SA Mathematics Challenge
Wiskunde Uitdaging

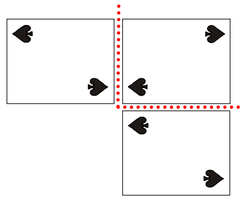
MEMORANDUM 2013

QUESTION	4(1)	4(F)	5(1)	5(F)	6(1)	6(F)	7(1)	7(F)	VRAAG
1	D	E	C	A	B	B	B	A	1
2	B	B	B	B	A	B	C	D	2
3	A	B	C	A	E	C	B	B	3
4	E	C	C	B	E	E	A	C	4
5	C	C	C	E	E	D	A	B	5
6	E	B	D	B	D	C	D	D	6
7	D	C	B	B	D	A	A	B	7
8	B	B	E	A	E	E	D	A	8
9	B	A	B	E	B	E	A	D	9
10	B	E	D	C	B	B	A	A	10
11	D	E	D	E	B	A	B	B	11
12	A	A	E	A	C	B	B	E	12
13	D	B	B	D	E	A	D	E	13
14	E	B	B	C	C	D	D	C	14
15	D	B	D	B	E	B	C	A	15
16	C	B	D	D	E	C	B	D	16
17	B	C	B	D	B	D	E	B	17
18	D	B	D	E	E	E	A	E	18
19	C	A	A	C	E	B	C	E	19
20	A	D	C	D	A	D	A	A	20
21	C	C	D	A	B	B	D	A	21
22	C	C	C	A	E	B	A	A	22
23	D	A	B	B	B	A	D	B	23
24	A	A	E	C	D	C	B	B	24
25	C	B	D	A	B	A	B	B	25
QUESTION	4(1)	4(F)	5(1)	5(F)	6(1)	6(F)	7(1)	7(F)	VRAAG

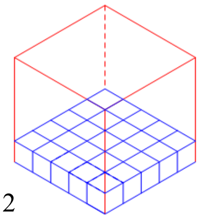
NOTES ON 2013 MEMORANDUM

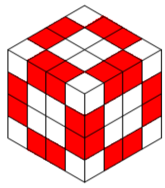
These notes are necessarily brief and often formal and symbolic.
Many questions could be answered using primitive methods, e.g. "If today is Wednesday, what day of the week will it be 100 days from now?" can be done by counting. That would be laborious, time-consuming and error-prone. The essence of a mathematical approach is to work more smartly by using appropriate representations to model the situation and to exploit the inherent structures and patterns in the situation.

GRADE 4(1)



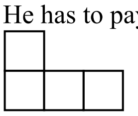
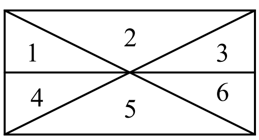
1. 5 + 3 = 8, while the others all have a sum of 7
2. Half of 8 × 8
6. There is a pattern of +14, +14, +14 in the numbers
7. $\frac{3}{4} \div \frac{1}{8} = \frac{6}{8} \div \frac{1}{8}$. How many $\frac{1}{8}$ are there in $\frac{6}{8}$?
8. B is a mirror-image in a horizontal or vertical line of symmetry, as shown





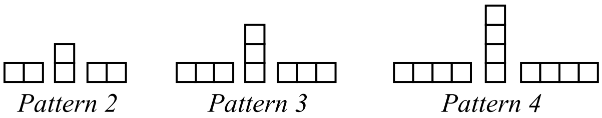
11. 35 000 mℓ ÷ 35 mℓ = 100
12. The figure can be divided into 32 equal triangles of which 16 are shaded.
13. 1, 4, 9, ... = 1×1, 2×2, 3×3, … So 8×8 = 64
14. 5 small cubes to a side. So 5×5 in bottom layer, with 5 layers, so 5×5×5
15. 274 – 246 + 1 = 29
16. 8 cubes on each of the 6 sides. But then they are all counted twice! So 6×8 ÷ 2
17. Bottom level: 3 × 3 = 9 blocks, Second level has 1 less: 8 blocks, Top level has 5 blocks
18. x – 4 + 5 – 6 = 3, so x – 5 = 3 so x = 8
19. Two people at the end + 2 people per table: 20 × 2 + 2 = 42 people
20. 58 – 2 = 56; 56 ÷ 2 = 28 tables
21. 120 km in 60 min, so 20 km in 10 min, so 200 km in 100 min, so the time is 11:40
22. 7, 17, 27, 37, … 77 (two!), 87, 97 is 11, plus 70, 71, 72, … ~~77~~, 78, 79 is another 9, so 20
23. M + M + 30 = 114, so 2 × M = 84, so Monde weighs 42 kg
24. 50 × 2 – 1 = 99
25. 11×3 = 33

