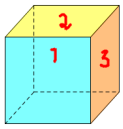
NOTES ON 2010 MEMORANDUM

These notes are necessarily brief and often formal and symbolic.
Many questions could be answered using primitive methods, e.g. "If today is Wednesday, what day of the week will it be 100 days from now?" can be done by counting. That would be laborious, time-consuming and error-prone. The essence of a mathematical approach is to work more smartly by using appropriate representations to model the situation and to exploit the inherent structures and patterns in the situation.

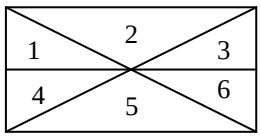
GRADE 4(1)

1. $\frac{1}{8} < \frac{1}{4} < \frac{1}{2}$
2. Check each answer, e.g. $9 = 3 \times 3$; $11 = 2 \times 3 + 5$; $13 = 2 \times 5 + 3$, etc. You know, or you learn from these calculations, that the sum of three odd numbers is odd, so 12 is not possible!
3. Straighten the string. Two loops of 1 cm make it 5 cm + 1 cm + 1 cm = 7 cm
4. 3 hours before 16:45 is 13:45, so 2 hours and 55 minutes (5 min less) is at 13:50
5. The watch gains 2 minutes every day (24 hours) for 7 days = 2 min/day× 7 days = 14 minutes
6. $10 \times 10 \times 10 = 1000$ or from 001 to 999 gives 999 combinations, plus 000 gives 1000
9. Start "painting" (numbering) the sides …



10. With 6 loose cubes, there would be 36 faces. Subtract the 10 non-visible faces ...
11. If Zuki has ♥ marbles, Zinkle has ♥ – 15. Together they have 2×♥ – 15 = 95 marbles. So ♥ = 55
12. The numbers must be different, so 99 + 98 + 97 = (100 – 1) + (100 – 2) + (100 – 3) = 300 – 6
13. $\frac{3}{4} + \frac{3}{4} \rightarrow 1\frac{1}{2} + \frac{3}{4} \rightarrow 2\frac{1}{4} + \frac{3}{4} \rightarrow 3 + \frac{3}{4} \rightarrow 3\frac{3}{4} + \frac{3}{4} \rightarrow 4\frac{1}{2}$
    ① ② ③ ④ ⑤ ⑥
14. A rings on the hour and half-hour. B rings at 08:00, 08:35, 09:10, 09:45, 10:20, 10:55 and 11:30
15. If the 21th is a Monday, then also the 14th, 7th and 0th are Mondays.
    The 0th is the last day of the previous month, so the next day is the 1st of this month, so it is a Tuesday
16.

		32+38		
	32		38	
15		17		21
10	5		12	9

17. For 6 milktarts she needs 8 cups of milk, so for 8 (6 + 2 = 6 + $\frac{1}{3}$ of 6) milktarts she needs $8 + \frac{1}{3}$ of $8 = 8 + \frac{1}{3}$ of $(6 + 2) =$ $8 + \frac{1}{3}$ of $6 + \frac{1}{3}$ of $2 = 8 + 2 + \frac{2}{3} = 10\frac{2}{3}$ cups of milk.
18. With 15 eggs (9 + 6) you can make $6 + \frac{2}{3}$ of 6 milktarts = 6 + 4 = 10 milktarts
19. 

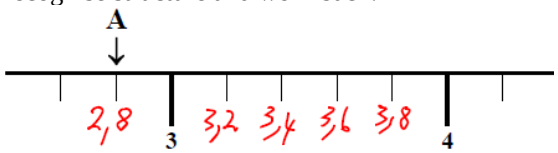
    Invent some notation and count systematically, e.g.:
    Areas 1, 2, 3, 4, 5 and 6 each form a triangle (6)
    Two areas 1-4 and 3-6 each form a triangle (2)
    Three areas 4-1-2, 2-3-6, 3-6-5 and 5-4-1each form a triangle (4)
20. $\frac{1}{5} = \frac{8}{40}$ and $\frac{1}{4} = \frac{10}{40}$, so $\frac{8}{40} < \frac{9}{40} < \frac{10}{40}$
21. Name the girls a, b and c, and make a systematic list: abc bac cab
    acb bca cba
22. List them systematically!
    South Africa vs. France
    South Africa vs. Uruguay
    South Africa vs. Mexico
    France vs. Uruguay
    France vs. Mexico
    Uruguay vs. Mexico
23. Let the children be A, B, C, D and E. List all the possibilities systematically, note patterns and structure:
    A vs B
    A vs C
    A vs D
    A vs E
    B vs C
    B vs D
    B vs E
    C vs D
    C vs E
    D vs E

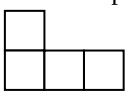
vs	A	B	C	D	E
A		X	X	X	X
B			X	X	X
C				X	X
D					X
E					

