

Modelling a Maths Competition Logo

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INTRODUCTION

At St Andrew's College and The Diocesan School for Girls we have recently introduced 'Single Focus Days'. Once every two weeks the whole Wednesday morning is devoted to a single subject for each of the five grades. The purpose of these Single Focus Days is twofold. Firstly, for subjects that have extended practical components, it means the entire morning can be given over to those subjects without pupils having to miss other classes. Secondly, it allows space for creativity with respect to cross-curricular project-based activities.

As part of a recent Single Focus Day our Grade 9s were tasked with designing a logo for a mathematics competition. We had just concluded a module on straight line graphs, and the idea behind the project was to draw together, in a practical context, the various concepts that had been covered in the module. The project worked particularly well, and in this article I share the activity itself and reflect on what made the process so successful.

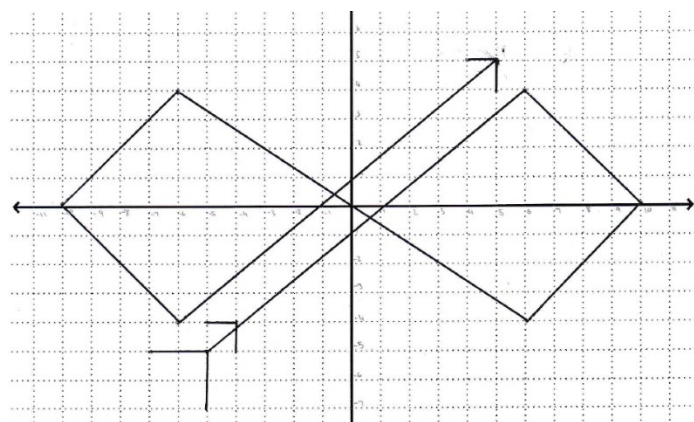
THE TASK

The brief, as provided to the students, was to design a logo for an upcoming mathematics competition. The logo had to be constructed from straight line segments only, and had to meet the following minimum requirements:

- The logo had to contain at least 6 oblique straight line segments
- The logo had to contain at least one vertical line segment
- The logo had to contain at least 1 horizontal line segment
- The logo had to extend into at least three quadrants of the Cartesian plane
- The coordinates of the end points of all line segments had to be clearly indicated

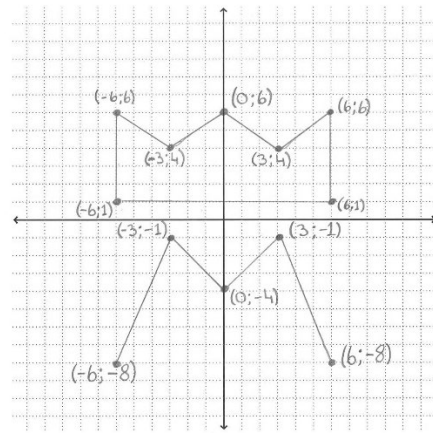
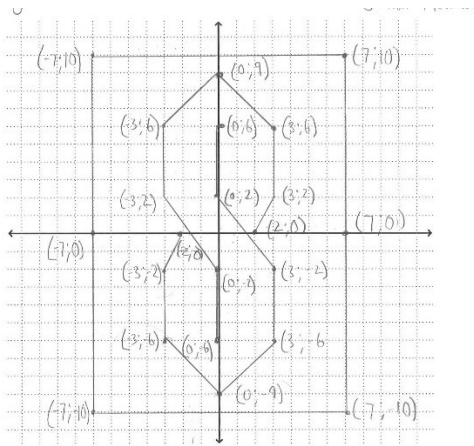
PART 1

Pupils were provided with a Cartesian plane for the initial design phase. The design itself could be as simple or complex as they wanted, provided the minimum requirements as outlined above were met. Pupils were encouraged to carry out this design phase using pencil so that they could make changes to their design as it developed.



PART 2

For the next part of the process, pupils needed to provide a clear set of instructions that a production company would be able to use to manufacture the logo. In order to do this, pupils were required to provide the exact equation of each straight line segment along with its domain (or range in the case of vertical lines).