GRADE 4(F)

1. The numbers are *halved*: 1; $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{16}$
2. 8×9
3. 1 pizza for 3 children
   15 pizzas × 3 children/pizza = 45 children
4. 33×58 = 1914
5. These are multiples of 6. Only 4182 = 6 ×697 is a multiple of 6
6. 09:47 to 10:18 = 31 minutes
   12:30 = 31 min. = 13:01
7. Jason has 2/3 of the stamps and Mary has 1/3 of the stamps
   96 ÷ 3 = 32 stamps
8. 438 – 257 = 181 km
9. 438 + 169 = 607 km
10. Thabo takes 4 out of 12; 4/12 = 1/3
    He has to pay 1/3 of R30 → R10
11.  The tower is on your left if you look at the object from the back
12. 6,8 ÷ 2 → 3,4 ÷ 2 → 1,7 ÷ 2 = 0,85
13. 24 – 24 = 0; 71 × 3 = 213 marbles
15. R35,60c ÷ 40 = R0,89c
    R0,89c × 15 = R13,35c
17. 8 × 2 + 3 × 2 = 22 m
18. $\frac{20}{36} = \frac{5}{9}$
21. 

    Invent some notation and system and count systematically, e.g.:
    One area, 1, 2, 3, 4, 5 and 6 each form a triangle (6)
    Two areas 1-4 and 3-6 each form a triangle (2)
    Three areas 4-1-2, 2-3-6, 3-6-5 and 5-4-1each form a triangle (4)

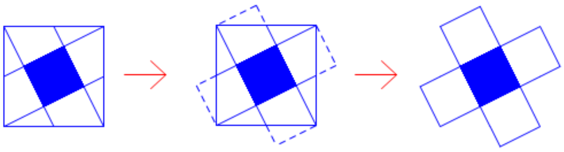
22. Do not count or calculate – look for *structure*, e.g.
    For Pattern 1: 3×1
    For Pattern 2: 3×2
    For Pattern 3: 3×3
    For Pattern 4: 3×4
    For Pattern 100: 3×100



23. Arrange them: O S (R) T (R) E (Ram can be between Siva and Temba or between Temba and Eby)
    Oscar is the shortest
24. 20c + 10c + 5c
    3 × 10c + 5c
25. Number of blocks = 1 + 2+ 3+ 4+ …. + 48 + 49 + 50 = (1+50) + (2 + 49) + … = 25 × 51 = 1275

GRADE 5(1)

1. There are 5 tiles in every metre because 1000 cm ÷ 200 cm = 5. So 15 × 10 = 150 tiles
2. The numbers inside the square *and* the circle are 2 and 3. 2 is not inside the triangle
3. Try and test each possible answer!
4. C – a rotation to the right through 90°
5. 4 reds – 10 greens – 3 purples. So 12 (3×4) reds – 9 (3×3) purples
6. n^{th} row has $2 \times n - 1$ dots, so 7^{th} row has 13 dots
7. n^{th} row has $2 \times n - 1$ dots, so 70^{th} row has $2 \times 70 - 1 = 139$ dots
8. 100 ÷ 24 = 4 rem 4, i.e. 4 full days bringing us to 10:00, plus 4 more hours, i.e. 11, 12, 13, 14:00
   Or 10 + 100 = 110, 110 ÷ 24 = 6 rem 14
9. *Height* = 12 cm + 1,5 cm/day × *days*. So *Height after 30 days* = 12 + 1,5×30 = 57 cm
10. (150 cm – 12 cm) ÷ 1,5 cm/day = 92 days
11. One more than a multiple of 6, so it is odd, so it cannot be A or B. Test the others: 4 182÷6 = 697
12. If a sack weighs S kg, then $3S = S + 30$, so $2S = 30$, so $S = 15$. So $3S = 45$ kg
13. 3, 6, 9, … is the 3-times table. So 50 × 3 = 150
14. Mathematics is $\frac{1}{4}$ of his time, and this is 2 hours. So $\frac{4}{4}$ of his time is 4×2 hours = 8 hours
15. In the bottom layer there are 8 × 4 = 32 blocks, so in two layers there are 64 blocks
16. All the blocks of the bottom layer (32) and all the blocks round the side of the top layer (20)
17. 3×2 + 2 =8; 7×2 + 2 = 16; so for rectangle with length 20: 20×2 + 2 = 42
18.