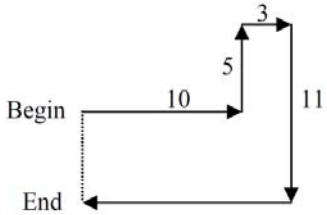
24. **A** vs B **B** vs C … **H** vs I **I** vs J
    A vs C B vs D … H vs J
    A vs D B vs E
    A vs E B vs F
    A vs F B vs G
    A vs G B vs H
    A vs H B vs I
    A vs I B vs J
    A vs J
    The structure is: 9 + 8 + ... + 2 + 1 = **45**
25. Look at the *structure* in the pictures!
    P_1: 1 = 1×1
    P_2: 1 + 3 = 4 = 2×2
    P_3: 1 + 3 + 5 = 9 = 3×3
    P_4: 1 + 3 + 5 + 7 = 16 = 4×4
    ….
    P_{50}: 1 + 3 + 5 + 7 + ... to 50 numbers = 50×50 = 2500

GRADE 4(F)

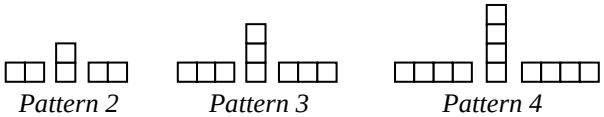
1. 11,23 < 11,32 < 11,4 So Peter Davids is third
2. Lucy's mother is 24 years older than Lucy
   16 + 8 = 24
   In 8 years Lucy's mother will be 48
   Lucy is 24 now.
3. 07:20 + 45 min = 08:05
4. Recognise structure and work back:



5. R35 ÷ 4 = (R32 ÷ 4) + (R3 ÷ 4) = R8 + R0,75 = R8,75
7. The pattern is 1 + **1** + 1 + **2** + 1 + **3** + 1 + **4** + **1** + **(5** + **1** + **6** + **1** + **5)** + **2** + 1 + 8 + 1
8. 438 – 257 = 181 km
9. 438 + 169 = 607 km
10. Thabo takes 4 out of 12; 4/12 = 1/3
    He has to pay 1/3 of R30 = R10
11.  The tower is on your left if you look at the object from the back
12. 6,8 ÷ 2 → 3,4 ÷ 2 → 1,7 ÷ 2 = 0,85
13. 24 – 24 = 0; 71 × 3 = 213 marbles
14. 37 will be opposite 38, therefore Con lives opposite Luke
15. R35,60 ÷40 = 89c so 89c × 15 = R13,35
16. 4 × 3 → 12 + 8 → 20 ÷ 2 → 10 – 6 = 4
17. 15 × 2 = 30; 15 × 1/5 = 15/5 = 3; so 30 + 3 = 33 viennas
18. 2 people sit at the end → 56 ÷ 2 (2 people per table) → 28 tables
19. If #T, then # people = 2 × T + 2 = 58, so T = (58 – 2) ÷ 2 = 28
20. Draw it physically! See diagram. It always helps to write!



21. Arrange them: O S (R) T (R) E (Ram can be between Siva and Temba or between Temba and Eby)
    Oscar is the shortest
22. Do not count or calculate – look for *structure*, e.g.
    For Pattern 1: 1×2
    For Pattern 2: 2×2
    For Pattern 3: 3×2
    For Pattern 4: 4×3
    For Pattern 100: 100×99



23. 

24. $0 \times 20 + 3 \times 10 + 1 \times 5$
    $1 \times 20 + 1 \times 10 + 1 \times 5$

25. List all the possibilities and be systematic:
    1 + 1 = 2 2 + 2 = 4 3 + 3 = 6 4 + 4 = 8 5 + 5 = 10 6 + 6 = 12
    1 + 2 = 3 2 + 3 = 5 3 + 4 = 7 4 + 5 = 9 5 + 6 = 11
    Any other combination will be a repetition – therefore 11 possible answers

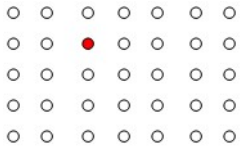
GRADE 5(1)

1. First pick up 7, then 1 then 6 etc.
3. Build a mental picture! B, E & F
4. The trip is 31 minutes, therefore 12:30 + 31 minutes → 13:01
5. Use trial and error, i.e. try each of the given answers one by one
6. 25 cones + 12 cones = 37 cones
7. (2 – 1) + (3 – 2) + (4 – 3) + (5 – 4) + (6 – 5) + … + (101 – 100)
   = 1 + 1 + 1 + 1 + 1 +… + 1 100 times
   = 100 × 1 = 100
8. R5 less for you and R5 more for her is R10
9. 274 – 246 + 1 = 29 (Check: if you read page 1 and 2, you have read 2 pages, not 2-1 = 1)
10. 3 + 3 + 1 + 3 + 3 + 1 + 3 + 1 + 3 + 3 = 24cm
11. 41000 g – 725 g = 40275 g = 40,275 kg
12. Divide 420 into 7 equal parts:420 ÷ 7 = 60. 3 of these parts are dresses, i.e. 3 × 60 = 180
13. The first (left) digit changes the least – it just changes from 0 to 1, from 1 to 2 and from 2 to 0. This happens when the clock changes from 09:59:59 to 10:00:00; from 19:59:59 to 20:00:00 and from 23:59:59 to 00:00:00.
14. Consider the possible choices from the top row:
    If I choose 1, then the options are 1, 5, 9 or 1, 6, 8 giving products 45 or 48 respectively.
    If I choose 2, the options are 2, 4, 9 or 2, 6, 7 with products 72 or 84 respectively.
    If I choose 3, the options are 3, 4, 8 or 3, 5, 7 with products 96 or 105.
    So 105 is the maximum possible product.