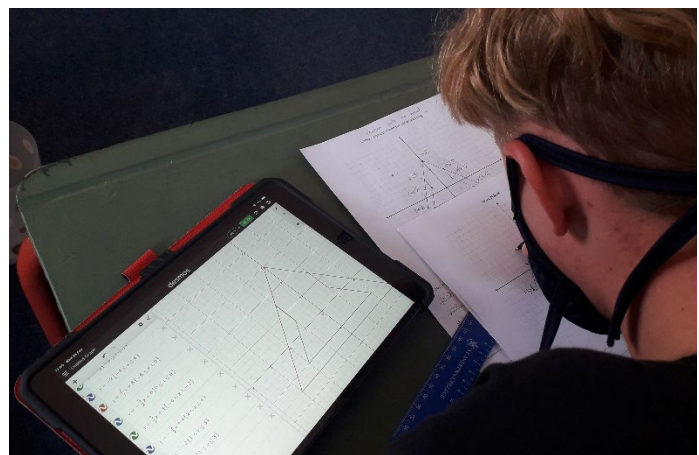
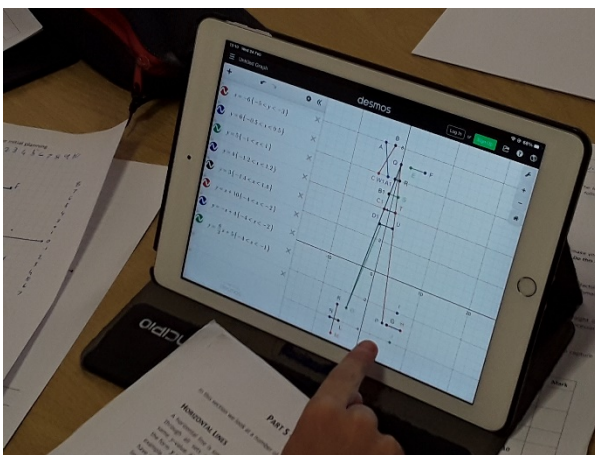


- Instructions:
- | | |
|---|------------------------------------|
| 1. $y = -1x + 9 \{ 0 < x < 3 \}$ | 9. $x = 3 \{ -6 < y < -2 \}$ |
| 2. $y = -\frac{4}{3}x + 12 \{ 0 < x < 3 \}$ | 10. $x = 0 \{ 2 < y < 6 \}$ |
| 3. $y = 2x - 4 \{ 2 < x < 3 \}$ | 11. $x = 0 \{ -6 < y < -2 \}$ |
| 4. $x = 3 \{ 2 < y < 6 \}$ | 12. $x = 7 \{ -10 < y < 10 \}$ |
| 5. $y = -1x + 9 \{ -3 < x < 0 \}$ | 13. $x = -7 \{ -10 < y < 10 \}$ |
| 6. $x = -3 \{ 2 < y < 6 \}$ | 14. $y = 10 \{ -7 < x < 7 \}$ |
| 7. $y = -\frac{4}{3}x - 2 \{ -3 < x < 0 \}$ | 15. $y = -10 \{ -7 < x < 7 \}$ |
| 8. $y = 1x - 9 \{ 0 < x < 3 \}$ | 16. $x = -3 \{ -6 < y < -2 \}$ |
| 17. $y = 2x + 4 \{ -3 < x < -2 \}$ | 18. $y = -1x - 9 \{ -3 < x < 0 \}$ |

- Instructions:
- | | |
|--|---|
| M: | Crown: |
| $y = -1x - 4 \{ -3 < x < 0 \}$ | $x = -6 \{ 1 < y < 6 \}$ |
| $y = \frac{7}{3}x + 6 \{ -6 < x < -3 \}$ | $y = 1 \{ -6 < x < 6 \}$ |
| $y = 1x - 4 \{ 0 < x < 3 \}$ | $x = 6 \{ 1 < y < 6 \}$ |
| $y = -\frac{7}{3}x + 6 \{ 3 < x < 6 \}$ | $y = \frac{7}{3}x + 2 \{ 3 < x < 6 \}$ |
| | $y = -\frac{7}{3}x + 6 \{ 0 < x < 3 \}$ |
| | $y = \frac{7}{3}x + 6 \{ -3 < x < 0 \}$ |
| | $y = -\frac{7}{3}x + 2 \{ -6 < x < -3 \}$ |

PART 3

The next part of the project involved a quality control process. Pupils first had to download *Desmos Graphing Calculator* onto their device. Once we had discussed how to enter the various equations, in particular the inequality notation required for restricted domains and ranges, pupils had to type in each of their equations and cross-check that this correctly produced their logo. If necessary, corrections were then made to the various equations until pupils were satisfied that their instructions were accurate.



PART 4

The last part of the project was to neatly capture the final version of the logo which would be submitted and marked according to the following rubric:

Component	Max	Mark
Creativity How creative is your design? How appealing is it from a visual perspective? How relevant is it to a Maths competition?	5	
Complexity How many different elements does your design contain? Are there parallel lines, perpendicular lines or elements of symmetry?	5	
Minimum requirements Have you met the five minimum requirements?	5	
Accuracy Does your set of instructions accurately produce your logo? Have you set out your instructions in a clear and methodical way?	10	
	25	

REFLECTING ON THE PROCESS

Feedback from staff and pupils alike was that they really enjoyed the project. For pupils who had struggled with some of the concepts while formally covering straight line graphs in class, the practical context afforded by the project allowed them to revisit these concepts in a tangible and meaningful manner. There were some wonderful “Aha!” moments when certain concepts finally fell into place.