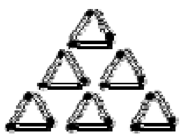
19. 75c more per week, so 12 × 75c =R9
20. There are 31 days in January , of which 15 are even (2, 4, …30). There are 28 or 29 days in February of which 14 are even. All other months have 15 even days. So the total in a year is 11×15 + 14
21. The number must start and end with 1 so list them systematically:
    101 111 121 131 141 151 161 171 181 191
22. Share 30 litres in ratio 5 to 1, i.e. 25 to 5
23. The ones digit of the product of the four numbers is equal to the product of the last digits of the numbers, i.e. 2×6×2×9 which is 6. So the remainder is 1.
24. List them systematically:
    4000
    3100, 3010, 3001
    2200, 2020, 2002
    2110, 2101, 2011
    ~~2020, 2002. 2011~~
    1300, 1030, 1003
    1210, 1201
    1120, 1102
    1111
    ~~1030, 1003~~
    1021, 1012
25. If a small pizza costs S rands and a large pizza costs L rands: $2S + 1L = 5S$, so $1L = 3S$, so the cost is $L = 3 \times$R11,50 = R34,50

GRADE 5(F)

1. There are nine 1×1 squares, four 2×2 squares, and one 3×3 square. Total = 9 + 4 + 1
3. Build a mental picture! B, E & F
4. The trip is 31 minutes long, so 12:40 + 31 minutes = 13:11
5. Use trial and error, i.e. try each of the given answers one by one
6. Sipho has 32 marbles, so Landi has 20 marbles. So together they have 32+20 = 52 marbles
7. (2 – 1) + (3 – 2) + (4 – 3) + … + (100 – 99) + (101 – 100) = 1 + 1 + 1 + 1 + ... 100 times = 100
8. If $\frac{2}{5}$ is 12 leaners, then $\frac{1}{5}$ is 12÷2 = 6 learners, and $\frac{5}{5}$ (the whole class) is 5×6 = 30 learners
9. $\frac{3}{4}+\frac{1}{4}+\frac{3}{4}+\frac{1}{2}=2\frac{1}{4}$. So there is $3-2\frac{1}{4}=\frac{3}{4}$ left for Oscar
10. 6 × 3 + 2 × 4 = 26 cm
11. 41000 g – 725 g = 40275 g = 40,275 kg
12. Divide 420 into 7 equal parts:420 ÷ 7 = 60. 3 of these parts are dresses, i.e. 3 × 60 = 180
13. If Kim has Δ stamps, then Jack has Δ+Δ stamps.
    So Δ + Δ+Δ + 40 = 220
    So Δ + Δ + Δ = 180
    So Δ = 60, so Kim has 60 stamps
14. Consider the possible choices from the top row:
    If I choose 1, then the options are 1, 5, 9 or 1, 6, 8 giving products 45 or 48 respectively.
    If I choose 2, the options are 2, 4, 9 or 2, 6, 7 with products 72 or 84 respectively.
    If I choose 3, the options are 3, 4, 8 or 3, 5, 7 with products 96 or 105.
    So 105 is the maximum possible product.

1	2	**3**
4	**5**	6
7	8	9

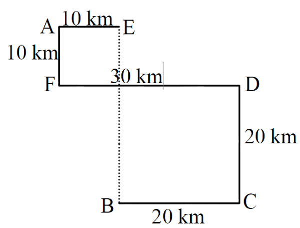
15. $T_n = 3 \times n + 1$. So $T_{50} = 3 \times 50 + 1 = 151$
16. $P_n = 3 \times n + 3$ or $3 \times (n + 1)$, so $P_{50} = 3 \times 51 = 153$
17. The first three may be blue, red and brown. Then the next one must match one of these colours
18. You can maybe take out, e.g. 10 red, then 10 brown, then 1 blue, then the next one is also blue
19. Look at the *structure* in the pictures!
    T_1: 1 = 1×1
    T_2: 1 + 3 = 4 = 2×2
    T_3: 1 + 3 + 5 = 9 = 3×3
    ….
    T_{10}: 1 + 3 + 5 + 7 + ... to 10 numbers = 10×10 triangles
20. Look at the *structure* in the pictures! Count the number of *triangles:*
    # triangles in T_1 = 1
    # triangles in T_2 = 1 + 2
    # triangles in T_3 = 1 + 2 + 3
    # *triangles* in T_{10} = 1 + 2 + 3 + 4 + ... + 9 + 10 = (1 + 10)×10/2 = 55
    So # *matches* = 55×3



