The South African Mathematics Olympiad – Third Round for Juniors

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INTRODUCTION

The South African Mathematics Olympiad (SAMO) began a little over 50 years ago in 1966. For the first 25 years or so the Olympiad consisted of two rounds – the top 100 pupils from the first round being invited to take part in the second and final round. The first round was already quite difficult, and the whole Olympiad was aimed at pupils who were especially mathematically adept. In the early 1990s the structure of the Olympiad was changed to three rounds. The rationale for this was to have a first round question paper that was accessible to a far greater number of pupils, thereby exposing many more pupils to Olympiad type thinking, and in turn stimulating interest and enthusiasm with respect to Olympiad involvement and mathematics in general. The challenge for the question setting committee is thus to come up with first round questions that are sufficiently accessible, while not being tied to the formal school curriculum, and which are interesting and require some real thinking. The first round consists of 20 questions, and every pupil who gets ten of more of the questions correct proceeds to the second round. Currently approximately 100 000 pupils take part in the first round each year. Of these about 10 000 juniors (Grades 8 & 9) and 8 500 seniors (Grades 10 – 12) make it through to the second round. Based on the second round results the top 100 juniors and the top 100 seniors are invited to take part in the third and final round.

Until approximately 15 years ago there was only a single third round paper, with juniors and seniors writing the same paper – six gruelling questions to be answered over a period of four hours. For most junior pupils this expectation was unreasonable, so a separate third round paper was introduced for Grade 8 & 9 pupils. The junior third round paper is also four hours long but consists of 15 shorter questions, for a total of 100 marks. Full solutions and working details need to be shown for all 15 questions. The challenge for the question setting committee is to come up with interesting and challenging questions that require genuine mathematical reasoning. As a guide, the question committee tries to include, amongst others, questions of the following types:

- Problems inspired by real life
- 'Easy' proof-type problems
- Games that involve a winning strategy
- Classic age-old problems (sometimes with a twist)
- Logic puzzles
- Questions whose solutions are unexpected or counterintuitive
- Interesting problems with clever solutions

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Over the fifteen or so years since the junior third round was introduced in 2004, the question committee has developed close on 250 questions tailor-made to the style of the junior third round. This represents a veritable treasure trove of fascinating, inspiring and thought-provoking material. It is therefore a little sad that so few people (100 juniors each year) get to enjoy these questions. The purpose of this article is to inspire teachers to explore this wonderful treasure chest and to incorporate the occasional question into their teaching. These are questions that require reflection, tinkering, experimentation, looking for patterns, conjecturing, visualising and generalising – all those wonderful mathematical habits of mind that we try to instil and nurture in our pupils. All the junior third round question papers, along with full written solutions, are available on the SAMO website (https://www.samf.ac.za/en/sa-mathematics-olympiad). All you need to do is register as an online user to gain access to all past question papers and solutions. You can follow the instructions on how to do this here:
https://www.samf.ac.za/content/files/FormsForSchools/Access_Olympiad_past_papers.pdf

To give you a taste of the wonderful wealth of questions on offer, here are a few to whet your appetite.

Problems inspired by real life

The picture shows a gift that you can buy in curio shops. It is a calendar which tells you the date and consists of two loose cubes which can be moved and rotated in any way. There must always be two numbers on display and in this case the date is 16 February. (Don't worry about the month which is displayed below the cubes)

What numbers must be on the six faces of each of the cubes so that all the necessary days of any month can be displayed?

(2012 Q8)

'Easy' proof-type problems

Did you know that if you form a four digit number using any four non-zero digits on the corners of any rectangle on a calculator the number will always be divisible by 11? In the example in the picture we have 7128.

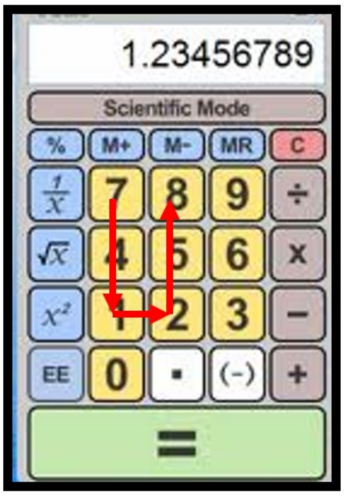
Look : $\frac{7128}{11} = 648$ which is an integer

5236 also works: $\frac{5236}{11} = 476$ which is an integer

(a) Prove it.