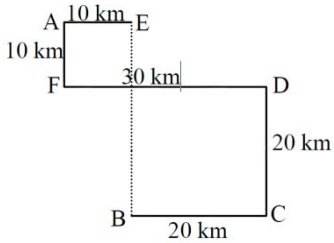
1	2	**3**
4	**5**	6
7	8	9

15. $T_n = 3 \times n + 1$
16. $X_n = 4 \times n + 1$
17. 4, 8, 12, 16, … are multiples of 4, so 6, 10, 14, 18, 22…are 2 more than a multiple of 4. The 100th multiple of 4 is 100×4, so 2 more is 402
18. If you divide by 4, they must all leave a remainder of 2, i.e. if you divide by 4 on a calculator, the answer must be *.5
    This is true for all, except 7576 ÷ 4 = 1894
19. Look at the *structure* in the pictures!
    T_1: 1 = 1×1
    T_2: 1 + 3 = 4 = 2×2
    T_3: 1 + 3 + 5 = 9 = 3×3
    ….
    T_{10}: 1 + 3 + 5 + 7 + ... to 10 numbers = 10×10 triangles
20. Look at the *structure* in the pictures!
    Count the number of *triangles:*
    # triangles in T_1 = 1
    # triangles in T_2 = 1 + 2
    # triangles in T_3 = 1 + 2 + 3
    # *triangles* in T_{10} = 1 + 2 + 3 + 4 + ... + 9 + 10 = (1 + 10)×10/2 = 55
    So # *matches* = 55×3

21. Make a sketch of the situation!
    "2nd from front, 4th from back" means there are 5 rows. "3rd from left, 5th from right" means there are 7 learners per row. So 7 learners/row × 5 row = 35 learners



22. Draw it! Fill in the information as you read. Re-read, bit by bit!



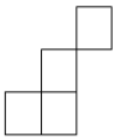
23. Be systematic, e.g.

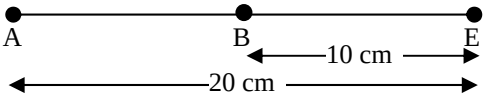
32	23	43	13
34	24	42	12
31	21	41	14

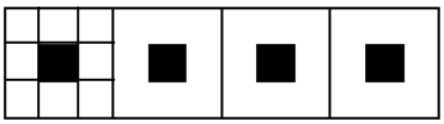
24: Be systematic, e.g.
143, 142; 134, 132; 124, 123
413, 412; 431, 432; 421, 423
314, 312; 341, 342; 321, 324
214, 213; 241, 243; 231, 234

25. Debbie is first, Peter is second, Tom is third and Robert is fourth.

GRADE 5(F)

1. In the bottom layer there are $8 \times 4 = 32$ blocks, so in two layers there are 64 blocks
2. All the blocks of the bottom layer (32) and all the blocks round the side of the top layer (20)
3. Try and test each possible answer!
4. $500 \div 12 = 41$ rem 8, so 42 cartons are needed
5. $\frac{120}{360} = \frac{22}{?}$ So $? = 66$ mm (The photo is enlarged 3 times)
6. If the loser had Δ votes, the winner had $\Delta + 1002$ votes. Together $2\times\Delta + 1002 = 39218$
7. These are multiples of 6. Only $4182 = 6 \times 697$ is a multiple of 6
8. 9, 18, 27, 36, … are multiples of 9, so 10, 19, 28, 37, …are 1 more than a multiple of 9. The 100th multiple of 9 is 100×9, and 1 more is 901.
9. $1639 \div 9 = 182$ remainder 1 (on the calculator 182.1111…). All the others leave different remainders.
10. 
11. Put the information in a sketch, fill in the details bit by bit, and extend the information. e.g.:
The distance from A to E is 20 cm
The distance from B to E is 10 cm
You can deduce that AB = 10 cm!

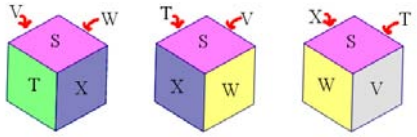


12. Peter worked 3 out of the 6 hours, so he should get 3/6 = 1/2 of R48 = R24
13. If Ashley's starting number is *S*, then he did $S \times 10 = 600$. So $S = 60$. So correct answer is $60 \div 10 = 6$
14. Look at *structure:*
$2 = 0 \times 5 + 2$
$7 = 1 \times 5 + 2$
$12 = 2 \times 5 + 2$
Generalise:
Output number = *Input number*$\times 5 + 2$
15. *Input number* = (*Output number* $- 2) \div 5$, so $(37 - 2) \div 5 = 7$
16. Look at the *structure:*
Length 3: $2\times3 + 2 = 8$
Length 7: $2\times7 + 2 = 16$
So for length 20: $2\times20 + 2 = 42$
17. $\frac{600}{500} = \frac{120}{100} = \frac{6}{5}$. So $\frac{300}{?} = \frac{6}{5}$, so $? = 250$
18. From 1-9 you use 9 digits. From 10 to 99 you use $90 \times 2 = 180$ digits. For 100 you use 3 digits. Total = 9 + 180 + 3 = 192
19. 
20. Study the structure:
$4 \boxtimes 3 = 15 = 4\times3 + 3 = 5\times3$
$6 \boxtimes 3 = 21 = 6\times3 + 3 = 7\times3$
$7 \boxtimes 5 = 40 = 7\times5 + 5 = 8\times5$
$8 \boxtimes 7 = 63 = 8\times7 + 7 = 9\times7$
So $6 \boxtimes 8 = 7\times8 = 56$