21. Because 365 = 52×7 + 1, the day of the week moves one day later each year (but remember leap years!)

2013	Wed
2014	Thu
2015	Fri
2016	Sun - 2016 is a leap year!
2017	Mon
2018	Tue
2019	Wed

22. Draw it! Fill in the information as you read. Re-read, bit by bit!



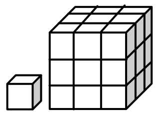
23. Be systematic, e.g.

32	23	43	13
34	24	42	12
31	21	41	14

24. There are 28 days in February of which 14 are odd (1, 3, …27)
    There are 30 days in Apr, Jun, Sep and Nov, of which 15 are odd (1, 3, …29)
    The other 7 months have 31 days of which 16 are odd days
    So the total odd dates in a year is 14 + 4×15 + 7×16 = 186
25. Debbie is first, Peter is second, Tom is third and Robert is fourth.

GRADE 6(1)

1. Make equal parts. Each small square is half of the next bigger square.
   So half of half of the big square is a quarter of the big square
2. There are 8 columns, each with 2 + 4 + 6 cubes. So 8 ×12 = 96 cubes
3. $\frac{1}{7} = \frac{5}{35}$ and $\frac{1}{5} = \frac{7}{35}$ so $\frac{6}{35}$ is exactly in between them. Or $(\frac{1}{5} + \frac{1}{7}) \div 2 = (\frac{7}{35} + \frac{5}{35}) \div 2 = \frac{6}{35}$
5. Use trial and error, i.e. try each of the given answers one by one
6. In middle row the missing number is 18 – (11+6) = 1, so in right column A = 18 – (1+10) = 7
8. Continue the patterns: 17, 22, 27, 32, 37, 42, 47, 52, … and 17, 24, 31, 38, 45, 52, …
9. For n dice, the number of visible faces is $n\times3 + 2$. So for 75 dice, $75\times3 + 2 = 227$
10. Imagine or draw the cube! If the side is 3 times as long, the big cube contains
    27 of the small cubes. So its mass is 27 times as large!

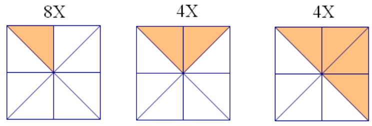


11. 0×20 + 3×10 + 1×5
    1×20 + 1×10 + 1×5
12. 102 ÷ 7 = 14 rem 4, so adding 3, we have 105 ÷ 7 = 15
14.

B	C	M
A	2	N
1	D	

In the middle row, N cannot be 2, so N is 1 or 3
Suppose N = 3. Then A = 1 which is impossible (already a 1 in left column).
So N = 1, A = 3. In left column B = 2. Then C = 1 (D ≠ 1), so M = 3, so M+N = 4

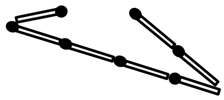
15. List the triangles systematically - *notation* and a *system* will help!



16. Count equal parts! There are 18 equal parts and 9 of them are shaded.
17. Vary the numbers systematically and note the behaviour of the product of the numbers:

1 + 17 = 18 and 1 × 17 = 17	6 + 12 = 18 and 6 × 12 = 72
2 + 16 = 18 and 2 × 16 = 32	7 + 11 = 18 and 7 × 11 = 77
3 + 15 = 18 and 3 × 15 = 45	8 + 10 = 18 and 8 × 10 = 80
4 + 14 = 18 and 4 × 14 = 56	9 + 9 = 18 and 9 × 9 = 81
5 + 13 = 18 and 5 × 13 = 65	10 + 8 = 18 and 10 × 8 = 80

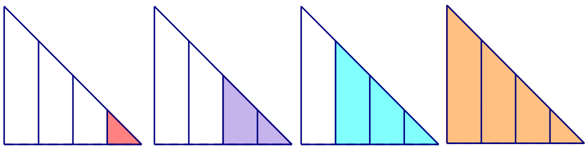
18. ? = 000ΔΔΔ = 0ΔΔΔ + $\frac{1}{2}$(0000ΔΔ) = 6□ + 4□ from first two balances



