 1 × 17 = 17; 2 × 16 = 32; 3 × 15 = 45; … 9 × 9 = 81, then the answer repeats, because the order does not matter (e.g. 2 × 16 = 16 × 2).
13. $\frac{8}{11} - \frac{5}{8} = \frac{9}{88}$ of tank is 135 ℓ. So $\frac{1}{88}$ of tank = 135 ℓ÷9 = 15 ℓ. So $\frac{88}{88}$ of the tank = 88 × $\frac{1}{88}$ of the tank = 88 × 15 ℓ
14. Volume = 15 cm × 8 cm × x cm = 120 cm^3, so x = 1. So area is (15 cm + 2 cm) × (8 cm + 2 cm) = 17 cm × 10 cm
15. In middle row the missing number is 18 – (11 + 6) = 1, so in right column z = 18 – (1 + 10) = 7

16. $p \times 1 \times \frac{1}{8} = 1$, so $p = 8$

p	q	r
s	1	t
u	4	$\frac{1}{8}$

$q \times 1 \times 4 = 1$, so $q = \frac{1}{4}$

$u \times 4 \times \frac{1}{8} = 1$, so $u = 2$

$p \times s \times u = 1$, so $8 \times s \times 2 = 1$, so $s = \frac{1}{16}$

$r + s = \frac{1}{2} + \frac{1}{16} = \frac{9}{16}$

17. You can draw it, or investigate numerical patterns for a triangle, square, pentagon, hexagon, etc.
Or you can reason it out: At each vertex of an n-gon there are n–3 diagonals because the point is connected to every other point, except to itself and to the two adjacent points (these are *sides* of the n-gon). So at n vertices there are $n \times (n–3)$ diagonals, counted twice. So the formula is $D(n) = n \times (n–3) \div 2$, so $D(8) = 8 \times (8 – 3) \div 2$
18. $D(80) = 80 \times (80 – 3) \div 2 = 3080$
19. List them systematically: 799; 979; 997; 889; 898; 988
20. Investigate the *structure* by finding a pattern in special cases*:*

# houses	1	2	3	4		n
# matches	5	9	13	17		$4 \times n – 1$

21. $4 \times n – 1 = 225$, so $n = (225 – 1) \div 4 = 56$
22. Multiply them all together: $(a \times b) \times (b \times c) \times (c \times a) = 2 \times 24 \times 3 = 144$
Transform: $(a \times b \times c)^2 = 12^2$, so $a \times b \times c = 12$
$a \times b \times c = 12$, and $b \times c = 24$, so $a \times 24 = 12$, so $a = \frac{1}{2}$
$a \times b \times c = 12$, and $a \times c = 3$, so $3 \times b = 12$, so $b = 4$
$a \times b \times c = 12$, and $a \times b = 2$, so $2 \times c = 12$, so $c = 6$
23. Look for structure!
$T_{50} = 1 + 2 + 3 + 4 + \dots + 49 + 50 = (1+50) + (2+49) + \dots + (25+26) = 25 \times 51 = 1275$
24. Look for structure and pattern!
$N_1 = 2 = 1 \times 1 + 1$
$N_2 = 5 = 2 \times 2 + 1$
$N_3 = 10 = 3 \times 3 + 1$
$N_4 = 17 = 4 \times 4 + 1$
Test the numbers! $30 \times 30 + 1 = 901$ is the only one which fits the pattern.
25. $\frac{1}{13} < \frac{5}{61} < \frac{1}{12}$ and $\frac{1}{5} < \frac{13}{57} < \frac{1}{4}$, therefore 12, 11, 10, 9, 8, 7, 6 and 5.