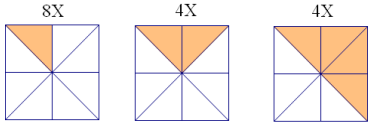
21. Be systematic, for example: 42 45 41 47
    Note: the order matters! 24 25 21 27
    54 52 51 57
    14 12 15 17
    There are 5×4 possibilities 74 72 75 71
22. List them systematically. Note there are 5×4×3 possibilities:
    425 421 427 245 241 247 542 541 547 142 145 147 742 745 741
    452 451 457 254 251 257 524 521 527 124 125 127 724 725 721
    412 415 417 214 215 217 514 512 517 154 152 157 754 752 751
    472 475 471 274 275 271 574 572 571 174 172 175 714 712 715
23. There are 5×4×3×2×1 possibilities
24. 100 – (3×10) = 70
25. The structure is 1 + 2 + 3 + 4 + 5 + 6 + ... + 48 + 49 + 50 = (1+50) + (2+49) + (3+48) + ... = 51 × 25

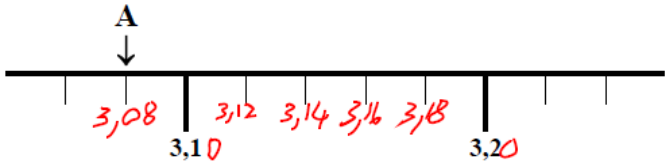
GRADE 6(1)

1. Form a mental picture!



2. List the triangles systematically - *notation* and a *system* will help!



3. Try trial and error, e.g. 8 + 9 + 10 + …
   Or test each of the given numbers …
   Or, if the smallest is x, then $x + (x + 1) + (x + 2) + \ldots + (x + 6) = 7 \times x + 21 = 63$, so $x = 6$
4. 

5. List them systematically: 1, 2, 4, 5, 10, 20, 25, 50, 100
6. 4002 ÷ 4 = 1000 rem 2
7. Count equal parts!
8. $\frac{5}{6} = \frac{40}{48}$ and $\frac{7}{8} = \frac{42}{48}$, so $\frac{40}{48} < \frac{41}{48} < \frac{42}{48}$
9. Vary the numbers systematically and note the behaviour of the product of the numbers:
   1 + 29 = 18 and 1 × 29 = 29 9 + 21 = 18 and 9 × 21 = 189
   2 + 28 = 18 and 2 × 28 = 46 10 + 20 = 18 and 10 × 20 = 200
   3 + 27 = 18 and 3 × 27 = 81 11 + 19 = 18 and 11 × 19 = 209
   4 + 26 = 18 and 4 × 26 = 56 12 + 18 = 18 and 12 × 18 = 216
   5 + 25 = 18 and 5 × 25 = 104 13 + 17 = 18 and 13 × 17 = 221
   6 + 24 = 18 and 6 × 24 = 144 14 + 16 = 18 and 14 × 16 = 224
   7 + 23 = 18 and 7 × 23 = 161 15 + 15 = 18 and 15 × 15 = 225
   8 + 22 = 18 and 8 × 22 = 176
10. Be systematic!
    South Africa vs. France France vs. Uruguay Uruguay vs. Mexico
    South Africa vs. Uruguay France vs. Mexico
    South Africa vs. Mexico
11. Write all the fractions as 1000ths: $\frac{399}{1000}; \frac{398}{1000}; \frac{410}{1000}; \frac{420}{1000}; \frac{300}{1000}$

    $\frac{2}{5} = \frac{400}{1000}$, so $\frac{399}{1000}$ is closest to $\frac{2}{5}$
12. $\frac{13}{20}$ is more than $\frac{12}{20}$ ($\frac{3}{5}$) and less than $\frac{16}{20}$ ($\frac{4}{5}$), so he is on side DE

13. He still has $\frac{7}{20}$ of the distance to go, so $\frac{7}{20}$ of 25 cm = (25 cm ÷ 20) × 7 = 8,75 cm

14. L(n) = n + (n – 1)×2, so L(100) = 100+ 99×2 = 298

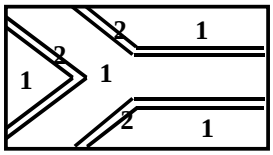
15. Trial and improvement: 30×31 = 930 is too small …. 35 × 36 = 1260 is too small …. 36 × 37 = 1332

16. Structure!

P_1	P_2	P_3	P_4	…	P_{50}
1×2	2×3	3×4	4×5	…	?