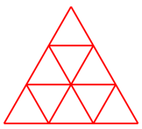
19. If the numbers are x and y: $6\times x + y = 17$. So $17 - y$ must be a multiple of 6, i.e. 12, so $y = 5$
    Note: If $17 - y = 6$, $y = 11$, which is not a *one*-digit number!
20. 331 and 322 (the sum of any two sides must be greater than the third side – why?)
21. 16 out of 24 marbles are not blue, so the probability of choosing a not-blue marble is $\frac{16}{24} = \frac{2}{3}$.
22. If the empty glass has a mass of g gram and the milk has a mass of m gram, then
    $g + m = 370$
    $g + \frac{1}{2}m = 290$
    So $\frac{1}{2}m = 370 - 290 = 80$ gram, so $m = 160$ gram and $g = 370 - 160 = 210$ gram
23. Each number is the sum of the two numbers above it, e.g. 6 = 1 + 5, 15 = 5 + 10
24. If a bubble gum cost B cents and a chocolate costs C cents:
    $B + C = 90$ and $10B + 5C = 470$, so $5B + 5(B + C) = 470$, so $5B + 5\times90 = 470$, so $B = 4$, so $C =$ R0,86
25. 1, 4, 9, … = 1×1, 2×2, 3×3, … 20×20

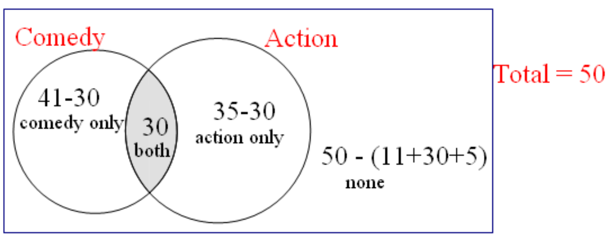
GRADE 6(F)

1. Try trial and improvement, e.g. $50 + 51 + 52 \neq 174$; …. But $57 + 58 + 59 = 174$
   Or test each of the given numbers …
   Or, if the smallest is x, then $x + (x + 1) + (x + 2) = 3 \times x + 3 = 174$, so $x = 57$
2. The average of the two numbers: $(7{,}8 + 7{,}85) \div 2 = 15{,}65 \div 2 = 7{,}825$
3. $(\frac{1}{4} + \frac{1}{3}) \div 2 = \frac{7}{12} \div 2 = \frac{7}{24}$
4. There are 4 different *sizes* of triangles as shown. Total = 8 + 8 + 2 + 2:



×8 ×8 ×2 ×2

5. Full lorry = 4653 kg; empty lorry = 2583 kg; 4653 – 2583 = 2070; 2070 kg ÷ 90 kg/bag = 23 bags
6. (2000 – 1999) + (1998 – 1997) + … + (2 – 1) = 1 + 1 + 1 + 1 + … + 1 (1000 times)
7. 
8. ½ of 3×3 cm^2 + 3×3 cm^2 + ½ of 3×1 cm^2 = 4, 5 cm^2 + 9 cm^2 + 1,5 cm^2 = 15 cm^2
9. Check each of them, e.g. for (A): $60\times2 + 10\times4 = 160$ does not give 140 legs. But (E) does: $30\times2 + 20\times4 = 160$
10. $100 \div \mathbf{20} = 5$; $3000 \div \mathbf{20} = 150$.
    Or $\frac{5}{100} = 5\%$. 5% of 3000 = 150
11. $\frac{2013+2012}{2013-2012}$
12. $\frac{1}{2} + \frac{1}{8} + \frac{1}{8} = \frac{3}{4}$; R15 is $\frac{1}{4}$; R60 = $\frac{4}{4}$
13. If the jersey costs Rx, the coat costs Rx + 150. Together they cost $x + x + 150 = 650$
    So $x = (650 - 150) \div 2 =$ R250
14. There is a general structure here: The denominators is twice the numerator + 1, i.e. $\frac{\lozenge}{2\times\lozenge+1}$
    We can therefore investigate a general pattern $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \frac{6}{13}, \frac{7}{15}, \ldots$
    Check with your calculator: $\frac{1}{3} = 0{,}333\ldots$, $\frac{2}{5} = 0{,}4$, … So $\frac{1}{3} < \frac{2}{5} < \frac{3}{7} < \frac{4}{9} < \frac{5}{11} < \frac{6}{13} < \frac{7}{15} < \ldots$
    Conclusion: the larger the denominator, the larger this kind of fraction, so $\frac{11}{23}$ is the largest
15. If the number is ◊, then ◊ + 1/3 of ◊ = 52, so 4/3 of ◊ = 52. So 1/3 of ◊ = 13, so 3/3 of ◊ = 39
16. Jane eats 2×12 sweets in 5 minutes; she eats 2×24 sweets in 10 minutes. Jane eats 48 sweets in 10 minutes
17. Make a representation of the situation (draw it):