(b) Prove that it even works if you rotate the calculator 90 degrees clockwise. (i.e. if you use numbers like 2365 or 4697)

(2012 Q10)



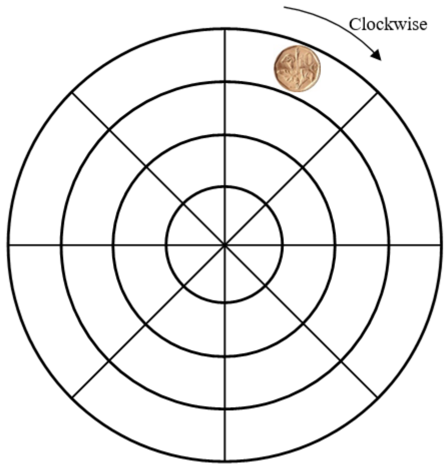
Games that involve a winning strategy

Zola and Ron play a game by alternately moving a single ten cent coin on a circular board. The game starts with the ten cent coin already on the board as shown. A player may move the coin either clockwise one position or one position toward the centre, but cannot move to a position that has been previously occupied. The last person who is able to move wins the game.

If Zola starts, which player can play in a way that guarantees a win?

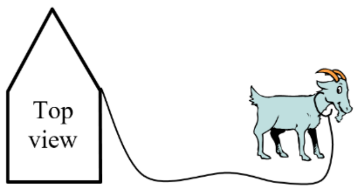
Explain this player's winning strategy.

(2016 Q10)

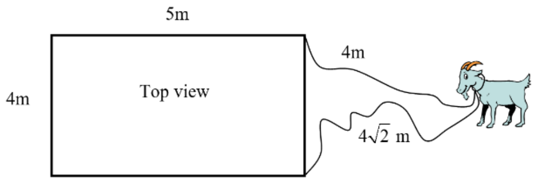


Classic age-old problems (with a twist)

(a) A goat is tethered to the corner of a shed which consists of a square and an equilateral triangle. The square has side length 2 m and the rope is 5 m long. What is the maximum area that the goat can graze outside of its shed? Give your answer in terms of π.



(b) This time the goat is tethered to the corner of a rectangular shed 4m by 5m, but with two ropes of length $4\sqrt{2}$ m and 4 m as shown. What is the area the goat can graze? Again, give your answer in terms of π.



(2012 Q14)

Logic puzzles

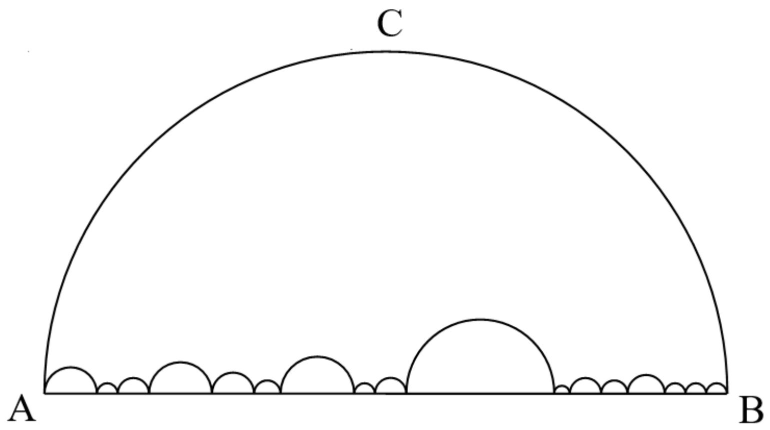
Prove that the alphanumeric, in which different letters represent different digits, does not have a solution.

$$\begin{array}{r} \text{TWENTY} \\ +\,\text{TWENTY} \\ \hline \text{CRICKET} \end{array}$$

(2015 Q13)

Questions whose solutions are unexpected or counterintuitive

When you travel from A to B, you can either travel along the big semi-circle (i.e. via C) or you can travel along all the smaller semi-circles. Which is the shorter route and why?

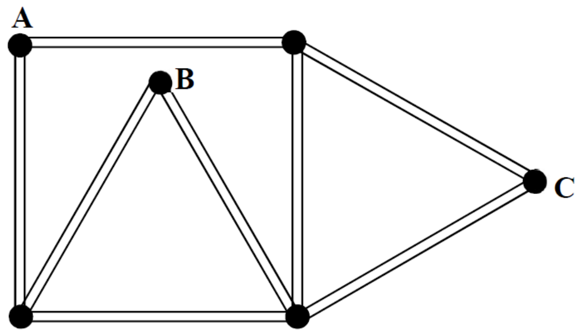


(2012 Q5)

Interesting problems with clever solutions

Matchsticks are arranged as shown.

Prove that the matchstick heads A, B and C lie on a straight line.



(2019 Q3)

CONCLUDING COMMENTS

The junior third round Olympiad papers contain a wealth of fascinating questions, intriguing scenarios, and problems which when solved leave one with a sense of "*Oh wow – that's really cool maths!*" Incorporating these types of questions into the classroom, when appropriate, has the potential to spark wonderful classroom discussions and expose pupils to genuine mathematical reasoning and problem solving. And it's not always arriving at a final solution that's the most important thing – the real payoff lies with the mathematical habits of mind that are fostered and developed along the way.