17. Make a systematic list, e.g. 3579; 3597 | 3759; 3795 | 3957; 3975 | 9375; 9357 | 9537 …
Or: He has 4 choices for the 1st number, then 3 choices for the 2nd, 2 for the 3rd and 1 for the 4th. So 4 × 3 × 2 × 1

18. Start with 2 colours (1 and 2) and draw it:



19. $\frac{4}{6} = \frac{6}{?}$ So ? = 9 cm

20. Introduce some notation, e.g. 1 = Father, 2 = Bride, 3 = Groom, 4 = Mother
Make a systematic list!
1234 ; 1243 ; 1324 ; 1342 ; 1423 ; 1432
2134 ; 2143 ; 2314 ; 2341 ; 2413 ; 2431
3124 ; 3142 ; 3214 ; 3241 ; 3412 ; 3421
4123 ; 4132 ; 4213 ; 4231 ; 4312 ; 4321
There are 4×6 = 4×3×2×1 different arrangements

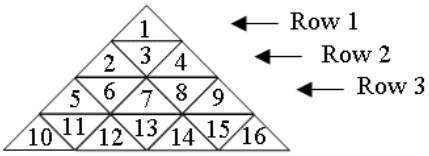
21. Through systematic elimination, e.g.
A in the top row must be 1, 8 or 6. But A in the right column cannot be 8 or 1, so A is 6
B in the bottom row must be 9, 4 or 2. But A in the right column cannot be 9 or 4, so B is 2
C in the top row must be 1 or 8. But C in the left column cannot be 1, so C is 8. So E is 1
D in the bottom row must be 9 or 4. But D in the left column cannot be 4, so D is 9. So F is 4
We only have 3, 5 and 7 left. But G cannot be 3 or 7, so G = 5. H cannot be 3, so H = 7 and X = 3

C	E	A
G	X	H
D	F	B

22. Investigate the *structure* by finding a pattern in special cases*:*

Row number	1	2	3	4		*n*
Number of numbers	1	3	5	7		2×*n* – 1

23. In question 23 you see that the last number in Row 49 is 49×49
So the first number in Row 50 is 49×49 + 1 = 2402

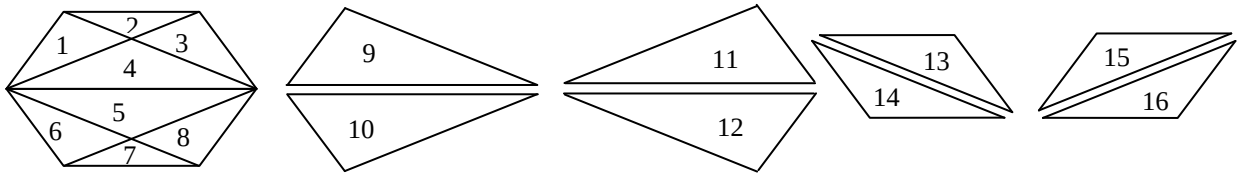


24. Investigate the *structure* by finding a pattern in special cases*:*

Row number	1	2	3	4		*n*
Last number	1	4	9	16		*n*×*n*

25. You can try to find a formula for the pattern 1, 3, 7, 13, ...
But note that the middle number is the *average* of the first and last numbers. So (2402+2500)/2 = 2451

GRADE 6(F)

1. Try trial and improvement, e.g. 50 + 52 + 54 ≠ 174; …. But 56 + 58 + 60 = 174
Or test each of the given numbers …
Or, if the smallest is *x*, then *x* + (*x* + 2) + (*x* + 4) = 3 × *x* + 6 = 174, so *x* = 56
2. (5,6 + 5,65) ÷ 2 = 11,25 ÷ 2 = 5,625
3. 3×3 – 3 + 3 = 9
4. 

5. Full lorry = 4653 kg; empty lorry = 2583 kg; 4653 – 2583 = 2070; 2070 kg ÷ 90 kg/bag = 23 bags
8. *Look* at the structure: For *n* dice, the number of visible faces is *n*×3 + 2. So for 30 dice, 30×3 + 2
9. If 50 faces are visible, *n*×3 + 2 = 50, so *n* = 16
10. From half to full in 1 minute. So, after 59 minutes it was half-full

11. 4 books = 2 books + 6 kg, so 2 books = 6 kg, so 1 book = 3 kg
12. $\frac{1}{2} + \frac{1}{8} + \frac{1}{8} = \frac{3}{4}$; R15 is $\frac{1}{4}$; R60 = $\frac{4}{4}$
13. If the book costs Rx, the CD costs Rx + 60. Together they cost $x + x + 60 = 230$
    So $x = (230 - 60) \div 2$ = R85
14. There is a general structure here: The denominators is twice the numerator + 1, i.e. $\frac{\Diamond}{2\times\Diamond+1}$

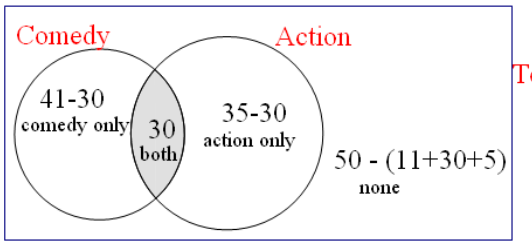
    We can therefore investigate a general pattern $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \frac{6}{13}, \frac{7}{15}, \ldots$