18. There are nine 1-digit numbers, ninety 2-digit numbers giving us $9\times1+90\times2 = 189$ digits. So we need $852-189 = 663$ more digits. $663 = 221\times3$ so we need 221 3-digit numbers, thus the numbers from 100 to 320. So there are 320 pages
19. For 19 wheels we can have: 1 tricycle and 8 bicycles = total of 9 (too much)
    3 tricycles and 10 bicycles = total of 13 (too much)
    5 tricycles and 2 bicycles = total of 7 (just right)
20. Look at the *structure* in the pictures!
    $P_1 = 4 \times 1 + 1 = 5$
    $P_2 = 4 \times 2 + 1 = 9$
    $P_3 = 4 \times 3 + 1 = 13$
    $P_{50} = 4\times50 + 1 = 201$

21. Work systematically!
    101, 111, 121, 131, 141, 151, 161, 171, 181, 191 – this is 10
    202, 212, 222, 232, 242, 252, 262, 272, 282, 292 – this is 10
    ….
    909, 999, 929, 939, 949, 959, 969, 979, 989, 999 – this is 10
    So $9\times10 = 90$
22. Make a list, varying the persons systematically. If the persons are a, b, c and d:
    abcd, abdc, acbd, acdb, adbc, adcb and similarly if the first person is b, c, and d. So $6 \times 4 = 24$. Or $4 \times 3 \times 2 \times 1 = 24$
23. Let the children be A, B, C, D and E. List all the possibilities systematic, note patterns and structure:

A vs B	**B** vs C	**C** vs D	**D** vs E
A vs C	B vs D	C vs E	
A vs D	B vs E		
A vs E			

vs	A	B	C	D	E
A		X	X	X	X
B			X	X	X
C				X	X
D					X
E					

24.

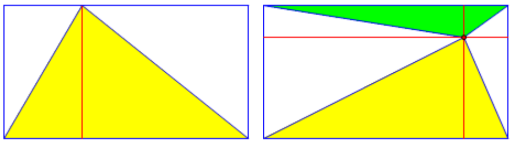
A vs B	**B** vs C	….	**H** vs I	**I** vs J
A vs C	B vs D	….	H vs J	
A vs D	B vs E	….		
A vs E	B vs F	….		
A vs F	B vs G	….		
A vs G	B vs H	….		
A vs H	B vs I	….		
A vs I	B vs J			
A vs J				

The structure is: $9 + 8 + ... + 2 + 1 = \mathbf{45}$

25. Look at the *structure* in the pictures!
    P_1: $1 = 1\times1$
    P_2: $1 + 3 = 4 = 2\times2$
    P_3: $1 + 3 + 5 = 9 = 3\times3$
    P_4: $1 + 3 + 5 + 7 = 16 = 4\times4$
    ….
    P_{50}: $1 + 3 + 5 + 7 + ...$ to 50 numbers $= 50\times50 = 2500$

GRADE 7(1)

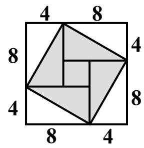
2. $3{\times}3 - 3 + 3 = 9 - 3 + 3 = 6 + 3 = 9$
3. nth number = $2{\times}n - 1$, so 83rd number = $2 \times 83 - 1 = 165$
4. & 5.