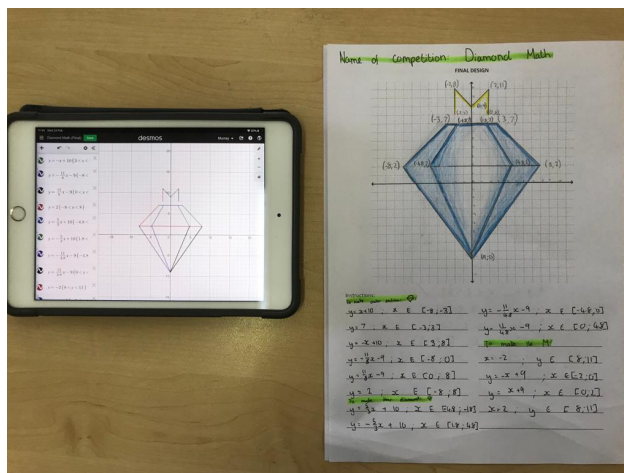
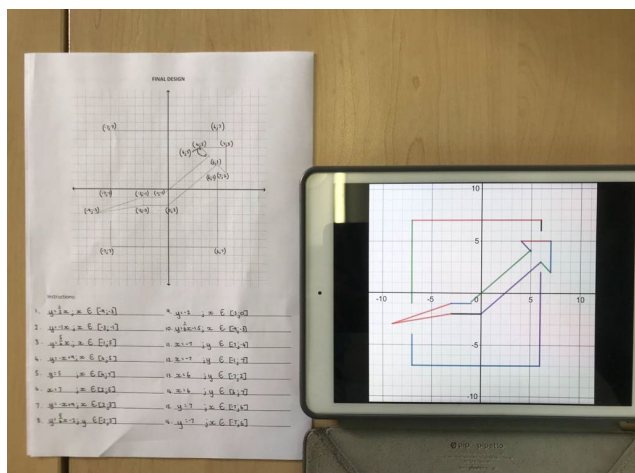
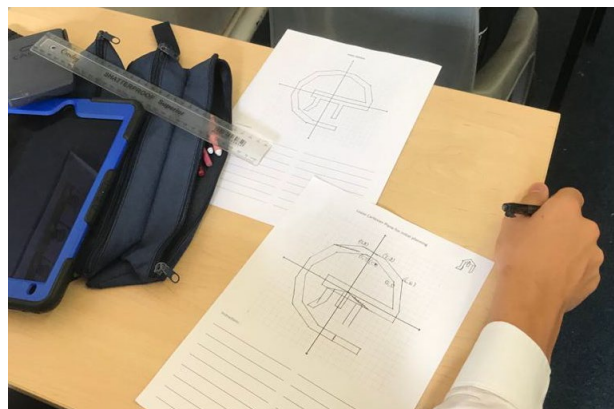
The process of getting pupils to cross-check the accuracy of their equations by typing them into appropriate graphing software worked particularly well. Pupils got immediate feedback, which they were able to interpret themselves, by seeing whether or not their equations accurately produced their design. Pupils were also able to ‘reverse-engineer’ their design by fiddling with the various parameters of their equations once they had been set up in *Desmos*. This afforded a wonderful opportunity to see how changing the different parameters affected their various straight line segments.

Incorporating restricted domains (or ranges in the case of vertical lines) into the design process gave this otherwise somewhat arcane concept a practical context. Pupils experienced first-hand why it might be necessary to restrict a domain, and for many pupils the concept of a restricted domain finally made sense. This also provided an opportunity to engage with both interval notation as well as inequality notation in a meaningful setting.

Many of the designs created by the pupils were symmetrical in structure – some had vertical and/or horizontal lines of symmetry, while others had oblique lines of symmetry. The concept of reflecting points and lines across the two axes thus arose naturally out of the design process. This led to some wonderfully serendipitous moments – for example when pupils became aware of the relationship between the gradients of lines that are reflections of one another across the vertical axis.

Perhaps one of the aspects of the project that I enjoyed the most was being able to wander around the class watching and listening to pupils engaged in the process. Pupils were discussing details and sharing ideas all the while making use of correct mathematical terminology. These discussions arose naturally over the course of the morning, as did the need to make use of correct mathematical terminology.

Because the design brief came with clear minimum requirements, this gave pupils the freedom to be as creative as they wanted provided the minimum requirements had been met. While some pupils who were still grappling with the mathematics of straight line graphs worked slowly on relatively simple designs, others who were more confident with the underlying mathematics were able to really shine and express their creativity.



CONCLUDING COMMENTS

As a way of rounding off the section on straight line graphs, this project worked particularly well. It allowed those pupils who were confident with the content to forge ahead, while it provided a practical context for other pupils to revisit certain concepts and to consolidate their understanding. While we used the project specifically in Grade 9 with straight line graphs, it could readily be extended to other grades where one could incorporate additional graphs such as parabolas and exponentials. We were fortunate to have an extended period of time where pupils could knuckle down, uninterrupted, and really engage with the process. It was time very well spent, and we will certainly do something similar in future years.