    Check with your calculator: $\frac{1}{3} = 0,333..., \frac{2}{5} = 0,4, \ldots$ So $\frac{1}{3} < \frac{2}{5} < \frac{3}{7} < \frac{4}{9} < \frac{5}{11} < \frac{6}{13} < \frac{7}{15} < \ldots$

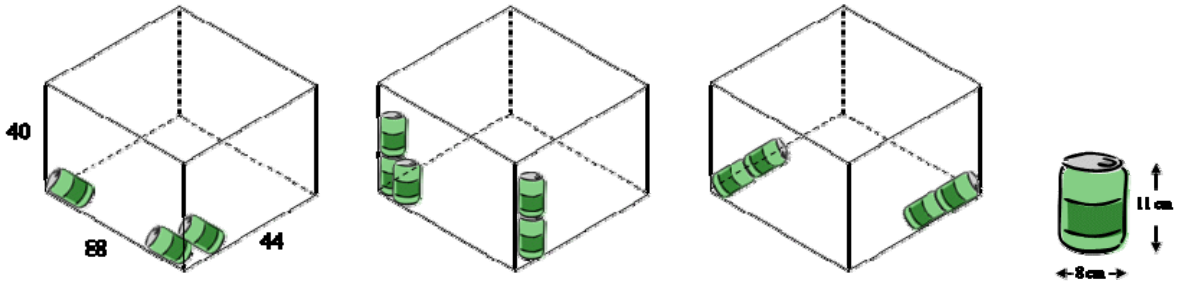
    Conclusion: the larger the denominator, the larger this kind of fraction, so $\frac{11}{23}$ is the largest
15. 365 days ÷ 7days/week = 52 weeks and 1 day …

2011 Thurs	2016 Thurs (Leap Year!)
2012 Sat (Leap Year!)	2017 Fri
2013 Sun	2018 Sat
2014 Mon	2019 Sun
2015 Tues	2020 Tues (Leap Year!)

16. Jane eats 2 × 12 sweets in 5 minutes; she eats 2 × 24 sweets in 10 minutes. Jane eats 48 sweets in 10 minutes
17. Make a representation of the situation (draw it):



18. Consider different possible packings:



    The number of cans in each of these packings is:
    8×5×5 = 200 11×5×3 = 165 11×4×5 = 220
19. For 19 wheels we can have: 1 tricycle and 8 bicycles = total of 9 (too much)
    3 tricycles and 10 bicycles = total of 13 (too much)
    5 tricycles and 2 bicycles = total of 7 (just right)
20. Look at the *structure* in the pictures!
    $P_1 = 4 \times 1 + 1 = 5$
    $P_2 = 4 \times 2 + 1 = 9$
    $P_3 = 4 \times 3 + 1 = 13$
    $P_{50} = 4 \times 50 + 1 = 201$
21. Work systematically!
    101, 111, 121, 131, 141, 151, 161, 171, 181, 191 – this is 10
    202, 212, 222, 232, 242, 252, 262, 272, 282, 292 – this is 10
    ….
    909, 999, 929, 939, 949, 959, 969, 979, 989, 999 – this is 10
    So 9×10 = 90
22. Be systematic!

South Africa vs France	France vs Uruguay	Uruguay vs Mexico
South Africa vs Uruguay	France vs Mexico	
South Africa vs. Mexico		

23. Let the children be A, B, C, D and E. List all the possibilities systematic, note patterns and structure:

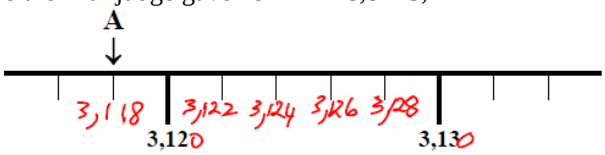
A vs B **B** vs C **C** vs D **D** vs E
A vs C B vs D C vs E
A vs D B vs E
A vs E

vs	A	B	C	D	E
A		X	X	X	X
B			X	X	X
C				X	X
D					X
E					

24. **A** vs B **B** vs C …. **H** vs I **I** vs J
A vs C B vs D …. H vs J
A vs D B vs E ….
A vs E B vs F ….
A vs F B vs G ….
A vs G B vs H ….
A vs H B vs I ….
A vs I B vs J
A vs J
The structure is: 9 + 8 + ... + 2 + 1 = **45**

25. Look at the *structure* in the pictures!
P_1: 1 = 1×1
P_2: 1 + 3 = 4 = 2×2
P_3: 1 + 3 + 5 = 9 = 3×3
P_4: 1 + 3 + 5 + 7 = 16 = 4×4
….
P_{50}: 1 + 3 + 5 + 7 + ... to 50 numbers = 50×50 = 2500

GRADE 7(1)

1. Square total area: (5 cm + 3 cm) × (5 m + 3 cm) = 8 cm × 8 cm = 64 cm²
Rectangles total area: 4×5 cm × 3 cm = 60cm²
Area of small square: 64 cm² - 60 cm² = 4 cm²
2. If we had 6 loose cubes, it would have 6×6 = 36 faces. In the given figure, 5×2 = 10 faces overlap, so there are 26 visible faces, each with an area of 1 cm²
3. (10 m – 2 m) × (4 m – 2m) = 8 m × 2 m = 16 m^2 (garden area)
4. 10 m × 4 m = 40 m^2 (whole area); 40 m^2 (garden area) – 16 m^2 = 24 m^2 (path area)
5. If her average score is 4,8, her total score is 5 × 4,8 = 24. The first four judges gave her 4,5 + 4,6 + 4,7 + 5 = 18,8
So the fifth judge gave her 24 – 18,8 = 5,2
6. 