6. We know: $\dfrac{\text{Sum of numbers}}{11} = 8$, so Sum of numbers = $11 \times 8 = 88$

   If the new number is x, then $\dfrac{88+x}{12} = 11$. So $x = 12 \times 11 - 88 = 44$

7. $1+\dfrac{1}{1+\frac{2}{3}} = 1+\dfrac{1}{\frac{5}{3}} = 1+\frac{3}{5}$
8. First fit the tiles in the width: If one tile is used in the width, it has a side of 2,5 m, and cannot cover the length exactly. If 2 tiles are used in the width the tiles have a length of 1,25 m, and then 3 of them can fit into the length. So $2{\times}3 = 6$ tiles are the minimum number of tiles.
9. 45 Ʊ 6: $45{\div}6 = 7$ res 3, dus is 45 Ʊ 6 = 3. Dan vir 123 Ʊ 3: $123{\div}3 = 41$ res 0, dus is 123 Ʊ 3 = 0
10. The largest, by guess-and-improvement is $31 \times 31 = 961$. So there are 31 squares smaller than 1000
11. # *Triangles* = 2×*squares* + 2, or 2×(*squares* + 1). So *Triangles (6)* = 2×6 + 2 = 14
12. *Triangles (60)* = 2×60 + 2 = 122
13. $2{\times}x + 2 = 60$, so $x = 29$
14. Make a list, varying the numbers systematically. If the digits are a, b, c and d:
    abcd, abdc, acbd, acdb, adbc, adcb and similarly if the first digit is b, c, and d. So $6 \times 4 = 24$

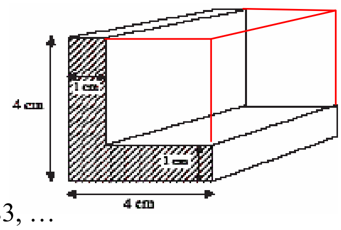


15. $2 \times (7 + 8 + 9) = 2 \times 24$
16.

	c	d
a	**12**	**20**
b	**21**	D

Using a representation like this, Area D = b × d
We know a × c = 12, b × c = 21, a × d = 20
Multiply them all together: $a^2 \times c^2 \times b{\times}d = 12 \times 20 \times 21$
But $a{\times}c = 12$, so $a^2{\times}c^2 = 144$, so $b{\times}d = 12{\times}20{\times}21 \div 144 = 35$

17. Volume = area of base × length = 7 cm^2 × 12 cm = 84 cm^3
    Or think of cutting out a rectangular prism:
    Volume = $4{\times}4 \times 12 - 3{\times}3 \times 12 = 7 \times 12$

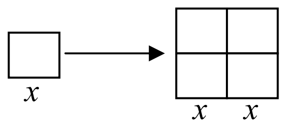


18. The first digit can be 2, 4, 6 or 8. The second digit can be 0, 2, 4, 6 or 8, which gives $4 \times 5 = 20$ possible combinations
19. The 6th column are multiples of 6, with formula 6 × row n.
    So the last number in row 80 is $6 \times 80 = 480$. Then row 81 is 481, 482, 483, …
20. T+F = R+17
    So 7+F=5+17
    So F=15
21. Fill in numbers in the calendar, and test each statement with the numbers.
22. Test specific cases, e.g. if $a = 5$, then $b = 6$, $c = 12$ and $d = 13$, the a+b+c+d = 36, which is not correct. Choose a better value for a …
    Or: We know $a + d = c + b$, so $a + b + c + d = a + d + c + b = 2{\times}(a + d) = 52$.
    So $a + d = 26$, so $a + (a + 8) = 26$, so $a = 9$
23. Choose different consecutive numbers, and test each statement with the numbers.
24. 3 lines from two corners divide the triangle in 4 × 4 sections
    10 lines from two corners will divide the triangle in 11 × 11 sections = 121
25. **6** pencils and **4** pens cost R62
    4 pencils and **6** pens cost R84
    Add them:
    10 pencils and **10** pens cost R146
    Divide by 2:
    5 pencils and **5** pens cost R73

GRADE 7(F)

1. The L-shaped region can be decomposed into a 4×1 rectangle and a 3×1 rectangle. So the total area is 7 cm^2
2. 4 + 1 + 3+ 3 + 1 + 4 = 16 cm
3. (2 – 1) + (3 – 2) + (4 – 3) + … + (100 – 99) + (101 – 100) = 1 + 1 + 1 + 1 + ... 100 times = 100
4. Numbers ending with 1, 2, or 5 have this property. They are 11, 12, 15, 21, 22, 25, 31, 32, 35, 41, 42, and 35
   In addition, we have 24, 33, 36, 44 and 48, for a total of 17
5. n^{th} number = $2\times n - 1$, so 83^{rd} number = $2 \times 83 - 1 = 165$
6. In middle row the missing number is 18 – (11+6) = 1, so in right column x = 18 – (1+10) = 7
7. Do not rush into calculation! *Look for structure!* $\frac{24\times18\times15+24\times18\times13+24\times18\times7}{24\times18}=\frac{24\times18\times(15+13+7)}{24\times18}=35$
8. b and c are both less than 1, so $b\times c$ is less than both b and c.
9. We know: $\frac{\text{Sum of numbers}}{11}=8$, so Sum of numbers = 11 × 8 = 88