7. 1+3+6 = 10
8. 6 cm^2 (the bottom row of cubes)
9. List them all, be systematic!
South Africa vs. France France vs. Uruguay Uruguay vs. Mexico
South Africa vs. Uruguay France vs. Mexico
South Africa vs. Mexico
10. If the length of each rectangle is x, and the width is y:
$3 \times y = 2 \times x$ and $x + y = 15$
So $2x + 2y = 30$, so $3y + 2y = 30$, so $5y = 30$, so $y = 6$, and $x = 9$.
11. Each of the 26 letters in the alphabet can be paired with itself (e.g. BB for Barry Brown) and paired with each of the other letters. Order matters - PG is different from GP! There are 26 × 26 combinations.
12. 4920 ÷ 100% = 49,2 × 80% (the difference is 20%) = 3936 visitors
13. $\frac{8}{11} - \frac{5}{8} = \frac{9}{88}$ of tank is 135 ℓ. So $\frac{1}{88}$ of tank = 135 ℓ÷9 = 15 ℓ. So $\frac{88}{88}$ of the tank = $88 \times \frac{1}{88}$ of the tank = 88 × 15 ℓ
14. Volume = 15 cm × 8 cm × x cm = 120 cm^3, so x = 1. So area is (15 cm + 2 cm) × (8 cm + 2 cm) = 17 cm × 10 cm
15. In middle row the missing number is 18 – (11 + 6) = 1, so in right column z = 18 – (1 + 10) = 7
16. $p \times 1 \times \frac{1}{8} = 1$, so $p = 8$ $q \times 1 \times 4 = 1$, so $q = \frac{1}{4}$ $p \times q \times r = 1$, so $8 \times \frac{1}{4} \times r = 1$, so $r = \frac{1}{2}$
$u \times 4 \times \frac{1}{8} = 1$, so $u = 2$ $p \times s \times u = 1$, so $8 \times s \times 2 = 1$, so $s = \frac{1}{16}$
$r + s = \frac{1}{2} + \frac{1}{16} = \frac{9}{16}$

p	q	r
s	1	t
u	4	$\frac{1}{8}$

17. You can draw it, or investigate numerical patterns for a triangle, square, pentagon, hexagon, etc.
    Or you can reason it out: At each vertex of an n-gon there are $n–3$ diagonals because the point is connected to every other point, except to itself and to the two adjacent points (these are *sides* of the n-gon). So at n vertices there are $n \times (n–3)$ diagonals, counted twice. So the formula is $D(n) = n \times (n–3) \div 2$, so $D(8) = 8 \times (8 – 3) \div 2$
18. $D(80) = 80\times (80 – 3) \div 2 = 3080$
19. If the numbers are x and y: $6\times x + y = 17$. Because $17 – y = 6\times x$, it means $17 – y$ must be a multiple of 6, i.e. 12.
    So $y = 5$, and $x = 5$.
20. Investigate the *structure* by finding a pattern in special cases*:*

# houses	1	2	3	4		n
# matches	5	9	13	17		$4\times n – 1$

21. $4\times n – 1 = 225$, so $n = (225 – 1) \div 4 = 56$
23. Make a systematic list. If the persons are A, B, C, D and E:

    ABCDE ABDCE ABECD
    ABCED ABDEC ABEDC

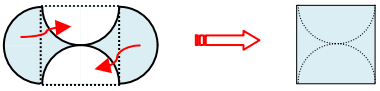
    ACBDE ACDBE ACEBD
    ACBED ACDEB ACEDB

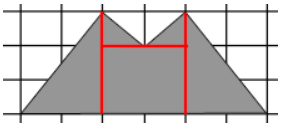
    ADBCE ADCBE ADEBC
    ADBEC ADCEB ADECB

    AEBCD AECBD AEDBC
    AEBDC AECDB AEDCB

    So, with A in the first position, there are $4 \times 3 \times 2$ possibilities. Likewise when B, C, D and E are in the first position.
    So, there are $5 \times 4 \times 3 \times 2 \times 1 = 120$ different arrangements.
24. Look for structure and pattern!
    $N_1 = 3 = 2\times 2 – 1$
    $N_2 = 8 = 3\times 3 – 1$
    $N_3 = 15 = 4\times 4 – 1$
    $N_4 = 24 = 5\times 5 – 1$
    None of the answers fit the pattern.
25. Trial and improvement! The nineteenth century is in the 1800s.
    $40^2 = 1600$ is too small
    $41^2 = 1681$ is too small
    $42^2 = 1764$ is too small
    $43^2 = 1849$ is right …
    $44^2 = 1936$ is too big!

GRADE 7(F)

1. $5 \times 3 – 2 \times 1 = 13$
   $6 \times 2 – 2 \times 1 = 10$
   $13 + 10 = 23$
2. $3 + 3 + 5 + 2 + 4 + 2 + 6 + 1 = 26$
3.  So 6 cm × 6 cm = 36 cm^{22}
4. Average = Total mass ÷ number of children = $(3\times 75 + 6\times 66)$ kg ÷ 9 = 69 kg
5. The “vertical” formula is $2\times a + 2$. Find a so that $2\times a + 2 = 64$
   Or the “horizontal” formula is $4 + 2\times(a – 1)$, so find a so that $4 + 2\times(a – 1) = 64$
6. If the price without VAT is Rx, then $1{,}14 \times x = 36{,}15$. So $x = 31{,}71$
7. Use trial and error to find that only $27 \times 28 = 756$. So $27 + 28 = 55$
8. Use fact that the area of a triangle is half of area of a rectangle …



9. $(2 + 4+ 6 + 8 + … + 98 + 100) – (1 + 3 + 5 + … + 97 + 99)$
   $= (2 – 1) + (4 – 3) + (6 – 5) + … + (98 – 97) + (100 – 99)$
   $= 1 + 1+ 1 + 1 + …$ 50 times $= 50$

10. If Sandy is x years old, Mandy is $x + 7$ years old. In 4 years time Sandy is $x + 4$ years old and Mandy is $x + 11$. Then $x + 4 = ½ \times (x + 11)$. So x, i.e. Sandy is 3 and Mandy is 10. $3 + 10 = 13$
11. The first digit can be 1, 3, 5, 7 or 9. The second digit can be 1, 3, 5, 7 or 9, which gives $5 \times 5 = 25$ possible combinations
12. $2^2 + 9^2 = 4 + 81 = 85$
    $6^2 + 7^2 = 36 + 49 = 85$
13. If the dimensions of the room is a by b by c, then the area to paint is $A = 2ab + 2ac + 2bc$
    Double the dimensions are $2a$ by $2b$ by $2c$, so the area to paint is $D = 2(2a)(2b) + 2(2a)(2c) + 2(2b)(2c) = 4 \times A$
14. Test all the cases systematically: $1 \times 17 = 17$; $2 \times 16 = 32$; $3 \times 15 = 45$; … $9 \times 9 = 81$, then the answer repeats, because the order does not matter (e.g. $2 \times 16 = 16 \times 2$).
15. List them all: 1, 2, 4, 5, 8, 10, 16, 20, 25, 40, 50, 80, 100, 125, 200, 250, 400, 500, 1000, 2000
16. Each position can be 26 letters of the alphabet characters (A to Z) or 10 digits (from 0 to 9).
    For 5 characters there are $36 \times 36 \times 36 \times 36 \times 36$ combinations
17. List them systematically: 799; 979; 997; 889; 898; 988
18. Make a list, varying the persons systematically. If the persons are a, b, c and d:
    abcd, abdc, acbd, acdb, adbc, adcb and similarly if the first person is b, c, and d. So $6 \times 4 = 24$. Or $4 \times 3 \times 2 \times 1 = 24$
19. $11 \times 8 = 88$; $11 \times 12 = 132$; $132 - 88 = 44$
20. $12 \div 4 = 3$; $3 \times 3 = 9$
21. Filling: In 1 minute $\frac{1}{12}$ of bath fills

    Emptying: In 1 minute $\frac{1}{18}$ of bath empties

    Together: In 1 minute $\frac{1}{12} - \frac{1}{18} = \frac{1}{36}$ of batch fills. So the whole bath $(\frac{36}{36})$ fills in 36 minutes
22. Row 1 = 1 dot; row 50 = 50 dots; row 1 + row 50 = 51 dots
    Row 2 = 2 dots; row 49 = 49 dots; row 2 + row 49 = 51 dots
    $51 \times 25 = 1275$ dots
23. Look for structure and pattern!
    $N_1 = 2 = 1 \times 1 + 1$
    $N_2 = 5 = 2 \times 2 + 1$
    $N_3 = 10 = 3 \times 3 + 1$
    $N_4 = 17 = 4 \times 4 + 1$
    Test the numbers! $30 \times 30 + 1 = 901$ is the only one which fits the pattern.
24. If they mine 5%, then 95% = 0,95 is left. So:
    After 1 year, 95% is left
    After 2 years, 95% of 95% = $0,95 \times 0,95$ is left
    After 3 years, 95% of 95% of 95% = $0,95 \times 0,95 \times 0,95 = 0,95^3$ is left
    After 10 years, $0,95^{10}$ is left. Use a calculator: $0,95^{10} = 0,598 = 59,8\%$ is left
    After 13 years, $0,95^{13}$ is left. $0,95^{13} = 0,513 = 51,3\%$, more than half, is left
    After 14 years, $0,95^{14}$ is left. $0,95^{14} = 0,487 = 48,7\%$, less than half, is left
25. $(1 + 1) \times (1 + \frac{1}{2}) \times \ldots \times (1 + \frac{1}{100}) = \frac{2}{1} \times \frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \ldots. \times \frac{100}{99} \times \frac{101}{100}$

    $= \frac{2 \times 3 \times 4 \times 5 \times \ldots \times 99 \times 100 \times 101}{2 \times 3 \times 4 \times 5 \times \ldots \times 99 \times 100}$

    $= 101$