   If the new number is x, then $\frac{88+x}{12}=11$. So $x = 12 \times 11 - 88 = 44$
10. $\frac{7}{8}-\frac{1}{2}=\frac{3}{8}$ = 420 litres, so $\frac{1}{8}$ = 420 litres ÷ 3 = 140 litres. So the full tank = $\frac{8}{8}$ = 140 litres × 8 = 1120 litres
11. *Look* at the structure: For n dice, the number of visible faces is $n\times3 + 2$. So for 30 dice, $30\times3 + 2$
12. If 50 faces are visible, $n\times3 + 2 = 50$, so $n = 16$
13. There are 8 possibilities: GGG, GGB, GBG, GBB, BGG, BGB, BBG, BBB. 'At least one girl' means 1, 2 or 3 girls, and only in the case of BBB is there no girl. So the probability is 7/8.
14. $\frac{1}{4}+(\frac{1}{4})^2+(\frac{1}{4})^3+(\frac{1}{4})^4=\frac{4^3+4^2+4^1+4^0}{4^4}=\frac{85}{256}$
15. To be divisible by 5, the last digit must be 5. But to be divisible by 2, the last digit must be 2 or 4. So none of these numbers can be divisible by 2 and 5, so none of them can be divisible by 1, 2, 3, 4, and 5.
16. Starting with the primes: 23, 29, 31, 37, 41, 43, 47,53, 59, 61, 67, 71, 73, 79, these are also prime 13, 73, 17, 37, 97, and of the five 31+1, 37+1, 71+1, 73+1 and 79+1, only 71+1 is a multiple of 3
17. Write as the product of factors, but do not repeat factors, e.g. do not write 6 = 2×3, because it is already there!
    $5\times7\times8\times9 = 5\times7\times2\times2\times2\times3\times3$
18. You must take all possible combinations of the numbers 19, 29, 59 and 79 (they are all prime). Do not calculate, simply systematically count all possible combinations. It is the same as taking all possible combinations of the letters A, B, C and D if the order does not matter:
    1 at a time: A, B, C, D – so 4
    2 at a time: AB, AC, AD, BC, BD, CD – so 6
    3 at a time: ABC, ABD, ACD, BCD – so 4
19. If the length of a square doubles (×2), then the area quadruples (x4), as illustrated in this simple example

    If the dimensions of the room is a by b by c, then the area to paint is A = $ab + 2ac + 2bc$
    Double the dimensions are $2a$ by $2b$ by $2c$, so the area to paint is D = $(2a)(2b) + 2(2a)(2c) + 2(2b)(2c) = 4\times$A
20. Add all together: 2A + 2B + 2C = 42, so A + B + C = 21
21. B + A + C = 21 and A + C = 16, so B + 16 = 21
22. Take special cases, be systematic, and notice the patterns:

    1 number: $\frac{1}{2}=\frac{1}{2}$

    2 numbers: $\frac{1+3}{2+4}=\frac{4}{6}=\frac{2}{3}$

    3 numbers: $\frac{1+3+5}{2+4+6}=\frac{9}{12}=\frac{3}{4}$

    50 numbers: $\frac{1+3+5+...+99}{2+4+6+...+100}=\frac{50}{51}$

    Alternatively, if you know or develop some formulas: $\frac{1+3+5+7+...+97+99}{2+4+6+8+...+98+100}=\frac{50^2}{50\times51}=\frac{50}{51}$
23. Suppose Xolile had x marbles. After giving 1/3 to Baba, he had 2/3 remaining; or 2/3 of x. After giving 1/4 of the remainder to Sam, he had 3/4 of them left, or 3/4 of 2/3 of x which equals 24.So 1/2 of x equals 24, so x = 48. This means she gave Baba 1/3 of 48 = 16 marbles
24. 1+3+6+10+15+21 = 56
25. Do not count or calculate, investigate the *structure:* 1, 4, 9, … = 1×1, 2×2, 3×3, … 20×20